Short-Term Debt Overhang*

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Abstract

We consider a dynamic asymmetric information model where firms face multiple investment opportunities and their capital structure is endogenous at all times. We identify a new economic force, short-term debt overhang, which leads firms to issue short-term debt and subsequently underinvest in growth options. This force, which arises at the optimal mechanism and is time-consistent, generates several new testable predictions. Strikingly, we find that greater retained earnings, or cash, can reduce the investment in positive net present value projects by firms with intermediate credit ratings, and that these firms are the most likely to issue short-term debt.

Keywords: debt overhang, adverse selection, capital structure, debt maturity, underinvestment, credit rating

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1 Introduction

Firms face multiple investment and financing decisions over their life cycle. On the one hand, today's choices depend on a firm's prediction about future investment opportunities and market conditions. On the other hand, these choices are shaped by the firm's past investment and financing decisions, which determine the size and composition of its liabilities and assets. The objective of our paper is to investigate this two-way feedback loop. We study the optimal choices firms make when they face multiple investment opportunities under persistent informational asymmetries, and their capital structure is endogenously determined at all times. In our model, when choosing its initial capital structure, a high-quality firm might reduce the overall cost of its external financing by distorting its future investment to reveal private information. However, the profitability of such a strategy depends critically on the dynamic incentives of low-quality firms, which can profit from the anticipated overpricing at the rollover stage. We obtain three main results, which reverse classic intuition obtained by shutting down one of the two directions of the loop.

First, high-quality firms benefit from issuing short-term debt, which gives them strong incentives to underinvest in the future and allows them to roll over their initial debt at more favorable prices. Thus, contrary to Myers (1977), the underinvestment problem (debt overhang) is more severe when the firm has *shorter* liabilities. Second, increasing a firm's retained earnings, or cash, can make the underinvestment problem *worse*, unlike in Myers and Majluf (1984). Cash increases a low-quality firm's skin in the game from investment, enabling high-quality firms to issue more short-term debt and, consequently, to profit more from the short-term debt overhang strategy. Third, due to the low-quality firms' dynamic incentives, the relation between debt maturity and credit ratings is U-shaped, and hence the *opposite* of that implied by exogenous learning models, such as Diamond (1991). That is, firms with intermediate credit ratings benefit the most from the short-term debt overhang strategy and are the most likely to issue short-term debt.

We model the dynamics of the firm's investment and financing decisions in a parsimonious two-period model with persistent private information. An entrepreneur, privately informed about the quality of an investment opportunity, raises external capital at date zero. Once funded, the return of the existing project can be increased via a growth option at date one that requires additional external financing. For a given initial capital structure, the date one investment problem is similar to Myers and Majluf (1984). Thus, the market interprets lack of investment in the growth option as a positive signal about the quality of the assets in place. The key to our analysis is the anticipation of the dynamic effect that the capital structure at date zero has on future investment incentives. A high-quality firm that finances its assets in place with short-term debt can strategically induce underinvestment to take advantage of the favorable market inference and roll over its initial debt at favorable terms. Had the firm issued longer-term debt, the benefits of the positive market updating would have been shared between debt and equity holders, and consequently, the firm would have had stronger incentives to invest in the growth option. As a result, while short-term debt makes the firm less likely to invest in the future—i.e., there is *short-term debt overhang*—issuing short-term debt is ex ante optimal as it maximizes shareholders' value in high-quality firms.

The following example helps with intuition. There are two dates $t = 0$ and $t = 1$. At $t = 0$, a firm needs to raise \$2 to finance a project that yields either \$10 or zero at the end of date $t = 1$. The probability of receiving \$10 is the firm's private information, and it can either be high $p_H = 70\%$ or low $p_L = 20\%$. Funds are raised from competitive investors, who know there is a 15% If the firm invests at $t = 0$, it can improve its probability of receiving \$10 by 25 percentage points, irrespective of its type, by investing another \$0.7 at the beginning of date $t = 1$.

If at $t = 0$ all firms raise \$2 via short-term debt, which matures at $t = 1$, then this debt will always be rolled over successfully at $t = 1$, and as a result, it is risk-free. At $t = 1$, either the high type invests further, in which case the low type does so as well, and the total promised repayment is $\frac{\$2+\$0.7}{15\%\cdot 0.95+(1-15\%)\cdot 0.45} = \$5.14$, which yields a high type's expected payoff equal to $(0.7 + 0.25)(\$10 - \$5.14) = \$4.62$. Or the high type does not invest further, signaling its type, and rolls over existing short-term debt at $\frac{\$2}{0.7} = \$2.86$. In this case, its payoff is $0.7(\$10 - \$2.86) = \$5$, which is higher than the payoff from undertaking the growth option. Whether the high type can separate itself from the low type depends on the incentives of the low type to mimic. If the low type mimics the high and does not invest as well, its payoff is $0.2(\$10 - \$2.86) = \$1.43$, while if it separates and invests, its payoff is $(0.2 + 0.25)\left(\$10 - \frac{\$2.7}{0.2+0.25}\right) = \$1.8 > \$1.43$. So, separation is incentive compatible. Therefore, if short-term debt is issued at $t = 0$, the high type underinvests.

Next, suppose that long-term debt was issued at $t = 0$. The face value of the long-term debt depends on equilibrium investment at $t = 1$. Raising long-term debt in anticipation of future underinvestment is suboptimal for the high type since the market underprices long-term debt from the high type's perspective. If pooling with investment is anticipated, then both types would raise \$2 via a long-term debt with a face value $\frac{\$2}{15\%\cdot 0.95+(1-15\%)\cdot 0.45} = \$3.81$ at $t = 0$ and additional \$0.7 via short-term date with a face value $\frac{\$0.7}{15\%\cdot 0.95+(1-15\%)\cdot 0.45} = \$1.33$ at $t = 1$. If the high type decides to invest at $t = 1$, its payoff is again $(0.7 + 0.25)(\$10 - \$5.14) = \$4.62$. However, if at date $t = 1$, the high-type defies the market expectations and forgoes the investment opportunity, it receives $0.7(\$10 - \$3.81) = \$4.33$, which is lower than the payoff from undertaking the growth option. Hence, if long-term debt is issued at $t = 0$, the high type prefers to invest at $t = 1$.

This example clarifies that, depending on the *duration* of its liabilities—which, in our model, is endogenous—the firm may or may not have incentives to invest at $t = 1$. Moreover, the firm might optimally issue short-term debt to induce future underinvestment. We show that such a short-term debt overhang arises even when the firm is not limited to choosing between short- and long-term debt, and arbitrary optimal mechanisms are used to allocate resources instead. The reason is that short-term debt minimizes the subsidy a high-type firm needs to pay to lower types to achieve separation. As a consequence, and unlike Myers' underinvestment story, the short-term debt overhang: (1) is *dynamically consistent*—that is, firms anticipate the overhang when issuing short-term debt; and (2) it maximizes the high-type firm's value—that is, *there does not exist a feasible mechanism that would improve its payoff relative to issuing short-term debt.*

The time-inconsistency of long-term debt overhang has received extensive scrutiny in the empirical literature, which points to the various contractual instruments that firms and lenders use to solve the long-term debt overhang problem in practice. These solutions include issuing short-term or callable debt, as well as inserting restricting covenants into debt agreements (see, e.g., Barclay and Smith (1995), Blouin and Macchiavello (2019) and Wittry (2021)). In sharp contrast, there *cannot* be a contractual solution to short-term debt overhang, as it is ex ante optimal for high-quality firms—i.e., it maximizes their ex ante shareholders' value. Thus, our work can rationalize the puzzling empirical evidence that, controlling for leverage, firms with a larger share of short-term debt in their capital structure invest less in the future (Hong, Hou and Nguyen (2023)).

In the example, the high type can separate issuing only short-term debt since the low type's payoff from mimicking (\$1.43) is lower than that from undertaking the growth option (\$1.8). However, this is only sometimes the case in our setting. When the net present value of the growth option is small relative to outstanding short-term debt, the low type prefers to forgo investment and capture the benefits of rolling over short-term debt at a high price. Thus, we introduce the notion of a firm's *duration floor*, the lowest duration of a firm's liabilities, such that the short-term debt part can be rolled over at the high-type full-information rate without violating incentive constraints. The duration floor is decreasing in the NPV of the growth option since the low type is less likely to mimic underinvestment when the growth option is more profitable.

The predictions of our model concerning investment and debt maturity depend critically on a firm's *credit rating* (i.e., the prior that the firm is a high type). Specifically, while highly-rated firms always invest for all parameter values and finance themselves through long-term debt, the other firms might underinvest to maximize shareholders' value and issue short-term debt. It is helpful to distinguish between medium- and low-rated firms, as the predictions differ across the two groups.

First, for medium-rated firms, an increase in retained earnings might reduce (and ultimately have a non-monotonic effect on) investment. This result stands in sharp contrast to Myers and Majluf (1984) and is an outcome of an interplay between two competing forces. On the one hand, an extra $1 of retained earnings makes investment more attractive since the high-type firm needs to raise $1 less dollar of external financing. On the other hand, an extra $1 of retained earnings makes underinvestment more attractive since it lowers the duration floor and allows the high-type firm to refinance a larger share of its initial debt at the rollover stage. The latter force is stronger due to an amplification effect. With an extra $1 of retained earnings, the high-type firm could issue $1 more of short-term debt and $1 dollar less of long-term debt. Such a re-balancing does not affect the rollover benefits for the low-type firm, but it increases the low-type's skin-in-the-game (due to lower long-term debt) and makes it more willing to invest. The high type can exploit this increase in the willingness to invest by issuing more than $1 extra short-term debt, which, in turn, reduces the amount of long-term debt and further increases the low type's willingness to invest, and so on.

This amplification effect makes medium-rated firms more likely to issue short-term debt and underinvest with higher retained earnings. Eventually, as cash increases further, pooling with investment maximizes firm value again since the investment is primarily financed using retained earnings. Thus, our analysis sheds new light on the investment-cash flow sensitivity relation (see Fazzari, Hubbard, Petersen et al. (1988), Kaplan and Zingales (1997), Moyen (2004) and Hadlock and Pierce (2010)). While for firms that underinvest cash relaxes financing constraints, as in existing models, we find that for firms that invest more cash *introduces* a financing constraint and leads to underinvestment: shareholders' value maximization leads to a switch from full investment to short-term debt overhang. For low-rated firms, the relation between investment and cash flows is monotonic, as in Myers and Majluf (1984). That is, low-rated firms invest if and only if they have enough cash, which unambiguously relaxes financing constraints.

We now turn to the relationship between debt maturity and a firm's credit rating. The predictions of our model debt maturity depend critically on a firm's cash holdings. Specifically, while cash-rich firms can rely exclusively on short-term debt when they underinvest (consequently, their debt maturity is increasing in credit rating) the other firms always issue some long-term debt.

For moderate-cash firms with low credit ratings, adverse selection is so severe that it is optimal for these firms to forgo their growth options and separate using a mixture of short- and long-term debt which corresponds to the duration floor. As the credit rating goes up, the duration floor decreases, and the optimal debt maturity falls. A higher credit rating increases the price of long-term debt and allows the high-type firm to reduce its face value. A lower long-term debt burden increases the low type's skin-in-the-game and makes it more willing to invest, allowing the high type to increase the amount of short-term debt issued. In turn, a higher amount of short-term debt reduces the need to rely on long-term debt and further increases the low type's willingness to invest, and so on. Eventually, when the rating is high enough, the mispricing is so low that high-growth firms switch to full investment and long-term debt again. This implies that the relation between debt maturity and credit rating is U-shaped, which is the opposite of that in the Diamond (1991) model: firms with intermediate credit ratings benefit the most from issuing short-term debt and underinvesting in their growth option. Consequently, our model can explain the puzzling empirical results in Berger, Espinosa-Vega, Frame and Miller (2005) concerning high-risk firms.

Cash-poor firms raise more capital than the initial required investment amount when they forgo their growth options. A high amount of long-term debt and substantial excess cash create asset substitution incentives for the low-type firm, allowing the high-type firms to separate by underinvesting. As the credit rating goes up, the optimal debt maturity falls initially due to a decrease in the duration floor and subsequently due to a switch to full investment with long-term debt. A negative relationship between credit rating and optimal debt maturity for cash-poor firms contrasts Flannery (1986) who predicts the opposite. Moreover, it offers an alternative to the precautionary savings explanation for why firms raise external capital while at the same time holding positive cash balances (e.g., see the review Almeida, Campello, Cunha and Weisbach (2014)).

Summary of the empirical implications. To sum up, or model generates the following novel predictions. First, underinvestment is associated with short liabilities, not long. Second, for low-rated firms, cash relaxes the financing constraint and leads to higher rates of investment and longer-maturity liabilities. Third, for medium-rated firms, cash has a non-monotonic effect on the financing constraint. Namely, full investment and long liabilities arise when cash is either low or high. For intermediate cash levels, these firms opt for the short-term debt overhang strategy, issue more short-term debt, and underinvest. Moreover, firms with intermediate credit ratings are the most likely to issue short-term debt and underinvest, so the relation between rating and maturity is U-shaped for moderate-cash firms. Finally, cash-poor firms are likely to raise excess external financing even when they choose to underinvest in the future.

Relation to the literature. Our paper contributes to several strands of the literature. Theoretically, we explicitly introduce multiple investment and financing dates in the Myers and Majluf (1984) static adverse selection model, which is similar to the Akerlof (1970) lemons problem. Consequently, we highlight the bite of two implicit assumptions of MM's model: (i) a firm's assets in place have been financed with *inside equity*, i.e., the owners' cash; and (ii) all financing and investment choices except the current one are exogenous.[1] Relaxing these two assumptions leads to the discovery of short-term debt overhang, which arises at the optimal mechanism and therefore changes our predictions on the relation between cash, ratings, and investment under asymmetric information. This exercise complements a growing literature that introduces different types of dynamics in MM's model. For instance, Daley and Green (2012), Zryumov (2015), Asriyan, Fuchs and Green (2017) and Martel, Mirkin and Waters (2022), among others, focus on the timing of investment, while Bond and Zhong (2016) and Bond, Yuan and Zhong (2019) on multiple share-trading rounds.

Our paper also relates to the literature that studies the relation between liabilities and investment, which started with Myers (1977). Several theoretical papers explored the consequences of Myers' long-term debt overhang in various settings—see, e.g., He (2011) and Philippon and Schnabl (2013). Like us, Diamond and He (2014) also explore investment distortions created by debt with small maturities. However, in their setting, debt always matures after the single investment decision the firm faces. More recently, debt overhang is at the heart of the leverage ratcheting papers, such as Admati, DeMarzo, Hellwig and Pfleiderer (2018), Demarzo (2019), and DeMarzo and He (2021). While in these settings issuing short-term (or callable) debt would solve the underinvestment problem, in our model, it is precisely the possibility of issuing short-term debt which drives underinvestment due to the presence of persistent asymmetric information.

Our theory has implications for the optimal debt maturity chosen by firms. Specifically, it predicts that asymmetric information drives high-quality firms towards issuing short-term liabilities because they benefit from signaling their type through underinvestment at the rollover stage. This channel differs from the Diamond (1991), where there is exogenous learning over time.[2] In

[1] In Akerlof, this corresponds to the implicit assumptions that (i) good cars are owned by the seller, as opposed to having been leased, for instance; and (ii) the buy vs. lease decision is not modeled.

[2] Se also the recent dynamic extension of Diamond's model which is studied in Geelen (2019).

our model, information generation is endogenous, and whether or not a high-quality firm's underinvestment signals the type critically depends on the dynamic incentives of low-quality firms. Consequently, we obtain the opposite relation between credit ratings and debt maturity relative to Diamond's model. Alternatively, it has been argued that short-term debt helps in resolving commitment problems (see, e.g., Calomiris and Kahn (1991), He and Milbradt (2016) and Hu, Varas and Ying (2021)), absent informational asymmetries, or that it solves a trade-off between early termination and incentives provisions (see Huang, Oehmke and Zhong (2019)).

The paper proceeds as follows. Section 2 presents both the primitives of the model and the game played. Section 3 offers a few preliminary results, which include a full characterization of the second investment and financing stage for an exogenous debt-maturity structure. This analysis allows us to compare the static MM model, which starts at the second date, with our dynamic extension that takes us one step back to financing a firm's future 'assets in place.' Section 4 characterizes the equilibrium of the game and the short-term debt overhang. Section 5 studies the determinants of a firm's debt maturity structure and investment policy. Section 6 provides a strong justification for our game, showing that it implements the competitive planner's allocation. As a result, the short-term debt overhang cannot be eliminated by using alternative, superior mechanisms for allocating resources without violating feasibility or incentive compatibility. Section 7 concludes.

2 The model

Environment. There are three dates: $t = 0, 1, 2$. At $t = 0$, a continuum of risk-neutral firms have a project that requires raising $I_0 > 0$ from external investors. The investment yields two cash flows: a certain $C \geq 0$ at $t = 1$ and a stochastic $\tilde{X} \in \{0, X > 0\}$ at $t = 2$. Each firm privately knows its probability of success $\Pr.[\tilde{X} = X] = p_\theta \in \{p_L, p_H\}$. The investors only know the fraction of each type in the population: $\Pr.[\tilde{\theta} = \theta_H] = \alpha_0 \in (0, 1)$. At $t = 1$, all firms receive a second investment opportunity, which requires investing $I_1 > 0$ to increase the probability of success to $p_\theta + \Delta$, for some $\Delta \in (0, 1 - p_H)$. With a slight abuse of notation, define $p_\theta(a) = p_\theta + \mathbb{1}\{a = i\} \cdot \Delta$ to be the probability of success following the firm's $t = 1$ action $a \in \{i, n\}$ to invest or not, i.e., $p_\theta(n) = p_\theta$ and $p_\theta(i) = p_\theta + \Delta$. To make the problem interesting we assume that: (i) investment at $t = 0$ is positive-NPV, irrespective of the firm's type: $C + p_H X > C + p_L X > I_0$; (ii) investment at $t = 1$ is positive-NPV, and it requires external financing: $\Delta X > I_1 > C$;[3] and (iii) when the

[3]While the NPV is independent of the firm's type, this is not needed for our qualitative results. Moreover, qualitatively our results would not change if the firm had access to a smoother investment technology, with variable

high type does not invest, it is more productive than a low type that invests: $p_H > p_L + \Delta$. Let $p_t \stackrel{def}{=} \alpha_t p_H + (1 - \alpha_t) p_L$, for $t = 0, 1$, where α_0 is the exogenous prior, whereas α_1 is the updated prior based on date-zero actions (as we shall discuss in details). Moreover, we normalize all agents' outside options as well as the risk-free rate to zero.

Game. At each investment date $t = 0, 1$ competitive investors and firms interact through the following three-stage screening game. In the first stage, the investors offer contracts to the firms. Each contract describes the cash flows that investors receive in return for their capital, and it may also specify a break-up option for investors, which we describe below. In the second stage, firms either select one contract, or reject all contracts. Whenever firms are indifferent between contracts, they choose each contract with equal probability. If the accepted contract has no break-up option, then the third stage is not played: both the firm and the investors proceed to the next period committed to the accepted contract. If the accepted contract comes with a break-up option, then in the third-stage of the game investors have the right to withdraw their offer, based on their updated belief about the firm's type. If investors exercise their break-up option, then the accepted contract is withdrawn and the firm stays at its endowment. If they do not exercise it, then both the firm and the investors proceed to the next period committed to the accepted contract.

This three-stage game resembles standard practice in financial markets, where lenders typically propose rates but do not commit to accept all applications that qualify for their offers. As we will show in the next section, considering this game is without loss of generality, as it implements the optimal allocation. The presence of the third stage ensures that an equilibrium always exists, which is not the case with a standard two-stage screening game.

Date-zero contracts. At $t = 0$, lenders can offer a mixture of short- and long-term debt in exchange for I_0. The face value of the short-term debt is D_1 and that of the long-term debt is $D_2 \geq 0$. Let this debt be senior to any future claim. Thus, a contract is a tuple $K_0 = (D_1, D_2, \gamma_0)$, where $\gamma_0 \in \{c, nc\}$ denotes whether the investor is committed to its offer ($\gamma_0 = c$), or whether it retains the break-up option ($\gamma_0 = nc$). A date-zero contract is *feasible* if $D_1 + D_2 \leq X + C$. We model rejection at $t = 0$ as the acceptance of a "zero contract", which does not provide funding to the firm and does not generate any cash flows for the investors. That is, all face values are zero

investment. Indeed, short-term debt overhang is not driven by the non-convexity of our investment technology, but by the implied repricing of debt claims at $t = 1$.

and the firm does not undertake the investment opportunity. Thus, the set of contracts offered by lenders at date zero is $\mathcal{K}_0 \stackrel{def}{=} \{K_0 | K_0 \text{ was offered at } t = 0\} \cup \{K_\emptyset\}$, where $K_\emptyset$ is the zero contract.

