Asset Pricing Implications of Heterogeneous Investment Horizons

November, 2022

Abstract

Short performance windows shrink mutual fund managers' investment horizons well below value investors' long-term investment mandates, and relative performance evaluations introduce benchmarking incentives among mutual fund managers' objectives. We show that these two asset management frictions, primarily through their interaction, can explain why the risk premium, volatility, and Sharpe ratio on short-term dividend strips are higher than on long-term dividend strips — predictions that leading equilibrium asset pricing models cannot generate. Our theory rationalizes the empirical findings on unconditional and conditional downward sloping risk premiums. We provide novel empirical evidence to further support the mechanism, which is set in continuous time, and admits closed-form expressions.

1 Introduction

A growing body of literature studies the impact of professional asset managers on asset markets. Asset managers' contracts are expected to align their incentives with those of the investors to address possible agency problems, but the resulting investment strategy affects asset prices. This literature relies on two established facts. First, mutual fund managers' performance is evaluated relative to a benchmark, and second, their performance is evaluated frequently. In particular, Ma, Tang, and Gómez (2019) have shown recently that 79% of the mutual fund managers in the US are evaluated relative to a prespecified benchmark with rolling performance windows of various lengths. The short performance evaluation windows are especially critical since they strongly incentivize asset managers to become short-term investors because they may get fired or demoted if they underperform their benchmark in the short term. Consequently, those asset managers have little interest in investment horizons beyond their minimum performance window.

The data shows that mutual fund asset managers' minimum rolling performance windows are excessively short across all the major investment mandates. In particular, Figure 1 shows that 83.8% of mutual fund managers have a minimum rolling performance window of one year or less, and the distribution median is 7 to 8 months, assuming uniform contract starting dates.

A different but equally significant group of investors, which we refer to as value investors, do not suffer from the frictions of the asset management industry and, therefore, has a much longer investment horizon objective than mutual fund managers' performance window of seven to eight months. Furthermore, they generally care about absolute performance rather than performance relative to a benchmark. The terms *asset managers* and *value investors* refer to these two groups of investors throughout the paper, even though the value investors group may consist of asset managers with value investors' incentives. Recently, Cochrane (2022) has emphasized the importance of assets' payout policy when considering optimal portfolios of long-term investors.

In this paper we conjecture that the heterogeneity of the investment horizons of these two groups of investors, combined with the relative performance objectives of mutual fund asset managers, might explain inconsistencies between short-term and long-term asset prices, given that the proportion of assets under management affected by these frictions is significant. To get some perspective, the assets under management of the US active mutual fund industry equals roughly five trillion dollars; that is, about three to four trillion dollars flow through the asset management frictions laid out here when 60% - 80% of the industry suffers from those frictions.

Despite the saliency of the heterogeneity of investment horizons, its asset pricing implications have yet to be explored. This paper proposes a dynamic asset pricing equilibrium model that captures the heterogeneous investment horizons in order to study the empirical regularities of short-term and long-term dividend strips. Dividend strips grant the buyer dividends between two specified future dates. Like a coupon bond that can be decomposed into a linear combination of zero-coupon bonds with different maturities, a stock price can

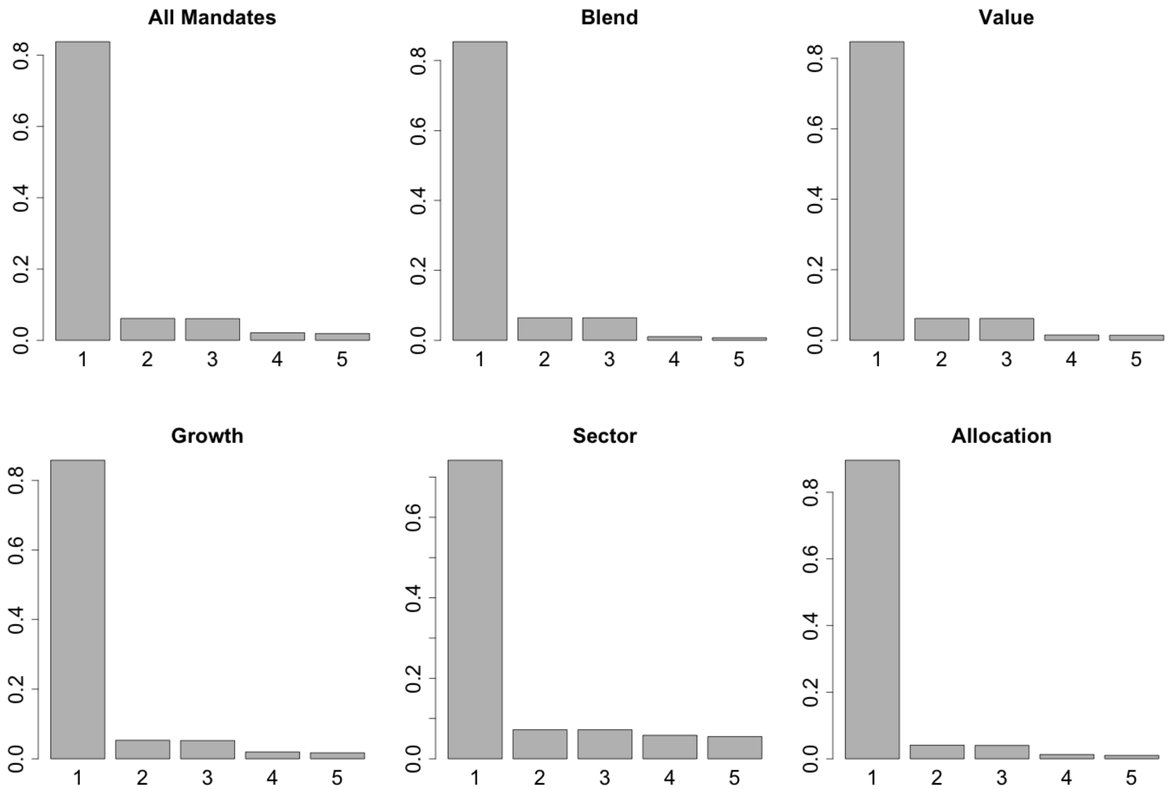

Figure 1: This figure plots the density (y-axis) as a function of the minimum performance evaluation window in years (x-axis). The top left figure represents the minimum performance evaluation window across all managers' mandates, and the remaining five represent the minimum performance evaluation window for mutual fund managers with different mandates. The minimum performance evaluation window across all mandates shows that 83.8% of managers have a minimum performance evaluation window below a year, and the outcome is consistent across the major mutual funds' mandates. The median is between 7-8 months when we assume a linear distribution of the contracts' starting dates. The data to construct these windows was graciously provided by Juan-Pedro Gómez, Linlin Ma, and Yuehua Tang.

be decomposed into a linear combination of short-term and long-term dividend strips as long as they are non-overlapping and cover the whole horizon. Dividend strips are very useful because they allow us to differentiate between the effect of long-term and short-term factors on stock prices.

Our work's main contribution is to show that the two asset management frictions generate a downward sloping risk premium, volatility, and Sharpe ratio, thereby providing an original theoretical foundation for what has been considered a puzzle. While other explanations for dividends' strips empirical regularities exist, this paper is the first to show that frictions tied to the mutual fund asset management industry can account, at least partially, for these empirical regularities. Our continuous-time setup admits precise closed-form expressions, allowing us to analyze cash-flow shocks' effects on prices. In particular, we find that short-term (*non-fundamental*) cash-flow news affects long-term asset prices more than (*fundamental*) long-term cash-flow news.[1]

The influential work of van Binsbergen, Brandt, and Koijen (2012) showed that short-term dividend strips have a higher risk premium, Sharpe ratio, and volatility than long-term dividend strips by analyzing options data. van Binsbergen, Hueskes, Koijen, and Vrugt (2013) and, more recently, van Binsbergen and Koijen (2017) extend this evidence using dividend futures data instead of options data to the US, Europe, Japan, and the UK. These empirical findings are at odds with the leading equilibrium asset pricing models such as the long-run risk, external habit formation, and rare disaster risk models.[2][3]

Our model implies that the downward sloping trend is stronger at the short end of the investment horizon. This result is consistent with Giglio, Kelly, and Kozak (2021), who show that the unconditional risk premium is downward sloping at the short end but flat or slightly upward-sloping at other horizons. In contrast, Gonçalves (2021a) shows that the equity risk premium is downward sloping in long maturities due to the reinvestment risk. Lastly, Weber (2018) and Gonçalves (2021b) have addressed the shortcomings of dividend futures by directly constructing a cash-flow duration measure; both papers provide evidence of downward sloping risk premiums and Sharpe ratios.

The literature agrees on the conditional analysis and concludes that the term struc-

[1] Short-term cash flow news is non-fundamental news about the long-term asset because this asset never pays in the short term, while long-term cash flow news is fundamental news about this asset's future cash flow.

[2] We point out that the evidence of unconditional downward sloping risk premium is contested. For instance, Bansal, Miller, Song, and Yaron (2021) argue that the relatively short data and the illiquidity of dividend futures imply that the evidence of unconditional downward sloping risk premiums is due to the high frequency of recessions in the sample, and the result flips when considering liquidity. Schulz (2016) explains the downward-sloping term structure of risk premia by the differential taxation between dividends and capital gains, and Boguth, Carlson, Fisher, and Simutin (2019) by the market microstructure noise. In contrast, Gormsen (2021) finds that the one-year unconditional risk premium slopes downward.

[3] Downward sloping risk premiums are not unique to equity markets and exist in other asset markets. For instance, evidence from Giglio, Maggiori, and Stroebel (2015) suggests a downward sloping risk premium in U.K. real estate over a very long horizon.

ture is downward sloping during recessions, as Aït-Sahalia, Karaman, and Mancini (2020), Bansal et al. (2021), and Giglio et al. (2021) recently document. Perhaps more importantly, Gormsen (2021) further shows that the conditional empirical evidence is inconsistent with the current theories generating an unconditional downward sloping trend, which provides further evidence to guide new theories explaining the phenomenon.

Consistent with the conditional empirical evidence, our theory predicts that the risk premium slopes down less as the benchmarking incentives become less severe and eventually slopes upward when there are no benchmarking incentives. Accordingly, we expect the downward sloping trend to attenuate or disappear in states with less severe benchmarking incentives. Cremers and Petajisto (2009) introduced the Active Share measure to investigate the share of portfolio holdings that differ from the benchmark index holdings. Perhaps not surprisingly, in the light of the predictions of our model, the Active Share in expansions is significantly larger than in recessions. Indeed, a panel regression shows that a fund reduces its Active Share in recessions by 1.4% below the fund's average, implying that an additional \$75 billion is invested in benchmarked assets in recessions out of the total \$5 trillion in active mutual fund assets under management.[4]

The model features two types of investors: (i) asset managers who have short-term horizons because their performance is evaluated in the short-term — annually or even quarterly; additionally, their performance is evaluated relative to indexes; (ii) value investors who do not pay attention to the indexes and have long term goals.[5] Two mechanisms drive the equilibrium in our model. The first mechanism results from the heterogeneous investment horizons *individually*: the short-term performance window relative to the value investor's long-term goals. The second mechanism results from the *combined* effect of the heterogeneous investment horizons and the relative performance objectives of asset managers: the interaction between the short-term performance window and benchmarking incentives. We explicitly disentangle these two channels and show how the individual and combined effect determine equilibrium.

[4]Stambaugh (2014)'s presidential address extensively discusses the Active Share. We preform the panel regression analysis using the data of Cremers and Petajisto (2009) and Petajisto (2013).

[5]The asset managers refer to the group of investors who suffers from the two asset management frictions like mutual fund asset managers, while the value investors refer to the group of investors who do not suffer from those frictions like hedge funds and retail investors. Of course, in reality, non-mutual fund asset managers may not suffer from the two asset management frictions and would follow value investing strategies.

The model has two assets that mimic dividend strips' payouts: the short-term asset pays dividends in the short term, and the long-term asset pays dividends in the long term; the dividends are independent, and their payouts' dates do not overlap. Furthermore, the asset manager cares only about the short-term investment horizon due to the short performance window. Therefore, optimality implies that he must hold the short-term asset eventually, at the end of his performance window, because the long-term asset does not pay off in the short term when the asset manager's performance is evaluated. Our theory addresses some of the concerns Cochrane (2022) recently raised about standard portfolio theories failing to consider long-term payout policies and the equilibrium implications of considering such payout policies. The equilibrium implications align with Campbell and Viceira (2002), who claim that long-term and short-term investors' portfolios are not the same because they evaluate risk differently.

The equilibrium objective is to set asset prices so that the asset manager optimally chooses to trade off the long-term asset and hold the short-term asset, while the value investor optimally chooses to trade off the short-term asset and hold the long-term asset. This objective does not depend on whether the benchmark is long-term or short-term or whether there is a benchmark. However, the equilibrium adjustments of assets' returns and market prices of risk substantially differ depending on whether there is a benchmark and the strength of asset managers' incentives to benchmark. In the primary analysis, we focus on a long-term asset benchmark and show that benchmarking is instrumental to the model's predictions. We then show that the model's predictions carry over to a general weighted average benchmark in the extension.

The equilibrium achieves this objective as follows. The value investor prefers the long-term asset due to the long-term risk aversion hedging desires. However, the long-term asset has higher overall risk exposure, as one would expect in a traditional setup where cash-flow news today significantly impacts assets that pay off in the long term. Equilibrium compensates the value investor for taking on that heightened long-term risk (relative to the short-term asset) by having a higher long-term risk premium, leading to an upward-sloping risk premium and volatility when there is no benchmark.

Benchmarking induces extra — unrelated to prices — demand to hold the benchmark asset. This excess demand pressure is unsustainable in the no-benchmark pricing environment. Prices counteract the benchmarking demand and restore equilibrium by negating the demand to hold the benchmark. Equilibrium achieves this goal by making the long-term asset more expensive when the benchmark is the long-term asset.

The price reaction makes the long-term asset too unattractive, so much so that the value investor prefers to have a risk free position in equilibrium, violating market clearing. To make the long-term asset more attractive, equilibrium decreases the long-term asset risk exposures to the point where the value investor optimally holds one unit while still negating the benchmarking demand of the asset manager. Equilibrium compensates the asset manager for taking on that heightened short-term risk (relative to the long-term asset) by having a higher short-term risk premium, inducing a downward-sloping risk premium and volatility.

The heterogeneity in investment horizons, through the individual effect, emphasizes the importance of non-fundamental news about the short-term payout — news unrelated to the long-term payout. We find that news unrelated to fundamentals affects long-term asset prices more than news about fundamentals. In contrast, in a homogenous investment horizon economy, the fundamental news is the main driving force behind asset pricing fluctuations, and news that does not directly affect dividends has minimal to no effect on prices.

While a common long-term and short-term positive shock positively affect prices, a positive long-term fundamental shock reduces the long-term asset price. Interestingly, this result provides a new testable implication: asset prices load negatively on long-term cash-flow risk factors when controlling for short-term cash-flow risk factors.

The individual effect introduces a negative fundamental risk exposure (news about the long-term payout), and a positive non-fundamental risk exposure (news about the short-term payout), meaning positive fundamental cash-flow news reduces the long-term asset price, and non-fundamental cash-flow news increases it in equilibrium. When a common shock (both fundamental and non-fundamental) arrives, the non-fundamental news effect would be stronger and increase the long-term asset despite having negative fundamental risk exposure.

When reflecting on the combined effect (the interaction of benchmarking incentives and heterogenous horizons) on the assets' return exposures, we find that in most aspects, but not all, the heterogeneous investment horizons economy is distinct from an economy with homogenous investment horizons. In a heterogeneous horizon economy, when the long-term asset belongs to the benchmark, its fundamental exposure increases but remains negative, and its non-fundamental short-term risk exposure decreases but remains positive. This result reduces the long-term asset return's total volatility, in line with Cuoco and Kaniel (2011) but opposite to Basak and Pavlova (2013)s' findings when the investment horizons are homogeneous. When the short-term asset belongs to the benchmark, its fundamental return exposure and total volatility do not change.

When reflecting on the combined effect (the interaction of benchmarking incentives and heterogenous horizons) on the market prices of risk, we find that in a heterogeneous horizons economy, when the long-term asset is included in the benchmark, the long-term (fundamental) market price of risk increases while the short-term (non-fundamental) market price of risk decreases. In contrast, in a homogenous horizon economy, benchmarking reduces the market price of benchmarked assets' fundamental risk while it does not affect non-fundamental market prices of risk. When the short-term asset is included in the benchmark, it entails another downforce on the short-term market price of risk, while it does not affect the long-term market price of risk, which aligns with the results of homogeneous investment horizon economies.[6]

Lastly, we provide novel empirical evidence to substantiate the importance of heterogeneous investment horizons. Our model predicts that the asset manager buys the benchmark assets due to the benchmark hedging and sells short the short-term asset due to the risk aversion hedging. Our regression analysis shows that the total net assets of active all-equity mutual funds load negatively on the short-term dividend strip while it loads positively on the long-term dividend strip, in line with simulated data. In a different regression specification, we verify a different prediction showing that as the size of the asset manager increases, the short-term asset price increases relative to the long-term asset price. The equilibrium predictions show up when measuring the total net assets of mutual actively managed funds within the Russell 3000 family of prospectus benchmarks.

Our regression specification excludes the effect of the mean-variance portfolio because empirical evidence suggests that mutual fund asset managers rebalance their hedging portfolios much more frequently than trading in and out of investment opportunities. Even more so, when mutual fund asset managers invest, these investments typically exhibit momentum. As a result, opportunistic investing is much less dynamic than rebalancing the hedging portfolios. For instance, Chan, Chen, and Lakonishok (2002) document that few funds take

[6] Please refer to Basak and Pavlova (2013), and Buffa and Hodor (2018) for a comprehensive analysis of the equilibrium effects of homogenous investment horizons.

positions away from their benchmark (despite having substantial turnover), and when they do, they favor past winners (indicative of slow-moving momentum strategies).

