Network Factors for Idiosyncratic Volatility Spillover*

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Abstract

This paper studies the unobservable network structure change of idiosyncratic volatility spillover among sectors. Changes in the network structure are captured by two asset pricing factors: Concentration factor and Magnitude factor. The two factors determine the node size distribution and linkage thickness distribution respectively in an idiosyncratic volatility spillover network and they contain distinct sources of systematic risk. Concentration factor measures the degree to which the contamination capacity is dominated by a few large sectors and Magnitude factor measures the average possibility of idiosyncratic volatility spillover. Sectors' risk exposures depend on their positions in the network and the exposures can predict their stock returns. The spillover network factors outperform the traditional production-based network factors, unlocking new avenues for understanding and leveraging network risk. Lastly, I give a multisector model to shed light on how the spillover network change affects aggregate volatility. A higher Concentration factor and a lower Magnitude factor can increase the conditional cross-sectional decay rate of aggregate volatility when sector number $n \to \infty$.

Keywords: Idiosyncratic volatility, contagion risk, dynamic network structure, supplier customer chain, cross-section of returns

JEL Classification: C67, D57, E32.

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1 Introduction

1.1 Introduction

The increasing interconnectedness among firms and industries has facilitated the transmission of idiosyncratic shocks from one sector to another. One well-known channel of interconnection is the production-based input-output chain. An idiosyncratic shock to one sector can be transmitted downstream to its customers or upstream to its suppliers, potentially contaminating the entire economy under specific conditions. This contagion risk, also known as "network risk," has been extensively studied within the context of production-based networks. For example, Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) examined the input-output relationship in a multi-sector model, demonstrating how small shocks are amplified and propagated through the economy, resulting in significant aggregate fluctuations. Herskovic (2018) focused on the network structure change within the input-output relationship, developing two network factors ("Concentration" and "Sparsity") to capture structural changes and establishing that network risk, as represented by these factors, constitutes systematic risk. Although the majority of existing network literature is based on production-based networks due to data accessibility and familiarity with sales relationships, the input-output relationship is not the sole driver of idiosyncratic volatility spillover. The network structure for idiosyncratic volatility spillover remains largely unexplored, with some studies highlighting the comovement among idiosyncratic volatility (e.g., Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) documented a common factor for the comovement of firm-level idiosyncratic volatility). However, no research has directly investigated the potential network structure of idiosyncratic volatility spillover. By viewing sectors as nodes and idiosyncratic shock spillover as directional linkages, a virtual network structure can be established, encompassing a wide range of underlying channels beyond production-based relationships (e.g., cross-holdings of assets, debt and liabilities, common investor channels, labor channels, and feedback and contagion). This paper aims to address several questions: How does this virtual network structure change over time? How does idiosyncratic shock influence aggregate volatility through the virtual network changes? Does this virtual structure change entail systematic risk? What are the similarities and differences between the idiosyncratic volatility spillover network and the production-based network?

In addition to complementing the production-based network literature, a second motivation stems from the contradiction within network literature. Although the input-output chain is closely related to first-order connections ("return connections"), which many researchers study, Diebold and Yilmaz (2009) argued that second-order connections ("volatility-based" networks) contain far more intriguing information than first-order connections. Their conclusion posits that return connections are distinct from volatility connections, with the former exhibiting little variation and a slight increasing trend over time and the latter demonstrating clear bursts during crises. In this paper, I directly compare second-order networks with first-order networks, indicating that the network structure change for first-order networks can be largely explained by CAPM, whereas the second-order network contains additional systematic risk.

Lastly, my third motivation is to bridge the gap between risk measurement literature and asset pricing. Numerous researchers have explored methodologies for measuring systemic risk, including equicorrelation by Engle and Kelly (2012), Expected Capital Shortfall (ECS) by Acharya, Engle, and Richardson (2012) and Brownlees, Engle, et al. (2012), CoVAR by Adrian and Brunnermeier (2011), and pairwise risk spillover measurement by Diebold and Yilmaz (2012), Diebold and Yılmaz (2014) and Demirer, Diebold, Liu, and Yilmaz (2018). However, these papers leave an open question: Which risks within the spillover structure are genuinely systematic and should therefore be priced? This paper aims to address this question by capturing spillover structure changes using two new asset pricing factors and demonstrating their validity in pricing. I will employ the methodology from Diebold and Yılmaz (2014) to construct a dynamic network for idiosyncratic volatility spillover. More specifically, the connectedness table will be generated from a generalized variance decomposition (GVD) of VAR's forecasting errors, and the two new asset pricing factors will be designed to capture the distribution of node size and linkage thickness throughout the entire system.

In this paper, I constructed two new network factors that can capture the network structure change for sector-level idiosyncratic volatility spillover, demonstrating that they embody additional systematic risk beyond market. Furthermore, I employ a multi-sector model to elucidate the impact of idiosyncratic shocks on aggregate volatility as mediated by alterations in the network structure. Envisioning sectors as nodes and idiosyncratic volatility spillover as connections, a network structure materializes. The following sample plot visualizes the static network structure for the NAICS 20 sectors spanning the period from 1969 to 2019.

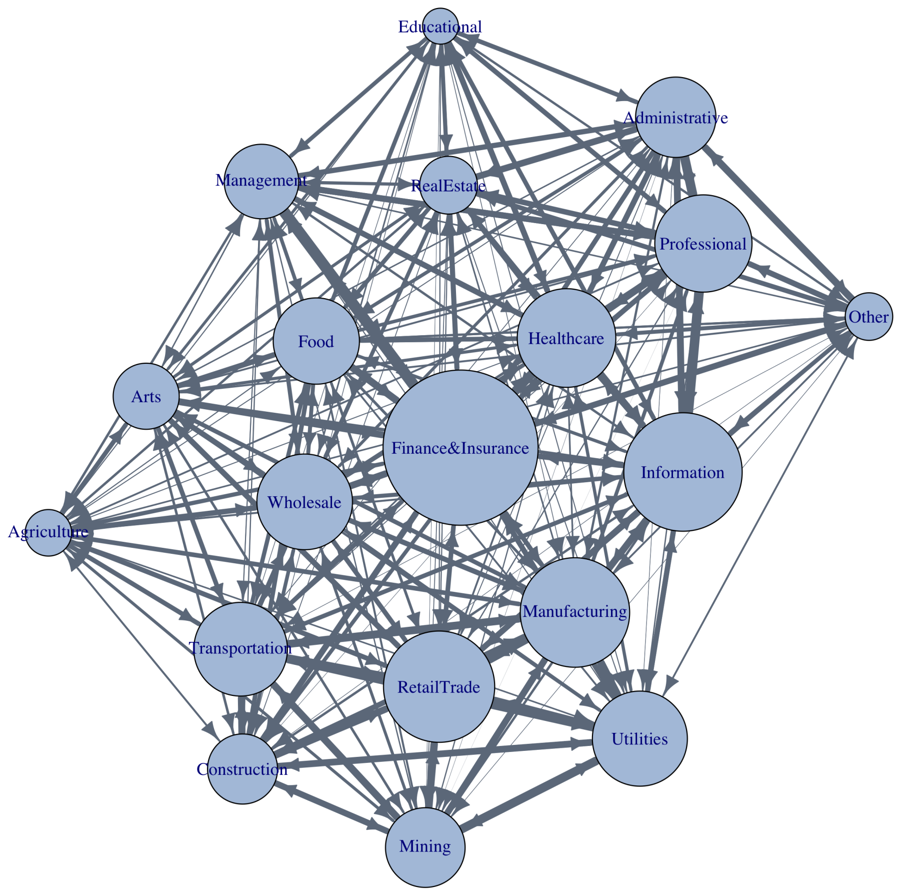


Figure 1. Idiosyncratic volatility spillover among NAICS 20 sectors. This figure contains the static idiosyncratic volatility spillover network of the NAICS 20 sectors from 1969 to 2019. An arrow from sector i to sector j means that i is propagating idiosyncratic volatility to sector j. The size of a node (sector) represents how much the sector is going to transmit its own idiosyncratic shock to the rest of the economy. The thickness of a linkage from node i to node j denotes how much percent that j's volatility is explained by the shock spillover from sector i.

The spillover network structure change is going to be captured by two asset pricing factors: Concentration factor and Magnitude factor. The two factors interact to make a difference for the aggregate market. In the sample plot provided, a larger node size signifies a more substantial risk contribution from a given sector to the overall economy. Consequently, idiosyncratic shocks to such critical nodes are more likely to permeate throughout the economy. The directional linkage serves as a metric for the magnitude of idiosyncratic shock spillover, with a thicker linkage suggesting a higher probability of spillover occurring in that direction. The two factors are subsequently employed to determine the distribution of node sizes and linkage thicknesses across the entire system. The Magnitude factor, representing the average thickness of the linkages, gauges the mean spillover likelihood for the economy. An economy with a higher MAG value is more susceptible to systematic risk, as idiosyncratic shock transmission is more likely to occur. The Concentration factor measures the extent to which the capacity for contamination is dominated by a few prominent sectors. A higher CON value suggests that, in a given economy, only idiosyncratic shocks to several key nodes are likely to contaminate the entire economy, whereas a lower CON value implies that almost all sectors—whether central or peripheral—contribute equally to market risk. Therefore, when the Magnitude factor is substantial, a smaller CON value is less desirable, as it indicates that every sector becomes a potential risk disseminator. The following diagram offers a visual representation of these concepts.

In the sample plot provided, economies 1 and 2 exhibit the same MAG values, yet economy 1 has a lower CON. Consequently, in economy 1, should either sector 1 or sector 2 experience an idiosyncratic shock, both are likely to transmit it to the entire economy. In contrast, in economy 2, only an idiosyncratic shock to sector 2 can contaminate the whole economy, while a shock to sector 1 would have no impact on the broader market. Intuitively, economy 1 represents an economy during bad times, with all sectors functioning as potential volatility propagators. Even

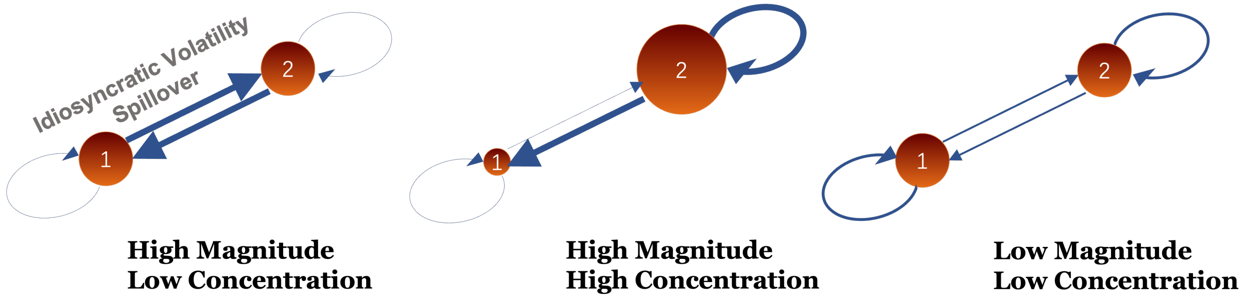


Figure 2. Sample Plot for Volatility-based Network Factors. There are 3 sample plots for different idiosyncratic volatility spillover structure. Panel 1 presents a network with a high magnitude factor and a low concentration factor. Panel 2 presents a network with a high magnitude factor and a high concentration factor. Panel 3 presents a network with a low magnitude factor and a low concentration factor.

small sectors' influence is amplified during recessions. Economy 2, on the other hand, signifies an economy during normal periods, wherein certain key intermediary sectors control volatility spillover while peripheral sectors exert less influence on the market.

Comparing economies 1 and 3, it is evident that they share the same CON values but differ in terms of MAG. Economy 1 is more susceptible to systematic risk, as shock spillover is more likely to occur. Intuitively, the average probability of volatility spillover is related to the collective market power of firms. In recent years, firms have gained increased market power and have become more adept at diversifying product-specific shocks, leading to a decrease in the overall probability of volatility spillover. As a result, in more recent years, our economy has gradually transitioned from economy 1 to economy 3.

In summary, a high CON value signifies fewer potential sources of contamination within the economy, while a high MAG value indicates a greater likelihood of idiosyncratic shock transmission. A high CON represents a favorable situation, whereas a high MAG reflects an unfavorable one. These two factors encapsulate distinct sources of systematic risk.

When I take the factors to the cross-section, I find that Concentration factor earns a positive price of risk while Magnitude factor earns a negative price of risk. Cross-sectional tests show that stocks more exposed to Concentration factor are riskier (with an annual return spread of +5%) and stocks more exposed to Magnitude factor are hedges (with an annual return spread of -4%). These return gaps cannot be explained by standard asset pricing models such as three/four/five-factor model. Moreover, more empirical tests show that second-order ("volatility level") network factors contain extra systematic risk than the corresponding first-order ("return level") network factors, which sheds light on the intuition that idiosyncratic volatility spillover covers more underlying interconnection mechanisms other than the ordinary input-output chain.

In order to demonstrate how idiosyncratic shock is linked to the aggregate volatility through this virtual network structure change, I come up with a multi-sector model where the idiosyncratic productivity shock transmits through the input-output linkage. I focus on the variation of the idiosyncratic stock return and form a spillover network as in the empirics. The theoretical results show that a higher CON and a lower MAG will lead to a higher cross-sectional (conditional) decay rate of aggregate volatility, which sheds more light on why a high CON is good news and a high MAG is bad news. From the microeconomic perspective, the story is that idiosyncratic shocks may lead to aggregate fluctuations. According to the classical diversification argument (Lucas Jr (1977)), idiosyncratic volatility averages out and the aggregate volatility concentrates to its mean at a very fast speed, proportional to $\sqrt{n}$ (i.e. the number of sectors). The aggregate volatility disseminates rapidly in a highly disconnected economy. But this argument does not take into account the interconnection between firms or sectors. When we turn to the network setting, Acemoglu et al. (2012) explores the production-based network and shows that higher variations in the degrees of different sectors imply lower rates of decay for aggregate volatility. This is actually consistent with my finding. From the following subsection, we will know that the volatility spillover factors have completely opposite macro implications with the production-based network factors. If the variation of production-based outdegree is high, it means that the Concentration of the input-output structure is high. A high CON in a production-based network slows down the cross-sectional dissemination speed of aggregate volatility. However, a high CON in a volatility-based network is going to speed up the cross-sectional dissemination speed of aggregate volatility. This validates that production-based factors and volatility-based factors are playing distinct roles in the macro finance. Additionally, Acemoglu et al. (2012) only looks at a static model while I am considering a dynamic framework in this paper by introducing dynamic productivity risk. Under this richer framework, we can look at the time series properties of the two new network factors. Through simple time series simulations, we can see that CON_t is 0.3 (p=0) correlated with aggregate output while MAG_t is -0.2 (p=0) correlated with aggregate output. Overall, the theoretical framework is consistent with the empirical results.

1.2 Volatility Spillover Networks vs. Production-based Networks

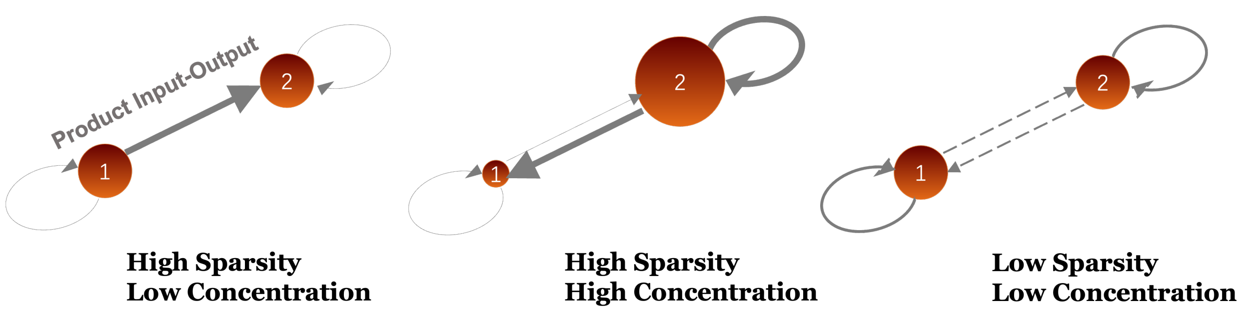


Figure 3. Sample Plot for Production-based Network Factors. There are 3 sample plots for production-based input-output structure. Panel 1 presents a network with a high sparsity factor and a low concentration factor. Panel 2 presents a network with a high sparsity factor and a high concentration factor. Panel 3 presents a network with a low sparsity factor and a low concentration factor.

In this subsection, I will directly compare the changes in production-based network structure with those in idiosyncratic volatility spillover structure, highlighting the differences between the two. In Herskovic (2018), the author devised the Concentration factor and the Sparsity factor to capture changes in the network structure within a production-based framework. Network concentration quantifies the extent to which equilibrium output is dominated by a few large sectors, whereas network sparsity assesses the average input specialization of the economy.

A high Concentration factor in the production-based network indicates that a small number of sectors dominate output, resulting in an economy that leans towards monopolistic conditions and away from a fully competitive environment. Given the diminishing returns to scale, a higher CON value leads to a decrease in aggregate output within the economy, rendering a higher CON undesirable. Conversely, a high Sparsity factor in the production-based network suggests that, on average, each sector receives input from only a few other sectors but enjoys a larger sales share. Although the linkages are sparse, they become more substantial. As sparsity increases, firms reoptimize inputs based on marginal product, substituting inputs with decreased marginal product for those with increased marginal product. This updated input allocation enables firms to achieve efficiency by utilizing more inputs with higher marginal products, thereby increasing final output. Thus, a higher Sparsity factor is beneficial.

The idiosyncratic volatility spillover factors discussed in this paper carry entirely different implications. A higher MAG factor is unfavorable, as idiosyncratic shock spillover is more likely to occur, while a higher CON factor is advantageous, as only a few nodes possess the potential to contaminate the entire economy. Interestingly, it is worth noting that in a production-based network, the network does not necessarily have to be fully connected. Some sectors utilize the output of only a few other sectors as input and maintain no trading relationships with the remaining sectors. In contrast, within a volatility spillover network, the network structure consistently forms a fully connected graph. Each sector either propagates or receives volatility spillover from other sectors, albeit with varying magnitudes of shock spillover. Additionally, it is important to recognize that the narrative of production-based network factors is contingent upon the assumption of decreasing returns to scale. In subsequent discussions, we will demonstrate that the volatility spillover factors can operate effectively under any return-to-scale assumption.

We have observed that network factors in production-based networks differ significantly from those in volatility spillover networks, both in terms of interpretation and pricing mechanisms. Moreover, the Fama-MacBeth analysis of the cross-section reveals that the production-based network factors exhibit entirely opposite signs for the price of risk compared to the signs of volatility spillover network factors. In the production-based network, CON carries a negative price of risk, while SPC exhibits a positive price of risk. Stocks with greater exposure to SPC yield a positive return spread, whereas stocks with higher exposure to CON generate a negative return spread.

In the volatility spillover network, however, CON displays a positive price of risk and MAG presents a negative price of risk. Stocks with greater exposure to the Concentration factor are riskier, exhibiting an annual return spread of +5%, while stocks with higher exposure to the Magnitude factor serve as hedges, featuring an annual return spread of -4%.

In the robustness check, when incorporating both production-based factors and volatility-based factors into the Fama-MacBeth test, the volatility-based network factors not only remain robust but also reverse the signs of the production-based networks. This finding further underscores the greater potency of volatility spillover factors compared to production-based network factors.

1.3 Mechanisms for idiosyncratic volatility spillover

Before delving into the empirical procedures, let us discuss the underlying mechanisms for idiosyncratic volatility spillover. Although my empirical work will treat idiosyncratic volatility spillover as given and directly study its network structure changes, understanding these mechanisms can help explain some of the intriguing results of this paper.

First, if two sectors are linked through a production-based chain, an idiosyncratic shock to sector 1 will influence its output, which will then be used as input by sector 2. The input change for sector 2 will subsequently affect its production. In this case, the idiosyncratic shock is transmitted from sector 1 to sector 2. Customer shocks can also propagate in the opposite direction. It is reasonable to assert that production-based trading relationships play a significant role in driving idiosyncratic volatility spillover. However, they are not the sole channel for such spillovers.

Second, idiosyncratic shocks can spillover through common investors' activities. For instance, if sectors 1 and 2 produce complementary goods and share customers, a negative idiosyncratic shock to sector 1 leading to a decline in product quality could also affect the sales of sector 2's products. In some cases, investors may even have indirect influence on the two sectors due to limited information and uncertainty about whether the idiosyncratic shock is sector-independent.

Third, the transmission of idiosyncratic industry shocks can occur through the asset channel. An industry-level shock could influence stock returns within that industry and then affect the stock returns of other firms holding portfolios of that industry's assets. As the industry market return is value-weighted firm return, returns of other industries may also be impacted. This constitutes a much broader transmission channel.

Fourth, there is a labor channel. According to Herskovic et al. (2016), idiosyncratic shocks have direct effects on labor income. For instance, a negative idiosyncratic shock to sector 1 could negatively affect its productivity and profitability, which would, in turn, decrease the wages of its employees. The employees in sector 1, experiencing a drop in their labor income and personal wealth, will also reduce their consumption of other goods.

Fifth, there is a credit channel. This channel is built on the supplier-customer relationship among sectors but differs from direct purchase or sale transactions. For example, some sectors may have accounts payable or receivable with their suppliers or customers, borrowing in advance based on their strong past relationships. Industry-level idiosyncratic shocks might render these accounts unrealizable, thereby affecting the value of another industry. In another scenario, many firms use "clientele credit," whereby suppliers request customers to pay before the delivery of goods or services. If the supplier sector experiences an idiosyncratic shock that leads to default, the input and production of the customer sectors could be affected.

Sixth, there is also a feedback and contagion channel through distressed competition. According to Chen, Dou, Guo, and Ji (2020), here is an illustration figure (figure 4).

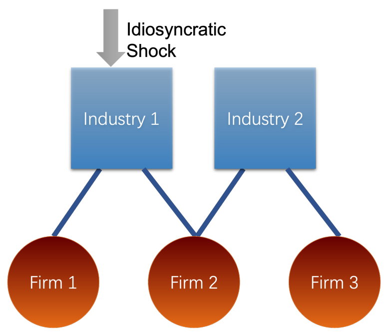


Figure 4. Feedback and contagion through distressed competition. This figure contains the possible idiosyncratic volatility spillover channel through feedback and contagion.

We assume Firm 1 operates in Sector 1, Firm 2 operates in both Sectors 1 and 2, and Firm 3 operates in Sector 3. An idiosyncratic shock to Sector 1 directly puts Firm 1 into financial distress. As Firm 1's default probability increases, it becomes more impatient, and its collusion incentive decreases. Consequently, it competes more aggressively, leading to a decline in profit margins for all firms in the same sector. Other firms, such as Firm 2, will also engage in more strategic competition. If there is a high entry barrier for Sector 1, then Firm 2 may choose to decrease its profit margin to drive Firm 1 out of the industry. Interestingly, such a conglomerate will intensify competition in both Sectors 1 and 2 to gain more profit. As competition in Sector 2 rises, financial distress for Firm 3 will also increase. Thus, idiosyncratic shocks can transmit across sectors. Of course, the transmission becomes more straightforward if an idiosyncratic shock to Sector 1 directly puts Firm 2 in distress.

