Factor and stock-specific disagreement and trading flows*

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Abstract

We study how disagreement on both factor and stock-specific risk exposures across many investors and securities impacts asset prices. Our theoretical analyses predict that disagreement about factor dynamics drives larger flows into portfolios that are more exposed to the factors. These concentrated bets on the factor lead to higher volatility and reduced diversification benefits. We then test these predictions using a novel empirical setting – exchange-traded funds (ETFs). We find that when factor disagreement rises, ETFs that mimic the factor see increased flows, higher forward-looking volatility risk, and a higher forward-looking correlation among the stocks in the ETF.

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The proliferation of Exchange-Traded Funds (ETFs) and index futures have made trading portfolios of assets easier. The perception is that these vehicles improve risk sharing by providing investors with cheap access to well-diversified portfolios. From an asset-pricing perspective, the greater diffusion of risks across agents should result in lower aggregate risk. However, many studies find that the introduction of index-linked assets, such as ETFs, have had the opposite effect of increasing excess volatility (see, inter alia Ben-David, Franzoni, and Moussawi, 2018). While prior studies attribute this finding to liquidity shocks from uninformed investors propagating between assets via an arbitrage channel, this study shows that an alternative mechanism — heterogeneous beliefs — is also at play. Namely, we provide evidence that speculative trading due to disagreement in subjective expectations lead to exorbitant shifts in wealth, inducing crowded trades, and higher volatility and correlation risk for the index and its underlying assets.

On the theoretical front, we answer *why* concentrated positions are created (flow generated from time-varying factor disagreement) and *how* the risks are embedded in asset prices (higher factor volatility and stock correlations). On the empirical front, we quantify the effects of disagreement-induced trading within a unique setting: ETFs. We thus use the term "factor" to refer to common components of returns that investors are concerned about, e.g., exposure to technology or healthcare. Our analysis also focuses on information in ETF-linked options, which have grown in recent prominence, to examine disagreement's ex-ante and long run impact on perceptions, rather than realizations, of risk.

The canonical asset pricing framework assumes that variations in the returns of individual securities are a function of factor risk exposures and stock-specific shocks. This sets up a dichotomy: with many investors and stocks, there are a near infinite number of dimensions along which investors can disagree on the idiosyncratic portion of returns. However, this form of disagreement (henceforth, *stock-specific* or *idiosyncratic disagreement*) can wash out in the aggregate. In contrast, investors can disagree on far fewer dimensions regarding the portion of returns attributable to factor exposure (henceforth, *factor* or *systematic disagreement*). This generates the possibility of concentrated exposures (both long and short) across investors. As each form of disagreement differs in its impact on asset prices, its consequence for financial risk also differs.

To demonstrate why, consider investors trading on their beliefs about the prospects of the U.S. technology sector. On the one hand, if beliefs are dispersed, meaning that investors believe that stock-specific shocks to firms are more likely to drive next period's returns, then investors will choose to take disparate positions in individual technology stocks, such as Microsoft Corporation and Apple Corporation. On the other hand, if investors believe that returns will be driven by exposure to a common "technology" factor, then they will choose to trade the portfolio of stocks. For instance, investors may trade the Invesco QQQ Trust Series 1 ETF (NASDAQ: QQQ), an ETF that tracks the returns of the Nasdaq 100 index. Consequently, a large flow into (out of) QQQ suggests that investors are predominately trading on the systematic (firm-specific) component of technology stock returns.

We first codify this intuition using a pure exchange economy with multiple Lucas trees that are exposed to both factor risk and stock-specific shocks. The model's innovation is that investors can now disagree along both dimensions of returns—i.e., there are periods with strong disagreement about the factor and periods with strong disagreement about stock-specific returns. While there is considerable disagreement in the periods when agents disagree about the stock-specific shocks (and thus individual stock volatility may be high), agents take uncorrelated bets on different stocks. Given the uncorrelated nature of both the shocks and positioning, the aggregate effect of this disagreement at the factor-mimicking portfolio level is muted. In contrast, high disagreement on the factor drives investors to take correlated bets on the systematic component of returns, inducing a large impact on the portfolios that mimic the factors.

