Real Options, Skewness, and the Pricing of Equity Options

March, 2023

Abstract

We provide a theoretical explanation for the negative relation between skewness of the equity distribution and measures of real option. We show how such relationship affects the pricing of equity options in the observed data and and in the model. A calibrated version of the model is able to match many unconditional financial characteristics of the average option-able stock and produce very flexible implied volatility surfaces: upward, downward sloping, and u-shaped.

JEL Classifications: G12, G32

Keywords: option pricing, future realized skewness, risk-neutral skewness, real options, leverage, investments

1. Introduction

There is a long and valued literature that studies how real options impact asset prices. Some of the most well known examples include, but are not limited to, McDonald and Siegel (1985), Carlson, Fisher, and Giammarino (2004), Kogan (2004a), Grullon, Lyandres, and Zhdanov (2012), Kogan and Papanikolaou (2014), Kuehn and Schmid (2014), and Gomes and Schmid (2021). Most of these papers focus on how real options (and their exercise through investments) affect current and future asset prices. Because they affect all future possible investment decisions, real options influence the shape of the entire distribution of future asset prices, and hence also impact the prices of state contingent claims. In particular, the skewness of the equity distribution has been linked to real options and plays an important role in determining the relative pricing of options, as in Bakshi, Kapadia, and Madan (2003).

Empirically, we show that, in the universe of optionable stocks, the relation between measures of real option value and measures of future realized skewness is negative. This has implications for the cross-section of option prices. Stocks with more valuable growth options have more negatively slopped implied volatility surface, or equivalently more negative risk-neutral skewness (see for example Morellec and Zhdanov, 2019, for a recent study that finds similar results).

A negative relation between real options and skewness is at odds with some papers that either find or assume a positive relation (see for example, Trigeorgis and Lambertides, 2014a; Del Viva, Kasanen, and Trigeorgis, 2017; Bali, Del Viva, Lambertides, and Trigeorgis, 2020; Panayiotis, Bali, Kagkadis, and Lambertides, 2021; Del Viva, Kothari, Lambertides, and Trigeorgis, 2021). The argument that these authors put forth is that, because they contribute some non-linearity to the firm's payoff, real-options should positively contribute to the skewness of the equity distribution. van Zwet (1964) in fact shows that the convexity of a function of a random variable is related to the skewness of the distribution of the function of the random variable, and that more convexity implies larger skewness. The work of van Zwet (1964) is commonly used to assume that the presence of real options would produce more positively skewed distributions, relative to a firm that has no such option. As a consequence, more valuable real options are assumed to lead to more positively skewed distributions. We show that this is not necessarily the case in a real option model in which firms have an infinite stream of chances to realize investments or disinvestments.

The intuition behind the analysis in our paper is simple, and rests on two arguments. First, all firms have paths to possible futures where they will want to either optimally increase or decrease their capital stock, and their current conditions determine how likely any of these paths are. In other words, it is hard to think that firms with no options to grow or shrink actually exist: *all firms* currently have some real options that are more or less likely to be exercised within a given time frame (i.e., they are more or less in-the-money). The convexity effect proposed by van Zwet (1964) thus applies to all firms but to a degree that depends on the "moneyness" of the real options considered (i.e., how close those options are to be realized), because that is what affects the non-linearity of the payoff. Using the Black and Scholes (1973) model as a reference, the convexity of the option price is greatest close to the exercise threshold (i.e., the ATM strike), where the option has maximum gamma (i.e., the second derivative relative to the underlying price). Thus, the relative convexity of different real options is what matters.

If we were to compare two firms with different real *options to grow*, and set aside contraction options, we could obtain a different ordering of option values and skewness depending on the characteristics of the real options considered. For example, relative to a firm that is equally likely to exercise or decline the growth option (say that the option is ATM), a firm that has a more valuable real option (i.e., the real option is ITM and the firm is very likely to invest in the future) would have a less skewed equity distribution because the real option payoff is similar to that of the asset in place (i.e., the delta of the real option is close to one and the gamma is close to zero). Similarly, a less valuable real option (i.e., OTM) that the firm is not very likely to convert in the future, would also contribute relatively little skewness to the equity distribution, because it also has a very low gamma. Thus, if we compare a firm which is very unlikely to invest to one that has some chances of doing so, we would get a positive relation between the value of the respective options and the skewness of the future equity distribution. But if we compare a firm which has some chances to invest to one that is almost certain to invest, we would obtain a negative relation between the value of the options and the skewness of the future equity distribution.

Second, firms have two types of real options: One can think of the option to increase the capital stock as a call option and the option to decrease the capital stock as a put option (as for example in the real option models of Gu, Hackbarth, and Johnson, 2017; Aretz and Pope, 2018). We will refer to the combination of the two as a real straddle. The real straddle is worth more when one of the two options is very likely to be exercised, but its convexity is still highest when there is uncertainty about which of the two options might be exercised (i.e., straddle is at the money). Thus, the relation between real straddle value and convexity of the payoffs should be negative. And since the relative convexity of the real option is what affects the skewness of the distribution of future equity values, the relation between real straddle value and skewness should be overall negative.

We provide theoretical validation of the intuition described above in the context of a model that incorporates both investment and financing decision. The basic set up follows the lead of Hennessy and Whited (2005, 2007) and Zhang (2005), and considers a very popular modeling choice in the dynamic corporate and investment literature. Our work is thus directly related to recent contributions such as Morellec and Zhdanov (2019), for example, who analyze the impact of product market competition on option prices within the confines of a structural model of firm decisions. Our work is also directly connected to structural models that have attempted to incorporate financial leverage in option pricing, such as Geske (1979) (i.e., the compound option model), Toft and Prucyk (1997), who price an equity option on a firm that faces taxes and bankruptcy costs as in Leland (1994), and Geske, Subrahmanyam, and Zhou (2016) who show that accounting for the leverage effect reduces pricing errors relative to the traditional model of Black and Scholes (1973). We present a stripped down version of the model, in which the firm can alter its capital stock only once, to highlight the main economic mechanisms. A fully dynamic version of the model is calibrated to the data to show how closely the model can get at reproducing salient empirical regularities.

We use the simulated economy to validate relationships between properties of option prices (i.e., risk-neutral skewness and implied volatilities curves) and firm characteristics that we observe in the data, focusing explicitly on the role of real options. Similarly to the data, we show that some cross-sectional and time-series variation in option prices can be explained by the choices the firm makes along the path of productivity shocks that it encounters. In particular, we show that, after controlling for size, level of volatility, and leverage, the risk-neutral skewness and the steepness of the slope of the implied volatility curve (in the simulated economy) are more negative for firms with higher values of the real option and for higher market-to-book ratios, which is often used as an empirical proxy for the former.

One attractive feature of the model is that, since the equilibrium choices of production capital and level of indebtedness are state-contingent and endogenous, the resulting equilibrium option prices are also state-contingent. By consequence, the model can generate different implied volatility curves depending on the current state: any form of implied volatility curve (upward sloping, downward sloping, u-shaped, or even inverted u-shape) and of implied distributions (fat, long and short tails). For example, firms that have very little leverage and have at-the-money real options (i.e., they are as likely to invest as they are to disinvest in the next period) will tend to have upward sloping implied volatility curves. The model flexibility in generating different forward looking distributions is important because it allows us to match the substantial amount of cross-sectional and time-series variability shown by the data. For example, at three months maturity, on average 73% of the times stocks exhibit an implied volatility smirk (i.e., implied volatility is monotonically decreasing with strikes) relative to 70% in the real data. 23% of the firm/period observations present a smile (i.e., implied volatility decreases and then increases with strikes) in the simulated economy relative to 19% in the data. In the remaining cases, the surface is either concave (i.e., an inverted smile that we call frown), or increasing with strikes. Over more than 20 years of data, the percentage of stocks that exhibit a smirk varies considerably from 41.3% to 87.7% in the data. The model can replicate much of that variability with a range of 58% to 82%. Substantial variation also occurs through contracts' maturities: Implied volatilities are on average decreasing with options maturities at similar paces both in the model and in the data; along the same dimension, the percentage of stocks that exhibit a smirk increases to over 80% when the implied surface is extracted from contracts that are approximately one year to maturity.

While it is possible to explain that much variation in equity option prices in the context of (reduced form) option pricing models that allow the underlying price to follow a non-gaussian distribution, as for example by introducing jumps and stochastic volatility, such an effort would require a substantial variation in parameters across firms and time (see for example, Geske, Subrahmanyam, and Zhou, 2016; Bakshi, Cao, and Zhong, 2021), leaving unexplained what originates such parameter variability. Our work attempts to answer that question by endogenizing options prices as function of investment and financing choices made by firms. Notably, structural models that are simpler than ours, such as Geske (1979) and of Toft and Prucyk (1997), can produce implied volatility surfaces that respond to changes in capital structure, but that are only downward sloping, and thus do not explain curves of other shapes.

As no model is perfect, so it is ours. There are many missing features that might be important in determining the relation between real option and asset prices: for example, our economy is characterized by a homogeneous technology (i.e., all firms have the same production function), and thus does not account for the fact that differences in forward looking distributions might arise from adoption of new technologies as in Garleanu, Panaceas, and Yu (2012). Also, all firms in the economy have access to a short-term zero-coupon bond, but recent literature, as for example Chaderina, Weiss, and Zechner (2021) and Friewald, Nagler, and Wagner (2021), suggests that heterogeneity in the maturity of debt contracts might have sizable asset pricing implications.

Finally, although our model makes predictions about the fundamental drivers of future skewness, it is silent to whether there should or should not be a skewness risk-premium (i.e., a difference between risk-neutral and future skewness in equity returns). Existing literature documents the existence of such risk-premium in the index market (see for example Kozhan, Neuberger, and Schneider, 2013). However, despite the large cross-sectional distribution of the difference between the two quantities in individual equity names, very little is known about the properties of such differences and why they originate (one rare example is Pederzoli, 2020). Because we show that the empirical relation between measures of real options and future skewness is negative, both in the physical and risk-neutral distributions, our theoretical results should still be valid if additional assumptions needed to generate a skewness risk premium do not affect the way firms accrue real options in the model.

2. Related literature

This paper is primarily related to the strand of literature that aims at explaining equity option prices in the cross-section of stocks. Starting from the seminal work of Merton (1974), there have been a few attempts at incorporating option pricing into a structural model of the firm. Geske (1979) offers a first attempt by producing a double compound option that allows one to price a call option on the equity of a levered firm. Toft and Prucyk (1997) extends this approach to the Leland (1994) economy, thus allowing for taxes and bankruptcy costs to determine the optimal leverage policy of the firm. Geske, Subrahmanyam, and Zhou (2016) show that accounting for the leverage effect greatly reduces option pricing errors relative to the Black and Scholes (1973) model. Bai, Goldstein, and Yang (2019) show that the leverage effect is essential to explain the spread between index and individual banks equity options. Morellec and Zhdanov (2019) show risk-neutral skewness is related to the competitive landscape that surround a firm. Following Hennessy and Whited (2005, 2007), we introduce a fully dynamic model where shareholders endogenously choose production capacity, financial leverage, and default. We show that these ingredients are essential to reproduce the heterogeneity in option prices present in the data.

The leverage effect introduced by Merton (1974) has been considered in a number of applications that link volatility to stock prices/returns. Engle and Siriwardane (2017) propose a structural GARCH model that embeds the leverage effect into equity volatility forecasting models. Because we introduce a model where firms are exposed both to systematic and idiosyncratic risk, our work is also related to studies such as Duan and Wei (2009), who show the differential impact of the two sources of risk. Similar to Duan and Wei (2009), our model also implies that large variation in the prices of individual equity options is produced by realizations of aggregate risk. The leverage effect is a fundamental mechanism of modern models of credit risk. Thus, because we share many model features and because we rely on some of the same intuition our paper is also related to the rather large literature that studies corporate credit risk: from Leland (1994) to more recent contributions such as Gomes and Schmid (2021).

Our paper is also related to the very large literature that studies the impact of growth option on asset prices. From the many contributions to the understanding of the role of dynamic investment policies, among the most directly related to our paper are the works of Berk, Green, and Naik (1999) who link the predictability of stock returns to firm characteristics in a model with dynamic investments. Kogan (2004b), Kogan (2004a), Zhang (2005), Cooper (2006), Ai and Kiku (2013), Kogan and Papanikolaou (2013), Kogan and Papanikolaou (2014), and Gu, Hackbarth, and Johnson (2017) discuss the role of complete and partial investment irreversibility in shaping the risk-return profile. Aretz and Pope (2018) consider a model with both investment and disinvestment options use it to propose a rational explanation of empirical regularities in the cross-section of stock returns. Trigeorgis and Lambertides (2014a) suggest an alternative measure of growth options to the book-to-market ratio and relate it to future stock returns. Del Viva, Kasanen, and Trigeorgis (2017) show that firms with more prevalent growth opportunities have more positive skewness in the return distribution. Cao, Simin, and Zhao (2008) show that growth options are mainly driven by idiosyncratic volatility. Lyandres and Zhdanov (2020) link miss-pricing to the presence of growth options, Bali, Del Viva, Lambertides, and Trigeorgis (2020) explains several stock return anomalies by linking them to options to alter the asset composition that are proxied by idiosyncratic skewness. Aguerrevere (2009) and Morellec and Zhdanov (2019) study how product market competition affect the optimal exercise of growth options.

Finally, our paper is related to the growing literature that studies the informational content of risk-neutral moments of the return distribution that can be obtained from option prices. Bakshi, Kapadia, and Madan (2003) introduce a feasible way to compute risk-neutral moments of the return distribution from option prices using a model-free approach. Bakshi, Kapadia, and Madan (2003) shows that while there is a one to one mapping between the risk-neutral distribution and the implied volatility surface, there is no unique mapping between each of the moments and implied volatilities: volatility, skewness, and kurtosis all combine to determine option prices. Dennis and Mayhew (2002) and Hansis, Schlag, and Vilkov (2010) study the relationship between risk-neutral moments and firm characteristics and find relatively contrasting results about the impact of the leverage effect.

A very large literature relates risk-neutral moments to realized and expected stock returns, including, but not limited to, Bali and Murray (2013), Conrad, Dittmar, and Chisels (2013), Amaya, Christoffersen, Jacobs, and Vasquez (2015), Kadan and Tang (2019), Martin and Wagner (2019), Schneider, Wagner, and Zechner (2020), and Christoffersen, Fournier, Jacobs, and Karoui (2021).

In summary, this paper follows an established literature that aims at measuring and understanding the impact of corporate policies on asset prices (among many others, see for example, Kuehn and Schmid, 2014, who use a similar model to analyze the pricing of corporate debt.) Similar to Toft and Prucyk (1997), Geske, Subrahmanyam, and Zhou (2016), and Morellec and Zhdanov (2019) we offer an alternative approach to option pricing studies that rely on exogenous specifications of stochastic properties of equity prices. While we do not believe that our approach could be as successful in delivering small pricing errors for each security as this last class of models, our calibration is remarkably close in pricing options on the average firm, and in producing, with a single set of parameters, a widespread cross-section that is entirely produced by optimal investment and capital structure decisions. That is also our main point of departure from studies such as Toft and Prucyk (1997), Geske, Subrahmanyam, and Zhou (2016), and Morellec and Zhdanov (2019): our model is calibrated to the data to obtain option prices and firm characteristics that are quantitatively close to the observed economy.

3. Data

We construct our sample of option-able stocks by combining CRSP and COMPUSTAT with OptionMetrics. To increase the frequency of observations we obtain quarterly balance sheet observations and match them to stock returns data using common filters (i.e., share code 10 and 11, no ADRs, total assets in excess of 10 million USD, etc.). We construct stock returns and accounting ratios (leverage, profitability, market-to-book) using standard definitions as in Fama and French (1992).

We then match the resulting sample with OpionMetrics. Because option data is recorded daily but firms accounting information comes at quarterly frequency, we lined up the data by averaging options data over the last three trading days of the last month of the earning reporting quarter. For each firm and quarterly reporting date we extract option prices and volatility surfaces that, at that point in time, have maturities closest to 90 days (one quarter), 180 days, and 360 days. Most of the empirical regularities do not change if we considered 30 days contract. Calibrating the model at monthly frequency is however incredibly challenging and produces unreliable simulated economies (i.e., very small changes in some parameters produce substantial changes in the properties of the simulated data). Hence we prefer to calibrate the model at quarterly frequency and consider 90 days options as the baseline. Having multiple maturities allows us to construct a term structure of option prices and volatilities.

We retain two sets of data which are used for different purposes. Measures that refer to the implied volatility surface (i.e., the implied volatility slopes and implied volatility shapes) are computed using OptionMetrics Volatility Surface files. Instead of averaging the volatility surface of call contracts with that of put contracts, for each firm date and contract maturity, we merge the implied volatilities obtained from calls with strike prices higher than the underlying, and from puts with strike prices below the underlying. In other words, only out-of-the money call and put contracts are used. We now have only one surface. For simplicity we refer to $IV(k)$ as the implied volatility corresponding to moneyness k: so that $IV(1)$ is the ATM implied volatility, $IV(0.8)$ is the OTM implied volatility, and $IV(1.2)$ is the ITM implied volatility. For each firm-quarter and option maturity we retain implied volatilities for the range of moneyness (i.e., ratio of strike to underlying) that goes from 0.8 to 1.2. We then construct three slope measures: the difference between $IV(1.2)$ and $IV(0.8)$ (total slope), the difference between $IV(0.8)$ and $IV(1)$ (left slope), and the difference between $IV(1.2)$ and $IV(1)$ (right slope). For each firm-quarter and option maturity we also classify the shape of the implied volatility curve into four types: the curve is downward sloping as $IV(0.8) > IV(1) > IV(1.2)$ (left smirk), the curve is upward sloping as $IV(0.8) < IV(1) < IV(1.2)$ (right smirk), the curve is u-shaped as $IV(0.8) > IV(1) < IV(1.2)$ (smile), and the curve is inverted u-shaped as $IV(0.8) < IV(1) > IV(1.2)$ (frown).