Due to these reasons, frequently rebalancing the benchmark and risk aversion hedging portfolios show up statically in a regression analysis with *simulated data*, while infrequently rebalancing the mean-variance portfolio does not show up.

The remainder of the paper is organized as follows. Section 2 summarizes the related literature; Section 3 sets up the economy with the benchmark and heterogeneous horizons; Section 4 solves for the equilibrium; Section 5 analyzes the equilibrium mechanism; Section 6 discusses the equilibrium implications; Section 7 introduces the novel empirical evidence; Section 8 extends the main setup to a general geometric average benchmark; and Section 9 concludes.

2 Related Literature

Our article builds upon the growing literature studying the effects of professional asset management on asset prices. The most relevant to our paper is a strand in that literature focusing on the asset pricing implications induced by an asset manager with a performance benchmark objective.

In this strand of literature, it is typical to embed the performance benchmark into the asset manager's objective function. Brennan (1993) and Gómez and Zapatero (2003) introduce a static setup with a mean-variance asset manager that maximizes the portfolio return relative to the return on a benchmark portfolio. Cuoco and Kaniel (2011) introduce a dynamic setup with a constant relative risk aversion asset manager that cares about the benchmark through performance-based fees. Basak and Pavlova (2013) and Basak and Pavlova (2016) introduce a dynamic setup with a reduced-form approach to incorporate the benchmark incentives into the manager's objective. They provide the salient features a reduced-form asset manager objective should satisfy and study the asset pricing implications of a single manager. Buffa and Hodor (2018) adopt their reduced-form asset manager objective and introduce a dynamic setup to study the asset pricing implications of multiple asset managers with different performance benchmarks.

Similarly, Basak, Pavlova, and Shapiro (2007) and, more recently, Sotes-Paladino and Zapatero (2019) embed the performance benchmark into the asset manager's objective function to study the asset manager's optimal behavior due to benchmarking and given asset prices. A common thread in all these papers is that there is no heterogeneous investment horizons, and performance windows align with long-term investment mandates. In contrast, this paper studies the asset pricing implications when there is heterogeneous investment horizons and fund managers' performance windows are strictly shorter than long-term investment mandates.

Our paper proposes another constructive addition to the literature by introducing an endogenous benchmark that depends on the equilibrium price rather than the terminal exogenous dividend, as the literature assumes. Despite the substantial complexity of considering an endogenous benchmark, our paper provides analytical closed-form solutions to the equilibrium quantities. We achieve this goal by considering Basak and Pavlova (2013)s' objective and letting the benchmarking importance parameter approach infinity.

A related strand of literature addresses the importance of benchmarks to align asset managers' incentives. Ou-Yang (2003), Cadenillas, Cvitanić, and Zapatero (2007), and Lioui and Poncet (2013) show that benchmarking is a part of an optimal contract given prices. Benchmarking is essential to align incentives even when considering the interplay between the equilibrium asset pricing and optimal contracting, as Cvitanić and Xing (2018) and Buffa, Vayanos, and Woolley (2019) show.

While there are other explanations for dividends' strips empirical regularities, our paper is the first that connects the frictions arising from asset managers' objectives to the empirical regularities of dividend strips. van Binsbergen and Koijen (2017) provides an extensive review of the different models that generate the dividend strips irregularities and classify their mechanisms into six broad categories: alternative models of preferences (Berrada, Detemple, and Rindisbacher (2013), Marfè (2014), Curatola (2015), Eisenbach and Schmalz (2016), Andries, Eisenbach, and Schmalz (2019), Andries (2021)), alternative models of technology (Gourio (2008), Nakamura, Steinsson, Barro, and Ursúa (2013), Belo, Collin-Dufresne, and Goldstein (2015), Lopez, Lopez-Salido, and Vazquez-Grande (2015), Hasler and Marfè (2016), Marfè (2013), Ai, Croce, Diercks, and Li (2018), Corhay, Kung, Schmid, and Nieuwerburgh (2020)), alternative models of beliefs (Croce, Lettau, and Ludvigson (2015)), heterogeneous agent models (Lustig and Nieuwerburgh (2006), Marfè (2017), Favilukis and Lin (2016)), asset pricing models with an exogenous stochastic discount factor (Lettau and Wachter (2007), Lettau and Wachter (2011), Lynch and Randall (2011)), market microstructure and tax effects (Boguth et al. (2019), Schulz (2016)). In their review, van Binsbergen and Koijen (2017) argue that frictions introduced by asset managers may cause the dividend strips empirical irregularities.

Lastly, our theory relates to a growing literature, connecting asset management's inelastic demand for assets as a determinant of asset pricing. Recently, Koijen and Yogo (2019) investigate the cross-sectional effects of inelastic institutional demand. Further, Chang, Hong, and Liskovich (2015) and Pavlova and Sikorskaya (2022) investigate the inelastic demand of the stock inclusion effects; Ben-David, Li, Rossi, and Song (2021) due to Morningstar ratings change, Peng and Wang (2021), Li (2021), and Gabaix and Koijen (2022) due to funds flow.[7]

3 The Economic Setup

This section lays out a simple and tractable model to study the mismatch between investors with short-term performance windows and long-term investment mandates. We consider a standard pure-exchange finite horizon economy. Time t is continuous and goes from zero to T. Uncertainty is driven by two independent Brownian motions, (Z_{1t}, Z_{2t}). This model has two investment horizons: a short-term investment horizon, $\frac{T}{N}$, and a long-term investment horizon, T, whereby $N \geq 2$ is a finite number. For the primary analysis, we set $N = 2$.[8]

There are two dividend payout dates: short-term and long-term. The dividend payout (supply of dividends) at the short-term is denoted by $D_{\mathcal{S}\frac{T}{2}}$ and at the dividend payout in long-term by $D_{\mathcal{L}T}$, where $\mathcal{S}$ and $\mathcal{L}$ stand for short- and long-term dividend payouts, respectively. These short-term and long-term dividend payouts are determined by the dynamics of

$$dD_{\mathcal{S}t} = D_{\mathcal{S}t} \left(\mu dt + \sigma dZ_{1t} \right), \tag{1}$$

$$dD_{\mathcal{L}t} = D_{\mathcal{L}t} \left(\mu dt + \sigma dZ_{2t} \right), \tag{2}$$

where μ and σ are positive constants. We refer to these processes as news about the short- and long-term dividend payouts. Notice that the short- and long-term dividend payouts are independent and have the same distributional properties for any given $t < T/2$. Therefore,

[7] Ben-David et al. (2021) and Gabaix and Koijen (2022) provide an extensive overview of this growing literature.

[8] A longer performance window, $N > 2$, affects the levels of the equilibrium quantities, but as long as there are two distinct investment horizons, the equilibrium mechanism remains unchanged.

any price difference arises due to the equilibrium mechanism that prices short-term (Z_{1t}) and long-term (Z_{2t}) risks differently.

Potential differences in the distributional properties of the short- and long-term dividend payouts may have adverse effects on our results. However, Belo et al. (2015) report that short-term dividend payouts have higher volatility than long-term dividend payouts — making our results stronger.

There are two risky assets. The first asset, $S_{\mathcal{S}t}$, represents a claim on the short-term dividend payout, and the second asset, $S_{\mathcal{L}t}$, represents a claim on the long-term dividend payout. We assume that both assets are in unit supply and follow

$$dS_{\mathcal{S}t} = S_{\mathcal{S}t} \left(\mu_{\mathcal{S}t} dt + \sigma_{\mathcal{S}1t} dZ_{1t} + \sigma_{\mathcal{S}2t} dZ_{2t} \right), \tag{3}$$

$$dS_{\mathcal{L}t} = S_{\mathcal{L}t} \left(\mu_{\mathcal{L}t} dt + \sigma_{\mathcal{L}1t} dZ_{1t} + \sigma_{\mathcal{L}2t} dZ_{2t} \right). \tag{4}$$

Prices, (instantaneous) expected returns, and (instantaneous) volatilities are endogenous and determined in equilibrium. To capture the short- and long-term assets' interdependencies, our analysis focuses on $t \leq T/2$, meaning the economy resets to $t = 0$ when time reaches $T/2$. In addition to the risky assets, investors can trade with a riskless bond. We assume the bond is in zero net supply and denote it by B_t, and for $t \leq T/2$, the bond pays a continuous, exogenous riskless interest payment r, which we set to zero to simplify the analysis.

3.1 Investors

Two types of investors populate the economy: long-term value and short-term asset managers. The long-term value investor, subject to *absolute* wealth concerns, evaluates his portfolio at the long-horizon date and has standard constant relative risk aversion preferences over the terminal value of his portfolio ($W_T^{\mathcal{V}}$):

$$E \left[\frac{\left(W_T^{\mathcal{V}} \right)^{1-R}}{1-R} \right]. \tag{5}$$

The short-term asset manager, subject to *relative* wealth concerns, evaluates the terminal value of his portfolio ($W^{\mathcal{I}}_{T/2}$) relative to a benchmark ($S_{\mathcal{L}T/2}$) at the short-horizon date:

$$E\left[\left(S_{\mathcal{L}T/2}\right)\frac{\left(W^{\mathcal{I}}_{T/2}\right)^{1-R}}{1-R}\right], \tag{6}$$

where $\mathcal{V}$ and $\mathcal{I}$ stand for the value and asset managers, respectively.

The benchmark is the long-term asset, and Section 8 discusses an extension in which the benchmark combines the short-term and long-term assets. Though, such a benchmark does not affect the main equilibrium implications. The economy further assumes that investors have the same risk aversion parameter, $R > 1$. Still, the economic setup allows for different risk aversion parameters while still providing closed-form precise expressions to all the equilibrium quantities. Qualitatively, the equilibrium outcomes remain the same when the asset manager is less risk-averse than the value investor.

The asset manager utility function captures the two essential frictions in the asset management industry: short-term performance window and relative performance objective, commonly referred to as benchmarking. First, the model captures the short-term performance window by setting the time the asset manager collects rewards shorter than the value investor ($T/2 < T$). Second, the model captures the relative performance objective by interacting the asset manager utility over end-of-period wealth with the benchmark. This interaction term captures the critical aspect of benchmarking laid out by Basak and Pavlova (2013), implying that the asset manager strives to post higher returns when the benchmark is high than when it is low. It captures asset managers' relative wealth concerns in a reduced form, allowing a highly tractable equilibrium outcome.

The asset manager utility function (6) introduces two variations to the established utility function of Basak and Pavlova (2013), introduced to study the equilibrium effects of asset managers on asset prices. We divide their utility function by the constant b and obtain the following equivalent preference representation

$$E\left[\left(\frac{1}{b}+I\right)\log\left(W\right)\right], \tag{7}$$

where b represents the benchmark's importance, and I represents the exogenous benchmark news in their model.

In our first modification of their utility function, we let $b \to \infty$ and assume that benchmarking is extremely important. It substantially simplifies the subsequent analysis and allows for analytical, closed-form solutions. Our second modification introduces a risk aversion parameter strictly bigger than one ($R > 1$) because myopic investors with $R = 1$ do not care about long-term shocks. Therefore, the financial markets with short- and long-term investment mandates are not dynamically complete. This assumption is innocuous because the model subsumes the myopic, $R = 1$ case. We can set $R = 1 + \epsilon$ with arbitrarily small $\epsilon > 0$ and investigate the myopic case with $R \to 1$. The asset manager's performance objective (6) captures the critical aspect of benchmarking laid out by Basak and Pavlova (2013): the asset manager strives to post higher returns when the benchmark is high than when it is low.

One constructive and essential deviation from the literature is that, in our model, the asset manager's benchmark is endogenous and determined in equilibrium. It includes the long-horizon asset price ($S_{\mathcal{L}T/2}$) and not its dividend news, as typically assumed in the literature.

We denote the total asset market at $t = 0$ by S_m and assume that at $t = 0$, the asset manager is endowed with λ shares of the total asset market, while the value investor with the residual $1 - \lambda$. Starting with these initial endowments, each investor dynamically chooses a portfolio π^k_{it}, where π^k_i represents the fraction of wealth investor k invests in security i, where $k = \mathcal{I},\ \mathcal{V}$ and $i = \mathcal{S},\ \mathcal{L}$. The wealth processes of the two investors then follow the dynamics

$$\frac{dW_t^{\mathcal{V}}}{W_t^{\mathcal{V}}} = \pi_t^{\mathcal{V}\prime} \begin{pmatrix} \mu_{\mathcal{S}t} \\ \mu_{\mathcal{L}t} \end{pmatrix} dt + \pi_t^{\mathcal{V}\prime} \Sigma_t \begin{pmatrix} dZ_{1t} \\ dZ_{2t} \end{pmatrix}, \quad \frac{dW_t^{\mathcal{I}}}{W_t^{\mathcal{I}}} = \pi_t^{\mathcal{I}\prime} \begin{pmatrix} \mu_{\mathcal{S}t} \\ \mu_{\mathcal{L}t} \end{pmatrix} dt + \pi_t^{\mathcal{I}\prime} \Sigma_t \begin{pmatrix} dZ_{1t} \\ dZ_{2t} \end{pmatrix}, \tag{8}$$

where we denote the vector of portfolio weights of investor k by π^k_t, and the matrix of return volatilities by Σ_t, such that

$$\pi_t^k \equiv \begin{bmatrix} \pi^k_{\mathcal{S}t} \\ \pi^k_{\mathcal{L}t} \end{bmatrix}, \quad \Sigma_t \equiv \begin{bmatrix} \sigma_{\mathcal{S}1t} & \sigma_{\mathcal{S}2t} \\ \sigma_{\mathcal{L}1t} & \sigma_{\mathcal{L}2t} \end{bmatrix}. \tag{9}$$

4 Equilibrium

This section unravels the heterogeneous investment horizons equilibrium mechanism. Our model introduces two frictions: (i) the heterogeneous investment horizons and (ii) the relative performance objective of asset managers. Accordingly, three potential equilibrium channels drive the equilibrium asset pricing quantities. There are two *individual* effects due to the heterogeneous investment horizons and the relative performance objective separately. One *combined* effect comes from the interaction of the heterogeneous investment horizons and the relative performance objective. However, notice that the individual relative performance objective effect cannot drive our equilibrium mechanism and explain the dividend strip irregularities because dividend payouts are simultaneous without heterogeneous investment horizons. Ultimately, we remain with one individual effect and the combined effect.

Throughout the analysis, we distinguish between these two potential channels and eventually show that the combined effect is responsible for the downward-sloping risk premium and volatility since, without benchmarking, the risk premium and volatility slope upward.

We define the equilibrium in a standard way: equilibrium prices and portfolio holdings are such that (i) both the asset manager and the value investor choose their optimal portfolios for given prices, and (ii) stocks, the bond, and consumption-good markets clear.

The separation between the short-term and the long-term assets' payouts and the separation between the short-term asset manager and long-term value investor objectives imply that equilibrium quantities are set up to ensure that the asset manager optimally trades off the long-term asset while the value investor trades off the short-term asset.

In the first step towards unraveling the equilibrium, we present investors' optimal risk exposures. It is a partial equilibrium result because investors choose their risk exposures (and portfolios) given prices. Still, it allows us to separate the different shock propagation channels and analyze how the heterogeneous investment horizons mechanism affects prices.

Lemma 1 (Risk Exposure). *The value and asset managers' risk exposures are given by*

$$\Sigma_t' \pi_t^{\mathcal{V}} = \theta_t + \begin{bmatrix} 0 \\ -(R-1)\sigma \end{bmatrix}, \tag{10}$$

$$\Sigma_t' \pi_t^{\mathcal{I}} = \theta_t + \begin{bmatrix} -(R-1)\sigma \\ 0 \end{bmatrix} + \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix}. \tag{11}$$

The Lemma shows that investors' wealth has three separate shock propagation channels that affect them. The first channel is the myopic mean-variance channel (θ_t). It states that investors buy and sell assets to correlate their wealth with the market prices of risk; in other words, risk exposure is proportional to excess return per unit of risk. We have yet to identify which asset has a better risk-return trade-off; it will be revealed explicitly in equilibrium.

The second channel is the risk aversion intertemporal hedge channel. Since the risk aversion parameter is strictly greater than one ($R > 1$), the risk aversion hedge states that investors want more wealth in bad economic states. Alternatively, wealth is less valuable when investment opportunities are better because it is easier to increase wealth through investments. The value investor hedges against long-term risk aversion, while the asset manager hedges against short-term risk aversion. The intertemporal risk aversion hedge goes back to Merton (1971).

The third remaining channel is an intertemporal benchmark hedge channel. It is entirely driven by the asset manager's benchmarking motives and disappears when there is no benchmark. The benchmark hedge states that the asset manager strives to do well when the benchmark does well. To achieve that goal, the asset manager optimally correlates his risk exposure with the benchmark's return and simply buys the benchmark to hedge against benchmark fluctuations. Accordingly, any deviation from holding the long-term benchmarked asset is due to either risk aversion hedging or mean-variance considerations.

Notice that the benchmark hedge is endogenous and depends on the equilibrium long-term benchmarked asset risk exposures. This outcome is unique to our economy and does not exist in a homogeneous investment horizon economies where the benchmark hedging exposure is exogenous. As we soon reveal, the endogenous benchmark hedging is a critical determinant of equilibrium prices.

Significantly, the two assets that mimic dividend strips' payouts: the short-term asset pays dividends in the short term, and the long-term asset pays dividends in the long term; the dividends are independent ($D_{\mathcal{L}t} \perp D_{\mathcal{S}t}$), and their payouts' dates do not overlap. Furthermore, the asset manager cares only about the short-term investment horizon due to the short performance window. Therefore, optimality implies that asset manager must hold the short-term asset eventually, at the end of his performance window, because the long-term asset does not pay off in the short term when the asset manager's performance is evaluated.