The model's key predictions are fourfold: first, higher (lower) factor (stock-specific) disagreement increases the exposure of investors to the factor. In the context of our empirical analysis, this translates into greater flow into securities that are primarily exposed to factor risk (e.g., ETFs). Second, higher (lower) factor (stock-specific) disagreement increases the return volatility of ETFs, as these instruments closely align with the systematic risk factor. Third, this increase in factor volatility causes the correlations between pairs of securities that have large loadings on the factor to increase, reflecting the reduced diversification benefits of holding an ETF. Finally, both higher factor and higher stock-specific disagreement increase volatility at the single stock level. The contrasting relationship between volatility and stock-specific disagreement at the portfolio versus single-stock level is important in differentiating our proposed mechanism from others. These predictions are, additionally, a product of a frictionless economy with time-varying subjective beliefs. The extant literature on ETFs derives predictions mainly from either micro-structure frictions (e.g., heterogeneity in liquidity needs) or limited participation (e,g., the notion that ETFs are pure retail products).

In any theoretical framework, changes in disagreement induce trading between agents due to shifting exposures. This, downstream, coincides with changes in the risk and return characteristics of assets thus linking disagreement to asset prices. Most papers in this area of research study the latter association—that of disagreement impacting asset prices—without providing evidence on the former (see, e.g., Buraschi, Trojani, and Vedolin (2014) and Daniel, Klos, and Rottked (2021)).

ETFs allow us to circumvent this shortcoming in that they provide high-frequency information on both their asset bases and return moments. For example, as many ETFs track specialized indices, and investment "themes" and "styles," market participants are incentivized to maintain the connection between the underlying asset value of the ETF and that of its component securities. From an asset flow perspective, if there is even an infinitesimally higher cost of trading a basket of securities than trading the index itself, than a factor disagreement induced increase (decrease) in aggregate demand for diversified factor exposure should lead to the creation (destruction) of ETFs.

Our first piece of analysis exploits this connection. We find that a one-standard-deviation higher factor versus stock-specific disagreement leads to a 0.09-standard-deviation higher flow into the ETF, confirming our model's theoretical prediction. This positive association is robust to controlling for both ETF and time fixed effects, and a variety of confounding variables that could drive ETF flows, e.g., lagged ETF returns.

Our second piece of analysis exploits the richness of the ETF options market, which today composes more than 40% of all option volume.[1] Most papers linking ETF activity to risk use realized returns for their analyses. ETF option prices allow us to link disagreement to forward and long-dated changes in investor perceptions, rather than noisy realizations, of volatility and correlations. We also document a

[1] See the Wall Street Journal article "Just as Hot as ETFs: Options on ETFs" from December 9, 2019.

significant amount of heterogeneity in disagreement, volatility, and correlations across ETFs, creating an ideal crucible in which to analyze our model's predictions.

We find that risk-neutral or forward-looking ETF volatility is related to the relative amount of factor versus single stock disagreement. A one-standard-deviation increase in factor versus single-stock disagreement leads to volatility rising by about 0.08 standard deviations, even when accounting for ETF and time fixed effects and a battery of ETF-level controls. As noted, the increased factor volatility also implies an increased correlation (i.e., reduction in diversification benefits) between the pairs of securities in the ETF. We show this to also be true in the data; a one-standard-deviation increase in factor versus single-stock disagreement predicts a 0.08 standard deviation higher average correlation between securities in an ETF.

Our final analysis tests the prediction that while changes in factor and stock-specific disagreement carry opposite predictive signs for flow, volatility and correlation at the portfolio (i.e., ETF) level, they both carry positive predict signs for volatility at the single stock level. Intuitively, all that matters to an investor at the single stock level is total risk. As such, whether the increase in disagreement of cash-flows is generated from the factor or stock-specific shock is irrelevant. As predicted, in the data, a one-standard-deviation increase in both factor and stock-specific disagreement increase the risk neutral volatility of the stock by approximately 0.02 standard deviations.