We construct model-free implied skewness and kurtosis directly from OTM call and OTM put option prices as in Bakshi, Kapadia, and Madan (2003). In particular we follow the procedure described in Hansis, Schlag, and Vilkov (2010), whose code is available on Grigory Vilkov's page. We impose several filters to limit the impact of liquidity. In particular we eliminate prices that violate arbitrage bounds, that have zero open interest, zero-bid quotes, and have both bid and ask quotes unchanged for two consecutive days. For each stock we select options that at a particular point in time have approximate maturity 90, 180, or 360 days, and have moneyness (strike divided by stock price) between 0.7 and 1.3. We interpolate their implied volatilities in order to obtain a dense grid of prices relative to moneyness. We then compute implied moments. On average, 8 option contracts enter the calculation of risk-neutral moments. Because we require firm-quarter observations to have valid measures of implied volatility and implied moments, the liquidity filters applied towards constructing implied moments are also implicitly applied to the implied volatility measures.

The realized skewness is calculate as follows: let t be the date at which a quarterly report is filed. For each stock and quarterly filing date, future realized skewness is measured from realized log returns from $t+1$ to $t+T$, where $T = \{90, 180, 360\}$. We record the observation only when a minimum of $\{30, 60, 90\}$ observations are present. Future realized idiosyncratic skewness is computed using the same data, but by imputing in the formula the residuals from a market model, where the market is the value weighted aggregate portfolio similar to Harvey and Siddique (2000). The final sample is composed of 3,536 stocks and includes quarterly observations between the years 1996 and 2019.

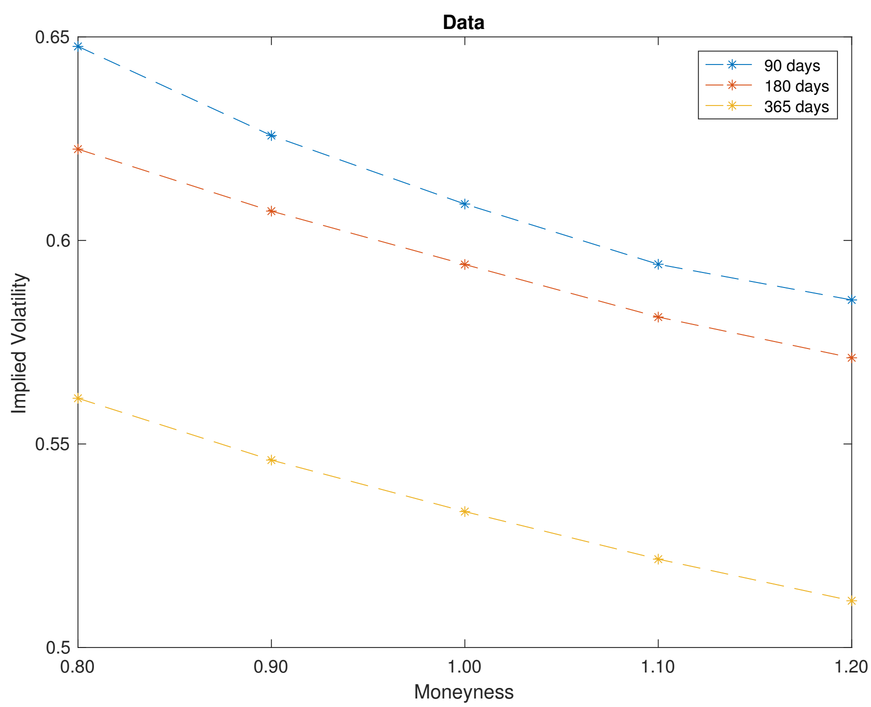
The option pricing literature has mainly focused on two different ways to organize option prices for different maturities and moneyness: implied volatility surfaces and implied risk-neutral moments. Mots of these efforts have been concentrated on index options, which offer a great way to understand aggregate risk premia and investor attitudes towards risk. We organize the data along the same lines but we focus on individual equity options.

The average implied volatility surface is downward sloping along both moneyness and maturity (see Figure 1), although less pronouncedly than the index option surface. At the shortest maturity (that we consider) of 90 days, the average difference between $IV(0.8)$ (i.e., moneyness of 0.8) and $IV(1)$ is 3.5%, while the average difference between $IV(1)$ and $IV(1.2)$ is about 1.2%. Along maturities, the average difference between 90 and 360 days varies between 5.3% for the moneyness of 0.8, to 4% for moneyness of 1.

There is a considerable amount of cross-sectional and time-series variability in implied volatility surfaces. For example, the cross-sectional average ATM implied volatility varies between 90% at the height of the internet bubble crash to 35% in the middle of 2005 (see Panel A of Appendix Figure A1). At the same time, there is a fair amount of cross-sectional dispersion: for example at the hight of the financial crisis, the 95^{th} percentile of implied volatility is higher than 120%, while the 5^{th} is as low as 40%. Similarly, the left tail of the implied volatility curve can be as high as 15% and as low as -5% (see also Panel B of Appendix Figure A1). The right tail varies even more from 10% to -10%. Similar variation can be seen even across maturities, where the slope of the volatility surface hovers around

Figure 1: Implied volatility surface

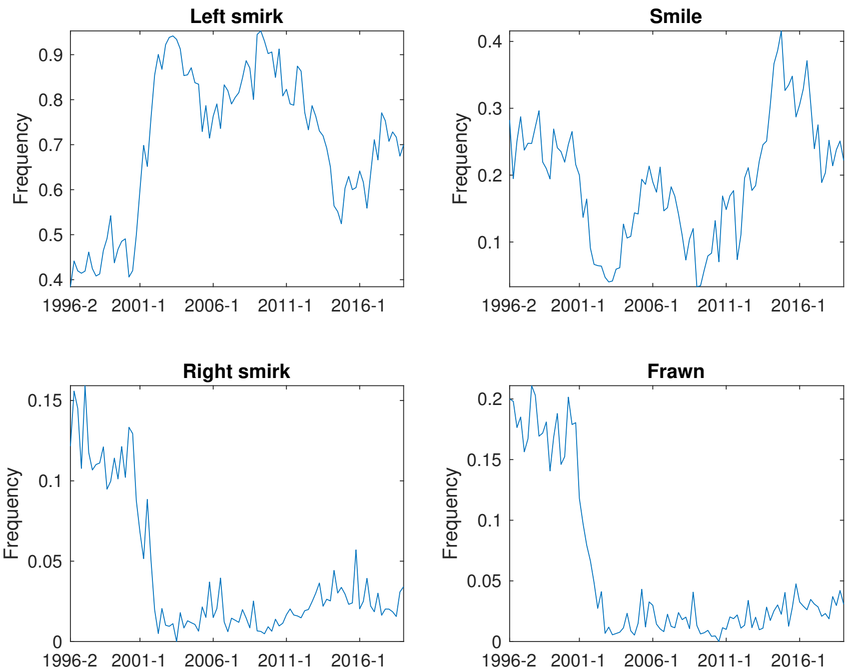
The figure plots the average implied volatility surface extracted from the data. The sample contains all industrial firms with options trading on their equity between 1996 and 2019. Data is sampled at quarterly frequency. A total of 3,536 firms are included.



4% but can be even negative (an upward sloping volatility term structure) for some stocks at particular points in time.

Figure 2: Surface types – time series

The figure plots the time series of frequencies of different implied volatility surface types extracted from the data. The sample contains all industrial firms with options trading on their equity between 1996 and 2019. Data is sampled at quarterly frequency. A total of 3,536 firms are included.



Variation in implied volatilities through time and across stocks produces also a very rich cross-section of different "shapes". We categorize the shape of the implied volatility curve into four mutually exclusive types: left smirk (i.e., implied volatility decreasing with moneyness), smile, right smirk (i.e., implied volatility rising with moneyness), and frown (i.e., inverted smile). We plot the cross-sectional frequency of each surface type for 90 days options in Figure 2.

The most predominant surface type is a left smirk, which is observed on average 73% of the times, with a large time-series variation between 40% and 95% (see Panel A). The second most frequent surface is a smile (Panel B), which is observed on average in 18% of the cases. Right smirks and frowns are less frequent on average; they however manifest in a significant number of stocks during the years of the internet bubble (Panels C and D).

An alternative way to summarize the information contained in option prices on the same stock is to extract risk-neutral moments. Cross-sectional and time-series variation in risk-neutral skewness and kurtosis generally follows that shown by implied volatility curves (see Appendix Figure A2).

4. Empirical evidence

We present here some empirical regularities that connect cross-sectional and time-series variability in statistics that summarize forward looking distributions to observable characteristics of the firm that proxy for the presence and the value of real options. Although we include a wider set of covariates in some specifications, our base-line set of controls comprises variables that can be also constructed in our quantitative model (i.e., size, leverage, profitability, and the level of volatility).

4.1. Real options and equity option prices

In Table 1 we present results of panel regressions of measures that summarize the relative pricing of equity options on several empirical measures of real options: the market to book ratio and the GO score of Trigeorgis and Lambertides (2014b). Each regression also includes firm fixed effects and clustering of standard errors at the firm level. As dependent variables we consider the risk-neutral skewness and the slope of the implied volatility curve (i.e., log difference of IV(1.2) and IV(0.8)). We tabulate results for 90 day maturity options and for a limited set of controls. We present results that include a larger set of controls and that also consider 180 and 360 days to maturity options in Appendix Table A2.

The table largely confirm the results reported in Table 3 of Morellec and Zhdanov (2019). After controlling for different fixed effects, the slope of the implied volatility curve and the risk-neutral skewness of the equity distribution are positively related to the level of volatility and profitability (in some specifications), and negatively related to size (i.e., natural logarithm of total assets), book leverage, and the market-to-book ratio. A similar negative relation is found with the GO ratio of Trigeorgis and Lambertides (2014b). We are quick to note that most of the variation in the data is absorbed by fixed effects. Thus one could conjecture that it is the type of firm, rather than variation in characteristics over time that drive changes in option prices (see for example Figlewski and Wang, 2000, and Doshi, Ericsson, Szaura, and Yu, 2022). However, we note that covariates remain largely economically (the point estimates do not change much) and statistically significant even in the firm fixed

Table 1: Option prices and real options

The table shows regression results of risk-neutral skewness and implied volatility slope on measures of real options value. Risk-neutral skewness and implied volatility slope, which is computed as the log difference of IV(1.2) and IV(0.8), are based on 90 day maturity options. Real options measures include the market-to-book ratio and the GO ratio of Trigeorgis and Lambertides (2014b). We control for size, book leverage, profitability, and the level of ATM IV. Regressions include time and firm fixed effects. A constant is included but not reported. All right hand side variables are winsorized at the first and 99^{th} percentile, and subsequently standardized to aid comparability across specifications. We report parameter estimates (multiplied by 100) and standard errors clustered at the firm level. The sample contains all non-financial firms with options trading on their equity between 1996 and 2019. Data is sampled at quarterly frequency. A total of 3,536 firms are included.

	RN-Skew		Slope	
	(1)	(2)	(3)	(4)
IV	12.43	12.26	4.14	4.10
	(27.90)	(27.11)	(28.95)	(28.80)
Size	-10.37	-8.47	-2.85	-2.49
	(-7.63)	(-6.30)	(-8.74)	(-7.94)
Leverage	-1.75	-1.76	-0.34	-0.34
	(-3.54)	(-3.49)	(-2.84)	(-2.85)
Profitability	-0.10	-3.43	0.21	-0.55
	(-0.37)	(-8.04)	(2.57)	(-4.92)
M2B	-6.17		-1.28	
	(-16.41)		(-12.01)	
GO		-4.68		-1.08
		(-12.33)		(-10.62)
Adj-R^2	0.50	0.50	0.54	0.54
FE Adj-R^2	0.48	0.48	0.52	0.52

effects regressions, thus suggesting that a relevant amount of variation in the data is attributable to time-series variation in the regressors: high value of real options, proxied by the market-to-book and GO ratios, present more negatively slopped implied volatility curves (more negative skewness). As Appendix Table A2 shows, the negative relation between implied skewness and measures of real options is also present in 180 and 360 day maturity options.

Some of these relationships are not easily explainable in the context of a model such as those of Toft and Prucyk (1997) and Geske, Subrahmanyam, and Zhou (2016), which can describe very well the impact of leverage and the level of volatility, but not how size and market-to-book values affect option prices.

Overall, implied volatility surfaces or risk-neutral moments present a pretty consistent picture of the cross-section (across strikes) of option prices: There is large time-series and cross-sectional variation in standardized option prices. While it is entirely possible that such variation can be explained by exogenously specifying the equity and volatility process as in Bakshi, Cao, and Zhong (2021), we propose a structural approach based on the idea that optimal firm decisions shape the physical and risk-neutral distributions of equity returns. The results reported in Table 1 are consistent with this approach.

4.2. Real options and future realized skewness

While our theoretical analysis is mostly centered around equity option pricing, the relation between the future distribution of equity prices and real options can be studied also in the physical measure. Thus, in this section we show how variability in future realized skewness relates to measures of real options. Table 2 reports results of panel regressions of future skewness realized at different time horizon of 90, 180, and 360 days. We present realized skewness at increasing future time horizons to mitigate the concern that measuring realized skewness from daily log returns is quite challenging, see for example Neuberger (2012) and Kozhan, Neuberger, and Schneider (2013). All specification includes firm and time fixed effects. We present results for realized idiosyncratic skewness (see Mitton and Vorkink, 2007, for the different implications of total and idiosyncratic measures) from the linear and quadratic market model (as in Harvey and Siddique, 2000) in Appendix Table A3.

Table 2: Future realized skewness and real options

The table shows regression results of future realized skewness on measures of real options value. Independent variables are measured at day t, while future realized skewness is extracted from log returns of days $t+1$ to $t+T$, where $T = 90, 180, \text{or } 360$. Real options measures include the market-to-book ratio and the GO ratio of Trigeorgis and Lambertides (2014b). We control for size, book leverage, profitability, and the level of ATM IV as in Table 1, as well as a lag of the dependent variable, turnover, cumulative returns from t-12 to t-1 (momentum), and a Nasdaq indicator variable as in Boyer, Mitton, and Vorkink (2010), and a measure of R&D intensity and investments as in Del Viva, Kasanen, and Trigeorgis (2017). A constant is included but not reported. All right hand side variables are winsorized at the first and 99^{th} percentile, and subsequently standardized to aid comparability across specifications. We report parameter estimates (multiplied by 100) and standard errors clustered at the firm level. The sample contains all non-financial firms with options trading on their equity between 1996 and 2019. Data is sampled at quarterly frequency. A total of 3,536 firms are included.

	90 days				180 days				360 days			
IV	2.93	2.65	7.01	7.15	1.06	0.47	5.85	6.23	0.82	0.03	9.18	9.39
	(2.61)	(2.34)	(4.24)	(4.32)	(0.72)	(0.32)	(2.60)	(2.78)	(0.42)	(0.02)	(3.19)	(3.26)
Size	-0.72	0.05	-2.58	-1.08	0.27	2.20	-5.35	-1.16	1.81	5.60	-3.35	1.61
	(-0.32)	(0.02)	(-0.94)	(-0.41)	(0.08)	(0.72)	(-1.24)	(-0.28)	(0.41)	(1.32)	(-0.52)	(0.26)
Leverage	1.81	1.23	3.38	3.24	3.46	3.21	4.33	4.25	3.72	4.00	6.75	6.71
	(1.99)	(1.37)	(2.90)	(2.83)	(2.63)	(2.47)	(2.47)	(2.48)	(2.09)	(2.26)	(2.78)	(2.80)
Profitability	2.74	-4.42	4.68	-3.77	0.76	-5.71	1.94	-4.87	1.45	-4.55	0.29	-5.66
	(3.30)	(-4.11)	(4.36)	(-2.75)	(0.70)	(-3.86)	(1.33)	(-2.36)	(1.08)	(-2.44)	(0.16)	(-2.02)
M2B	-8.33		-9.67		-10.95		-14.66		-14.46		-16.20	
	(-9.16)		(-7.68)		(-8.43)		(-7.27)		(-7.95)		(-5.68)	
GO		-11.28		-12.36		-10.17		-8.99		-8.85		-6.73
		(-11.44)		(-10.22)		(-8.29)		(-5.53)		(-5.47)		(-3.10)
Lag Skew			-4.63	-4.70			-8.38	-8.67			-15.96	-16.19
			(-6.40)	(-6.50)			(-8.94)	(-9.22)			(-10.89)	(-11.06)
Turnover			-2.98	-3.27			-3.42	-4.06			-6.05	-6.85
			(-2.66)	(-2.94)			(-2.18)	(-2.60)			(-3.01)	(-3.41)
Momentum			-2.73	-4.71			-1.37	-4.80			1.76	-2.08
			(-3.64)	(-6.92)			(-1.27)	(-4.92)			(1.24)	(-1.65)
Nasdaq			-0.95	-0.31			-2.03	-1.99			-2.44	-2.66
			(-0.36)	(-0.12)			(-0.49)	(-0.49)			(-0.44)	(-0.48)
R&D			-0.63	-1.12			-5.16	-6.19			-8.31	-10.01
			(-0.45)	(-0.84)			(-2.32)	(-2.86)			(-2.23)	(-2.74)
Investment			-2.59	-2.66			-1.47	-1.62			-1.85	-1.99
			(-3.78)	(-3.89)			(-2.34)	(-2.58)			(-2.85)	(-3.07)
Adj-R^2	0.03	0.03	0.04	0.04	0.09	0.08	0.11	0.10	0.18	0.18	0.20	0.19
FE Adj-R^2	0.03	0.03	0.03	0.03	0.08	0.08	0.09	0.09	0.18	0.18	0.18	0.18

The regression results reported in Table 2 are in line with those reported in Table 1: realized skewness is negatively associated with the size of the firm (i.e., large firms tend to have more negative skewed distributions and steeper implied volatility curves), positively related to the level of implied volatility (i.e., total risk), and show uncertain signs relative to profitability depending on the set of variables included in the regression. The coefficients related to real-option values are negative and largely significant. Differently from its risk-neutral expectation, realized skewness is not negatively associated with leverage, as its coefficients are positive and significant. Including other controls that have been used to study similar measures of future realized skewness (as in Boyer, Mitton, and Vorkink, 2010, and Del Viva, Kasanen, and Trigeorgis, 2017) leaves the magnitude and significance of the real options measures unchanged.