Lastly, if a financial sector exists, idiosyncratic volatility spillover between the financial sector and other sectors can occur due to lending issues or collateral constraints. On one hand, financial firms hold claims, so their prospects depend on other sectors. On the other hand, their lending to other sectors affects the latter's financial well-being.

The seven channels detailed above demonstrate the richness and complexity of the idiosyncratic volatility spillover structure. Different channels may be more prominent in explaining different findings about the virtual network. In this paper, I will take the idiosyncratic volatility spillover structure as given and attempt to measure the dynamic structure changes. In future works, I plan to study each spillover channel explicitly.

1.4 Literature

For instance, Ahern (2013) is an early paper that indicates stocks in more central industries have greater market risk because they have greater exposure to sectoral shocks that transmit from one industry to another through intersectoral trade and Ahern and Harford (2014) finds that stronger product market connections lead to a greater incidence of cross-industry mergers. Aobdia, Caskey, and Ozel (2014) provides evidence that the intensity of shock transfers depends on industries' positions within the economy and Menzly and Ozbas (2010) shows that stocks that are in economically related supplier and customer industries cross-predict each other's returns. Cohen and Frazzini (2008) explores the "customer momentum": news to customers' can permanently influence suppliers' stock returns with a lag. This paper is more related to Herskovic (2018). In this paper, he developed two network factors ("Concentration" and "Magnitude" factor) to describe the structure change in sectors' input-output network and he proved that the network risk is a systematic risk. Related to his paper, we are imagining idiosyncratic volatility spillover as a "virtual" network and then try to build network factors to capture systematic risk. What's more, I also want to explore the relationship between production-based network and idiosyncratic volatility spillover network. Intuitively, the factors could have completely different asset pricing implications. For instance, if Concentration Factor is high in production-based network, then a few sectors are dominating the output and the society is more close to monopoly and is far away from a completely competitive one. So a high production-based Concentration Factor may be a bad news for the economy. However, when we think of idiosyncratic volatility spillover, if the Concentration Factor is low, then it means even some small/unimportant sectors are having the same probability to propagate the idiosyncratic shock to the whole economy as the large/important sectors do. This would be a disaster for the society. Therefore, a low volatility-based Concentration Factor (conditional on the spillover magnitude is large) may be a bad news for the economy. Therefore, it is appealing to systematically explore the commons and differences between production-based network factors and volatility-based network factors.

We know that volatility spillover can occur not only through input-output linkages, but also through cross-holdings of assets, debts and liabilities. So in a more broad sense, the volatility network contains something more than the production chain. For instance, Jackson and Pernoud (2021) presents a taxonomy of different types of systemic risk, differentiating between direct externalities between financial organizations (e.g., defaults, correlated portfolios and firesales), and perceptions and feedback effects (e.g., bank runs, credit freezes) and Elliott, Golub, and Jackson (2014) studies how cascades of failures depend on the diversification and integration of the network structure. Although in practice, cascades of failures may occur all at once, but it is useful to distinguish the sequence of dependencies in order to figure out how they might be avoided.

My work is closely related to Herskovic et al. (2016), which comes up with a "CIV" factor to capture the comovement of firms' idiosyncratic volatility. This factor is constructed by the cross-sectional average of firm-level idiosyncratic volatility and it is a proxy for the dispersion of labor income and consumption growth. However, they did not look into the granular network structure of the comovement. It is hard to see whether some important firms are dominating the comovement, etc. Therefore my paper is trying to explore the details of this volatility network. Also, Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2020) is relating firm volatility distribution to empirical firm size distribution through network effects and Irvine and Pontiff (2009) shows that increased competition in the economy can explain increases in idiosyncratic risk over time.

Besides studying the sources and channels of volatility spillover, there are also many researchers looking at how to measure the volatility spillover. There are a bunch of methodology papers, including Engle and Kelly (2012), Diebold and Yilmaz (2012). This paper applies the methodology from Diebold and Yılmaz (2014). In this paper, the authors derived a volatility connectedness table through generalized variance decomposition (GVD) of VAR's forecasting error (and Demirer et al. (2018) adopted GVD of LASSO VAR's forecasting error). This methodology even implicitly unifies MES and CoVar and can be applied to study many open questions related to connectedness. For example, which risks are truly systematic and hence should be priced? And my paper is designed to answer this question.

Network theory are studied a lot in economic contexts. For instance, Long Jr and Plosser (1983) is an early paper to use muti-sector models to demonstrate how certain very ordinary economic principles lead maximizing individuals to choose consumption-production plans that display many of the characteristics commonly associated with business cycles. Acemoglu et al. (2012) studied how small shocks are amplified and propagated through the economy to cause sizable aggregate fluctuations through a multi-sectoral model framework and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2017) used a multisector general equilibrium model to explain how the interplay of idiosyncratic microeconomic shocks and sectoral heterogeneity result in macro tail risk. While Gabaix (2011) showed that firm-level idiosyncratic shocks translate into aggregate fluctuations when the firm size distribution is sufficiently heavy-tailed and the law of large numbers does not hold, Acemoglu et al. (2012) showed that even when the law of large numbers holds, the existence of the network structure can lower the decay rate of aggregate volatility. We know from Lucas Jr (1977) that the "diversification argument" implies that the idiosyncratic shock will be diversified and aggregate volatility decays at a very quick speed (around $\sqrt{n}$). And Acemoglu et al. (2012) showed that the first and higher-order network structure can lead to different convergence rate of aggregate volatility. My work is also closely related to this paper and we are going to show how idiosyncratic shocks influence aggregate volatility's decay rate through the network factors. Some other related literatures may include Carvalho (2008), which derived analytical expressions linking directly the degree of fat-tailness of the distribution of input-supply links with the strength of the propagation mechanism in a multisector economy, and Carvalho and Gabaix (2013) shows that changes in the microeconomic composition of the economy during the post-war period can account for the evolution of macro volatility including the “great moderation” and its unraveling. A working paper from Grant and Yung (2019) developed a multi-sector DSGE model to calculate upstream and downstream industry exposure networks and explored their relationships with asset prices. Branger, Konermann, Meinerding, and Schlag (2021) used a general equilibrium model to show links in cash flow networks affect the cross-section of price exposures and market prices of risk in equilibrium and their cash-flow connectedness table is built through mutual exciting process.

The rest of the paper is organized as follows. Section 2 discusses the empirical methodology and results. Section 3 shows the robustness tests and further discussions about the evidence. Section

4 presents the model and simulations. Section 5 concludes. Section 6 shows the appendix for theoretical proof and some empirical results.

2 Empirical Evidence

2.1 Database

I use all CRSP stocks from 1969-01-01 to 2019-12-31. Delisting account is taken into consideration. I remove penny stocks ($abs(PRC) < 5$) and only look at stocks with share code 10,11,12.

2.2 Data Preprocessing

First, I calculate the daily idiosyncratic return for each sector. On a daily basis, the industry market return is computed from value-weighted firm returns, with the FF48 industries serving as the benchmark. Then, for each month, I estimate a factor model utilizing all observations within that month and employ the residual return as the idiosyncratic return.

$$r_{i,t} = \gamma_{0,i} + \gamma_i' F_t + \epsilon_{i,t}$$

where t denotes a daily observation in month τ. I have two different specifications for F_t. In the benchmark, I use daily Fama-French three factors. In robustness, I am going to use the first five principal components.

Next, I calculate the monthly idiosyncratic return volatility by determining the standard deviation of daily idiosyncratic returns. Since volatility is typically assumed to follow a log-normal distribution, it can be characterized using its first two moments. Therefore, I employ $log(vol)$ in subsequent calculations. Although volatility exhibits self-correlation, this can be accounted for using a VAR model, which will be implemented later in the analysis.

2.3 Construction of connectedness table for idiosyncratic volatility spillover

Following Diebold and Yılmaz (2014) , I first run a VAR(1) model for the volatility panel.

$$\begin{bmatrix} logvol_{1,t} \\ logvol_{2,t} \\ logvol_{3,t} \\ \dots \\ logvol_{48,t} \end{bmatrix} = \begin{bmatrix} \phi^0_{1,t} \\ \phi^0_{2,t} \\ \phi^0_{3,t} \\ \dots \\ \phi^0_{48,t} \end{bmatrix} + \begin{bmatrix} \phi_{1,1} & \dots & \phi_{1,48} \\ \phi_{2,1} & \dots & \phi_{2,48} \\ \phi_{3,1} & \dots & \phi_{3,48} \\ & \dots & \\ \phi_{48,1} & \dots & \phi_{48,48} \end{bmatrix} \begin{bmatrix} logvol_{1,t-1} \\ logvol_{2,t-1} \\ logvol_{3,t-1} \\ \dots \\ logvol_{48,t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \\ \dots \\ \epsilon_{48,t} \end{bmatrix}$$

Still according to Diebold and Yılmaz (2014), I build connectedness table by doing generalized variance decomposition(GVD) of the forecasting error ϵ. Mechanically, the element d_{ij} of the connectedness table is derived from

$$d_{ij} = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} \left(e_i' \Theta_h \Sigma e_j\right)^2}{\sum_{h=0}^{H-1} \left(e_i' \Theta_h \Sigma \Theta_h' e_i\right)}$$

e_i is the vector with jth element 1 and the rest zero. Σ is the variance-covariance matrix of the residual and σ_{jj} is the jth element of the diagonal. Θ_h is the coefficient matrix Φ multiplied by the h-lagged shock vector in the infinite MA representation. [1]

[1]GVD is more attractive compared to the standard variance decomposition, which is Cholesky-based(Diebold and Yilmaz (2012),Diebold and Yilmaz (2009)). That's because GVD invariant to ordering. In a Cholesky-factor orthogonalization, the first variable in the ordering is affected contemporaneously only by its own innovations, the second variable in the ordering is affected contemporaneously only by innovations of the first and second variables, and so on. GVDs, in contrast, effectively treat each variable as "first in the ordering". They do so not by attempting to orthogonalize shocks, but rather by allowing for correlated shocks while simultaneously accounting for the correlation among them observed historically, under a normality assumption.

Table 1. Adjacency matrix A_0

	IND_1	IND_2	...	IND_{48}	From others (Indegree)	
IND_1	$d_{1,1}$	$d_{1,2}$	...	$d_{1,48}$	$In_1 = \sum_{j=1}^{48} d_{1,j}$,	$j \neq 1$
IND_2	$d_{2,1}$	$d_{2,2}$	...	$d_{2,48}$	$In_2 = \sum_{j=1}^{48} d_{2,j}$,	$j \neq 2$
...		...				...
IND_{48}	$d_{48,1}$	$d_{48,2}$	...	$d_{48,48}$	$In_{48} = \sum_{j=1}^{48} d_{48,j}$,	$j \neq 48$
To others (Outdegree)	$O_1 = \sum_{i=1}^{48} d_{i,1}$ $i \neq 1$	$O_2 = \sum_{i=1}^{48} d_{i,2}$ $i \neq 2$	... ..	$O_{48} = \sum_{i=1}^{48} d_{i,48}$ $i \neq 48$	total connectedness	$\frac{1}{48} \sum_{i=1}^{48} d_{i,j}$ $i \neq j$

We denote the fraction of H-month ahead forecast error variance of industry i's volatility explained by shocks in industry j's volatility by $d_{i,j}$. We normalize each row to be of sum 1.

According to Diebold and Yılmaz (2014), the outdegree (which represents the propagation of sector j's shock to the rest of the economy) is defined as the sum of each column, excluding the diagonal. Similarly, the indegree is defined as the sum of each row, excluding the diagonal. The outdegree is analogous to risk contribution, while the indegree is analogous to risk exposure. The total connectedness of the economy is defined as the sum of all elements, excluding the diagonals. This network table is also referred to as a spillover measurement. The direction of shock spillover d_{ij} is from sector j to sector i. Consequently, each row of the table represents the shock receiving channel, and each column represents the shock spreading channel.

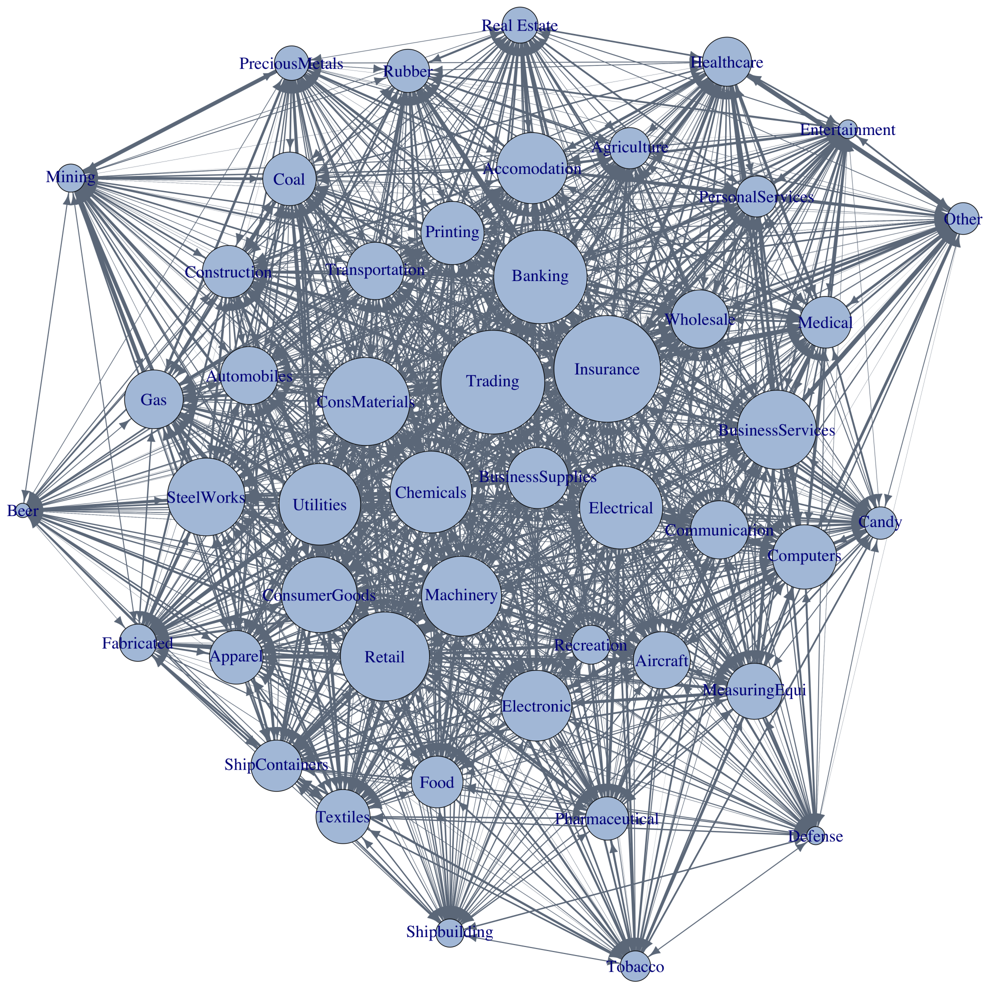


Figure 5. Idiosyncratic volatility spillover among FF48. This figure contains the static idiosyncratic volatility spillover network of the FF 48 sectors from 1969 to 2019. An arrow from sector i to sector j means that i is propagating idiosyncratic volatility to sector j. The size of a node (sector) represents how much the sector is going to transmit its own idiosyncratic shock to the rest of the economy. The thickness of a linkage from node i to node j denotes how much percent that j's volatility is explained by the shock spillover from sector i.

Figure 5 illustrates the connectedness table for idiosyncratic volatility spillover among the FF48 sectors from 1969 to 2019. Throughout the entire sample period, the Finance, Trading, and Banking sectors demonstrate a strong capacity for shock propagation. It is intuitive that every sector purchases insurance or has connections with banking or trading. If these three sectors experience shocks, the rest of the economy would be significantly impacted.

2.4 Construction of network factors: Concentration and Magnitude

In accordance with information theory, the negative entropy of degree distributions signifies the concentration of a network structure. A statistical explanation and a straightforward illustration example are provided in the appendix.[2] I construct the Concentration Factor using *"Negative Entropy"* of normalized outdegree.

$$CON = \sum_{i=1}^{48} O_i \times \log O_i$$

I construct the Magnitude Factor as the specialization ("concentration") of each sector's "shock receiving channel" and then take average cross-sectionally.

$$MAG = \frac{1}{48} \sum_{i=1}^{48} \sum_{j=1}^{48} d_{ij} \times \log d_{ij}$$

For sector i, its shock receiving channel is the ith row of the adjacency matrix, which is $[d_{i,1}, d_{i,2}, ..., d_{i,48}]$.

$$\left[\frac{1}{48}, \frac{1}{48}, \frac{1}{48}, \frac{1}{48}, ..., \frac{1}{48}\right] \text{vs.} \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, ..., 0\right]$$

[2]Alternative measurements, such as the Herfindahl-type concentration measurement, are used in the robustness analysis. It is important to note that the objective of this paper is not to develop a sufficient statistic for concentration, but rather to provide a thorough description of the changes in network structure and demonstrate the significant implications these changes have on asset pricing.

If the shock receiving channel is concentrated, then the sector is only influenced by several important nodes, but the influential magnitude is large. Therefore, the MAG is designed to capture the average magnitude of the shock spillover in the economy.

We recognize that the role of each sector within the network changes over time. As a result, I will transition from a static network to a dynamic one using a rolling window approach. For every 90-month interval, we re-estimate the VAR, construct the spillover table, and compute a single value for CON and MAG. Ultimately, the two factors become time series. However, I will employ the elastic net method for estimating the VAR. When constructing the dynamic connectedness table, the number of sectors (48) is relatively large compared to the 90-month window length. This raises concerns about prediction error, as in such a scenario, the linear regression fit would exhibit nearly zero bias, with the majority of the prediction error stemming from the large variance of coefficient estimates. Consequently, I have opted for the LASSO VAR approach to estimate the coefficient Φ. In accordance with Demirer et al. (2018) and Zou and Zhang (2009), I utilize the elastic net, which combines LASSO and Ridge methods. For each of the 48 regressions, the coefficient vector ϕ is estimated through

$$\hat{\phi} = arg \min_{\phi} \left[\sum_{t=1}^{T} \left(logvol_t - \sum_i \phi_i logvol_{it} \right)^2 + \lambda \sum_i \left(\frac{1}{2} \|\phi_i\| + \frac{1}{2} \phi_i^2 \right) \right]$$

where λ is selected through 5-fold cross validations.

2.5 Empirical results

2.5.1 Time series of network factors

FIGURE 5 ABOUT HERE

In Figure 5, the factor level plot reveals that MAG has a greater variation (ranging from -3.6 to -2.4) compared to CON (-3.85 to -3.70). The CON level is low during recessions, while the MAG level is less clear. The CON innovation has a 0.15 (p=0.00) correlation with the PD ratio, whereas the MAG innovation exhibits a -0.01 (p=0.80) correlation with the PD ratio.[3] This suggests that the concentration factor is procyclical. During recessionary periods, the influence of smaller sectors is amplified, and even peripheral sectors become significant shock transmitters, resulting in a decline in CON. One possible explanation is that the financial sector engages in numerous transactions with other peripheral sectors (e.g., mining, arts, and agriculture) or holds claims on them. Consequently, the diminished prosperity of these other sectors during recessions may amplify their backward influence on the financial sector, reducing its central position.

Furthermore, if we examine the overall trend, a level shift is observed for the two factors. After 2002, the MAG level shifts downward, and CON shifts upward.[4] This is consistent with the market power narrative. Post-2004, firms gain more market power, and one underlying channel is through decreasing product concentration and expanding product scope, as documented by Clara, Corhay, and Kung (2021). After 2004, the industry HHI increases, while the PCM (average product concentration measurement within firm) decreases. As a result, firms become more adept at diversifying product-specific shocks. The declining MAG and increasing CON are consistent with this intuition. From 2018 to 2020, the MAG level shifts up again, indicating the overall idiosyncratic volatility spillover possibility is increasing for the overall market. This suggests that firms and industries are becoming increasingly interconnected. Additionally, during the most recent years, big tech companies' consolidation, and the growth of renewable energy firms represent a stark contrast to the decline of traditional retail. The idiosyncratic volatility then becomes more

[3]The PD ratio is sourced from Welch and Goyal (2008).

[4]The annual plot 16 in the appendix makes it clearer that CON increases monotonically and MAG declines monotonically after 2004.

concentrated, and in accordance with my theoretical model framework, the aggregate volatility declines at a sluggish rate during this period. [5]

By using the AR(1) residual as the factor innovation, we observe that the two innovations are not correlated, indicating that they contain different sources of risk.

2.5.2 Sorting portfolio (Single Sort)

I use a trailing window of 80 months to sort cross-sectional stocks.

$$R^e_{i,t} = \alpha_{i,t} + \beta_{1,t} \times \Delta MAG_t + \beta_{2,t} \times \Delta CON_t + Controls_t + \epsilon_{i,t}$$

Left-hand side is all CRSP firm-level stock returns (no penny). Benchmark controls are Market, Size, Value & Previous Factor Levels. The controls are not crucial for the results. In the robustness check, removing controls or adding other controls like market volatility innovation would not influence the result. From time t-79 to t, I estimate the factor betas and form value-weighted or equal-weighted portfolio from time t to t+1.

Table 4 About Here

Here I show single sort results. The risk premium, volatility and Sharpe ratio are annualized. I also show the monthly α for CAPM, FF3 factors, 4factors (FF3+ Momentum from Carhart (1997)) and 5factors (4 factor + Liquidity from Pástor and Stambaugh (2003)).

From the tercile sort, we can see there is a significant H-L spread for both factors. Stocks more exposed to MAG are hedges and they earn a lower risk premium. The return spread is -0.04

[5] According to Gaspar and Massa (2006), market power serves as a hedging instrument that smooths out idiosyncratic fluctuations. Additionally, market power reduces information uncertainty for investors, thereby lowering return volatility. Both channels could explain the decreasing spillover effect magnitude.

annually. Stocks more exposed to CON are riskier and they earn a higher risk premium. The return spread is +0.05 annually. Additionally, the High-Low $alpha$s are not explained by the factor models.

The two factor betas are -0.3 correlated with each other, so the two factors are containing different sources or risk.

2.5.3 Fama-MacBeth Analysis

We do two-step cross-sectional regression test to test for price of risk.

$$\mathbf{R_t^e} = \alpha + \beta \mathbf{f} + \epsilon$$
$$\mathbb{E}\left[\mathbf{R_t^e}\right] = \lambda_\mathbf{0} + \lambda\beta + \mathbf{u}$$

On the left-hand side, we put different combinations of anomaly portfolios. The portfolios include: 10 Concentration-beta sorted portfolio, 10 Magnitude-beta sorted portfolio, 10 Industry portfolio, 30 Industry portfolio, 10 BM portfolio, 10 Investment portfolio, 10 Momentum portfolio, 10 Operating profitability portfolio and 10 Size portfolio from French's website. On the right-hand side, I consider two sets of asset pricing factors: (1) market excess return which is the CAPM model, and (2) the factor-mimicking portfolios along with the market excess returns. I estimate the two-step cross-sectional regression through GMM with lag 1 Newey-West error.