Contributions to the literature. Our paper is related to the literature on general equilibrium models with heterogeneous beliefs such as Harrison and Kreps (1978), Detemple and Murthy (1994), Zapatero (1998), and Basak (2000), among others.[2] This literature has mainly focused on economies with a single stock or Lucas tree.[3] In contrast, we consider an economy featuring multiple Lucas trees with a factor structure to study the implications of disagreement on factor versus stock-specific risk.

Our paper also adds to the literature on the empirical relations between disagreement and asset

[2]Models with disagreement include Shalen (1993), Scheinkman and Xiong (2003), Basak (2005), Berrada (2006), Buraschi and Jiltsov (2006), Jouini and Napp (2007), David (2008), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010), Cvitanic and Malamud (2011), Cvitanic et al. (2012), Bhamra and Uppal (2014), Buraschi, Trojani, and Vedolin (2014), Cujean and Hasler (2017), Ehling et al. (2018), and Atmaz and Basak (2018).

[3]For a few exceptions see Fedyk, Heyerdahl-Larsen, and Walden (2013) who study the survival of agents with biased beliefs in an economy with multiple assets, Buraschi, Trojani, and Vedolin (2014) who consider an economy with two different consumption goods, and Hansen (2015) who study asset prices in an economy with heterogeneous beliefs and preferences with multiple Lucas trees.

prices (e.g., Berkman et al. (2009); Chen, Hong, and Stein (2002); Diether, Malloy, and Scherbina (2002); Park (2005)) and disagreement and trading activity (e.g., Ajinkya, Atiase, and Gift (1991); Bessembinder, Chan, and Seguin (1996); Goetzmann and Massa (2005)). In the spirit of this literature, we construct measures of disagreement using the dispersion of analysts' earnings estimates. However, unlike many of the these studies, we use these analysts' estimates to construct measures of disagreement that are specific to each ETF in our sample. Our ETF-specific measures capture both the extent to which brokers disagree about the common factor underlying an ETF and the extent to which analysts disagree about the idiosyncratic component of an ETF's earnings. These two proxies map to our notions of factor and stock-specific disagreement, respectively.

Another contribution of our study is to elucidate a distinct and novel driver of index and ETF activity — i.e., time-varying heterogeneous beliefs and its relationship to the demand for factor exposure. Ben-David, Franzoni, and Moussawi (2018) analyzes how the level of ETF ownership predicts single security volatility and mispricing, and Da and Shive (2018) analyze how ETF trading activity induces changes in physical correlation. Baltussen, van Bekkum, and Da (2019) provide evidence that the availability of easy to access index products has altered the structure of serial dependence in index returns (see also Agarwal et al., 2018).[4]

This literature argues that shocks transmitted from ETFs to their constituent stocks via an arbitrage channel lead to excess volatility and correlation effects, focusing on the incentives of arbitrageurs to trade on the difference in net asset values. Critically, the shocks to demand for indexation in these frameworks are *non-fundamental* in nature (e.g., Brown, Davies, and Ringgenberg, 2021, for a parsimonious example of the mechanism). In contrast, our paper provides evidence of a *fundamental* origin of trades. ETFs complete the market only if it is cheaper to trade them (due to the services provided by authorized participants) than a basket of individual assets. From this perspective, our work develops an understanding of the upstream drivers of index-oriented demand, elucidating the reasons that arbitrage trading may be happening in the first place.

A concurrent literature has shown that ETFs increase informational efficiency. For example,

[4]Our analysis is also aligned with other work on the effects of indexation (see, e.g. Barberis, Shleifer, and Wurgler, 2005; Bond and Garcia, 2022; Brogaard, Ringgenberg, and Sovich, 2019).

Huang, O'Hara, and Zhong (2021) show that hedge funds strategically deploy a long-short strategy using single stocks versus their related industry ETFs. A higher prevalence of this strategy predicts higher unexpected earnings and lower post-earnings-announcement drift. Antoniou et al. (2022) provide follow-on evidence that higher price-efficiency induced by ETF ownership leads to higher sensitivity of real investment decisions to a firms own stock price. Similar to our work, this strand of the literature suggests a fundamental role for ETF trading activity.