The equilibrium objective is to set asset prices so that the asset manager optimally chooses to trade off the long-term asset and hold the short-term asset, while the value investor optimally chooses to trade off the short-term asset and hold the long-term asset. This objective does not depend on whether the benchmark is long-term or short-term or whether there is a benchmark. However, the equilibrium adjustments of assets' returns and market prices of risk substantially differ depending on whether there is a benchmark and the strength of asset managers' incentives to benchmark.

To identify the individual effect, we solve for an equilibrium without the relative performance objective but with the heterogeneous investment horizons and denote the equilibrium quantities by the upper bar ($\bar{X}$) throughout the analysis. We then compare the asset pricing quantities arising from these two equilibriums and identify whether the individual channel or the combined channel drives the difference between the short-term and long-term equilibrium quantities.

The economic setup allow us to differentiate between the effect of long-term and short-term factors on stock prices. In particular, short-term cash flow news is *non-fundamental news* about the long-term asset because this asset never pays in the short-term. Similarly, long-term cash flow news is *non-fundamental news* about the short-term asset because this asset pays dividends only in the short term. In what follows we characterize the equilibrium return volatilities and identify the effects of non-fundamental cash-flow news.

Proposition 1 (**Volatility**). *The short-term asset has short-term risk exposure and no long-term risk exopsure,*

$$\sigma_{\mathcal{S}1t} = \bar{\sigma}_{\mathcal{S}1t} = \sigma, \quad \sigma_{\mathcal{S}2t} = \bar{\sigma}_{\mathcal{S}2t} = 0, \tag{12}$$

where $\bar{\sigma}_{\mathcal{S}1t}$ and $\bar{\sigma}_{\mathcal{S}2t}$ are the equilibrium risk exposures without benchmarking motives. The long-term asset has a positive exposure to the short-term risk and a negative exposure to the long-term risk,

$$\sigma_{\mathcal{L}1t} = \bar{\sigma}_{\mathcal{L}1t} - \frac{R}{2}\sigma > 0, \quad \sigma_{\mathcal{L}2t} = \bar{\sigma}_{\mathcal{L}2t} + \frac{R-1}{2}\sigma < 0, \tag{13}$$

where $\bar{\sigma}_{\mathcal{L}1t}$ and $\bar{\sigma}_{\mathcal{L}2t}$ are the equilibrium risk exposures without benchmarking motives, given by

$$\bar{\sigma}_{\mathcal{L}1t} = R\sigma > 0, \quad \bar{\sigma}_{\mathcal{L}2t} = -\left(R-1\right)\sigma < 0. \tag{14}$$

The individual effect of the heterogeneous investment horizons implies *(i) that the long-term asset has a negative fundamental exposure,* $(\bar{\sigma}_{\mathcal{L}2t}, \sigma_{\mathcal{L}2t} < 0)$*, and a positive non-fundamental exposure,* $(\bar{\sigma}_{\mathcal{L}1t}, \sigma_{\mathcal{L}1t} > 0)$*.*

(ii) that non-fundamental cash-flow news has a bigger impact on the long-term price than fundamental cash-flow news, $(|\sigma_{\mathcal{L}1t}| > |\sigma_{\mathcal{L}2t}|)$ *and* $(|\bar{\sigma}_{\mathcal{L}1t}| > |\bar{\sigma}_{\mathcal{L}2t}|)$*;*

(iii) that a common shock to short-term and long-term cash-flow news has a positive effect on the long-term price, $(\sigma_{\mathcal{L}1t} + \sigma_{\mathcal{L}2t} > 0)$*,* $(\bar{\sigma}_{\mathcal{L}1t} + \bar{\sigma}_{\mathcal{L}2t} > 0)$*.*

The combined effect of the heterogeneous investment horizons and the relative performance objective

(i) increases the long-term asset (fundamental) long-term exposure $(\sigma_{\mathcal{L}2t})$ *and decreases the (non-fundamental) short-term exposure* $(\sigma_{\mathcal{L}1t})$*; however, their sign remains the same.*

(ii) reduces the long-term asset total volatility: $\sqrt{\bar{\sigma}^2_{\mathcal{L}1t} + \bar{\sigma}^2_{\mathcal{L}2t}} > \sqrt{\sigma^2_{\mathcal{L}1t} + \sigma^2_{\mathcal{L}2t}}$*;*

(iii) reduces the effect of a common shock $(\sigma_{\mathcal{L}1t} + \sigma_{\mathcal{L}2t} < \bar{\sigma}_{\mathcal{L}1t} + \bar{\sigma}_{\mathcal{L}2t})$*;*

In a traditional economy with homogenous investment horizons and without benchmarking incentives, fundamental cash-flow news is the main and, in some cases, the only driving force behind asset pricing fluctuations. Basak and Pavlova (2013) show that this outcome persists in an economy with homogenous investment horizons and benchmarking incentives. However, Proposition 1 reveals that the individual effect (the heterogeneous investment horizons) reverses this outcome by showing that the non-fundamental cash-flow news (news about the short-term payout) impacts prices more than the fundamental cash-flow news (news about the long-term payout), $(|\sigma_{\mathcal{L}1t}| > |\sigma_{\mathcal{L}2t}|)$. This equilibrium outcome can potentially rationalize key asset pricing findings that traditional models struggle to justify, such as why asset prices fluctuate much more than dividends, as was first revealed by Shiller (1981).

While a common positive shock positively affects prices $(\bar{\sigma}_{\mathcal{L}1t} + \bar{\sigma}_{\mathcal{L}2t} > 0)$, a positive long-term fundamental cash-flow news reduces the long-term asset price $(\bar{\sigma}_{\mathcal{L}2t} < 0)$. Interestingly, this result provides a new testable implication: asset prices load negatively on long-term cash-flow risk factors when controlling for short-term cash-flow risk factors.

Notice that these two outcomes are *not* due to the combined effect (the interaction of the heterogeneous investment horizons and relative performance objective) since they persist in an economy without benchmarking incentives: (i) $\bar{\sigma}_{\mathcal{L}2t} < 0$, and (ii) $|\bar{\sigma}_{\mathcal{L}1t}| > |\bar{\sigma}_{\mathcal{L}2t}|$.

An essential feature of the equilibrium arising from the combined effect is that the total volatility of a benchmarked asset *decreases* in the presence of benchmarking aligning with Cuoco and Kaniel (2011) but reversing Basak and Pavlova (2013)s' findings when the investment horizons are homogeneous, as item (i) of the combined effect in Proposition 1 reveals.

Lastly, before discussing the equilibrium mechanism, we characterize the market prices of risk.

Proposition 2 (**Market Price of Risk**). *The short- and long-term market prices of risk are given by*

$$\theta_{1t} = \frac{R}{2}\sigma > 0, \qquad \theta_{2t} = \frac{R-1}{2}\sigma > 0, \tag{15}$$

where $\bar{\theta}_{1t} = R\sigma$ and $\bar{\theta}_{2t} = 0$ are the equilibrium market prices of risk without benchmarking motives.

The individual effect of the heterogeneous investment horizons implies that the short-term market price of risk is higher than the long-term market price of risk

$$\theta_{1t} > \theta_{2t}, \quad \bar{\theta}_{1t} > \bar{\theta}_{2t}. \tag{16}$$

The combined effect of the heterogeneous investment horizons and the relative performance objective

(i) increases the long-term market price of risk ($\theta_{2t} > \bar{\theta}_{2t}$);

(ii) and decreases the short-term market price of risk ($\bar{\theta}_{1t} > \theta_{1t}$);

In a homogenous horizon economy, benchmarking reduces the market price of benchmarked assets' fundamental risk while it does not affect non-fundamental market prices of risk. To ensure market clearing, equilibrium depresses the market price of risk so that a retail investor without benchmarking motives finds the benchmarked asset unattractive due to its low risk-return trade-offs. The asset manager is willing to forgo the worse risk-return trade-off because the benchmarked asset satisfies the benchmark hedging desires. Following the homogenous horizon economy's logic, we expect the long-term market price of risk to decrease and to have no impact on the short-term market price of risk. However, the equilibrium's workings are different in the heterogeneous investment horizon, and the results from the homogenous case do not follow.

Interestingly, item (i) in Proposition 2 reveals that when the long-term asset belongs to the benchmark, the market price of the long-term risk increases. Further, item (ii) in

Proposition 2 indicates that the market price of the short-term risk decreases even though the short-term asset does not belong to the benchmark. These two outcomes run against the theoretical findings with homogenous investment horizons, such as Basak and Pavlova (2013) and Buffa and Hodor (2018).[9]

However, despite the opposing price pressures reducing the short-term and increasing the long-term market prices of risk, the short-term market price risk always remains higher than the long-term market price of risk.

5 Equilibrium Mechanism

So far, we have identified the market prices of risk and return volatilities. Next, we revisit investors' partial equilibrium risk exposures, given in Lemma (1), introduces the optimal portfolios, and analyze the mechanism.

We start with the optimal portfolios. By inverting the volatility matrix transpose $\left(\Sigma_t^{'}\right)$, we convert investors' risk exposures, (10) and 11), to portfolio holdings.

$$\pi_t^{\mathcal{V}} = \left(\Sigma_t^{'}\right)^{-1} \theta_t + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} 0 \\ -(R-1)\,\sigma \end{bmatrix}, \tag{17}$$

$$\pi_t^{\mathcal{I}} = \left(\Sigma_t^{'}\right)^{-1} \theta_t + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} -(R-1)\,\sigma \\ 0 \end{bmatrix} + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix}. \tag{18}$$

There are three reasons to hold assets: (i) mean-variance risk exposure, (ii) risk aversion hedging, and (iii) benchmark hedging. To stay consistent with these different risk exposures, we represent the value investor's portfolio ($\pi_t^{\mathcal{V}}$) as the sum of two separate portfolios: (i) the first is the mean-variance portfolio ($\phi_{\text{m.v.}}$), and (ii) the second is the short-term risk aversion hedging portfolio ($\phi_{\text{r.a.}}^{\mathcal{V}}$). Similarly, we represent the asset manager's portfolio as the sum of three separate portfolios: (i) the first is the mean-variance portfolio ($\phi_{\text{m.v.}}$), (ii) the second is the long-term risk aversion hedging portfolio ($\phi_{\text{r.a.}}^{\mathcal{I}}$), and (iii) the third is the benchmark hedging portfolio ($\phi_{\text{b}}^{\mathcal{I}}$). The following Proposition characterizes these portfolios.

[9] In Section 8, we show that when only the short-term asset belongs to the benchmark, the short-term market price of risk decreases while the long-term price of risk is unaffected, aligning with the findings of homogenous investment horizon economies.

Proposition 3 (Portfolios). *The asset manager benchmark hedging portfolio and long-term risk aversion hedging portfolio are given by*

$$\phi_b^{\mathcal{I}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \phi_{r.a.}^{\mathcal{I}} = \begin{bmatrix} 1-R \\ 0 \end{bmatrix}, \tag{19}$$

while the value investor short-term risk aversion hedging portfolio is given by

$$\phi_{r.a.}^{\mathcal{V}} = \begin{bmatrix} -R \\ 2 \end{bmatrix}. \tag{20}$$

The mean-variance portfolio is given by

$$\phi_{m.v.} = \begin{bmatrix} R \\ -1 \end{bmatrix} \tag{21}$$

and $\phi_b^{\mathcal{I}} + \phi_{r.a.}^{\mathcal{I}} + \phi_{m.v.} = \pi_t^{\mathcal{I}} = \bar{\pi}_t^{\mathcal{I}}$ *and* $\phi_{r.a.}^{\mathcal{V}} + \phi_{m.v.} = \pi_t^{\mathcal{V}} = \bar{\pi}_t^{\mathcal{V}}$.

Proposition 3 reveals that the asset manager has a one-leg long portfolio and a one-leg short portfolio. Specifically, in the benchmark hedge portfolio, the asset manager buys the long-term asset, and in the short-term risk aversion hedge portfolio, sells the short-term asset. The mean-variance portfolio is a two-leg long-short portfolio that offsets the short position in the short-term asset and the long position in the long-term asset.

The equilibrium objective is to set asset prices so that the asset manager optimally chooses to trade off the long-term asset and hold the short-term asset, while the value investor optimally chooses to trade off the short-term asset and hold the long-term asset. This objective does not depend on whether the benchmark is long-term or short-term or whether there is a benchmark. However, the equilibrium adjustments of assets' returns and market prices of risk substantially differ depending on whether there is a benchmark and the strength of asset managers' incentives to benchmark.

We start the discussion of the mechanism by analyzing the risk exposures of the individual effect when there is no benchmark. By plugging the market prices of risk for the individual effect ($\bar{\theta}$), we obtain

$$\text{Individual Effect:} \begin{cases} \bar{\Sigma}_t' \bar{\pi}_t^{\mathcal{V}} / \sigma = \begin{bmatrix} R \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -(R-1) \end{bmatrix} = \begin{bmatrix} R \\ -(R-1) \end{bmatrix} \Rightarrow \begin{bmatrix} \bar{\pi}_{\mathcal{S}t}^{\mathcal{V}} = 0 \\ \bar{\pi}_{\mathcal{L}t}^{\mathcal{V}} = 1 \end{bmatrix}, \\ \bar{\Sigma}_t' \bar{\pi}_t^{\mathcal{I}} / \sigma = \begin{bmatrix} R \\ 0 \end{bmatrix} + \begin{bmatrix} -(R-1) \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \bar{\pi}_{\mathcal{S}t}^{\mathcal{I}} = 1 \\ \bar{\pi}_{\mathcal{L}t}^{\mathcal{I}} = 0 \end{bmatrix}. \end{cases} \tag{22}$$

The equilibrium outcome when there is no benchmark aligns with a traditional setup where cash-flow news today has a more significant impact on assets that pay off in the long term (as opposed to the short term). Equilibrium compensates for the extra risk in the long-term asset by having a higher risk premium, leading to an upward-sloping risk premium and volatility.

To see why this is the case, observe that the value investor requires a negative exposure to long-term shocks due to the long-term risk aversion hedge, $-(R-1)$. Equilibrium ensures the value investor buys one unit of the long-term asset ($\bar{\pi}_{\mathcal{L}t}^{\mathcal{V}} = 1$) by positively correlating the risk aversion hedge with the long-term asset risk exposure ($\bar{\sigma}_{\mathcal{L}2t} = -(R-1) < 0$).

In addition, equilibrium ensures that the asset manager does not desire to hold (or short) the long-term asset ($\bar{\pi}_{\mathcal{L}t}^{\mathcal{I}} = 0$) by setting the long-term risk price to zero ($\bar{\theta}_{2t} = 0$); equilibrium set ($\bar{\theta}_{1t} = R$) so that the asset manager desire to hold one unit of the short-term asset.

Lastly, the value investor requires positive exposure to short-term shocks (R). To ensure that the value investor does not obtain this exposure with the short-term asset ($\bar{\pi}_{\mathcal{S}t}^{\mathcal{V}} = 0$), equilibrium sets the long-term asset risk exposure high enough to match the value investor desired exposure, ($\bar{\sigma}_{\mathcal{L}1t} = R$). This outcome implies that the long-term asset has higher short-term risk exposure ($\bar{\sigma}_{\mathcal{L}1t} > \bar{\sigma}_{\mathcal{S}1t}$).

Equilibrium compensates the value investor for taking on that risk by having a higher long-term risk premium, leading to an upward-sloping risk premium ($\bar{\mu}_{\mathcal{L}t} > \bar{\mu}_{\mathcal{S}t}$) and volatility ($\sqrt{\bar{\sigma}_{\mathcal{L}1t}^2 + \bar{\sigma}_{\mathcal{L}2t}^2} > \sqrt{\bar{\sigma}_{\mathcal{S}1t}^2 + \bar{\sigma}_{\mathcal{S}2t}^2}$).

Next, we focus on the combined effect of the heterogeneous investment horizons and the relative performance objective. By plugging the market prices of risk (θ) and the long-term benchmarked asset equilibrium exposures ($\sigma_{\mathcal{L}1t}^2$, $\sigma_{\mathcal{L}2t}^2$), we obtain

$$\text{Combined Effect:} \quad \begin{cases} \Sigma_t' \pi_t^{\mathcal{V}} / \sigma = \begin{bmatrix} R/2 \\ (R-1)/2 \end{bmatrix} + \begin{bmatrix} 0 \\ -(R-1) \end{bmatrix} = \begin{bmatrix} R/2 \\ -(R-1)/2 \end{bmatrix} \Rightarrow \begin{bmatrix} \pi_{\mathcal{S}t}^{\mathcal{V}} = 0 \\ \pi_{\mathcal{L}t}^{\mathcal{V}} = 1 \end{bmatrix}, \\ \Sigma_t' \pi_t^{\mathcal{I}} / \sigma = \begin{bmatrix} R/2 \\ (R-1)/2 \end{bmatrix} + \begin{bmatrix} -(R-1) \\ 0 \end{bmatrix} + \begin{bmatrix} R/2 \\ -(R-1)/2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \pi_{\mathcal{S}t}^{\mathcal{I}} = 1 \\ \pi_{\mathcal{L}t}^{\mathcal{I}} = 0 \end{bmatrix}. \end{cases} \tag{23}$$

Benchmarking induces extra — unrelated to prices — demand to hold the benchmark asset. This excess demand pressure is unsustainable in the no-benchmark pricing environment (individual effect).