The economic magnitude of the relation between real option value and skewness is relatively stable across different time horizons (i.e., future period used to construct realized skewness), and is larger than that reported for option implied skewness: one standard deviation change in the GO index is related to a change of -0.12 if future realized skewness and of -0.04 in implied skewness at 90 days horizon (all variables are standardized and tabulated coefficients are multiplied by 100).

4.2.1. Revisiting previous findings

Del Viva, Kasanen, and Trigeorgis (2017) report a positive correlation between future idiosyncratic skewness and the GO index. We repeat their exercise in the same sample period (i.e., between 1983 and 2011), but using our quarterly sampled data. Each regression includes firm and time fixed effects. Standard errors are clustered at the firm level, and results are presented in Table 3. Results for future realized skewness are reported in Appendix Table A4.

Column (1) of Table 3, which uses the same controls as Column (1) of Table 2 of Del Viva, Kasanen, and Trigeorgis (2017), shows a positive relation between future idiosyncratic skewness and GO. The coefficient is smaller than the one reported by Del Viva, Kasanen, and Trigeorgis (2017) and less statistically significant. The difference in the economic magnitude could be due to a number of choices in constructing the sample and the fact that our data is sampled at quarterly frequency. Statistical significance is also lower and that is likely due to the fact that we cluster standard errors at firm level. Adding additional controls turn the GO coefficient to insignificant first and then negative and significant. Adding an interaction term between GO and an indicator variable that is set to one when equity options are available for a firm-quarter observation points to the negative relation between GO and skewness being more negative for optionable stocks. Since the sample includes a period before 1996, when Optionmetrics become first available, this indicates that the more negative correlation is either due to stocks that are optionable after 1996 or to a difference in the entire sample before and after 1996. Since the coefficient reported in Table 3 is about four times as large as that reported in the equivalent regression specification in Appendix Table A3 (i.e., -40 relative to -11), which only includes optionable stocks after 1996, we conjecture that the difference in mostly due to the sample period.

In summary, the evidence presented in Table 3 suggests that the positive relation between GO and future idiosyncratic skewness reported by Del Viva, Kasanen, and Trigeorgis (2017) is due to a multitude of factors, and cannot be found for optionable stocks in the most recent period (after 1996).

5. Basic intuition

As described in the previous section, the data displays a very large amount of variability in the cross-section and in the time series. While it is possible to explain that much variation in equity option prices in the context of (reduced form) option pricing models that allow the underlying price to follow a non-gaussian distribution, as for example by introducing jumps and stochastic volatility, such an effort would require a substantial variation in parameters across firms and time (see for example, Geske, Subrahmanyam, and Zhou, 2016; Bakshi, Cao, and Zhong, 2021). The goal of our theoretical analyses is to understand whether some variability in the data, and hence in the parameters of reduced form models, can be interpreted from the point of view of a structural model of a firm, in the spirit of Merton (1974).

In this section, we convey the main economic intuition by means of a stripped-down version of the quantitative model that we present in Section 6. We start from the neoclassical model of Zhang (2005) and make some simplifications.

As in Zhang (2005), the model has infinite horizon and is in discrete time. The firm's productivity shock, z, follows a log-AR(1) process with parameters ρ and σ. The cash flow from operations is the result of applying the current productivity shock to a production function where capital exhibits decreasing returns to scale and accrues some operating costs:

$$\pi(z, k) = e^z k^\alpha - fk,$$

Table 3: Future idiosyncratic skewness and real options in sample period of Del Viva, Kasanen, and Trigeorgis (2017)

The table shows regression results of future realized idiosyncratic skewness on the GO ratio of Trigeorgis and Lambertides (2014b). Independent variable is measured at day t, while future idiosyncratic skewness is extracted from the residuals of regressions of log returns on the corresponding log market return of days $t+1$ to $t+360$. We control for size, book leverage, profitability as in Table 1, as well as a lag of the dependent variable, past realized volatility from the previous 90 days, quarterly average turnover, cumulative returns from $t-12$ to $t-1$ (momentum), a Nasdaq indicator variable as in Boyer, Mitton, and Vorkink (2010), and a measure of R&D intensity and investments as in Del Viva, Kasanen, and Trigeorgis (2017). In the last specification we also include an indicator variable that is set to one when equity options are available for a firm-quarter observation. A constant is included but not reported. All right hand side variables are winsorized at the first and 99^{th} percentile, and subsequently standardized to aid comparability across specifications. We report parameter estimates (multiplied by 100) and standard errors clustered at the firm level. The sample contains all non-financial firms with options trading on their equity between 1983 and 2011. Data is sampled at quarterly frequency. A total of 5,616 firms are included.

	(1)	(2)	(3)	(4)	(5)
Past Skew		-9.87	-10.06	-10.39	-10.20
		(-10.58)	(-10.44)	(-10.75)	(-10.57)
Vol		10.31	13.93	10.41	10.15
		(8.74)	(11.30)	(8.33)	(8.15)
Size				-10.99	-7.65
				(-3.18)	(-2.23)
Leverage				5.09	4.46
				(3.57)	(3.13)
Profitability				-1.38	-1.35
				(-0.99)	(-0.97)
GO	3.68	1.08	-1.02	-3.47	-1.89
	(3.38)	(0.87)	(-0.80)	(-2.42)	(-1.26)
Optionable					13.54
					(1.88)
Optionable × GO					-39.97
					(-4.31)
Turnover			-13.55	-11.28	-9.38
			(-10.28)	(-8.25)	(-6.81)
Momentum			-9.27	-9.91	-9.42
			(-11.32)	(-11.80)	(-11.23)
Nasdaq			-3.34	-6.42	-5.77
			(-1.21)	(-2.35)	(-2.09)
R&D	-1.82	-1.61	-0.84	-1.71	-1.48
	(-1.21)	(-0.92)	(-0.46)	(-0.93)	(-0.81)
Investment	-3.59	-3.25	-2.20	-2.16	-1.98
	(-6.43)	(-5.26)	(-3.45)	(-3.35)	(-3.06)
Adj-R^2	0.09	0.11	0.11	0.12	0.12
FE Adj-R^2	0.09	0.10	0.10	0.10	0.10

where $0 < \alpha < 1$ and $f \geq 0$. We model the operating cost as proportional to capital to capture the effects of mechanisms that are present in the main model such as financial leverage, and depreciation, which, contrary to a fix operating leverage, have an impact on the investment/disinvestment choice (i.e., a fix cost would not affect the investment decision).

At time $t = 1$, the equity holder invests $i = k' - k$ (i.e., there is no depreciation), and investment/disinvestment entails a quadratic asymmetric capital adjustment cost

$$h(i,k) = \frac{1}{2}\left(\theta_1 \mathbb{1}_{\{i>0\}} + \theta_2 \mathbb{1}_{\{i<0\}}\right)\left(\frac{i}{k}\right)^2 k.$$

The firm makes no other investment decisions after period 1 and lives in perpetuity. We refer to this one time ability to change the stock of productive capital as the real option (i.e., to distinguish it from the financial option on the equity), with the general understanding it comprises the ability to both increase or decrease capital stock depending on economic conditions (i.e., the realization of the productivity shock). Investment is financed either with internal or external equity. There are no transaction costs when raising external equity financing. We assume investors do not require any risk premia and therefore securities are priced by a constant discount factor $0 < \beta < 1$.

Because we are interested in pricing financial options written on the firm's equity, we assume the following timeline: at $t = 0$, European call (and put) options are written, with strike price X, and maturity $t = 1$. The options are written after z_0 has been observed and k has been decided. At $t = 1$, the realization z_1 comes from a log-normal distribution with mean ρz_0 and standard deviation σ, and that determines the choice of k'. Hence, the value of equity at $t = 1$ is

$$S(z_1) = \max_{k'} \left\{\pi(z_1, k) - i - h(i,k) + V(z_1, k')\right\}$$

where $V(z_1, k')$ is the continuation value under the assumption that investment in all future periods is 0 (i.e., capital remains at level k'). [1] Therefore, one can think about the firm has having some capital in place plus one real (investment/disinvestment) option.

[1]Given the distributional assumptions on the productivity shock,

$$V(z_1, k') = \mathbb{E}_1\left[\sum_{j=1}^{\infty} \beta^j \pi(z_{1+j}, k')\right] = \Phi(z_1)(k')^\alpha - \frac{\beta}{1-\beta} f k'$$

where

$$\Phi(z_1) = \sum_{j=1}^{\infty} \beta^j \mathbb{E}_1\left[e^{z_{1+j}}\right] = \sum_{j=1}^{\infty} \beta^j \exp\left\{\rho^j z_1\right\} \exp\left\{\frac{\sigma^2}{2}\frac{1-\rho^{2j}}{1-\rho^2}\right\} < \infty$$

Denoting the solution of the equity program contingent on z_1 as $k^*(z_1)$, and the optimal investment as $i^*(z_1) = k^*(z_1) - k$, the equity value is

$$S(z_1) = \pi(z_1, k) - i^*(z_1) - h(i^*(z_1), k) + V(z_1, k^*(z_1)).$$

Hence, the price at $t = 0$ of a European call option with strike X maturing at $t = 1$ is

$$C_0(X) = \beta \int \max\{S(z_1) - X, 0\} \varphi(z_1) dz_1,$$

where $\varphi(\cdot)$ is the standard normal density.

Aside from the adjustment to the capital stock at time 1, some other elements of the model have implications for the pricing of options. We detail those first to create a baseline. In Figure 3 we present some comparative statics of the impact of relevant model parameters on the distribution of equity returns and the implied volatility surface, once we remove the investment decision. In this version of the model the firm starts with an initial capital that cannot be changed.

In the top row, we vary the coefficient of autocorrelation of the productivity shock. The case with $\rho = 1$ provides a good comparison case, as it essentially reduces to a Black and Scholes economy: the equity return is equal to 3% (i.e., $r - 0.5\sigma^2,\ r = 5\%$), equity volatility is equal to 20% (i.e., the volatility of the productivity shock), equity skewness is equal to 0, and the implied volatility curve is flat and leveled at 20%. As the autocorrelation decreases the continuation value of the firm becomes less valuable and less volatile (as a function of z_1). Because the drop in the second moment is much larger than the drop in the third moment, a decrease in autocorrelation leads to an increase in skewness, thus shifting mass to the right. As a consequence the implied volatility curve lowers and slightly steepens (positively) with moneyness.

---

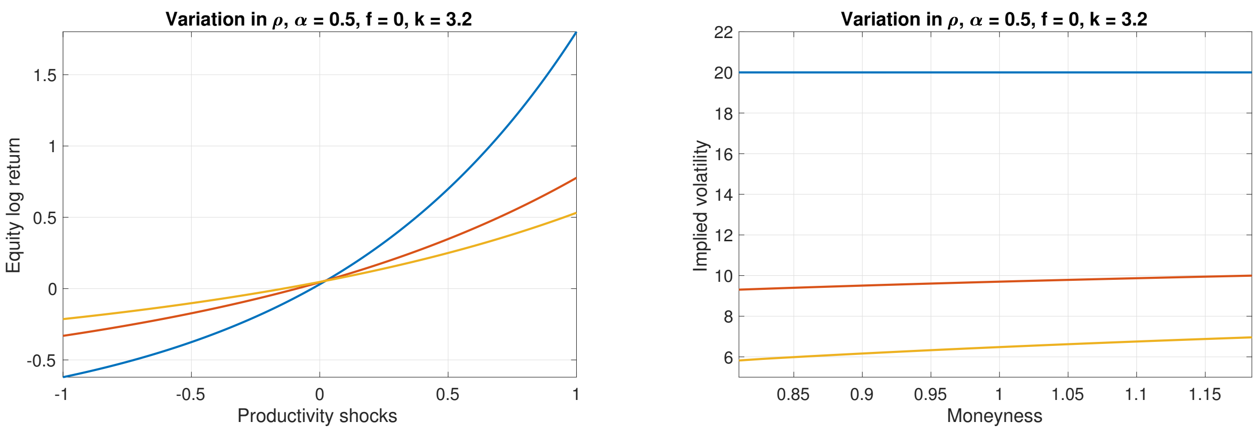
can be easily calculated. In the equity program, the optimal k' is found solving the first order condition

$$\alpha(k')^{\alpha-1}\Phi(z_1) - \frac{\beta}{1-\beta} f = 1 + \left(\theta_1 \mathbb{1}_{\{k'>k\}} + \theta_2 \mathbb{1}_{\{k'<k\}}\right)\left(\frac{k'}{k} - 1\right)$$

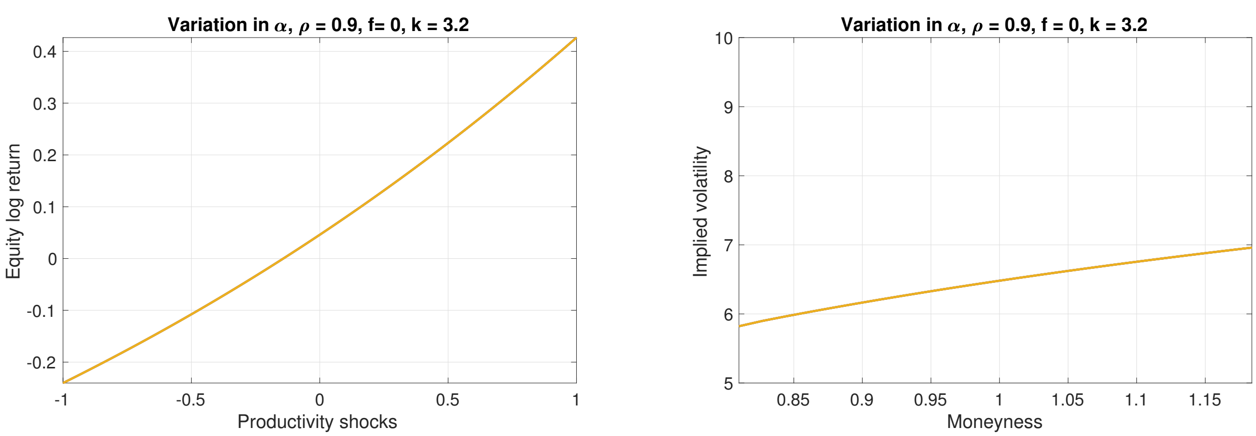
which can be found numerically using Newton's method.

Figure 3: Model with no investments

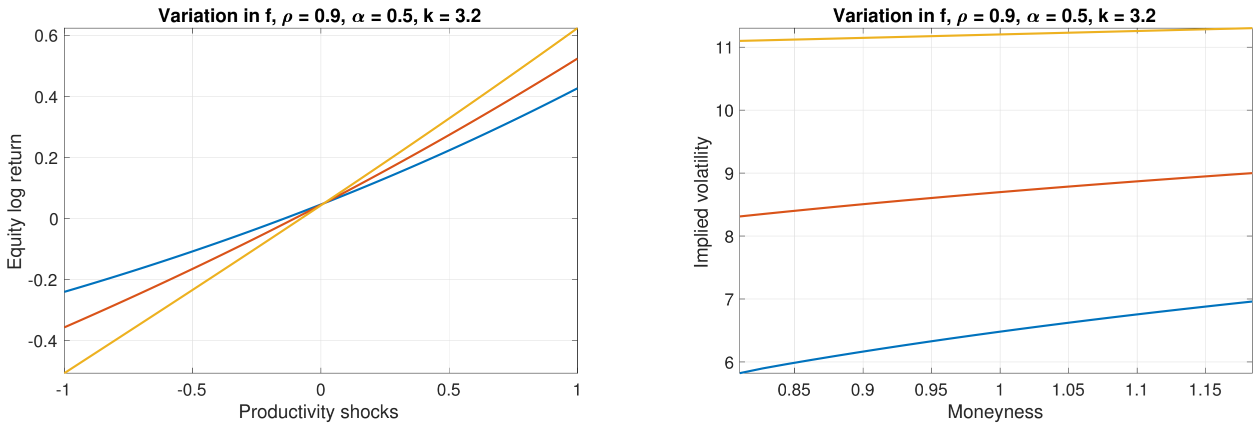
In the left column, the figure plots the log equity returns between 0 and 1 for different realizations of the log productivity shock at time 1. For each case, we include the expected value, volatility, and skewness in the figure's legend. The right column displays the implied volatility of the call option prices at time 1 for moneyness levels (i.e., ratio of strike to underlying price) between 0.8 and 1.2 . For each case, we include the slope of the implied volatility curve (i.e., IV calculated for a moneyness of 1.2 minus the IV for a moneyness of 0.8), the risk-neutral skewness, and the market to book ratio. We vary the autocorrelation of the productivity shock, $\rho \in \{1, 0.95, 0.9\}$, the curvature of the production function, $\alpha \in \{0.4, 0.5, 0.6\}$, and the leverage parameter, $f = \{0, 0.15, 0.25\}$. We fix the rest of the parameters as follows: $\sigma = 20\%$, $k = 3.2$, $\theta_1 = 0$, $\theta_2 = 0$, $\beta = e^{-0.05}$.