Notice that the face values D_1 and D_2 are not contingent on the future investment action a. Therefore, in our implementation we need to consider the subgame perfection constraint which ensures that the investment decision at date one, a, is ex post optimal for a firm that has a capital structure in place with long- and short-term debt. This makes the dynamics of our game relevant, and it shows that such commitment is not needed to implement the efficient allocation.

Date-one contracts. At $t = 1$, the firm raises capital using short-term debt. Formally, a $t = 1$ contract is a tuple (Q_1, F_2, a, γ_1) that specifies the amount of capital Q_1 raised at $t = 1$, the face value of the short-term debt $F_2 \geq 0$ to be repaid at $t = 2$, the investment action $a \in \{i, n\}$ at $t = 1$, and break-up option $\gamma_1 \in \{c, nc\}$. A date-one contract is *feasible* if the capital raised is sufficient to cover the firm's short-term liabilities, as well as its investment needs $Q_1 = \mathbb{1}\{a = i\} I_1 - C + D_1$, and the promised face value to be repaid when the project succeeds $F_2 \leq X - D_2$.[4] We allow date-one investors to offer menus of contracts $K_1 = \{(Q_1^a, F_2^a, a, \gamma_1^a)\}_{a \in \{i,n\}}$ with two options, one per investment action. Denote the set of menus offered by competitive lenders at date one by $\mathcal{K}_1 \stackrel{def}{=} \{K_1 | K_1 \text{ was offered at } t = 1\} \cup \{K_\emptyset\}$ where, with a slight abuse of notation, $K_\emptyset = (0, 0, n, c)$ denotes a zero contract at $t = 1$. That is, a contract in which the firm does not receive any capital from $t = 1$ investors, the face value of the date-one debt is zero and the firm does not invest.

Payoffs. When a type-θ firm selects a sequence of non-zero contracts (K_0, K_1) and an investment option a, the firm's payoff (in the absence of future withdrawal by investors) is

$$U_\theta(K_0, K_1, a) \stackrel{def}{=} p_\theta(a)[X - D_2 - F_2^a]$$

The expected profits for date-zero investors are

$$\pi_{\theta,0}(K_0, K_1, a) \stackrel{def}{=} D_1 + p_\theta(a) D_2 - I_0.$$

The above equation reflects the fact that the short-term debt with face value D_1, in this case, is risk-free. That is, if the firm does not default this face value is always repaid. The expected profits

[4] Notice that, whenever $a = n$ and $D_1 < C$, taking the no-investment option requires the firm to pay some positive amount of cash to the lenders $C - D_1 > 0$ at date one, as well as another positive amount of cash at date two $F_2^n \geq 0$. Thus, whenever $D_1 < C$ this option is clearly dominated by taking the zero contract, which is always feasible.

for date-one investors in this case are

$$\pi_{\theta,1}(K_0, K_1, a) \stackrel{def}{=} p_\theta(a)F_2 - Q_1.$$

When a type θ firm selects a sequence of contracts $(K_0, K_\emptyset)$, or when the investors exercise the break-up option at $t = 1$, the expected insiders' payoff is

$$U_\theta(K_0, K_\emptyset, a) \stackrel{def}{=} \begin{cases} p_\theta(n)[X + C - D_1 - D_2] + (1 - p_\theta(n))[C - D_1 - D_2]^+ & \text{if } D_1 \leq C; \\ 0 & \text{if } D_1 > C, \end{cases}$$

while the expected profits for date-zero investors are

$$\pi_{\theta,0}(K_0, K_\emptyset, a) \stackrel{def}{=} \begin{cases} p_\theta(n)(D_1 + D_2) + (1 - p_\theta(n)) \cdot \min[C, D_1 + D_2] - I_0 & \text{if } D_1 \leq C; \\ C + p_\theta(n)X - I_0 & \text{if } D_1 > C. \end{cases}$$

In the above payoffs, we implicitly assumed that the failure to pay D_1 at $t = 1$ results in the firm defaulting and transferring all assets to time $t = 0$ debt holders. When a type θ firm selects a zero contract at $t = 0$, or the time $t = 0$ investors exercise their break-up option, all parties receive 0.

Equilibrium. As our screening game has three stages, investors observe the firm's choices in the second stage and they might update their beliefs about the firm's quality before exercising the break-up option. Thus, the appropriate equilibrium concept is Perfect Bayesian Equilibrium. Due to the dynamic nature of the model, we define equilibrium recursively, starting at $t = 1$.

Definition. *For any given set of offered contracts $\mathcal{K}_0$ and chosen contract K_0 at $t = 0$, which implements the initial investment, a* **date-one equilibrium** *consists of a set of menus $\mathcal{K}_1^*$, a chosen menu K_1^*, firm actions (a_H^*, a_L^*) and withdrawal policy w_1^* that satisfy:*

1. *Contract optimality: there does not exist another set $\mathcal{K}_1'$, a menu $K_1' \in \mathcal{K}_1'$ with associated firm actions (a_H', a_L') and withdrawal policy w_1' which is weakly more attractive to at least one type of the firm, and generates strictly higher expected profits to investors than K_1^*, given their updated belief $\alpha_1 = Pr.[\theta = H|\mathcal{K}_0, K_0]$.*

2. *Firm's optimality: $U_\theta(K_0, K_1^*, a_\theta^*) \geq U_\theta(K_0, K_1, a)$ for every a and every $K_1 \in \mathcal{K}_1$, anticipating contract K_1 possible future withdrawal w_1^*.*

3. *Break-up optimality: for every menu $K_1 \in \mathcal{K}_1^*$, and every chosen action within the menu a, if the menu has a break-up option, then $w_1^* = 1$ (i.e., there is no withdrawal) if and only if*

$$\alpha_2 \pi_{H,1}(K_0, K_1, a) + (1 - \alpha_2)\pi_{L,1}(K_0, K_1, a) \geq 0,$$

where $\alpha_2 = Pr.[\theta = H|\mathcal{K}_0, K_0, \mathcal{K}_1^*, (K_1, a)]$ denotes the investors' posterior belief at $t = 1$, given that the firm chose a contract K_0 from $\mathcal{K}_0$, and chose (K_1, a) from $\mathcal{K}_1^*$.

The date-one equilibrium has the following features: menus are offered optimally by competitive investors; firms choose the optimal menu among those that have been offered; investors exercise optimally their break-up option (if any such option is part of the optimal menu that has been chosen by the firm). We can now define equilibrium at date zero.

Definition. *At $t = 0$, a* **date-zero equilibrium** *consists of a set of offered contracts $\mathcal{K}_0^*$, a chosen contract K_0^* and a withdrawal policy w_0^* that satisfy:*

1. *Contract optimality: there does not exist another set $\mathcal{K}_0'$, a menu $K_0' \in \mathcal{K}_0'$ and a withdrawal policy w_0' which is weakly more attractive to at least one type of the firm, and generates strictly higher expected profits to investors than K_0^*, given the date-one equilibrium induced by $(\mathcal{K}_0^*, K_0^*)$, which is $(\mathcal{K}_1^*, K_1^*, a^*, w_1^*)$, and that induced by $(\mathcal{K}_0', K_0')$, which is $(\mathcal{K}_1', a', w_1', K_1')$.*

2. *Firm's optimality: $U_\theta(K_0^*, K_1^*, a_\theta^*) \geq U_\theta(K_0, K_1, a)$ for every $K_0 \in \mathcal{K}_0$, and every (K_1, a) induced by K_0, anticipating contract withdrawals (w_0^*, w_1^*).*

3. *Break-up optimality: for every contract chosen by the firm $K_0 \in \mathcal{K}_0^*$, if the contract contains a break-up option, then $w_0^* = 1$ (i.e., there is no withdrawal) if and only if*

$$\alpha_1 \pi_{H,0}(K_0, K_1, a) + (1 - \alpha_1)\pi_{L,0}(K_0, K_1, a) \geq 0,$$

where $\alpha_1 = Pr.[\theta = H|\mathcal{K}_0, K_0]$, as defined before.

The date-zero equilibrium concept mirrors that for date one. The key difference is that choices are made anticipating what equilibrium will be played subsequently.

3 Preliminary Analysis

In this section, we characterize the mapping between each possible history at date zero—which consists of $(\mathcal{K}_0, K_0)$, as well as a posterior belief α_1—and the corresponding equilibrium at $t = 1$, assuming that investment at $t = 0$ takes place. Given that we allow for $D_1 = D_2 = 0$, this analysis nests the Myers and Majluf case, in which the firm finances itself at date zero with inside equity.

The following Lemma states that: (i) the equilibrium allocation maximizes the payoff of the best firm type, subject to incentive compatibility and feasibility; (ii) lender profits must be zero in expectation (ZP); and (iii) lenders cannot make strictly positive profits on low types (NP$_L$).

Lemma 1. *An equilibrium pair $(\mathcal{K}_1^*, K_1^*)$, an action profile (a_H^*, a_L^*) and a withdrawal policy w_1^* solve the following problem, for any given non-zero contract K_0 accepted at $t = 0$:*

$$(\mathcal{K}_1^*, K_1^*, a_H^*, a_L^*, w_1^*) \in \text{argmax}_{\mathcal{K}', K', a_H', a_L', w'} \; U_H(K_0, K', a_H') \; \textit{subject to:} \tag{1}$$

$$U_\theta(K_0, K', a_\theta') \geq U_\theta(K_0, K', \hat{a}), \qquad \forall \theta \; \textit{and} \; \forall (K', \hat{a}) \; \textit{s.t.} \; K' \in \mathcal{K}' \tag{IC_θ}$$

$$\alpha_1 \cdot \pi_{H,1}(K_0, K', a_H') + (1 - \alpha_1) \cdot \pi_{L,1}(K_0, K', a_L') = 0 \tag{ZP}$$

$$\pi_{L,1}(K_0, K', a_L') \leq 0 \tag{NP_L}$$

$$(\mathcal{K}', K') \; \textit{is feasible}$$

Proof. All proofs are in the Appendix. □

Competition among investors and the fact that all firm types have a positive net present value project drives investor profits to zero in equilibrium.[5] Moreover, low-quality firms can always achieve their full-information payoff, because the incentive compatibility constraint of high-quality firms does not bind. It follows that investors cannot make profits on low-quality firms in any zero-profit equilibrium menu. Finally, the equilibrium maximizes the high-quality firms' payoff because of competition among investors, coupled with the fact any other allocation would offer high types deviations that signal their type, breaking the equilibrium.

We now consider a few cases separately, depending on the characteristics of the date-zero capital structure, which is exogenous in the date-one game. We start from the case in which the firm did not issue any outside debt at date zero, which is the MM's case.

[5] Notice that we do not impose zero-profits on a contract by contract basis, but rather we allow for cross-subsidization across types.

Lemma 2 (Myers-Majluf)**.** *If $D_1 = D_2 = 0$, then, the date-one equilibrium features investment by all types when*

$$\underbrace{\Delta X - I_1}_{NPV_1} > \underbrace{\frac{(p_H - p_1)}{(p_1 + \Delta)}(I_1 - C)}_{Investment\ mispricing}. \tag{2}$$

If inequality (2) is strictly reversed, the equilibrium features investment only by low types, while high types take the zero contract. If (2) holds as an equality, both allocations are equilibria.

Inequality (2) reproduces the analysis by Myers and Majluf in our setting. In equilibrium, the growth option is undertaken by high-type firms if and only if its net present value exceeds the mispricing associated with financing it for high types. Notice that, when a firm has higher retained earnings, which corresponds to a higher cash flow C at t=1, inequality (2) is relaxed, which stimulates investment. That is, MM predicts a *positive investment-cash flow sensitivity.*

From this viewpoint, however, the case of $D_1 = D_2 = 0$ appears rather special: very few firms are able to grow without requiring any external financing early on. Thus, the next Lemma considers the more plausible case in which a firm contracted some liability $t = 0$ and new forces are at play:

Lemma 3. *Suppose that $D_1 \leq C$. Then, in a date-one equilibrium all types invest when*

$$\underbrace{\Delta(X - D_2) - I_1}_{NPV_1} > \underbrace{\frac{(p_H - p_1)}{(p_1 + \Delta)}(I_1 - (C - D_1))}_{Investment\ Mispricing} - \underbrace{(1 - p_H)\min(D_2, C - D_1)}_{Dilution\ of\ LT\ debt} \tag{3}$$

If inequality (3) is strictly reversed, high types take the zero contract and low types invest when

$$\underbrace{\Delta(X - D_2) - I_1}_{NPV_1} > -\underbrace{(1 - p_L)\min(D_2, C - D_1)}_{Dilution\ of\ LT\ debt}. \tag{4}$$

If both (3) and (4) are strictly reversed, then all types take the zero contract and do not invest.

The presence of a small amount of short-term debt D_1 increases the external capital required to invest at $t = 1$. As a result, it increases the hurdle for the growth option to be undertaken by high-type firms. In contrast, as highlighted in inequalities (3) and (4), long-term debt D_2 affects investment incentives in two opposite ways, regardless of the firm's type. On the one hand, it reduces the incentives of the equity holders to invest through Myers (1977) long-term debt overhang channel. On the other hand, it makes investment more attractive through the dilution channel. That is, by investing the remaining cash $C - D_1$ in the growth option, equity holders increase the riskiness of the long-term debt outstanding, and benefit from this shifting of risk.

The two opposing forces created by long-term debt can make the equilibrium level of investment non-monotone in D_2. With a small amount of long-term debt, the dilution effect is stronger than the overhang because $\Delta < 1 - p_H$. Thus, an increase in the face value of the long-term debt makes investment more attractive for high types, as the l.h.s. of (3) is decreasing in D_2. In contrast, for large values of D_2 (i.e., such that $D_2 > C - D_1$), the overhang effect dominates and an increase in D_2 makes the growth option less attractive for high types, which become less likely to invest.

The next lemma characterizes the $t = 1$ continuation equilibrium when the level of short-term debt D_1 exceeds C and, as a consequence, the amount $D_1 - C$ needs to be rolled over regardless of the investment decision, as otherwise the firm defaults and the inside equity holders get zero.

Lemma 4. *Suppose that $D_1 > C$. Then, equilibrium investment can be broken down in two cases:*

1. *If $\Delta(X - D_2) - I_1 > 0$, then the date-one equilibrium features investment by all types when*

$$\underbrace{\Delta(X-D_2)-I_1}_{NPV_1} \geq \underbrace{\frac{(p_H - p_1)}{(p_1 + \Delta)}(I_1+D_1-C)}_{\text{Investment mispricing}} - \underbrace{\frac{(1-\alpha)p_H}{p_1}\left[\frac{p_H-p_L}{p_H}(D_1-C)-(\Delta(X-D_2)-I_1)\right]^+}_{\text{ST debt rollover subsidy to L type}}. \quad (5)$$

   Otherwise, if inequality (5) is reversed, then only low type firms invest.

2. *If $\Delta(X-D_2)-I_1 \leq 0$, then the date-one equilibrium features no investment by any firm type.*

The characterization of the equilibrium investment depends on the amount of long-term debt issued at date zero D_2. In case (1), the Myers' long-term debt overhang is not too strong, and as a consequence efficient investment is possible. In contrast, in case (2) the face value of long-term debt is so high that the Myers' long-term debt overhang channel prevents further investment.

Case (1) has two sub-cases, depending on the amount of short-term debt $D_1 - C$ that needs to be rolled over with external funds at $t = 1$. As in MM, lack of investment in equilibrium is a positive signal about the quality of the assets in place. However, whether or not investment occurs is determined by inequality (5), which differs from MM's inequality (2) in three important ways.

First, the face value of long-term debt D_2 makes investment relatively less profitable through the Myers' long-term debt overhang channel. Second, the amount of short-term debt that needs to be rolled over with external funds $D_1 - C$ reduces the incentives to invest by increasing the profitability of debt rollover without investment. Third, $D_1 - C$ affects the incentives of low-type firms. When $D_1 - C$ is small relative to the NPV of the growth option, net of long-term debt, the short-term debt rollover subsidy that is required for incentive compatibility to hold is zero. As a result, high types can separate by forgoing the growth option and rolling over the short-term debt at its full-information price. In contrast, when $D_1 - C$ is large enough, high types are forced to rollover their short-term debt at a discount relative to the full information price, in order to deter low types from mimicking. This discount makes pooling with investment more attractive, as can be seen in (5). Mimicking high types is more attractive when either $D_1 - C$ is high—i.e., when there is a large amount of liabilities to be rolled over at date one—or when the NPV of the growth option is low. Thus, the rollover discount is increasing in $D_1 - C$ and decreasing in the NPV.

4 Equilibrium and Short-term Debt Overhang

Having characterized all possible continuation equilibria in the sub-game at date $t = 1$, we now turn our attention to date $t = 0$. We begin by showing three useful properties of the date-zero equilibrium, summarized in the next Lemma, that simplify the characterization.

Lemma 5. *Any date-zero equilibrium must be pooling, i.e., $\alpha_1 = \alpha_0$. Moreover, the equilibrium contract delivers zero profits to investors and maximizes the payoff of the high-type firm.*

The intuition behind pooling at date $t = 0$ is straightforward, as date-zero separation can only be achieved via the participation constraint of one of the types.[6] However, the firm's outside option is zero, irrespective of its type, while all investment opportunities (at $t = 0$ and $t = 1$) have a positive net present value. Thus, there always exists a contract—e.g., fairly priced long-term debt—which generates a strictly positive payoff to the firm, a non-negative payoff to the investors, and avoids defaults at $t = 1$. Thus, separation at $t = 0$ is impossible.

That lenders break even follows immediately from the absence of menus at date $t = 0$ and competition. A different lender would undercut any contract that makes positive profits due to free entry. The zero-profit condition, together with the pooling belief α_0, creates a link between the time $t = 0$ financing decision which results in some debt mixture (D_1, D_2) and a date $t = 1$ continuation game. This link, together with Lemmas (3) and (4), allow us to characterize the equilibrium outcome of the date $t = 0$ game.

[6] Because at date $t = 0$, investors offer single contracts, not menus.

Finally, as standard in screening games, the equilibrium contract maximizes the payoff of high-type firms. Intuitively, if this was not the case, then there would exist a profitable deviation for high types that would still generate zero profits for investors. A slightly modified version of this contract would still be more profitable for high types relative to the equilibrium contract, and it would generate positive profits for investors. Because of free entry of investors, any strictly positive profits would attract them to post such a contract, which destroys the conjectured equilibrium.

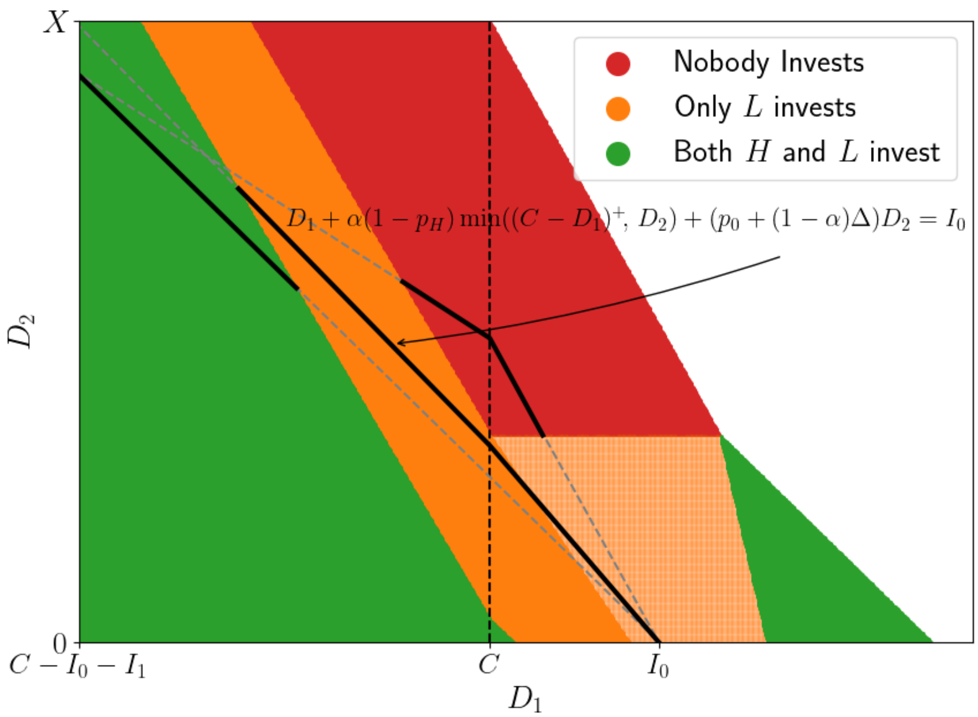


Figure 1: Date $t = 1$ equilibrium outcomes together with date $t = 0$ zero profit lines. The lower zero profit line is under the expectation of full investment; it is time consistent in the green region. The middle zero profit line is under the expectation of partial investment; it is time consistent in the orange region. The top zero profit line is under the expectation of no investment; it is time consistent in the red region. Model parameters are $X = 10$, $p_H = 0.65$, $p_L = 0.2$, $\alpha = 0.35$, $I_0 = 2.75$, $C = 1.0$, $\Delta = 0.3$, $I_1 = 2.0$.