We break the equilibrium pricing of the combined effect into two separate effects: (i) the partial combined effect of the price reaction ($\hat{\theta}_{1t}$, $\hat{\theta}_{2t}$), given that price dynamics are unchanged ($\bar{\sigma}_{\mathcal{L}t}, \bar{\sigma}_{\mathcal{S}t}$); (ii) the complete combined effect of both price reaction and the price dynamics ($\sigma_{\mathcal{L}t}, \sigma_{\mathcal{S}t}$). We pin down the price reaction in the partial combined effect ($\hat{\theta}_{1t}$, $\hat{\theta}_{2t}$) by imposing the same equilibrium restriction that prices counteract the benchmarking demand and restore equilibrium by negating the demand to hold the benchmark.

$$\text{Partial Combined Effect (i):} \quad \begin{cases} \Sigma_t' \pi_t^{\mathcal{V}} / \sigma = \begin{bmatrix} \hat{\theta}_1 = 0 \\ \hat{\theta}_2 = R-1 \end{bmatrix} + \begin{bmatrix} 0 \\ -(R-1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \hat{\pi}_{\mathcal{S}t}^{\mathcal{V}} = 0 \\ \hat{\pi}_{\mathcal{L}t}^{\mathcal{V}} = 0 \end{bmatrix}, \\ \Sigma_t' \pi_t^{\mathcal{I}} / \sigma = \begin{bmatrix} \hat{\theta}_1 = 0 \\ \hat{\theta}_2 = R-1 \end{bmatrix} + \begin{bmatrix} -(R-1) \\ 0 \end{bmatrix} + \begin{bmatrix} \bar{\sigma}_{\mathcal{L}1} = R \\ \bar{\sigma}_{\mathcal{L}2} = -(R-1) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \hat{\pi}_{\mathcal{S}t}^{\mathcal{I}} = 1 \\ \hat{\pi}_{\mathcal{L}t}^{\mathcal{I}} = 0 \end{bmatrix}. \end{cases} \tag{24}$$

In the partial effect, equilibrium induces the long-term (benchmarked) asset to be more expensive:

$$\begin{aligned} \hat{\theta}_{1t} < \bar{\theta}_{1t} \text{ and } \hat{\theta}_{2t} > \bar{\theta}_{2t} &\Rightarrow \mathbb{C}\mathrm{ov}(-d\hat{\xi}_t, d\bar{S}_{\mathcal{L}t}) < \mathbb{C}\mathrm{ov}(-d\bar{\xi}_t, d\bar{S}_{\mathcal{L}t}) \\ &\Rightarrow \bar{S}_{\mathcal{L}t}(\hat{\theta}) > \bar{S}_{\mathcal{L}t}, \end{aligned} \tag{25}$$

where $\bar{S}_{\mathcal{L}t}(\hat{\theta})$ is the long-term asset price considering the partial effect (i), given that price dynamics are unchanged (24). However, the partial price effect alone cannot restore both optimal portfolios and market clearing conditions.

The partial price reaction makes the long-term asset too expensive, so much so that the value investor prefers to have a risk-free position in equilibrium than holding the long-term or the short-term assets, violating the market clearing condition. To make the long-term asset more attractive, equilibrium decreases the long-term asset risk exposures ($|\sigma_{\mathcal{L}1t}| < |\bar{\sigma}_{\mathcal{L}1t}|$, $|\sigma_{\mathcal{L}2t}| < |\bar{\sigma}_{\mathcal{L}2t}|$) to the point where the value investor optimally holds one unit while still negating the benchmarking demand of the asset manager, as (23) reveals. In doing so, the short-term asset has a more pronounced risk exposure ($\sigma_{\mathcal{S}1t} > \sigma_{\mathcal{L}1t}$) and, eventually, is more volatile than the long-term asset.

Equilibrium compensates the asset manager for taking on that heightened short-term risk (relative to the long-term asset) by having a higher short-term risk premium, inducing a downward-sloping risk premium ($\mu_{\mathcal{S}t} > \mu_{\mathcal{L}t}$) and volatility ($\sqrt{\sigma^2_{\mathcal{S}1t} + \sigma^2_{\mathcal{S}2t}} > \sqrt{\sigma^2_{\mathcal{L}1t} + \sigma^2_{\mathcal{L}2t}}$).

To conclude, equilibrium strives to negate the benchmark demand by offsetting it entirely since the asset manager exposure remains unchanged regardless of whether the asset manager has a benchmark. Equilibrium cannot achieve its goal only through a partial price reaction since the value investor finds the long-term asset too expensive. As a result, equilibrium reduces the long-term asset risk exposures until it becomes attractive enough for the value investor to buy one unit, inducing a downward sloping risk premium and volatility.

6 Equilibrium Implications

There are two primary equilibrium implications. We first discuss the downward-sloping term structure of risk, as measured by risk premium, total volatility, and Sharpe ratio. We follow with the optimal portfolios and conclude with a discussion of the equilibrium dynamics.

6.1 Downward Sloping Term Structure of Risk

This section introduces the downward sloping trend in three terms structure of risk. The following Proposition verifies that this model generates the three empirical regularities first documented by van Binsbergen et al. (2012).

Proposition 4 (**Risk Premium, Total Volatility, and Sharpe Ratio**).

(i) **Risk Premium:** *The short- and long-term risk premiums are given by*

$$\mu_{\mathcal{S}t} = \sigma^2 \frac{R}{2} > 0, \qquad \mu_{\mathcal{L}t} = \sigma^2 \frac{1}{2}\left(R - \frac{1}{2}\right) > 0. \tag{26}$$

The short-term asset risk premium is higher than the long-term asset risk premium,

$$\mu_{\mathcal{S}t} > \mu_{\mathcal{L}t}, \tag{27}$$

while the reverse is true without benchmarking incentives: $\bar{\mu}_{\mathcal{S}t} < \bar{\mu}_{\mathcal{L}t}$*, where* $\bar{\mu}_{\mathcal{S}t} = \sigma^2 R$ *and* $\bar{\mu}_{\mathcal{L}t} = \sigma^2 R^2$*. The combined effect further implies*

(a) A reduction in expected returns of both the benchmarked and non-benchmarked assets

$$\bar{\mu}_{\mathcal{S}t} > \mu_{\mathcal{S}t}, \quad \bar{\mu}_{\mathcal{L}t} > \mu_{\mathcal{L}t}. \tag{28}$$

The reduction in the long-term asset expected return is more pronounced

$$\bar{\mu}_{\mathcal{L}t} - \mu_{\mathcal{L}t} > \bar{\mu}_{\mathcal{S}t} - \mu_{\mathcal{S}t}. \tag{29}$$

(ii) **Total Volatility:** *The short-term asset volatility is higher than the long-term asset volatility if and only if investors' risk aversion is sufficiently low, such that*

$$\sqrt{\sigma^2_{\mathcal{L}1t} + \sigma^2_{\mathcal{L}2t}} < \sqrt{\sigma^2_{\mathcal{S}1t} + \sigma^2_{\mathcal{S}2t}} \iff R < \bar{R}, \tag{30}$$

where $\bar{R} < 2$ *is given in (A.59). The reverse is true without benchmarking incentives and for any risk aversion parameter* $R > 1$*.*

(iii) **Sharpe Ratio:** *The short-term asset Sharpe ratio is higher than the long-term asset Sharpe ratio*

$$\frac{\mu_{\mathcal{S}t}}{\sqrt{\sigma^2_{\mathcal{S}1t} + \sigma^2_{\mathcal{S}2t}}} > \frac{\mu_{\mathcal{L}t}}{\sqrt{\sigma^2_{\mathcal{L}1t} + \sigma^2_{\mathcal{L}2t}}}. \tag{31}$$

The equilibrium market price of the short-term risk is always higher than that of the long-term risk (16). However, it does not immediately imply that the short-term risk premium is higher than the long-term risk premium. In fact, without benchmarking incentives, the short-term risk premium is lower than the long-term risk premium ($\bar{\mu}_{\mathcal{S}t} < \bar{\mu}_{\mathcal{L}t}$). By decomposing the risk premium into the sum of the short-term and the long-term values of risks multiplied by their respective risk prices, we show that the reason is that the long-term asset has a bigger value of short-term risk than the short-term asset ($\bar{\sigma}_{\mathcal{L}1t} > \bar{\sigma}_{\mathcal{S}1t}$), which leads to a higher long-term risk premium when there are no benchmarking motives.

$$\bar{\theta}_{1t}\bar{\sigma}_{\mathcal{S}1t} + \underbrace{\bar{\theta}_{2t}}_{=0}\bar{\sigma}_{\mathcal{S}2t} < \bar{\theta}_{1t}\bar{\sigma}_{\mathcal{L}1t} + \underbrace{\bar{\theta}_{2t}}_{=0}\bar{\sigma}_{\mathcal{L}2t}. \tag{32}$$

With benchmarking incentives, the price of the short-term risk drops ($\theta_{1t} < \bar{\theta}_{1t}$) while the price of the long-term risk increases ($\theta_{2t} > \bar{\theta}_{2t}$) due to the benchmark hedging portfolio, which suggests an even higher long-term asset risk premium if the values of risk are unchanged. However, equilibrium reveals that changes in the values of short-term and long-term risks are the underlying reason the short-term risk premium is higher than the long-term risk premium with benchmarking incentives. First, the value of the short-term risk drops for the long-term asset ($\bar{\sigma}_{\mathcal{L}1t} > \sigma_{\mathcal{L}1t}$) but remains unchanged for the short-term asset ($\bar{\sigma}_{\mathcal{S}1t} = \sigma_{\mathcal{S}1t}$), which depresses the long-term risk premium. Second, the value of the long-term risk is, in fact, negative for the long-term asset ($\sigma_{\mathcal{L}2t} < 0$), which depresses the long-term risk premium further due to the positive and higher long-term risk price ($\theta_{2t} > \bar{\theta}_{2t} > 0$). Eventually, the two forces lead to a negative sloping risk premium.

$$\theta_{1t}\underbrace{\sigma_{\mathcal{S}1t}}_{=\bar{\sigma}_{\mathcal{S}1t}} > \theta_{1t}\underbrace{\sigma_{\mathcal{L}1t}}_{<\bar{\sigma}_{\mathcal{L}1t}}, \qquad \theta_{2t}\underbrace{\sigma_{\mathcal{S}2t}}_{=0} > \theta_{2t}\underbrace{\sigma_{\mathcal{L}2t}}_{<0} \tag{33}$$

$$\mu_{\mathcal{S}t} = \theta_{1t}\sigma_{\mathcal{S}1t} + \theta_{2t}\sigma_{\mathcal{S}2t} > \theta_{1t}\sigma_{\mathcal{L}1t} + \theta_{2t}\sigma_{\mathcal{L}2t} = \mu_{\mathcal{L}t}. \tag{34}$$

The Proposition further shows that the total volatility (30) of the long-term asset ($S_{\mathcal{L}t}$) is, in fact, lower than the short-term asset ($S_{\mathcal{S}t}$) when risk aversion is not too high.

Significantly, the combined effect plays a crucial role in these two outcomes since both the risk-premium and volatility results reverse without the benchmarking incentives when only the individual effect exists.

When looking at the Sharpe ratio, it is not immediately apparent that the short-term asset Sharpe ratio is higher than the long-term asset Sharpe ratio because both the risk premium and the total volatility of the short-term asset are higher than the long-term counterparts. When dividing the risk premium by the total volatility, the Sharpe ratio of the short-term asset may turn out to be smaller. Even more so, whether the combined effect of heterogeneous investment horizons and relative performance objective drives this result or the heterogeneous investment horizons individually is not entirely clear. Proposition 4 concludes that the combined effect drives the risk-premium and volatility results; however, the individual effect drives the Sharpe ratio result.

Lastly, Proposition 4 reveals that the risk premium and the Sharpe ratio results do not depend on the model's parameters, while the total volatility result requires that investors' risk aversion parameter is not too high and roughly below two. This parameter restriction is in line with established asset pricing models such as the external habit formation of Campbell and Cochrane (1999) and others.

6.2 Equilibrium Dynamics

We conclude this section with an analysis of the equilibrium dynamics. When summing the three propagation channels affecting investors' risk exposures in (10) and (11), we find that overall, the value investor risk exposure correlates with the long-term asset, while the asset manager risk exposure correlates with the short-term asset.

$$\Sigma_t' \pi_t^{\mathcal{V}} = \theta_t + \begin{bmatrix} 0 \\ -(R-1)\,\sigma \end{bmatrix} = \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix}, \tag{35}$$

$$\Sigma_t' \pi_t^{\mathcal{I}} = \theta_t + \begin{bmatrix} -(R-1)\,\sigma \\ 0 \end{bmatrix} + \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix} = \begin{bmatrix} \sigma_{\mathcal{S}1t} \\ \sigma_{\mathcal{S}2t} \end{bmatrix}. \tag{36}$$

Suppose that the short-term asset gets positive fundamental cash-flow news ($D_{\mathcal{S}t} \uparrow$). The mean-variance portfolio states that the value investor and asset manager's wealth increases because they both purchase R units of the short-term asset for their mean-variance portfolio ($\phi_{\text{m.v.}} = [R, -1]$), and since the short-term asset price increases following fundamental short-term news, as (21) reveals.

The asset manager's benchmark hedge ($\phi_{\text{b}}^{\mathcal{I}} = [0, 1]$) correlates the risk exposure with the long-term asset return, and since the long-term asset price increases after non-fundamental news about the short-term payout ($\sigma_{\mathcal{L}1t} > 0$), the asset manager's wealth increases further due to the benchmark hedge, as (19) reveals. However, the asset manager's wealth decreases following news about the short-term payout due to the risk aversion hedge ($\phi_{\text{r.a}}^{\mathcal{I}} = [1-R, 0]$), which reverses some of the positive impacts of the mean-variance and the benchmark hedge but not all of it, as (19) reveals. The value investor sells R units of the short-term asset to hedge against risk aversion, as ($\phi^{\mathcal{V}}_{\text{r.a}} = [-R, 2]$) indicates. The risk aversion short-term position completely offsets the mean-variance short-term position, leaving the value investor with no exposure to the short-term asset.

Overall, the value investor and asset managers' wealth increase following news about the short-term payout. However, the asset manager's wealth increases more than the value investor's wealth ($\sigma_{\mathcal{S}1t} > \sigma_{\mathcal{L}1t}$), implying that the short-term asset increases more than the long-term asset following news about the short-term payout.

Alternatively, suppose that the long-term asset gets positive fundamental cash-flow news ($D_{\mathcal{L}t} \uparrow$). The mean-variance portfolio states that both the value and asset managers' wealth increase because they both sell 1 unit of the long-term asset for their mean-variance portfolio ($\phi_{\text{m.v.}} = [R, -1]$), and since the long-term asset price decreases after fundamental long-term news ($\sigma_{\mathcal{L}2t} < 0$), as (21) reveals.

The asset manager's wealth decreases due to the benchmark hedge portfolio ($\phi^{\mathcal{I}}_{\text{b}} = [0, 1]$) since the long-term benchmarked asset price decreases after fundamental long-term news ($\sigma_{\mathcal{L}2t} < 0$). These two effects cancel each other, and overall, the asset manager has no long-term cash-flow news exposure.

The value investor purchases two units of the long-term asset for the risk aversion hedging portfolio ($\phi^{\mathcal{V}}_{\text{r.a}} = [-R, 2]$). The negative (long position) exposure due to the risk aversion hedge is more pronounced than the mean-variance positive (short position) exposure, and overall, the value investor holds one unit of the long-term asset, inducing a negative exposure to fundamental long-term news.

Overall, the value investor's wealth decreases following long-term cash-flow news, while the asset manager's wealth remains unchanged ($0 = \sigma_{\mathcal{S}2t}, 0 > \sigma_{\mathcal{L}2t}$), implying that the short-term asset remains unaffected by long-term cash-flow news, while the long-term asset decreases following long-term cash-flow news.

More importantly, common positive cash-flow news to both the short-term and long-term assets ($D_{\mathcal{L}t} \uparrow, D_{\mathcal{S}t} \uparrow$) increases both prices, implying that non-fundamental news (news about the short-term payout) has a stronger effect on the long-term asset (a claim to long-term payout) than fundamental news (news about the long-term payout). The following Proposition explicitly presents the short- and long-term security prices in closed form.

Proposition 5 (Security Prices). *The short- and long-term asset prices are given by*

$$S_{\mathcal{L}t} = \bar{S}_{\mathcal{L}t} \left(D_{\mathcal{L}t}\right)^{\frac{R-1}{2}} \left(D_{\mathcal{S}t}\right)^{-\frac{R}{2}} A_{\mathcal{L}t}, \qquad S_{\mathcal{S}t} = \bar{S}_{\mathcal{S}t} A_{\mathcal{S}t}, \tag{37}$$

where $\bar{S}_{\mathcal{L}t}$ and $\bar{S}_{\mathcal{S}t}$ are the equilibrium prices without benchmarking incentives given by

$$\bar{S}_{\mathcal{L}t} = \left(\frac{1-\lambda}{\lambda}\right) \left(D_{\mathcal{L}t}\right)^{1-R} \left(D_{\mathcal{S}t}\right)^{R} \bar{A}_{\mathcal{L}t}, \qquad \bar{S}_{\mathcal{S}t} = D_{\mathcal{S}t} \bar{A}_{\mathcal{S}t}, \tag{38}$$

where $t \leq \frac{T}{2}$. The functions $\bar{A}_{\mathcal{L}t}$, $\bar{A}_{\mathcal{S}t}$, $A_{\mathcal{L}t}$, and $A_{\mathcal{S}t}$ are deterministic and positive functions of time, defined in (A.36), (A.37), (A.38), and (A.39), respectively.
The heterogeneous investment horizons individual effect (38) implies that

- *(i) the short-term asset price increases relative to the long-term asset price as the asset manager size increases (λ).*
- *(ii) the long-term asset price is convex and increases in the short-term news, indicating that short-term news is more critical in good short-term states.*
- *(iii) the long-term asset price is convex and decreases in the long-term news, indicating that long-term news is more critical in bad long-term states.*

The combined effect of heterogeneous investment horizons and relative performance objective (37) reverses item (ii) above and implies that

- *(iv) the long-term asset price becomes concave and increases in the short-term news, indicating that short-term news is more critical in bad states.*

Proposition 5 reveals several critical features of the individual effect of the heterogeneous investment horizons. Item (i) reveals that the long-term asset price drops as the asset manager size increases relative to the value investor ($\lambda \uparrow$), while the short-term asset price remains unchanged.