	μ	σ	Skew	Slope	RN-Skew	M2B
$\rho = 1.00$	3.00	20.00	0.00	0.00	-0.00	11.12
$\rho = 0.95$	4.52	9.77	0.11	0.61	0.09	12.58
$\rho = 0.90$	4.78	6.61	0.17	1.00	0.11	11.90



	μ	σ	Skew	Slope	RN-Skew	M2B
$\alpha = 0.4$	4.78	6.61	0.17	1.00	0.11	10.59
$\alpha = 0.5$	4.78	6.61	0.17	1.00	0.11	11.90
$\alpha = 0.6$	4.78	6.61	0.17	1.00	0.11	13.37



	μ	σ	Skew	Slope	RN-Skew	M2B
$f = 0.00$	4.78	6.61	0.17	1.00	0.11	11.90
$f = 0.15$	4.61	8.77	0.11	0.61	0.08	8.97
$f = 0.25$	4.37	11.22	0.03	0.18	0.02	7.02

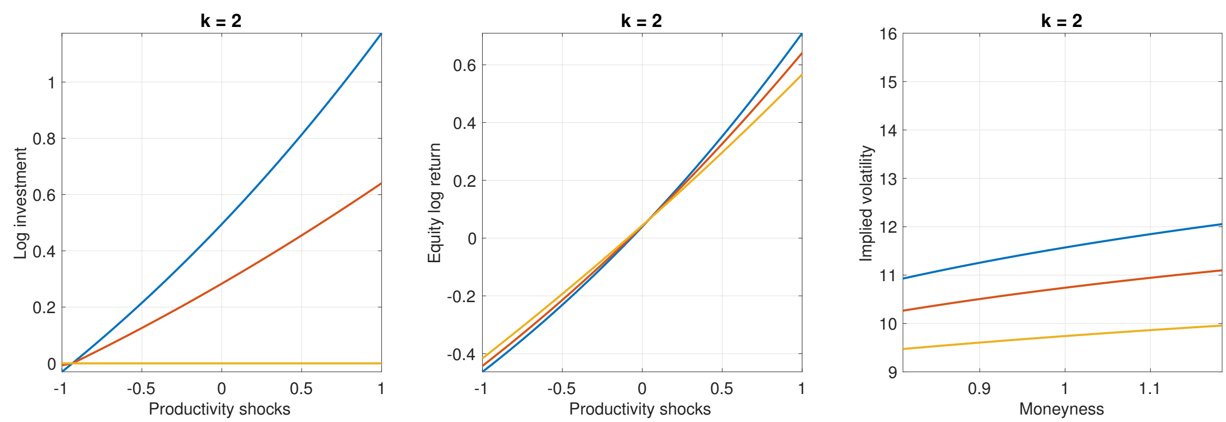
Increasing the curvature of the productivity function (shown in the middle two plots of the figure) lowers the equity volatility, but not the skewness: the impact of α on the first and second moment of the equity distribution is proportional. Increasing leverage (i.e., f) increases equity volatility and shifts the equity distribution to the left (shown in the bottom two plots in the figure). Similarly to Geske (1979), Toft and Prucyk (1997), and Morellec and Zhdanov (2019) this reduces the slope of implied volatility curve, and for sufficiently larger values of f turns the slope negative. Thus, for a constant production technology and dynamic of productivity, leverage impacts the pricing of financial options.

Because adding the real option increases the convexity of the equity value, it also affects its distribution. van Zwet (1964) shows that the convexity of a function of a random variable is related to the skewness of the distribution of the function of the random variable, and that more convexity implies larger skewness. van Zwet (1964) is often cited in the growth option literature as a motivation for the idea that firms with more growth options (a convex function of the underlying) have more positively skewed distributions (see for example, Del Viva, Kasanen, and Trigeorgis, 2017; Bali, Del Viva, Lambertides, and Trigeorgis, 2020). We detail how the skewness of the equity distribution is induced by a real option that consist of a combination of an option to buy and an option to sell (i.e., a “real straddle”).

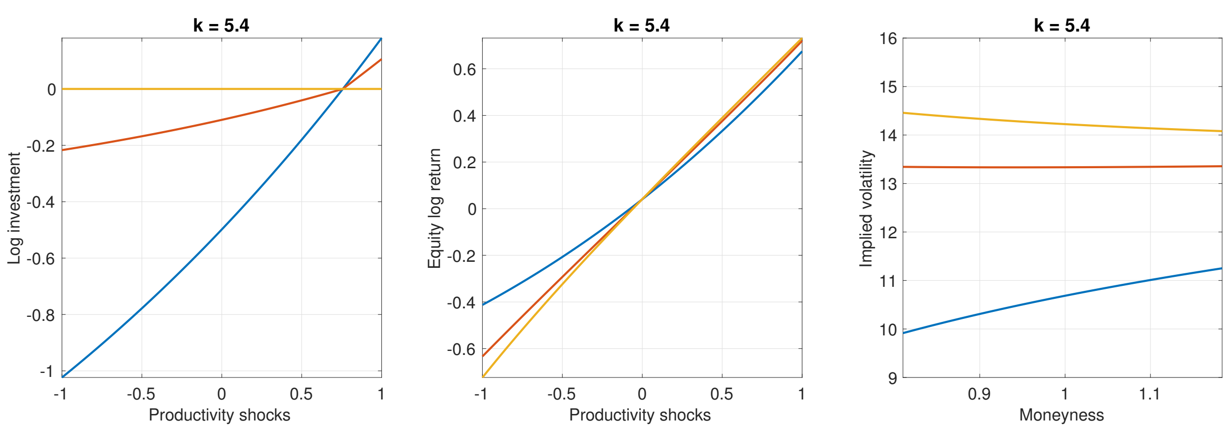
In Figure 4 we show how introducing the real option affects the skewness of equity distribution and the related implied measures, in the general case where $\rho < 1$. Cognizant that investments and disinvestments do not have symmetric effects, because the capital adjustment cost function is asymmetric, we consider a firm that, given the current shock,

Figure 4: Model with investments

The figure plots the log investment and equity returns between 0 and 1 for different realizations of the log productivity shock at time 1 (i.e., x' or x_1), as well as the implied volatility curve at time 0 derived from options that matures at time 1. We compare the case with no investments to the solution of the model with the investment/disinvestment option with and without capital adjustment costs. We show results for a firm that will mostly invest at time 1 (i.e., $k = 2$), and for one that will mostly disinvest (i.e., $k = 6$). In the legend of the central figure we also report the present value of the real option, PVRO (i.e., present value of real option), which combines both the investment and the disinvestment options, and is defined as the difference between the market-to-book ratio of the solution with investment and the market-to-book ratio of the constrained model.When the firm faces adjustment costs we set $\theta_1 = 2$ and $\theta_2 = 10$. We fix the rest of the parameters as follows: $\rho = 0.9$, $x_0 = 0$, $\sigma = 20\%$, $f = 0.25$, $\beta = e^{-0.05}$.



	μ_i	σ_i	Skew	PVRO	Slope	RN-Skew
No Adj Costs	0.50	0.12	0.17	0.49	1.13	0.16
Adj Costs	0.28	0.07	0.13	0.27	0.83	0.12
No Inv			0.08		0.49	0.06



	μ_i	σ_i	Skew	PVRO	Slope	RN-Skew
No Adj Costs	-0.50	0.12	0.20	0.28	1.34	0.19
Adj Costs	-0.11	0.03	0.00	0.07	0.01	-0.00
No Inv			-0.06		-0.38	-0.06

is investing for almost all future realization of the productivity shock (i.e., $k = 2$ top row) and one that almost always disinvest (i.e., $k = 5.4$ bottom row). For each case, we compare the case with constant capital, to the case with the real option with and without capital adjustment costs.

We start by considering the base case scenario where the firm cannot change its capital stock at time 1 (yellow line). Going directly to the implied volatility plots, we observe that there is a size effect: The curve for the small firm is upward sloping, while the opposite is true for the large firm. This is the result of modeling operating leverage as a proportional cost, which produces the same effect of debt (without requiring further complications). The intuition is as follow: the implied volatility (at any strike) represents the volatility of the log equity price. Assume that the firm is liquidated at date 1, after the shock is realized. Then future equity value is simply the value of the cash flow realized at that point in time: $\pi_1 = e^{z_1}k^{\alpha} - fk$. This function is always convex relative to z_1. However, the convexity of $\ln(\pi_1)$ depends on how large k is (for a fixed α and f): it becomes concave for a sufficiently large k. Adding back the continuation value, so that the firm is not liquidated at time 1, makes the concavity of the value function even more parameter dependent. The convexity/concavity of the log equity price function is what determines the slope of the implied volatility curve. We can see this by comparing the yellow lines in the top and bottom right most panels of Figure 4, where as the log equity curve becomes more concave/less convex the slope of implied volatility curve becomes less positive/more negative. The leverage effect is a function of size, which is largely in line with the empirical evidence presented in Table 1, and with the existing literature.

As mentioned above, whether it implies increasing or decreasing the capital stock, the real option always adds (positive) skewness. Focus on the central panels of Figure 4. Consider first the case without adjustment costs (blue line): for the small firm (top), relative to the firm that does not invest, the ability to increase productive capital increases equity returns in good states and decreases them during bad times. However the effect is more prominent in good times, so that the net result is to shift the distribution to the right. Conversely, for the large firm (bottom), the ability to downsize increases equity returns during bad realization of the profitability shock, and decreases them in good times. The effect is much more pronounced in negative states of nature: the result is again to push more mass to the right. The resulting implied volatility curves exhibit an upward slope. Adding adjustment costs reduces future investment and disinvestments and their contribution to the equity value: since costs are asymmetric, downsizing produces a more pronounced effect. As they constrain the optimal policy, adjustment costs also dampen the skewness in the equity value function and thus produce a less skewed equity return distribution and a lower positively slopped implied volatility curve. The larger the costs, the less skewed the distribution, the lower the expected present value of the real option value (PVRO). PVRO is the difference between the equity value, at time 1, when the firm can adjust the capital ratio and the corresponding value when it cannot, scaled by the current capital stock: in expected value terms, this is the difference between the respective *current* equity values scaled by the current capital stock. In other words, this is the component of the market-to-book ratio that captures only the value of the real option, controlling away for how leveraged the firm is and how large its current capital stock is.

As Table 1 shows, controlling for leverage, larger firms and firms with high market-to-book ratios tend to have more negative skewed risk-neutral distributions. We have showed above how the size effects works. We focus here on the negative relation between PVRO and RN-skewness/slope of the IV curve. To illustrate how one could obtain such relation, we compare firms of different initial productive capital, k_0, and different future prospects, which depends on the initial state x (as the profitability shock is mean-reverting). Because the production function exhibits decreasing returns to scale, small firms, everything else equal, will invest more in the future when economic conditions are good, while large firms will have an incentive to reduce capital in place during bad times. Moreover, because productivity shocks are persistent, firms that face negative productivity shocks at time 0, x (i.e., or x_0), face worse future conditions than firms that are exposed to positive shocks at time 0.

The impact of these two effects on the relation between PVRO and the shape of the future equity distribution depends on the magnitude of the adjustment costs parameters, which alter the future investment policy as shown in Figure 4. We provide here an example of how a particular choice of parameters can produce the economic forces necessary to reproduce, qualitatively at least, the relationships described in Table 1. We plot investments and implied volatility curves in Panel A and B of Figure 5 for different levels of k and of x.

Consider first Panel A. The small firm (top row) mostly invests, and it does so even when future economic conditions are not positive. For example, consider the future scenarios that stems from the current state $x = 0$: even when $x' < 0$, as long as it is not too negative, the optimal decision is to increase the capital stock. The large firm (bottom row) mostly disinvests, and does so even when future shocks are positive. In contrast the intermediate firm has a much more mixed optimal policy: it invests when the future shocks are positive and disinvests when they are negative (we chose k in order to produce this precise split at $x' = 0$).

Both investment and disinvestment options are valuable. However, the extent to which they contribute to the current value of the equity depends on their "moneyness". We can think of the small firm as currently having a deep in-the-money investment option and an out-of-the-money disinvestment option, since it will mostly invest in period 1, regardless of future economic conditions. Similarly, the large firm has an out of the money investment option and an in-the-money disinvestment option, as it most likely disinvest. In both cases, the value of the bundle is driven by the option that is probably going to be exercised. We can instead think of the intermediate firm has having both an option to invest and one to disinvest, that are almost exactly at-the-money, and thus on average produce a small expected investment that does not move the value of the firm away from the value of the constrained firm. Since, one can think about the real option as a combination of a call (invest) and a put (disinvest), the value of the combination (i.e., a straddle) depends on the absolute value of the future investment. We can see the value being very large when either the average investment or the average disinvestment are very large in the top right and bottom left plots, respectively. The current value of the real option is instead lower, when the expected investment is small (central panel).

Table 4: Distributional properties of PVRO

The table reports expected value, standard deviation, and skewness of PVRO, the present value of the real option, defined as the difference between the equity value when the firm can adjust the capital ratio and the corresponding value when it cannot, scaled by the current capital stock. We vary the initial capital stock $k \in \{2, 3.2, 6\}$, and the productivity shock $x \in \{-0.5, 0, 0.5\}$. We fix the rest of the parameters as follows: $\rho = 0.9$, $\sigma = 20\%$, $f = 0.25$, $\theta_1 = 2$, $\theta_2 = 10$, $\beta = e^{-0.05}$.

	$x = -0.5$	$x = 0.0$	$x = 0.5$
	Expected value		
$k = 2$	0.07	0.28	0.76
$k = 3.2$	0.02	0.01	0.12
$k = 6$	0.17	0.10	0.03
	Standard deviation		
$k = 2$	0.06	0.15	0.30
$k = 3.2$	0.01	0.02	0.10
$k = 6$	0.04	0.03	0.02
	Skewness		
$k = 2$	1.62	1.10	0.92
$k = 3.2$	0.96	4.13	1.64
$k = 6$	0.02	0.23	0.82

The real option affects the equity distribution not only through how much value it produces, but also by moving value across different states of nature. As Table 4 shows, the expected PVRO is not only higher when firms have more defined investment policies (current situations that lead them to either almost always invest or almost always disinvest in the future), but it is also more volatile in the same situations. For example, when the small firm currently has just observed a very good productivity shock (i.e., $k_0 = 2,\ x_0 = 0.5$), the expected PVRO is large at 0.76, but so is its volatility, at 0.3. The combined effect is to produce a positive skewness of 0.92. On the opposite end, when the same small firm has just observed a negative shock, the expected PVRO is low and so is the volatility, but the probabilistic reallocation of value from negative to positive states of the world is larger, with a positive skewness of 1.62. Looking at the intermediate firm, we see that the highest PVRO skewness corresponds to the case $x_0 = 0$, when the investment policy is split exactly in half: half the times it invest and half the times it disinvests. Thus, for this particular choice of parameters and distribution of initial capital, there is a marked negative correlation between the expected PVRO and its skewness. In this example, we learn that, controlling for leverage and size, the real options add (positive) skewness to the distribution of the equity value, but they do so more prominently when there is more curvature in the real option function. To create a parallel to the Black-Scholes model, the real option can be thought of as a straddle, a combination of a call and a put. The curvature of each of the two functions (i.e., gamma) is highest when the options are exactly ATM.

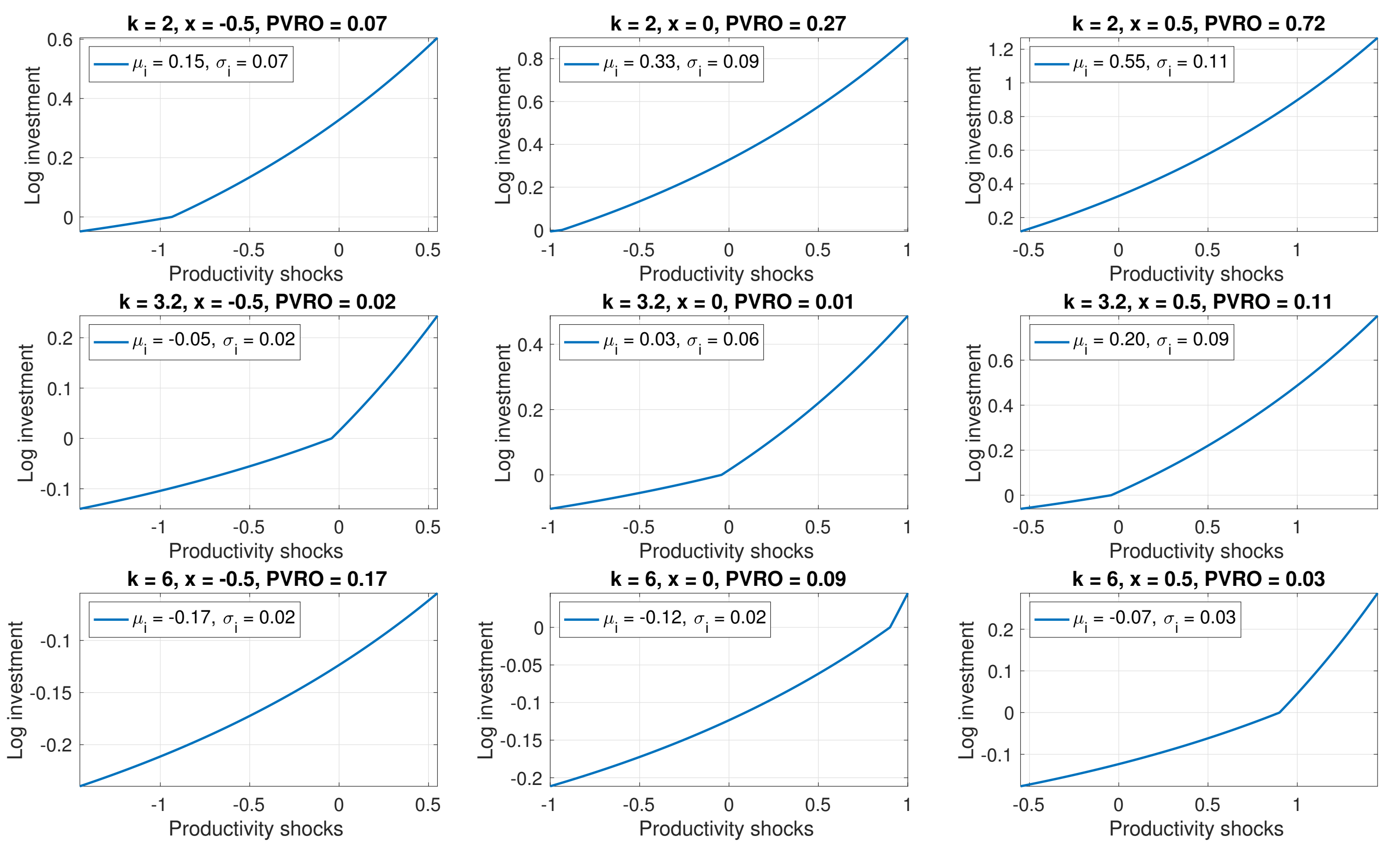
We now turn to Panel B of Figure 5. There are two economic forces affecting the shape of the implied volatility curve: leverage and the real option. Leverage induces negative skewness and a negative slope of the IV curve, the real option does the exact opposite. However, the impact of the real option on skewness is inversely related to how "in-the-money" that option is (the real option is a combination of a call and a put). Moreover, the effect of leverage is decreasing with economic conditions: when prospects are good, leverage induces less negative skewness. As the figure shows, combining those two effects together leads to a situation where, for small and large firms, there is a negative relation between the slope of the IV curve (or the risk-neutral skewness) and the PVRO. The same relation is u-shaped for the intermediate firm.