Finally, another important innovation of our paper versus previous work is in the use of ETFs and ETF options to analyze the dynamic effects of *disagreement* on ETF and individual security flows, volatility, and correlations. Option-based measures are uniquely suited to speak to the forward-looking and long-dated implications of this activity. The excess volatility and correlation literature finds primarily short-run and mean-reverting effects, which suggests that mispricing drives the results. In contrast, our results are driven by a fundamental shock that affects risk premia.

1 Model and theoretical predictions

To study how factor and stock-specific disagreement impact investors' decisions to trade either an ETF or individual securities, we consider a pure exchange economy with incomplete information about cash flow dynamics. The investors in the economy have different beliefs about the contribution of factor versus stock-specific shocks to observable dynamics of dividends. For the purpose of simplicity, we assume that there is only one common factor, and we interpret the value-weighted portfolio of all the individual stocks as a passive ETF. Extending the model to multiple factors or "ETFs" would be simple, but numerically intensive to solve. Moreover, to focus on the main economic intuition, we relegate many of the details of the model to the Online Appendix A.2.

1.1 Preferences and cash flows

Preferences. Our setting is a standard continuous time pure exchange economy populated by J investors. We assume that each agent has power utility so that the lifetime expected utility of investor

j is

$$\mathbb{E}^j \left[\int_0^\infty e^{-\rho t} \frac{c_{j,t}^{1-\gamma}}{1-\gamma} dt \right]. \tag{1}$$

Here, ρ represents the time discount factor, γ is the coefficient of relative risk aversion, and the expectation is taken with respect to the *subjective* belief of investor j. Heterogeneity in beliefs, which we describe in section 1.2, is the only type of investor heterogeneity we consider.

Cash flows. The economy features N stocks, with stock n paying a flow of dividends $D_{n,t}$ at time t. We interpret a claim on aggregated dividends, i.e., $D_t = \sum_{n=1}^{N} D_{n,t}$, as representing the ETF in our setting. For example, D_t represents the claim to the total dividends of all NASDAQ stocks at time t, while $D_{n,t}$ represents the claim to the dividends of a single stock in the index. Since NASDAQ dividends are a small fraction of total consumption and our model abstracts from multiple factors, we also assume that there is an additional endowment stream paying E_t at time t. This additional endowment can be interpreted as including labor income and the dividends of stocks that we do not analyze or that are not part of the specific ETF of interest. We include this additional endowment to break the strong link between dividends and aggregate consumption.[5]

Aggregate consumption in the economy is therefore $C_t = E_t + D_t$. The dynamics of the endowment stream, E_t, is

$$dE_t = \mu_E E_t dt + \sigma_E E_t dw_{E,t}. \tag{2}$$

The dividend of stock n at time t is

$$D_{n,t} = D_{n,0} E_t e^{z_t + \epsilon_{n,t}}. \tag{3}$$

Here, $D_{n,0}$ is the initial dividend of stock n at time zero, E_t reflects how the average level of dividends changes over time (e.g., the fact that the level of dividends is often higher in good times than bad), z_t reflects the component of dividend growth that is common across all stocks (i.e., the common factor),

[5] Our model and its predictions are related to the composition of disagreement and its effects on a single ETF. Our empirical analysis utilizes the cross-section of ETFs. We verify that our empirical results are robust to the inclusion of both ETF and time fixed effects.

and $\epsilon_{n,t}$ represents a stock-specific component of dividend growth. This common factor z_t and the stock-specific components are assumed to evolve according to

$$dz_t = \mu_z dt + \sigma_z dw_{z,t} \tag{4}$$

and

$$d\epsilon_{n,t} = \mu_n dt + \sigma_n dw_{n,t}, \tag{5}$$

for $n = 1, \ldots, N$, where $w_{z,t}$ and $w_{n,t}$ are mutually independent standard Brownian motions.