The convexity and positive price reaction of the long-term asset to short-term news (item ii) imply that news about the short-term payout is more critical for long-term prices in good short-term states; a one percent increase in the news about the short-term payout ($D_{\mathcal{S}t} \uparrow$) has a more substantial effect on the long-term price when prices are high than when they are low. In contrast, the convexity and negative price reaction of the long-term asset to long-term news (item iii) imply that long-term cash-flow news is more critical for long-term

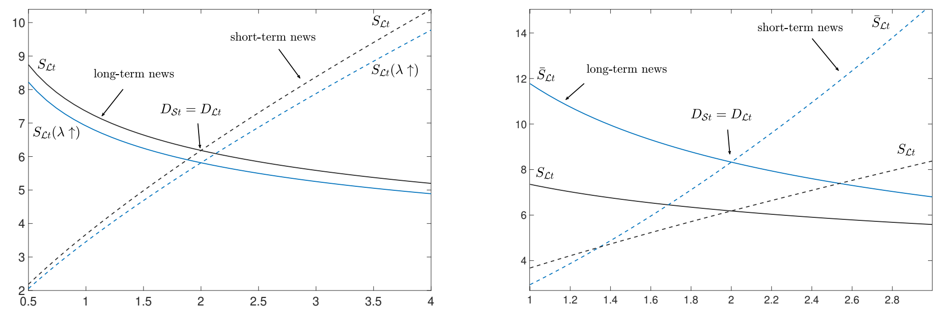

Figure 2: This figure plots asset prices as a function of cash-flow news. The left figure shows that long-term assets are convex in the news about the long-term payout and concave in the news about the short-term payout. Further, as λ increases from 0.2 to 0.21, the long-term asset price drops from the black to the blue lines. The solid line plots the long-term price as a function of the long-term news while short-term news remains fixed ($D_{\mathcal{S}t} = 2$). The dashed line plots the long-term price as a function of the short-term news while long-term news remains fixed ($D_{\mathcal{L}t} = 2$). The right figure plots the changes in asset prices due to the combined effect. The long-term asset shifts from concavity to convexity in the short-term news once we introduce the benchmark (the combined effect, $\bar{S}_{\mathcal{L}t} \to S_{\mathcal{L}t}$), as the black and blue dashed lines indicate. The rest of the parameters are $R = 1.5$, $\mu = 0.1$, $\sigma = 0.2$, $T = 3$, $t = 0.5$, $\lambda = 0.2$, $D_{\mathcal{L}0} = D_{\mathcal{S}0} = 1$.

prices in bad long-term states; a one percent increase in long-term cash-flow news ($D_{\mathcal{L}t} \uparrow$) has a more substantial effect on the long-term price when prices are high than when they are low. Overall, the effect of non-fundamental short-term news is more critical for the long-term price when the long-term price is high, while the effect of fundamental long-term news is more critical for the long-term price when the price is low.

Due to the interaction between heterogeneous investment horizons and relative performance objectives (the combined effect in item iv), short-term and long-term news become more critical for the long-term price in bad states. The concavity with respect to the short-term news implies that, following positive short-term news, the increase in the long-term price is more substantial for low levels of news about the short-term payout. Figure 2 illustrates these results.

Finally, we present the closed-form expression for the discount factor.

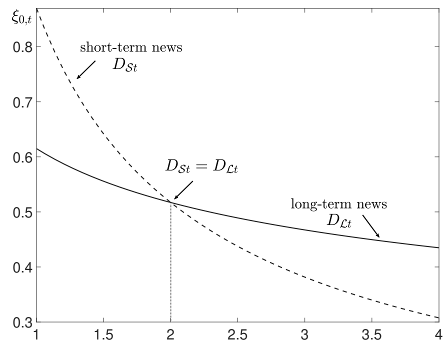

Figure 3: This figure plots the discount factor ($\xi_{0,t}$) as a function of news. In the solid line, we plot the discount factor as a function of the long-term news, while short-term news remains fixed ($D_{\mathcal{S}t} = 2$). In the dashed line, we plot the discount factor as a function of the short-term news, while long-term news remains fixed ($D_{\mathcal{L}t} = 2$). The rest of the parameters are as in Figure 2.

Proposition 6 (**Discount Factor**). *The equilibrium discount factor is given by*

$$\xi_{0,t} = \frac{(D_{\mathcal{L}t}/D_{\mathcal{L}0})^{\frac{1-R}{2}}}{(D_{\mathcal{S}t}/D_{\mathcal{S}0})^{\frac{R}{2}}} \frac{1}{\mathcal{E}_{0,t}\left(\frac{1-R}{2}\right)\mathcal{E}_{0,t}\left(-\frac{R}{2}\right)}, \quad t \leq T/2, \tag{39}$$

where $\mathcal{E}_{0,t}\left(\frac{1-R}{2}\right)\mathcal{E}_{0,t}\left(-\frac{R}{2}\right)$ *are deterministic functions (A.3), and (A.4).*

The discount factor is inversely related to both the short- and long-term cash-flow news. A feature similar to a traditional asset pricing model: an asset that pays off in states with a low cash flow gets a high value. Notice that the long-term asset price pays off when fundamental long-term cash-flow news is low; indeed, its price is inflated in this bad state, as Figure 2 illustrates.

Interestingly, the discount factor is more sensitive to short-term news than long-term news, implying that news about the short-term cash flow has a higher impact on prices. The differential sensitivity of the discount factor is another manifestation of the equilibrium mechanism inducing better risk-return trade-off for assets that pay off in the short-term asset ($\theta_{1t} > \theta_{2t}$). Figure 3 illustrates this idea.

7 Empirical Evidence

So far, we have analyzed the equilibrium mechanism and showed that the equilibrium volatility, risk premium, and Sharpe ratio are downward sloping, supporting the recent empirical evidence. This section provides novel empirical evidence to further substantiate the heterogeneous investment horizons mechanism further and support other model predictions.

Our model predicts that the asset manager buys the benchmark assets due to the benchmark hedging and sells short the short-term asset due to the risk aversion hedging. The mean-variance portfolio is a two-leg portfolio that offsets the hedging portfolios, as Proposition 3 reveals.

Empirical evidence suggests that mutual fund asset managers rebalance their hedging portfolios much more frequently than trading in and out of investment opportunities. Even more so, when mutual fund asset managers invest, these investments typically exhibit momentum. As a result, opportunistic investing is much less dynamic than rebalancing the hedging portfolios. For instance, Chan et al. (2002) document that few funds take positions away from their benchmark (despite having substantial turnover), and when they do, they favor past winners (indicative of slow-moving momentum strategies).

Due to these reasons, frequently rebalancing the benchmark and risk aversion hedging portfolios show up statically in a regression analysis with *simulated data*, while infrequently rebalancing the mean-variance portfolio does not show up. The following empirical evidence is consistent with the regression analysis of simulated data.

We estimate the asset manager's wealth using the total net assets (TNA) of mutual actively managed funds, and we estimate the short-term and long-term asset prices using van Binsbergen et al. (2012) dividend prices data. They employ the European put-call parity to extract the prices of dividends of the S&P 500 index between the quoted time and the options' maturity time. For instance, the dividend price in January 1996 for dividends paid out until June 1997 was $20. They derive a time series of monthly dividend prices for 6-month, 12-month, 18-month, and 24-month maturities.

Giglio et al. (2021) have shown that the term structure of risk premium slopes downward at the short end and slightly upward at the long end, so we expect the mechanism to show up when considering dividend strips in short to medium terms. Perhaps more importantly, the realized dividend price data is highly volatile. The longer the dividend price maturity, the more volatile its price realization, so explaining a 1.5 to 2-year ahead dividend price variations in such a volatile environment is highly challenging.

Accordingly, we define the short-term asset price as the 6-month dividend price and the long-term asset price as the difference between the 24-month and 18-month dividend prices. In other words, our short-term asset is a dividend strip paying dividends within zero to six months, while our long-term asset is a dividend strip paying dividends within one and a half and two years.[10] Then, we conduct an OLS regression of the total net assets (TNA) on the short-term and long-term dividend strip prices,

$$\text{TNA}_t = \alpha + \beta_s \times (S_{\mathcal{S}t}/\text{S\&P } 500_t) + \beta_l \times (S_{\mathcal{L}t}/\text{S\&P } 500_t) + \epsilon_t. \tag{40}$$

Based on the predictions from simulated data with infrequent mean-variance portfolio rebalance, we hypothesize that an increase in the short-term asset price decreases the asset manager's wealth due to its risk aversion hedge ($\beta_S < 0$), while an increase in the long-term asset price increases the asset manager's wealth ($\beta_l > 0$) due to its benchmark hedge.

We focus our regression analysis on four different groups of funds. The first is the broadest group, consisting of all the mutual actively managed funds with a Morningstar category of US Equity. These funds invest in all the possible styles and market caps. The remaining three groups consist of mutual actively managed funds within the Russell 3000 family of prospectus benchmarks: (i) Russell 3000 ; (ii) Russell 3000 Growth; (iii) and the Russell 3000 Value.

Table 1 reports that the asset manager loads negatively on the 0-to-6 months (short-term) dividend strip and positively on the 18-to-24 months (long-term) dividend strip. The result is robust to different specifications of asset managers' benchmarks, as the four columns in Table 1 show. For instance, the first column in Table 1 states that one standard deviation increase in the 0-to-6 months dividend strip decreases the US Equity TNA by 0.72 standard deviations, while one standard deviation increase in the 18-to-24 months dividend strip increases the US Equity TNA by 0.93 standard deviations.

The empirical analysis in this section abstracts away from essential features of the asset management industry. In particular, hedging against benchmark fluctuations means a differ-

[10] The TNA monthly data is from Morningstar and our sample period runs from 31/12/1998 until 30/10/2009.

ent composition of long- and short-term assets for asset managers with different benchmark mandates. Still, the equilibrium predictions show up in various specifications of well-known benchmarks, as Figure 1 indicates. Lastly, one should not confuse an asset manager with a value index mandate and a value investor without a performance benchmark mandate. As long as the manager has a performance benchmark objective, the manager behaves as the model describes.

Proposition 5 reveals that the short-term asset price increases relative to the long-term asset price as the asset manager size increases relative to the value investor. To support this hypothesis, we carry out the following OLS regression

$$\left(S_{\mathcal{S}t} - S_{\mathcal{L}t}\right)/\text{S\&P } 500_t = \alpha + \beta \times \lambda_t + \epsilon_t \tag{41}$$

and hypothesize that as the asset manager size increases relative to the value investor size ($\lambda_t \uparrow$), the short-term asset price increases relative to the long-term asset price ($\beta > 0$). We approximate the equity asset market fluctuations by fluctuations in the S&P 500 index and changes to the asset manager's size by fluctuations in the total TNA of actively managed mutual funds with a particular benchmark. Using these approximations, we measure λ_t by comparing the cumulative log return in the asset manager's TNA relative to the cumulative log return in the S&P 500 index. For instance, if the asset manager's TNA increases by 2% while the S&P 500 index increases by 5% over the same period, the asset manager's size and λ shrink. Intuitively, when the equity asset market grows faster than the asset manager size, it is because the value investor size increases faster than the asset manager size, indicating a shrinking λ.

Table 1: TNA on long-term and short-term dividend strips' prices

Dependent Variable: TNA$_t$	US Equity	Russell 3000	Russell 3000 Growth	Russell 3000 Value
constant	4.3071(***)	-0.06457	5.7401(***)	2.8293(***)
	(0.48127)	(0.372213)	(1.34844)	(0.59541)
$S_{\mathcal{L}t}$/S&P 500	0.9268(***)	0.81502(***)	0.3685	1.0897(***)
	(0.22124)	(0.199148)	(0.26738)	(0.28014)
$S_{\mathcal{S}t}$/S&P 500	-0.7180(***)	-0.10453	-0.6679(***)	-0.6940(**)
	(0.28694)	(0.253569)	(0.20143)	(0.26584)
R^2	0.1871	0.5232	0.1427	0.149

Notes: The table presents the OLS regression of dollar TNA on the long-term and short-term dividend strip prices. The first column consists of all the actively managed mutual funds with a US Equity Morningstar category. The remaining three columns reflect the actively managed funds with the indicated prospectus benchmark. Newey-West standard errors in parenthesis. To avoid non-stationarity issues, we divide the S&P 500 index dividend strip prices by the S&P 500 index level, similar to van Binsbergen et al. (2012). Variables are standardized. (***),(**),(*) corresponds to 1%, 5%,10% confidence levels, respectively.

Table 2 reports that as the asset manager size increases, the short-term asset price increases relative to the long-term asset price. The result is robust whether we measure asset manager size with the cumulative log return of (i) the TNA across all funds with the US Equity Morningstar category, (ii) the TNA of funds with the Russell 3000 prospectus benchmark, or (iii) the TNA of funds with the Russell 3000 Growth prospectus benchmark. For instance, the third column in Table 2 states that a 1% increase in the Russell 3000 Growth funds size increases the difference between the short- and long-term dividend strip prices by 1.64 standard deviations. The result attenuates when measuring the asset manager size with the Russell 3000 Value prospectus benchmark TNA, but also, the R-squared vanishes.

Table 2: Short-term minus long-term dividend strips' prices on λ.
Dependent Variable: $(S_{\mathcal{S}t} - S_{\mathcal{L}t})$/S&P 500

constant	-0.8718	-0.1671	-1.7472(*)	1.0696(**)
	(1.13946)	(0.3067)	(1.00677)	(0.50250)
λ (US Equity)	1.2443	—	—	—
	(0.925)	—	—	—
λ (Russell 3000)	—	0.5031(*)	—	—
	—	(0.269)	—	—
λ (Russell 3000 Growth)	—	—	1.6399(**)	—
	—	—	(0.68835)	—
λ (Russell 3000 Value)	—	—	—	-0.3215
	—	—	—	(0.42552)
R^2	0.03281	0.063	0.09824	0.004122

Notes: The table presents the OLS regression of the difference between the short-term and the long-term dividend strip prices on the size of the asset manager. We measure the relative size by comparing the cumulative log return in the asset manager's TNA relative to the cumulative log return in the S&P 500 index. The different specifications refer to the US actively managed mutual funds with various mandates are similar to Table 1. To avoid non-stationarity issues, we divide the dividend strip prices by the S&P 500 index level, similar to van Binsbergen et al. (2012), and to adjust for potential heteroskedasticity, we use robust standard errors in parenthesis. The dependent variable is standardized. (***),(**),(*) corresponds to 1%, 5%,10% confidence levels, respectively.

We have presented two novel and straightforward pieces of evidence showing that frictions tied to the mutual fund asset management industry can account, at least partially, for the empirical evidence originally discovered by van Binsbergen et al. (2012).

8 Extensions and Further Discussion

This section extends the primary analysis and allows for a benchmark that composes both the short-term and long-term assets. By doing so, the model introduces another equilibrium channel from short-term benchmarking incentives. Still, the additional channel corroborates the findings in the primary analysis and supports the main empirical findings.

Specifically, we consider a benchmark, a geometric weighted average of the short- and long-term assets. The short-term asset weight is α, and the long-term asset weight is β, $0 \leq \alpha, \beta \leq 1$; the rest of the economic setup remains unchanged. other ceteris paribus. Accordingly, extending the risk exposures in Lemma (1), we find that the asset manager's benchmark hedging portfolio consists of α units of the long-term asset and β units of the short-term asset.

$$\Sigma_t' \pi_t^{\mathcal{I}} = \theta_t + \begin{bmatrix} -(R-1)\sigma \\ 0 \end{bmatrix} + \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix} \alpha + \begin{bmatrix} \sigma_{\mathcal{S}1t} \\ \sigma_{\mathcal{S}2t} \end{bmatrix} \beta. \tag{42}$$

The value investor risk exposure remains unchanged. Unlike the primary analysis, in this case, the asset manager desires to get exposure to the short-term asset due to benchmarking incentives. Equilibrium achieves market clearing by negating the benchmarking hedging desires.

Proposition 7 (**Extension: Volatility, Market Prices of Risk, and Portfolios**). *The short-term and long-term assets' risk exposures are given by*

$$\begin{bmatrix} \sigma_{\mathcal{S}1t} \\ \sigma_{\mathcal{S}2t} \end{bmatrix} = \begin{bmatrix} \sigma \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix} = \sigma \begin{bmatrix} \frac{R-\beta}{1+\alpha} \\ \frac{1-R}{1+\alpha} \end{bmatrix}. \tag{43}$$

The short- and long-term market prices of risk are given by

$$\theta_{1t} = \frac{R-\beta}{1+\alpha}\sigma, \qquad \theta_{2t} = \frac{\alpha(R-1)}{1+\alpha}\sigma, \qquad \theta_{1t} > \theta_{2t} > 0. \tag{44}$$

The effects introduced in the original analysis remains unchanged. The asset manager benchmark hedging portfolio and long-term risk aversion hedging portfolio are given by

$$\phi_b^{\mathcal{I}} = \alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \phi_{r.a.}^{\mathcal{I}} = \begin{bmatrix} 1-R \\ 0 \end{bmatrix}, \tag{45}$$

while the value investor short-term risk aversion hedging portfolio is given by

$$\phi_{r.a.}^{\mathcal{V}} = \begin{bmatrix} -(R-\beta) \\ 1+\alpha \end{bmatrix}. \tag{46}$$

The mean-variance portfolio is given by

$$\phi_{m.v.} = \begin{bmatrix} R - \beta \\ -\alpha \end{bmatrix} \tag{47}$$

and $\phi_b^{\mathcal{I}} + \phi_{r.a.}^{\mathcal{I}} + \phi_{m.v.} = \pi_t^{\mathcal{I}} = \bar{\pi}_t^{\mathcal{I}}$ *and* $\phi_{r.a.}^{\mathcal{V}} + \phi_{m.v.} = \pi_t^{\mathcal{V}} = \bar{\pi}_t^{\mathcal{V}}$*.*

By plugging the market prices of risk and the benchmark exposures, we obtain

$$\text{Combined Effect:} \quad \begin{cases} \Sigma_t' \pi_t^{\mathcal{V}} / \sigma = \begin{bmatrix} \frac{R-\beta}{1+\alpha} \\ \frac{\alpha(R-1)}{1+\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ -(R-1) \end{bmatrix} = \begin{bmatrix} \frac{R-\beta}{1+\alpha} \\ -\frac{R-1}{1+\alpha} \end{bmatrix} \Rightarrow \begin{bmatrix} \pi_{\mathcal{S}t}^{\mathcal{V}} = 0 \\ \pi_{\mathcal{L}t}^{\mathcal{V}} = 1 \end{bmatrix}, \\ \Sigma_t' \pi_t^{\mathcal{I}} / \sigma = \begin{bmatrix} \frac{R-\beta}{1+\alpha} \\ \frac{\alpha(R-1)}{1+\alpha} \end{bmatrix} + \begin{bmatrix} -(R-1) \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} \frac{R-\beta}{1+\alpha} \\ -\frac{R-1}{1+\alpha} \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \pi_{\mathcal{S}t}^{\mathcal{I}} = 1 \\ \pi_{\mathcal{L}t}^{\mathcal{I}} = 0 \end{bmatrix}. \end{cases} \tag{48}$$

Similar to the primary analysis, benchmarking induces extra — unrelated to prices — demand to hold the benchmark asset. This excess demand pressure is unsustainable in the no-benchmark pricing environment (individual effect).