While the qualitative impact of the two effects is unchanged by parameter choices (i.e., leverage always induces negative skewness and the real option always induces positive skewness, especially when the option value is the most convex), the way they combine together to create decreasing or increasing implied volatility curves is state (in the simple model that is k_0 and z_0) and also parameter dependent. It is therefore reasonable to ask whether the same net effect can be obtained in the contest of a model that is calibrated to match other features of the data. We present an extended model, its calibration, and the relationship between properties of the risk-neutral distribution and firm characteristics within the simulated economy, in the next two sections.

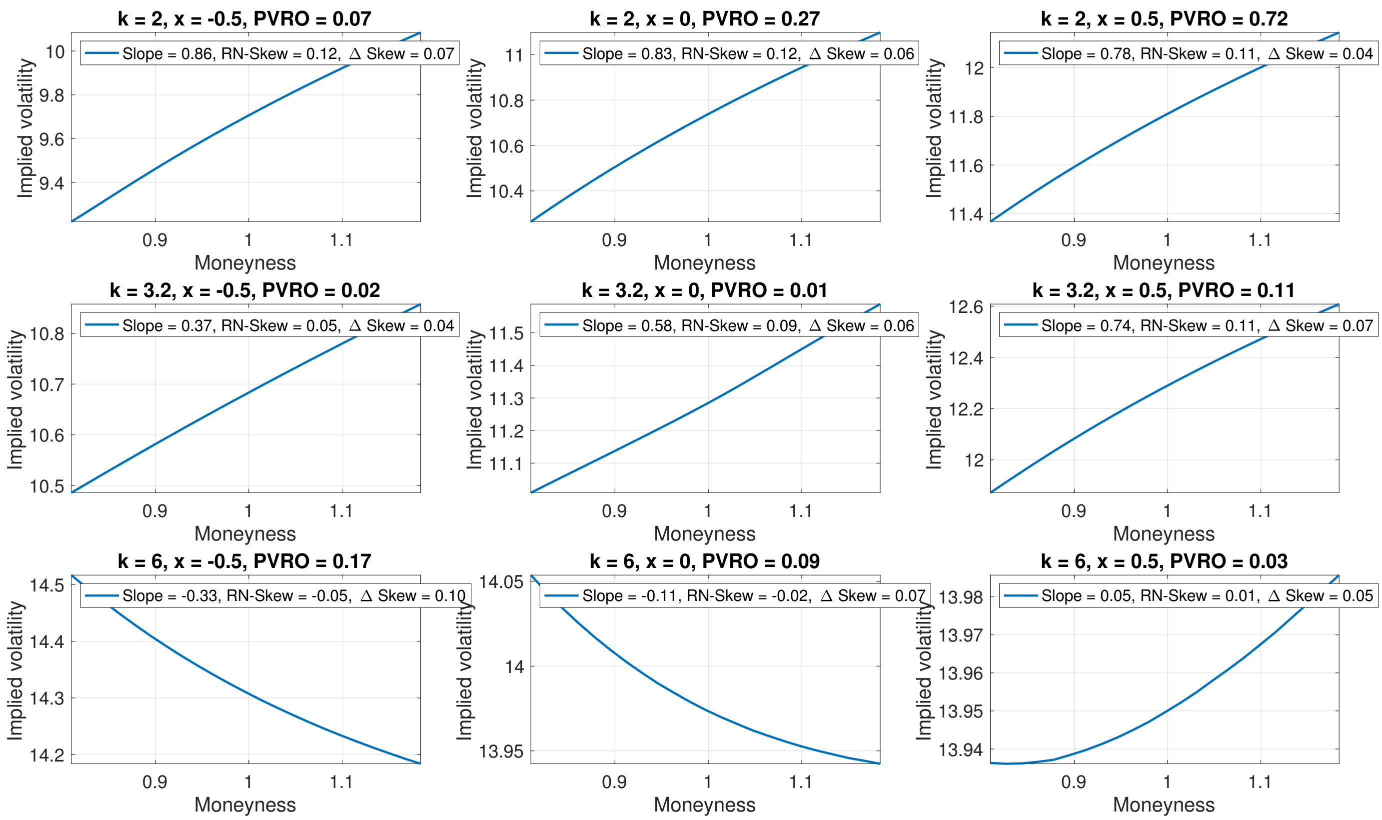
Figure 5: Investments and implied volatilities

The figure plots investments at time 1 and implied volatility curves at time 0 derived from options that matures at time 1. We vary the initial capital stock $k_0 \in \{2, 3.2, 6\}$, and the productivity shock $x_0 \in \{-0.5, 0, 0.5\}$. We fix the rest of the parameters as follows: $\rho = 0.9$, $\sigma = 20\%$, $f = 0.25$, $\theta_1 = 2$, $\theta_2 = 10$, $\beta = e^{-0.05}$.

Panel A: Investments at $t = 1$



Panel B: Implied volatilities at $t = 0$



6. Quantitative model

While the basic intuition developed in the previous section confirms the basic patterns in the data, whether a model of endogenous real options can give a quantitative description of the data is a question that can only be answered by calibrating a model that includes more realistic features such as; heterogeneity, endogenous default, corporate taxes, real adjustment costs, external equity financing frictions, debt adjustment costs, and considers countercyclical risk premia. The model is therefore similar, in spirit, to that of Hennessy and Whited (2007) in the description of the firm's decisions, and to those of Berk, Green, and Naik (1999), Zhang (2005) and Gomes and Schmid (2010) in the choice of a reasonably simple (exogenously specified) pricing kernel.

6.1. The economy

Information is revealed and decisions are made at a set of discrete dates $\{0, 1, \ldots, t, \ldots\}$. The time horizon is infinite. The economy is composed by a utility maximizing representative agent and a fixed number of heterogenous firms ($j = 1, \ldots, J$) that produce the same good. Firms make dynamic investment and financing decisions and are allowed to default on their obligations. Defaulted firms are restructured and then continue operations, so as to guarantee a constant number of firms in the economy. The agent consumes the dividends paid by the firms and saves by investing in the financial market. We do not close the economy and derive the equilibrium, but instead choose an exogenously specified stochastic discount factor.

There are two sources of risk that capture variation in the firm's productivity. The first, z_j, captures variations in productivity caused by firms' specific events. Idiosyncratic shocks are independent across firms, and have a common transition function $Q_z(z_j, z_j')$. z_j denotes the current (or time–t) value of the variable, and z_j' denotes the next period (or time–$(t+1)$) value.

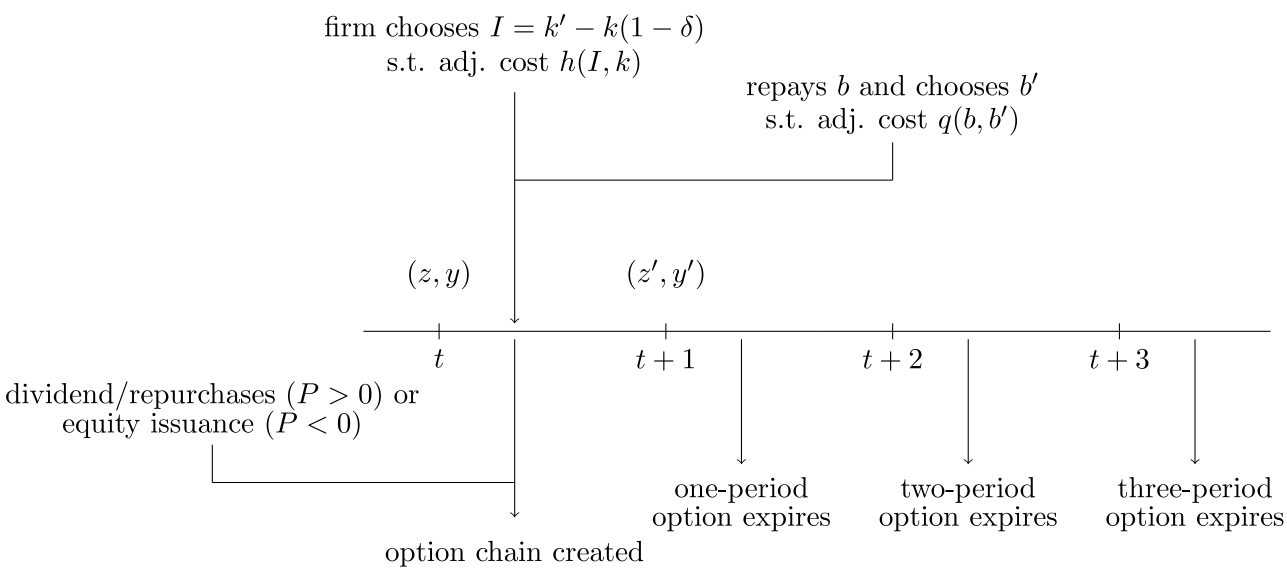
The second source of risk, y, captures variations in productivity caused by macroeconomics events. The aggregate risk is independent of the idiosyncratic shocks and has transition function $Q_y(y, y')$. Q_z and Q_y are stationary and monotonic Markov transition functions that satisfy the Feller property. z and y have compact support. For convenience of exposition, we define the state variable $x = (y, z)$, whose transition function, $Q(x, x')$, is the product of Q_y and Q_z. As there is no risk of confusion, we drop the index j in the rest of the section.

6.2. Firm policies

We assume that firm's decisions are made to maximize shareholders' value. An intuitive description of the chronology of the firm's decision problem is presented in Figure 6. At t, the two shocks $x = (y, z)$ are realized and the firm cash flow is determined based on current capital stock, k, and total face value of debt, b. Immediately after that, the firm simultaneously chooses the new set of capital, k', and debt, b' for the period $]t, t + 1]$. This decision determines P, the payout to shareholders, which can be positive (dividends and/or share repurchases) or negative (an injection of equity capital by issuing new shares).

Figure 6: Model time line

This figure offers a description of the chronology of the firm's recursive decision problem. At t, the shocks $x = (y, z)$ are realized, and the firm's cash flow is determined based on the capital stock k and the debt b, or $a = (k, b)$. Immediately after t, the firm chooses the new set of capital and debt, as the combination $a' = (k', b')$ that maximizes the value of the equity, given by the sum of the current cash flow plus the continuation value.



At t, the cash flow from operations (EBITDA) depends on the idiosyncratic and aggregate shocks, and on the current level of asset in place, $\pi = \pi(y, z, k) = e^{y+z}k^{\alpha} - f$, where $\alpha < 1$ models decreasing returns to scale and $f \geq 0$ is a operating cost parameter that summarizes all operating expenses excluding interest on debt.

The capital stock of the firm might change over time. The asset depreciates both economically and for accounting purposes at a constant rate $\delta > 0$. After observing the realization of the shocks at time t, the firm chooses the new capital stock k', which will be in operation during the period $]t, t + 1]$. The firm can either increase or decrease the capital stock, and the net investment equals to $I = k' - k(1-\delta)$. Similar to Abel and Eberly (1994) and many others after them, we assume that the change in capital entails an asymmetric and quadratic adjustment cost $h(I,k) = \left(\lambda_1 \mathbf{1}_{\{I>0\}} + \lambda_2 \mathbf{1}_{\{I<0\}}\right) I^2/(\delta k)$, where $0 < \lambda_1 < \lambda_2$ model costly reversibility, and $\mathbf{1}_{\{.\}}$ is the indicator function. The economic interpretation of λ_i, $i = 1, 2$, is straightforward: it is the per cent cost of a (dis)investment $I = \delta k$.

The debt level might also change over time. At any date, the firm can issue a one–period zero–coupon unsecured debt. As is shown in Figure 6, at time t the firm chooses the face value of the debt, b', that will be repaid at $t+1$. If the firm is solvent, the market value of the debt, $B(x, a')$, depends on the current state x and on the choices of the face value and the capital stock, $a' = (k', b')$, that are made after observing the shocks.

Changing the debt level entails a proportional adjustment cost, $\theta|b' - b|$, with $\theta \geq 0$. Since the issuance decision is contemporaneous to repayment of the nominal value of old debt b, the debt decision generates a net cash flow equal to $B(x, a') - b - \theta|b' - b|$.

We assume a linear corporate tax function with rate τ. The tax code allows deduction from the taxable income of the depreciation of assets in place, δk, and of interest expenses. Modeling deduction of the interest at maturity of the bond would entail keeping track of the value of the debt at issuance, therefore increasing the number of state variables. For the sake of numerical tractability, we assume that the expected present value of the end-of-period interest payment $b' - B(x, a')$, which we denote $H(x, a')$, can be expensed when the new debt is issued at time t. In case of linear corporate tax, and assuming knowledge of the equilibrium conditional default probability, this is equivalent to the standard case of deduction at $t+1$. The after–tax cash flow from operations plus the net proceeds from the debt decision is

$$v = v(x, a, a') = (1-\tau)\pi + \tau\delta k + \tau H(x, a') + B(x, a') - b - \theta|b' - b|. \tag{1}$$

The cash flow to equity is therefore equal to $w = w(x, a, a') = v - I - h(I, k)$ where, on the right-hand side, the first term is the after–tax cash flow from operations and the other terms are the net proceeds from (dis)investment. If the cash flow to equity is positive, the firm pays dividends and/or repurchases shares from the current shareholders; if the cash flow to equity is negative the firm issues new shares. In the latter case, the company incurs a proportional issuance cost $\zeta \geq 0$, as only w is the actual inflow to the corporation

$$P = P(x, a, a') = w \cdot (1 + \zeta \mathbf{1}_{\{w<0\}}). \tag{2}$$

6.3. The value of corporate securities

Following Berk, Green, and Naik (1999), Zhang (2005), and Gomes and Schmid (2010), we exogenously define a pricing kernel that depends on the aggregate source of risk, y. The associated one–period stochastic discount factor $M(x, x')$ defines the risk-adjustment corresponding to a transition from the current state y to state y'. We assume that M is a continuous function of both arguments.

The firm can issue two types of securities, debt and equity, whose equilibrium prices are determined under rational expectations in a competitive market. The cum–dividend price of equity, $S(x, a)$, is the sum of current payout, P, and the present value of the expected future optimal distributions, which is equal to the next period price $S(x', a')$. Since this sum can be negative, a limited liability provision is also included (i.e., default on a value basis), in which case the firm's equity is worthless:

$$S(x, a) = \max \left\{ 0, \max_{a'} \left\{ P(x, a, a') + \mathbb{E}_x \left[M(x, x') S(x', a') \right] \right\} \right\}. \tag{3}$$

The value function, S, is the solution of functional equation (3). We define $\omega = \omega(x, a)$ as in indicator function that captures the event of default. Note that, if $\omega = 0$, the optimal investment and financing decision is $\varphi(x, a) = a^*$, where $a^* = (k^*, b^*)$ is the optimal choice of the second argument in the max in (3). The optimal policy is therefore summarized by (ω, φ).

As for the debt contract, the end-of-period payoff to debt holders, $u(x', a')$, depends on the current policy, $a' = (k', b')$, the new realization of the shocks x', and on whether the firm is in default:

$$u(x', a') = b'(1 - \omega(x', a')) + \left[\pi' + \tau \delta k' + k'(1 - \delta) \right] (1 - \eta) \omega(x', a'). \tag{4}$$

In case of default, similarly to Hennessy and Whited (2007), the bondholders receive the sum of the cash flow from operations, the depreciated book value of the asset, and the tax shield from depreciation, all net of a proportional bankruptcy cost, η. Hence, at issuance the debt value is

$$B(x, a') = \mathbb{E}_x \left[M(x, x') u(x', a') \right]. \tag{5}$$

One final item that needs to be evaluated is the expected present value of the interest payment, $H(x, a')$, which enters the determination of the after tax cash flow in (1):

$$H(x, a') = [b' - B(x, a')] \mathbb{E}_x [M(x, x')(1 - \omega(x', a'))] . \tag{6}$$

Because the interest is deductible only if the firm is not in default, the expectation term is the conditional price of a default contingent claim.

6.4. Option prices

We assume options are on a single share of equity, rather than on the entire equity value. The assumption is necessary to allow us to compute the price of options with maturity longer than one period, in the context of our model, where investment and debt decisions affect the residual payout at the end of each period. In essence, we derive the option prices from the stock price, under the assumption that distribution to equity holders do not happen in the form of a cash dividend but are either a share repurchase or an equity issuance (when negative).[2] Thus, we track the number of shares outstanding and must compute the option price on a single share.

Denote with $n(x, a)$ the number of outstanding shares before the current payout decision is made. The stock price of one share is

$$s(x, a) = \frac{S(x, a)}{n(x, a)}.$$

Define $S'(x, a) = S(x, a) - P(x, a, a')$ the equity value after the payout, where $a' = \varphi(x, a)$ is the optimal policy from (3).

After a payout, the firm changes the number of shares for next period to $n'(x, a)$. In particular, if $P(x, a, a') > 0$, some shares are repurchased; if $P(x, a, a') < 0$ new share are issued. The new number of shares is

$$n'(x, a) = \frac{S'(x, a)}{s(x, a)} = \frac{S'(x, a)}{S(x, a)} \; n(x, a). \tag{7}$$

While n and n' are integer numbers in real life, we assume here that $n, n' \in \mathbb{R}$.