TABLE 5 ABOUT HERE

From the table 5, we can see CON has a positive price of risk and MAG has a negative price of risk. They are both significant in different anomaly portfolios. If we compare the CAPM with the three factor model ("Mkt+CON+MAG"), it is clear that the mean absolute error drops a lot and R^2 increases significantly. Therefore, the two network factors contain extra systematic risk other than market. [6]

If we plot the real average risk premium versus predicted risk premium (figure 8), it is obvious that in the three-factor model, portfolios line more close to the 45 degree.

FIGURE 8 ABOUT HERE

3 Robustness and further validation

We do the following robustness check: 1. We show the robustness of Fama-MacBeth Analysis by using other anomaly test portfolios. 2. We show the robustness of Fama-MacBeth analysis by considering a richer factor structure. We consider different combinations of factors including two network factors, FF3 factors and the CIV factor (the "common factor of idiosyncartic volatility") from Herskovic et al. (2016). 3. We show the robustness of Fama-MacBeth analysis by including the production-based network factors constructed according to Herskovic (2018). 4. We show the robustness of portfolio sorting by double sorting the two network factors or by double sorting one of our network factors with one other anomaly factors. 5. We show the robustness of benchmark portfolio sorting by considering alternative measurements of network factors, by adding controls and by changing alternative definitions of industries. 6. We show the robustness of benchmark portfolio sorting by considering different parameters (e.g. rolling window for constructing network factors, forecasting period for GVD and trailing window for portfolio sorting).

Additionally, in order to justify why volatility-based network is more interesting than the well-known return-based network (or production-based network), We compare the portfolio sorting result

[6] In different test assets, the mean CON beta is around 0.2. The mean MAG beta is around -0.1. Cross-sectionally, the betas vary a lot. For the complete cross-sectional distribution of the factor betas (from FM analysis using 48 industry portfolios), please refer to figure 10 and figure 11.

of first-order ("return-based") network factors with that of second-order (benchmark "volatility-based") network factors. Futhermore, we check the correlation between the network factor betas and the systematic risk measurements ("outdegree" and "indegree") derived directly from the adjacency table and show that our network factor betas are other ways to measure systemic risk. Additionally, we explore the robustness of the benchmark results by excluding the intermediary sectors and show that our results are not purely driven by intermediary models.

3.1 Fama-MacBeth test of other anomaly portfolios

First, we put some double-sorted portfolios on the left-hand side and do FM analysis.

TABLE 6 ABOUT HERE

Second, we also tried to use some other anomaly portfolios to see whether the two factos priced. The first test asset is 10 GID-β sorted portfolios from Guvenen, Ozkan, and Song (2014). Guvenen, Ozkan, and Song (2014) posted the quintile sort result of log earnings growth, and we obtain GID-β by regressing yearly excess returns on yearly H-L mimicking factor. Then we sort monthly excess return based on the yearly β. This is exactly what Herskovic et al. (2016) did. The second test asset is 5×3 Aggregate volatility and Market double sorted portfolios from Ang, Hodrick, Xing, and Zhang (2006).

TABLE 7 ABOUT HERE

3.2 Fama-MacBeth test with CIV factor

Herskovic et al. (2016) constructed a comovement factor for firm-level idiosyncratic shock spillover ("CIV" factor). However, they did not look at the granular network structure of the comovement (e.g. whether the comovement is led by some important factors or not, etc.) Here we try to do FM analysis by including CIV factor and the FF3 factors, so that we can compare richer factor specifications.

<u>Table 8 About Here</u>

In table 8, using all 98 anomaly portfolios, we can see that the CON and MAG factors survive well and the CIV factor is reversed sign. In most cases, adding all of those factors will significantly increase the R^2.

3.3 Fama-MacBeth test with production-based network factors

In Herskovic (2018), the author came up with two (annual) factors (Concentration factor and Sparsity factor) to capture the network structure change in production-based network. The production-based CON should have a negative price of risk and SPC should have a positive price of risk. If we go back to the illustration plot 2 and imagine them as network structures as input-output relationship. A higher CON means in economy 2, sector 2 dominates the output and with a decreasing return to scale, the aggregate output would decrease. Economy 2 is a monopolistic society while economy 1 is a equally competitive society. Also, a higher specialization in input channels would lead to higher aggregate output. A higher SPC means that each sector is only receiving input from several important nodes. The linkages are more sparse but the thickness is greater. It is obvious that in a production-based network, a high CON is bad news and a high MAG is good news. This indicates the volatility-based network factors have completely opposite asset pricing implications with the production-based network factors.

Here I compare two (annual) factor specifications in Fama-MacBeth: "Mkt+CON_{prod}+SPC_{prod}" vs. "Mkt+CON_{prod}+SPC_{prod}+ CON_{vol}+MAG_{vol}". In this way, I try to check whether our factors contain something more than the production-based factors.[7]

TABLE 9 ABOUT HERE

Our volatility-based CON should have a positive price of risk and MAG should have a negative price of risk. For most of the cases, we can see that at least one of our factors is significant, driving the production-based factors totally insignificant or "sign-reverting". Also, our volatility-based factors are improving R^2 a lot.

3.4 Robustness of double sorting

We double sort CON with MAG, or we double sort one of the network factors with other factors. It is nice to see both factors survive well through double-sort.

TABLE 10 ABOUT HERE

TABLE 11 ABOUT HERE

TABLE 12 ABOUT HERE

3.5 Robustness of different alternative measurements

TABLE 13 ABOUT HERE

Table 13 reports the tercile sorted results for Magnitude and Concentration by using different measurements of factors, different controls or different definitions of industries. Line 1-3: We consider different measurements for constructing the CON factor. (line 1-3). We discuss the

[7]The annual factor innovations for CON and MAG are annualized mimicking factors. Due to the limitation of the methodology, I can not directly construct annual factors since the time series is not long enough.

details in the next paragraph. Line 4-7: We consider different controls in portfolio sorting (line 4-7). Line 4 reports results by adding benchmark controls + momentum + liquidity. Line 5 reports results by adding benchmark controls + market volatility innovation. The market volatility is calculated as the monthly standard deviation of market return. Line 6 reports results by adding previous factor level as controls. Line 7 reports results by adding no control. Line 8: We consider sorting portfolios using all CRSP return including penny stocks. Line 9: We consider idiosyncratic volatility from PC5 factors. Line 10: We consider idiosyncratic volatility spillover for NAICS 20 sectors. Line 11: We consider idiosyncratic volatility spillover for 3 digit SIC code (line 11). It is obvious that the return spread remains significant for both factors under different alternative measurements and under different controls. The anomaly αs are also significant, suggesting that the return spread can not be explained by the factor models. The only interesting thing exists in line 11. In line 11, we retained the sectors with nonmissing values. There are 151 sectors in total from 1969 to 2019. We can see the H-L α is significant for MAG but not for CON. This results make sense to me. When we go to a very granular level, the size difference between nodes becomes much smaller. It is harder to capture the concept of "dominance". But the magnitude is still about the connection level for all sectors.

Here we elaborate the alternative measurements for the concentration factor (line 1-3). First, besides Outdegree, we can also use out-Herfindahl index in graph theory to represent volatility spreading capacity.

$$\begin{aligned} Herfout &= \begin{bmatrix} \frac{\sum_{i=1,i\neq 1}^{48} d_{i,1}}{\sum_{i=1}^{48} d_{i,1}} & \frac{\sum_{i=1,i\neq 2}^{48} d_{i,2}}{\sum_{i=1}^{48} d_{i,2}} & \dots & \frac{\sum_{i=1,i\neq 48}^{48} d_{i,48}}{\sum_{i=1}^{48} d_{i,48}} \end{bmatrix} \\ &= \begin{bmatrix} \frac{O_1}{\sum_{i=1}^{48} d_{i,1}} & \frac{O_2}{\sum_{i=1}^{48} d_{i,2}} & \dots & \frac{O_{48}}{\sum_{i=1}^{48} d_{i,48}} \end{bmatrix} \end{aligned}$$

We normalize it and compute Negative Entropy as the first alternative measurement for Concentration Factor. [8]

For the second and third measurement, we first give a symmetric approximation of the original adjacency matrix and then capture the concentration of the network using negative entropy of the eigenvector centrality.

In the network literature, it is attractive to construct "undirected" or symmetric approximation for the original network structure. It is known that the "direction" is an important part in the network: The shock from industry i to industry j is not equal to the shock from industry j to industry i, but only the thicker edge has important asset pricing implications for both i and j.

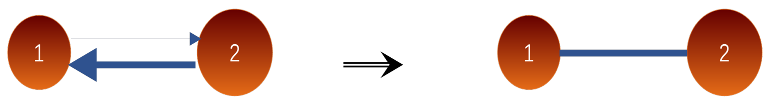


Figure 6. From directional to undirectional

So naturally, the first symmetric approximation for A_0 is to assign each pair of diagonal-symmetric elements to be the larger one of them. i.e.

$$d^*_{i,j} = \max_{i,j}\{d_{i,j}, d_{j,i}\}$$

where$A_0 = \left[d_{i,j}\right]$ and $A^* = \left[d^*_{i,j}\right]$. This is also the method adopted by Ahern (2013).

Next, I construct a second symmetric approximation of A_0 from a statistical perspective. In network literature, we often require the adjacency matrix to be irreducible and positive(because we often want to apply Perron-Frobenius theorem to extract eigenvector centrality etc.) So the approximation must also be irreducible and positive.

For convenience, we construct a 1-dim symmetric approximation(instead of a 48-dim). It is

[8]Since each row is already normalized in the benchmark. We do not have an alternative ratio measurement for the Magnitude factor.

reasonable as it still produces an irreducible matrix. In network language, "irreducible" means that the network is strongly connected and every node is reachable by other nodes. This is guaranteed by the nature property of volatility spillover. Hence, we assume the symmetric approximation $A^* = xx'$ and try to find vector x to minimize the Frobenius-norm of $\|A_0 - xx'\|_F$. The solution is then $x'H = \|x\|^2 x'$ where $H = \frac{A_0 + A_0'}{2}$ (the Hermitian matrix of A_0). Then we find all largest eigenvalue λ and the corresponding eigenvector μ and set x as $\pm\frac{\lambda^{1/2}}{\|\mu\|}\mu$. (Please refer to the appendix for the detailed solution.)

In the following empirical results, we denote the statistical approximation as A_1^* and the approximation using Ahern's method as A_2^*. Since the two alternative adjacency matrices implicitly unite indegree and outdegree, both degree can be represented using the "eigenvector centrality degree" and we are going to construct an alternative measurement of "Concentration" factor through Negative Entropy of normalized eigenvector centrality.

"eigenvector centrality degree" is constructed according to Bonacich (1972) and Bonacich and Lloyd (2001). If we have a symmetric and nonnegative adjacency matrix, then we can do the Perron-Frobenius-type eigenvector decomposition(EVD) and eigenvector centrality is simply defined as the eigenvector corresponding to the absolutely largest eigenvalue. All of the elements in the eigenvector centrality are nonnegative (according to Perron-Frobenius theorem).[9] [10]

[9]The Perron-Frobenius theorem says that there exists an eigenvector with only non-negative components. The Perron-Frobenius theorem also says that the non-negative eigenvector is associated with the absolutely largest eigenvalue (called the spectral radius) which is also non-negative. The eigenvectors related to all other eigenvalues must contain negative entries.

[10]Since the symmetric approximation makes the shock receiving channel (i.e. each row) obscure, it is difficult to construct an alternative measurement for the Magnitude factor. So we keep the Magnitude factor the same for line 1-3 of table 13.

3.6 Robustness of different different parameters

We show the portfolio sorting is robust by change different parameter choices: trailing window for sorting portfolio, rolling window for constructing dynamic network and forecasting period for estimating VAR forecasting error.

In table 14, in line 1-5, we are using different trailing window length to sort portfolio. In line 6-9, we are using different rolling window length to construct the dynamic networks. In line 10-13, we are using different forecasting period for doing generalized variance decomposition of the forecasting error. The portfolio sorting results are very robust under different parameter choices.

TABLE 14 ABOUT HERE

3.7 First-order vs. second-order network factors

Supplier-customer relationship has been well studied and this trading-based relationship has direct influence on firms' productivity and thus firms' stock returns. People are more familiar with this ordinary connections (i.e. return connection or the so-called "first-order" connection). For instance, Herskovic (2018) has directly explored the network structure change of the supplier-customer relationship. It seems not so natural why we would like to skip the first-order connection and directly explore the second-order network (i.e. "volatility-based" network)? One answer may be idiosyncratic volatility spillover is not well understood since the spillover channel does not rely solely on the trading relationship. But still, people may wonder what would things be like if we construct network factors for the return-based network and compare them with the idiosyncratic volatility based network.

Diebold and Yilmaz (2009) has done some comparison between the first-order connectedness table and second-order connectedness table for the global stock market. In that paper, they showed that total connectedness of the economy computed from the volatility network has clear bursts around crisis while that computed from the return network has very little variation and no clear bursts, indicating the second-order connection is much more interesting to look at. But still, we need to notice that Diebold at al. only compared the total connectedness of the economy and they did not explore the granular structure change of the network. Would situations still be more interesting for second-order network if we directly compare the CON factors and MAG factors? The answer is still yes.

This time, I directly constructed CON and MAG factors for the return matrix instead of the idiosyncratic volatility matrix. [11]

FIGURE 9 ABOUT HERE

TABLE 15 ABOUT HERE

From Figure 9, we can see the two MAG factors and two CON factors are not significantly correlated with each other. And the MAG and CON from the second-order network have much larger variations than those from the first-order network. I also sort portfolio using first-order factors.

From Table 15, we can see that the the MAG and CON factors from the first-order network are completely explained by CAPM. This is actually not surprising. MAG reflects average magnitude of return comovement, which should be proportional to the market return level. And according to Ahern (2013), in the ordinary connections, stocks in more central industries ("higher CON

[11] I construct the return matrix for FF48 industries and redo every step in the benchmark (i.e. rolling window GVD of the forecasting error) to construct two network factors for the first-order connection.

beta") have greater market risk because they have greater exposure to sectoral shocks that transmit from one industry to another through intersectoral trade. Therefore, the CON and MAG factors from a first-order connection doesn't seem to be as interesting as the factors from a second-order connection.

3.8 Are network factors other ways to measure systemic risk?

This paper indicates that sectors' risk exposure depends on their positions in the network and this exposure ("beta") can predict their stock returns. Sectors' factor loadings should pick up their positions. We should take a look at the correlation between beta and the Outdegree/Indegree measurement. From the connetedness table, Diebold et al. have directly constructed Outdegree/Indegree to measure systemic risk. Then it is interesting to see whether the factors are other ways to measure systemic risk. [12]

In the benchmark portfolio sorting, I am using All CRSP stock returns so the beta matrix is N*k (thousands*3). In order to verify this conjecture, I am now sorting portfolio on 48 industry excess returns so each industry can have factor exposures ("beta"). The betas are time-series. Now I plot the distribution of Outdegree, Indegree and two factor betas. And I check the correlations.

FIGURE 10 ABOUT HERE

FIGURE 11 ABOUT HERE

In the figure 10 and figure 11, we are looking at the cross-sectional distribution of the four time series. We can notice that the during crisis times, the Outdegree is a little bit right-skewed and the Indegree is a little bit left-skewed, which indicates some industries are receiving little but

[12]Thanks to Prof. Timothy Johnson for this valuable point!

transmitting much. But the variation for Outdegre and Indegree is relatively small. When we turn to look at the beta distributions, we can find that the variations are much larger. The CON beta varies from -10 to +10 and the MAG beta varies from -2 to +2. The MAG beta is a little bit left skewed during crisis, which indicates many sectors are having negative exposures to the MAG factor. Since benchmark results show that stocks more exposed to MAG are hedges ("earn lower risk premium"), here the plot indicates that many sectors are riskier during crisis time.

TABLE 16 ABOUT HERE

If we directly look at the correlations (Table 16), we can see that Outdegree, Indegree and two factor betas are significantly correlated with each other. This suggests that the factor exposures can be other measurements for systemic risk.

3.9 The role of intermediary sectors

We can clearly notice from the static plot that financial sectors are playing a central role in the structure. People may wonder whether results would still hold by excluding the financial sectors. The answer is: Yes, the results are not solely driven by the intermediary sectors. I excluded the finance sectors ("banking", "trading" and "insurance") and showed the sorting portfolio results.

TABLE 17 ABOUT HERE

In the table 17, the H-L return spreads for both both factors drop a little but they are still significant. The H-L α for both factors are still not explained by the factor models (at 0.1 significance for Concentration Factor and at 0.05 significance for Magnitude Factor), indicating our results are not solely driven by the intermediary sectors. [13]

[13]This is the reason why we are considering a production-based multi-sector model instead of an intermediary

The interesting thing is that we can still shed some light on the role of the intermediary sectors since the Concentration factor can be explained more by market after excluding the financial sectors. Besides the common channels discussed in the introduction, financial sectors are still special due to collateral constraint, etc.

Fan, Masini, and Medeiros (2021) suggests there is a potential "financial factor" that causes firms in other sectors to depend more strongly on financial firms even after taking away the common factors. Rampini and Viswanathan (2018) shows that financial intermediaries act as collateralization specialist. Economic activity and spreads are determined by firm and intermediary net worth jointly. The accumulation of intermediary net worth after a credit crunch is very slow (compared to firms) even it continues to pay dividends. Therefore, a credit crunch has persistent real effects on other firms since they need to wait for the intermediary to accumulate enough capital to make investments. Rampini and Viswanathan (2019) said something about the insurance sector. From the demand side, insurance demand is globally increasing in net worth and income. From the supply side, the insurers' balance sheets consist of assets (collateralized loans) and liabilities (diversified portfolio of insurance claims) as a consequence of intertemporal aspect to insurance. The collateral constraint (e.g. collateral scarcity) lowers the equilibrium interest rate, reduces insurance and increases inequality. Therefore, the insurance sector is somewhat playing an important role in influencing the whole economy.

4 Theoretical Framework

In empirical studies, network factors that capture changes in the network structure of idiosyncratic volatility spillover have been shown to have significant asset pricing implications. Furthermore, the

model in the model section.

idiosyncratic volatility spillover encompasses a more complex structure than the production-based network. However, it remains unclear how this spillover structure can influence aggregate quantities (e.g., aggregate volatility), and thus can influence cross section of stocks. In this section, I examine a multi-sector production-based model to elucidate how idiosyncratic shocks can influence aggregate volatility through changes in the spillover network structure. This analysis aims to provide further insight into why a high CON is considered advantageous, and a high MAG is deemed unfavorable.

4.1 Notations

Given two sequences of positive real numbers $\{f_n\}_{n\in\mathbb{N}}$ and $\{g_n\}_{n\in\mathbb{N}}$, we define

$$
\begin{aligned}
&f_n = O(g_n) \text{ iff there exists } C > 0 \text{ and } k \geq 1 \text{ such that } f_n \leq C g_n \text{ for every } n \geq k. \\
&f_n = \Omega(g_n) \text{ iff there exists } C > 0 \text{ and } k \geq 1 \text{ such that } f_n \geq C g_n \text{ for every } n \geq k. \\
&f_n = \Theta(g_n) \text{ iff } f_n = O(g_n) \text{ and } f_n = \Omega(g_n). \\
&f_n = o(g_n) \text{ iff } \lim_{n\to\infty} \frac{f_n}{g_n} = 0. \\
&f_n = \omega(g_n) \text{ iff } \lim_{n\to\infty} \frac{f_n}{g_n} = \infty.
\end{aligned}
$$

4.2 Model Setting

I consider a model closely related to Acemoglu et al. (2012), Acemoglu, Akcigit, and Kerr (2016) and Long Jr and Plosser (1983), which considers Cobb-Douglas production function.

Consider a perfectly competitive economy with n industries, and each industry follows a Cobb-Douglas production function:

$$
y_{i,t} = e^{z_{i,t}} \zeta_i l_{i,t}^{\alpha} \prod_{j=1}^{n} x_{ij,t}^{(1-\alpha)\omega_{ij}}
$$

Here $z_{i,t}$ is the idiosyncratic productivity shock to industry i and ζ_i is a normalization constant.

We assume the productivity shocks $z_{i,t}$ are independent across industries. Specifically, $z_{i,t} = \rho_z z_{i,t-1} + \sigma_z \epsilon_{i,t}$ where $\epsilon_{i,t} \sim IIDN(0,1)$. $l_{i,t}$ is the labor and we assume the market has a unit labor ($\sum_{i=1}^{n} l_{i,t} = 1$). $x_{ij,t}$ is the quantity of goods produced by industry j used as inputs by industry i and ω_{ij} indicates how much of the input quantity would be transferred to output. We assume $\sum_{j=1}^{n} \omega_{ij} = 1$, so that there is a constant return to scale. We define the input-output matrix as $\mathbf{W} \equiv \left[\omega_{ij}\right]$.

All sectors maximize their time-separable profits with respect to condition (4.2).

$$D_{i,t} = \max_{\{x_{ij,t}\},\{l_{i,t}\}} P_{i,t} y_{i,t} - \sum_{j=1}^{n} P_{j,t} x_{ij,t} - h_t l_{i,t}$$

where h_t is the market wage and $P_{i,t}$ is the market price of good i. We also denote the cum-dividend value of firm i to be $V_{i,t}$. It is computed recursively by

$$V_{i,t} = D_{i,t} + \mathbb{E}_{\approx} \left[M_{t+1} V_{i,t+1}\right]$$

.

The representative household has Epstein-Zin recursive preferences with respect to a consumption aggregator:

$$U_t = \left[(1-\beta)\, C_t^{1-\rho} + \beta \left(\mathbb{E}_t \left(U_{t+1}^{1-\gamma} \right) \right)^{\frac{1-\rho}{1-\gamma}} \right]^{\frac{1}{1-\rho}}$$

where γ is the risk aversion and ρ is the inverse of the elasticity of intertemporal substitution. C_t is a Cobb-Douglas consumption aggregator:

$$C_t = \prod_{i=1}^{n} c_{i,t}^{\alpha_i}$$

where $c_{i,t}$ is the consumption of good i at time t and α_i is the taste preference on good i. We assume

$\sum_{i=1}^{n} \alpha_i = 1$.

The household maximization problem is

$$J_t\left(P_t, s_t, Q_t, h_t\right) = \max_{\left\{c_{i,t}, s_{i,t+1}\right\}} \left[(1-\beta)\, C_t^{1-\rho} + \beta \left(\mathbb{E}_t \left(J_{t+1}^{1-\gamma} \right) \right)^{\frac{1-\rho}{1-\gamma}} \right]^{\frac{1}{1-\rho}}$$

where $Q_{i,t} = V_{i,t} - D_{i,t}$ is the pre-dividend value of firm i and $s_{i,t}$ is the share of firm i at time t.

The intertemporal budget constraint is

$$\sum_{i=1}^{n} P_{i,t} c_{i,t} + \sum_{i=1}^{n} \left(V_{i,t} - D_{i,t}\right) s_{i,t+1} = \sum_{i=1}^{n} V_{i,t} s_{i,t} + h_t$$

That is to say, total expenditure in consumption goods and firms' share net of dividends must be equal to the gains from shares value and labor income.

The market clearing condition satisfies

$$\begin{cases} y_{i,t} = c_{i,t} + \sum_{j=1}^{n} x_{ji,t} \\ \sum_{i=1}^{n} s_{i,t} = 1 \\ \sum_{i=1}^{n} l_{i,t} = 1 \end{cases}$$

4.3 Competitive equilibrium

A competitive equilibrium consists of spot market prices $\left\{P_{i,t}\right\}$, consumption of different goods $\left\{c_{i,t}\right\}$, shares holding $\left\{s_{i,t}\right\}$ and input choices $\left\{x_{ij,t}\right\}$. For every period t, the sectors and household maximize their own problems and the market clearing conditions hold.