We break the equilibrium pricing of the combined effect into two separate effects: (i) the partial combined effect of the price reaction ($\hat{\theta}_{1t}$, $\hat{\theta}_{2t}$), given that price dynamics are unchanged ($\bar{\sigma}_{\mathcal{L}t}, \bar{\sigma}_{\mathcal{S}t}$); (ii) the complete combined effect of both price reaction and the price dynamics ($\sigma_{\mathcal{L}t}, \sigma_{\mathcal{S}t}$). We pin down the price reaction in the partial combined effect ($\hat{\theta}_{1t}$, $\hat{\theta}_{2t}$) by imposing the same equilibrium restriction that prices counteract the benchmarking demand and restore equilibrium by negating the demand to hold the benchmark.

$$\text{Partial Combined Effect (i):} \quad \begin{cases} \Sigma_t' \pi_t^{\mathcal{V}} / \sigma = \begin{bmatrix} \hat{\theta}_1 = R(1-\alpha) - \beta \\ \hat{\theta}_2 = \alpha(R-1) \end{bmatrix} + \begin{bmatrix} 0 \\ -(R-1) \end{bmatrix} = \begin{bmatrix} R(1-\alpha) - \beta \\ -(R-1)(1-\alpha) \end{bmatrix} \Rightarrow \begin{bmatrix} \hat{\pi}_{\mathcal{S}t}^{\mathcal{V}} = -\beta \\ \hat{\pi}_{\mathcal{L}t}^{\mathcal{V}} = 1-\alpha \end{bmatrix}, \\ \Sigma_t' \pi_t^{\mathcal{I}} / \sigma = \begin{bmatrix} \hat{\theta}_1 = R(1-\alpha) - \beta \\ \hat{\theta}_2 = \alpha(R-1) \end{bmatrix} + \begin{bmatrix} -(R-1) \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} R \\ -(R-1) \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \hat{\pi}_{\mathcal{S}t}^{\mathcal{I}} = 1 \\ \hat{\pi}_{\mathcal{L}t}^{\mathcal{I}} = 0 \end{bmatrix}. \end{cases} \tag{49}$$

The partial effect induces the short-term and long-term assets to be more expensive:

$$\begin{aligned}
&\hat{\theta}_{1t} < \bar{\theta}_{1t} \quad \text{and} \quad \hat{\theta}_{2t} > \bar{\theta}_{2t} \\
&\qquad \Rightarrow \mathbb{C}\text{ov}(d\hat{\xi}_t, d\bar{S}_{\mathcal{L}t}) < \mathbb{C}\text{ov}(d\bar{\xi}_t, d\bar{S}_{\mathcal{L}t}) \Rightarrow \bar{S}_{\mathcal{L}t}(\hat{\theta}) > \bar{S}_{\mathcal{L}t}, \\
&\qquad \Rightarrow \mathbb{C}\text{ov}(d\hat{\xi}_t, d\bar{S}_{\mathcal{S}t}) < \mathbb{C}\text{ov}(d\bar{\xi}_t, d\bar{S}_{\mathcal{S}t}) \Rightarrow \bar{S}_{\mathcal{S}t}(\hat{\theta}) > \bar{S}_{\mathcal{S}t},
\end{aligned} \tag{50}$$

where $\bar{S}_{\mathcal{S}t}(\hat{\theta})$ and $\bar{S}_{\mathcal{L}t}(\hat{\theta})$ are the short-term and long-term asset price considering the partial effect (i), given that price dynamics are unchanged (49). However, the partial price effect alone cannot restore both optimal portfolios and market clearing conditions.

The partial price reaction renders the short-term asset too expensive, so much so that the value investor takes a short position on that asset in that pricing environment. The long-term asset is also not attractive, inducing the value investor to have a long position of $(1-\alpha)$ which is less than one unit. In the general benchmark case as well, the partial effect cannot clear the financial markets, as (49) reveals. To make the long-term asset more attractive, equilibrium decreases the long-term asset risk exposures ($|\sigma_{\mathcal{L}1t}| < |\bar{\sigma}_{\mathcal{L}1t}|,\ |\sigma_{\mathcal{L}2t}| < |\bar{\sigma}_{\mathcal{L}2t}|$) to the point where the value investor optimally holds one unit of the long-term asset and no position in the short-term asset while still negating the benchmarking demand of the asset manager, as (48) reveals. In doing so, the short-term asset has a more pronounced risk exposure ($\sigma_{\mathcal{S}1t} > \sigma_{\mathcal{L}1t}$) and, eventually, is more volatile than the long-term asset.

Equilibrium compensates the asset manager for taking on that heightened short-term risk (relative to the long-term asset) by having a higher short-term risk premium, inducing a downward-sloping risk premium ($\mu_{\mathcal{S}t} > \mu_{\mathcal{L}t}$) and volatility ($\sqrt{\sigma_{\mathcal{S}1t}^2 + \sigma_{\mathcal{S}2t}^2} > \sqrt{\sigma_{\mathcal{L}1t}^2 + \sigma_{\mathcal{L}2t}^2}$).

Noticeably, equilibrium is restored by depressing the long-term risk exposures, while the short-term risk exposures remain unaffected, regardless of the benchmark composition. This outcome runs against the homogenous investment horizon with benchmarking economy in which benchmarking increases the fundamental volatility. Moreover, the market price of the short-term risk drops due to the short-term benchmark ($\beta > 0$), similar to the homogenous investment horizon with benchmarking economy. However, this section shows that the mechanism driving this outcome is different.

To see why the short-term risk premium is higher than the long-term risk premium, notice that $\sigma_{\mathcal{L}1t} < \sigma_{\mathcal{S}1t}$ if and only if $R < 1 + \alpha + \beta$, equaling 2 when $\alpha + \beta = 1$ (the geometric average case). Following the primary analysis and representing the risk premiums as their values of risk times prices of risk (33) leads to

$$\theta_{1t}\sigma_{\mathcal{S}1t} > \theta_{1t}\sigma_{\mathcal{L}1t}, \qquad \theta_{2t}\sigma_{\mathcal{S}2t} > \theta_{2t}\sigma_{\mathcal{L}2t} \qquad \Rightarrow \qquad \mu_{\mathcal{S}t} > \mu_{\mathcal{L}t}. \tag{51}$$

The following proposition ensures that the short-term risk premium, short-term volatility, and short-term Sharpe ratio are higher than their long-term counterparts in the general geometric benchmark case.

Proposition 8 (**Extension: Risk Premium, Total Volatility, and Sharpe Ratio**).

(i) **Risk Premium:** *The short-term asset risk premium is higher than the long-term asset risk premium if and only if investors' risk aversion is sufficiently low, such that*

$$\mu_{\mathcal{S}t} > \mu_{\mathcal{L}t} \iff R < 2. \tag{52}$$

The effects introduced in the original analysis remains unchanged.

(ii) **Total Volatility:** *The short-term asset volatility is higher than the long-term asset volatility if and only if investors' risk aversion is sufficiently low. The threshold $\bar{R}$ depends on the composition of the benchmark, but the condtion $\bar{R} < 2$ remains.*

(iii) **Sharpe Ratio:** *The short-term asset Sharpe ratio is higher than the long-term asset Sharpe ratio.*

One may consider an additional investor with a passive inelastic investment mandate. Inelastic passive demand to hold the benchmark reduces the remaining supply of the benchmarked assets to other investors. Consequently, the passive investment mandate translates to a typical supply shift from the value investor and asset manager's perspective. Such a supply shift increases the benchmarked asset prices and decreases their risk premiums. As long as the benchmark composition of assets tilts towards the long-term asset, the passive index investment will not affect the equilibrium implications. Notice that passive investment with exchange-traded funds (ETFs) is small in the equity market; the average proportion of the US stock market held by all equity ETFs is roughly 6%; therefore, their effects are inconsequential.

9 Conclusion

Empirical evidence points to a critical difference between the asset managers' short-term performance window and value investors' long-term investment mandates. The heterogeneity in their investment horizons combined with the relative performance objectives of asset managers has the potential explanatory power of asset pricing anomalies because mutual funds' assets under management affected by these frictions are enormous. Despite its economic significance, the asset pricing effects of the heterogeneity in investment horizons have yet to be explored. Campbell and Viceira (2002), and more recently, Cochrane (2022), emphasize the importance of these effects to portfolio theory and equilibrium prices.

The data shows that mutual fund asset managers' minimum performance windows are excessively short across all the primary investment mandates. In particular, 83.8% of mutual fund managers have a minimum rolling performance window of one year or less, and the distribution median is 7 to 8 months, assuming uniform contract starting dates. The short performance windows are especially critical since they strongly incentivize mutual fund asset managers to become short-term investors because they may get fired or demoted if they underperform their benchmark in the short term. Consequently, those mutual fund asset managers have little interest in investment horizons beyond their minimum performance window.

Our main contribution shows that these two mutual fund asset management frictions generate a downward-sloping risk premium, volatility, and Sharpe ratio, thereby providing an original theoretical foundation for recent empirical evidence. This paper is the first to show that frictions tied to the mutual fund asset management industry can account, at least partially, for these empirical regularities.

Our theory predicts that the risk premium slopes down less as benchmarking incentives become less severe and eventually slopes upward without benchmarking incentives. Accordingly, our theory aligns with the conditional empirical evidence since the Active Share in expansions is significantly larger than in recessions, indicating that the downward-sloping risk premium is more severe in recessions due to stronger benchmarking motives.

Our continuous-time setup admits precise closed-form expressions, allowing us to analyze cash-flow shocks' effects on prices. In particular, we find that news about the short-term payout affects long-term asset prices more than news about the long-term payout. The closed-form precise equilibrium characterization allows us to introduce novel empirical evidence to support other predictions. Our model predicts that the asset manager buys the long-term asset due to the benchmark hedging and sells short the short-term asset due to the risk aversion hedging. Further, as the size of the asset manager increases, the short-term asset price increases relative to the long-term asset price. Our empirical analysis verifies those predictions.

Methodologically, our setup introduces a benchmark that depends on endogenous prices rather than exogenous dividend news as currently assumed in the literature.

There are several avenues for future research. Theoretically, it would be interesting to extend the model to allow for variations in the risk premium and observe if the mechanism addresses other well-known empirical phenomena, such as momentum and reversal. Empirically, data on short- and long-term dividend strips is scarce and, therefore, explored by relatively few papers. More work remains to alleviate the data shortcomings.

A Proofs

In this section we show how to derive the equilibrium quantities. We introduce two generalizations to the short-term asset manager utility function in the main setup.

$$E\left[\left(S_{\mathcal{S}T/N}\right)^{\beta}\left(S_{\mathcal{L}T/N}\right)^{\alpha}\frac{\left(W^{\mathcal{I}}_{T/N}\right)^{1-R}}{1-R}\right]. \tag{A.1}$$

First, the short-term period is T/N for a general $N \geq 2$, and second, the parameters α and β respectively control the importance of the long-term and short-term assets to asset manager's incentives. We assume that $\alpha \geq 0$ and $\beta \geq 0$. When $\alpha = \beta = 0$, there is no benchmark and the equilibrium coincide with a traditional two consumption dates setup. When $\alpha = 1$ and $\beta = 0$, we obtain the main setup, and when $\beta > 0$, the short-term asset becomes important for benchmarking. We do not restrict $\alpha, \beta \leq 1$, and analyze cases in which the benchmark is extremely important for incentive purposes. Throughout the analysis, the long-term value investor utility function remains identical to the main setup (5). The proofs' order reflects the most convinient way to solve for equilibrium and does not reflect the order in which the propositons appear in the text.

Before we begin with the formal proofs, we solve the following expected values.

$$\mathcal{E}_{t_1,t_2}\left(1-R\right) \equiv E_{t_1}\left[\left(D_{\mathcal{L}t_2}\right)^{1-R}\right] / \left(D_{\mathcal{L}t_1}\right)^{1-R} = e^{(1-R)\left(\mu-\frac{1}{2}\sigma^2 R\right)(t_2-t_1)}, \tag{A.2}$$

$$\mathcal{E}_{t_1,t_2}\left(\frac{(1-R)\,\alpha}{1+\alpha}\right) \equiv E_{t_1}\left[\left(D_{\mathcal{L}t_2}\right)^{\frac{(1-R)\alpha}{1+\alpha}}\right] / \left(D_{\mathcal{L}t_1}\right)^{\frac{(1-R)\alpha}{1+\alpha}} = e^{\frac{(1-R)\alpha}{1+\alpha}\left(\mu-\frac{\sigma^2}{2}\frac{1+\alpha R}{1+\alpha}\right)(t_2-t_1)}, \tag{A.3}$$

$$\mathcal{E}_{t_1,t_2}\left(\frac{\beta-R}{1+\alpha}\right) \equiv E_{t_1}\left[\left(D_{\mathcal{L}t_2}\right)^{\frac{\beta-R}{1+\alpha}}\right] / \left(D_{\mathcal{L}t_1}\right)^{\frac{\beta-R}{1+\alpha}} = e^{\frac{\beta-R}{1+\alpha}\left(\mu-\frac{\sigma^2}{2}\frac{1+\alpha-\beta+R}{1+\alpha}\right)(t_2-t_1)}, \tag{A.4}$$

$$\mathcal{E}_{t_1,t_2}\left(1+\frac{\beta-R}{1+\alpha}\right) \equiv E_{t_1}\left[\left(D_{\mathcal{L}t_2}\right)^{1+\frac{\beta-R}{1+\alpha}}\right] / \left(D_{\mathcal{L}t_1}\right)^{1+\frac{\beta-R}{1+\alpha}} = e^{\left(1+\frac{\beta-R}{1+\alpha}\right)\left(\mu-\frac{\sigma^2}{2}\frac{R-\beta}{1+\alpha}\right)(t_2-t_1)}. \tag{A.5}$$

We reference to these expected values throughout the proofs.