Equilibrium. The properties highlighted in Lemma 5 allow us to evaluate equilibrium outcomes from the point of view of the high-type firm and to reduce the space of potential continuations.

First, we argue that it is (weakly) suboptimal to issue short-term debt below $\min(I_0, C)$. Any amount of short-term debt below C moves cash from date $t = 1$ to date $t = 0$ and allows the high type to reduce its reliance on costly long-term debt. Thus, a continuation game with $D_1 < C$, as characterized in Lemma 3, is only possible when the initial amount of investment I_0 is below C. Whenever $I_0 > C$, a high type always prefers the continuation game of Lemma 4, with $D_1 \geq C$.

Second, we show that the low-type firm always undertakes the growth option in any equilibrium. That is, the equilibrium contract (D_1, D_2) cannot be in the red area of Figure 1. Surprisingly, the intuition for this result stems from the incentives of the high-type firm. As shown in Lemmas 3 and 4, any continuation outcome in which the low type firm does not invest at date $t = 1$ requires a substantial amount of long-term debt to create long-term debt overhang for low types. Within the no-investment region, the high type can swap long-term debt for short-term debt and weakly increase its payoff. Such a reduction of debt maturity can be performed until one reaches the boundary of the no-investment region—i.e., when the long-term debt overhang just binds $\Delta(X - D_2) - I_1 = 0$. The date $t = 0$ zero-profit line that sustains investment in the growth option by low types is necessarily flatter than the one where no firm invests. That is, it requires a lower face-value of long-term debt for the same amount of short-term debt. Moreover, as the high-type firm strictly prefers a lower D_2, the no-investment outcome is dominated by either both types undertaking the growth option, or only the low type doing it. It follows that, when both directions of the two-way feedback loop are explicitly considered, *there is no long-term debt overhang* in our model.

Having excluded the no-investment outcome, the remaining candidate allocations are (a) pooling with both types undertaking the growth option, and (b) separating where only the low type invests at $t = 1$. Conditional on date $t = 1$ investment, the equilibrium payoff of the high-type firm depends only on the total quantity of debt $I_0 + I_1 - C$ issued. Thus, if in the continuation equilibrium all types invest, the substitution of long- for short-term debt does not affect the final payoff of high-type firms, and it is therefore irrelevant.

In contrast, debt maturity plays a crucial role when the high-type firm does not invest at date $t = 1$. In this case, the high type would want to reduce the quantity of long-term debt issued at date $t = 0$ and raise short-term debt $D_1 > C$, whenever the excess short-term debt $D_1 - C$ can be rolled over without a discount. The maximal quantity of short-term debt and, consequently, the *duration floor* of the high-type firm's liabilities is disciplined by the incentives of low-type firm. Lemma 4 shows that if high types issue too much short-term debt, they need to roll it over at a discount to deter the low types from mimicking. This point is illustrated in Figure 1: as the amount of short-term debt rises, the zero-profit line crosses from the orange region, where short-term debt can be rolled over without a discount, into the light-orange region, where the discount on short-term is unavoidable.

We define the *duration floor* $MacD$ as the lowest (Macaulay) duration of the date $t = 0$ liabilities along the lender's zero-profit curve $D_1 + (p_0 + (1-\alpha)\Delta)D_2 = I_0$ that allows the high-type firm to roll over its excess debt $D_1 - C$ without a discount.[7] Lemma 4 shows that the excess debt can be rolled over without subsidy whenever $(1 - p_L/p_H)(D_1 - C) \leq \Delta(X - D_2) - I_1$. Thus, the shortest duration is given by

$$MacD \stackrel{def}{=} 1 + \frac{1}{I_0} \cdot \frac{p_H(p_0 + (1-\alpha)\Delta)}{p_0(p_H - p_L - \Delta)} \cdot \left[\frac{p_H - p_L}{p_H}(I_0 - C) - (\Delta X - I_1)\right]^+ . \tag{6}$$

Notice that the duration floor is equal to one when the incentive constraint for low types to mimic is not binding, in which case the square bracket in the above definition equals zero. Otherwise, the duration floor raises above one, which implies that a strictly positive amount of long-term debt is issued.

The duration floor $MacD$ plays an essential role in characterizing the equilibrium level of investment, because it affects the maximal payoff the high-type firm can achieve when it chooses to separate from low types by not investing in the growth option.

When the amount of long-term debt D_2 that gives rise to the duration floor $MacD$ is large enough, the low type might forgo investment all together due to the long-term debt overhand. In order to incentivize investment of the low type firm, the high type needs to lower the duration of liabilities down to

$$\overline{MacD} \stackrel{def}{=} 1 + \frac{1}{I_0} \cdot (p_0 + (1-\alpha)\Delta) \cdot \left(X - \frac{I_1}{\Delta}\right). \tag{7}$$

Since $\overline{MacD}$ entails issuing more short-term debt relative to $MacD$, the short-term debt can no longer be rolled over at fair terms on date $t = 1$. Hence, the high-type firm will have to accept to a discount on both its long-term debt at date $t = 0$ and short-term at the rollover stage. However, as the next proposition shows, the total mispricing is independent of the split and is pinned down by $MacD$ only.

Proposition 1. *In any date $t = 0$ equilibrium the low-type firm always invests in the date $t = 1$ growth option. Whether the high-type firm undertakes the growth option is determined by the*

[7] With any arbitrarily small amount of rollover risk the duration floor uniquely pins down the optimal composition of long- and short- term debt in equilibrium.

following inequality:

$$\underbrace{\Delta X - I_1}_{NPV_1} \geq \underbrace{\frac{(p_H - p_0)}{(p_0 + \Delta)}(I_1 + I_0 - C)}_{Investment\ mispricing} - \underbrace{I_0 \cdot (MacD - 1) \cdot \frac{p_H - p_0 - (1-\alpha)\Delta}{p_0 + (1-\alpha)\Delta}}_{Joint\ mispricing\ of\ LT\ and\ ST\ debt}. \tag{8}$$

When inequality (8) does not hold, we have the following two cases:

1. *When $MacD = 1$, then both firms issue* ***only*** *short-term debt at date $t = 0$. At $t = 1$, high types separate by repaying $\min(I_0, C)$ and rolling over the remaining short-term debt $(I_0 > C)^+$ without investment, at fair terms;*

2. *When $MacD > 1$, then both firms issue a* ***mixture*** *of short- and long-term debt with duration $\min(MacD, \overline{MacD})$ at date $t = 0$. At $t = 1$, high types separate by repaying C and rolling over the remaining short-term debt without investment, at fair terms (at a discount) when $MacD < \overline{MacD}$ (when $MacD > \overline{MacD}$).*

Proposition 1 describes how the date $t = 1$ equilibrium investment decision of high type firms depends on the primitive model parameters, under the *optimal* date $t = 0$ capital structure. When the duration floor $MacD$ equals 1, inequality (8) differs from MM's comparison of NPV in (2) because of the presence of one additional term, which reflects short-term debt overhang. This stems from the fact that, absent investment, the high-type firm would issue $D_1 = I_0$ short-term debt at date $t = 0$, and subsequently roll it over at fair terms. Therefore, the presence of short-term debt in the optimal capital structure creates an overhang and might preclude the high-type firm from undertaking an ex-post efficient investment.

When the duration floor $MacD$ is greater than 1, a new determinant of the high-type firm's investment arises. Lack of investment necessitates using some long-term debt, in order to prevent low-type firms from mimicking. Long-term debt is too expensive to issue for high types, and so it makes separating without investment less attractive. The higher is the duration floor $MacD$, the larger the aggregate mispricing of long-term securities required for separation and underinvestment, and the easier it is to satisfy the investment constraint (8). Thus, in sharp contrast with the Myers' long-term debt overhang channel, this channel implies that underinvestment occurs when a firm's liabilities have *shorter* maturity.

Short-term debt overhang. Thus far, we have identified parameter conditions that lead to underinvestment in equilibrium and have highlighted the novel role played by short-term debt.

However, underinvestment does not immediately imply a presence of a debt overhang. Due to adverse selection in our model, the high-type firm might not undertake the growth option regardless of the securities issued at date zero.

To clarify that our model does indeed generate a short-term debt overhang, in the following proposition, we characterize the conditions under which (i) in equilibrium, the high type issues some short-term debt at $t = 0$ and does not want to invest at $t = 1$; and (ii) if a high type were to issue enough long-term debt at $t = 0$ (which would be suboptimal), it would subsequently prefer to invest at $t = 1$. In other words, underinvestment is driven precisely by the short maturity of the firm's debt, as in Myers (1977). There is short-term debt overhang in the sense that high-quality firms issued *too much short-term debt* to be able to invest in their growth options later on. However, unlike in Myers' case, facing this kind of overhang ex post is actually ex ante optimal for high-quality firms, as it maximizes their shareholders' value. Thus, it cannot be contracted away.

Proposition 2 (Short Term Debt Overhang)**.** *Whenever the model parameters are such that the firm underinvests (i.e., inequality (8) does not hold), then there is short-term debt overhang if the following inequality holds:*

$$\Delta \left(X - \frac{I_0 + I_1 - C}{p_0 + \Delta} \right) - I_1 \geq -(1 - p_H) \min \left(\frac{I_0 + I_1 - C}{p_0 + \Delta}, I_1 \right). \tag{9}$$

To see the intuition behind Proposition 2, notice that the firm can issue at date zero an amount of long-term debt such that it raises all the required external funds $I_0 + I_1 - C$, by setting $D_1 = C - I_1 < 0$. In this case, if investment in the growth option is anticipated at date one, the face value of the date-zero long-term debt is $D_2 = \frac{I_0+I_1-C}{p_0+\Delta}$. If we also have $D_2 \leq I_1$, then according to our Lemma 3 high types invest if and only if $\Delta X - I_1 \geq D_2(\Delta + p_H - 1)$.[8] Because $\Delta X > I_1$, $D_2 > 0$ and $\Delta + p_H \leq 1$, the inequality always holds and there is short-term debt overhang. In this case, issuing long-term debt with face value D_2 does not induce any long-term debt overhang at date one, and hence the liability structure matters for investment. In contrast, if $D_2 > I_1$, it is possible that raising such an amount of long-term debt induces overhang at date one, in which case underinvestment does not depend on the firm's liability structure.

Discussion. The previous analysis uncovers how, under persistent informational asymmetries, the short-term debt overhang strategy can be optimal and maximize the date-zero shareholders'

[8] Lemma 3 is the relevant preliminary result to rely on here, due to the fact that $D_1 < C$ in our case.

value. This strategy consists in financing early investments with as much short-term debt as it is incentive compatible, and then subsequently underinvest in growth options, which signals the quality of the firm's assets in place and allows a high-quality firm to rollover the short-term debt at more favorable terms. The economic driving forces of short-term debt overhang clearly apply to a broader set of environments, with potentially multi-dimensional signaling opportunities. Whenever the signal structure available to high-quality firms does not lead to a crossing of zero-profit curves across types, high types need to distort future actions to credibly generate information. In all cases in which a signal that induces single crossing can be generated at a cost, issuing short-term debt enables high-type firms to fully profit from the future repricing of securities, which maximizes the returns for these firms from costly signaling, and as a result it is optimal. One example of such a model could feature variable investment, another i.i.d. projects over time.

It follows from the optimality of short-term debt overhang that standard contractual solutions proposed to mitigate either the Myers or the Myers-Majluf underinvestment problems cannot work in our setting. Specifically, in Myers (1977) a series of potential solutions to the long-term debt overhang problem is offered, which includes restricting dividends, introducing protective covenants, shortening debt maturity and relying on impartial mediators. None of these instruments can 'solve' the short-term debt overhang problem, because it implements the ex ante optimal allocation.

5 Comparative Statics

We have defined $MacD$ as the duration floor, that is, the shortest duration that allows high-type firms to separate from low types by rolling over short-term debt without investment. Next, we show how the duration floor changes with the model parameters. Such comparative statics allows us to investigate the properties of the optimal debt maturity (or capital structure) in the absence of high-type investment, and it is crucial for understanding whether investment happens in equilibrium.

Proposition 3. *The duration floor $MacD$ is*

(i) decreasing in the NPV of the date $t = 1$ growth option, i.e., decreasing in X and Δ and increasing in I_1;

(ii) increasing in the date $t = 0$ net debt issuance, i.e., increasing gross date $t = 0$ debt issuance I_0 and decreasing in date $t = 1$ retained earnings C;

(iii) increasing in the severity of the adverse selection, i.e., decreasing in α and p_L.

To see the intuition behind Proposition 3, recall that the duration floor is pinned down by the incentives of low-type firms (IC_L) and the investor s' zero profit condition (ZP_0):

$$D_1 + [\alpha p_H + (1-\alpha)(p_L + \Delta)]D_2 = I_0, \qquad (ZP_0)$$

$$\frac{p_H - p_L}{p_H}(D_1 - C) \leq \Delta(X - D_2) - I_1. \qquad (IC_L)$$

A higher NPV of the time $t = 1$ project makes the low type more willing to undertake the growth option—that is, it increases its skin in the game and relaxes (IC_L). Thus, a deviation to rollover without investment becomes relatively less attractive for a low type. High types can exploit this slack in the low type's incentive constraint by increasing the amount of short-term debt in their capital structure, reducing long-term debt. As a result, $MacD$ decreases the NPV of the growth option. This intuition exactly describes the economic forces associated with an increase in the project payoff X and a decrease in the project cost I_1. The probability of success Δ has an additional driving force, which operates through the $t = 0$ zero-profit constraint (ZP_0). A higher Δ lowers the face value of long-term debt D_2 for any given D_1. A smaller D_2 alleviates the long-term debt overhang problem, further increasing the low type's incentives to undertake the growth option. Thus, an increase in Δ decreases the duration floor via the NPV and long-term debt overhang channels.

When a firm has higher retained earnings C, the excess short-term debt $D_1 - C$ that needs to be rolled over is lower. As a result, mimicking high types by rolling over short-term debt without investment becomes relatively less attractive for low types. Thus, high types can increase the face value of short-term debt one-for-one with retained earnings without attracting low types. Moreover, an increase in the face value of short-term debt reduces the amount of long-term debt D_2 required at $t = 0$ through (ZP_0), which alleviates the long-term debt overhang problem and makes undertaking the growth option more appealing for low types, i.e., relaxes (IC_L). Slack in the (IC_L) allows high types to further shorten their debt maturity. This virtuous feedback effect implies that an increase of \$1 in retained earnings reduces the duration floor, allowing low types to increase the face value of short-term by more than \$1.

Finally, the level of adverse selection affects the duration floor through the $t = 0$ zero-profit constraint of investors. A higher fraction of high types α, or a higher quality of low types p_L, lowers the face value of long-term debt D_2 and alleviates the long-term debt overhang problem. Moreover, a higher p_L reduces the incentives for low types to mimic high types, as the benefits of pooling are proportional to $p_H - p_L$. Stronger incentives for low types to undertake the growth option imply that high types can shorten their debt maturity, which implies that $MacD$ is decreasing in α.

Next, we characterize how the equilibrium level of investment depends on the firm's retained earnings.

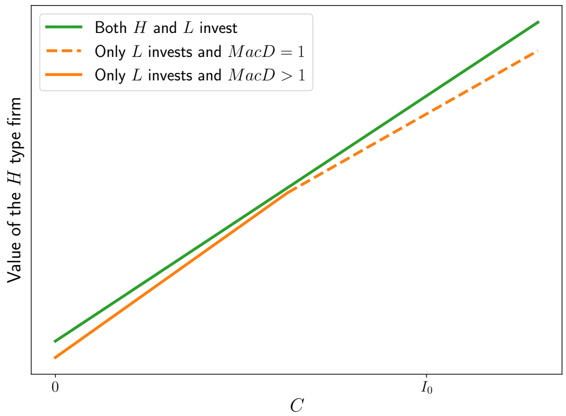
Lemma 6. *A marginal increase in retained earnings C makes the growth option more likely to be undertaken if $MacD = 1$ and less likely to be undertaken if $MacD > 1$.*

As low-type firms always undertake the growth option, the equilibrium level of investment is determined by the incentives of high-type firms. Consequently, to uncover the impact of retained earnings C on investment, one must understand whether a marginal increase in C tightens or relaxes the constraint (8).

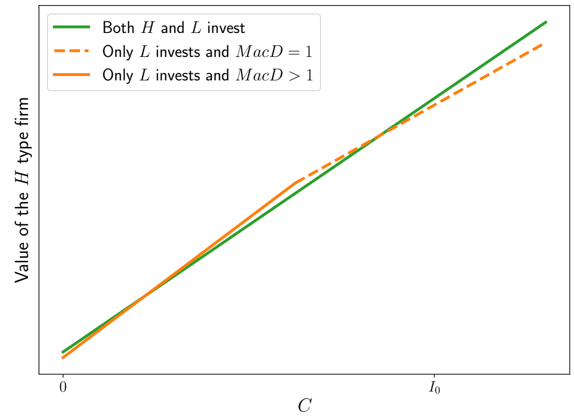
When the duration floor $MacD$ equals 1, a variation in retained earnings affects only the investment mispricing. With higher retained earnings, the high-type firm needs to raise less external costly capital and is more likely to invest. This is the standard intuition that is present in MM.

However, when the duration floor $MacD$ is greater than 1, a variation in retained earnings affects both the investment mispricing and the duration floor itself. As shown in Proposition 3, higher retained earnings C reduce the duration floor. This allows the high-type firm to use less long-term debt if it separates by not investing. A smaller burden of costly long-term debt makes not investing more attractive. Thus, the overall effect of retained earnings on investment depends on whether the mispricing or duration-floor channel dominates. Figure 2 shows numerically, and Proposition 4 argues analytically that the second channel is the dominant one.

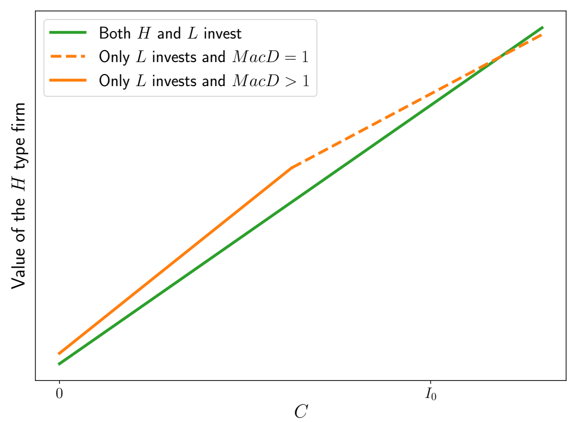
The critical intuition for why an increase in retained earnings C might increase investment stems from the way retained earnings affect the total *amount* of debt issued as well as the *composition* of debt. The impact on the total amount of debt is straightforward: an extra \$1 of retained earnings reduces the need for outside financing precisely by \$1. The impact on the composition of debt features a virtuous cycle. When the firm has an extra \$1 of retained earnings, it can increase the amount of short-term debt by \$1 and reduce the amount of long-term debt by \$1. Such a rebalancing does not affect the low-type firm's benefits of mimicking the high-type firm because the excess short-term debt $D_1 - C$ remains constant. However, it reduces the long-term debt overhang and increases the low-type firm's incentives to undertake the growth option. This extra slack in the incentive constraint implies that high-type firms can further shorten their debt maturity and improve their separating payoff. As a result, an extra \$1 of retained earnings allows the firm to



(a) High Rating



(b) Medium Rating



(c) Low Rating

Figure 2: Payoff of the high-type firm depending on the investment decision for various α. Model parameters are $X = 10$, $p_H = 0.6$, $p_L = 0.2$, $I_0 = I_1 = 2$, $\Delta = 0.25$.

increase the amount of short-term debt by more than \$1.