The first order conditions for firms' problems are given by

$$\begin{cases} x_{ij,t} = \dfrac{(1-\alpha)\,\omega_{ij} P_{i,t} y_{i,t}}{P_{j,t}} \\ l_{i,t} = \dfrac{\alpha P_{i,t} y_{i,t}}{h_t} \end{cases}$$

The intra-period of consumption for the household's problem is given by

$$c_{i,t} = \frac{\alpha_i \xi_t}{P_{i,t}}$$

where ξ_t is the GDP at time t, i.e. the wealth added to the economy at time t. The first order condition for household's problem also yields

$$\mathbb{E}_t \left[\underbrace{\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{\frac{\partial C_{t+1}}{\partial c_{1,t+1}} / P_{1,t+1}}{\frac{\partial C_t}{\partial c_{1,t}} / P_{1,t}} \left(\frac{J_{t+1}}{\mathbb{E}_t \left(J_{t+1}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma}}_{\equiv M_{t+1}} \underbrace{\frac{D_{i,t+1} + Q_{i,t+1}}{Q_{i,t}}}_{\equiv R_{i,t+1}} \right] = 1$$

where M_{t+1} is the stochastic discount factor and $R_{i,t+1}$ is the one-period return of holdding firm i's share from t to t+1. In the first order condition, the firm share $s_{i,t+1}$ is chosen such that the equation (4.3) holds for each asset i.

In addition, the spot prices are normalized such that

$$\prod_{j=1}^{n} P_{j,t}^{\alpha_j} = \prod_{j=1}^{n} \alpha_{j,t}^{\alpha_j}$$

Under this normalization, the aggregate consumption can be written as

$$C_t = \prod_{j=1}^{n} C_{j,t}^{\alpha_j} = \prod_{j=1}^{n} \left(\frac{\alpha_j}{P_{j,t}}\right)^{\alpha_j} \xi_t = \xi_t$$

The market clearing condition further yields

$$\frac{P_{i,t} y_{i,t}}{\xi_t} = \alpha_i + (1-\alpha) \sum_{j=1}^{n} \omega_{ji} \left(\frac{P_{j,t+1} y_{j,t+1}}{\xi_{t+1}}\right)$$

We denote $\frac{P_{i,t} y_{i,t}}{\xi_t}$ as $v_{i,t}$. $v_{i,t}$ is the output share of each sector or the ratio of contribution to the wealth added. It is often referred to as the "Domar weight" in literature. The Domar weight is decided by the network structure as follows:

$$v_{i,t} = \sum_{j=1}^{n} \alpha_j \tilde{l}_{ji}$$

where $\tilde{\mathbf{L}} \equiv \left[\tilde{l_{ij}}\right] = \left(\mathbf{I} - (1-\alpha)\,\mathbf{W}\right)^{-1}$ is the Leontief inverse. Also, all Domar weight sums up to 1.

4.3.1 Proposition 1

The spot price vector is given by

$$\ln \mathbf{P}_t = \tilde{\mathbf{L}} \left[-ln\alpha^{\alpha} (1-\alpha)^{1-\alpha} - (1-\alpha) \sum_{j=1}^{n} \omega_{ij} \ln \omega_{ij} + \alpha \ln h_t \mathbf{1} - \mathbf{z}_t \right]$$

We can see the aggregate market wage drives simultaneously for the cross-sectional price. The idiosyncratic part of the stock price lies in the last productivity shock component. Therefore we can approximate the idiosyncratic return as follows:

$$d \ln \mathbf{P}^*_{t \to t+1} = \tilde{L} \left[\mathbf{z}_t - \mathbf{z}_{t+1} \right]$$

The conditional variation of the idiosyncratic return is decided by the diagonal element of the var-cov matrix.

$$Cov_t \left(d \ln \mathbf{P}^*_{t \to t+1} \right) = \sigma_z^2 \tilde{L} \tilde{L}'$$

Because all of the eigenvalues of $(1-\alpha)W$ lie within the unit circle, we can write the Leontief inverse as follows:

$$(I - (1-\alpha) W)^{-1} = I + (1-\alpha) W + (1-\alpha)^2 W^2 + (1-\alpha)^3 W^3 + \ldots$$

Here we then use $I + (1-\alpha) W$ as a first-order approximation of the Leontief inverse.

For each firm i, its conditional variation of the idiosyncratic return is determined by

$$Var_t \left(d \ln P^*_{i,t \to t+1} \right) = \sigma_z^2 \left[\left((1-\alpha) \omega_{i,1} \right)^2 + \left((1-\alpha) \omega_{i,2} \right)^2 + \ldots + \left(1 + (1-\alpha) \omega_{i,i} \right)^2 + \ldots + \left((1-\alpha) \omega_{i,n} \right)^2 \right]$$

From this result, we know that for each sector i, its idiosyncratic return variation is not only explained by its own idiosyncratic shock (i.e. $\left(1 + (1-\alpha) \omega_{i,i}\right)^2$), but also explained by spillover of other sectors' idiosyncratic shocks (i.e. $\left((1-\alpha) \omega_{i,j}\right)^2$). Therefore we can approximate the conditional volatility spillover matrix $\Delta_{t \to t+1}$ according to the coefficients. Also, we normalize each row to be of length 1.

$$\Delta_{t\to t+1} \equiv \left[\delta_{ij}\right]$$

$$= \begin{bmatrix} \frac{\left(1+(1-\alpha)\,\omega_{1,1}\right)^2}{denom_1} & \frac{\left((1-\alpha)\,\omega_{1,2}\right)^2}{denom_1} & \cdots & \frac{\left((1-\alpha)\,\omega_{1,n}\right)^2}{denom_1} \\ \frac{\left((1-\alpha)\,\omega_{2,1}\right)^2}{denom_2} & \frac{\left(1+(1-\alpha)\,\omega_{2,2}\right)^2}{denom_2} & \cdots & \frac{\left((1-\alpha)\,\omega_{2,n}\right)^2}{denom_2} \\ & \cdots & \cdots & \\ \frac{\left((1-\alpha)\,\omega_{n,1}\right)^2}{denom_n} & \frac{\left((1-\alpha)\,\omega_{n,2}\right)^2}{denom_n} & \cdots & \frac{\left(1+(1-\alpha)\,\omega_{n,n}\right)^2}{denom_n} \end{bmatrix}$$

where $denom_i = \left((1-\alpha)\,\omega_{i,1}\right)^2 + \ldots + \left(1+(1-\alpha)\,\omega_{i,i}\right)^2 + \ldots + \left((1-\alpha)\,\omega_{i,n}\right)^2$ is the normalization constant.

Each element δ_{ij} represents how much percentage of sector i's volatility is explained by idiosyncratic shock spillover from sector j. Therefore, the matrix Δ corresponds to the adjacency matrix $\left[d_{ij}\right]$ in the empirical part.

We define normalized outdegree as the (normalized) column sum of Δ except the diagonal to represent the volatility spillover from each sector i to the rest of the economy. *Concentration* is defined as the negative entropy of the normalized outdegree to capture the "dominance" of the outdegree. *Magnitude* is defined as the average of the shock spillover magnitude. All of the definitions are consistent with the empirical procedures.

$$\begin{cases} Out_{i,t\to t+1}\left(\omega_{ij}\right) \triangleq \dfrac{\sum\limits_{j=1}^{n}\delta_{ji}}{\left(\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\delta_{ji}\right)} \\ CON_{t\to t+1}\left(\omega_{ij}\right) \triangleq \sum\limits_{i=1}^{n} Out_i \ln\left(Out_i\right) \\ MAG_{t\to t+1}\left(\omega_{ij}\right) \triangleq \dfrac{1}{n}\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\delta_{ij}\ln\left(\delta_{ij}\right) \end{cases}$$

4.3.2 Proposition 2

Aggregate output is a linear combination of the idiosyncratic productivity shock:

$$\ln \xi_t = \alpha' \ln \alpha + \alpha' \tilde{\mathbf{L}} \mathbf{z_t}$$

where α is the vector of good preference.

4.3.3 Proposition 3

When we assume the household has equal preference for the n goods (i.e. $\alpha_i = \frac{1}{n}$), the conditional aggregate volatility has a slower cross-sectional decay rate ("diversification speed") if the CON is smaller and MAG is larger.

$$\sqrt{var\left(\xi_{t\to t+1}\right)} = \Omega\left(\frac{1}{n}\sqrt{\frac{CON_{t\to t+1}}{MAG_{t\to t+1}}}\right)$$

Here *CON* and *MAG* are negative values by definition. When *Magnitude* of the volatility network is large (i.e. the average magnitude of idiosyncratic shock spillover is large) or *Concentration* is low (i.e. spillover capacity of each sector is similar), the limit infimum of variance of aggregate output is bounded by a greater value (i.e., aggregate volatility decays more slowly).

This proposition shows that the existence of the idiosyncratic volatility network structure has influence on the (cross-sectional) decay rate of the aggregate volatility. From the microeconomic perspective, the story is that idiosyncratic shocks may lead to aggregate fluctuations. According to the classical diversification argument (Lucas Jr (1977)), idiosyncratic volatility averages out and the aggregate volatility concentrates to its mean at a very fast speed, proportional to $\sqrt{n}$. But this argument does not take into account the interconnection between firms or sectors. When we turn to the network setting, this proposition is indicating that the decay rate of aggregate volatility is possible to be much slower than $\sqrt{n}$. It depends on the distribution of the CON and MAG factors.

4.3.4 Model Implied Asset Pricing Moments

Along the time series, we evaluate the variance-covariance matrix of the idiosyncratic return using a rolling window. That is to say, from time $t-\tau$ to t, the $n \times \tau$ idiosyncratic return matrix is expressed as

$$\mathbf{R} = \mathbf{L}\boldsymbol{\Lambda}$$

$\mathbf{L}$ is $n \times n$ Leontief inverse and $\boldsymbol{\Lambda}$ is the $n \times \tau$ matrix where the element $\Lambda_{i,t} = (1-\rho_z)\, z_{i,t-1} - \sigma_z \epsilon_{i,t}$. We then compute the variance-covariance matrix as follows:

$$\begin{aligned} Cov\left(\mathbf{R}_t\right) &= \mathbf{L}\frac{\boldsymbol{\Lambda}\boldsymbol{\Lambda}'}{\tau}\mathbf{L}' \\ &\equiv \mathbf{L}\widehat{\boldsymbol{\Lambda}}\mathbf{L}' \\ &\equiv \left[l_{ij}\right]\left[\lambda_{\mathbf{ij}}\right]\left[l_{ij}\right]' \\ &= \left[\sum_m \left(\sum_k k_{ik}\lambda_{km}\right) l_{jm}\right] \end{aligned}$$

where under the first-order approximation

$$l_{ij} = \begin{cases} 1 + (1-\alpha)\,\omega_{ii} & i = j \\ (1-\alpha)\,\omega_{ij} & i \neq j \end{cases}$$

The diagonal element of the variance-covariance matrix is the variance of the idiosyncratic return for each firm i.

$$Var\left(R_{i,t}\right) = \left[\sum_m \left(\sum_k k_{ik}\lambda_{km}\right) l_{im}\right]$$

Say if we have three sectors (i.e. $n = 3$). The idiosyncratic return variance for firm i is expressed as follows:

$$\begin{aligned} Var\left(R_{i,t}\right) = {} & l_{i1}^2\lambda_{11} + l_{i2}\lambda_{21}l_{i1} + l_{i3}\lambda_{31}l_{i1} \\ & + l_{i1}\lambda_{12}l_{i2} + l_{i2}^2\lambda_{22} + l_{i3}\lambda_{32}l_{i2} \\ & + l_{i1}\lambda_{13}l_{i3} + l_{i2}\lambda_{23}l_{i3} + l_{i3}^2\lambda_{33} \end{aligned}$$

It is hard to distinguish between the contribution of different idiosyncratic shocks since there are many interaction terms (e.g. $l_{i2}l_{i1}$ term).

Naturally one way to approximate a shock spillover matrix is to look at the terms that come purely from one idiosyncratic shock (e.g. l_{i1}^2 term) and we define the shock spillover matrix to be

$$\Delta = \left[\delta_{ij}\right] = \left[l_{ij}^2\lambda_{jj}\right]$$

We normalize each row to be of sum 1. We then compute the outdegree, CON and MAG using the same definitions as before.

In such case we are dropping the interaction term. Another natural thinking is to separate the interaction term equally to two different sources of idiosyncratic shock. That is to say $\frac{l_{i2}\lambda_{21}l_{i1}}{2}$ is contributed by firm 2's idiosyncratic shock and $\frac{l_{i2}\lambda_{21}l_{i1}}{2}$ is contributed by firm 1's idiosyncratic shock. Under this assumption, the spillover matrix looks like

$$\Delta = \left[\delta_{ij}\right] = \left[\sum_{k=1}^{n} \frac{\lambda_{kj} + \lambda_{jk}}{2} l_{ij} l_{ik}\right]$$

We normalize each row to be of sum 1. However, we can not rely on such approach. One reason is that the Δ matrix is not a nonnegative matrix (since it comes from the covariance matrix). It is then not consistent with our empirical definition. Also, if we do simple simulation using the parameters in table 2 and look at the absolute average ratio of the l_{i1}^2 term over $\sum_{k=1}^{n} \frac{\lambda_{kj}+\lambda_{jk}}{2} l_{ij} l_{ik}$ term along the whole sample period, we can clearly see that the square term accounts for 65.5% of the total variation. So it is reasonable to define the spillover matrix based on equation (4.3.4).

Table 2. Parameters

	Parameters	Value	Target
γ	Risk aversion	10	BY
$\frac{1}{\rho}$	EIS	1.5	BY
β	Discount (monthly)	0.996	Average Risk-free rate
α	Labor proportion to scale	0.65	Labor Literature
σ_z	Volatility for productivity shock	0.088	Herskovic: 0.0124, Consumption std: 0.025, Palomino: 0.102
ρ_z	Autocorrelation for productivity shock	0.97	Li&Palomino=0.95
n	Sector number	48	Consistent with empirics
τ	Rolling window for return covariance estimate	80	Consistent with empirics
α_i	Preference for good i	$\frac{1}{48}$	Literature

We set $\rho_z = 0.97$ and $\sigma_z = 0.105$, which implies an average annual volatility of individual stock returns of 45%, close to the data counterpart at 46%.

Simulation is at a monthly frequency, consistent with empirical works. We simulate an initial weight matrix $[\omega_{ij}]$ through uniform distribution (and normalize it). We simulate $12 \times 43 + 2000$ months and discard the first 2000 months to neutralize the impact of initial conditions. The 516-month panel is consistent with the sample length in the data. Through this process, we can get time series aggregate output, CON and MAG, and thus two factor innovations. Additionally, we simulate 2000 stock returns according the formula (4.3.4) and sort portfolio using a trailing window of 80. The model implied asset pricing moments are reported in table 3.

Finally, the simulated aggregate output is 0.3 (p=0) correlated with CON factor and -0.2 (p=0) correlated with MAG factor. This is consistent with the empirical results where the price of risk of CON is around 0.3 and the price of risk of MAG is around -0.3.

Panel A: Summary Statistics of Simulated Factor Innovations										
	Magnitude Factor					Concentration Factor				
(Data)	Mean	-0.0035	Std	0.0372		Mean	0.0031	Std	0.0026	
(Model)	Mean	0.0001	Std	0.0072		Mean	-0.0005	Std	0.0015	
Panel B: Equal-weighted portfolio sorting										
	Magnitude Factor					Concentration Factor				
	1	2	3	H-L	t	1	2	3	H-L	t
(Data) ERP	0.132	0.103	0.092	-0.040	-2.539	0.091	0.101	0.134	0.043	2.001
(Model) ERP	0.033	0.000	-0.020	-0.053	-1.995	-0.024	-0.020	0.030	0.054	1.989
(Data) Volatility	0.164	0.137	0.163	0.093		0.153	0.141	0.171	0.096	
(Model) Volatility	0.130	0.124	0.140	0.094		0.131	0.130	0.135	0.196	
(Data) Sharpe Ratio	0.804	0.748	0.563	-0.434		0.595	0.715	0.786	0.447	
(Model) Sharpe Ratio	0.252	0.002	-0.141	-0.270		-0.182	-0.152	0.226	0.277	

Table 3. Model Implied Asset Pricing Moments. Panel A reports the summary statistics of simulated factor innovations and Panel B reports the equal-weighted portfolio sorting results. Average excess return, volatility and Sharpe ratio are all annualized.

5 Conclusion

In this paper, I investigate changes in the network structure of sector-level idiosyncratic volatility spillover. These changes are captured by two asset pricing factors: the Concentration factor and the Magnitude factor. The factors respectively determine the distribution of node sizes and linkage thickness in an idiosyncratic volatility spillover network, containing distinct sources of systematic risk. The Concentration factor measures the extent to which the contamination capacity is dominated by a few large sectors, while the Magnitude factor measures the average probability of idiosyncratic volatility spillover.

The Concentration factor is associated with a positive price of risk, whereas the Magnitude factor is related to a negative price of risk. Cross-sectional tests reveal that stocks with greater exposure to the Concentration factor are riskier (with an annual return spread of +5%), while stocks with higher exposure to the Magnitude factor serve as hedges (with an annual return spread of -4%). These return differentials cannot be explained by standard asset pricing models, such as three-, four-, or five-factor models. The second-order ("volatility level") network factors contain more systematic risk than their corresponding first-order ("return level") network factors, highlighting the idea that idiosyncratic volatility spillover encompasses more underlying interconnection mechanisms beyond the standard input-output chain.

Lastly, I propose a multi-sector model to link changes in idiosyncratic structure with aggregate volatility. Conditionally, higher Concentration and lower Magnitude levels lead to an increased cross-sectional decay rate of aggregate volatility, illustrating the mechanism through which the virtual volatility spillover network influences aggregate quantities.

There are two takeaways for investors. First, sectors' risk exposures depend on their positions in the network and the exposures ("beta") can predict their stock returns. This discovery underscores the importance of network position when assessing risk exposures and forecasting stock performance. Second, this paper expands the notion of network risk. My research challenges the prevailing notion that network risk is solely rooted in input-output relationship. Instead, I advocate for a broader perspective that includes the examination of high-order network structural changes. By transcending the confines of traditional data sources, such as bilateral trade data, we can unlock new avenues for understanding and leveraging network risk. These higher-order network changes offer fresh insights and opportunities for market participants and policymakers.

There are many future works to be done. In this paper, I accept the idiosyncratic volatility spillover as given and seek to highlight the significance of its dynamic structure changes. In future research, I intend to examine each specific channel of idiosyncratic volatility spillover and provide empirical evidence unique to that channel. Additionally, I plan to investigate network structure changes under extreme circumstances, such as micro- and macro-level tail risks.

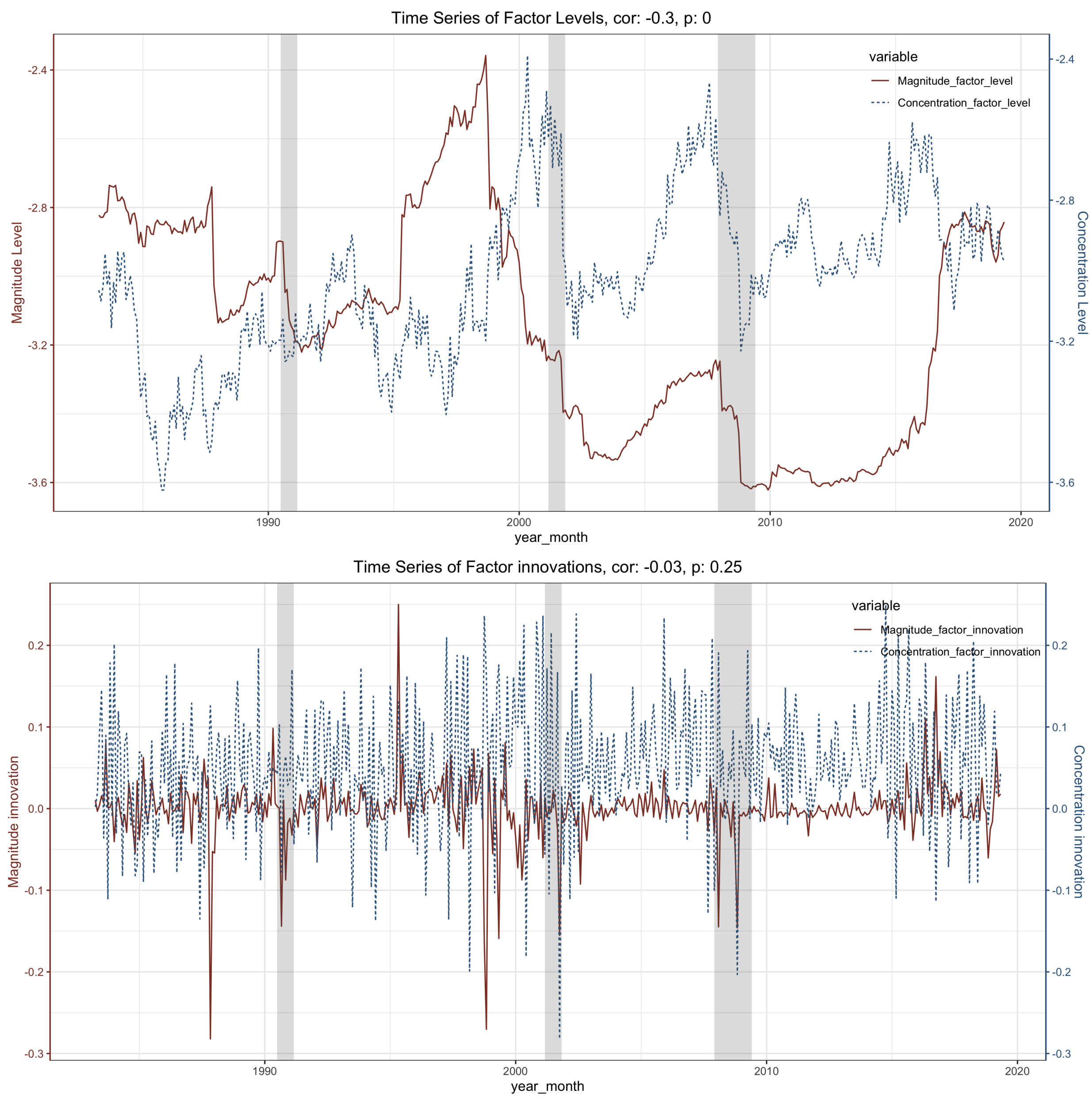


Figure 7. Time Series Plots of the Two Network Factors. This Figure plots the monthly time series of magnitude (red solid line) and concentration (blue dashed line) computed from the CRSP. The first panel plots both factors in level, and the second one plots the innovations (AR(1) residual).