1.2 Disagreement

As noted in equation (1), investors in this economy have heterogeneous beliefs and disagree about the dynamics of the common factor, z_t, and the firm-specific components of dividends, $\epsilon_{n,t}$ for $n \in N$. That is, we assume that while agents can perfectly observe the *realized* values of z_t and $\epsilon_{n,t}$, they *disagree* about the dynamics of these shocks. To focus on disagreement related to the stocks in this economy and not the underlying fundamentals of the economy itself, we do not allow for any disagreement about the dynamics of the additional endowment process E_t.[6]

Factor disagreement. We assume that each agent $j = 1, \ldots, J$ believes that the factor evolves according to

$$dz_t = \mu^j_{z,t} dt + \sigma_z dw^j_{z,t}, \quad \text{where} \quad \mu^j_{z,t} = \mu_z + \sigma_z \Delta^j_z s_t. \tag{6}$$

Here, Δ^j_z is a scalar that reflects whether agent j is optimistic ($\Delta^j_z > 0$) or pessimistic ($\Delta^j_z < 0$) about the factor. Moreover, and as we elaborate below, $s_t \in (0,1)$ is a time-varying process that captures the fraction of total disagreement that is attributed to the factor at time t.

[6]To focus on the role of disagreement related to the securities market, we do not allow for any correlation between E_t and the dividends beyond the explicit dependence of the dividends on E_t. Introducing an additional correlation would not change our theoretical predictions.

Since the investors observe z_t we must have

$$\underbrace{\mu^j_{z,t}dt + \sigma_z dw^j_{z,t}}_{\text{Agent } j\text{'s perception}} = \underbrace{\mu_z dt + \sigma_z dw_{z,t}}_{\text{Realization}}, \tag{7}$$

In other words, Equation (7) implies that each agent j attributes the observed variation in the factor to either the mean ($\mu^j_{z,t}dt$) or innovation ($\sigma_z dw^j_{z,t}$) components. This decomposition is determined by the degree to which the agent is either optimistic or pessimistic about the factor. Specifically, we can use the definition of $\mu^j_{z,t}$ from Equation (7) to relate the shock perceived by investor j to the true shock under the objective measure as

$$dw^j_{z,t} = dw_{z,t} - \Delta^j_z s_t dt. \tag{8}$$

If investor j is *optimistic* about the factor z_t, such that $\Delta^j_z > 0$, then the investor is likely to perceive a positive change in the factor as representing an increase in the expected component (since $\mu^j_{z,t} > \mu_z$) and a small (or even negative) innovation (since $dw^j_{z,t} < dw_{z,t}$). The converse holds true if the investor is *pessimistic* about the factor, i.e., when $\Delta^j_z < 0$.

Stock-specific disagreement. We also assume that agent j believes that the dynamics of the idiosyncratic component of stock n's dividends are

$$d\epsilon_{n,t} = \mu^j_{n,t}dt + \sigma_n dw^j_{n,t}, \quad \text{where} \quad \mu^j_{n,t} = \mu_n + \sigma_n \Delta^j_n (1 - s_t). \tag{9}$$

Here, investor j is optimistic (pessimistic) about the dynamics of stock n if $\Delta^j_n > 0$ ($\Delta^j_n < 0$). Using the fact that all agents can observe $\epsilon_{n,t}$ and following the logic underlying Equation (8) allows us to express the degree to which agent j's perception of the idiosyncratic shock to stock n differs from the objective realization as

$$dw^j_{n,t} = dw_{n,t} - \Delta^j_n(1 - s_t)dt. \tag{10}$$

Intuitively, this type of disagreement allows agents to have different beliefs about the *relative* performance of the individual constituents of an index or ETF.