Lemma 6 shows that retained earnings can either positively or negatively affect the high-type firm's incentives to undertake the growth option. However, it does not specify whether those two cases are mutually exclusive or can coexist for the same set of parameters. The following result shows that the impact of retained earnings on investment depends on the level of retained earnings and the prior α, which in the literature, following Diamond (1991), is referred to as the firm's credit rating.

Proposition 4. *When the NPV of the growth option $\Delta X - I_1 \in (m, M)$, i.e., is neither too high nor too low, there exist two credit rating thresholds $\bar{\alpha} > \underline{\alpha}$ such that the high-type firms with:*

(a) *High credit rating, $\alpha \geq \bar{\alpha}$, invest for all levels of C;*

(b) *Intermediate credit rating, $\alpha \in (\underline{\alpha}, \bar{\alpha})$, invest if retained earnings C are either high or low, and do not invest otherwise.*

(c) *Low credit rating, $\alpha \leq \underline{\alpha}$, invest only if retained earnings are high.*

For high-rated firms, shown in Figure 2(a), the adverse selection discount is low relative to the size of the growth option, and they optimally invest regardless of retained earnings C.

For intermediate-rated firms, shown in Figure 2(b), the adverse selection discount is high, and investment does not happen for all values of retained earnings C. When retained earnings are low, the benefits of underinvestment for the high type are limited by the high duration of its liabilities needed to achieve separation (high duration floor). Consequently, the high-type firm optimally invests if retained earnings C are low. As retained earnings C increase, the duration floor falls and separating becomes relatively more attractive (as discussed in Lemma 6). Hence, the high-type firm optimally underinvests whenever it can separate via a relatively low-duration debt. When retained earnings are high, the total amount of external capital needed is low since the investment can be primarily financed internally. Even though the high-type firm can separate via issuing exclusively short-term debt, it prefers to invest since the total investment mispricing is low. Such non-monotonicity of investment with respect to retained earnings stands in sharp contrast with classical adverse selection intuition of Myers and Majluf (1984).

The adverse selection discount is the highest for low-rated firms, shown in Figure 2(c). Such firms underinvest even for low values of retained earnings despite the high duration floor limiting the benefits of underinvestment. Low-rated firms invest only when retained earnings C are sufficiently high, and there is little need to rely on external financing.

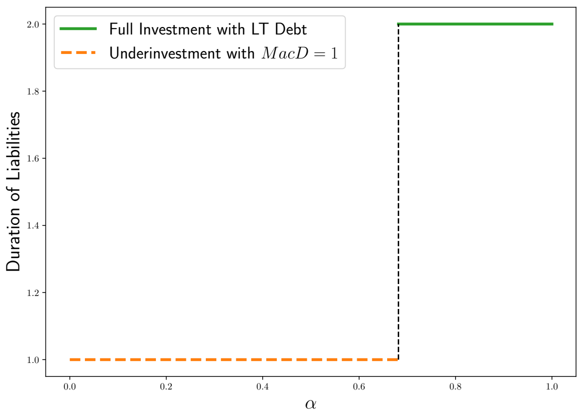
Finally, we show how the firms' optimal debt maturity depends on their credit ratings retained earnings. In order to better link our measure of debt maturity to those utilized in the empirical literature, we do not directly rely on $MacD$ as a measure of debt maturity. The reason is that $MacD$ has three components: short-term debt, long-term debt and cash investments, whenever the firm raises more than I_0 at date zero. In this case, $MacD$ does not capture the duration of a firm's liabilities net of cash investments, as do the empirical measures. Thus, we define the duration of a firm's liabilities as $D := \min(MacD, 2)$, to rule out cash investments.

Proposition 5. *When the NPV of the growth option $\Delta X - I_1 \in (m, M)$, i.e., is neither too high nor too low, there exist two cutoffs $\bar{C} > \underline{C}$ such that the firm's optimal debt maturity D:*

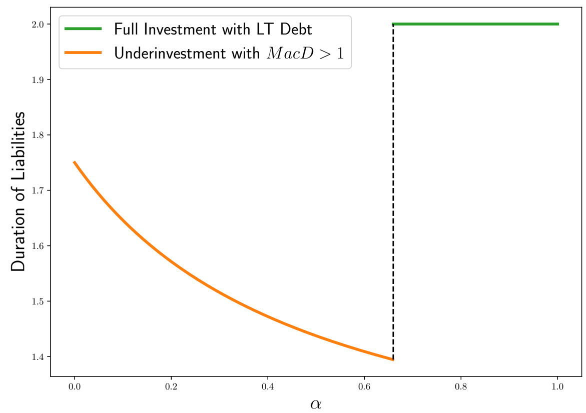
(a) weakly increases in the credit rating α for cash-rich firms $C > \bar{C}$;

(b) is U-shaped in the credit rating α for moderate-cash firms $C \in (\underline{C}, \bar{C})$;

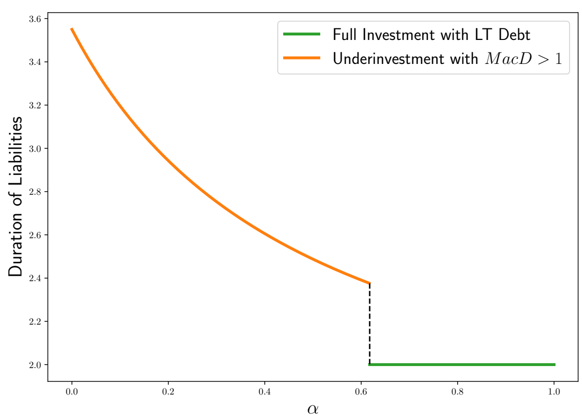
(c) is decreasing in the credit rating α for cash-poor firms $C < \underline{C}$.



(a) Cash-Rich Firms



(b) Moderate-Cash Firms



(c) Cash-Poor Firms

Figure 3: Optimal debt maturity as a function of credit rating α for various C. Model parameters are $X = 10$, $p_H = 0.6$, $p_L = 0.2$, $I_0 = I_1 = 2$, $\Delta = 0.25$.

First, notice that whether a firm's duration floor $MacD$, defined in (6), is above one or not does not depend on its credit rating. Moreover, there exists $\bar{C}$ such that the duration floor of cash-rich, i.e., $C > \bar{C}$, firms equals one. Thus, cash-rich firms, shown in Figure 3(a), can rely exclusively on short-term debt to separate. Consequently, their optimal debt maturity is driven exclusively by the investment decision. Such firms underinvest (and rely on short-term debt) when their credit rating is sufficiently low and invest with long-term debt when their credit rating is high enough. Increasing debt maturity prediction of our model in this region is consistent with Flannery (1986).

Moderate-cash firms, shown in Figure 3(b), have a duration floor above one. They underinvest due to the high adverse selection discount associated with the investment when the credit rating is low. The duration floor falls as the firm's credit rating increases (recall Proposition 3). Hence, intermediate-rated firms that underinvest can separate using a relatively low duration of their liabilities. When the credit rating increases even further, the firm optimally switches to investment with long-term debt due to the negligible adverse selection discount. Our U-shape prediction for the optimal debt maturity in the setting where information revelation is endogenous contrasts sharply with the results of Diamond (1991), which relies on exogenous information revelation.

Finally, low-cash firms, shown in Figure 3(c), have a duration floor above two. A high duration of liabilities is achieved by borrowing more than the required initial investment I_0 using long-term debt and investing the excess into safe short-term debt.[9] Excessive borrowing at date zero is required for the viability of separating at date one. If the high type were to borrow only I_0 using long-term debt, the low type would find it suboptimal to undertake the growth option due to long-term debt overhang. Borrowing more pushes the low-type firm to risk-shift: it invests at date one to reduce its cash holdings and ultimately dilute the long-term debt holders. As the credit rating increases, the duration of the firm's liabilities first decreases due to the falling duration floor and then because of the switch to investment using only long-term debt. Our model's decreasing debt maturity prediction in this region contrasts with the results in Flannery (1986). Moreover, our model provides an alternative explanation for why the firms might raise external financing and hold cash, which differs from the precautionary savings story (see, e.g., Almeida et al. (2014)).

6 Optimal Allocation

In this section, we show that the restriction to just short- and long-term debt, or the inability of the firm to commit to the time $t = 1$ investment decision at date $t = 0$, are without loss of generality. To do so, we consider the problem of a planner that seeks to maximize the expected payoff of a high-quality firm, subject to incentive compatibility, feasibility and participation constraints. We

[9] Formally, these firms have $D_1 < 0$, which we interpret as either cash savings or short-term risk-free investments.

show that the solution to this relaxed problem, in which the planner can offer arbitrary contracts, coincides with the equilibrium allocation characterized in Proposition 1.

In full generality, a contract for the planner is a triple (z, s, a) of (i) cash transfers $z = (z_0, z_1, z_2)$ from the investors to the firm[10], where the transfer z_t is paid at date $t = 0, 1, 2$; (ii) payment $s \in [0, X]$ from the firm to investors when the project succeeds at $t = 2$; and (iii) prescribed investment choice $a \in \{i, n\}$, where $a = i$ denotes investment at $t = 1$.

Definition. *A contract is* feasible *if $s \in [0, X]$ and the cash transfers z satisfy limited liability. That is: (a) $z_0 \geq I_0$; (b) $z_1 \geq \mathbb{1}(a = i)I_1 - C - (z_0 - I_0)$; (c) $z_2 \geq -C - (z_0 - I_0) - (z_1 - \mathbb{1}; (a = i)I_1)$;*[11]

To understand conditions $(a) - (c)$ in the above definition, note that any feasible contract that implements investment at date zero must satisfy $z_0 \geq I_0$, as otherwise the firm does not have enough resources to invest. Moreover, we need $z_1 \geq \mathbb{1}(a = i)I_1 - C - (z_0 - I_0)$, because the firm receives earnings at date one equal to C and carries a cash balance from date zero equal to $z_0 - I_0 \geq 0$, so it cannot be required to pay lenders more than this amount of cash at $t = 1$. For similar reasons, we must have that, at date two, $z_2 \geq -C - (z_0 - I_0) - (z_1 - \mathbb{1}(a = i)I_1)$.

The planner offers a (possibly degenerate) menu of contracts to the firm, which we denote by $M = \{(z^a_\theta, s^a_\theta, a)\}^{a \in \{i,n\}}_{\theta \in \{H,L\}}$. A menu consists of four contracts indexed by type θ and investment action a. Upon observing all offered menus, the firm either accepts one, or rejects all of them. If the firm accepts a menu, it then gets to pick a contract within the menu by sending a message $m = (\hat{\theta}, \hat{a})$ that reports it's type $\hat{\theta}$ and preferred investment action $\hat{a}$. Thus, the firm can effectively commit to investment action $\hat{a}$ at date $t = 0$.

When a type θ firm accepts a menu M and picks a contract by sending a message $m = (\hat{\theta}, \hat{a})$, the expected payoff of the firm's insiders is

$$U_\theta(M, \hat{\theta}, \hat{a}) \overset{def}{=} (p_\theta + \mathbb{1}(\hat{a} = i)\Delta)[X - s^{\hat{a}}_{\hat{\theta}}] + C + z^{\hat{a}}_{\hat{\theta},0} + z^{\hat{a}}_{\hat{\theta},1} + z^{\hat{a}}_{\hat{\theta},2} - I_0 - \mathbb{1}(\hat{a} = i)I_1,$$

while expected investors' profits are

$$\pi_\theta(M, \hat{\theta}, \hat{a}) \overset{def}{=} (p_\theta + \mathbb{1}(\hat{a} = i)\Delta) \cdot s^{\hat{a}}_{\hat{\theta}} - z^{\hat{a}}_{\hat{\theta},0} - z^{\hat{a}}_{\hat{\theta},1} - z^{\hat{a}}_{\hat{\theta},2}.$$

[10]These are the net payments from the investors to the firm. We allow the payments to be negative, i.e., the firm might be paying investors rather than the other way around.

[11]The indicator variable $\mathbb{1}(a = i)$ equals to 1 if the firm takes the investment action i and equals 0 if the firms takes investment action is n.

Observe that, because of competition, the outside option of a low-quality firm is to secure its full-information payoff $U_L^{FI} = (p_L + \Delta)X + C - I_0 - I_1$.

We define the planner's problem (PP) as follows:

$$\max_{M,a_H,a_L} U_H(M, \theta, a_H) \tag{PP}$$

subject to:

$$U_\theta(M, \theta, a_\theta) \geq U_\theta(M, \hat{\theta}, \hat{a}) \qquad \forall \theta, \hat{\theta}, \hat{a} \tag{IC_θ}$$

$$U_L(M, L, a_L) \geq U_L^{FI} \tag{IR_L}$$

$$\alpha \cdot \pi_H(M, \theta, a_H) + (1 - \alpha) \cdot \pi_L(M, L, a_L) \geq 0 \tag{BB}$$

$$M \text{ is feasible}$$

That is, the planner is maximizing the payoff of the high-type firm subject to three constraints: (a) incentive constraints, i.e., that each type reveals itself truthfully and takes the proposed action, (b) individual rationality, i.e., the low-type firm should do no worse than its outside option, and (c) budget balance, i.e., the planner cannot make losses on the proposed menu.

The next Propositions shows that solution to the planner's problem coincides with the equilibrium allocation described in Proposition 1.

Proposition 6. *The high-type optimal allocation that solves the program (PP) coincides with the equilibrium allocation of Proposition 1.*

Proposition 6 shows that allowing for arbitrary contracts and dynamic commitment does not affect equilibrium investment decisions and payoffs. It highlights that underinvestment that Proposition 1 generates through the reliance on short-term debt is the optimal behavior of the firm, and that issuing short-term debt is one way of achieving the highest possible payoff without undertaking the growth option. Proposition 6 also hints that other securities, e.g. callable debt, that implement the optimal allocation would also generate underinvestment. Such securities would reduce the effective maturity of the firm's liabilities and would allow it to take advantage of the favorable repricing in absence of investment.

7 Conclusion

We study a dynamic adverse selection model in which firms make multiple investment decisions and optimally choose their capital structure along the way. We find that underinvestment is associated with the issuance of short-, not long-term debt because it is driven by the favorable repricing of short-term debt at the rollover stage when high-quality firms convey information about their type to the market by underinvesting. Had these firms chosen longer-term debt, they would have continued to make positive net present value investments. Thus, these firms have issued *too much* short-term debt for them to have an incentive to take on positive NPV projects, and there is *short-term debt overhang*. In contrast to Myers' story, in our model, having a short-term debt overhang ex-post might be optimal from an ex-ante standpoint, as it maximizes the firm owner's payoff. Thus, this type of overhang cannot be contracted around, and it arises under the optimal mechanism.

The amount of short-term debt that can be issued to credibly convey private information is determined by the low-type's incentive constraint. More cash (or retained earnings) relax this incentive constraint and consequently lower the duration floor of a firm's liabilities. For firms with intermediate credit rating the duration floor effect creates a non-monotonic relationship between firm's cash and investment levels. That is, an increase in retained earnings reduces investment (when the initial level of cash is low) and increases investment (when the initial level of cash is high). Moreover, an increase in firms' credit rating simultaneously lowers the duration floor and makes investment more attractive. As a result, firms with intermediate credit rating that underinvest have she shortest duration of their liabilities. Therefore, our findings have implications relevant to both the investment-cash flow sensitivity literature and the literature on debt maturity.

References

Admati, Anat R, Peter M DeMarzo, Martin F Hellwig, and Paul Pfleiderer, "The leverage ratchet effect," *The Journal of Finance*, 2018, *73* (1), 145–198.

Akerlof, George A, "The Market for "Lemons": Quality Uncertainty and the Market Mechanism," *The Quarterly Journal of Economics*, 1970, *84* (3), 488–500.

Almeida, Heitor, Murillo Campello, Igor Cunha, and Michael S Weisbach, "Corporate liquidity management: A conceptual framework and survey," *Annu. Rev. Financ. Econ.*, 2014, *6* (1), 135–162.

Asriyan, Vladimir, William Fuchs, and Brett Green, "Information spillovers in asset markets with correlated values," *American Economic Review*, 2017, *107* (7), 2007–40.

Barclay, Michael J and Clifford W Smith, "The maturity structure of corporate debt," *the Journal of Finance*, 1995, *50* (2), 609–631.

Berger, Allen N, Marco A Espinosa-Vega, W Scott Frame, and Nathan H Miller, "Debt maturity, risk, and asymmetric information," *The Journal of Finance*, 2005, *60* (6), 2895–2923.

Blouin, Arthur and Rocco Macchiavello, "Strategic default in the international coffee market," *The Quarterly Journal of Economics*, 2019, *134* (2), 895–951.

Bond, Philip and Hongda Zhong, "Buying high and selling low: Stock repurchases and persistent asymmetric information," *The Review of Financial Studies*, 2016, *29* (6), 1409–1452.

_ , Yue Yuan, and Hongda Zhong, "Share Issues versus Share Repurchases," *Available at SSRN 3489555*, 2019.

Calomiris, Charles W and Charles M Kahn, "The role of demandable debt in structuring optimal banking arrangements," *The American Economic Review*, 1991, pp. 497–513.

Daley, Brendan and Brett Green, "Waiting for News in the Market for Lemons," *Econometrica*, 2012, *80* (4), 1433–1504.

Demarzo, Peter M, "Presidential address: Collateral and commitment," *The Journal of Finance*, 2019, *74* (4), 1587–1619.

DeMarzo, Peter M and Zhiguo He, "Leverage dynamics without commitment," *The Journal of Finance*, 2021, *76* (3), 1195–1250.

Diamond, Douglas W, "Debt maturity structure and liquidity risk," *the Quarterly Journal of economics*, 1991, *106* (3), 709–737.

_ **and Zhiguo He**, "A theory of debt maturity: the long and short of debt overhang," *The Journal of Finance*, 2014, *69* (2), 719–762.

Fazzari, Steven M, R Glenn Hubbard, Bruce C Petersen et al., "Financing Constraints and Corporate Investment," *Brookings Papers on Economic Activity*, 1988, *19* (1), 141–206.

Flannery, Mark J, "Asymmetric information and risky debt maturity choice," *The Journal of Finance*, 1986, *41* (1), 19–37.

Geelen, Thomas, "Information dynamics and debt maturity," *Swiss finance institute research paper*, 2019.

Hadlock, Charles J and Joshua R Pierce, "New evidence on measuring financial constraints: Moving beyond the KZ index," *The review of financial studies*, 2010, *23* (5), 1909–1940.

He, Zhiguo, "A model of dynamic compensation and capital structure," *Journal of Financial Economics*, 2011, *100* (2), 351–366.

_ **and Konstantin Milbradt**, "Dynamic debt maturity," *The Review of Financial Studies*, 2016, *29* (10), 2677–2736.

Hong, Claire Yurong, Kewei Hou, and Thien T Nguyen, "Debt Maturity Structure and Corporate Investment," *Fisher College of Business Working Paper*, 2023, p. 03.

Hu, Yunzhi, Felipe Varas, and Chao Ying, "Debt maturity management," Technical Report, Working Paper 2021.

Huang, Chong, Martin Oehmke, and Hongda Zhong, "A theory of multiperiod debt structure," *The Review of Financial Studies*, 2019, *32* (11), 4447–4500.

Kaplan, Steven N and Luigi Zingales, "Do investment-cash flow sensitivities provide useful measures of financing constraints?," *The quarterly journal of economics*, 1997, *112* (1), 169–215.

Martel, Jordan, Kenneth Mirkin, and Brian Waters, "Learning by owning in a lemons market," *The Journal of Finance*, 2022.

Moyen, Nathalie, "Investment–cash flow sensitivities: Constrained versus unconstrained firms," *The Journal of finance*, 2004, *59* (5), 2061–2092.

Myers, Stewart C, "Determinants of corporate borrowing," *Journal of Financial Economics*, 1977, *5* (2), 147–175.

_ **and Nicholas S Majluf**, "Corporate financing and investment decisions when firms have information that investors do not have," *Journal of Financial Economics*, 1984, *13* (2), 187–221.

Philippon, Thomas and Philipp Schnabl, "Efficient recapitalization," *The Journal of Finance*, 2013, *68* (1), 1–42.

Wittry, Michael D, "(Debt) Overhang: Evidence from Resource Extraction," *The Review of Financial Studies*, 2021, *34* (4), 1699–1746.

Zryumov, Pavel, "Dynamic adverse selection: Time-varying market conditions and endogenous entry," *Available at SSRN 2653129*, 2015.

A Appendix

A.1 Proof of Lemma 1

Proof. The proof that lender profits must be zero and that lenders cannot make profits on low types are straightforward. Formal arguments can be found in the proof of Lemma B.1 . Thus, we now proceed in proving that the equilibrium must maximize the utility of high types subject to constraints.