	Magnitude Factor							Concentration Factor						
Panel A: Value-weighted														
	1	2	3	H-L	t	grs stat	grs p	1	2	3	H-L	t	grs stat	grs p
Ave.Exc.Ret	0.129	0.107	0.091	-0.038	-2.558			0.089	0.103	0.135	0.046	2.023		
volatility	0.166	0.139	0.159	0.092				0.155	0.141	0.167	0.090			
Sharpe ratio	0.779	0.767	0.569	-0.419				0.571	0.728	0.807	0.513			
α_{CAPM}	0.005	0.004	0.002	-0.003	-2.139	4.576	0.033	0.002	0.003	0.005	0.003	2.533	6.414	0.012
α_{FF3}	0.003	0.002	0.000	-0.003	-2.389	5.710	0.017	0.000	0.001	0.003	0.003	2.296	5.273	0.022
$\alpha_{4factor}$	0.004	0.002	0.000	-0.004	-2.900	8.412	0.004	0.001	0.002	0.005	0.004	3.014	9.087	0.003
$\alpha_{5factor}$	0.004	0.002	0.001	-0.004	-2.765	7.645	0.006	0.001	0.002	0.005	0.004	2.988	8.933	0.003
Panel B: Equal-weighted														
	Magnitude Factor							Concentration Factor						
	1	2	3	H-L	t	grs stat	grs p	1	2	3	H-L	t	grs stat	grs p
Ave.Exc.Ret	0.132	0.103	0.092	-0.040	-2.539			0.091	0.101	0.134	0.043	2.001		
volatility	0.164	0.137	0.163	0.093				0.153	0.141	0.171	0.096			
Sharpe ratio	0.804	0.748	0.563	-0.434				0.595	0.715	0.786	0.447			
α_{CAPM}	0.005	0.003	0.002	-0.003	-2.382	5.672	0.018	0.002	0.003	0.005	0.003	2.093	4.380	0.037
α_{FF3}	0.004	0.002	0.000	-0.004	-2.721	7.406	0.007	0.001	0.001	0.003	0.003	1.968	3.925	0.038
$\alpha_{4factor}$	0.005	0.002	0.000	-0.004	-3.158	9.974	0.002	0.001	0.002	0.004	0.003	2.076	4.310	0.039
$\alpha_{5factor}$	0.005	0.002	0.000	-0.004	-2.899	8.407	0.004	0.001	0.002	0.004	0.003	2.198	4.831	0.029

Table 4. Single Sorted Portfolios. This Table reports the tercile sorted results for Magnitude and Concentration. Panel A reports the value-weighted results and Panel B reports the equal-weighted results. Left panel shows the results for Magnitude factor and right panel shows the results for Concentration Factor. Average excess return, volatility and Sharpe ratio are all annualized and the anomaly alphas are reported as monthly. α_{FF3} refers to the intercept by regressing sorted portfolios onto $Fama - French$ 3 factors. $\alpha_{4factor}$ refers to the intercept by regressing sorted portfolios onto 4 factors (FF3+ Momentum from Carhart (1997)). $\alpha_{5factor}$ refers to the intercept by regressing sorted portfolios onto 5 factors (4 factor + Liquidity from Pástor and Stambaugh (2003)).

	CON10 +MAG10		CON10 +SZ10		CON10 +IND30		CON10 +BM10	
Const	-0.10	0.38	0.11	0.19	1.01	1.09	-0.15	0.07
t	-0.30	1.37	0.35	0.62	4.22	4.48	-0.47	0.23
Mkt.Rf	1.12	0.57	0.89	0.79	-0.01	-0.12	1.22	0.96
t	2.69	1.47	2.22	1.97	-0.04	-0.37	3.34	2.75
CON		0.35		0.34		0.23		0.35
t		2.85		2.41		2.86		2.47
MAG		-0.33		-0.34		-0.35		-0.31
t		-3.44		-2.43		-2.70		-2.22
MAPE%	0.11	0.02	0.09	0.03	0.15	0.12	0.11	0.07
χ^2	27.13	17.03	36.21	26.77	71.11	60.66	40.45	31.35
p value	0.10	0.45	0.17	0.48	0.02	0.09	0.08	0.26
R^2	0.30	0.97	0.43	0.93	0.00	0.29	0.34	0.70
	MAG10 +INV10		MAG10 +MOM10		MAG10 +OP10		ALL100	
Const	0.65	0.80	1.11	1.23	0.94	1.06	0.88	0.96
t	2.17	2.75	3.72	4.13	3.29	3.72	3.43	3.70
Mkt.Rf	0.33	0.15	-0.20	-0.35	-0.02	-0.16	0.09	-0.03
t	0.91	0.42	-0.50	-0.90	-0.06	-0.46	0.25	-0.07
CON		0.60		0.37		0.33		0.30
t		2.59		1.83		2.31		2.11
MAG		-0.31		-0.29		-0.35		-0.35
t		-2.30		-2.21		-2.57		-2.69
MAPE%	0.14	0.09	0.15	0.10	0.14	0.08	0.13	0.08
χ^2	57.06	47.15	58.61	48.67	58.37	48.08	47.14	37.02
p value	0.00	0.01	0.00	0.01	0.00	0.01	0.02	0.09
R^2	0.04	0.55	0.02	0.39	0.00	0.60	0.01	0.59

Table 5. Two-Step Cross-sectional Regression. This Table reports the prices of risk estimated from Fama MacBeth regression through GMM (lag 1 Newey-West error). I consider two sets of asset pricing factors: (1) market excess return which is the CAPM model, and (2) the network factor innovations along with the market excess returns. In terms of test assets, I consider 8 different test assets: (a) 10 Concentration-beta sorted portfolios + 10 Magnitude-beta sorted portfolios, (b) 10 size portfolios, (c) 30 industry portfolios (French's website), (d) 10 Concentration-beta sorted portfolios + 10 book to market portfolios, (e) 10 investment portfolios , (f) 10 momentum portfolios (French's website), (g) 10 Magnitude-beta sorted portfolios + 10 operating profitability portfolios (French's website), (h) All 100 portfolios listed in previous columns. The sample is from Jan 1969 to December 2019 on monthly frequency.

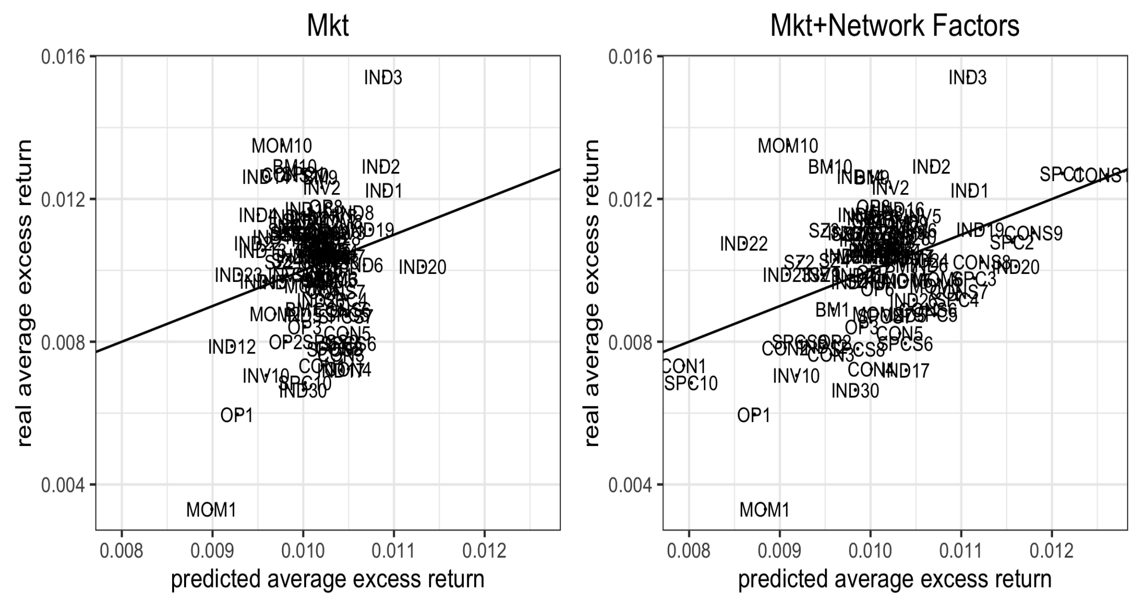


Figure 8. Predicted Risk Premium vs. Expected Risk Return. This figure shows the monthly predicted risk premium versus expected risk premium from Fama MacBeth Analysis. I consider two sets of asset pricing factors: (1) market excess return which is the CAPM model, and (2) the network factor innovations along with the market excess returns. In terms of test assets, I consider combinations of all 100 different test assets: 10 Concentration-beta sorted portfolios + 10 Magnitude- beta sorted portfolios + 30 industry portfolios (French's website) + 10 book to market portfolios (French's website) + 10 investment portfolios (French's website) + 10 momentum portfolios + 10 operating profitability portfolios (French's website) + 10 size portfolios (French's website). The sample is from July 1976 to December 2019 on monthly frequency.

	SZBM25		SZOP25		SZINV25		SZBMOP32	
Const	0.95	0.98	0.91	0.95	0.88	0.91	1.24	1.27
t	2.51	2.73	2.96	3.23	2.89	3.16	3.51	3.83
Mkt.Rf	0.06	0.01	0.09	0.04	0.15	0.10	-0.21	-0.27
t	0.13	0.02	0.24	0.09	0.38	0.25	-0.49	-0.66
CON		0.41		0.39		0.38		0.41
t		2.87		2.71		2.67		2.83
MAG		-0.25		-0.31		-0.25		-0.25
t		-1.73		-2.18		-1.71		-1.68
MAPE%	0.18	0.15	0.17	0.14	0.18	0.16	0.20	0.18
R^2	0.00	0.23	0.00	0.30	0.01	0.23	0.02	0.16

Table 6. Fama-MacBeth Analysis of other multi-sorted portfolios. This Table reports the prices of risk estimated from Fama MacBeth regression through GMM (lag 1 Newey-West error). I consider two sets of asset pricing factors: (1) market excess return which is the CAPM model, and (2) the network factor innovations along with the market excess returns. In terms of test assets, I consider 4 different multiple-sorted test assets: (a) 25 size-BM double sorted portfolios (French's website), (b) 25 size-operating profitability double sorted portfolios (French's website), (c) 25 size-investment double sorted portfolios (French's website), (d) 32 size, BM and OP three-way sorted portfolios (French's website). The sample is from July 1976 to December 2019 on monthly frequency.

	10 GID-beta portfolios		15 AggVol*Mkt portfolios	
Const	-0.17	-0.04	1.1	0.44
t	-0.52	-0.09	3.23	0.95
Mkt.Rf	1.27	1.07	-0.24	0.10
t	3.11	2.15	-0.45	0.19
CON		0.3		0.43
t		1.86		1.76
MAG		-1.98		-3.41
t		-1.98		-2.01
MAPE%	0.13	0.10	0.17	0.06
R^2	0.45	0.74	0.07	0.92

Table 7. Fama-MacBeth Analysis of other anomaly portfolios. This Table reports the prices of risk estimated from Fama MacBeth regression through GMM (lag 1 Newey-West error). I consider two sets of asset pricing factors: (1) market excess return which is the CAPM model, and (2) the network factor innovations along with the market excess returns. In terms of test assets, I consider 4 different multiple-sorted test assets: (a) 10 GID-β sorted portfolios from Guvenen, Ozkan, and Song (2014). Guvenen, Ozkan, and Song (2014) posted the quintile sort result of log earnings growth, and we obtain GID-β by regressing yearly excess returns on yearly H-L mimicking factor. Then we sort monthly excess return based on the yearly β. This is exactly what Herskovic et al. (2016) did. (b) The second test asset is 5×3 Aggregate volatility and Market double sorted portfolios from Ang et al. (2006). The sample is from July 1976 to December 2019 on monthly frequency.

	CON10 +MAG10				IND48				OP10			
Const	0.42	0.35	-1.86	0.35	1.01	0.93	0.82	0.93	1.18	1.57	0.16	1.40
t	1.42	1.08	-1.43	1.07	3.27	3.71	3.16	3.70	3.04	2.98	0.26	2.37
Mkt.Rf	0.50	0.30	3.79	0.30	0.03	0.15	0.27	0.15	-0.24	-0.59	0.77	-0.43
t	1.17	0.50	2.17	0.50	0.07	0.44	0.81	0.45	-0.51	-1.03	1.24	-0.70
SMB		0.31	-1.55	0.31		-0.47	-0.45	-0.48		-0.27	-0.63	-0.40
t		0.80	-1.74	0.76		-2.14	-2.05	-2.18		-1.29	-2.88	-1.67
HML		0.36	-0.34	0.35		-0.13	-0.04	-0.13		-0.45	0.62	-0.42
t		0.79	-0.40	0.77		-0.60	-0.18	-0.62		-1.38	1.82	-1.10
CON	0.35	0.35		0.35	0.32	0.32		0.32	0.34	0.35		0.36
t	2.45	2.48		2.48	1.97	2.11		2.05	2.40	2.45		2.45
MAG	-0.33	-0.33		-0.33	-0.71	-0.64		-0.63	-1.99	-1.80		-1.83
t	-2.44	-2.41		-2.41	-1.96	-1.44		-1.99	-2.20	-2.04		-1.90
CIV	-0.98		-3.87	-0.73	1.99		-0.77	-0.31	1.43		-2.31	-3.85
t	-0.55		-1.09	-0.37	1.42		-0.61	-0.26	0.64		-1.02	-1.46
MAPE%	0.02	0.02	0.07	0.02	0.13	0.11	0.12	0.11	0.07	0.06	0.11	0.06
χ^2	16.89	14.87	25.50	14.84	59.12	55.95	78.96	55.69	36.04	34.13	41.05	30.16
p value	0.39	0.46	0.06	0.39	0.29	0.36	0.02	0.34	0.00	0.00	0.00	0.01
R^2	0.97	0.97	0.74	0.97	0.31	0.51	0.34	0.52	0.72	0.79	0.30	0.82
	INV10				MOM10				ALL 98			
Const	1.02	-0.43	-0.36	-0.37	1.14	2.17	2.19	1.87	1.11	1.11	0.98	1.09
t	2.36	-0.82	-0.68	-0.63	2.60	3.09	3.52	2.60	3.82	4.85	4.16	4.74
Mkt.Rf	0.00	1.40	1.35	1.34	-0.10	-1.01	-1.00	-0.74	-0.09	-0.06	0.07	-0.05
t	-0.01	2.45	2.38	2.18	-0.20	-1.44	-1.59	-1.05	-0.23	-0.20	0.22	-0.15
SMB		-0.68	-0.83	-0.47		0.09	-0.03	0.08		-0.37	-0.35	-0.34
t		-2.52	-2.88	-1.77		0.45	-0.15	0.43		-1.85	-1.83	-1.75
HML		0.76	0.68	0.72		-1.06	-1.07	-0.80		-0.08	0.00	-0.07
t		2.60	2.41	2.08		-2.40	-2.55	-1.84		-0.43	-0.01	-0.35
CON	1.72	1.35		1.32	0.27	1.12		1.11	0.35	0.34		0.35
t	2.55	2.02		1.99	0.35	1.92		2.03	2.05	2.18		2.23
MAG	-0.32	-0.32		-0.32	-0.30	-0.33		-0.32	-0.40	-0.36		-0.37
t	-2.36	-2.33		-2.35	-2.19	-2.41		-2.38	-2.55	-2.28		-2.39
CIV	4.93		-6.34	4.21	4.64		-1.26	2.90	2.47		0.29	1.10
t	1.98		-1.44	1.30	2.22		-0.51	1.55	1.81		0.24	0.93
MAPE%	0.07	0.05	0.08	0.04	0.09	0.06	0.09	0.06	0.12	0.12	0.13	0.12
χ^2	30.51	23.08	30.43	18.77	31.19	31.12	39.21	28.95	189.89	187.99	198.71	187.97
p value	0.02	0.08	0.02	0.17	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00
R^2	0.69	0.86	0.53	0.90	0.70	0.87	0.69	0.88	0.34	0.38	0.22	0.40

Table 8. Fama MacBeth Analysis with richer factor models. This Table reports the prices of risk estimated from Fama MacBeth regression through GMM (lag 1 Newey-West error). I consider four sets of asset pricing factors: (1) Mkt + CON + MAG + CIV (where the CIV factor innovation is the common factor in idiosyncratic volatility from Herskovic et al. (2016), (2) FF3 + CON + MAG, (3) FF3+ CIV and (4) FF3 + CON + MAG + CIV. In terms of test assets, I consider 6 different test assets: (a) 10 Concentration-beta sorted portfolios + 10 Magnitude- beta sorted portfolios, (b) 48 industry portfolios, (c)30 operating profitability portfolios, (d) 10 Magnitude-beta sorted portfolios + 10 investment portfolios, (e) 10 Magnitude-beta sorted portfolios + 10 momentum portfolios, (f) all 98 test assets mentioned in previous columns. The sample is from Jan 1969 to December 2019 on monthly frequency.

	MAG10 +OP10		CON10 +SZ10		IND30		INV10		ALL80	
Const	16.69	12.96	11.73	5.34	9.33	10.22	11.54	14.44	9.81	10.11
t	3.29	3.30	2.75	1.27	1.05	0.89	1.87	3.27	2.41	3.05
Mkt.Rf	-4.73	-1.21	0.46	8.45	1.19	-2.08	2.17	-2.48	3.92	2.19
t	-0.91	-0.26	0.09	1.59	0.15	-0.17	0.36	-0.46	0.81	0.51
CON_{prod}	-8.41	1.32	-0.36	2.46	18.06	26.51	-10.77	0.77	-7.68	3.01
t	-1.88	0.25	-0.09	0.72	0.68	1.98	-2.26	0.26	-1.87	0.75
SPC_{prod}	2.44	6.95	8.38	-1.36	28.28	35.41	8.76	6.69	5.47	6.35
t	0.23	1.00	1.87	-0.38	2.10	2.86	1.09	1.42	1.99	1.01
CON_{vol}		2.21		5.63		9.84		4.62		4.46
t		2.44		2.24		2.83		1.99		2.96
MAG_{vol}		-3.80		-2.41		-3.24		-6.77		-3.39
t		-3.97		-2.46		-2.96		-2.54		-3.88
MAPE%	1.07	0.73	1.19	0.41	2.30	1.72	1.37	0.68	1.37	0.64
R^2	0.42	0.76	0.26	0.93	0.34	0.62	0.37	0.87	0.23	0.80

Table 9. Fama MacBeth Analysis with production-based network factors. This Table reports the prices of risk estimated from Fama MacBeth regression through GMM (lag 1 Newey-West error). I consider 2 sets of asset pricing factors: (1) Mkt + production-based CON factor + production-based SPC factor, (2) Mkt + production-based CON factor + production-based SPC factor + idiosyncratic volatility spillover based CON + idiosyncratic volatility spillover based MAG. In terms of test assets, I consider 5 different test assets: (a) 10 Magnitude-beta sorted portfolios + 10 operating profitability sorted portfolios, (b) 10 Concentration-beta sorted portfolios + 10 size portfolios, (c) 30 industry portfolios, (d) 10 investment portfolios, (e) all 80 test assets mentioned in previous columns. The sample is from 1984 to December 2014 on annual frequency.

Panel A: Average Excess Return					
		MAG			
		1	2	3	H-L
CON	1	0.102	0.087	0.077	-0.025
	2	0.122	0.099	0.087	-0.035
	3	0.155	0.128	0.122	-0.033
	H-L	0.053	0.041	0.045	

Panel B: α_{FF3}						
		MAG				
		1	2	3	H-L	t-stat
CON	1	0.002	0.001	-0.001	-0.003	-1.989
	2	0.003	0.002	0.000	-0.003	-2.385
	3	0.005	0.003	0.003	-0.002	-1.967
	H-L	0.003	0.002	0.004		
	t-stat	2.173	1.765	2.596		

Panel C: $\alpha_{4factor}$						
		MAG				
		1	2	3	H-L	t-stat
CON	1	0.002	0.001	-0.001	-0.003	-2.332
	2	0.004	0.002	0.000	-0.003	-2.529
	3	0.006	0.004	0.003	-0.003	-1.917
	H-L	0.004	0.003	0.005		
	t-stat	2.727	2.614	3.246		

Panel D: $\alpha_{5factor}$						
		MAG				
		1	2	3	H-L	t-stat
CON	1	0.002	0.001	-0.001	-0.003	-2.159
	2	0.004	0.002	0.000	-0.003	-2.466
	3	0.006	0.004	0.004	-0.003	-1.783
	H-L	0.004	0.003	0.005		
	t-stat	2.714	2.602	3.198		

Table 10. Double Sorting portfolios based on two network factors. This Table reports the tercile-tercile sorted results for Magnitude and Concentration. The portfolios are formed by value-weighted. Panel A reports the annualized average risk premium. Panel B reports the monthly α_{FF3} which refers to the intercept by regressing sorted portfolios onto $Fama - French$ 3 factors. Panel C reports the monthly $\alpha_{4factor}$ which refers to the intercept by regressing sorted portfolios onto 4 factors (FF3+ Momentum from Carhart (1997)). Panel D reports the monthly $\alpha_{5factor}$ which refers to the intercept by regressing sorted portfolios onto 5 factors (4 factor + Liquidity from Pástor and Stambaugh (2003)).

		Returns	α_{FF3}		$\alpha_{4factor}$		$\alpha_{5factor}$	
		H-L	H-L	t-stat	H-L	t-stat	H-L	t-stat
Mkt.RF	L	-0.032	-0.003	-2.007	-0.003	-2.340	-0.003	-2.247
	M	-0.044	-0.004	-2.498	-0.004	-2.986	-0.004	-2.859
	H	-0.040	-0.004	-2.268	-0.005	-2.869	-0.004	-2.710
SMB	L	-0.043	-0.004	-2.816	-0.005	-3.441	-0.004	-3.348
	M	-0.044	-0.004	-2.598	-0.004	-3.002	-0.004	-2.861
	H	-0.028	-0.002	-1.513	-0.003	-1.950	-0.003	-1.802
HML	L	-0.035	-0.002	-1.607	-0.004	-2.359	-0.003	-2.278
	M	-0.043	-0.004	-2.630	-0.004	-2.941	-0.004	-2.818
	H	-0.038	-0.004	-2.519	-0.004	-2.856	-0.004	-2.690

Table 11. Double Sorting portfolios based on MAG and other factors. This Table reports the tercile-tercile sorted results for Magnitude and other anomaly factor. The portfolios are formed via value-weighted. I reported the annualized High-Low risk premium, the monthly α_{FF3}, the monthly $\alpha_{4factor}$ and the monthly $\alpha_{5factor}$ for each category of the anomaly factors.

		Returns	α_{FF3}		$\alpha_{4factor}$		$\alpha_{5factor}$	
		H-L	H-L	t-stat	H-L	t-stat	H-L	t-stat
Mkt.RF	L	0.044	0.003	2.543	0.004	3.285	0.004	3.276
	M	0.046	0.003	2.222	0.004	2.930	0.004	2.854
	H	0.048	0.003	1.803	0.004	2.384	0.004	2.393
SMB	L	0.052	0.004	2.658	0.004	3.304	0.004	3.303
	M	0.043	0.003	1.913	0.004	2.687	0.004	2.726
	H	0.043	0.003	1.972	0.004	2.573	0.004	2.478
HML	L	0.044	0.003	2.263	0.004	2.592	0.004	2.543
	M	0.048	0.003	2.378	0.004	2.935	0.004	2.930
	H	0.047	0.003	1.788	0.004	2.865	0.004	2.849

Table 12. Double Sorting portfolios based on CON and other factors. This Table reports the tercile-tercile sorted results for Concentration and other anomaly factor. The portfolios are formed via value-weighted. I reported the annualized High-Low risk premium, the monthly α_{FF3}, the monthly $\alpha_{4factor}$ and the monthly $\alpha_{5factor}$ for each category of the anomaly factors.