[2]It is possible to solve the model and compute prices even when the firm pays an exogenous dividend. In that case, we are also able to price an American option.

The evolution of the number of shares is given by the application of the current optimal policy, $a' = \varphi(x, a)$, and the state transition from x to x', so that at the new state (x', a') following from (x, a),

$$n(x', a') = n'(x, \varphi(x, a)), \tag{8}$$

with $n'(x, a)$ from (7).

For definiteness, we consider a European call option with strike κ, with payoff at maturity $\max\{s(x, a) - \kappa, 0\}$, which is based on the convention that the dividend has been paid before the option expires, and therefore the payoff is based on the ex dividend price.

Because the shares number is endogenous (i.e., it depends on the payout policy), option pricing by straightforward backward induction is numerically intractable. The drawback introduced by path dependency is due to the fact that the option price at the current state, and the stock price $s(x, a)$, is

$$c(x, a; \kappa) = \mathbb{E}_x \left[M(x, x') \max\{s(x', a') - \kappa, 0\}\right],$$

in which $a' = \varphi(x, a)$. To determine $s(x', a')$, the underlying asset of the option in state (x', a'), from $S(x', a')$ we need $n(x', a')$. However, as one can see from equation (8), $c(x', a'; \kappa)$ also depends on $n(x, a)$.

We avoid the issue of path dependency by observing that

$$\begin{aligned} c(x, a; \kappa) &= \mathbb{E}_x \left[M(x, x') \max\left\{\frac{S(x', a')}{n(x', a')} - \kappa, 0\right\}\right] \\ &= \frac{1}{n(x', a')} \mathbb{E}_x \left[M(x, x') \max\left\{S(x', a') - \kappa\ n(x', a'), 0\right\}\right]. \end{aligned}$$

From the expression above, defining the sum of prices of all options with strike k written on the firm's stock, $C(x, a; \kappa\ n(x', a')) = c(x, a; \kappa) n(x', a')$, we can write

$$C(x, a; \kappa\ n(x', a')) = \mathbb{E}_x \left[M(x, x') \max\{S(x', a') - \kappa\ n(x', a'), 0)\}\right],$$

which shows that we use backward induction to price total equity options on a predetermined set of strike prices $\mathcal{K} = \{K_1, \ldots, K_N\}$, such that for each $K \in \mathcal{K}$ we solve

$$C(x, a; K) = \mathbb{E}_x \left[M(x, x') \max\{S(x', a') - K, 0\}\right],$$

working backward from the option maturity to the current period. Given these prices, we can determine the current price of a European call option with strike price κ, by interpolating $\widehat{C}(x, a; \kappa\ n'(x, a))$ on the grid $\mathcal{K}$, and then

$$\widehat{c}(x, a; \kappa) = \frac{1}{n'(x, a)}\ \widehat{C}(x, a; \kappa\ n'(x, a)).$$

Using (7), the previous equation becomes

$$\widehat{c}(x, a; \kappa) = \frac{1}{n(x, a)} \frac{S^{ex}(x, a)}{S'(x, a)}\ \widehat{C}\left(x, a; \kappa\ n(x, a)\frac{S'(x, a)}{S^{ex}(x, a)}\right). \tag{9}$$

Given the current equity value, $S(x, a)$, our goal is to calculate the price of options on equity value at $t = 0$ with maturity T and moneyness $m \in \{m_1, m_2, \ldots, m_N\}$. Where the strikes are $\mathcal{K} = \{S(x, a)m_i, i = 1, \ldots, N\}$. Because the current number of shares is arbitrary, we choose $n(x, a) = S(x, a)$, which is equivalent to assuming that the current (ex dividend) stock price is \$1. Then our goal is met by solving the pricing problem

$$\widehat{c}(x, a; m) = \frac{1}{S'(x, a)}\ \widehat{C}\left(x, a; m\ S'(x, a)\right),$$

where $\widehat{c}(x, a; m)$ is the price of an European call option on a stock with current price \$1 and strike m.

6.5. Stochastic discount factor

We assume that the idiosyncratic shock z and the aggregate shock, y, follow auto-regressive processes of first order, $z' = (1 - \rho_z)\overline{z} + \rho_z z + \sigma_z \varepsilon'_z$ and $y' = (1 - \rho_y)\overline{y} + \rho_y y + \sigma_y \varepsilon'_y$, respectively. In the above equations, for $i = y, z$, $|\rho_i| < 1$ and ε_i are i.i.d. and obtained from a truncated standard normal distribution, so that the actual support is compact around the unconditional average. We assume that ε_z are uncorrelated across firms and time and are also uncorrelated with the aggregate shock, ε_y. The parameters ρ_z, σ_z, and $\overline{z}$ are the same for all the firms in the economy, $\overline{z}$ and $\overline{y}$ denote the long term mean of idiosyncratic risk and of macroeconomic risk, respectively, $(1 - \rho_i)$ is the speed of mean reversion, and σ_i is the conditional standard deviation. With this specification, the transition function Q satisfies all the assumptions required for the existence of the value function.

Finally, we adopt the stochastic discount factor proposed by Jones and Tuzel (2013):

$$M(y, y') = \beta e^{-g(y)\sigma_y \varepsilon'_y - \frac{1}{2} g(y)^2 \sigma_y^2},$$

with $\beta \in (0,1)$, and where the state-dependent coefficient of risk-aversion is $g(y) = \exp(\gamma_1 + \gamma_2 y)$, with $\gamma_1 > 0$ and $\gamma_2 < 1$. With this choice, the coupon is equal to the state-independent real risk-free rate, $r = 1/\beta - 1$.

Following the literature, the aggregate risk parameters are taken from Cooley and Prescott (1995) and converted to quarterly frequency. We obtain a value for the persistence of the systematic risk (ρ_x) and the aggregate volatility (σ_x) of 0.979 and 0.0072, respectively. The personal discount factor (β) is set to 0.9851, and the SDF parameters (γ_1 and γ_2) to 3.22 and -15.3, respectively. These parameters produce an annualized average real interest rate of 6.1%.

6.6. Calibration

We fix the five parameters that describe the aggregate source of risk and the SDF, equity and debt floatation costs, and the depreciation rate (as for example, Warusawitharana and Whited, 2016). We calibrate the remaining parameters by minimizing the sum of square deviations of a set of quantities that are observable in the data and in the simulated economy (which is characterized by 5,000 repetitions of a panel composed by 1,000 firms that are observed for 92 "quarterly" periods).

Important objectives of the calibration exercise are that the model captures the outcomes of the decisions that firms make and that affect the relationship between the asset and the equity volatility. The model should therefore match the average (book and market) leverage ratio and the average investment as the real economy. As the relevant sources of total risk match up with the economy, firms should exhibit similar market to book ratios, and similar equity distributions in the physical measure (i.e., average, standard deviation, skewness and kurtosis of equity percentage returns). We also calibrate the model to fit the average ATM 90 days implied volatility, as well the frequency of each implied volatility surface (i.e., left smirk, smile, right smirk, frown).

We report parameter values and quantities used for calibration in Panel A of Table 5. The firm-specific productivity shock is less persistent (0.91 versus 0.98) and more volatile (0.19 versus 0.01) than the aggregate shock. The estimated marginal corporate tax rate, τ, is 0.120, close to the estimates produced by Graham (1996a) and Graham (1996b) (i.e., average of approximately 13% for our sample). The estimate for the production function parameter α is 0.56. There are large bounds around figures reported in the literature, which are largely affected by the frequency at which models are calibrated and what type of fixed costs (proportional or not) are considered. Our value is close to the 0.3 figure used in Zhang (2005) and Gomes (2001). We estimate the operating cost to 4.32 (unit of capital), which

Table 5: Model calibration

This table presents the calibration results of the firm model. In Panel A, we report the list of model parameters. In Panel B, we compare the quantities that are weighted to calibrate the model. In the left column (*Data*) we report the value of the moment conditions computed from the observed empirical sample, while in the right column (*Model*) we report the moment conditions computed from the simulated sample. Data is from various sources and spans the period between January 1996 throughout December 2019.

Panel A: Parameters		
Aggregate		
Systematic Productivity Autocorrelation	ρ_x	0.970
Systematic Productivity Volatility	σ_x	0.013
Discount Factor	β	0.985
Constant Price of Risk Parameter	γ_1	3.220
Time-varying Price of Risk Parameter	γ_2	-15.300
Firm Specific		
Depreciation	δ	0.050
Equity Issuance Cost	ζ	0.018
Debt Adjustment Cost	θ	0.009
Idiosyncratic Productivity Autocorrelation	ρ_y	0.918
Idiosyncratic Productivity Volatility	σ_y	0.197
Production Function	α	0.560
Fix Cost	f	4.327
Cost of Expansion	λ_1	0.272
Cost of Contraction	λ_2	0.820
Corporate Taxes	τ	0.120
Bankruptcy Cost	η	0.284

translates to an annualized value of approximately 35% of the average capital. The calibrated value of the bankruptcy cost parameter, η, is 0.284, which is in the range of the firm's average default costs estimated by Glover (2016). Capital adjustment costs are lower than those in Zhang (2005).[3]

Panel B: Calibrated quantities		
Option Prices (90 days to maturity):	Data	Simulation
Implied Volatility OTM	0.648	0.647
Implied Volatility ATM	0.609	0.603
Implied Volatility ITM	0.585	0.589
Percentage Left Smirk	0.703	0.723
Percentage Smile	0.193	0.239
Percentage Right Smirk	0.044	0.001
Percentage Frown	0.062	0.037
Stock Return:		
Average	0.025	0.037
Standard Deviation	0.345	0.363
Skewness	0.649	0.540
Kurtosis	4.167	4.135
Firm characteristics:		
Market-to-Book	2.570	2.022
Leverage	0.504	0.479
Investments	0.043	0.053

In Panel B of Table 5, we compare the simulated economy to the real data along the dimensions used to calibrate the mode. The investment and financing choices of the average simulated firm reflects well those of real firms (investment and leverage are really close). Valuations are also appropriately close, as well the physical distribution of percentage equity returns. Average option prices are also relatively well matched as is the frequency of implied volatility shapes: the average implied volatility curve at 90 days ATM is close to the equivalent quantity in the data. Moreover, the model can create enough heterogeneity in the IV curve shapes that it matches closely what is observed in the data: about 70% of the time the curve is downward sloping with smiles, and about 20% of the time it is "smiling". Thus, similarly to Geske (1979) and Toft and Prucyk (1997) who both incorporate leverage, the model can generate average downward sloping curves across moneyness levels. Differently from those other models, our set up can also create other IV surfaces.

[3]A direct comparison to the parameters in Zhang (2005) requires dividing the parameter that we report by the depreciation rate. We obtain 5.4 and 16.4 for the expansion and contraction costs, while Zhang (2005) reports 15 and 150, for the same cost function.

7. Comparison of simulated and observed option prices

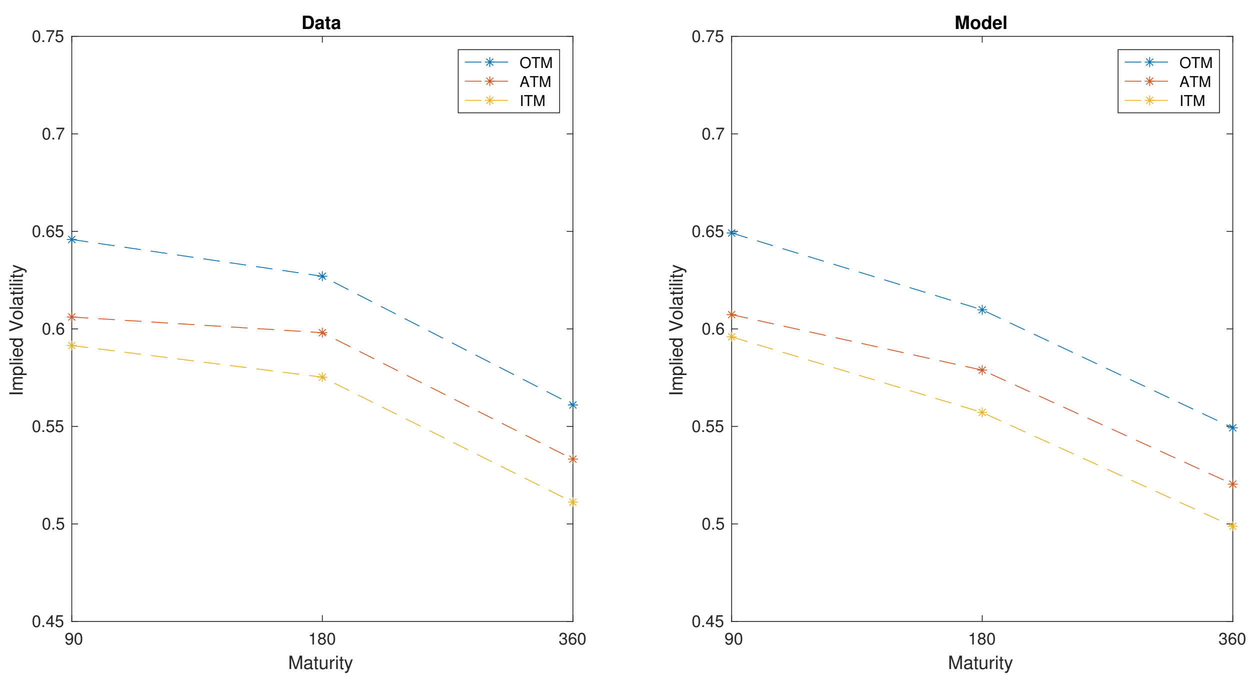
While it is remarkable that the model can match the 90 days IV curve and the frequencies of various IV shapes, it is also true that we used those quantities as part of the calibration exercise. In this section, we present comparisons of the simulated economy with the real one along other dimensions.

7.1. Term structure of implied volatilities

We start by comparing the average IV surfaces across all maturities considered (90, 180, and 360 days). Please remember that the model is only calibrated to fit the 90 days curve. Figure 7 juxtaposes the curves extracted from the data (left panel) to those extracted from the simulation. To obtain each curve, we first average across time, then across firms, and eventually across simulated economies.

Figure 7: Average implied volatility surface comparison

The figure plots the average implied volatility surface extracted from the data (left panel) and from the simulation (right panel). The sample contains all industrial firms with options trading on their equity between 1996 and 2019. Data is sampled at quarterly frequency. A total of 3,536 firms are included.



Interestingly, the model can also generate a downward sloping surface across maturities without exogenously imposing a term-structure of volatility. Productivity shocks that affect the firm's value at short horizon tend to revert towards long run values, and as that happens the relationship between asset and equity volatility flattens. The total effect is to decrease prices for options at longer maturities, and henceforth producing a decreasing volatility surface. As Figure 7 shows, the model is able to replicate this feature of the data quite well.

7.2. Risk-neutral moments across different maturities

As Panel B of Table 5 shows, the moments of the physical distribution of stock returns match quite well with the corresponding quantities in the data. Table 6 confirms that the implied higher moments of the risk-neutral distribution match as well. While skewness is relatively flat across maturities (i.e., slowly decreasing in the data and moderately increasing in the model), the model can replicate the downward sloping feature of implied kurtosis, almost perfectly. As one might expect, there is more heterogeneity in the data, as evidenced by larger standard deviations. Nonetheless the ranges of the variables compare quite favorably.

Table 6: Model free risk-neutral moments

The table compares summary statistics for model free risk-neutral skewness and kurtosis extracted from the data (left side) and from the simulated economy (right side). The sample contains all industrial firms with options trading on their equity between 1996 and 2019. Data is sampled at quarterly frequency. A total of 3,536 firms are included.

	Data				Model			
	90 days maturity							
	Aver.	S.Dev	5^{th} perc	95^{th} perc	Aver.	St.Dev	5^{th} perc	95^{th} perc
Skewness	-0.51	0.38	-1.15	0.01	-0.45	0.15	-0.73	-0.31
Kurtosis	3.76	1.28	2.66	5.84	3.79	0.46	2.76	4.35
	180 days maturity							
	Aver.	S.Dev	5^{th} perc	95^{th} perc	Aver.	St.Dev	5^{th} perc	95^{th} perc
Skewness	-0.42	0.35	-1.01	0.08	-0.48	0.19	-0.71	-0.27
Kurtosis	3.32	0.93	2.28	4.83	3.44	0.59	1.94	3.94
	360 days maturity							
	Aver.	S.Dev	5^{th} perc	95^{th} perc	Aver.	St.Dev	5^{th} perc	95^{th} perc
Skewness	-0.41	0.39	-1.03	0.20	-0.50	0.28	-0.73	0.03
Kurtosis	3.10	0.96	1.94	4.75	3.11	0.74	1.44	3.84

7.3. Cross-sectional regressions

As the firm parameters are determined by the calibration exercised, variability in the simulated economy in terms of implied volatility shapes is dictated by the optimal choices made by firms relative to the realizations of the exogenous variables and the current state of capital and debt. Ultimately those choices determine the equity value relative to the capital in place and optimal amount of leverage. We estimate here the same linearized relationships that we presented in Section 4.1.

Table 7: Option prices and firm characteristics

The table presents regression results in the simulated economy, that mirrors those in the data presented in Table 1. Left hand side variables include the risk-neutral skewness, the slope (i.e., log difference of IV(1.2) and IV(0.8)) of 90 day maturity options. Dependent variables include the ATM IV, the natural logarithm of assets (Size), book leverage, profitability, the market to book ratio, and the ratio of the present value of the real option to assets (PVRO2A). PVRO is constructed by decomposing the equity into the value derived from keeping current capital level constant and the value derived from making adjustments, by either investing or disinvesting. Since fixed effects are not very meaningful in simulated data, reported coefficient are obtained from a simulated Fama-MacBeth regression, where slopes obtained from cross-sectional regressions (all firms observations in one period in one simulated economy) are averaged first through time and then through simulated economies. Standard errors are obtained from considering deviations around the mean across simulated economies.