Clearly, any date-one equilibrium $(\mathcal{K}_1^*, K_1^*, a_H^*, a_L^*, w_1^*)$ must satisfy all the constraints of program (1). Suppose, however, that the equilibrium $(\mathcal{K}_1^*, K_1^*, a_H^*, a_L^*, w_1^*)$ does not solve (1), i.e. that it does not maximize the payoff of the high type. Let $(\mathcal{K}', K', a_H', a_L', w')$ be a solution to (1). Evidently, this is possible only when $U_H(K_0, K', a_H') > 0$, as otherwise every contract must be equivalent for the high type, keeping it at its participation constraint. Then, anticipating the future equilibrium withdrawal policies associated to both contracts, we must have that

$$U_H(K_0, K', a_H') > U_H(K_0, K_1^*, a_H^*).$$

Case 1: First, it could be that $U_L(K_0, K_1^*, a_L^*) \geq U_L(K_0, K', a_H')$. It follows from ($ZP$) and ($NP_L$) that in K' investors are making either positive or zero profits on the high type. Consider a menu with commitment $\hat{K}$ in which the (a_H') contract is the ϵ modified contract from K' (to deliver $\epsilon > 0$ more profits to investors) and the other option is a zero contract. We know that this is feasible because of our Conjecture. The menu $\hat{K}$ attracts the high type because

$$U_H(K_0, \hat{K}, a_H') = U_H(K_0, K', a_H') - \epsilon > U_H(K_0, K_1^*, a_H^*),$$

as long as $\epsilon > 0$ is small enough, while it does not attract low types because

$$U_L(K_0, \hat{K}, a_H') \overset{\epsilon}{<} U_L(K_0, K', a_H') \overset{\text{case 1}}{\leq} U_L(K_0, K_1^*, a_L^*).$$

Moreover, the menu $\hat{K}$ is guaranteed to deliver at least $\epsilon > 0$ profits to investors. As a result, its existence contradicts K_1^* being an equilibrium.

Case 2: Now consider the case in which $U_L(K_0, K_1^*, a_L^*) < U_L(K_0, K', a_H')$. We have two sub-cases.

Case 2.1: First, suppose that $U_L(K_0, K', a'_L) > U_L(K_0, K', a'_H)$. In this case, consider a deviation menu $\hat{K}$ constructed as follows. The contract (a'_H) is the same as in the K' menu. The option (a'_L) is an ϵ modified contract from K' that generates $\epsilon > 0$ higher profits for the investors. The menu $\hat{K}$ attracts the high type who chooses (a'_H) as

$$U_H(K_0, \hat{K}, a'_L) \overset{\epsilon}{<} U_H(K_0, K', a'_L) \overset{IC}{\leq} U_H(K_0, K', a'_H) = U_H(K_0, \hat{K}, a'_H).$$

Moreover, the menu $\hat{K}$ attracts the low type who picks (a'_L) for small enough ϵ because

$$U_L(K_0, \hat{K}, a'_L) = U_L(K_0, K', a'_L) - \epsilon \overset{\text{case 2.1}}{>} U_L(K_0, K', a'_H) \overset{\text{case 2}}{>} U_L(K_0, K_1^*, a_L^*).$$

Finally, the menu $\hat{K}$ is guaranteed to deliver strictly higher profits to investors that the menu K' (which itself is a zero profit menu). Hence, the existence of $\hat{K}$ contradicts K_1^* being an equilibrium.

Case 2.2: Otherwise, the only remaining is the case $U_L(K_0, K', a'_L) = U_L(K_0, K', a'_H)$. In this case, consider a deviation menu (with commitment) $\hat{K}$, constructed as follows. The contract (a'_L) is the same as in K'. Option (a'_H) is an ϵ-modified contract from the menu K' that generates $\epsilon > 0$ higher profits for investors. $\hat{K}$ attracts the high type who chooses (a'_H) as

$$U_H(K_0, \hat{K}, a'_L) = U_H(K_0, K', a'_H) - \epsilon > U_H(K_0, K_1^*, a_H^*),$$

as long as $\epsilon > 0$ is sufficiently small. Moreover, $\hat{K}$ attracts the low type who picks (a'_L) as

$$U_L(K_0, \hat{K}, a'_L) = U_L(K_0, K', a'_L) \overset{\text{case 2.2}}{=} U_L(K_0, K', a'_H) \overset{\epsilon}{>} U_L(K_0, \hat{K}, a'_H)$$
$$U_L(K_0, \hat{K}, a'_L) = U_L(K_0, K', a'_L) \overset{\text{case 2.2}}{=} U_L(K_0, K', a'_H) \overset{\text{case 2}}{>} U_L(K_0, K_1^*, a_L^*)$$

Finally, $\hat{K}$ is guaranteed to deliver strictly higher profits to investors than K' (which itself is a zero profit menu). Thus, the existence of $\hat{K}$ contradicts K_1^* being an equilibrium. □

A.2 Proof of Lemma 2

Proof. The payoff for a type-θ firm associated to investment is $(p_\theta+\Delta)[X-F_2^i]$, while that associated to taking the zero contract is $p_\theta X+C$. Thus, type θ invests if and only if $(p_\theta+\Delta)[X-F_2^i] \geq p_\theta X+C$, or $\Delta X - C \geq (p_\theta + \Delta)F_2^i$. As the left-hand side is independent of θ, while the right-hand side increases in θ, if the inequality holds for the high type it holds for the low type. Thus, competition implies that $F_2^i = \frac{I_1 - C}{p_0 + \Delta}$, and investment takes place if and only if inequality (2) holds weakly.

Now, suppose that inequality (2) is strictly reversed. Then, the high type chooses the zero contract and gets a payoff of $p_H X - C$. If the equilibrium is such that a low type does not invest, then the low type must be taking the zero contract as well, receiving a payoff of $p_L X + C$. Consider now a deviation (with commitment) in which a date-one lender offers to a low type only an investment option with face value $F_2^i = \frac{I_1 - C}{p_L + \Delta} + \epsilon$, for a small $\epsilon > 0$. A low type deviates from the zero contract when $p_L X + C \leq (p_L + \Delta)\left[X - \frac{I_1 - C}{p_L + \Delta} - \epsilon\right]$, or $(p_L + \Delta)\epsilon \leq \Delta X - I_1$, which is possible given that $\Delta X > I_1$. Therefore, irrespective of what a high type does, lenders make strictly positive profits at the deviation, which contradicts the presumption that a low type does not invest. □

A.3 Proof of Lemma 3

We prove this Lemma using by splitting splitting the space $D_1 \leq C$ into two regions and verifying the statement of the lemma in each region separately.

Lemma A.1. *If $0 < D_1 + D_2 \leq C$, then, the equilibrium features investment by all types when*

$$\underbrace{\Delta X - I_1}_{NPV_1} + \underbrace{(1 - (p_H + \Delta))D_2}_{Dilution\ of\ D_2} > \underbrace{\frac{(p_H - p_1)}{(p_1 + \Delta)}(I_1 - C)}_{Investment\ Mispricing} + \underbrace{\frac{(p_H - p_1)}{(p_1 + \Delta)}D_1}_{Rollover\ Mispricing} . \tag{A.1}$$

If inequality (A.1) is strictly reversed, the equilibrium features investment only by low types, while high types take the zero contract. If (A.1) holds as an equality, both allocations are equilibria.

Proof. The payoff for a type-θ firm associated to investment is $(p_\theta + \Delta)[X - D_2 - F_2^i]$, while that associated to taking the zero contract is $p_\theta X + C - D_1 - D_2$. Therefore, as we have argued before, investment requires pooling and $F_2^i = \frac{I_1 - C + D_1}{p_0 + \Delta}$. Thus, the payoff associated to investment reads $(p_\theta + \Delta)\left[X - D_2 - \frac{I_1 - C + D_1}{p_1 + \Delta}\right]$. A high type prefers to invest only if $(p_H + \Delta)\left[X - D_2 - \frac{I_1 - C + D_1}{p_1 + \Delta}\right] \geq p_H X + C - D_1 - D_2$, or $\Delta X - C + (1 - (p_H + \Delta))D_2 \geq (p_H + \Delta)\frac{I_1 - C + D_1}{p_1 + \Delta} + \frac{p_H - p_1}{p_1 + \Delta}D_1$, as in (A.1). The case in which (A.1) does not hold mirrors previous analysis for the case $D_1 = D_2 = 0$, and indifference arises when (A.1) holds as an equality, in which case we have two equilibrium allocations. □

Lemma A.2. *If $D_1 + D_2 \geq C$ and $D_1 \leq C$, then all types invest in equilibrium when*

$$\underbrace{\Delta X - I_1}_{NPV_1} - \underbrace{\Delta D_2}_{Overhang} + \underbrace{(1 - p_H)[C - D_1]}_{Partial\ dilution\ of\ D_2} > \underbrace{\frac{(p_H - p_1)}{(p_1 + \Delta)}(I_1 - C)}_{Investment\ mispricing} + \underbrace{\frac{(p_H - p_1)}{(p_1 + \Delta)}D_1}_{Rollover\ mispricing}. \tag{A.2}$$

If (A.2) is strictly reversed, then low types invest and high types take the zero contract when

$$\underbrace{\Delta X - I_1}_{NPV_1} - \underbrace{\Delta D_2}_{Overhang} + \underbrace{(1 - p_L)[C - D_1]}_{Partial\ dilution\ of\ D_2} > 0, \tag{A.3}$$

If both (A.2) and (A.3) are strictly reversed, then all types take the zero contract. Whenever (A.2) and/or (A.3) hold as equality, there are multiple equilibria.

Proof. The payoff for a type-θ firm associated to investment is $(p_\theta + \Delta)[X - D_2 - \frac{I_1 - C + D_1}{p_1 + \Delta}]$, while that associated to taking the zero contract is $p_\theta[X + C - D_1 - D_2]$, as all cash goes to date-zero debt holders if the firm does not get X. Thus, investment requires $(p_H + \Delta)[X - D_2 - \frac{I_1 - C + D_1}{p_1 + \Delta}] \geq p_H[X + C - D_1 - D_2]$, or $\Delta[X - D_2] - C + (1 - p_H)[C - D_1] \geq (p_H + \Delta)\frac{I_1 - C}{p_1 + \Delta} + \frac{p_H - p_1}{p_1 + \Delta}D_1$. If this inequality does not hold, then high types do not invest. As for low types, they prefer investment under full information rather than taking the zero contract whenever inequality (A.3) holds, and this is always feasible. To check feasibility, observe that investment at date one requires $X - D_2 \geq \frac{I_1 - C + D_1}{p_L + \Delta}$. This constraint binds before inequality (A.3) only if $\frac{I_1 + D_1 - C}{p_L + \Delta} > \frac{I_1 - (1 - p_L)(C - D_1)}{\Delta}$, which can be rewritten as $0 > I_1 - C + D_1 + (p_L + \Delta)(C - D_1) \geq 0$, which is a contradiction. □

Together Lemmas A.1 and A.2 constitute the proof of Lemma 3 since they cover the whole space $D_1 < C$. Inequalities (A.1) and (A.2) are collapsed into inequality (3), and inequality (A.3) is equivalent to (4).

A.4 Proof of Lemma 4

We prove this Lemma using by splitting splitting the space $D_1 > C$ into two regions and verifying the statement of the lemma in each region separately.

Lemma A.3. *Suppose that $D_1 > C$ and $\Delta(X - D_2) - I_1 > 0$. Then the date-one equilibrium* *features investment by all types when*

$$\Delta X - I_1 \geq \Delta D_2 + \frac{(p_H - p_1)}{(p_1 + \Delta)}(I_1 - C) + \frac{(p_H - p_1)}{(p_1 + \Delta)} D_1 - \frac{(1-\alpha)p_H}{p_1}\left[\frac{p_H - p_L}{p_H}(D_1 - C) - (\Delta(X - D_2) - I_1)\right]^+ . \tag{A.4}$$

Otherwise, if inequality A.4 is reversed, then only low type firms invest.

Proof. In this region we have (at most) 4 different types of allocations which result in the following payoffs to the High (H) and Low (L) type firms

1. Both H and L roll over their debt, in which case a type-θ firm's payoff reads

$$U_\theta = p_\theta \left(X - D_2 - \frac{D_1 - C}{p_1} \right)$$

2. Both H and L invest, receiving a payoff equal to

$$U_\theta = (p_\theta + \Delta) \left(X - D_2 - \frac{I_1 + D_1 - C}{p_1 + \Delta} \right)$$

3. H rolls over and L invests. In this case, we have two sub-cases, depending on whether the firms are offered a pooling menu with cross-subsidies, or two zero-profit separating contracts.

3a. In the event of separating contracts without cross-subsidization, payoffs read

$$U_H = p_H \left(X - D_2 - \frac{D_1 - C}{p_H} \right)$$
$$U_L = (p_L + \Delta) \left(X - D_2 - \frac{I_1 + D_1 - C}{p_L + \Delta} \right),$$

and the incentive constraint for a low type not to mimic the high type reads

$$IC_L: \qquad (p_L + \Delta) \left(X - D_2 - \frac{I_1 + D_1 - C}{p_L + \Delta} \right) \geq p_L \left(X - D_2 - \frac{D_1 - C}{p_H} \right).$$

3b In the case of a zero-profit menu with cross-subsidization, payoffs are

$$U_\theta = p_\theta \left(X - D_2 - \frac{D_1 - C - (1-\alpha)[\Delta(X - D_2) - I_1]}{p_1} \right)$$

where the utilities above follow after solving for F_2^n and F_2^i from

$$\begin{cases} \alpha(D_1 - C) + (1-\alpha)(I_1 + D_1 - C) = \alpha p_H F_2^n + (1-\alpha)(p_L + \Delta)F_2^i \\ (p_L + \Delta)(X - D_2 - F_2^i) = p_L(X - D_2 - F_2^n) \end{cases}$$

4. H invests and L rolls over. Again, we have to consider two sub-cases:

4a Without cross-subsidization

$$\begin{aligned} &U_H = (p_H + \Delta)\left(X - D_2 - \frac{I_1 + D_1 - C}{p_H + \Delta}\right) \\ &U_L = p_L\left(X - D_2 - \frac{D_1 - C}{p_L}\right) \\ &IC_L: \qquad p_L\left(X - D_2 - \frac{D_1 - C}{p_L}\right) \geq (p_L + \Delta)\left(X - D_2 - \frac{I_1 + D_1 - C}{p_H + \Delta}\right). \end{aligned}$$

4b With cross-subsidization

$$U_\theta = (p_\theta + \Delta)\left(X - D_2 - \frac{I_1 + D_1 - C + (1-\alpha)[\Delta(X - D_2) - I_1]}{p_1 + \Delta}\right).$$

Notice that the allocation 4a is not feasible, since the IC_L constraint does not hold when $\Delta(X - D_2) - I_1 > 0$. Observe that $U_H(4b) < U_H(2)$ and $U_H(1) < U_H(3b)$. Moreover, whenever 4b is feasible, so is 2; and whenever 1 is feasible, so is 3b. Hence, only 3 allocations can deliver the highest possible payoff to a high type U_H: (2), (3a), or (3b).

The allocation (3a) exists whenever the IC_L is satisfied. Incentive constraint of the low type can be rewritten as

$$\Delta(X - D_2) - I_1 \geq \frac{p_H - p_L}{p_H}(D_1 - C).$$

Whenever this inequality holds, it can be checked that allocation 3b cannot be an equilibrium, as investors would make strictly positive profits on low types (which we know from Lemma B.1 is impossible). If allocation (3a) exists, then the H type prefers to invest, i.e., prefers allocation (2) to allocation (3a) whenever

$$\Delta(X - D_2) - I_1 \geq \frac{p_H - p_1}{p_1}(I_1 + D_1 - C), \tag{A.5}$$

otherwise the allocation (3a) delivers the highest utility. Notice that the inequality above is equivalent to (A.4) since the last term is zero.

If allocation (3a) does not exist, then the H type prefers to invest, i.e., prefers allocation (2) to allocation (3b) whenever

$$\frac{p_H(1-\alpha)}{p_1}\left[\Delta(X-D_2)-I_1-\frac{p_H-p_L}{p_H}(D_1-C)\right]\leq \Delta(X-D_2)-I_1-\frac{p_H-p_1}{p_1}(I_1+D_1-C). \quad \text{(A.6)}$$

Notice that the inequality above is equivalent to (A.4). ☐

Lemma A.4. *Suppose that $D_1 > C$ and $\Delta(X-D_2)-I_1<0$. Then the date-one equilibrium features no investment by either type.*

Proof. As $D_1 > C$, taking the zero contract leads to default and a firm payoff equal to zero.

Using the notation from the previous lemma we notice the following. First, allocation $3b$ is dominated by allocation 1

$$\begin{aligned}U_H(3b) &= p_H\left(X-D_2-\frac{D_1-C-(1-\alpha)[\Delta(X-D_2)-I_1]}{p_1}\right)\\ &< p_\theta\left(X-D_2-\frac{D_1-C}{p_1}\right)=U_H(1)\end{aligned}$$

and whenever allocation $3b$ is feasible then allocation 1 is also feasible. Hence, allocation $3b$ cannot be an equilibrium.

Second, allocation 2 is dominated by allocation $4b$

$$\begin{aligned}U_H(2) &= (p_H+\Delta)\left(X-D_2-\frac{I_1+D_1-C}{p_1+\Delta}\right)\\ &< (p_H+\Delta)\left(X-D_2-\frac{I_1+D_1-C+(1-\alpha)[\Delta(X-D_2)-I_1]}{p_1+\Delta}\right)=U_H(4b)\end{aligned}$$

and whenever allocation 2 is feasible then allocation $4b$ is also feasible. Hence, allocation 2 cannot be an equilibrium.

Third, the IC_L constraint for allocation $3a$ is never satisfied: the low type does not want to invest in the negative NPV project at the full information price and would prefer to roll over existing short-term debt by pretending to be the high type.

Hence, the only candidate allocations are (1) - both type roll over, (4a) and (4b) - high type invests and low type rolls over.

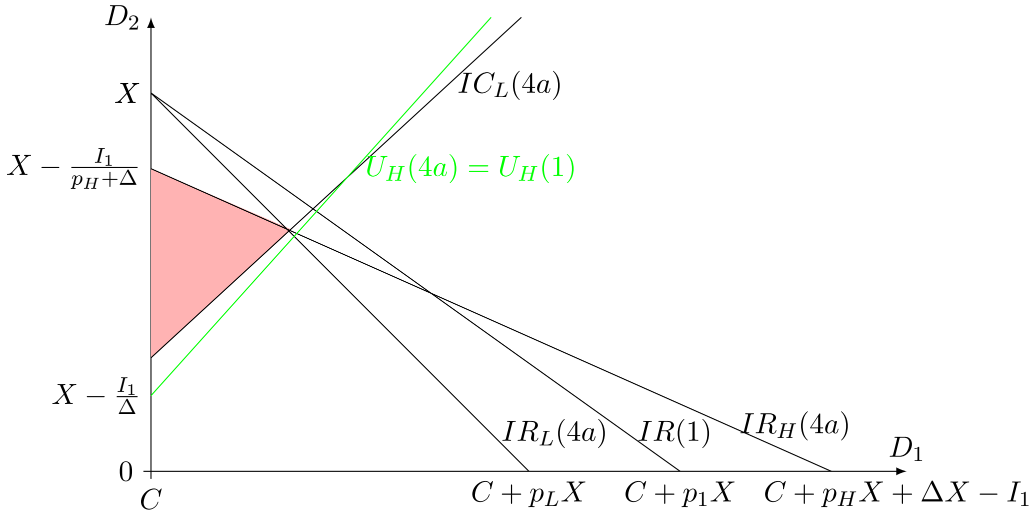


Figure 4: Region where allocation (4a) exists.

We next claim that allocation 1 exists and dominates allocation $4a$ whenever $4a$ exists. We begin by looking at the feasibility (IR) constraints of the two allocations:

$$X - D_2 \geq \frac{D_1 - C}{p_1} \tag{$IR(1)$}$$

$$X - D_2 \geq \frac{D_1 - C}{p_L} \tag{$IR_L(4a)$}$$

$$X - D_2 \geq \frac{I_1 + D_1 - C}{p_H + \Delta} \tag{$IR_H(4a)$}$$

All three lines here have a negative slope in D_1.