Proof of Proposition 6 (Discount Factor). The security market is dynamically complete. As such, there exists a unique state price density process, ξ, and the no arbitrage relations

$$\xi_t S_{\mathcal{L}t} = E_t\left[\xi_T D_{\mathcal{L}T}\right], \quad t \in [0,T], \tag{A.6}$$

$$\xi_t S_{\mathcal{S}t} = E_t\left[\xi_{T/N} D_{\mathcal{S}T/N}\right], \quad t \in [0,T/N], \tag{A.7}$$

are always satisfied. In our setup, we set $r = 0$ for $t \leq T/N$, and thus the state price density evolves according to

$$d\xi_t = -\xi_t\left(\theta_{1t} dZ_{1t} + \theta_{2t} dZ_{2t}\right), \quad t \leq T/N. \tag{A.8}$$

The processes θ_{1t} and θ_{2t} are the market prices of risk of the short-term and long-term shocks, respectively. Restating the dynamic budget constraints as

$$\xi_t W_t^{\mathcal{V}} = E_t\left[\xi_T W_T^{\mathcal{V}}\right], \quad t \in [0,T], \tag{A.9}$$

$$\xi_t W_t^{\mathcal{I}} = E_t\left[\xi_{T/N} W_{T/N}^{\mathcal{I}}\right], \quad t \in [0,T/N], \tag{A.10}$$

and maximizing each investor's objective function (5) and (A.1), subject to (A.9) and (A.10) at time $t = 0$, respectively, we obtain the first order conditions

$$\left(\frac{S_{\mathcal{S}T/N}^{\beta} S_{\mathcal{L}T/N}^{\alpha}}{y^{\mathcal{I}}\xi_{T/N}}\right)^{\frac{1}{R}} = W_{T/N}^{\mathcal{I}}, \tag{A.11}$$

$$\left(y^{\mathcal{V}}\xi_T\right)^{-\frac{1}{R}} = W_T^{\mathcal{V}}, \tag{A.12}$$

where $y^{\mathcal{S}}$ and $y^{\mathcal{L}}$ are the corresponding Lagrange multipliers. By utilizing the budget constraints at $t = 0$, (A.9), and (A.10), we find that the Lagrange multipliers satisfy

$$\left(\frac{1}{y^{\mathcal{I}}}\right)^{\frac{1}{R}} = \frac{\xi_0 \lambda S_m}{E\left[\left(S^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}} \left(\xi_{T/N}\right)^{1-\frac{1}{R}}\right]}, \tag{A.13}$$

$$\left(\frac{1}{y^{\mathcal{V}}}\right)^{\frac{1}{R}} = \frac{\xi_0 \left(1-\lambda\right) S_m}{E\left[\left(\xi_T\right)^{1-\frac{1}{R}}\right]}. \tag{A.14}$$

We obtain the state price density at the long- and short-term by utilizing the market clearing conditions and observing that $D_{\mathcal{S}T/N} = S_{\mathcal{S}T/N}$ due to the no arbitrage condition. By doing so, we obtain

$$\xi_{T/N} = \left(\frac{\xi_0 \lambda S_m}{D_{\mathcal{S}T/N}}\right)^R \frac{D^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}}{E\left[\left(D^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}} \left(\xi_{T/N}\right)^{1-\frac{1}{R}}\right]^R}, \tag{A.15}$$

$$\xi_T = \left(\frac{\xi_0 \left(1-\lambda\right) S_m}{D_{\mathcal{L}T} E\left[\left(\xi_T\right)^{1-\frac{1}{R}}\right]}\right)^R. \tag{A.16}$$

Next, to pin down the benchmark, we derive the long-term security price at the short-term date, $S_{\mathcal{L}T/N}$. We do so by utilizing the no-arbitrage condition, given in (A.6), at $t = T/N$.

$$\xi_{T/N} S_{\mathcal{L}T/N} = E_{T/N}\left[\xi_T D_{\mathcal{L}T}\right]. \tag{A.17}$$

Plugging $\xi_{T/N}$ and ξ_T from (A.15) and (A.16) leads to an equation for $S_{\mathcal{L}T/N}$ given by

$$\frac{S^{1+\alpha}_{\mathcal{L}T/N}}{\left(D_{\mathcal{S}T/N}\right)^{R-\beta}} \left(\frac{\lambda S_m \xi_0}{E\left[\left(D^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}} \left(\xi_{T/N}\right)^{1-\frac{1}{R}}\right]}\right)^R = \left(D_{\mathcal{L}T/N}\right)^{1-R} \mathcal{E}_{T/N,T} \left(1-R\right) \left(\frac{\left(1-\lambda\right) S_m \xi_0}{E\left[\left(\xi_T\right)^{1-\frac{1}{R}}\right]}\right)^R. \tag{A.18}$$

To ease notation we define $\bar{\xi}_{\mathcal{S}}$ and $\bar{\xi}_{\mathcal{L}}$ by

$$\bar{\xi}_{\mathcal{S}} \equiv \xi_0 \left(\frac{\lambda S_m}{E\left[\left(D^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}} \left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\right]}\right)^R, \quad \bar{\xi}_{\mathcal{L}} \equiv \xi_0 \left(\frac{\left(1-\lambda\right) S_m}{E\left[\left(\xi_{0,T}\right)^{1-\frac{1}{R}}\right]}\right)^R, \tag{A.19}$$

where $\xi_{s,t} \equiv \frac{\xi_t}{\xi_s}$. We obtain that $S_{\mathcal{L}T/N}$ is given by

$$S_{\mathcal{L}T/N} = \left(D_{\mathcal{L}T/N}\right)^{\frac{1-R}{1+\alpha}} \left(D_{\mathcal{S}T/N}\right)^{\frac{R-\beta}{1+\alpha}} \left(\mathcal{E}_{T/N,T} \left(1-R\right) \frac{\bar{\xi}_{\mathcal{L}}}{\bar{\xi}_{\mathcal{S}}}\right)^{\frac{1}{1+\alpha}}, \tag{A.20}$$

where $\mathcal{E}_{T/N,T}\left(1-R\right)$ is given in (A.2), evaluated at $(t_1,t_2)=(\frac{T}{N},T)$. Plugging $S_{\mathcal{L}T/N}$ back to $\xi_{T/N}$ we find

$$\xi_{T/N}=\left(D_{\mathcal{L}T/N}\right)^{\frac{(1-R)\alpha}{1+\alpha}}\left(D_{\mathcal{S}T/N}\right)^{\frac{\beta-R}{1+\alpha}}\left(\mathcal{E}_{T/N,T}\left(1-R\right)\bar{\xi}_{\mathcal{L}}\right)^{\frac{\alpha}{1+\alpha}}\left(\bar{\xi}_{\mathcal{S}}\right)^{\frac{1}{1+\alpha}}. \tag{A.21}$$

Further, because $\xi_{T/N}$ is a martingale it is given by the relationship $\xi_t=E_t\left[\xi_{T/N}\right]$, which leads to

$$\xi_t=\left(D_{\mathcal{L}t}\right)^{\frac{(1-R)\alpha}{1+\alpha}}\left(D_{\mathcal{S}t}\right)^{\frac{\beta-R}{1+\alpha}}\left(\mathcal{E}_{T/N,T}\left(1-R\right)\bar{\xi}_{\mathcal{L}}\right)^{\frac{\alpha}{1+\alpha}}\left(\bar{\xi}_{\mathcal{S}}\right)^{\frac{1}{1+\alpha}}\mathcal{E}_{t,T/N}\left(\frac{(1-R)\,\alpha}{1+\alpha}\right)\mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right), \tag{A.22}$$

for $t\le T/N$, where both $\mathcal{E}_{t,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right)$ and $\mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right)$ are deterministic functions of time given in (A.3) and (A.4), respectively, and evaluated at $(t_1,t_2)=(t,\frac{T}{N})$. Dividing ξ_t by ξ_0 (A.22), we obtain

$$\xi_{0,t}=\frac{\left(D_{\mathcal{L}t}\right)^{\frac{(1-R)\alpha}{1+\alpha}}\left(D_{\mathcal{S}t}\right)^{\frac{\beta-R}{1+\alpha}}}{\left(D_{\mathcal{L}0}\right)^{\frac{(1-R)\alpha}{1+\alpha}}\left(D_{\mathcal{S}0}\right)^{\frac{\beta-R}{1+\alpha}}}\frac{1}{\mathcal{E}_{0,t}\left(\frac{(1-R)\alpha}{1+\alpha}\right)\mathcal{E}_{0,t}\left(\frac{\beta-R}{1+\alpha}\right)},\quad t\le T/N. \tag{A.23}$$

□

Proof of Proposition 2 (Market Price of Risk). We take Itô's Lemma on $\xi_{0,t}$ (A.23) and obtain

$$\theta_{1t}=\frac{R-\beta}{1+\alpha}\sigma,\qquad\theta_{2t}=\frac{\alpha\left(R-1\right)}{1+\alpha}\sigma, \tag{A.24}$$

Further, we find that $\theta_{1t}>\theta_{2t}$ if, and only if

$$R\left(1-\alpha\right)>\beta-\alpha, \tag{A.25}$$

which is always satisfied because $R\left(1-\alpha\right)>\left(1-\alpha\right)>\beta-\alpha$ since $R>1$ and $0<\alpha,\beta\le 1$. We obtain the no benchmark case by setting $\alpha=\beta=0$, and we obtain (15) by setting $\alpha=1$ and $\beta=0$. Addressing the remaining items in the proposition, it is straightforward to see that $\bar{\theta}_{1t}>\bar{\theta}_{2t}$, $\bar{\theta}_{1t}>\theta_{1t}$, and $\theta_{2t}>\bar{\theta}_{2t}$, when $0<\alpha,\beta\le 1$. □

Proof of Proposition 5 (Security Prices). To find the long-term security price, we again utilize the no-arbitrage condition

$$S_{\mathcal{L}t}=\frac{E_t\left[\xi_{T/N}S_{\mathcal{L}T/N}\right]}{\xi_t},\quad t<T/N, \tag{A.26}$$

and find that

$$S_{\mathcal{L}t}=\left(D_{\mathcal{L}t}\right)^{\frac{1-R}{1+\alpha}}\left(D_{\mathcal{S}t}\right)^{\frac{R-\beta}{1+\alpha}}\left(\frac{\mathcal{E}_{T/N,T}\left(1-R\right)\bar{\xi}_{\mathcal{L}}}{\bar{\xi}_{\mathcal{S}}}\right)^{\frac{1}{1+\alpha}}\frac{\mathcal{E}_{t,T/N}\left(1-R\right)}{\mathcal{E}_{t,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right)\mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}. \tag{A.27}$$

By plugging $\bar{\xi}_{\mathcal{S}}$ and $\bar{\xi}_{\mathcal{L}}$, given (A.19), we find that

$$S_{\mathcal{L}t} = \left(\frac{1-\lambda}{\lambda}\right)^{\frac{R}{1+\alpha}} \left(\frac{E\left[\left(D^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}} \left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\right]}{E\left[\left(\xi_{0,T}\right)^{1-\frac{1}{R}}\right]}\right)^{\frac{R}{1+\alpha}} \left(D_{\mathcal{L}t}\right)^{\frac{1-R}{1+\alpha}} \left(D_{\mathcal{S}t}\right)^{\frac{R-\beta}{1+\alpha}} \frac{\left(\mathcal{E}_{T/N,T}\left(1-R\right)\right)^{\frac{1}{1+\alpha}} \mathcal{E}_{t,T/N}\left(1-R\right)}{\mathcal{E}_{t,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right) \mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}, \quad \text{(A.28)}$$

for $t \leq T/N$. To characterize $E\left[\left(D^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}} \left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\right]$, we evaluate the above equation at $t = T/N$, raise it to the power of $\frac{\alpha}{R}$, multiply both sides by $D^{\frac{\beta}{R}}_{\mathcal{S}T/N} \left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}$, and take expectations. By doing so, we get

$$\begin{aligned} & E\left[\left(D^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}} \left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\right] \\ & = \left(\frac{1-\lambda}{\lambda}\right)^{\alpha} \left(\frac{1}{E\left[\left(\xi_{0,T}\right)^{1-\frac{1}{R}}\right]}\right)^{\alpha} \left(E\left[\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}} \left(D_{\mathcal{L}T/N}\right)^{\frac{1-R}{R}\frac{\alpha}{1+\alpha}} \left(D_{\mathcal{S}T/N}\right)^{\frac{\alpha+\frac{\beta}{R}}{1+\alpha}}\right]\right)^{1+\alpha} \left(\mathcal{E}_{T/N,T}\left(1-R\right)\right)^{\frac{\alpha}{R}}. \quad \text{(A.29)} \end{aligned}$$

By plugging this identity back to $S_{\mathcal{L}t}$ (A.28) and rearranging, we find

$$S_{\mathcal{L}t} = \left(\frac{1-\lambda}{\lambda}\right)^{R} \left(\frac{E\left[\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}} \left(D_{\mathcal{L}T/N}\right)^{\frac{1-R}{R}\frac{\alpha}{1+\alpha}} \left(D_{\mathcal{S}T/N}\right)^{\frac{\alpha+\frac{\beta}{R}}{1+\alpha}}\right]}{E\left[\left(\xi_{0,T}\right)^{1-\frac{1}{R}}\right]}\right)^{R} \left(D_{\mathcal{L}t}\right)^{\frac{1-R}{1+\alpha}} \left(D_{\mathcal{S}t}\right)^{\frac{R-\beta}{1+\alpha}} \frac{\mathcal{E}_{T/N,T}\left(1-R\right) \mathcal{E}_{t,T/N}\left(1-R\right)}{\mathcal{E}_{t,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right) \mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}, \quad \text{(A.30)}$$

We are left to evaluate the two unconditional expected values in this last formulation of $S_{\mathcal{L}t}$. By evaluating the first expected value, we obtain

$$E\left[\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}} \left(D_{\mathcal{L}T/N}\right)^{\frac{1-R}{R}\frac{\alpha}{1+\alpha}} \left(D_{\mathcal{S}T/N}\right)^{\frac{\alpha+\frac{\beta}{R}}{1+\alpha}}\right] = \frac{\left(D_{\mathcal{L}0}\right)^{\frac{1-R}{R}\frac{\alpha}{1+\alpha}} \left(D_{\mathcal{S}0}\right)^{\frac{\alpha+\frac{\beta}{R}}{1+\alpha}} \left(\mathcal{E}_{0,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right)\right)^{\frac{1}{R}} \mathcal{E}_{0,T/N}\left(\frac{1+\alpha+\beta-R}{1+\alpha}\right)}{\left(\mathcal{E}_{0,T/N}\left(\frac{\beta-R}{1+\alpha}\right)\right)^{\frac{R-1}{R}}}, \quad \text{(A.31)}$$

where $\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}$ is obtained by evaluating (A.23) at $t = T/N$ and by raising the expression to the power of $1 - \frac{1}{R}$. The function $\mathcal{E}_{0,T/N}\left(\frac{1+\alpha+\beta-R}{1+\alpha}\right)$ is given in (A.5), evaluated at $(t_1, t_2) = (0, \frac{T}{N})$. We continue with the evaluation of the second expected value. By dividing ξ_T (A.16) by ξ_0 (A.22), we obtain

$$\frac{\xi_T}{\xi_0} = \left(\frac{\bar{\xi}_{\mathcal{L}}}{\bar{\xi}_{\mathcal{S}}}\right)^{\frac{1}{1+\alpha}} \frac{\left(D_{\mathcal{L}T}\right)^{-R}}{\left(D_{\mathcal{L}0}\right)^{\frac{(1-R)\alpha}{1+\alpha}} \left(D_{\mathcal{S}0}\right)^{\frac{\beta-R}{1+\alpha}} \left(\mathcal{E}_{T/N,T}\left(1-R\right)\right)^{\frac{\alpha}{1+\alpha}} \mathcal{E}_{0,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right) \mathcal{E}_{0,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}. \quad \text{(A.32)}$$

By plugging $\bar{\xi}_{\mathcal{L}}$, $\bar{\xi}_{\mathcal{S}}$ from (A.19), by raising the expression to the $1-\frac{1}{R}$ power of both sides, taking expectations, and rearranging we obtain

$$E\left[(\xi_{0,T})^{1-\frac{1}{R}}\right]=\left(\frac{1-\lambda}{\lambda}\right)^{\frac{(R-1)}{\alpha+R}}\left(E\left[\left(D^{\beta}_{\mathcal{S}T/N}S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}}\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\right]\right)^{\frac{(R-1)}{\alpha+R}}$$

$$\left(\frac{(D_{\mathcal{L}0})^{1-R}\,\mathcal{E}_{0,T}\left(1-R\right)}{\left[(D_{\mathcal{L}0})^{\frac{(1-R)\alpha}{1+\alpha}}\left(D_{\mathcal{S}0}\right)^{\frac{\beta-R}{1+\alpha}}\left(\mathcal{E}_{T/N,T}\left(1-R\right)\right)^{\frac{\alpha}{1+\alpha}}\mathcal{E}_{0,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right)\mathcal{E}_{0,T/N}\left(\frac{\beta-R}{1+\alpha}\right)\right]^{\frac{R-1}{R}}}\right)^{\frac{1+\alpha}{\alpha+R}}.$$

By plugging $E\left[\left(D^{\beta}_{\mathcal{S}T/N}S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}}\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\right]$, given in (A.29), using (A.31), and rearranging, we obtain

$$E\left[(\xi_{0,T})^{1-\frac{1}{R}}\right]=\left(\frac{1-\lambda}{\lambda}\right)^{\frac{R-1}{R}}\left(D_{\mathcal{L}0}\right)^{\frac{1-R}{R}}\left(D_{\mathcal{S}0}\right)^{\frac{R-1}{R}}\left(\frac{\mathcal{E}_{0,T/N}\left(\frac{1+\alpha+\beta-R}{1+\alpha}\right)}{\mathcal{E}_{0,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}\right)^{\frac{R-1}{R}}\left(\mathcal{E}_{0,T}\left(1-R\right)\right)^{\frac{1}{R}}. \quad \text{(A.33)}$$

By plugging these two expected values back to $S_{\mathcal{L}t}$ (A.30), we obtain

$$S_{\mathcal{L}t}=\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{D_{\mathcal{L}t}}{D_{\mathcal{L}0}}\right)^{\frac{1-R}{1+\alpha}}\left(\frac{D_{\mathcal{S}t}}{D_{\mathcal{S}0}}\right)^{\frac{R-\beta}{1+\alpha}}\left(D_{\mathcal{S}0}\right)\frac{\mathcal{E}_{0,t}\left(\frac{(1-R)\alpha}{1+\alpha}\right)\mathcal{E}_{0,T/N}\left(\frac{1+\alpha+\beta-R}{1+\alpha}\right)}{\mathcal{E}_{0,t}\left(1-R\right)\mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}. \quad \text{(A.34)}$$

Similarly, the short-term security price is given by

$$S_{\mathcal{S}t}=\left(D_{\mathcal{S}t}\right)\frac{\mathcal{E}_{t,T/N}\left(\frac{1+\alpha+\beta-R}{1+\alpha}\right)}{\mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}. \quad \text{(A.35)}$$

We get $\bar{S}_{\mathcal{S}t}$ and $\bar{S}_{\mathcal{L}t}$ by setting $\alpha=\beta=0$, and $S_{\mathcal{S}t}$ and $S_{\mathcal{L}t}$ by setting $\alpha=1$ and $\beta=0$. Further, we define $\bar{A}_{\mathcal{L}t}$ and $\bar{A}_{\mathcal{S}t}$ to decompose the deteminstic parts affecting prices, as follows.

$$\bar{A}_{\mathcal{L}t}\equiv\left(\frac{1}{D_{\mathcal{L}0}}\right)^{1-R}\left(\frac{1}{D_{\mathcal{S}0}}\right)^{R}D_{\mathcal{S}0}\frac{\mathcal{E}_{0,t}\left(0\right)\mathcal{E}_{0,T/N}\left(1-R\right)}{\mathcal{E}_{0,t}\left(1-R\right)\mathcal{E}_{t,T/N}\left(-R\right)}, \quad \text{(A.36)}$$

$$\bar{A}_{\mathcal{S}t}\equiv\frac{\mathcal{E}_{t,T/N}\left(1-R\right)}{\mathcal{E}_{t,T/N}\left(-R\right)}. \quad \text{(A.37)}$$