Composition of disagreement. In the above, s_t is a process that governs the composition of the disagreement in the economy. Specifically, we assume that

$$s_t = \frac{1}{1+e^{-\delta_t}}, \quad \text{where } \; d\delta_t = \kappa_\delta \left(\bar{\delta} - \delta_t\right) dt + \sigma_\delta dw_{\delta,t}, \tag{11}$$

and $w_{\delta,t}$ is a standard Brownian motion that is independent of all other shocks. Since s_t is bounded between zero and one, it can be interpreted as the amount of systematic versus idiosyncratic disagreement in the economy. As s_t approaches one, there is less disagreement about idiosyncratic processes $\epsilon_{n,t}$ and more disagreement about the factor process z_t.

To focus on the mix between factor and stock-specific disagreement, we have assumed a simple structural form for the beliefs of the investors in the economy. First, we keep the total disagreement constant. One could include an additional factor that changes the total level of disagreement in addition to the composition of the disagreement. Second, we assume that the composition effect is driven by an independent source of uncertainty. One could potentially generalize this by allowing the composition of disagreement to be correlated with the fundamental shocks in the economy.[7] One way to endogenize the belief structure is by assuming that agents learn or follow a specific updating rule. The outcome would in most cases lead to total disagreement varying over time and a composition effect, s_t, that correlates with the fundamental shocks to the dividends.

1.3 Model Intuition

To build intuition about the building blocks of the model and their link to our empirical results, in this section we present a concrete example of the mechanism. Our starting point is an economy with a single stock and two agents (a pessimist and optimist). To illustrate and without loss of generality, we assume that the stock is expected to earn 6% per period under the *objective* probability measure. Investor O is an optimist who subjectively believes that the stock will yield 8% per period, whereas

[7] An untabulated extension of the model shows that separating the factor and idiosyncratic aspects of disagreement and correlating disagreement with fundamentals shocks lead to similar results.

Investor P is a pessimist who subjectively believes that the stock will only yield 4% per period, i.e., the two agents are symmetric in their disagreement about the mean return. Both agents have equal wealth at time 0, but due to their disagreement, Investor O allocates a larger proportion of their wealth to the stock than Investor P.

If the stock returns a positive 10% in the next period, then the stock's price will increase for two reasons. First, the positive shock will increase the amount of dividends the firm pays, thereby increasing the firm's price. Second, and more importantly, the fact that the investors disagree about the value of the firm will amplify the price increase. The positive shock will cause Investor O to become wealthier than investor P due to their higher initial position in the stock. Thus, from a wealth-weighted perspective, the stock will become priced closer to Investor O's 8% expected return than Investor P's 4% expected return. The reverse, however, is also true. If the stock were to fall by 10%, then the stock's price would fall to reflect the (relatively wealthier) pessimist's view more than the (relatively poorer) optimist's view. This amplification effect on the stock's price from disagreement (i.e., higher highs after positive shocks; lower lows after negative) will lead to higher stock return volatility.

For this example, we will refer to the situation described above as "scenario one," and sequentially incorporate the additional features of our model starting with (i) $N \to \infty$ stocks, (ii) then the common factor z_t, and finally (iii) the composition of disagreement s_t.

(i) $N \to \infty$ stocks and agents. Suppose the economy is now populated by the same two investors but features $N \to \infty$ stocks with idiosyncratically varying dividends. If the investors' beliefs are randomly distributed across the stocks (i.e., each investor is optimistic about one subset of stocks, pessimistic about another subset, and neutral in regards to a third subset), then each agent's wealth is distributed across many stocks, and each agent is asymmetrically exposed to many of these stocks' orthogonal shocks. As a result, their exposure to any single shock will be extremely small. Thus, if the same 10% shock hits one stock in the economy, this will have a muted effect on an agent's wealth. This is because agents have economically small exposures to each stock, and any positive shock to one stock is likely to be offset by a negative shock to another. This type of dispersed stock-specific disagreement effectively decreases the amplification effect described in the previous example with one stock.

If this economy were to also feature multiple agents, then we would obtain the same conclusion as the two-agent case above: each individual agent's wealth would be exposed to a large number of uncorrelated shocks, and any amplification effect of disagreement on return volatility would be low by the virtue of the agents' diversified holdings across the large number of securities. We refer to this situation as "scenario two."