Next, for allocation $4a$ to exist, the IC_L constraint needs to be satisfied as well:

$$I_1 - \Delta(X - D_2) \geq \frac{p_H - p_L}{p_H + \Delta}(I_1 + D_1 - C)$$

$$D_2 \geq X - \frac{I_1}{\Delta} + \frac{p_H - p_L}{\Delta(p_H + \Delta)}(I_1 + D_1 - C) \tag{$IC_L(4a)$}$$

This IC constrain has a positive slope in D_1. Moreover, this line goes through the intersection of $IR_H(4a)$ and $IR_L(4b)$ since at that point the low type gets exactly 0 through separating and not investing, or though mimicking the high type and investing, and is indifferent as a result.

Hence, the allocation $4a$ exits within the triangular shaped region shown in Figure 4. Next, check when the allocation $4a$ dominates 1, i.e.

$$U_H(1) = p_H\left(X - D_2 - \frac{D_1 - C}{p_1}\right) < (p_H + \Delta)\left(X - D_2 - \frac{I_1 + D_1 - C}{p_H + \Delta}\right) = U_H(4a)$$
$$I_1 - \Delta(X - D_2) < \frac{p_H - p_1}{p_1}(D_1 - C)$$
$$D_2 < X - \frac{I_1}{\Delta} + \frac{p_H - p_1}{\Delta \cdot p_1}(D_1 - C)$$

Notice that when $D_1 = C$ then it simplifies to $D_2 < X - \frac{I_1}{\Delta}$, the region which is strictly below the red triangle. To see whether any part of the triangle lies inside of the half-space $U_H(4a) > U_H(1)$ it is necessary and sufficient to check whether the right vertex of the triangle (intersection of $IR_L(4a)$, $IR_H(4a)$, and $IC_L(4a)$) lies in that half-space. But at that point $U_H(4a) = 0$ and $U_H(1) > 0$. Hence, the triangle where $4a$ is feasible and the half-space $U_H(4a) > U_H(1)$ do not intersect. This proves that whenever $4a$ is feasible, the high type prefers allocation 1.

Finally, we prove that allocation 1 exists and dominates allocation $4b$ whenever $4b$ exists. We

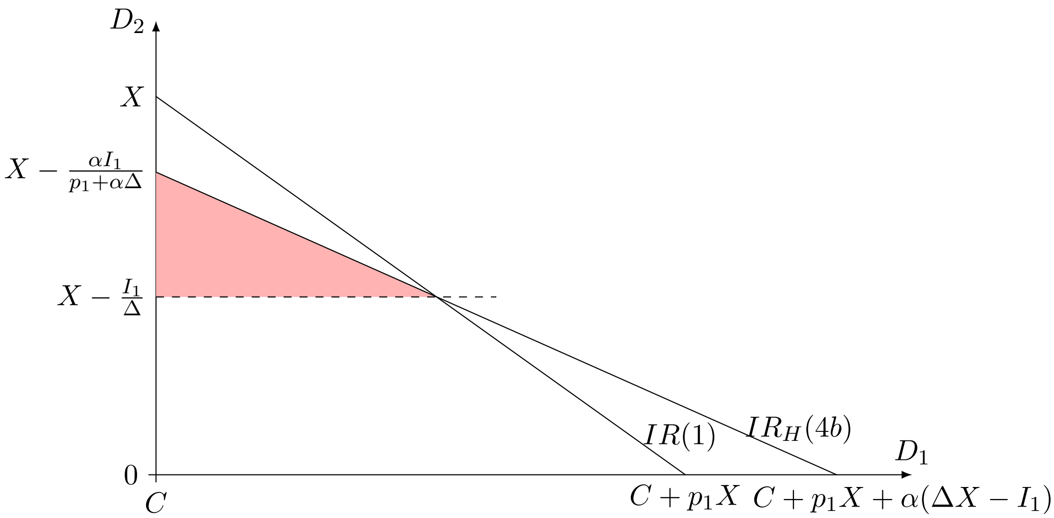


Figure 5: Region where allocation (4b) exists.

begin by looking at the feasibility (IR) constraints of the two allocations. Allocation 1 has $IR(1)$ as its feasibility constraint and allocation $4b$ has

$$X - D_2 \geq \frac{I_1 + D_1 - C + (1 - \alpha)[\Delta(X - D_2) - I_1]}{p_L}. \qquad (IR_H(4b))$$

Both lines here have a negative slope in D_1 and $IR(1)$ is steeper than $IR_H(4b)$ and these lines cross exactly at $\Delta(X - D_2) = I_1$

Since we are interested in the parametric region $\Delta(X - D_2) < I_1$ the region where allocation $4b$ exists is the shaded triangle shown in Figure 5. Hence, when the allocation $4b$ exits allocation 1 exists as well.

Next, check when the allocation $4b$ dominates 1, i.e.

$$\begin{aligned} U_H(1) &= p_H\left(X - D_2 - \frac{D_1 - C}{p_1}\right) \\ &< (p_H + \Delta)\left(X - D_2 - \frac{I_1 + D_1 - C + (1-\alpha)[\Delta(X - D_2) - I_1]}{p_1 + \Delta}\right) = U_H(4b) \end{aligned}$$

To check whether the shaded triangle in Figure 5 has a non-empty intersection with the half-space $U_H(1) < U_H(4b)$ it is necessary and sufficient that at least one vertex of the triangle lies inside of that half-space. First, check the top left vertex: there $U_H(4b) = 0 < UH_(1)$, hence it lies outside of the half-space. Second, check the right vertex: there $U_H(4b) = 0 = UH_(1)$ hence it lies on the edge of the half-space. Finally, check the bottom left vertex where $D_1 = C$ and $\Delta(X - D_2) = I_1$:

$$\begin{aligned} U_H(1) &\quad vs. \quad U_H(4b) \\ p_H(X - D_2) &\quad vs. \quad (p_H + \Delta)\left(X - D_2 - \frac{I_1}{p_1 + \Delta}\right) \\ 0 &\quad vs. \quad \Delta(X - D_2) - \frac{p_H + \Delta}{p_1 + \Delta} I_1 \\ 0 &\quad > \quad I_1 - \frac{p_H + \Delta}{p_1 + \Delta} I_1. \end{aligned}$$

Hence this vertex also lies outside of the half-plane. As a result, the whole triangle lies outside of the half-plane and the allocation 1 always dominates allocation $4b$ whenever the latter exists. □

Lemmas A.3 and A.4 jointly cover all cases of Lemma 4.

A.5 Proof of Lemma 5

Proof. Suppose, by contradiction, that the date-zero equilibrium was separating. Then, given that K_0 consists of single contracts, it follows that the equilibrium strategy of one type, say θ' is to reject all offers and not invest at date zero. This strategy yields a payoff equal to zero to type θ'. As for the other type, θ'', investment yields a payoff equal to $p_{\theta''}\left(X - D_2 - \frac{D_1 - C}{p_{\theta''}}\right)$ if this type anticipates that it will rollover at date one, while it gives a payoff of $(p_{\theta''} + \Delta)\left(X - D_2 - \frac{I_1 + D_1 - C}{p_{\theta''} + \Delta}\right)$ if this type anticipates investment. If either of these two payoffs is positive, then default can be avoided.

Moreover, it cannot be a date-zero equilibrium to induce default at date one, as this is dominated by a feasible pooling offer $F_2^i = \frac{I_1+D_1-C}{p_0+\Delta} + \epsilon$, for some $\epsilon > 0$ that represents lender profits. Further, the payoff of the investing type is strictly positive at F_2^i as all projects have strictly positive net present value. Given that the equilibrium payoff of the investing type must be strictly positive, type θ' mimics, incentive compatibility fails and there cannot be separation at date zero.

Zero profits and maximization of the high-type firm's payoff can be shown using similar arguments to Lemmas 1 and B.1. □

A.6 Proof of Proposition 1

We again break down the proof of into several lemmas.

Lemma A.5. *If $I_0 \leq C$ then in any equilibrium the low type always invests and the high-type invests if*

$$\Delta X - I_1 \geq \frac{(p_H - p_0)}{(p_0 + \Delta)}(I_1 + I_0 - C) \tag{A.7}$$

Proof. If the high-type firm invests at date $t = 0$ the zero profit condition reads $D_1 + (p_0 + \Delta)D_2 = I_0$. Plugging the former condition in the expected payoff of a high type under full investment yields

$$U_H = (p_H + \Delta)\left(X - \frac{I_0 - D_1}{p_0 + \Delta} - \frac{I_1 + D_1 - C}{p_0 + \Delta}\right) = (p_H + \Delta)\left(X - \frac{I_0 + I_1 - C}{p_0 + \Delta}\right).$$

Observe that this expression does not depend on D_1.

If only the low type invests, the zero profit condition reads $D_1 + (\alpha + (1-\alpha)(p_L + \Delta))D_2 = I_0$. Clearly in this case the high-type is better off issuing $D_1 = I_0$ and avoiding costly long-term debt. Separating with short-term debt generates

$$U_H = p_H X + C - I_0,$$

Comparison between the pooling and the separating payoffs gives the inequality (A.7). □

Case $I_0 > C$. There are two possible ways for the firm to finance its time-zero investment in this event. First, the firm might consider issuing riskless short-term debt $D_1 \leq C$. Lemma A.6 shows that in this case the firm would want to issue $D_1 = C$. Next, the firm might want to issue $D_1 \geq C$. In this case, Lemma A.7 shows that, without loss, the firm might go all in on the short-term debt, i.e. put $D_1 = I_0$. Date zero equilibrium immediately follows from Lemma A.7 via comparing the payoffs of the high type firm with $D_1 = I_0$ and $D_2 = 0$.

Lemma A.6. *When $I_0 > C$, the only candidate date-zero equilibrium allocation such that $D_1 \leq C$ is one in which $D_1 = C$. It is never optimal for both types to rollover, and there is investment by all types if and only if*

$$\Delta X - I_1 > \frac{p_H - p_0}{p_0 + \Delta} I_1 + (I_0 - C) \frac{\Delta(1-\alpha)(p_L + \Delta)}{(p_0 + \Delta)(p_0 + (1-\alpha)\Delta)} \tag{A.8}$$

Proof. Observe that, because $I_0 > C$, we need $D_1 + D_2 > C$, as otherwise date-zero lenders could not break even. Therefore, when $D_1 \leq C$ we have the following possibilities. First, it could be that there is investment by all types, which implies that the zero-profit condition for lenders read $D_1 + (p_0 + \Delta)D_2 = I_0$. In this case, the expected payoff of a high type reads

$$U_H = (p_H + \Delta) \left(X - \frac{I_0 - D_1}{p_0 + \Delta} - \frac{I_1 + D_1 - C}{p_0 + \Delta} \right) = (p_H + \Delta) \left(X - \frac{I_0 + I_1 - C}{p_0 + \Delta} \right),$$

and it is independent of the firm's debt maturity structure. Therefore, we can set without loss of generality $D_1 = C$ in this case. Second, it could be that only the low type invests, while the high type takes the zero contract. The zero-profit condition reads $D_1 + (1-\alpha)(p_L + \Delta)D_2 + \alpha[p_H D_2 + (1 - p_H)(C - D_1)] = I_0$. Solving for D_2 and plugging in the utility function of a high type that takes the zero contract yields

$$U_H = p_H(X + C - D_1 - D_2) = p_H(X + C) - p_H D_1 - p_H \frac{I_0 - (1-\alpha)\alpha C - D_1(1 - \alpha(1 - p_H))}{p_0 + (1-\alpha)\Delta}.$$

Taking the derivative with respect to D_1 yields $-p_H + p_H \frac{(1-\alpha(1-p_H))}{p_0+(1-\alpha)\Delta}$. The derivative is positive if and only if $-1 + \frac{(1-\alpha(1-p_H))}{p_0+(1-\alpha)\Delta} > 0$, or $\frac{1-(p_0+\Delta)}{1-(p_H+\Delta)} > \alpha$. We know that this inequality must hold because we have both $\frac{1-(p_0+\Delta)}{1-(p_H+\Delta)} > 1$ and $\alpha < 1$. Thus, in this case it is strictly optimal for a high type to choose $D_1 = C$ and $D_2 = \frac{I_0 - C}{p_0+(1-\alpha)\Delta}$, and the payoff received by a high type reads $U_H = p_H \left(X - \frac{I_0 - C}{p_0+(1-\alpha)\Delta} \right)$. Finally, all types might pool and roll over. The zero-profit condition in this case reads $p_0(D_1 + D_2) + (1 - p_0)C = I_0$, and the high-type's payoff at the zero contract becomes $U_H = p_H \left(X + C - \frac{I_0 - (1-p_0)C}{p_0} \right) = p_H \left(X - \frac{I_0 - C}{p_0} \right)$, which is again independent of D_1. As a result, we can set $D_1 = C$ without loss of generality, as claimed in the Lemma.

Comparing the high type's payoff at the pooling allocation in which all types roll over and do not invest at date one (which is $U_H = p_H \left(X - \frac{I_0 - C}{p_0} \right)$), with the one achieved when low types invest and high types roll over (which is $U_H = p_H\left(X - \frac{I_0 - C}{p_0 + (1-\alpha)\Delta}\right)$), it is immediate that a high type prefers the latter, and as feasibility never binds pooling with rollover cannot be an equilibrium.

So, there are two possible allocations remaining: pooling with investment and separating in which only low types invest. The high type prefers pooling with investment if and only if

$$(p_H + \Delta)\left(X - \frac{I_0 + I_1 - C}{p_0 + \Delta}\right) > p_H\left(X - \frac{I_0 - C}{p_0 + (1-\alpha)\Delta}\right),$$

which can be rewritten as in inequality A.8. $\square$

Finally, we need to consider the case in which $I_0 > C$ and the firm chooses to raise risky short-term debt $D_1 > C$.

Lemma A.7. *When $I_0 > C$, the candidate date-zero equilibrium when $D_1 > C$ is such that:*

1. *If $\Delta X - I_1 \geq \frac{p_H - p_L}{p_H}(I_0 - C)$, then there is investment by all types if and only if inequality (A.7) holds. If (A.7) does not hold, then only low type invests. All agents receive the full information payoff associated to their chosen investment;*

2. *If $\Delta X - I_1 < \frac{p_H - p_L}{p_H}(I_0 - C)$, then there exist investment by all types if and only if*

$$(p_H + \Delta)\left(X - \frac{I_1 + I_0 - C}{p_0 + \Delta}\right) \geq p_H\left(X - \frac{I_0 - C - (1-\alpha)(\Delta X - I_1)}{p_0}\right). \tag{A.9}$$

   Otherwise only the low type invests.

Moreover, regardless of parameter values, the date-0 equilibrium payoff can be achieved with only issuing short-term debt, i.e., $D_1 = I_0$ and $D_2 = 0$.

Proof. First we show that no investment by both types cannot be an equilibrium. To see that, notice along the zero profit line with no investment

$$D_1 + p_0 D_2 = I_0 \tag{$ZP_\emptyset$}$$

the payoff of the high type

$$U_H^\emptyset = p_H\left(X - D_2 - \frac{D_1 - C}{p_0}\right) = p_H\left(X - \frac{I_0 - C}{p_0}\right)$$

is constant. Next, consider the point $(D_1, D_2) = (I_0, 0)$ that is on the $ZP_\emptyset$ and on

$$D_1 + (p_0 + (1-\alpha)\Delta)D_2 = I_0 \tag{ZP_L}$$

$$D_1 + (p_0 + \Delta)D_2 = I_0 \tag{ZP_{HL}}$$

simultaneously. At this point pooling with no investment is a feasible allocation. However, it is dominated by either pooling with investment or one of the separating allocations because $\Delta(X - D_2) > I_1$. Since $(D_1, D_2) = (I_0, 0)$ is on all zero-profits lines simultaneously, one of the separating allocations or the pooling with investment one can be supported as a time 1 equilibrium with higher profits to the high-type than the pooling with no investment. Hence, pooling with no investment cannot be a time-0 equilibrium.

Now that we have ruled out pooling with no investment, we can limit our analysis to only one of the two zero profit conditions ZP_L and ZP_{HL}. In separation without cross-subsidy equilibrium region (in the (D_1, D_2) space) the payoff of the high type along ZP_L is

$$\begin{aligned}
U_H^{sep-no-cs} &= p_H\left(X - D_2 - \frac{D_1 - C}{p_H}\right) \\
&= p_H\left(X - \frac{I_0 - D_1}{p_0 + (1-\alpha)\Delta} - \frac{D_1 - C}{p_H}\right) \\
&\sim D_1\left(\frac{1}{p_0 + (1-\alpha)\Delta} - \frac{1}{p_H}\right) \\
&\sim D_1[p_H - p_0 - (1-\alpha)\Delta] \\
&\sim D_1(1-\alpha)[p_H - p_L - \Delta]
\end{aligned}$$

and this payoff is increasing in D_1.

In separation with cross-subsidy equilibrium region the payoff of the high type along ZP_L is

$$\begin{aligned}
U_H^{sep-with-cs} &= p_H\left(X - D_2 - \frac{D_1 - C - (1-\alpha)[\Delta(X - D_2) - I_1]}{p_0}\right) \\
&= \frac{p_H}{p_0}\left(p_0 X - p_0 D_2 - (D_1 - C) + (1-\alpha)[\Delta(X - D_2) - I_1]\right) \\
&= \frac{p_H}{p_0}\left(p_0 X - (p_0 + (1-\alpha)\Delta)D_2 - D_1 + C + (1-\alpha)[\Delta X - I_1]\right) \\
&= \frac{p_H}{p_0}\left(p_0 X - I_0 + C + (1-\alpha)[\Delta X - I_1]\right)
\end{aligned}$$

and this payoff is constant.

Hence, when we increase D_1 along ZP_L the separating payoff of the high type is either strictly, or weakly increases. Consequently, along the ZP_L the highest separating payoff for the high type is achieved at $D_1 = I_0$ and $D_2 = 0$.

Consider two cases. Case 1: suppose that $(D_1, D_2) = (I_0, 0)$ is in the separating region, then the separating payoff to the high type at this point is

$$U_H^{sep} = \begin{cases} p_H\left(X - \frac{I_0 - C}{p_H}\right) & \text{if } \Delta X - I_1 \geq \frac{p_H - p_L}{p_H}(I_0 - C) \\ p_H\left(X - \frac{I_0 - C - (1-\alpha)[\Delta X - I_1]}{p_0}\right) & \text{if } \Delta X - I_1 < \frac{p_H - p_L}{p_H}(I_0 - C) \end{cases}$$

Clearly, the pooling with investment is a feasible allocation at $(D_1, D_2) = (I_0, 0)$. Since the point with $(D_1, D_2) = (I_0, 0)$ is in the separating equilibrium region it must be that the separating payoff to the high type at $(D_1, D_2) = (I_0, 0)$ is higher than the pooling with investment payoff at $D_1 = I_0$ and $D_2 = 0$

$$U_H^{pool-inv} = (p_H + \Delta)\left(X - \frac{I_1 + I_0 - C}{p_0 + \Delta}\right).$$

But pooling with investment payoff does not change along the ZP_{HL}, hence, the separating payoff to the high type at $(D_1, D_2) = (I_0, 0)$ is higher than the pooling payoff anywhere along ZP_{HL}. Hence, the separating payoff to the high type at $(D_1, D_2) = (I_0, 0)$ is the highest among all payoffs consistent with time zero ZP conditions and, therefore, it is the equilibrium payoff.

Case 2: suppose that $(D_1, D_2) = (I_0, 0)$ is in the pooling region. Then at this point either separating with or without cross-subsidy is a feasible payoff. Hence, U_H^{sep} is feasible for the high type. Moreover, U_H^{sep} is also the highest separating payoff among all consistent with ZP_L. However, it is dominated by the $U_H^{pool-inv}$ at $(D_1, D_2) = (I_0, 0)$ and this payoff can be achieved since $(D_1, D_2) = (I_0, 0)$ is on the ZP_{HL}. Hence, $U_H^{pool-inv}$ (which does not change along ZP_{HL}) is the highest among all payoffs consistent with time zero ZP conditions and, therefore, it is the equilibrium payoff. □

It is immediate to see that the statement of the Proposition 1 follows.

A.7 Proof of Proposition 2

Proof. First, notice that the largest amount of funds the firm can borrow at date zero which affects incentive constraints is $I_0 + I_1 - C$. Raising any additional dollar would amount to a risk-free transfer across time, as the firm cannot invest it. Thus, to identify whether there is debt overhang or not (i.e., if the liability structure drives investment or not), we can focus on the case in which

(1) the firm borrows $I_0 + I_1 - C$ at date zero, and (2) the firm underinvests in equilibrium, which means that inequality (8) fails.