Similarly, we define

$$A_{\mathcal{L}t}\equiv\frac{1}{\bar{A}_{\mathcal{L}t}}\left(\frac{1}{D_{\mathcal{L}0}}\right)^{\frac{1-R}{2}}\left(\frac{1}{D_{\mathcal{S}0}}\right)^{\frac{R}{2}}D_{\mathcal{S}0}\frac{\mathcal{E}_{0,t}\left(\frac{1-R}{2}\right)\mathcal{E}_{0,T/N}\left(1-\frac{R}{2}\right)}{\mathcal{E}_{0,t}\left(1-R\right)\mathcal{E}_{t,T/N}\left(-\frac{R}{2}\right)}, \quad \text{(A.38)}$$

$$A_{\mathcal{S}t}\equiv\frac{1}{\bar{A}_{\mathcal{S}t}}\frac{\mathcal{E}_{t,T/N}\left(1-\frac{R}{2}\right)}{\mathcal{E}_{t,T/N}\left(-\frac{R}{2}\right)}. \quad \text{(A.39)}$$

$\square$

Proof of Lemma 1 (Risk Exposure). We utilize the no arbitrage conditions (A.9) and (A.10), the market clearing conditions in the consumption good, and write

$$\xi_t W_t^{\mathcal{V}} = E_t\left[\xi_T D_{\mathcal{L}T}\right], \quad t \in [0,T], \tag{A.40}$$

$$\xi_t W_t^{\mathcal{I}} = E_t\left[\xi_{T/N} D_{\mathcal{S}T/N}\right], \quad t \in [0,T/N]. \tag{A.41}$$

By plugging ξ_T (A.16) and $\xi_{T/N}$ (A.21) we find that

$$\xi_t W_t^{\mathcal{V}} \propto \left(D_{\mathcal{L}t}\right)^{1-R}, \tag{A.42}$$

$$\xi_t W_t^{\mathcal{I}} \propto \left(D_{\mathcal{S}t}\right)^{1+\frac{\beta-R}{1+\alpha}} \left(D_{\mathcal{L}t}\right)^{\frac{(1-R)\alpha}{1+\alpha}} = \left(D_{\mathcal{S}t}\right)^{1-R} \left(D_{\mathcal{S}t}\right)^{R\frac{\alpha+\frac{\beta}{R}}{1+\alpha}} \left(D_{\mathcal{L}t}\right)^{\frac{(1-R)\alpha}{1+\alpha}} \propto \left(D_{\mathcal{S}t}\right)^{1-R} \left(S_{\mathcal{L}t}\right)^{\alpha} \left(S_{\mathcal{S}t}\right)^{\beta}. \tag{A.43}$$

In the third term of (A.43), we separate the benchmarking from the non-benchmarking components by setting $\alpha = \beta = 0$, which leads to a non benchmarking component that equals $\left(D_{\mathcal{S}t}\right)^{1-R}$ and a benchmakring component that equals $\left(D_{\mathcal{S}t}\right)^{R\frac{\alpha+\frac{\beta}{R}}{1+\alpha}} \left(D_{\mathcal{L}t}\right)^{\frac{(1-R)\alpha}{1+\alpha}}$. We obtain the last term of (A.43) by observing that the benchmarking component equals $\left(S_{\mathcal{L}t}\right)^{\alpha} \left(S_{\mathcal{S}t}\right)^{\beta}$, as (A.34) and (A.35) reveals. We finish by applying Itô's Lemma to both sides of the above equations, (A.42), (A.43) and find that

$$\Sigma_t' \pi_t^{\mathcal{V}} - \theta_t = \begin{bmatrix} 0 \\ -(R-1)\sigma \end{bmatrix}, \tag{A.44}$$

$$\Sigma_t' \pi_t^{\mathcal{I}} - \theta_t = \begin{bmatrix} -(R-1)\sigma \\ 0 \end{bmatrix} + \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix} \alpha + \begin{bmatrix} \sigma_{\mathcal{S}1t} \\ \sigma_{\mathcal{S}2t} \end{bmatrix} \beta, \tag{A.45}$$

which leads to the desired result when setting $\alpha = 1$ and $\beta = 0$. $\square$

Proof of Proposition 1 (Volatility). We obtain the volatility coefficients by taking Itô's Lemma of $S_{\mathcal{L}t}$ (A.34) and $S_{\mathcal{S}t}$ (A.35) as follows.

$$\begin{bmatrix} \sigma_{\mathcal{S}1t} \\ \sigma_{\mathcal{S}2t} \end{bmatrix} = \begin{bmatrix} \sigma \\ 0 \end{bmatrix}, \qquad \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix} = \sigma \begin{bmatrix} \frac{R-\beta}{1+\alpha} \\ \frac{1-R}{1+\alpha} \end{bmatrix}. \tag{A.46}$$

We set $\alpha = 1$ and $\beta = 0$ to get the results for the benchmark case and $\alpha = \beta = 0$ for the no benchmark case. Following (A.46), it is strightforward to see that when $\alpha = \beta = 0$, we have $(\bar{\sigma}_{\mathcal{L}1t}) > 0$, $(\bar{\sigma}_{\mathcal{L}2t}) < 0$, $|\bar{\sigma}_{\mathcal{L}2t}| < |\bar{\sigma}_{\mathcal{L}1t}|$ and when $0 < \alpha, \beta < 1$ we have $|\sigma_{\mathcal{L}2t}| < |\sigma_{\mathcal{L}1t}|$ to verify item (i) and item (ii) of the individual effect. Verifying item (iii) and item (i) of the combined effect, it is strightforward to see that $0 < \sigma_{\mathcal{L}1t} < \bar{\sigma}_{\mathcal{L}1t}$ and $0 > \sigma_{\mathcal{L}2t} > \bar{\sigma}_{\mathcal{L}2t}$ when $0 < \alpha, \beta < 1$. Lastly, since $\sigma_{\mathcal{L}2t}$ is closer to zero than $\bar{\sigma}_{\mathcal{L}2t}$, and $\sigma_{\mathcal{L}1t}$ is closer to zero than $\bar{\sigma}_{\mathcal{L}1t}$ it must be that the total volatility shrinks when $0 < \alpha, \beta < 1$. $\square$

Proof of Proposition 4 (Risk Premium, Total Volatility, and Sharpe Ratio). We start the proof by showing that the risk premium of the short-term asset is higher than the long-term asset. To find the risk premiums we use the identity

$$\begin{bmatrix}\mu_{\mathcal{S}t}\\ \mu_{\mathcal{L}t}\end{bmatrix}=\begin{bmatrix}\sigma_{\mathcal{S}1t} & \sigma_{\mathcal{S}2t}\\ \sigma_{\mathcal{L}1t} & \sigma_{\mathcal{L}2t}\end{bmatrix}\begin{bmatrix}\theta_{1t}\\ \theta_{2t}\end{bmatrix}=\sigma^2\begin{bmatrix}1 & 0\\ \frac{R-\beta}{1+\alpha} & -\frac{R-1}{1+\alpha}\end{bmatrix}\begin{bmatrix}\frac{R-\beta}{1+\alpha}\\ \frac{R-1}{1+\alpha}\alpha\end{bmatrix}=\sigma^2\begin{bmatrix}\frac{R-\beta}{1+\alpha}\\ \left(\frac{R-\beta}{1+\alpha}\right)^2-\left(\frac{R-1}{1+\alpha}\right)^2\alpha\end{bmatrix}. \tag{A.47}$$

We find that $\mu_{\mathcal{S}t}>\mu_{\mathcal{L}t}$ if and only if

$$\frac{R-\beta}{1+\alpha}>\left(\frac{R-\beta}{1+\alpha}\right)^2-\left(\frac{R-1}{1+\alpha}\right)^2\alpha. \tag{A.48}$$

Let us define $X\equiv\frac{R-1}{R-\beta}$, divide both sides by $\left(\frac{R-\beta}{1+\alpha}\right)^2$, and rewrite the inequality such that

$$X^2\alpha>1-\frac{1+\alpha}{R-\beta}=\frac{R-\beta-1-\alpha}{R-\beta}=\frac{R-1}{R-\beta}-\frac{\beta+\alpha}{R-\beta}=X-\frac{\beta+\alpha}{R-\beta}. \tag{A.49}$$

Transferring the X to the left hand side leads to

$$X\left(X\alpha-1\right)=\frac{R-1}{R-\beta}\left(X\alpha-1\right)>-\frac{\beta+\alpha}{R-\beta}. \tag{A.50}$$

Rearranging, we finally obtain

$$\left(R-1\right)X\alpha>\left(R-1\right)-\left(\beta+\alpha\right). \tag{A.51}$$

It is clear from (A.51) that if $\alpha=\beta=0$ the inequality is not satisfied, implying that the short-term asset expected return is lower than the long-term asset expected return without benchmarking when $R>1$. To show that (A.51) is satified with benchmarking incentives, we assume that $0<\alpha+\beta\leq 1$. We want to show that for a given $0\leq\alpha\leq 1$ there exists a threshold $0\leq\underline{\beta}<1-\alpha$, such that for any $\beta\in[\underline{\beta},1-\alpha]$ the inequality (A.51) is satisfied. Towards that goal, let $R=1+\epsilon<2$ for $0<\epsilon<1$. Then, for any given $\alpha\in[0,1]$, we set $\underline{\beta}=\max\{\epsilon-\alpha,0\}$. It is apparent that $\underline{\beta}<1-\alpha$ because $\epsilon<1$, so $[\underline{\beta},1-\alpha]$ is a non empty set. Eventually, we obtain

$$\left(R-1\right)X\alpha>0\geq\epsilon-\left(\underline{\beta}+\alpha\right)\geq\epsilon-\left(\beta+\alpha\right)=\left(R-1\right)-\left(\beta+\alpha\right), \tag{A.52}$$

where the right inequality holds because $\beta\geq\underline{\beta}$. We conclude that with benchmarking, the short-term asset expected return is higher than the long-term asset expected return. Notice that (A.51) always holds when $\alpha=1,\beta=0$, but holds when $\alpha=0,\beta=1$ if and only if $R<2$. We finish with the risk premiums by verifying (28) and (29). Following (A.47), it is apparent that

$$\bar{\mu}_{\mathcal{S}t}=\frac{R-0}{1+0}>\frac{R-\beta}{1+\alpha}=\mu_{\mathcal{S}t}, \tag{A.53}$$

and

$$\bar{\mu}_{\mathcal{L}t} = \left(\frac{R-0}{1+0}\right)^2 - \left(\frac{R-1}{1+0}\right)^2 0 > \left(\frac{R-\beta}{1+\alpha}\right)^2 - \left(\frac{R-1}{1+\alpha}\right)^2 \alpha = \mu_{\mathcal{L}t}, \tag{A.54}$$

verifying (28). To verify (29), notice that $\bar{\mu}_{\mathcal{L}t} > \bar{\mu}_{\mathcal{S}t}$ and $\mu_{\mathcal{S}t} > \mu_{\mathcal{L}t}$ (A.47). Adding these two ineqaulities leads to

$$\bar{\mu}_{\mathcal{L}t} + \mu_{\mathcal{S}t} > \bar{\mu}_{\mathcal{S}t} + \mu_{\mathcal{L}t}. \tag{A.55}$$

Algebraic manipulation leads to the desired result.

Next, we show that the total volatility of the short-term asset is higher than the long-term asset. Following the definition of total volatility (30) and the volatility coefficients (A.46) from the proof Proposition 1, we find that the total volatility of the short-term asset is higher than the long-term asset, if, and only if,

$$1 > \left(\frac{R-\beta}{1+\alpha}\right)^2 + \left(\frac{R-1}{1+\alpha}\right)^2. \tag{A.56}$$

An algebraic manipulation leads to

$$2R^2 - 2R\left(1+\beta\right) + \left[\left(1+\beta^2\right) - \left(1+\alpha\right)^2\right] < 0. \tag{A.57}$$

We solve for the roots of R and find

$$R = \frac{2\left(1+\beta\right) \pm \sqrt{4\left(1+\beta\right)^2 - 4 \times 2\left[\left(1+\beta^2\right) - \left(1+\alpha\right)^2\right]}}{4}. \tag{A.58}$$

Another algebraic manipulation finally leads to

$$R = \frac{\left(1+\beta\right) \pm \sqrt{\left[1-\beta\left(\sqrt{2}-1\right)\right]\left[1+\beta\left(\sqrt{2}+1\right)\right] + 2\alpha\left(2+\alpha\right)}}{2}. \tag{A.59}$$

This quadratic equation has two solutions. We define $\bar{R}$ as the upper solution. It is immediately observable that when $\alpha = \beta = 0$, the upper solution equals 1, implying that the short-term asset volatility is lower than the long-term asset volatility for $R > 1$ without benchmarking. When there is benchmarking, $1 \geq \alpha + \beta > 0$, we left to show that $1 < \bar{R} < 2$. Following (A.59), we find that $\bar{R} > 1$ if, and only if

$$\alpha\left(2+\alpha\right) > \beta\left(\beta-2\right), \tag{A.60}$$

which is satisfied for any $\beta, \alpha \in (0, 1]$, since the right hand side is negative and the left hand side is positive.

Further, we find that $\bar{R} < 2$ if, and only if

$$\alpha\,(2+\alpha) < (2-\beta)^2\,. \tag{A.61}$$

Adding and subtracting 1 from the left hand side, and rearranging leads to

$$-1 < (1-\alpha-\beta)\,(3+\alpha-\beta)\,, \tag{A.62}$$

which is satisfied for $\alpha+\beta \leq 1$.

Lastly, we find that the short-term asset Sharpe ratio is higher than the long-term asset Sharpe ratio if, and only if,

$$\frac{R-\beta}{1+\alpha} > \frac{\left(\frac{R-\beta}{1+\alpha}\right)^2 - \left(\frac{R-1}{1+\alpha}\right)^2 \alpha}{\sqrt{\left(\frac{R-\beta}{1+\alpha}\right)^2 + \left(\frac{R-1}{1+\alpha}\right)^2}}. \tag{A.63}$$

Let us define $X \equiv \frac{R-1}{R-\beta}$ and rewrite the inequality above in terms of X, leading to

$$\sqrt{1+X^2} > 1 - X^2\alpha. \tag{A.64}$$

This inequality is always satisfied because $X > 0$. Notice that the inequality holds for $\alpha = \beta = 0$ as well.

□

Proof of Proposition 3 (Portfolios). It is easy to verify from (A.47) that

$$\left(\Sigma_t^{'}\right) = \begin{bmatrix} 1 & \frac{R-\beta}{1+\alpha} \\ 0 & -\frac{R-1}{1+\alpha} \end{bmatrix} \sigma, \qquad \left(\Sigma_t^{'}\right)^{-1} = \begin{bmatrix} 1 & \frac{R-\beta}{R-1} \\ 0 & -\frac{1+\alpha}{R-1} \end{bmatrix} \frac{1}{\sigma}, \tag{A.65}$$

where Σ_t is defined in (9). Multiplying the risk exposures, given in (A.44) and (A.45), by the inverse of $\left(\Sigma_t^{'}\right)$, we obtain

$$\pi_t^{\mathcal{V}} = \left(\Sigma_t^{'}\right)^{-1} \theta_t + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} 0 \\ -(R-1)\,\sigma \end{bmatrix}, \tag{A.66}$$

$$\pi_t^{\mathcal{I}} = \left(\Sigma_t^{'}\right)^{-1} \theta_t + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} -(R-1)\,\sigma \\ 0 \end{bmatrix} + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix} \alpha + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} \sigma_{\mathcal{S}1t} \\ \sigma_{\mathcal{S}2t} \end{bmatrix} \beta. \tag{A.67}$$

We define the portfolios of the individual equilibrium channels, which also appears in the text above Proposition 3) as

$$\phi_{\text{m.v.}} \equiv \left(\Sigma_t^{'}\right)^{-1} \theta_t, \quad \phi_{\text{r.a.}}^{\mathcal{V}} \equiv \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} 0 \\ -(R-1)\,\sigma \end{bmatrix}, \tag{A.68}$$

$$\phi_{\text{r.a.}}^{\mathcal{I}} \equiv \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} -(R-1)\,\sigma \\ 0 \end{bmatrix}, \quad \phi_{\text{b}}^{\mathcal{I}} \equiv \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix} \alpha + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} \sigma_{\mathcal{S}1t} \\ \sigma_{\mathcal{S}2t} \end{bmatrix} \beta. \tag{A.69}$$

By taking the inner products we obtain

$$\phi_{\text{m.v.}} = \begin{bmatrix} R-\beta \\ -\alpha \end{bmatrix}, \quad \phi_{\text{r.a.}}^{\mathcal{V}} = \begin{bmatrix} -(R-\beta) \\ 1+\alpha \end{bmatrix}, \quad \phi_{\text{r.a.}}^{\mathcal{I}} = \begin{bmatrix} -(R-1) \\ 0 \end{bmatrix}, \quad \phi_{\text{b}}^{\mathcal{I}} = \alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{A.70}$$

leading to our desired results in (19), (20) and (21) when $\beta = 0$ and $\alpha = 1$. When there is no benchmark, we plug $\alpha = \beta = 0$ and find that $\pi_t^{\mathcal{V}} = \bar{\pi}_t^{\mathcal{V}}$ and $\pi_t^{\mathcal{I}} = \bar{\pi}_t^{\mathcal{I}}$. □

Proof of Proposition 7 (Extension: Volatility and Market Prices of Risk). The proof follows the steps of the proof of Proposition 1 and Proposition 2. □

Proof of Proposition 8 (Extension: Risk Premium, Total Volatility, and Sharpe Ratio). The proof follows the steps of the proof of Proposition 4. □

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