	RN-Skew		Slope	
IV	0.41	0.41	-0.17	-0.24
	(115.15)	(119.50)	(-9.09)	(-13.69)
Size	0.01	0.00	-0.83	-0.92
	(34.73)	(5.35)	(-71.93)	(-73.73)
Book Leverage	-0.03	-0.03	-0.20	-0.47
	(-42.51)	(-116.96)	(-17.44)	(-55.80)
Profitability	0.07	0.05	2.29	1.93
	(16.30)	(16.01)	(20.47)	(25.27)
M2B	-0.07		-3.33	
	(-12.22)		(-33.40)	
PVRO2A		-0.05		-2.00
		(-10.91)		(-59.89)

From a qualitative point of view, the results reported in Table 7 largely mirror those reported in Table 1. In the simulated economy, the skewness of the risk-neutral distribution and the slope of the IV curve are negatively related to leverage, as in Toft and Prucyk (1997), and to the value of the real option, whether that is proxied by the market to book ratio, or measured exactly by the present value of the real option, which we obtain in the model by separating the contribution to the equity value of assets in place and of future investments and disinvestments.

8. Conclusions

Traditional option pricing models often requires very strong assumptions about investor preferences and the dynamic of equity prices. We show that equity options can be priced in a production economy where we do not make strong exogenous assumptions about equity and volatility. In our set up the relation between risk and value arises endogenously through a dynamic sequence of optimal decisions that maximize the value of the firm. We derive option prices that match many properties of those observed in the cross-section of US equities starting from a different set of assumptions that specify the functional forms of corporate trade-offs.

Our approach is not a better option pricing model, but rather an attempt to provide a link between fundamentals and derivative pricing. We think that such link is important as it relates the primitives of the most successful finance models (i.e., those that price financial derivatives) to a large body of well understood economic mechanisms that describe the decision-making process within a typical firm.

Ultimately, we hope to provide an explanation for why option prices contain forward looking information about stock prices and corporate policies, despite being classically derived in models where such links should be uninformative unless one assumes some form of market segmentation.

References

Abel, Andrew B., and Janice C. Eberly, 1994, A unified model of investment under uncertainty, *American Economic Review* 85, 1369–1384 doi:10.3386/w4296.

Aguerrevere, Felipe, 2009, Real options, product market competition, and asset returns, *Journal of Finance* 44, 957–983 doi:10.1111/j.1540-6261.2009.01454.x.

Ai, Hengjie, and Dana Kiku, 2013, Growth to value: Option exercise and the cross section of equity returns, *Journal of Financial Economics* 107, 325–349 doi:10.1016/j.jfineco.2012.08.009.

Amaya, Diego, Peter Christoffersen, Kris Jacobs, and Aurelio Vasquez, 2015, Does realized skewness predict the cross-section of equity returns?, *Journal of Financial Economics* 118, 135 – 167 doi:10.1016/j.jfineco.2015.02.009.

Aretz, Kevin, and Peter Pope, 2018, Real options models of the firm, capacity overhang, and the cross section of stock returns, *Journal of Finance* 73, 1363–1415 doi:10.1111/jofi.12617.

Bai, Jennie, Robert Goldstein, and Fan Yang, 2019, The leverage effect and the basket-index spread, *Journal of Financial Economics* 131, 180–205 doi:10.1016/j.jfineco.2018.07.015.

Bakshi, Gurdip, Charles Cao, and Zhaodong (Ken) Zhong, 2021, Assessing models of individual equity option prices, *Review of Quantitative Finance and Accounting* 57 doi:10.1007/s11156-020-00951-4.

Bakshi, G., N. Kapadia, and D. Madan, 2003, Stock return characteristics, skew laws, and the differential pricing of individual equity options, *Review of Financial Studies* 16, 637–659 doi:10.1093/rfs/16.1.0101.

Bali, Turan, and Scott Murray, 2013, Does risk-neutral skewness predict the cross-section of equity option portfolio returns?, *Journal of Financial and Quantitative Analysis* 48, 1145–1171 doi:10.1017/S0022109013000410.

Bali, Turan G., Luca Del Viva, Neophytos Lambertides, and Lenos Trigeorgis, 2020, Growth options and related stock market anomalies: profitability, distress, lotteryness, and volatility, *Journal of Financial Quantitative Analysis* 55, 2150–2180 doi:10.1017/S0022109019000619.

Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553–1607 doi:10.1111/0022-1082.00161.

Black, Fisher, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–659 doi:10.1086/260062.

Boyer, Brian, Todd Mitton, and Keith Vorkink, 2010, Expected idiosyncratic skewness, *Review of Financial Studies* 23, 169–202 doi:10.1093/rfs/hhp041.

Cao, Charles, Tim Simin, and Jing Zhao, 2008, Can growth options explain the trend in idiosyncratic risk?, *Review of Financial Studies* 21, 2599–2633 doi:10.1093/rfs/hhl039.

Carlson, M., A. Fisher, and R. Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross–section of returns, *Journal of Finance* 59, 2577–2603 doi:10.1111/j.1540-6261.2004.00709.x.

Chaderina, Maria, Patrick Weiss, and Josef Zechner, 2021, The maturity premium, *Journal of Financial Economics* forthcoming doi:10.1016/j.jfineco.2021.07.008.

Christoffersen, Peter, Mathieu Fournier, Kris Jacobs, and Mehdi Karoui, 2021, Option-based estimation of the price of coskewness and cokurtosis risk, *Journal of Financial and Quantitative Analysis* 56, 65–91 do:10.1017/S002210902000023X.

Conrad, Jennifer, Robert Dittmar, and Eric Chisels, 2013, Ex ante skewness and expected stock returns, *Journal of Finance* 68, 85–124 doi:10.1111/j.1540-6261.2012.01795.x.

Cooley, T. F., and E. C. Prescott, 1995, Economic growth and business cycles, in Thomas F. Cooley, ed.: *Frontiers of Business Cycle Research* . pp. 1–38 (Princeton University Press: Princeton - NJ).

Cooper, Ilan, 2006, Asset pricing implications of non-convex adjustment costs and irreversibility of investment, *The Journal of Finance* 61, 139–170 doi:10.1111/j.1540-6261.2006.00832.x.

Del Viva, Luca, Eero Kasanen, and Lenos Trigeorgis, 2017, Real options, idiosyncratic skewness, and diversification, *Journal of Financial and Quantitative Analysis* 52, 215–241 doi:10.1017/S0022109016000703.

Del Viva, Luca, S.P. Kothari, Neophytos Lambertides, and Lenos Trigeorgis, 2021, Asymmetric returns and the economic content of accruals and investment, *Management Science* 67, 3321–3984 doi:10.1287/mnsc.2020.3676.

Dennis, Patrick, and Stewart Mayhew, 2002, Risk-neutral skewness: Evidence from stock options, *Journal of Financial and Quantitative Analysis* 37, 471–493 doi:10.2307/3594989.

Doshi, Hitesh, Jan Ericsson, Stephen Szaura, and Fan Yu, 2022, Accounting transparency and the implied volatility skew, Discussion paper University of Houston.

Duan, Jin-Chuan, and Jason Wei, 2009, Systematic risk and the price structure of individual equity options, *Review of Financial Studies* 22, 1981–2006 doi:10.1093/rfs/hhn057.

Engle, Robert F., and Emil N. Siriwardane, 2017, Structural garch: The volatility-leverage connection, *The Review of Financial Studies* 31, 449–492 doi:10.1093/rfs/hhx099.

Fama, Eugene Francis, and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465 doi:10.1111/j.1540-6261.1992.tb04398.x.

Figlewski, Stephen, and Xiaozu Wang, 2000, Is the "leverage effect" a leverage effect?, Discussion paper New York University.

Friewald, Nils, Florian Nagler, and Christian Wagner, 2021, Debt refinancing and equity returns, *Journal of Finance* forthcoming doi:10.2139/ssrn.2684462.

Garleanu, Nicolae, Stavros Panaceas, and Jianfeng Yu, 2012, Technological growth and asset pricing, *Journal of Finance* 67, 1265–1292 doi:10.1111/j.1540-6261.2012.01747.x.

Geske, Robert, 1979, The valuation of compound options, *Journal of Financial Economics* 7, 63–81 doi:10.1016/0304-405X(79)90022-9.

———, Avanidhar Subrahmanyam, and Yi Zhou, 2016, Capital structure effects on the prices of equity call options, *Journal of Financial Economics* 121, 231–253 doi:10.1016/j.jfineco.2016.03.009.

Glover, Brent, 2016, The expected cost of default, *Journal of Financial Economics* 119, 284–299 https://doi.org/10.1016/j.jfineco.2015.09.007.

Gomes, Joao F., 2001, Financing investment, *American Economic Review* 91, 1263–1285 doi:10.1257/aer.91.5.1263.

———, and Lukas Schmid, 2010, Levered returns, *Journal of Finance* 65, 467–494 doi:10.1111/j.1540-6261.2009.01541.x.

———, 2021, Equilibrium asset pricing with leverage and default, *Journal of Finance* 76, 977–1018 doi:10.1111/jofi.12987.

Graham, John R., 1996a, Debt and the marginal tax rate, *Journal of Financial Economics* 41, 41–73 doi:10.1016/0304-405X(95)00857-B.

———, 1996b, Proxies for the corporate marginal tax rate, *Journal of Financial Economics* 42, 187–221 doi:10.1016/0304-405X(96)00879-3.

Grullon, Gustavo, Evgeny Lyandres, and Alexey Zhdanov, 2012, Real options, volatility, and stock returns, *Journal of Finance* 62, 1499–1537 doi:10.1111/j.1540-6261.2012.01754.x.

Gu, Lifeng, Dirk Hackbarth, and Tim Johnson, 2017, Inflexibility and stock returns, *Review of Financial Studies* 31, 278–321 doi:10.1093/rfs/hhx092.

Hansis, A., C. Schlag, and G. Vilkov, 2010, The dynamics of risk-neutral implied moments: evidence from individual options, Discussion paper Frankfurt School of Finance.

Harvey, Campbell, and Akhtar Siddique, 2000, Conditional skewness in asset pricing tests, *Journal of Finance* 55, 1263–1295 doi:10.1111/0022-1082.00247.

Hennessy, Christopher A., and Tony Whited, 2005, Debt dynamics, *Journal of Finance* 60, 1129–1165 doi:10.1111/j.1540-6261.2005.00758.x.

Hennessy, Christopher A., and Tony M. Whited, 2007, How costly is external financing? evidence from a structural estimation, *Journal of Finance* 62, 1705–1745 doi:10.1111/j.1540-6261.2007.01255.x.

Jones, Christopher S., and Selale Tuzel, 2013, Inventory investment and the cost of capital, *Journal of Financial Economics* 107, 557–579 doi:10.1016/j.jfineco.2012.09.001.

Kadan, Ohad, and Xiaoxiao Tang, 2019, A bound on expected stock returns, *The Review of Financial Studies* 33, 1565–1617 doi:10.1093/rfs/hhz075.

Kogan, Leonid, 2004a, Asset prices and real investment, *Journal of Financial Economics* 73, 411–431 doi:10.1016/j.jfineco.2003.12.002.

———, 2004b, An equilibrium model of irreversible investment, *Journal of Financial Economics* 73, 411–431 doi:10.1016/S0304-405X(01)00077-0.

———, and Dimitris Papanikolaou, 2013, Firm characteristics and stock returns: the role of investment-specific shocks, *Review of Financial Studies* 26, 2718–2759 doi:10.1093/rfs/hht026.

———, 2014, Growth opportunities, technology shocks, and asset prices, *Journal of Finance* 69, 675–718 doi:10.1111/jofi.12136.

Kozhan, Roman, Anthony Neuberger, and Paul Schneider, 2013, Skew risk premia in the equity index market, *Review of Financial Studies* 26, 2174–2203 doi:10.1093/rfs/hht039.

Kuehn, Lars-Alexander, and Lukas Schmid, 2014, Investment-based corporate bond pricing, *Journal of Finance* 69, 2741–2776 doi:10.1111/jofi.12204.

Leland, Hayne E., 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252 doi:10.2307/2329184.

Lyandres, Evgeny Matveyev, Egor, and Alexey Zhdanov, 2020, Does the market correctly value investment options?, *Review of Finance* 62, 1159–1201 doi:10.1093/rof/rfaa002.

Martin, Ian W. R., and Christian Wagner, 2019, What is the expected return on a stock?, *The Journal of Finance* 74, 1887–1929 doi:10.1111/jofi.12778.

McDonald, Robert, and Daniel Siegel, 1985, Investment and valuation of firms when there is an option to shut down, *International Economic Review* 26, 331–349 doi:10.2307/2526587.

Merton, Robert, 1974, On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance* 4, 449–470 doi:10.2307/2978814.

Mitton, Todd, and Keith Vorkink, 2007, Equilibrium under-diversification and the preference for skewness, *Review of Financial Studies* 20, 1255–1288 doi:10.1093/revfin/hhm011.

Morellec, Erwan, and Alexey Zhdanov, 2019, Product market competition and option prices, *Review of Financial Studies* 32, 4343–4386 doi:10.1093/rfs/hhz027.

Neuberger, Anthony, 2012, Realized skewness, *Review of Financial Studies* 25, 3423–3455 doi:10.1093/rfs/hhs101.

Panayiotis, Andreou, Turan G. Bali, Anastasios Kagkadis, and Neophytos Lambertides, 2021, Firms growth potential and option returns, Discussion paper Georgetown University.

Pederzoli, Paola, 2020, Crash risk in individual stocks, Discussion paper University of Houston.

Schneider, Paul, Christian Wagner, and Josef Zechner, 2020, Low risk anomalies?, *Journal of Finance* 75, 2673–2718 doi:10.1111/jofi.12910.

Toft, Klauss Bjerre, and Brian Prucyk, 1997, Options on leverage equity: theory and empirical tests, *Journal of Finance* 52, 1151–1180 doi:10.2307/2329519.

Trigeorgis, Lenos, and Neophytos Lambertides, 2014a, The role of growth options in explaining stock returns, *Journal of Financial and Quantitative Analysis* 49, 749–771 doi:10.1017/S0022109014000118.

———, 2014b, The role of growth options in explaining stock returns, *Journal of Financial and Quantitative Analysis* 49, 749–771.

van Zwet, Willem, 1964, Convex transformations: A new approach to skewness and kurtosis, *Statistica Neerlandica* 18, 433–441 doi:10.1111/j.1467-9574.1964.tb00530.x.

Warusawitharana, Missaka, and Tony Whited, 2016, Equity market misvaluation, financing, and investment, *Review of Financial Studies* 29, 603–634 doi:10.1093/rfs/hhv066.

Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67–103 doi:10.1111/j.1540-6261.2005.00725.x.

A. Appendix tables and figures

Figure A1: Implied volatility surface – time series

The figure plots the time series of cross sectional averages, as well fifth and ninety-fifth percentiles, of implied volatility surface extracted from the data. The sample contains all industrial firms with options trading on their equity between 1996 and 2019. Data is sampled at quarterly frequency. A total of 3,536 firms are included.

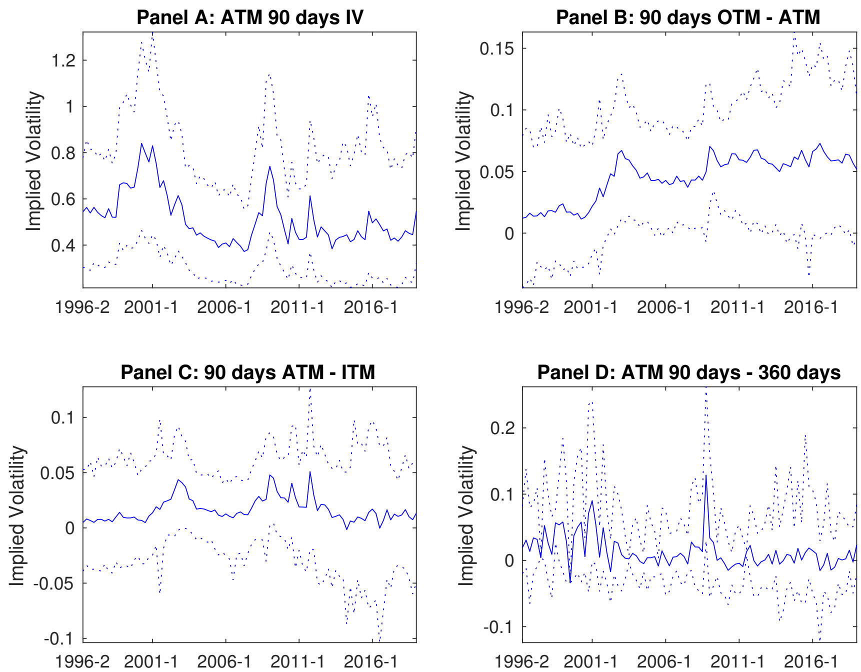


Figure A2: Risk neutral moments – time series

The figure plots the time series of cross sectional averages, as well fifth and ninety-fifth percentiles, of risk-neutral skewness and kurtosis extracted from the data. The sample contains all industrial firms with options trading on their equity between 1996 and 2019. Data is sampled at quarterly frequency. A total of 3,536 firms are included.