If the firm is anticipated to invest at date one, then the face values of the date-zero long- and short-term debt are, respectively: $D_2 = \frac{I_0+I_1-C}{p_0+\Delta}$ and $D_1 = C - I_1$. Plugging these face values into the investment condition for high types in Lemma 3 yields:

$$\Delta\left(X - \frac{I_0 + I_1 - C}{p_0 + \Delta}\right) - I_1 \geq -(1 - p_H)\min\left(\frac{I_0 + I_1 - C}{p_0 + \Delta}, I_1\right).$$

We have two cases, depending on the relation between D_2 and I_1. If $D_2 \leq I_1$, then the inequality can be re-written as

$$\Delta X - I_1 \geq \frac{I_0 + I_1 - C}{p_0 + \Delta}(\Delta + p_H - 1),$$

which always holds because $D_2 > 0$, $\Delta X - I_1 > 0$ and $\Delta + p_H - 1 \leq 0$. Otherwise, investment requires:

$$\Delta X - I_1 \geq \frac{\Delta(I_0 + I_1 - C)}{p_0 + \Delta} - (1 - p_H)I_1,$$

which may or may not hold. Specifically, using the condition $D_2 > I_1$, we obtain a sufficient condition for the inequality to hold which is

$$\Delta X \geq \frac{p_H + \Delta}{p_0 + \Delta}(I_0 + I_1 - C).$$

□

Proof of Proposition 3.

Proof. Recall that $MacD$ is defined as

$$I_0 \cdot MacD = 2I_0 - D_1 = I_0 + \frac{p_H(p_0 + (1-\alpha)\Delta)}{p_0(p_H - p_L - \Delta)}\left[\frac{p_H - p_L}{p_H}(I_0 - C) - (\Delta X - I_1)\right]^+.$$

It is trivial to see that $I_0 \cdot MacD$, and hence $MacD$ is decreasing in X, increasing in I_1, decreasing in C. To see the dependence of I_0 notice that

$$MacD = 2 - \frac{D_1}{I_0} = 1 + \frac{p_H(p_0 + (1-\alpha)\Delta)}{I_0 \cdot p_0(p_H - p_L - \Delta)}\left[\frac{p_H - p_L}{p_H}(I_0 - C) - (\Delta X - I_1)\right]^+$$

$$= 1 + \frac{p_H(p_0 + (1-\alpha)\Delta)}{p_0(p_H - p_L - \Delta)} \left[\frac{p_H - p_L}{p_H} - \frac{p_H - p_L}{p_H} \cdot \frac{C}{I_0} - \frac{\Delta X - I_1}{I_0} \right]^+ .$$

Since $-(1 - p_L/p_H)C - (\Delta X - I_1) < 0$, $MacD$ is increasing in I_0.

To see the dependence on α notice that

$$\begin{aligned}
\frac{d}{d\alpha}(I_0 \cdot MacD) &\sim \frac{d}{d\alpha}\left(\frac{p_0 + (1-\alpha)\Delta}{p_0}\right) \\
&\sim \frac{d}{d\alpha}\left(\frac{(1-\alpha)}{p_0}\right) \\
&= \frac{-p_0 - (1-\alpha)(p_H - p_L)}{p_0^2} \\
&= \frac{-p_H}{p_0} < 0.
\end{aligned}$$

Hence, $MacD$ is decreasing in α or, equivalently, increasing in $1-\alpha$.

Finally, to see the dependence of Δ notice that $MacD$ comes from the indifference of the low type and zero profit conditions, i.e.

$$\begin{aligned}
\Delta(X - D_2) - I_1 &= \frac{p_H - p_L}{p_H}(D_1 - C) \\
\Delta\left(X - \frac{I_0 - D_1}{p_0 + (1-\alpha)\Delta}\right) - I_1 &= \frac{p_H - p_L}{p_H}(D_1 - C) \\
\Delta\left(X - \frac{I_0}{p_0 + (1-\alpha)\Delta}\right) - I_1 &= \left(\frac{p_H - p_L}{p_H} - \frac{\Delta}{p_0 + (1-\alpha)\Delta}\right) D_1 - \frac{p_H - p_L}{p_H} C \\
\Delta\left(X - \frac{I_0}{p_0 + (1-\alpha)\Delta}\right) - I_1 &= \frac{p_0(p_H - p_L - \Delta)}{p_H(p_0 + (1-\alpha)\Delta)} D_1 - \frac{p_H - p_L}{p_H} C
\end{aligned}$$

The l.h.s. of the last equation is increasing in Δ, and the r.h.s. is decreasing in Δ. Hence, to make the equation hold D_1 has to rise in response to higher Δ, i.e. D_1 is increasing in Δ. Since $I_0 MadD = 2I_0 - D_1$, the shortest separating maturity in decreasing in Δ.

Comparative statics w.r.t. p_L easily follows from examining the date $t = 0$ zero profit constraint and the low type IC constraint separately.

□

Proof of Lemma 6

Proof. When $MacD = 1$ an increase in C first keeps $MacD = 1$ (due to the fact that $MacD$ is weakly decreasing in C, see Proposition 3) and second relaxes the inequality (8) since it only affects the $\frac{(p_H - p_0)}{(p_0+\Delta)}(I_1 - C)$ term.

When $MacD > 1$ a marginal increase in C keeps $MacD > 1$ and affects r.h.s. the inequality (8) by

$$-\frac{p_H - p_0}{p_0 + \Delta} + \frac{(1-\alpha)p_H}{p_0} \cdot \frac{p_H - p_L}{p_H} = (p_H - p_0)\left(-\frac{1}{p_0 + \Delta} + \frac{1}{p_0}\right) > 0$$

□

Proof of Proposition 4

Proof. First, we require for the kink at the separating value function $\hat{C}$, i.e., the point where $MacD = 1$ to be positive, which implies that

$$\Delta X - I_1 < \frac{p_H - p_L}{p_H} I_0 =: M.$$

Next, by Lemma 6, we know that if a $\theta = H$ firm invests when $C = \hat{C}$, then it invests for all values of C (since $MacD > 1$ for $C < \hat{C}$ and $MacD = 1$ for $C > \hat{C}$). Moreover, observe that the kink point $C = \hat{C}$ does not depend on the firm's credit rating α. Therefore, we can plug $C = \hat{C}$ into (A.9) and solve for α to find the threshold $\overline{\alpha}$ that separates high- and medium-rated firms, which yields:

$$\overline{\alpha} := \frac{\Delta X(p_H - p_L) - \Delta(\Delta X - I_1)}{\Delta X(p_H - p_L) + p_H(\Delta X - I_1)}.$$

Notice that $\overline{\alpha} < 1$ if and only if $p_H + \Delta > 0$, which always holds, and $\overline{\alpha} > 0$ if and only if $(p_H - p_L - \Delta)X > -I_1$, which holds whenever $p_H > p_L + \Delta$, as we assumed. It is immediate that every firm with a rating $\alpha \geq \overline{\alpha}$ invests for all parameter values.

Next we turn to lower-rated firms. The pooling payoff crosses the separating payoff at most twice. The right crossing point at $C > \hat{C}$ is clearly increases as α goes down. To derive the comparative statics of the left intersection at $C < \hat{C}$ we need to sign the derivative of (A.9) with respect to α whenever the inequality (A.9) is tight. The lower crossing point decreases as α goes down whenever:

$$\frac{p_H + \Delta}{(p_0 + \Delta)^2}(p_H - p_L)(I_0 + I_1) \geq \left(\frac{p_H}{p_0}\right)^2 \left[\frac{p_H - p_L}{p_H}(I_0 - C) - (\Delta X - I_1)\right].$$

Whenever (A.9) is tight we also have

$$(\Delta X - I_1) - \frac{p_H - p_0}{p_0 + \Delta}(I_0 + I_1 - C) = \frac{(1-\alpha)p_H}{p_0}\left[(\Delta X - I_1) - \frac{p_H - p_L}{p_H}(I_0 - C)\right]$$

Solving for $(\Delta X - I_1) - \frac{p_H - p_L}{p_H}(I_0 - C)$ and substituting it back into the derivative of (A.9) we get

$$\Delta X - I_1 \geq \frac{\Delta(p_H - p_0)^2}{p_H(p_0 + \Delta)^2} \cdot (I_0 + I_1 - C)$$

This inequality is the most tight at $\alpha = 0$ and $C = 0$, hence it is sufficient to require that

$$\Delta X - I_1 \geq \frac{\Delta(p_H - p_L)^2}{p_H(p_L + \Delta)^2} \cdot (I_0 + I_1) \tag{A.10}$$

When this inequality holds, then the right intersection point of the pooling and separating payoffs is decreasing as α goes down.

We also need to check whether this threshold will move all the way to zero, i.e., whether at $\alpha = 0$ and $C = 0$ separating will dominate pooling. This happens whenever

$$\begin{aligned}(p_H + \Delta)\left(X - \frac{I_0 + I_1}{p_L + \Delta}\right) &\leq p_H\left(X - \frac{I_0 - (\Delta X - I_1)}{p_L}\right)\\ \frac{\Delta}{p_L + \Delta}I_0 - I_1\frac{p_L}{p_L + \Delta} &\leq \Delta X - I_1\end{aligned}$$

Finally, put

$$m := \max\left[\frac{\Delta}{p_L + \Delta}I_0 - I_1, \frac{\Delta(p_H - p_L)^2}{p_H(p_L + \Delta)^2} \cdot (I_0 + I_1)\right]. \tag{A.11}$$

Whenever $\Delta X - I_1 \in (m, M)$ we have full investment for all $\alpha > \bar{\alpha}$. For $\alpha < \bar{\alpha}$ the the cash interval of no investment is monotonically increasing as alpha goes down and at $\alpha = 0$ the no-investment region includes $C = 0$. Hence, there exists $\underline{\alpha}$ for $\alpha \in (\underline{\alpha}, \bar{\alpha})$ the investment happens for low or high values of cash (no-investment region does not contain $C = 0$), and for $\alpha < \underline{\alpha}$ the investment happens only for high values of C (no-investment region contains $C = 0$).

□

B Optimal Allocation Proofs

Lemma B.1 (High-Type Optimal Allocation)**.** *An equilibrium menu of contracts M^* and a firm investment action profile (a_H^*, a_L^*) implement the high-type firm's optimal allocation. That is, they solve the following problem:*

$$\max_{M,a_H,a_L} U_H(M,\theta,a_H) \tag{PP}$$

subject to:

$$U_\theta(M,\theta,a_\theta) \geq U_\theta(M,\hat{\theta},\hat{a}) \qquad \forall\theta,\hat{\theta},\hat{a} \tag{IC_θ}$$

$$U_L(M,L,a_L) \geq U_L^{FI} \tag{IR_L}$$

$$\alpha \cdot \pi_H(M,\theta,a_H) + (1-\alpha)\cdot\pi_L(M,L,a_L) \geq 0 \tag{BB}$$

$$M \text{ is feasible}$$

That is, the planner is maximizing the payoff of the high-type firm subject to three constraints: (a) incentive constraints, i.e., that each type reveals itself truthfully and takes the proposed action, (b) individual rationality, i.e., the low-type firm should do no worse than its outside option, and (c) budget balance, i.e., the planner cannot make losses on the proposed menu.

Proof of Lemma B.1

Proof. First, note that if the equilibrium menu M^* makes strictly positive profits, another lender could enter and offer an menu $\hat{M}$ that is identical to M in all respects, with the only difference that now we increase $\hat{z}^a_{\theta,2} = z^a_{\theta,2} + \epsilon$ by $\epsilon > 0$ for all $\theta \in \{H, L\}$ and $a \in \{i, n\}$. Evidently, this has no effects on incentive constraints, and if ϵ is small it will be profitable for the entrant, who anticipates that all types will prefer the deviation contract relative to the equilibrium contract.

Second, suppose by contradiction that $\pi_L(M^*, L, a_L^*) > 0$. Consider a deviation menu $\hat{M}$ in which the (L, a_L^*) option differs from the original (L, a_L^*) contract only in z_2 such that $\hat{z}^{a_L^*}_{L,2} = z^{a_L^*}_{L,2} + \epsilon$. And all other options in $\hat{M}$ are zero contracts. That is, in (L, a_L^*) contract the investors pay the firm an $\epsilon > 0$ more at $t = 2$ relative to the original (L, a_L^*) contract and in all other options the investors provide just enough funds for to cover I_0 and I_1 receive all generated cash flows.

Clearly such menu attracts the low type since $U_L(\hat{M}, L, a_L^*) = U_L(M, L, a_L^*) + \epsilon$. Moreover, the low type firm would prefer to pick the (L, a_L^*) contract in the menu $\hat{M}$ since all other options deliver zero payoff. If only the low type firm switches to the new menu $\hat{M}$ then investors would make positive profits on it since $\pi_L(\hat{M}, L, a_L^*) = \pi_L(M^*, L, a_L^*) - \epsilon > 0$ for sufficiently small $\epsilon > 0$.

If the high type also switches to $\hat{M}$ then (a) it will pick the (L, a_L^*) and (b) investors would make even more profits since $\pi_H(\hat{M}, L, a_L^*) > \pi_L(\hat{M}, L, a_L^*) = \pi_L(M^*, L, a_L^*) - \epsilon > 0$.

Thus, we have proved that the constraints to the program must hold. Now we show that the equilibrium must maximize the utility of a high type. Suppose, by contradiction, that the equilibrium (M^*, a_H^*, a_L^*) does not solve (1), i.e. that it does not maximize the payoff of the high type firm. Let (M, a'_H, a'_L) be a solution to (1). Then we must have

$$U_H(M', H, a'_H) > U_H(M^*, H, a_H^*).$$

Case 1: First, it could be that $U_L(M^*, L, a_L^*) \geq U_L(M', H, a'_H)$. It follows from ($ZP$) and ($NP_L$) that in M' investors are making either positive or zero profits on the high type. Consider a menu $\hat{M}$ in which the (H, a'_H) contract is the ϵ modified contract from M' (to deliver $\epsilon > 0$ more profits to investors) and all other options are zero contracts. The menu $\hat{M}$ attracts the high type because

$$U_H(\hat{M}, H, a'_H) = U_H(M', H, a'_H) - \epsilon > U_L(M^*, H, a_H^*),$$

as long as $\epsilon > 0$ is small enough, while it does not attract low types because

$$U_L(\hat{M}, H, a'_H) \overset{\epsilon}{<} U_L(M', H, a'_H) \overset{\text{case 1}}{\leq} U_L(M^*, L, a_L^*).$$

Moreover, the menu $\hat{M}$ is guaranteed to deliver at least $\epsilon > 0$ profits to investors. As a result, its existence contradicts M^* being an equilibrium.

Case 2: Now consider the case in which $U_L(M^*, L, a_L^*) < U_L(M', H, a'_H)$. We have two sub-cases.

Case 2.1: First, suppose that $U_L(M', L, a'_L) > U_L(M', H, a'_H)$. In this case, consider a deviation menu $\hat{M}$ constructed as follows. The contract (H, a'_H) is the same as in the M' menu. The option (L, a'_L) is an ϵ modified contract from M' that generates $\epsilon > 0$ higher profits for the investors. The two other options are zero contracts. The menu $\hat{M}$ attracts the high type who chooses (H, a'_H) as

$$U_H(\hat{M}, H, a'_L) \overset{\epsilon}{<} U_H(M', H, a'_L) \overset{IC}{\leq} U_H(M', H, a'_H) = U_H(\hat{M}, H, a'_H).$$

Moreover, the menu $\hat{M}$ attracts the low type who picks (L, a'_L) for small enough ϵ because

$$U_L(\hat{M}, L, a'_L) = U_L(M', L, a'_L) - \epsilon \overset{\text{case 2.1}}{>} U_L(M', H, a'_H) \overset{\text{case 2}}{>} U_L(M^*, L, a^*_L).$$

Finally, the menu $\hat{M}$ is guaranteed to deliver strictly higher profits to investors that the menu M' (which itself is a zero profit menu). Hence the existence of $\hat{M}$ contradicts M^* being an equilibrium.

Case 2.2: Otherwise, the only remaining is the case $U_L(M', L, a'_L) = U_L(M', H, a'_H)$. In this event, consider a deviation menu $\hat{M}$ constructed as follows. The contract (L, a'_L) is the same as in M'. Option (H, a'_H) is an ϵ-modified contract from the menu M' that generates $\epsilon > 0$ higher profits for investors. All other options are zero contracts. $\hat{M}$ attracts the high type who chooses (H, a'_H) as

$$U_H(\hat{M}, H, a'_L) = U_H(M', H, a'_H) - \epsilon > U_H(M^*, H, a{*}_H),$$

as long as $\epsilon > 0$ is sufficiently small. Moreover, $\hat{M}$ attracts the low type who picks (L, a'_L) as

$$U_L(\hat{M}, L, a'_L) = U_L(M', L, a'_L) \overset{\text{case 2.2}}{=} U_L(M', H, a'_H) \overset{\epsilon}{>} U_L(\hat{M}, H, a'_H)$$
$$U_L(\hat{M}, L, a'_L) = U_L(M', L, a'_L) \overset{\text{case 2.2}}{=} U_L(M', H, a'_H) \overset{\text{case 2}}{>} U_L(M^*, L, a^*_L)$$

Finally, $\hat{M}$ is guaranteed to deliver strictly higher profits to investors than M' (which itself is a zero profit menu). Thus, the existence of $\hat{M}$ contradicts M^* being an equilibrium. $\square$

Proof of Proposition 6

We prove the proposition via a sequence of lemmas that increasingly better characterize the optimal allocation.

Lemma B.2. *Suppose that M^* and $a^*_H = a^*_L = i$ is an equilibrium. Then, we must have that $\pi_H(M^*, H, i) > 0 > \pi_L(M^*, L, i)$. That is, the equilibrium must involve subsidization across types.*

Proof. Suppose the contrary, i.e., that $\pi_H(M^*, H, i) = 0 = \pi_L(M^*, L, i)$. An L-type firm's expected payoff after sending a message $(\hat{\theta}, i)$ is

$$U_L(M^*, \hat{\theta}, i) = (p_L + \Delta)X - I_0 - I_1 - \pi_L(M^*, \hat{\theta}, i).$$

Incentive compatibility ensures that $U_L(M^*, L, i) \geq U_L(M^*, H, i)$, which, in turn, implies that

$\pi_L(M^*, H, i) \geq \pi_L(M^*, L, i) = 0$. We have two cases to consider.

Case 1: Suppose first that $U_L(M^*, L, i) > U_L(M^*, H, i)$. It follows immediately $\pi_L(M^*, H, i) > \pi_L(M^*, L, i) = 0$. However, we know that $\pi_H(M^*, H, i) \overset{s_H^i \geq 0}{\geq} \pi_L(M^*, H, i) > 0$, which contradicts the fact that investors must make zero profits on the high type.

Case 2: Alternatively, suppose that $U_L(M^*, L, i) = U_L(M^*, H, i)$. It follows immediately that $\pi_L(M^*, H, i) = \pi_L(M^*, L, i) = 0$. Moreover, the fact that $\pi_H(M^*, H, i) = \pi_L(M^*, H, i) = 0$ implies that $s_H^i = 0$. However, then the contract option (H, i) cannot break even for investors because $C < I_0 + I_1$ and, as a result, $z_{H,0}^i + z_{H,1}^i + z_{H,2}^i > 0$. Thus, another contradiction is reached. □

Lemma B.3. *There does not exists an equilibrium (M^*, a_H^*, a_L^*) with $a_H^* = i$ and $a_L^* = n$.*

Proof. We break down our argument in two main cases.

Case 1: first, suppose that there is no cross-subsidy across types i.e. that $\pi_H(M^*, H, i) = 0 = \pi_L(M^*, L, n)$. In this case, consider a deviation menu $\hat{M}$ with the (L, i) contract being characterized by $\hat{z}_{L,0}^i = I_0$, $\hat{z}_{L,1}^i = I_1$, $\hat{z}_{L,2}^i = 0$ and $\hat{s}_L^i$ such that

$$U_L(\hat{M}, L, i) = (p_L + \Delta)(X - \hat{s}_L^i) + C = U_L(M^*, L, n) + \epsilon = p_L X - I_0 + C + \epsilon,$$

while all other options in the menu $\hat{M}$ are zero contracts. By construction, this menu attracts low type firms who choose the option (L, i). Moreover, investors make profits on low types in $\hat{M}$ as

$$\pi_L(\hat{M}, L, i) = (p_L + \Delta)\hat{s}_L^i - I_0 - I_1 = \Delta X - I_1 - \epsilon > 0$$

for a sufficiently small $\epsilon > 0$. In addition, as $\pi_H(\hat{M}, L, i) > \pi_L(\hat{M}, L, i) > 0$, this menu makes profits for investors regardless of whether the H type accepts it or not. Thus, the existence of $\hat{M}$ contradicts M^* being an equilibrium.