(ii) Common factor z_t. The second feature we add is a common factor that drives part of the return variation across all individual stocks. When stock returns depend on a common factor, and agents are either optimistic or pessimistic only about the factor's prospects, then there are, in reality, far fewer dimensions along which agents disagree for this subset of stocks. This is because, in contrast to scenario two above, speculating on the common factor may have large wealth effects, since factor innovations induce groups of stocks to move in unison. Thus, as disagreement about the common factor increases, the economy is closer to the single-stock case described under scenario one above. The "single stock," however, is now a well-diversified portfolio of N stocks and moves in response to fluctuations in the factor (i.e., z_t). Thus, inter-stock correlations also rise when agents wish to speculate on factor exposures.

(iii) Composition of disagreement s_t. The final, and perhaps most novel, feature of our economy is that the share of disagreement can vary continuously between the two scenarios described above. This quantity, s_t, not only allocates the fixed amount of disagreement across agents between the idiosyncratic and factor components of returns but also elicits trading activity in our model.

When s_t is close to zero, then agents primarily disagree about stocks' idiosyncratic shocks, and the economy is like that described under scenario two. In contrast, when s_t is close to one, then disagreement across agents is occurring largely along the factor dimension and agents act in almost unison as either optimists or pessimists in regard to the factor. This mimics the single-stock economy represented by scenario one, where disagreement amplifies volatility.

The key is that to transition between the two extremes as s_t moves, the agents must rebalance or trade portions of their portfolios. For example, when s_t is near one, agents want to hold the same well-diversified portfolio (diversifying away idiosyncratic risks), but to varying degrees depending on their level of optimism or pessimism. However, as s_t transitions to zero, then agents desire very disparate positions in individual stocks, reflecting their randomly distributed disagreement on idiosyncratic portions of returns.

1.4 Asset-pricing moments

We are primarily interested in how disagreement s_t affects (i) the return dynamics of the ETF and (ii) the return correlations between pairs of individual stocks in the ETF. To analyze how disagreement affects these asset-pricing moments, we first need to define the relevant state variables. With these in hand, we can write the returns associated with each individual security and the ETF, which we interpret as a "passive" value-weighted index of the N individual stocks. Finally, given these asset returns, we can compute the volatilities and correlations of interest.

To start, the state variables in the economy are (i) the N dividend shares, (ii) the $J-1$ consumption shares, and (iii) the fraction of factor disagreement, s_t. We collectively define state variables (i) and (ii) as X_t. Since our focus is on the share of factor disagreement, we will generally examine how key equilibrium quantities, such as asset-pricing moments, evolve as a function of s_t. Next, we assume that the equilibrium price of stock n represents the claim to the stream of the firm's dividends. This allows us to express the price of stock n at time t as

$$P_{n,t} = P_n\left(X_t, s_t\right) = \mathbb{E}_t\left[\int_t^{\infty} \frac{M_u}{M_t} D_{n,u} du\right], \tag{12}$$

where M_t is the equilibrium stochastic discount factor under the objective belief (i.e., the probability measure of the true data generating process). An application of Ito's lemma to Equation (12) yields the return process for each stock.

$$dR_{n,t} = \frac{dP_{n,t} + D_{n,t}dt}{P_{n,t}} = \mu_{R_n,t}dt + \sigma'_{R_n,t}dw_t, \tag{13}$$

Here, $w_t = (w_{E,t}, w_{\delta,t}, w_{z,t}, w_{1,t}, \ldots, w_{N,t}) \in \mathbb{R}^{N+3}$ is a vector of all of the Brownian shocks in the economy. Hence, the equilibrium loading of stock n onto each of the $N+3$ shocks in the economy is given by the vector of diffusion coefficients $\sigma_{R_n,t} = \sigma_{R_n}(X_t, s_t) \in \mathbb{R}^{N+3}$. We collect these loadings in the matrix Σ, the n^{th} column of which is $\sigma_{R_n,t}$. Finally, we define $\omega_p \in \mathbb{R}^N$, where $\sum_{n=1}^{N} \omega_{p,n} = 1$ are the portfolio weights of an arbitrary portfolio p. In the special case that $\omega_{p,n} = P_{n,t} / \sum_{k=1}^{N} P_{k,t}$ for all n stocks, then ω_p corresponds to the ETF. This is because we define the ETF to represent the value-weighted portfolio of the N individual securities.