Panel A: Average Excess Returns

		Magnitude Factor					Concentration Factor				
		1	2	3	H-L	t-stat	1	2	3	H-L	t-stat
1	$Concentration_{herfout}$	0.134	0.106	0.086	-0.048	-2.698	0.083	0.100	0.123	0.040	2.011
2	$Concentration_{eigvec1}$	0.128	0.105	0.094	-0.034	-2.534	0.087	0.105	0.134	0.046	2.032
3	$Concentration_{eigvec2}$	0.126	0.103	0.097	-0.029	-2.001	0.101	0.102	0.123	0.022	1.974
4	Benchmark controls+MOM+LIQ	0.129	0.105	0.093	-0.036	-2.535	0.093	0.102	0.132	0.039	2.001
5	Benchmark controls+ΔMktVol	0.130	0.102	0.094	-0.036	-2.537	0.088	0.103	0.135	0.046	2.202
6	Control Prev Levels	0.134	0.101	0.091	-0.043	-2.600	0.089	0.102	0.134	0.045	2.032
7	No Control	0.122	0.102	0.101	-0.021	-1.985	0.103	0.103	0.126	0.023	1.985
8	All CRSP stocks	0.143	0.098	0.085	-0.058	-2.996	0.081	0.095	0.151	0.069	2.236
9	PC5 Idio Vol	0.130	0.106	0.090	-0.040	-2.598	0.089	0.103	0.134	0.044	2.228
10	NAICS20 sectors	0.095	0.090	0.056	-0.039	-2.591	0.070	0.084	0.098	0.029	1.998
11	SIC3digit	0.107	0.080	0.058	-0.050	-2.701	0.067	0.083	0.098	0.032	2.000

Panel B: α_{FF3}

		Magnitude Factor					Concentration Factor				
		1	2	3	H-L	t-stat	1	2	3	H-L	t-stat
1	$Concentration_{herfout}$	0.004	0.002	-0.001	-0.004	-3.229	0.001	0.001	0.003	0.001	1.963
2	$Concentration_{eigvec1}$	0.003	0.002	0.000	-0.003	-2.461	0.000	0.002	0.003	0.003	1.962
3	$Concentration_{eigvec2}$	0.003	0.002	0.000	-0.003	-1.969	0.002	0.001	0.002	0.001	1.927
4	Benchmark controls+MOM+LIQ	0.003	0.002	0.000	-0.003	-2.152	0.001	0.001	0.003	0.003	2.063
5	Benchmark controls+ΔMktVol	0.003	0.002	0.000	-0.003	-2.099	0.002	0.001	0.002	0.000	2.225
6	Control Prev Level	0.004	0.001	0.000	-0.005	-3.099	0.001	0.001	0.003	0.002	2.532
7	No Control	0.003	0.002	0.000	-0.003	-2.033	0.002	0.002	0.002	0.000	1.949
8	All CRSP stocks	0.004	0.001	-0.001	-0.005	-2.533	-0.001	0.001	0.004	0.005	2.122
9	PC5 Idio Vol	0.004	0.002	0.000	-0.004	-2.774	0.001	0.001	0.003	0.003	2.169
10	NAICS20 sectors	0.002	0.002	-0.002	-0.004	-2.665	-0.001	0.001	0.002	0.003	2.654
11	SIC3digit	0.002	0.001	-0.002	-0.004	-2.270	-0.001	0.001	0.002	0.003	1.499

Panel C: $\alpha_{4factor}$

		Magnitude Factor					Concentration Factor				
		1	2	3	H-L	t-stat	1	2	3	H-L	t-stat
1	$Concentration_{herfout}$	0.005	0.002	0.000	-0.005	-3.423	0.002	0.002	0.004	0.002	2.003
2	$Concentration_{eigvec1}$	0.005	0.002	0.000	-0.004	-2.952	0.001	0.002	0.004	0.003	2.064
3	$Concentration_{eigvec2}$	0.004	0.002	0.001	-0.003	-2.192	0.002	0.002	0.003	0.000	2.310
4	Benchmark controls+MOM+LIQ	0.004	0.002	0.001	-0.003	-2.187	0.001	0.002	0.004	0.003	2.381
5	Benchmark controls+ΔMktVol	0.004	0.002	0.001	-0.003	-2.218	0.003	0.002	0.003	0.000	2.112
6	Control Prev Level	0.005	0.002	0.000	-0.005	-3.521	0.001	0.002	0.004	0.003	1.985
7	No Control	0.004	0.002	0.001	-0.003	-2.225	0.002	0.002	0.003	0.000	1.917
8	All CRSP stocks	0.006	0.002	0.000	-0.006	-3.018	0.001	0.001	0.006	0.005	2.287
9	PC5 Idio Vol	0.005	0.002	0.000	-0.004	-3.163	0.001	0.002	0.004	0.003	2.509
10	NAICS20 sectors	0.002	0.002	-0.002	-0.004	-2.337	0.000	0.001	0.002	0.002	2.262
11	SIC3digit	0.003	0.001	-0.002	-0.005	-2.659	-0.001	0.001	0.002	0.002	1.267

Panel D: $\alpha_{5factor}$

		Magnitude Factor					Concentration Factor				
		1	2	3	H-L	t-stat	1	2	3	H-L	t-stat
1	$Concentration_{herfout}$	0.005	0.002	0.000	-0.005	-3.233	0.002	0.002	0.004	0.002	2.374
2	$Concentration_{eigvec1}$	0.004	0.002	0.001	-0.004	-2.686	0.001	0.002	0.004	0.003	2.137
3	$Concentration_{eigvec2}$	0.004	0.002	0.001	-0.003	-1.974	0.002	0.002	0.003	0.001	2.675
4	Benchmark controls+MOM+LIQ	0.004	0.002	0.001	-0.003	-1.944	0.001	0.002	0.004	0.003	2.398
5	Benchmark controls+ΔMktVol	0.004	0.002	0.001	-0.003	-2.018	0.002	0.002	0.003	0.001	2.383
6	Control Prev Level	0.005	0.002	0.000	-0.005	-3.230	0.001	0.002	0.004	0.003	1.952
7	No Control	0.004	0.002	0.001	-0.003	-1.972	0.002	0.002	0.003	0.001	1.966
8	All CRSP stocks	0.006	0.002	0.000	-0.006	-2.856	0.001	0.001	0.006	0.006	2.419
9	PC5 Idio Vol	0.004	0.002	0.000	-0.004	-2.918	0.001	0.002	0.004	0.003	2.483
10	NAICS20 sectors	0.002	0.001	-0.002	-0.004	-2.017	0.000	0.001	0.003	0.003	2.176
11	SIC3digit	0.003	0.001	-0.002	-0.004	-2.359	0.000	0.001	0.001	0.002	0.991

Table 13. Single sorted portfolios under alternative measurements or controls.

Panel A: Average Excess Returns

		Magnitude Factor					Concentration Factor				
		1	2	3	H-L	t-stat	1	2	3	H-L	t-stat
1	trailing60	0.134	0.109	0.088	-0.046	-2.558	0.090	0.106	0.136	0.046	2.258
2	trailing70	0.140	0.107	0.094	-0.046	-2.562	0.096	0.109	0.136	0.040	2.166
3	trailing90	0.127	0.104	0.093	-0.033	-2.533	0.092	0.101	0.131	0.039	2.164
4	trailing95	0.129	0.107	0.101	-0.028	-2.388	0.094	0.106	0.137	0.043	2.230
5	trailing100	0.127	0.106	0.102	-0.025	-2.233	0.092	0.105	0.138	0.045	2.233
6	rolling80	0.133	0.111	0.110	-0.023	-2.232	0.100	0.112	0.142	0.042	2.226
7	rolling85	0.131	0.105	0.105	-0.036	-2.546	0.090	0.109	0.141	0.051	2.414
8	rolling95	0.126	0.099	0.096	-0.031	-2.565	0.090	0.100	0.132	0.042	2.173
9	rolling100	0.122	0.104	0.099	-0.024	-2.222	0.097	0.100	0.129	0.032	2.044
10	forecasting7	0.128	0.100	0.097	-0.032	-2.535	0.094	0.103	0.128	0.034	2.086
11	forecasting8	0.125	0.104	0.097	-0.029	-2.314	0.097	0.101	0.128	0.032	2.047
12	forecasting9	0.129	0.104	0.093	-0.036	-2.644	0.092	0.101	0.133	0.041	2.166
13	forecasting10	0.131	0.102	0.094	-0.037	-2.645	0.091	0.100	0.135	0.044	2.228

Panel B: α_{FF3}

		Magnitude Factor					Concentration Factor				
		1	2	3	H-L	t-stat	1	2	3	H-L	t-stat
1	trailing60	0.004	0.002	0.000	-0.004	-2.635	0.000	0.002	0.003	0.003	1.990
2	trailing70	0.004	0.002	0.000	-0.004	-2.986	0.001	0.002	0.003	0.002	1.989
3	trailing90	0.003	0.002	0.000	-0.003	-2.588	0.001	0.001	0.003	0.002	1.968
4	trailing95	0.003	0.002	0.000	-0.003	-2.245	0.001	0.002	0.003	0.003	1.963
5	trailing100	0.003	0.002	0.000	-0.003	-2.093	0.001	0.002	0.003	0.003	2.099
6	rolling80	0.003	0.002	0.001	-0.003	-1.967	0.000	0.002	0.004	0.004	2.658
7	rolling85	0.003	0.002	0.001	-0.003	-1.967	0.000	0.002	0.004	0.005	3.573
8	rolling95	0.003	0.002	0.001	-0.002	-2.047	0.000	0.001	0.004	0.003	2.311
9	rolling100	0.003	0.002	0.000	-0.003	-2.081	0.001	0.001	0.003	0.002	1.991
10	forecasting7	0.003	0.001	0.000	-0.003	-2.183	0.001	0.002	0.003	0.002	1.999
11	forecasting8	0.003	0.002	0.000	-0.003	-1.968	0.001	0.001	0.003	0.002	2.064
12	forecasting9	0.004	0.002	0.000	-0.004	-2.486	0.001	0.001	0.003	0.002	2.001
13	forecasting10	0.004	0.002	0.000	-0.004	-2.484	0.001	0.001	0.003	0.003	2.103

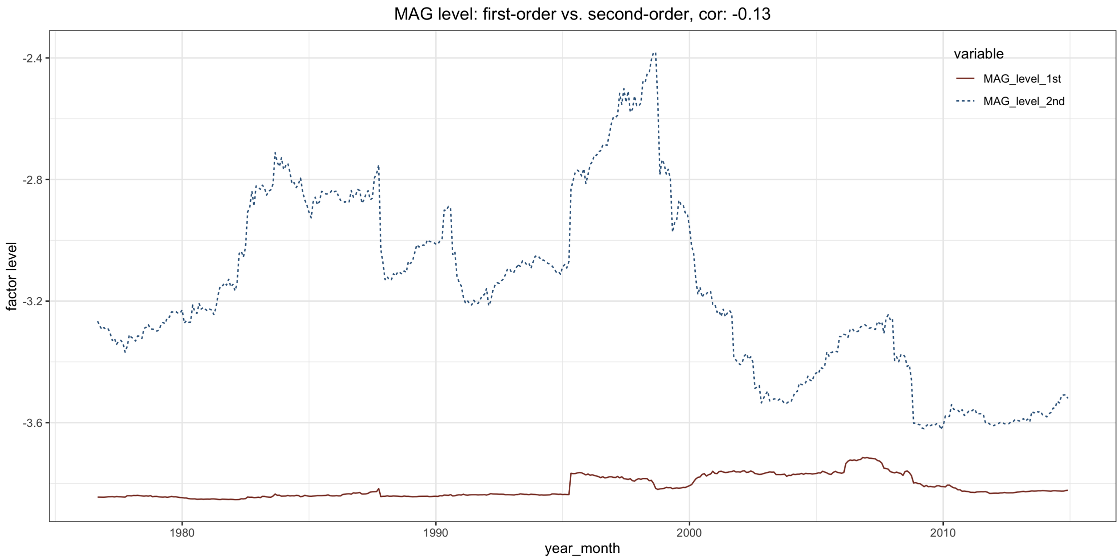
Panel C: $\alpha_{4factor}$

		Magnitude Factor					Concentration Factor				
		1	2	3	H-L	t-stat	1	2	3	H-L	t-stat
1	trailing60	0.005	0.003	0.000	-0.005	-3.127	0.001	0.002	0.005	0.004	2.436
2	trailing70	0.005	0.002	0.000	-0.005	-3.311	0.001	0.002	0.004	0.003	1.947
3	trailing90	0.004	0.002	0.000	-0.004	-2.978	0.001	0.002	0.004	0.003	2.053
4	trailing95	0.004	0.002	0.001	-0.003	-2.628	0.001	0.002	0.004	0.003	2.319
5	trailing100	0.004	0.002	0.001	-0.003	-2.459	0.001	0.002	0.004	0.003	2.459
6	rolling80	0.004	0.002	0.001	-0.002	-1.782	0.001	0.002	0.004	0.003	2.413
7	rolling85	0.004	0.002	0.001	-0.004	-2.467	0.000	0.002	0.005	0.005	3.619
8	rolling95	0.004	0.002	0.001	-0.003	-2.271	0.001	0.002	0.004	0.003	2.053
9	rolling100	0.004	0.002	0.001	-0.003	-2.289	0.001	0.002	0.004	0.003	2.196
10	forecasting7	0.005	0.002	0.001	-0.004	-2.686	0.001	0.002	0.004	0.003	2.376
11	forecasting8	0.004	0.002	0.001	-0.003	-2.386	0.002	0.002	0.004	0.002	2.032
12	forecasting9	0.005	0.002	0.000	-0.004	-3.033	0.001	0.002	0.004	0.003	1.989
13	forecasting10	0.005	0.002	0.001	-0.004	-2.759	0.001	0.002	0.005	0.004	2.544

Panel D: $\alpha_{5factor}$

		Magnitude Factor					Concentration Factor				
		1	2	3	H-L	t-stat	1	2	3	H-L	t-stat
1	trailing60	0.005	0.003	0.000	-0.005	-2.996	0.001	0.002	0.005	0.004	2.631
2	trailing70	0.005	0.002	0.000	-0.005	-3.122	0.001	0.002	0.004	0.003	2.106
3	trailing90	0.004	0.002	0.001	-0.004	-2.749	0.001	0.002	0.004	0.003	2.100
4	trailing95	0.004	0.002	0.001	-0.003	-2.409	0.001	0.002	0.004	0.003	2.351
5	trailing100	0.004	0.002	0.001	-0.003	-2.232	0.001	0.002	0.004	0.003	2.444
6	rolling80	0.004	0.002	0.002	-0.002	-1.719	0.001	0.002	0.005	0.004	2.478
7	rolling85	0.004	0.002	0.001	-0.003	-2.182	0.000	0.002	0.005	0.005	3.759
8	rolling95	0.004	0.002	0.001	-0.003	-2.049	0.001	0.002	0.004	0.003	2.170
9	rolling100	0.004	0.002	0.001	-0.003	-2.064	0.001	0.002	0.004	0.003	2.026
10	forecasting7	0.004	0.002	0.001	-0.003	-2.389	0.001	0.002	0.004	0.003	2.280
11	forecasting8	0.004	0.002	0.001	-0.003	-2.162	0.002	0.002	0.004	0.002	2.076
12	forecasting9	0.004	0.002	0.001	-0.004	-2.758	0.001	0.002	0.004	0.003	2.164
13	forecasting10	0.004	0.002	0.001	-0.004	-2.496	0.001	0.002	0.005	0.004	2.544

Table 14. Single sorted portfolios under different parameter choices.



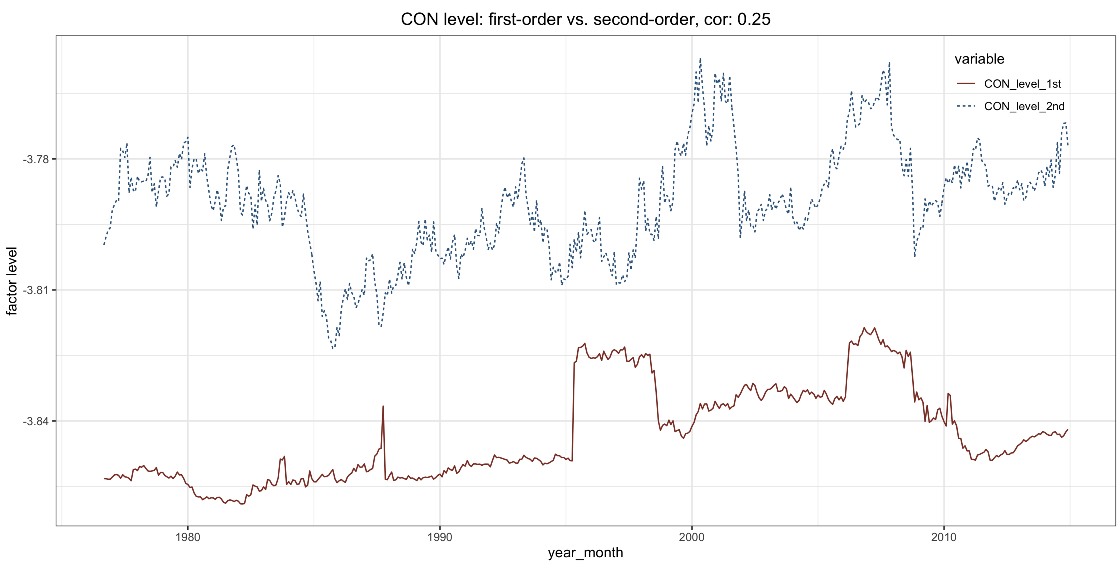


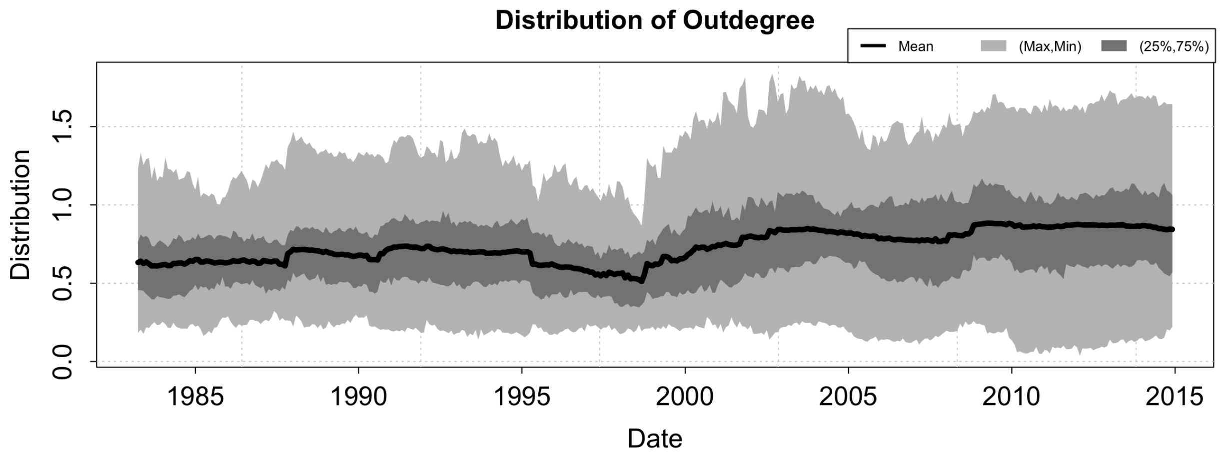
Figure 9. Time series plots of the first-order-based and the second-order-based network factors. The first panel plots both magnitude factors in level, with the red line denoting the first-order case and the blue line denoting the second-order case. The second panel plots both concentration factors in level, with the red line denoting the first-order case and the blue line denoting the second-order case.

Value-weighted portfolio for first-order network factors														
	Magnitude Factor							Concentration Factor						
	1	2	3	H-L	t	grs stat	grs p	1	2	3	H-L	t	grs stat	grs p
Ave.Exc.Ret	0.103	0.102	0.121	0.018				0.113	0.101	0.112	-0.001			
volatility	0.162	0.141	0.163	0.098				0.163	0.142	0.164	0.107			
Sharpe ratio	0.636	0.726	0.742	0.186				0.693	0.714	0.682	-0.005			
α_{CAPM}	0.003	0.003	0.004	0.001	0.965	0.931	0.335	0.002	0.003	0.003	0.001	-0.043	0.002	0.966
α_{FF3}	0.001	0.002	0.003	0.001	0.996	0.992	0.320	0.002	0.001	0.002	0.000	-0.034	0.001	0.973
$\alpha_{4factor}$	0.002	0.002	0.003	0.001	0.750	0.563	0.454	0.002	0.002	0.003	0.001	0.237	0.056	0.813
$\alpha_{5factor}$	0.002	0.002	0.003	0.001	0.707	0.500	0.480	0.002	0.002	0.003	0.001	0.381	0.145	0.704

Table 15. Single Sorted Portfolios for first-order factors. This Table reports the tercile sorted results for Magnitude and Concentration factors from first-order connection. Left panel shows the results for Magnitude factor and right panel shows the results for Concentration Factor. Average excess return, volatility and Sharpe ratio are all annualized and the anomaly alphas are reported as monthly. α_{FF3} refers to the intercept by regressing sorted portfolios onto $Fama - French$ 3 factors. $\alpha_{4factor}$ refers to the intercept by regressing sorted portfolios onto 4 factors (FF3+ Momentum from Carhart (1997)). $\alpha_{5factor}$ refers to the intercept by regressing sorted portfolios onto 5 factors (4 factor + Liquidity from Pástor and Stambaugh (2003)).

Correlation between systemic risk measurements									
	Cross-sectional Mean					Cross-sectional Std			
	Outdegree	Indegree	MAG	CON		Outdegree	Indegree	MAG	CON
Outdegree	1.00	0.63^{***}	-0.37^{***}	0.74^{***}	Outdegree	1.00	-0.48^{***}	0.54^{***}	0.42^{***}
Indegree	0.63^{***}	1.00	-0.57^{***}	0.74^{***}	Indegree	-0.48^{***}	1.00	-0.32^{***}	-0.49^{***}
MAG	-0.37^{***}	-0.57^{***}	1.00	-0.30^{***}	MAG	0.54^{***}	-0.32^{***}	1.00	0.67^{***}
CON	0.74^{***}	0.74^{***}	-0.30^{***}	1.00	CON	0.42^{***}	-0.49^{***}	0.67^{***}	1.00

Table 16. Correlation between four measurements. The left panel reports the correlation between cross-sectional mean of the four measurements: outdegree, indegree, concentration beta and magnitude beta. The right panel reports the correlation between cross-sectional standard deviation of the four measurements: outdegree, indegree, concentration beta and magnitude beta.