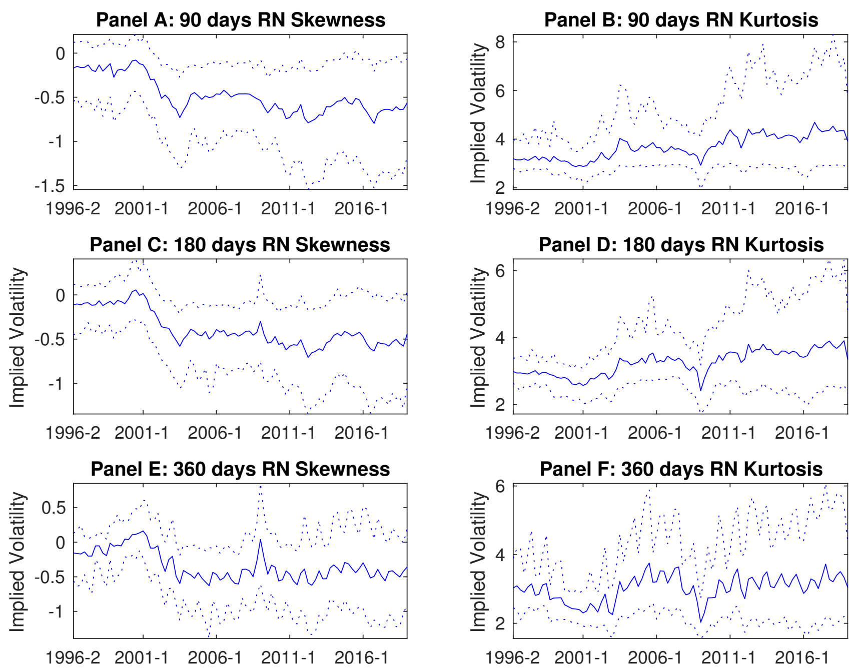


Table A1: Option prices and real options — 180 and 360 day maturity

The table shows regression results of risk-neutral skewness and implied volatility slope against measures of real options. Risk-neutral skewness and implied volatility slope, which is computed as the log difference of IV(1.2) and IV(0.8), are based on 180 and 360 day maturity options. Real options measures include the market-to-book ratio and the GO ratio of Trigeorgis and Lambertides (2014b). We control for size, book leverage, profitability, and the level of ATM IV. We report parameter estimates and standard errors clustered at the firm level. Regressions include time and firm fixed effects. A constant is included but not reported. The sample contains all non-financial firms with options trading on their equity between 1996 and 2019. Data is sampled at quarterly frequency. A total of 3,536 firms are included.

	180 day maturity				360 day maturity			
	RN-Skew		Slope		RN-Skew		Slope	
IV	16.07	16.06	3.72	3.72	27.68	27.48	3.50	3.48
	(37.68)	(37.10)	(29.21)	(29.12)	(35.48)	(34.95)	(17.01)	(16.86)
Size	-10.84	-7.17	-1.54	-0.90	-7.64	-4.51	-1.02	-0.64
	(-8.57)	(-5.85)	(-4.73)	(-2.99)	(-3.71)	(-2.41)	(-2.26)	(-1.56)
Leverage	-1.60	-1.42	-0.22	-0.18	-1.45	-1.09	-0.29	-0.25
	(-3.54)	(-3.10)	(-1.89)	(-1.60)	(-1.94)	(-1.48)	(-1.62)	(-1.39)
Profitability	0.48	-2.41	0.27	-0.29	0.43	-2.51	0.15	-0.20
	(1.80)	(-6.03)	(3.57)	(-2.85)	(0.94)	(-4.18)	(1.19)	(-1.27)
M2B	-6.08		-1.09		-5.42		-0.66	
	(-16.20)		(-10.32)		(-8.92)		(-4.44)	
GO		-3.32		-0.66		-3.15		-0.37
		(-9.68)		(-7.34)		(-6.22)		(-2.91)
Time FE	X	X	X	X	X	X	X	X
Firm FE	X	X	X	X	X	X	X	X
Adjusted-R^2	0.62	0.61	0.59	0.58	0.73	0.73	0.60	0.60
FE R^2	0.58	0.58	0.56	0.56	0.66	0.66	0.57	0.57

Table A2: Option prices and real options — 90, 180, and 360 day maturity

The table shows regression results of risk-neutral skewness and implied volatility slope against measures of real options. Risk-neutral skewness and implied volatility slope are based on 90, 180, and 360 day maturity options. Besides the control described in Table 1, we include a lag of the dependent variable, turnover, cumulative returns from t-12 to t-1 (momentum), and a Nasdaq indicator variable as in Boyer, Mitton, and Vorkink (2010) as well as a measure of R&D intensity and investments as in Del Viva, Kasanen, and Trigeorgis (2017). We report parameter estimates and standard errors clustered at the firm level. Regressions include time and firm fixed effects. A constant is included but not reported. The sample contains all non-financial firms with options trading on their equity between 1996 and 2019. Data is sampled at quarterly frequency. A total of 3,536 firms are included.

	90 day maturity				180 day maturity				360 day maturity			
	RN-Skew		Slope		RN-Skew		Slope		RN-Skew		Slope	
IV	10.94	10.97	3.22	3.22	12.87	12.96	2.60	2.62	16.69	16.56	2.21	2.20
	(18.69)	(18.70)	(17.02)	(17.13)	(24.07)	(24.13)	(16.12)	(16.24)	(14.27)	(13.90)	(6.23)	(6.13)
Size	-6.27	-4.83	-1.79	-1.50	-6.27	-4.79	-1.48	-1.18	-1.83	-0.71	-0.16	0.01
	(-5.09)	(-4.04)	(-6.26)	(-5.57)	(-5.67)	(-4.46)	(-5.26)	(-4.52)	(-0.86)	(-0.37)	(-0.46)	(0.04)
Leverage	-1.58	-1.52	-0.28	-0.27	-1.44	-1.37	-0.34	-0.33	-1.67	-1.44	-0.22	-0.20
	(-4.03)	(-3.84)	(-2.93)	(-2.79)	(-3.95)	(-3.78)	(-3.87)	(-3.65)	(-2.00)	(-1.78)	(-1.61)	(-1.46)
Profitability	1.03	-1.34	0.31	-0.27	0.60	-0.91	0.15	-0.17	0.26	-1.48	0.16	0.01
	(3.95)	(-3.52)	(4.27)	(-2.67)	(2.40)	(-2.50)	(2.39)	(-1.93)	(0.43)	(-2.11)	(1.43)	(0.08)
M2B	-4.55		-0.98		-3.70		-0.77		-2.82		-0.34	
	(-12.70)		(-9.58)		(-10.69)		(-7.99)		(-4.16)		(-2.93)	
GO		-3.16		-0.79		-1.85		-0.40		-2.16		-0.15
		(-9.86)		(-8.91)		(-6.42)		(-5.40)		(-3.97)		(-1.33)
Lag Skew	19.75	19.92	5.26	5.29	18.96	19.09	5.08	5.11	25.68	25.74	2.24	2.25
/Slope	(37.13)	(37.52)	(42.89)	(43.20)	(32.43)	(32.69)	(38.33)	(38.60)	(26.12)	(26.29)	(15.25)	(15.32)
Turnover	-2.24	-2.39	-0.12	-0.15	-1.69	-1.79	-0.04	-0.06	-0.29	-0.25	0.00	0.01
	(-6.18)	(-6.59)	(-1.16)	(-1.47)	(-5.49)	(-5.78)	(-0.42)	(-0.66)	(-0.47)	(-0.41)	(0.02)	(0.06)
Momentum	-0.91	-1.95	-0.03	-0.25	-0.56	-1.44	0.01	-0.17	-2.01	-2.65	-0.07	-0.15
	(-5.31)	(-12.81)	(-0.71)	(-5.94)	(-3.42)	(-10.19)	(0.22)	(-4.37)	(-5.94)	(-8.33)	(-1.12)	(-2.43)
Nasdaq	-0.88	-0.77	-0.12	-0.09	-1.32	-1.30	-0.08	-0.07	1.10	0.96	-0.19	-0.20
	(-0.96)	(-0.83)	(-0.50)	(-0.36)	(-1.42)	(-1.43)	(-0.31)	(-0.29)	(0.62)	(0.56)	(-0.41)	(-0.45)
R&D	0.52	0.23	0.12	0.06	0.34	0.15	0.05	0.01	-0.79	-0.66	-0.18	-0.17
	(1.78)	(0.80)	(1.44)	(0.73)	(1.23)	(0.54)	(0.71)	(0.13)	(-1.59)	(-1.43)	(-2.19)	(-1.94)
Investment	-0.03	-0.06	-0.10	-0.11	-0.02	-0.05	-0.06	-0.07	0.16	0.03	0.12	0.12
	(-0.17)	(-0.40)	(-2.19)	(-2.33)	(-0.14)	(-0.34)	(-1.58)	(-1.72)	(0.34)	(0.07)	(1.17)	(1.07)
Adj-R^2	0.58	0.58	0.62	0.61	0.68	0.68	0.67	0.67	0.79	0.79	0.60	0.60
FE Adj-R^2	0.48	0.48	0.54	0.54	0.59	0.59	0.58	0.58	0.66	0.66	0.54	0.54

Table A3: Future realized idiosyncratic skewness and real options

The table shows regression results of future realized idiosyncratic skewness on measures of real options value. Independent variables are measured at day t, while future idiosyncratic skewness is extracted from the residuals of regressions of log returns on linear and quadratic functions of the corresponding log market return (as in Bali, Del Viva, Lambertides, and Trigeorgis, 2020). All control variables are as in Table 2. A constant is included but not reported. Regressions include time and firm fixed effects, and standard error reported in parenthesis are clustered at firm level. The sample contains all non-financial firms with options trading on their equity between 1996 and 2019. Data is sampled at quarterly frequency. A total of 3,536 firms are included.

	Idiosyncratic skewness from linear market model											
	90 days				180 days				360 days			
IV	3.23	2.94	8.49	8.68	1.07	0.43	6.35	6.83	0.69	-0.54	9.83	10.04
	(2.61)	(2.35)	(4.65)	(4.75)	(0.65)	(0.26)	(2.52)	(2.72)	(0.32)	(-0.25)	(3.05)	(3.12)
Size	-0.70	0.05	-3.04	-1.37	0.66	2.79	-6.74	-1.72	1.40	5.35	-8.07	-1.60
	(-0.27)	(0.02)	(-0.96)	(-0.45)	(0.17)	(0.78)	(-1.33)	(-0.35)	(0.27)	(1.09)	(-1.08)	(-0.22)
Leverage	2.03	1.24	3.88	3.64	4.38	3.99	5.98	5.85	4.99	5.18	9.54	9.46
	(1.97)	(1.21)	(2.92)	(2.80)	(2.88)	(2.65)	(2.91)	(2.91)	(2.43)	(2.53)	(3.28)	(3.31)
Profitability	3.84	-5.35	6.58	-4.33	0.87	-7.25	2.70	-6.17	0.81	-6.39	-0.28	-8.68
	(4.07)	(-4.45)	(5.37)	(-2.84)	(0.70)	(-4.33)	(1.60)	(-2.62)	(0.53)	(-2.99)	(-0.13)	(-2.60)
M2B	-9.96		-11.73		-12.83		-17.84		-17.35		-21.34	
	(-9.74)		(-8.10)		(-8.56)		(-7.50)		(-8.32)		(-6.28)	
GO		-14.46		-16.00		-12.74		-11.81		-11.32		-9.79
		(-12.73)		(-11.30)		(-9.16)		(-6.33)		(-6.15)		(-3.80)
Lag Skew			-6.33	-6.37			-10.26	-10.56			-17.50	-17.74
			(-7.76)	(-7.83)			(-9.40)	(-9.65)			(-10.57)	(-10.71)
Turnover			-4.76	-5.10			-5.29	-6.07			-7.83	-8.85
			(-3.81)	(-4.10)			(-2.99)	(-3.45)			(-3.44)	(-3.88)
Momentum			-2.86	-5.21			-1.35	-5.47			1.98	-3.04
			(-3.35)	(-6.77)			(-1.10)	(-4.89)			(1.21)	(-2.07)
Nasdaq			-1.62	-0.79			-2.96	-2.84			-3.63	-3.77
			(-0.52)	(-0.25)			(-0.61)	(-0.59)			(-0.53)	(-0.56)
R&D			0.17	-0.35			-5.04	-6.23			-9.37	-11.59
			(0.12)	(-0.25)			(-2.15)	(-2.73)			(-2.30)	(-2.92)
Investment			-3.11	-3.19			-1.86	-2.03			-2.13	-2.30
			(-3.90)	(-4.01)			(-2.52)	(-2.76)			(-2.96)	(-3.18)
Adj-R^2	0.03	0.02	0.03	0.03	0.08	0.07	0.09	0.09	0.17	0.17	0.18	0.18
FE Adj-R^2	0.02	0.02	0.02	0.02	0.07	0.07	0.08	0.08	0.16	0.16	0.16	0.16

	Idiosyncratic skewness from quadratic market model											
	90 days				180 days				360 days			
IV	3.10	2.82	8.34	8.53	0.83	0.23	5.87	6.34	0.34	-0.88	9.06	9.28
	(2.55)	(2.30)	(4.65)	(4.74)	(0.51)	(0.14)	(2.35)	(2.54)	(0.16)	(-0.41)	(2.81)	(2.88)
Size	-0.79	-0.02	-3.40	-1.76	0.37	2.48	-7.01	-2.05	1.15	5.10	-8.58	-2.14
	(-0.32)	(-0.01)	(-1.09)	(-0.58)	(0.10)	(0.70)	(-1.39)	(-0.43)	(0.22)	(1.04)	(-1.15)	(-0.30)
Leverage	1.94	1.17	3.78	3.55	4.58	4.19	6.15	6.04	5.37	5.55	10.06	9.99
	(1.91)	(1.16)	(2.89)	(2.76)	(3.04)	(2.81)	(3.02)	(3.02)	(2.62)	(2.72)	(3.47)	(3.50)
Profitability	3.66	-5.35	6.41	-4.24	0.77	-7.29	2.63	-6.10	0.79	-6.49	-0.21	-8.67
	(3.92)	(-4.53)	(5.28)	(-2.82)	(0.62)	(-4.38)	(1.57)	(-2.61)	(0.52)	(-3.04)	(-0.10)	(-2.60)
M2B	-9.82		-11.46		-12.64		-17.59		-17.36		-21.26	
	(-9.70)		(-8.02)		(-8.52)		(-7.45)		(-8.36)		(-6.27)	
GO		-14.17		-15.61		-12.62		-11.62		-11.44		-9.86
		(-12.72)		(-11.20)		(-9.15)		(-6.28)		(-6.25)		(-3.83)
Lag Skew			-6.30	-6.34			-10.22	-10.53			-17.58	-17.83
			(-7.87)	(-7.94)			(-9.33)	(-9.60)			(-10.72)	(-10.88)
Turnover			-4.84	-5.18			-5.34	-6.11			-7.82	-8.83
			(-3.94)	(-4.22)			(-3.06)	(-3.52)			(-3.45)	(-3.90)
Momentum			-2.85	-5.14			-1.37	-5.42			1.90	-3.08
			(-3.38)	(-6.78)			(-1.12)	(-4.88)			(1.16)	(-2.11)
Nasdaq			-1.85	-1.04			-3.00	-2.88			-3.75	-3.88
			(-0.59)	(-0.34)			(-0.62)	(-0.60)			(-0.55)	(-0.58)
R&D			0.14	-0.36			-5.00	-6.17			-9.40	-11.58
			(0.10)	(-0.26)			(-2.16)	(-2.74)			(-2.31)	(-2.93)
Investment			-3.04	-3.12			-1.90	-2.07			-2.23	-2.40
			(-3.86)	(-3.97)			(-2.58)	(-2.82)			(-3.12)	(-3.33)
Adj-R^2	0.03	0.02	0.03	0.03	0.08	0.07	0.09	0.09	0.17	0.17	0.19	0.18
FE Adj-R^2	0.02	0.02	0.02	0.02	0.07	0.07	0.08	0.08	0.16	0.16	0.16	0.16

Table A4: Future realized skewness and real options in sample of Del Viva, Kasanen, and Trigeorgis (2017)

The table shows regression results of future realized idiosyncratic skewness on the GO ratio of Trigeorgis and Lambertides (2014b). Independent variable is measured at day t, while future realized skewness is extracted from days $t+1$ to $t+360$. We control for size, book leverage, profitability as in Table 1, as well as a lag of the dependent variable, past realized volatility from the previous 90 days, quarterly average turnover, cumulative returns from $t-12$ to $t-1$ (momentum), and a Nasdaq indicator variable as in Boyer, Mitton, and Vorkink (2010), and a measure of R&D intensity and investments as in Del Viva, Kasanen, and Trigeorgis (2017). In the last specification we also include an indicator variable that is set to one when equity options are available for a firm-quarter observation. A constant is included but not reported. All right hand side variables are winsorized at the first and 99^{th} percentile, and subsequently standardized to aid comparability across specifications. We report parameter estimates (multiplied by 100) and standard errors clustered at the firm level. The sample contains all non-financial firms with options trading on their equity between 1983 and 2011. Data is sampled at quarterly frequency. A total of 5,616 firms are included.

	(1)	(2)	(3)	(4)	(5)
Past Skew		-9.07	-9.36	-9.73	-9.59
		(-10.59)	(-10.63)	(-11.03)	(-10.88)
Vol		11.09	14.79	11.20	11.01
		(10.21)	(13.04)	(9.73)	(9.60)
Size				-10.90	-8.33
				(-3.48)	(-2.68)
Leverage				5.18	4.70
				(3.99)	(3.61)
Profitability				-1.28	-1.25
				(-0.97)	(-0.95)
GO	4.70	2.35	0.35	-2.07	-0.69
	(4.59)	(2.05)	(0.30)	(-1.55)	(-0.49)
Optionable					12.82
					(1.97)
Optionable × GO					-33.45
					(-3.94)
Turnover			-13.02	-10.72	-9.22
			(-10.82)	(-8.63)	(-7.33)
Momentum			-8.25	-8.91	-8.53
			(-11.03)	(-11.59)	(-11.12)
Nasdaq			-3.37	-6.27	-5.74
			(-1.35)	(-2.54)	(-2.31)
R&D	-2.11	-1.54	-0.66	-1.62	-1.49
	(-1.45)	(-0.92)	(-0.38)	(-0.92)	(-0.85)
Investment	-3.27	-3.20	-1.98	-1.94	-1.81
	(-6.33)	(-5.66)	(-3.41)	(-3.30)	(-3.07)
Adj-R^2	0.09	0.11	0.11	0.12	0.12
FE Adj-R^2	0.09	0.10	0.10	0.10	0.10