Case 2: suppose now that there is a subsidy across types, i.e. that $\pi_H(M^*, H, i) > 0 > \pi_L(M^*, L, n)$. We now divide the argument further in two sub-cases.

Case 2.1: suppose that $U_L(M^*, L, n) > U_L(M^*, H, i)$. In this case, consider a deviation menu $\hat{M}$ where the (H, i) contract is an ϵ-modification of the (H, i) contract from M^* that delivers $\epsilon > 0$ *less* profits to investors, for example by increasing z_2. All other options in $\hat{M}$ are zero contracts. $\hat{M}$ attracts the H type as $U_H(\hat{M}, H, i) = U_H(M^*, H, i) + \epsilon$. It does not attract the low type for a sufficiently small $\epsilon > 0$, because $U_L(\hat{M}, H, i) = U_L(M^*, H, i) + \epsilon \overset{\text{case 2.1}}{<} U_L(M^*, L, n)$. Finally, $\hat{M}$

makes positive profits for investors as $\pi_H(\hat{M}^*, H, i) = \pi_H(M^*, H, i) - \varepsilon > 0$, for a sufficiently small $\epsilon > 0$. Thus, the existence of $\hat{M}$ contradicts M^* being an equilibrium.

Case 2.2: the only remaining case is $U_L(M^*, L, n) = U_L(M^*, H, i)$, which implies that $\pi_L(M^*, H, i) = \Delta X - I_1 + \pi_L(M^*, L, n)$. In this event, consider again a deviation menu $\hat{M}$ where the (H, i) contract is an ϵ-modification of the (H, i) contract from M^* that delivers $\epsilon > 0$ *less* profits to investors, for example by increasing z_2. All other options in $\hat{M}$ are zero contracts. $\hat{M}$ attracts the H type as $U_H(\hat{M}, H, i) = U_H(M^*, H, i) + \epsilon$. It attracts the low type as $U_L(\hat{M}, H, i) = U_L(M^*, H, i) + \epsilon \overset{\text{case 2.2}}{=} U_L(M^*, L, n) + \epsilon > U_L(M^*, L, n)$. Investor profits from $\hat{M}$ are

$$\begin{aligned}\alpha \cdot \pi_H(\hat{M}, H, i) + (1-\alpha) \cdot \pi_L(\hat{M}, H, i) &= \alpha \cdot \pi_H(M, H, i) + (1-\alpha) \cdot \pi_L(M, H, i) - \epsilon \\ &= \alpha \cdot \pi_H(M, H, i) + (1-\alpha) \cdot [\pi_L(M, L, n) + \Delta X - I_1] - \epsilon \\ &= (1-\alpha)[\Delta X - I_1] - \epsilon > 0,\end{aligned}$$

where the last inequality holds for sufficiently small $\epsilon > 0$. Thus, the existence of $\hat{M}$ contradicts M^* being an equilibrium. □

At this point, we show that there cannot be an equilibrium in which none of the firm types invest at date one. This and the previous Lemma jointly imply that low types must be investing in any equilibrium, and that the only remaining investment choice to be characterized pertains high types.

Lemma B.4. *There does not exists an equilibrium (M^*, a_H^*, a_L^*) with $a_H^* = a_L^* = n$.*

Proof. We break down the argument in two cases.

Case 1: Suppose first that $\pi_H(M^*, H, n) = 0 = \pi_L(M^*, L, n)$ The low type's expected payoff after sending a message $(\hat{\theta}, n)$ is

$$U_L(M^*, \hat{\theta}, n) = p_L X - I_0 - \pi_L(M^*, \hat{\theta}, n).$$

Incentive compatibility ensures that $U_L(M^*, L, n) \geq U_L(M^*, H, n)$, which, in turn, implies that $\pi_L(M^*, H, n) \geq \pi_L(M^*, L, n) = 0$. We consider two sub-cases separately.

Case 1.1: Suppose first that $U_L(M^*, L, n) > U_L(M^*, H, n)$. It follows that $\pi_L(M^*, H, n) > \pi_L(M^*, L, n) = 0$, and therefore $\pi_H(M^*, H, n) \overset{s_H^n \geq 0}{\geq} \pi_L(M^*, H, n) > 0$ which contradicts the assumption that investors make zero profits on the high type.

Case 1.2: Alternatively, suppose that $U_L(M^*, L, n) = U_L(M^*, H, n)$. It follows that $\pi_L(M^*, H, n) = \pi_L(M^*, L, n) = 0$. The fact that $\pi_H(M^*, H, n) = \pi_L(M^*, H, n) = 0$ implies that $s_H^n = 0$. However, then the contract option (H, n) cannot break even since $C < I_0$ and so $z_{H,0}^n + z_{H,1}^n + z_{H,2}^n > 0$.

Case 2: Otherwise, we must have $\pi_H(M^*, H, n) > 0 > \pi_L(M^*, L, n)$. Again, we consider two sub-cases depending on whether the incentive constraint of the low type is slack or it binds.

Case 2.1: Suppose first that $U_L(M^*, L, n) > U_L(M^*, H, n)$. In this case, consider a deviation menu $\hat{M}$ where the (H, n) contract is an ϵ-modification of the (H, n) contract from M^* that delivers $\epsilon > 0$ *less* profits to investors, for example by increasing z_2. All other options in $\hat{M}$ are zero contracts. $\hat{M}$ attracts the H type as $U_H(\hat{M}, H, n) = U_H(M^*, H, n) + \epsilon$. It does not attract the low type for sufficiently small $\epsilon > 0$, as $U_L(\hat{M}, H, n) = U_L(M^*, H, n) + \epsilon \overset{\text{case 2.1}}{<} U_L(M^*, L, n)$, and it makes positive profits for investors because $\pi_H(\hat{M}^*, H, n) = \pi_H(M^*, H, n) - \varepsilon > 0$ for sufficiently small $\epsilon > 0$. Thus, the existence of $\hat{M}$ contradicts M^* being an equilibrium.

Case 2.2: The only remaining case is $U_L(M^*, L, n) = U_L(M^*, H, n)$. First, we will construct a modification of the (H, n) contract from M^* where $z_0 = I_0$, $z_1 = -C$, and $z_2 = z_{H,0}^n + z_{H,1}^n + z_{H,2}^n - (I_0 - C)$. Notice that this modification is feasible (it simply moves all the transfers in excess of investment needs to period $t = 2$) and does not affect the incentive constraints. Next we argue that $z_2 = 0$. Suppose the contrary, i.e. that $z_2 > 0$ ($z_2 < 0$ is not feasible). Then we can lower z_2 by $\epsilon > 0$ and simultaneously lower s_H^n by $\epsilon/(p_0 + \Delta)$ (this is always possible since $C < I_0$ implies $s_H^n > 0$ for the investors to make positive profits on the high type). This modification increases the utility of the high type (since $p_H > p_0$) and decreases the utility of the low type (since $p_0 > p_L$). As a result, only the high type would be attracted to this contract. Since investors made strictly positive profits on the high type in M^*, they would also make positive profits with the modified contract for sufficiently small $\epsilon > 0$. So, we can construct a menu $\hat{M}$ that consists of the modified contract and zero contracts and which attracts only the high type, making positive profits for investors. Thus, M^* cannot be an equilibrium, and we can pin down s_H^n through the zero-profit condition

$$
\begin{aligned}
\alpha\pi_H(M^*, H, n) + (1-\alpha)\pi_L(M^*, L, n) &\overset{\text{case 2.2}}{=} \alpha\pi_H(M^*, H, n) + (1-\alpha)\pi_L(M^*, H, n) \\
&= p_0 \cdot s_H^n - (I_0 - C) = 0
\end{aligned}
$$

Finally, offer a deviation menu $\hat{M}$ with the (θ, i) investment contract being $z_0 = I_0$, $z_1 = I_1 - C$, $z_2 = 0$ and $s = (I_0 + I_1 - C)/(p_0 + \Delta) + \epsilon$, and all other (θ, n) being the zero options. Evidently, such menu delivers a higher payoff to both high and low types (because $\Delta X - I_1 > 0$) when $\epsilon > 0$ is sufficiently small. Moreover, it generates positive profits for investors, contradicting the presumption that M^* was an equilibrium. $\square$

Lemmas B.2 and B.4 restrict the possible equilibrium investment policies to: (1) both types investing at both dates under a pooling contract; or (2) the low type investing at both dates, while the high type only invests at date zero. Henceforth, we refer to the former case as implementing the *Full Investment* allocation, while the latter case features *Partial Investment.* We now characterize the optimal contracts for these two possible allocations separately. This will then allow us to run a horse-race between these contracts and pin down optimal allocations. We begin with case (2), in which only the low types invest at $t = 1$.

Lemma B.5. *If an equilibrium features Full Investment (i.e., $a_H^* = a_L^* = i$) then*

$$U_\theta^{FI} \stackrel{def}{=} U_\theta(M^*, \theta, i) = (p_\theta + \Delta)\left(X - \frac{I_0 + I_1 - C}{p_0 + \Delta}\right) \qquad \forall\, \theta \in \{H, L\}. \tag{B.1}$$

Moreover, without loss, the menu M^* consists of $z^i_{\theta,0} = I_0$, $z^i_{\theta,1} = I_1 - C$, $z^i_{\theta,2} = 0$, $s^i_\theta = \frac{I_0+I_1-C}{p_0+\Delta}$ for $\theta \in \{H, L\}$, and zero contracts for $a = n$

Proof. First, for any menu M we can construct a modification of the (θ, i) contract, for each θ, where $z_{\theta,0} = I_0$, $z_{\theta,1} = I_1 - C$, and $z_{\theta,2} = z^i_{\theta,0} + z^i_{\theta,1} + z^i_{\theta,2} - (I_0 + I_1 - C)$, while s is unchanged. Notice that such modification is feasible (it simply moves all the transfers in excess of investment needs to period $t = 2$) and does not affect the incentive constraints. From now onward, we restrict attention to menus of this sort without loss of generality. For convenience, from now onward (in this proof) we omit the superscript i as both types invest.

Second, we claim that a high-type contract must be such that $z_{H,2} = 0$ Suppose the contrary, i.e. there exists an equilibrium menu M^* in which $z_{H,2} > 0$ ($z_{H,2} < 0$ is infeasible). Then we can lower $z_{H,2}$ by ϵ and simultaneously lower s_H by $\epsilon/(p_0 + \Delta)$ (this is always possible since $C < I_0$ implies $s_H > 0$ for the investors to make positive profits on the high type). This modification increases the utility of the high type (since $p_H > p_0$) and decreases the utility of the low type (since $p_0 > p_L$). As a result, only the high type would be attracted to this modified contract. Since investors made strictly positive profits on the high type in the menu M^*, they would also make positive profits with the modified contract for sufficiently small $\epsilon > 0$. Hence we can construct a menu $\hat{M}$ that consists of the modified contract and zero contracts that attracts only the high type and makes positive profits to investors - a contradiction to M^* being an equilibrium.

Incentive constraints read:

$$(p_H + \Delta)[X - s_H] + z_{H,2} \geq (p_H + \Delta)[X - s_L] + z_{L,2}$$

$$(p_L + \Delta)[X - s_L] + z_{L,2} \geq (p_L + \Delta)[X - s_H] + z_{H,2}$$

Adding up the two constraints yields: $-p_H s_H - p_L s_L \geq -p_L s_H - p_H s_L$, or simply $s_L - s_H \geq 0$.

If $s_L - s_H = 0$, then incentive compatibility requires that $z_{L,2} = z_{H,2}$, and so we have a pooling contract where the investment option is the same across types, and we can restrict attention to a degenerate menu with just one contract, which leads to investment. In this case, lenders make zero profits if $(p_0 + \Delta) \cdot s - z_0 - z_1 - z_2 = 0$. Therefore, the utility of a high type reads: $U_H = (p_H + \Delta)[X - s] + C + (p_0 + \Delta) \cdot s - I_0 - I_1$. Taking the derivative $\partial U_H / \partial s = p_0 - p_H < 0$, which implies that to maximize the utility of a high type, one needs to minimize s. As a result, we need to minimize the sum of the zs, which, by feasibility, implies that we have $z_0 = I_0$, $z_1 = I_1 - C$ and $z_2 = 0$, while from the lender's zero profit condition we get $s = \frac{I_0 + I_1 - C}{p_0 + \Delta}$.

Now, suppose that $s_L - s_H > 0$. Incentive compatibility implies that $z_L - z_H < 0$, but this is impossible since $z_H = 0$ and feasibility requires $z_L \geq 0$. $\square$

In order to characterize the the optimal contract that implements investment at $t = 1$ only by the low type, it is useful to break the analysis in two separate lemmas, depending on whether the incentive constraint for a low type to mimic a high type binds or not.

Lemma B.6. *If an equilibrium features Partial Investment (i.e., $a_H^* = n$, $a_L^* = i$) and the incentive constraint of the low type is slack, then*

$$\begin{aligned} U_H^{PI-slack} &\stackrel{def}{=} U_H(M^*, H, n) = p_H X - I_0 + C, \\ U_L^{PI-slack} &\stackrel{def}{=} U_L(M^*, L, i) = (p_L + \Delta)X - I_0 - I_1 + C. \end{aligned} \tag{B.2}$$

Moreover, without loss, the menu M^* consists of

$$\begin{aligned} z_{H,0}^n &= I_0, & z_{H,1}^n &= -C, & z_{H,2}^n &= 0, & s_H^n &= \frac{I_0 - C}{p_H}, \\ z_{L,0}^i &= I_0, & z_{L,1}^i &= I_1 - C, & z_{L,2}^i &= 0, & s_L^i &= \frac{I_0 + I_1 - C}{p_L + \Delta}, \end{aligned}$$

and zero contracts for (H, i) and (L, n)

Proof. First we argue that in such equilibrium investors should break even on a type-by-type basis, i.e, that $\pi_H(M^*, H, n) = \pi_L(M^*, L, i) = 0$. If it is not the case, then $\pi_H(M^*, H, n) > 0 > \pi_L(M^*, L, i)$, i.e., investors make positive profits on the high type. Then consider a menu $\hat{M}$ where the (H, n) contract is an ϵ modification of the (H, n) contract from M^* that delivers ϵ less profits to investors, for example by increasing z_2. All other options in the menu $\hat{M}$ are zero contracts.

The menu $\hat{M}$ attracts the H type firm since $U_H(\hat{M}, H, n) = U_H(M^*, H, n) + \epsilon$. It does not attract the low type firm for sufficiently small $\epsilon > 0$ since $U_L(\hat{M}, H, n) = U_L(M^*, H, n) + \epsilon < U_L(M^*, L, n)$, and it makes positive profits for investors since $\pi_H(\hat{M}^*, H, n) = \pi_H(M^*, H, n) - \varepsilon > 0$ for sufficiently small $\epsilon > 0$. Hence the existence of $\hat{M}$ contradicts M^* being an equilibrium.

Since the zero-profits hold type-by-type the expected payoff to firm insiders is simply

$$U_H(M^*, H, n) = p_H X - I_0 + C \qquad U_L(M^*, L, i) = (p_L + \Delta)X - I_0 - I_1 + C.$$

Without loss, we can move all the transfers above the investment needs to the period $t = 2$, i.e. set $z^n_{H,0} = I_0, z^n_{H,1} = -C$ and $z^i_{L,0} = I_0, z^i_{L,1} = I_1 - C$. Zero profit conditions become

$$p_H s^n_H = z^n_{H,2} - (I_0 - C) \qquad \text{and} \qquad (p_L + \Delta)s^i_L = z^i_{L,2} - (I_0 + I_1 - C).$$

In the relevant case $p_H > p_L + \Delta$ parameters $z^n_{H,2} = 0$ and $s^n_H = (I_0 - C)/p_H$ maximize the parameter range for which the IC constraint of the low type is slack. Higher $z^n_{H,2}$ and, consequently higher s^n_H, would increase the low type deviation payoff $U_L(M^*, H, n)$ and, hence, reduce the likelihood that $U_L(M^*, H, n) < U_L(M^*, L, i) = (p_L + \Delta)X - I_0 - I_1 + C$.

□

Finally, we consider the case in which the incentive constraint for a low type to mimic the high type is binding in equilibrium.

Lemma B.7. *If an equilibrium features investment only by the low type (i.e., $a^*_H = n$, $a^*_L = i$) and* *the IC constraint of the low type is tight, then*

$$\begin{aligned}U_H^{PI-binds} &\stackrel{def}{=} U_H(M^*, H, n) = p_H\left(X - \frac{I_0 - C - (1-\alpha)(\Delta X - I_1)}{p_0}\right),\\ U_L^{PI-binds} &\stackrel{def}{=} U_L(M^*, L, i) = p_L\left(X - \frac{I_0 - C - (1-\alpha)(\Delta X - I_1)}{p_0}\right).\end{aligned} \tag{B.3}$$

Moreover, without loss, the menu M^* consists of

$$\begin{aligned}&z_{H,0}^n = I_0, \qquad z_{H,1}^n = -C, \qquad z_{H,2}^n = 0, \qquad s_H^n = \frac{I_0 - C - (1-\alpha)(\Delta X - I_1)}{p_0},\\ &z_{L,0}^i = I_0, \qquad z_{L,1}^i = I_1 - C, \qquad z_{L,2}^i = 0, \qquad s_L^i = (p_L + \Delta)^{-1}\left[\frac{\alpha p_H}{p_0}(\Delta X - I_1) + \frac{p_L}{p_0}(I_0 - C)\right],\end{aligned}$$

and zero contracts for (H, i) and (L, n)

Proof. In this case, we have $U_L(M^*, L, i) = U_L(M^*, H, n)$. We again use the fact that, for any menu M, we can construct a modification of the (θ, a) contract, for each θ and a, where $z_{\theta,0} = I_0$, $z_{\theta,1} = \mathbb{1}(\hat{a} = i)I_1 - C$, and $z_{\theta,2} = z_{\theta,0}^a + z_{\theta,1}^a + z_{\theta,2}^a - (I_0 + \mathbb{1}(\hat{a} = i)I_1 - C)$, while s is unchanged. Notice that such modification is feasible (it simply moves all the transfers in excess of investment needs to period $t = 2$) and does not affect the incentive constraints. From now onward, we restrict attention to menus of this sort without loss of generality, and we have that $U_L(M^*, L, i) = U_L(M^*, H, n) \iff (p_L + \Delta)[X - s_L^i] + z_{L,2}^i = p_L[X - s_H^n] + z_{H,2}^n$, and the zero profit condition reads:

$$\begin{aligned}\alpha \cdot (p_H s_H^n - I_0 + C - z_{H,2}^n) + &(1-\alpha) \cdot ((p_L + \Delta)s_L^i - I_0 + C - I_1 - z_{L,2}^i) =\\ &= p_0 s_H^n + C - I_0 + (1-\alpha)(\Delta X - I_1) - z_{H,2}^n = 0.\end{aligned}$$

Therefore, the utility function of a high type reads $U_H = p_H(x - s_H^n) + p_0 s_H^n + C - I_0 + (1 - \alpha)(\Delta X - I_1)$, and we obtain that: $\partial U_H / \partial s_H^n = -p_H + p_0 < 0$. Therefore, optimal contracts will minimize s_H^n, setting $z_{H,2}^n = 0$, which yields: $s_H^n = \frac{I_0 - C - (1-\alpha)(\Delta X - I_1)}{p_0}$. From the utility function of a low type we get $U_L = (p_L + \Delta)[X - s_L^i] + z_{L,2}^i = p_L(X - \frac{I_0 - C - (1-\alpha)(\Delta X - I_1)}{p_0})$, independently of the specific choice of s_L^i and $z_{L,2}^i$, and there always exists a feasible pair that can be chosen, because the low type has a positive net present value investment project, at both $t = 0$ and $t = 1$. $\square$

One final thing to notice regarding the two separating allocations in Lemmas B.6 and B.7 is that whenever $U_H^{PI-slack} > U_H^{PI-binds}$ the allocation $PI - slack$ is infeasible because the IC constraint of the low type is violated. Similarly, whenever $U_H^{PI-slack} < U_H^{PI-binds}$ the $PI - binds$

allocation is impossible. Hence, the payoff of the high-type firm in the separating allocation is $\min(U_H^{PI-binds}, U_H^{PI-slack})$.

Consequently, optimal allocation features investment by both types of firms whenever

$$U_H^{FI} \geq \min(U_H^{PI-binds}, U_H^{PI-slack}), \tag{B.4}$$

which is exactly inequality (8).