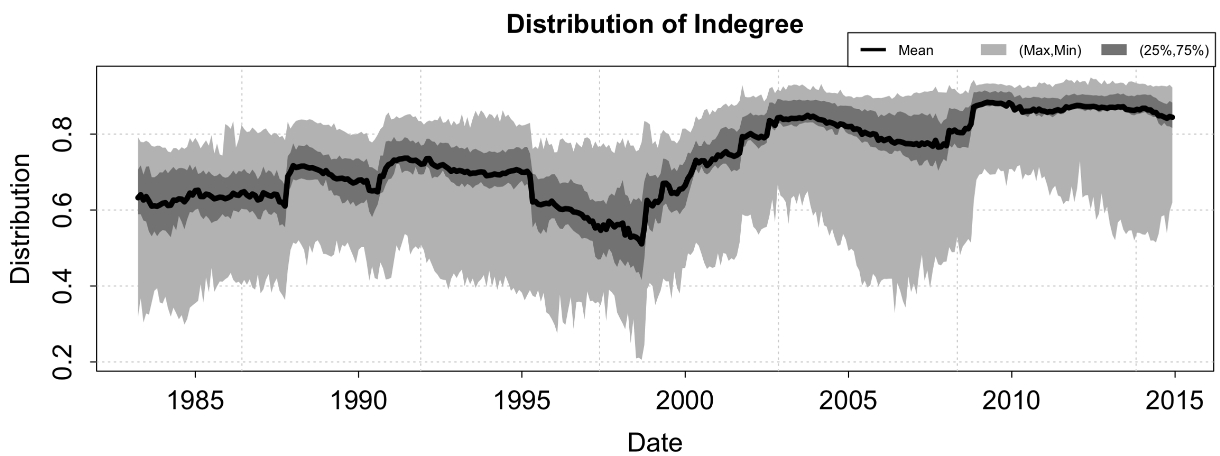


Figure 10. Cross sectional distribution of systemic risk measurements. This figure shows the cross-sectional distribution for the two systematic risk measurements derived directly from the adjacency matrix (according to Diebold and Yılmaz (2014)) and cross-sectional distribution of two network factor betas. The bold line denotes the cross-sectional mean. The shaded areas indicate the min/max and 25/75 percentile. The first panel shows the distribution of industry-level outdegree. The second panel shows the distribution of industry-level indegree.

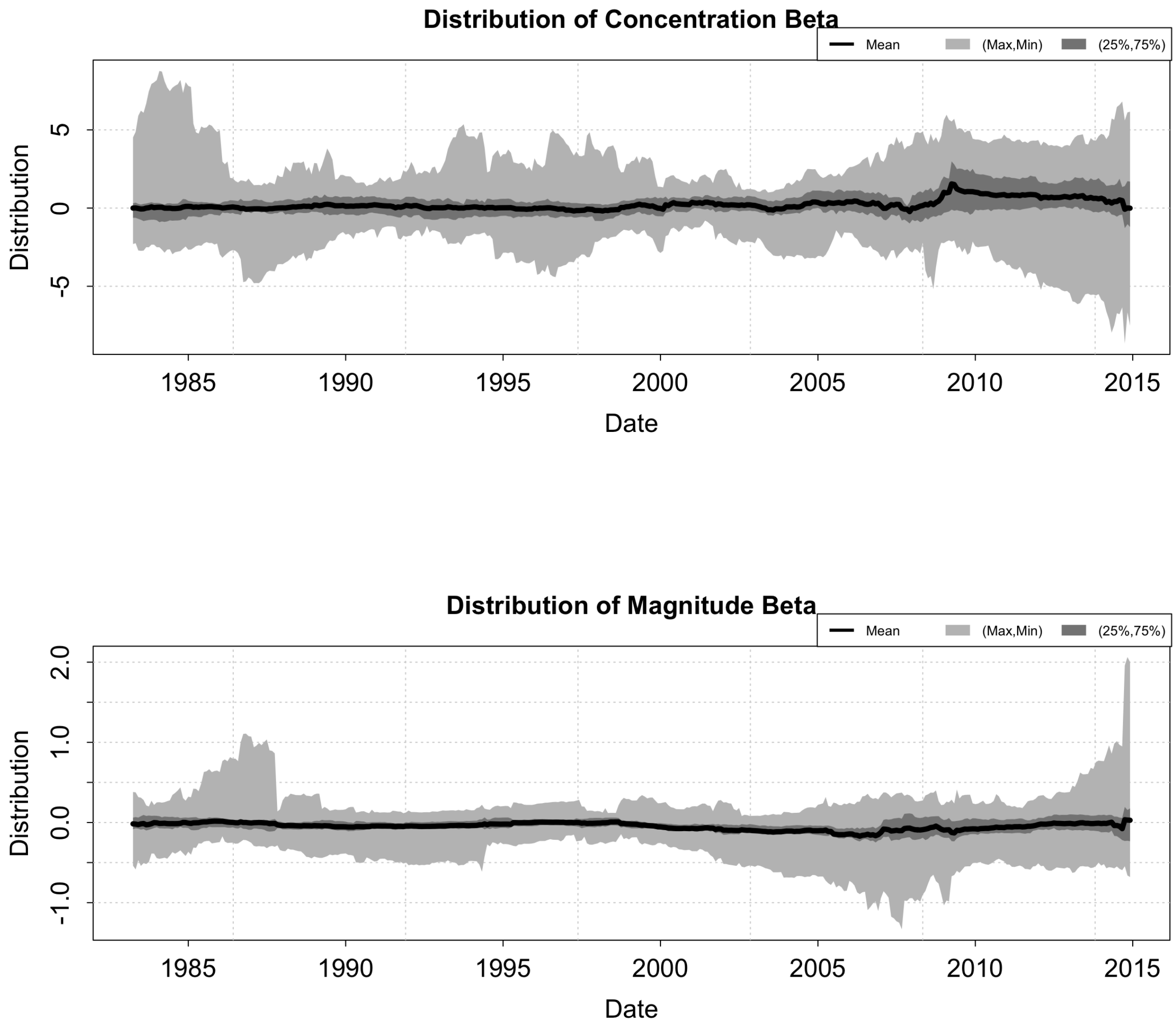


Figure 11. Cross sectional distribution of beta measurements. This figure shows the cross-sectional distribution for the two systematic risk measurements derived directly from the adjacency matrix (according to Diebold and Yılmaz (2014)) and cross-sectional distribution of two network factor betas. The bold line denotes the cross-sectional mean. The shaded areas indicate the min/max and 25/75 percentile. The first panel shows the distribution of industry-level Concentration Beta. The second panel shows the distribution of industry-level Magnitude Beta.

Value-weighted														
	Magnitude Factor							Concentration Factor						
	1	2	3	H-L	t	grs stat	grs p	1	2	3	H-L	t	grs stat	grs p
Ave.Exc.Ret	0.131	0.102	0.093	-0.038	-2.331			0.092	0.105	0.129	0.037	2.215		
volatility	0.164	0.138	0.161	0.093				0.155	0.138	0.171	0.094			
Sharpe ratio	0.799	0.734	0.577	-0.414				0.595	0.758	0.755	0.393			
α_{CAPM}	0.005	0.003	0.002	-0.003	-2.189	4.791	0.029	0.002	0.003	0.004	0.002	1.777	3.156	0.076
α_{FF3}	0.004	0.002	0.000	-0.003	-2.394	5.730	0.017	0.001	0.002	0.003	0.002	1.651	2.726	0.100
$\alpha_{4factor}$	0.004	0.002	0.001	-0.004	-2.677	7.168	0.008	0.001	0.002	0.004	0.003	2.290	5.245	0.023
$\alpha_{5factor}$	0.004	0.002	0.001	-0.003	-2.399	5.755	0.017	0.001	0.002	0.004	0.003	2.197	4.829	0.029

Table 17. Single Sorted Portfolios by excluding intermediary sectors. This Table reports the tercile sorted results for Magnitude and Concentration factors for the 45 sectors (excluding banking, trading and insurance sectors). Left panel shows the results for Magnitude factor and right panel shows the results for Concentration Factor. Average excess return, volatility and Sharpe ratio are all annualized and the anomaly alphas are reported as monthly. α_{FF3} refers to the intercept by regressing sorted portfolios onto $Fama - French$ 3 factors. $\alpha_{4factor}$ refers to the intercept by regressing sorted portfolios onto 4 factors (FF3+ Momentum from Carhart (1997)). $\alpha_{5factor}$ refers to the intercept by regressing sorted portfolios onto 5 factors (4 factor + Liquidity from Pástor and Stambaugh (2003)).

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Appendix

A Model Proof

A.1 Solve Firm's Problem

$$D_{i,t} = \max_{\{x_{ij,t}\},\{l_{i,t}\}} P_{i,t}y_{i,t} - \sum_{j=1}^{n} P_{j,t}x_{ij,t} - h_t l_{i,t}$$

s.t.

$$y_{i,t} = e^{z_{i,t}}\zeta_i l_{i,t}^{\alpha} \prod_{j=1}^{n} x_{ij,t}^{(1-\alpha)\omega_{ij}}$$

FOC with respect to $\{x_{ij,t}\}$ and $l_{i,t}$ yields that

$$\begin{cases} x_{ij,t} = \dfrac{(1-\alpha)\,\omega_{ij}P_{i,t}y_{i,t}}{P_{j,t}} \\ l_{i,t} = \dfrac{\alpha P_{i,t}y_{i,t}}{h_t} \end{cases}$$

A.1.1 Solve Household's Problem

The household maximization problem is

$$J_t\left(P_t, s_t, Q_t, h_t\right) = \max_{\left\{c_{i,t}, s_{i,t+1}\right\}} \left[(1-\beta)\, C_t^{1-\rho} + \beta\left(\mathbb{E}_t\left(J_{t+1}^{1-\gamma}\right)\right)^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1}{1-\rho}}$$

with respect to the intertemporal budget constraint:

$$\sum_{i=1}^{n} P_{i,t}c_{i,t} + \sum_{i=1}^{n}\left(V_{i,t} - D_{i,t}\right) s_{i,t+1} = \sum_{i=1}^{n} V_{i,t}s_{i,t} + h_t$$

Let λ_t be the Lagrange multiplier.

$$\max_{\left\{c_{i,t}, s_{i,t+1}\right\}} \left[(1-\beta)\, C_t^{1-\rho} + \beta\left(\mathbb{E}_t\left(J_{t+1}^{1-\gamma}\right)\right)^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1}{1-\rho}} - \lambda_t\left[\sum_{i=1}^{n} P_{i,t}c_{i,t} + \sum_{i=1}^{n}\left(V_{i,t} - D_{i,t}\right) s_{i,t+1} - \sum_{i=1}^{n} V_{i,t}s_{i,t} - h_t\right]$$

FOC with respect to $c_{i,t}$ yields $(1-\beta) J_t^{\rho} C_t^{-\rho} \frac{\partial C_t}{\partial c_{i,t}} = \lambda_t P_{i,t}$ where $\frac{\partial C_t}{\partial c_{i,t}} = \frac{\alpha_i}{c_{i,t}} C_t$. This further leads to

$$
\begin{aligned}
\frac{\frac{\partial C_t}{\partial c_{i,t}}}{\frac{\partial C_t}{\partial c_{j,t}}} &= \frac{P_{i,t}}{P_{j,t}} \\
c_{i,t} &= \frac{\alpha_i (1-\beta) J_t^{\rho} C_t^{-\rho}}{\lambda_t P_{i,t}} \\
\frac{c_{i,t} P_{i,t}}{\alpha_i} &= \frac{c_{j,t} P_{j,t}}{\alpha_j} \equiv \xi_t \\
\sum_i c_{i,t} P_[i,t] &= (\sum_i \alpha_i) \xi_t = \xi_t \\
\therefore\ c_{i,t} &= \frac{\alpha_i \xi_t}{P_{i,t}}
\end{aligned}
$$

FOC with respect to $s_{i,t+1}$ yields $J_t^{\rho} \beta \mathbb{E}_t \left[J_{t+1}^{1-\gamma} \right]^{\frac{\gamma-\rho}{1-\gamma}} \mathbb{E}_t \left[J_{t+1}^{-\gamma} \frac{\partial J_{t+1}}{\partial s_{i,t+1}} \right] = \lambda_t Q_{i,t} = 0.$
Following the envelop theorem:

$$
\begin{aligned}
\frac{\partial J_t}{\partial s_{i,t}} &= \frac{\partial J_t}{\partial s_{i,t+1}} \frac{\partial s_{i,t+1}}{\partial s_{i,t}} \\
&= \lambda_t Q_{i,t} \frac{V_{i,t}}{Q_{i,t}} \quad \text{(due to budget constraint)} \\
&= \lambda_t V_{i,t} \\
&= (1-\beta) J_t^{\rho} C_t^{-\rho} \frac{\partial C_t}{\partial c_{1,t}} \frac{1}{P_{1,t}} \left(D_{i,t} + Q_{i,t} \right)
\end{aligned}
$$

Evaluate it one period ahead and plug it into the FOC:

$$
J_t^{\rho} \beta \mathbb{E}_t \left[J_{t+1}^{1-\gamma} \right]^{\frac{\gamma-\rho}{1-\gamma}} \mathbb{E}_t \left[J_{t+1}^{-\gamma} (1-\beta) J_{t+1}^{\rho} c_{t+1}^{-\rho} \frac{\partial C_{t+1}}{\partial c_{1,t+1}} \frac{1}{P_{1,t+1}} \left(D_{i,t+1} + Q_{i,t+1} \right) \right] = \lambda_t Q_{i,t} = (1-\beta) J_t^{\rho} C_t^{-\rho} \frac{\partial C_t}{\partial c_{1,t}} \frac{1}{P_{1,t}} Q_{i,t}
$$

Reorganizing the equation yields the pricing function:

$$\mathbb{E}_t \left[\underbrace{\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{\frac{\partial C_{t+1}}{\partial c_{1,t+1}} / P_{1,t+1}}{\frac{\partial C_t}{\partial c_{1,t}} / P_{1,t}} \left(\frac{J_{t+1}}{\mathbb{E}_t \left(J_{t+1}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma}}_{\equiv M_{t+1}} \underbrace{\frac{D_{i,t+1} + Q_{i,t+1}}{Q_{i,t}}}_{\equiv R_{i,t+1}} \right] = 1$$

Here we normalize the spot price: $\frac{\partial C_t}{\partial c_{i,t}} / P_{i,t} = 1$. This normalization leads to the following relationship

$$\begin{aligned}
\frac{\partial C_t}{\partial c_{i,t}} / P_{i,t} &= \frac{\alpha_i}{P_{i,t}} \prod_{j=1}^{n} \left(\frac{\alpha_j P_{i,t}}{\alpha_i P_{j,t}} \right)^{\alpha_j} \\
&= \prod_{j=1}^{n} \left(\frac{\alpha_j}{P_{j,t}} \right)^{\alpha_j} \\
&= 1 \\
\therefore \prod_{j=1}^{n} P_{j,t}^{\alpha_j} &= \prod_{j=1}^{n} \alpha_{j,t}^{\alpha_j} \\
\therefore C_t &= \prod_{j=1}^{n} C_{j,t}^{\alpha_j} \\
&= \prod_{j=1}^{n} \left(\frac{\alpha_j}{P_{j,t}} \right)^{\alpha_j} \xi_t \\
&= \xi_t
\end{aligned}$$

A.1.2 Domar weight

The market clearing condition is $y_{i,t} = c_{i,t} + \sum_{j=1}^{n} x_{ji,t}$. Therefore, we can plug in the expression for ξ_t:

$$\begin{aligned}
y_{i,t} &= \frac{\alpha_i \xi_t}{P_{i,t}} + \sum_{j=1}^{n} \frac{(1-\alpha)\, \omega_{ji} P_{j,t} y_{j,t}}{P_{i,t}} \\
\therefore \frac{P_{i,t} y_{i,t}}{\xi_t} &= \alpha_i + (1-\alpha) \sum_{j=1}^{n} \omega_{ji} \left(\frac{P_{j,t+1} y_{j,t+1}}{\xi_{t+1}} \right)
\end{aligned}$$

$$\therefore v_{i,t} \equiv \frac{P_{i,t} y_{i,t}}{\xi_t} = \sum_{j=1}^{n} \alpha_j \tilde{l}_{ji}$$

where $\tilde{L} \equiv \left[\tilde{l_{ij}}\right] = (\mathcal{I} - (1-\alpha) W)^{-1}$ is the Leontief inverse. Also, all Domar weight sums up to 1.

$$\because \sum_{i=1}^{n} v_{i,t} = 1$$

$$\therefore \xi_t = \sum_{i=1}^{n} P_{i,t} y_{i,t} = C_t$$

A.1.3 Proof of Proposition 1

From the production function (4.2) and the FOC, we can derive the following:

$$\begin{aligned}
\ln y_{i,t} &= z_{i,t} + \ln \zeta_i + \alpha \ln l_{i,t} + (1-\alpha) \sum_{j=1}^{n} \omega_{ij} \ln x_{ij,t} \\
\ln y_{i,t} &= z_{i,t} + \ln \zeta_i + \alpha \ln \alpha + \alpha \ln P_{i,t} + \alpha \ln y_{i,t} - \alpha \ln h_t \\
&\quad + (1-\alpha) \sum_{j=1}^{n} \omega_{ij} \left[\ln (1-\alpha) + \ln \omega_{ij} + \ln P_{i,t} + \ln y_{i,t} - \ln P_{j,t}\right] \\
\ln \zeta_i &\equiv -\alpha \ln \alpha - (1-\alpha) \ln (1-\alpha) \\
\therefore \ln P_{i,t} &= (1-\alpha) \sum_{j=1}^{n} \omega_{ij} \ln P_{j,t} - z_{i,t} + \alpha \ln h_t - (1-\alpha) \sum_{j=1}^{n} \omega_{ij} \ln \omega_{ij}
\end{aligned}$$

Writing both sides into matrices, we can have:

$$\ln \mathbf{P}_t = \tilde{\mathbf{L}} \left[-ln\alpha^{\alpha} (1-\alpha)^{1-\alpha} - (1-\alpha) \sum_{j=1}^{n} \omega_{ij} \ln \omega_{ij} + \alpha \ln h_t \mathbf{1} - \mathbf{z}_t \right]$$

We can see the aggregate market wage drives simultaneously for the cross-sectional price. The idiosyncratic part of the stock price lies in the last productivity shock component. Therefore we can approximate the idiosyncratic return as follows:

$$d \ln \mathbf{P}^*_{t \to t+1} = \tilde{L} \left[\mathbf{z}_t - \mathbf{z}_{t+1}\right]$$

The conditional variation of the idiosyncratic return is decided by the diagonal element of the var-cov matrix.

$$Cov_t\left(d\ln \mathbf{P}^*_{t\to t+1}\right) = \sigma_z^2 \tilde{L}\tilde{L}'$$

Because all of the eigenvalues of $(1-\alpha)W$ lie within the unit circle, we can write the Leontief inverse as follows:

$$(I-(1-\alpha)W)^{-1} = I + (1-\alpha)W + (1-\alpha)^2 W^2 + (1-\alpha)^3 W^3 + \ldots$$

Here we then use $I + (1-\alpha)W$ as a first-order approximation of the Leontief inverse.

For each firm i, its conditional variation of the idiosyncratic return is determined by

$$Var_t\left(d\ln P^*_{i,t\to t+1}\right) = \sigma_z^2\left[\left((1-\alpha)\,\omega_{i,1}\right)^2 + \left((1-\alpha)\,\omega_{i,2}\right)^2 + \ldots + \left(1+(1-\alpha)\,\omega_{i,i}\right)^2 + \ldots + \left((1-\alpha)\,\omega_{i,n}\right)^2\right]$$

From this result, we know that for each sector i, its idiosyncratic return variation is not only explained by its own idiosyncratic shock (i.e. $\left(1+(1-\alpha)\,\omega_{i,i}\right)^2$), but also explained by spillover of other sectors' idiosyncratic shocks (i.e. $\left((1-\alpha)\,\omega_{i,j}\right)^2$). Therefore we can approximate the conditional volatility spillover matrix $\Delta_{t\to t+1}$ according to the coefficients. Also, we normalize each row to be of length 1.

$$\begin{aligned}\Delta_{t\to t+1} &\equiv \left[\delta_{ij}\right]\\ &= \begin{bmatrix} \frac{\left(1+(1-\alpha)\,\omega_{1,1}\right)^2}{denom_1} & \frac{\left((1-\alpha)\,\omega_{1,2}\right)^2}{denom_1} & \cdots & \frac{\left((1-\alpha)\,\omega_{1,n}\right)^2}{denom_1}\\ \frac{\left((1-\alpha)\,\omega_{2,1}\right)^2}{denom_2} & \frac{\left(1+(1-\alpha)\,\omega_{2,2}\right)^2}{denom_2} & \cdots & \frac{\left((1-\alpha)\,\omega_{2,n}\right)^2}{denom_2}\\ & \cdots & \cdots & \\ \frac{\left((1-\alpha)\,\omega_{n,1}\right)^2}{denom_n} & \frac{\left((1-\alpha)\,\omega_{n,2}\right)^2}{denom_n} & \cdots & \frac{\left(1+(1-\alpha)\,\omega_{n,n}\right)^2}{denom_n} \end{bmatrix}\end{aligned}$$

where $denom_i = \left((1-\alpha)\,\omega_{i,1}\right)^2 + \ldots + \left(1+(1-\alpha)\,\omega_{i,i}\right)^2 + \ldots + \left((1-\alpha)\,\omega_{i,n}\right)^2$ is the normalization constant.

A.1.4 Proof of Proposition 2

FOC for firms yields

$$\begin{cases} x_{ij,t} = \dfrac{(1-\alpha)\,\omega_{ij} v_i y_{j,t}}{v_j} \\ l_{i,t} = \alpha v_i \end{cases}$$

Substitute them into the production function:

$$\ln y_{i,t} = z_{i,t} + \ln v_i - (1-\alpha)\sum_j \omega_{ij} \ln v_j + (1-\alpha)\sum_j \omega_{ij} \ln y_{j,t}$$

We write it into matrices:

$$\ln \mathbf{y}_{i,t} = \tilde{\mathbf{L}}\mathbf{z_t} + \ln \mathbf{v}$$

$$\begin{aligned}
&\because \xi_t v_i = P_{i,t} y_{i,t} \\
&\therefore \ln \xi_t = \ln P_{i,t} + \ln y_{i,t} - \ln v_i \\
&\therefore \sum_i \alpha_i \ln \xi_t = \sum_i \alpha_i \ln P_{i,t} + \sum_i \alpha_i \ln y_{i,t} - \sum_i \alpha_i \ln v_i \\
&\therefore \ln \xi_t = \sum_i \alpha_i \ln \alpha_i + \sum_i \alpha_i \ln y_{i,t} - \sum_i \alpha_i \ln v_i \\
&\therefore \ln \xi_t = \sum_i \alpha_i \ln \alpha_i - \sum_i \alpha_i \ln v_i + \sum_i \alpha_i \left[\sum_k L_{ik} z_{k,t} + \ln v_i\right] \\
&\therefore \ln \xi_t = \sum_i \alpha_i \ln \alpha_i + \sum_i \alpha_i \sum_k L_{ik} z_{k,t}
\end{aligned}$$

The we write it into matrices:

$$\begin{aligned}
\ln \xi_t &= \alpha' \ln \alpha + \alpha' \tilde{\mathbf{L}}\mathbf{z_t} \\
&= \alpha' \ln \alpha + \mathbf{v}'\mathbf{z_t}
\end{aligned}$$

This proposition shows that aggregate output is a linear combination of the idiosyncratic productivity shock.

A.1.5 Proof of Proposition 3

Since the constant $(1-\alpha)$ does not affect the final result, we left it out in the following proof. We assume the preference as $\alpha_i = \frac{1}{n}$.

$$d \ln \xi_{t \to t+1} = \mathbf{v}' \left(\mathbf{z_{t+1}} - \mathbf{z_t} \right)$$
$$Var_t \left(d \ln \xi_{t \to t+1} \right) = \Theta \left(\|v\|_2 \right)$$

For each row of Δ, the denominator is smaller than or equal to $\left(1 + \sum_{j=1}^{n} \omega_{i,j}\right)^2 = 2^2$ and the denominator is greater than 1.

So by definition,

$$\Delta = \Theta \left(\begin{bmatrix} \omega_{1,1}^2 & \omega_{1,2}^2 & \dots & \omega_{1,n}^2 \\ \omega_{2,1}^2 & \omega_{2,2}^2 & \dots & \omega_{2,n}^2 \\ & \dots\dots & & \\ \omega_{n,1}^2 & \omega_{n,2}^2 & \dots & \omega_{n,n}^2 \end{bmatrix} \right) = \Theta \left(W \odot W \right)$$

Therefore,

$$Out_i = \Theta \left(\frac{\sum_{j=1}^{n} \omega_{ji}^2}{\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ji}^2} \right)$$
$$CON = \sum_{i=1}^{n} Out_i \ln Out_i$$
$$MAG = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{ij}^2 \ln \omega_{ij}^2$$
$$\text{Also, we define } \; d_i \triangleq \sum_{j=1}^{n} \omega_{ji}$$

The denominator of (B2) is $< \left(\sum_i \sum_j \omega_{ji}\right)^2 = n^2$, and it is $\geq \frac{1}{n^2} \left(\sum_i \sum_j \omega_{ji}\right)^2 = 1$. The second boundary is due to the property that $\sqrt{n}\ \|z\|_2 \geq\ \|z\|_1$ any vector z. Similarly, the nominator is $\in [\frac{d_i^2}{n}, d_i^2)$. Therefore, $Out_i \in \left(\frac{d_i^2}{n^3}, d_i^2\right)$.

The CON and MAG are all negative values according to the definitions. We compute $\frac{CON}{MAG}$ as $\frac{-CON}{-MAG}$ below in order to avoid any sign confusion when dealing with the nominator and denominator separately. (In the proof below, we may leave out some constants for simplicity.)

$$
\begin{aligned}
\frac{CON}{MAG} &= \frac{-CON}{-MAG} \\
&= \frac{\sum_{i=1}^{n} Out_i \ln \frac{1}{Out_i}}{\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} \omega_{ij}^2 \ln \frac{1}{\omega_{ij}}} \\
&\geq \frac{\sum_{i=1}^{n} Out_i \ln \frac{1}{Out_i}}{\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} \omega_{ij}} \\
&= \frac{\sum_{i=1}^{n} Out_i \ln \frac{1}{Out_i}}{1} \\
\therefore \frac{CON}{MAG} &= \Omega\left(\sum_{i=1}^{n} Out_i\right) \\
&= \Omega\left(\sum_{i=1}^{n} d_i^2\right)
\end{aligned}
$$

By assuming $\alpha_i = \frac{1}{n}$, we have

$$
\begin{aligned}
\mathbf{v} &= \frac{1}{n}\,[I - W]^{-1} \\
&= \frac{1}{n}\,[I + W + \ldots] \\
\|\mathbf{v}\|_2^2 &\geq \frac{1}{n^2}\mathbf{1}'\,[I + W]\,[I + W]'\,\mathbf{1} \\
&= \Theta\left(\frac{1}{n^2}\sum_{i=1}^{n} d_i^2\right) \quad \left(\because \frac{1}{n^2}\sum_{i=1}^{n} d_i^2 \text{ dominates } \frac{1}{n} \text{ when n is large}\right)
\end{aligned}
$$

From (B6), we know that $\frac{1}{n^2}\sum_{i=1}^{n} d_i^2$ is dominated by $\frac{1}{n^2}\frac{CON}{MAG}$.

Therefore, $\sqrt{var\left(\xi_{t\to t+1}\right)} = \Omega\left(\frac{1}{n}\sqrt{\frac{CON_{t\to t+1}}{MAG_{t\to t+1}}}\right)$.

B Time Series plots of representative sectors

We plot the outdegree, indegree, netdegree for several representative sectors. We can clearly see that in financial crisis, financial sectors like Insurance sector and Trading sector are influencing the rest of the economy a lot. And Computers sector is also having an increasing influence on other sectors in recent years. The interesting thing is that the Precious Metals is neither spreading nor receiving many shocks during 2008 crisis. We know that gold is a "safe heaven" in 2008. We can recall that gold did fall in the initial shock of the 2008 financial crisis. But while the S&P continued to decline, gold rebounded and ended the year up 5.5 percent. Over the total 18-month stock market selloff, gold rose more than 25 percent.

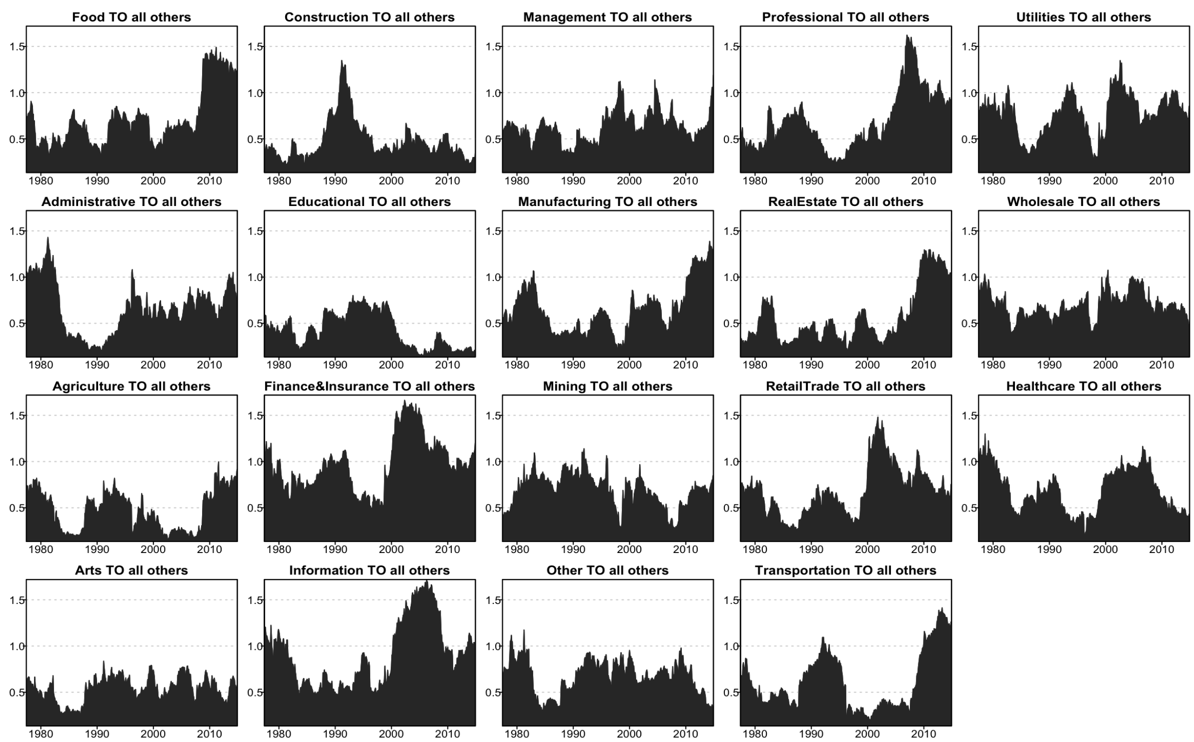

Figure 12. Time series of outdegree

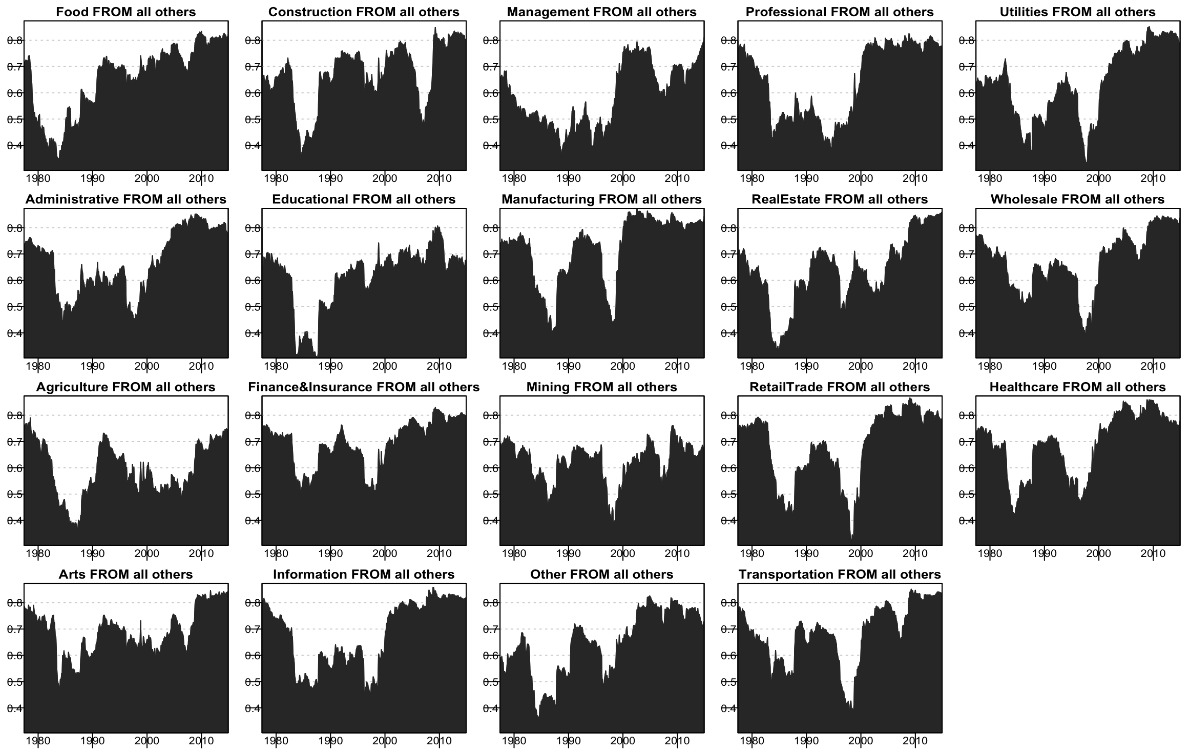

Figure 13. Time series of indegree

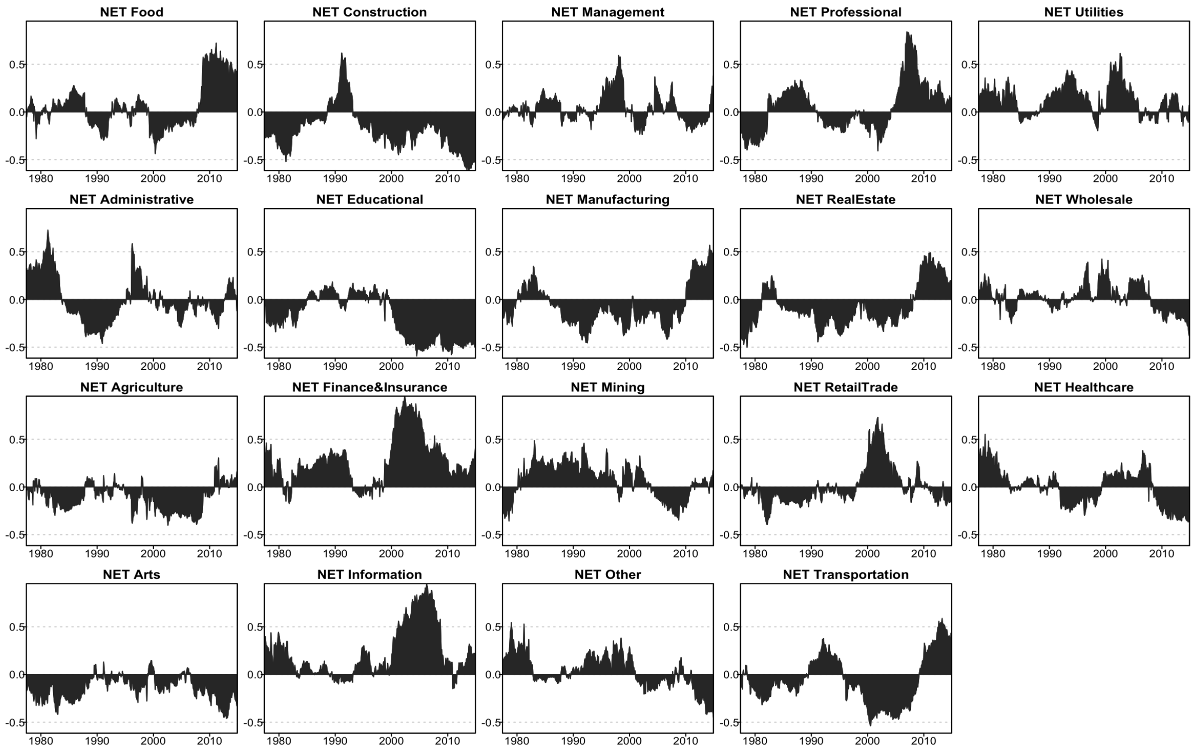

Figure 14. Time series of netdegree

And it's also interesting to look at the pairwise shock propagation between the representative sectors.

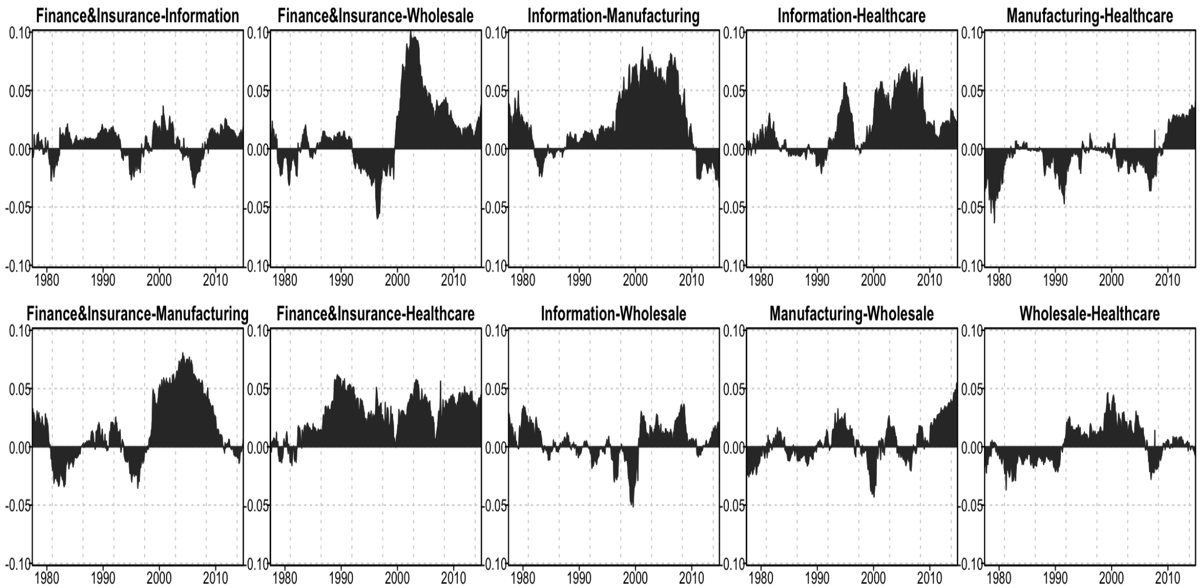

Figure 15. Pairwise Plots for representative sectors

C Annual Plots for Factor Levels

I first annualized monthly mimicking factors to mimic factor innovations and then cumulatively sum up the innovations to get the factor levels.

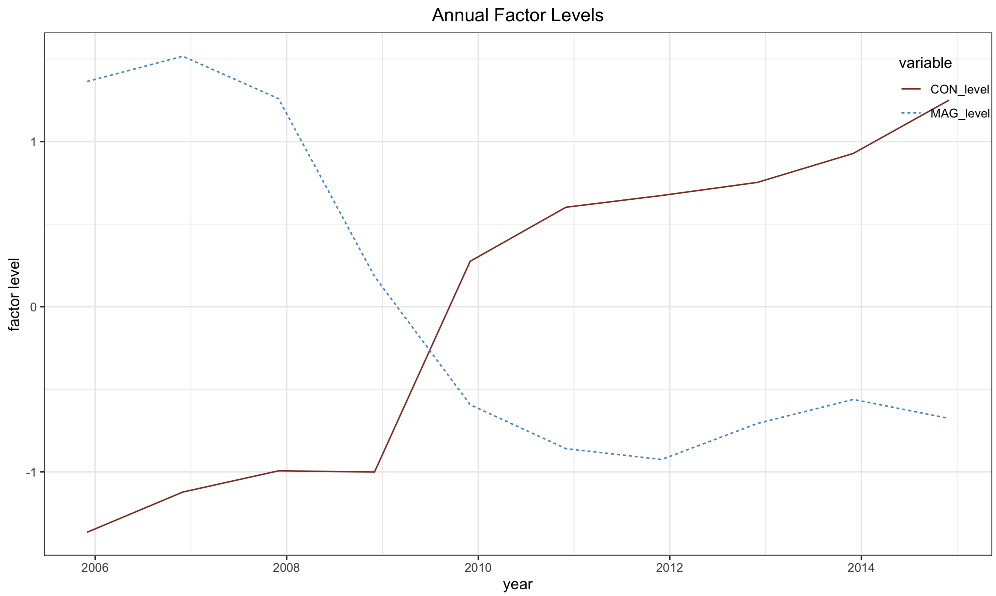

Figure 16. Annual time series plots of the network factor levels.

We can see from the figure that the CON is increasing after 2006 since firms are gaining more market power and MAG is decreasing since firms are better at diversifying the product-specific risks.

D Why Negative Entropy = Concentration in network context?

We rely heavily on some statistics and entropy literature. The network factors are trying to capture the characteristics of the whole economy and we use the idea of "Negative Entropy" to capture a network system's concentration. This mainly relies on some statistical knowledge that entropy is roughly opposite of concentration in a network context. A lot of literatures including Bianconi (2007), Bianconi (2009) and Robert (1990) showed that the smallest is the entropy of an ensemble network, the greater the information content in the constraints that define it. For example a network with Poisson degree distribution have a much larger entropy than scale-free networks with a power-law degree distribution, but generally speaking, networks with a power-law degree distribution are more concentrated as it has some very large degree while the rest are degrees are small.

Here is another intuitive explanation. We know that higher entropy means higher complexity(i.e. it is hard for the Turing Machine to reproduce a graph given some properties of the graph such as node size, linkage numbers etc...). In the network graph, higher complexity means lower concentration. We illustrate the reason using the two figures below.

For the left figure, there are 20 nodes in total. Every node is equivalently important and every directional linkage is a non negligible element of the structure. In order for the Turing Machine to reproduce such network, it needs to simulate C_{20}^{2} path and find the structure closest to the left figure. So the left figure has high complexity, but zero concentration. For the right figure, only node 1 is influencing the rest 19 nodes and there is no other linkages. It's easy to understand that such figure is much more easier to be reproduced. Thus the right has low complexity but high concentration.

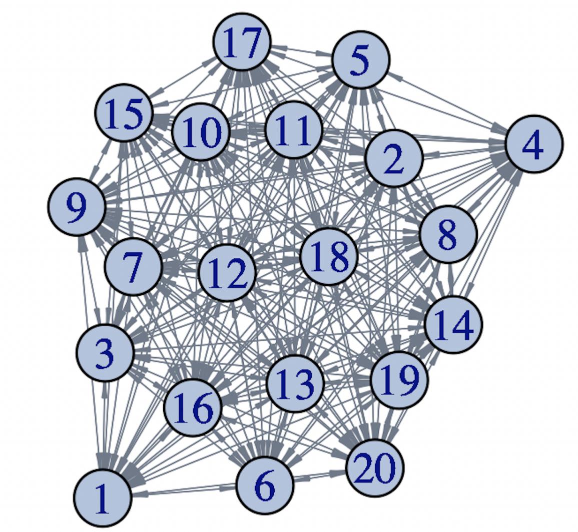

Figure 17. low concentration

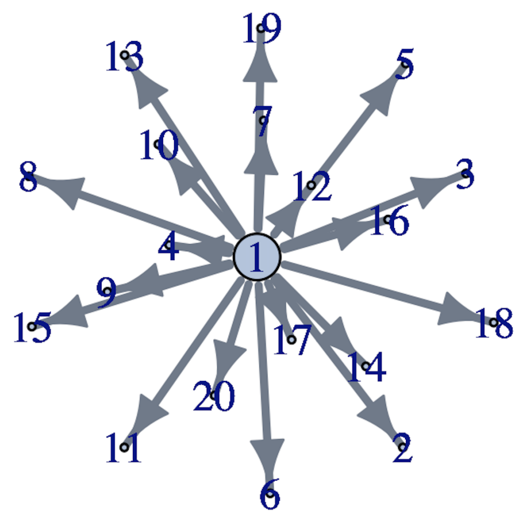

Figure 18. high concentration

Thus higher "negative entropy" means higher concentration.

E Solution to the second symmetric approximation of the adjacency matrix

For convenience, we construct a 1-dim symmetric approximation(instead of a 48-dim). It is reasonable as it still produces an irreducible matrix. In network language, "irreducible" means that the network is strongly connected and every node is reachable by other nodes. This is guaranteed by the nature property of volatility spillover. Hence, we assume the symmetric approximation

$A^* = xx'$ and try to find vector x to minimize the Frobenius-norm of $\|A_0 - xx'\|_F$. We assume $Y = A_0 - xx'$ and J to be the Frobenius norm of Y.

$$\begin{aligned}
dY &= -dxx' - xdx' \\
dJ &= tr(Y + dY)'(Y + dY) - trY'Y \\
&= tr(Y - dxx' - xdx')'(Y - dxx' - xdx') - trY'Y \\
&= tr(-Y'dxx' - Y'xdx' - xdx'Y - dxx'Y) \\
&= tr(-x'Y'dx - x'Ydx - x'Y'dx - x'Ydx) \\
&= tr(-2x'(Y + Y')dx) \\
J'(x) &= -2x'(Y + Y') \\
&= -2x'(A_0 + A_0' - 2xx') \\
&= -2x'(A_0 + A_0') + 4\|x\|^2 x'
\end{aligned}$$

We set $J'(x)$ to be 0 to solve for x. Or alternatively, if we directly realized that

$$\begin{aligned}
J(x) &= tr(A_0 - xx')'(A_0 - xx') \\
&= tr(A_0 A_0' - xx'A_0 - A_0'xx' + xx'xx') \\
&= \|A_0\|^2 - 2tr(x'\frac{A_0 + A_0'}{2}x) + \|x\|^2
\end{aligned}$$

The solution is then $x'H = \|x\|^2 x'$ where $H = \frac{A_0 + A_0'}{2}$ (the Hermitian matrix of A_0). Then we find all largest eigenvalue λ and the corresponding eigenvector μ and set x as $\pm\frac{\lambda^{1/2}}{\|\mu\|}\mu$.