With this notation in hand, the instantaneous standard deviation of portfolio p is then

$$std\,(dR_{p,t}) = \sqrt{\omega_p' \Sigma' \Sigma \omega_p}. \tag{14}$$

Similarly, the instantaneous correlation of the returns of portfolios p and q is

$$corr(dR_{p,t}, dR_{q,t}) = \frac{\omega_p' \Sigma' \Sigma \omega_q}{std\,(dR_{p,t})\, std\,(dR_{q,t})}. \tag{15}$$

1.5 Factor exposure

Another goal is to understand how factor versus individual-stock disagreement impacts the trading of the investors in the economy. In a frictionless model such as ours, there are two key features that complicates the comparison to the data, and therefore require additional assumptions. First, the ETF is a redundant security in our setting.[8] Hence, there is no intrinsic demand for the ETF in the model since investors could, in principle, trade the individual stocks underlying the ETF. Therefore, we assume that agents prefer to trade the ETF instead of the underlying stocks if the agents' goal is to take on factor exposure. The economic intuition underlying this argument is that in a model with even a small transaction cost for trading individual stocks, the investors would prefer buying the relatively cheap-to-trade ETF over the individual stocks to gain exposure to the common factor shock.

Second, trading volume is difficult to define and generally depends on the asset structure. For instance, if investors can trade claims that replicate their optimal consumption path, then trading

[8]The ETF is a value weighted portfolio of the individual stocks, and hence it can be replicated.

volume is trivially zero. In contrast, if investors can only trade individual stocks, then trading volume will typically be non-zero. Instead of focusing on trading *volume*, we therefore focus on the total *exposure* of each agent to the factor. It is natural to assume that changes in an agent's exposure to the factor arise from the agent trading securities related to the factor. Thus, if we define the equilibrium wealth of investor j as $W_{j,t} = W_j(X_t, s_t)$, then Ito's lemma lets us express the investor's wealth dynamics as

$$dW_{j,t} = \ldots dt + \sigma'_{W_j,t} dw_t. \tag{16}$$

From equation (16), the exposure of agent j to each of the underlying shocks in the economy can be represented by the diffusion coefficients, $\sigma_{W_j,t} \in \mathbb{R}^{N+3}$. These exposures can either be positive or negative. Since we are interested in the total amount of exposure (either long or short) to the factor, we define the total amount of factor exposure in this economy as

$$TE_{ETF,t} = \sum_{j=1}^{J} |\sigma_{W_j,z,t}|, \tag{17}$$

where $\sigma_{W_j,z,t}$ is the loading on the shock to the factor. While we employ this equation as our primary measure of exposure to the factor shock, Section A.4 in the Online Appendix shows that the key model predictions defined below are robust to alternative measures of exposure and trading activity also.

1.6 Model predictions for volatility, correlation, and factor exposure

We obtain testable predictions from the model by conducting Monte Carlo simulations aimed at capturing the equilibrium relations between the degree of factor disagreement (s_t) and key quantities such as (i) total factor exposure, (ii) the risk of the ETF, and (iii) the average correlation (i.e., the diversification benefits) within the ETF.

Simulation details. Our main simulation of the model considers an economy in which there are $N = 10$ individual stocks and $J = 2N = 20$ agents. Our baseline analysis considers a "symmetric" economy in which each stock is followed by an equal number of pessimists and optimists. Moreover, we assume that N agents are optimistic (pessimistic) about the factor. While not necessary, this assumption ensures that the model's predictions are not simply driven by an ex ante imbalance between the proportions of optimists and pessimists in the economy. The key feature we need in regard to beliefs is that there is sufficient "dispersion" in the beliefs about the individual stocks. This ensures that stock-specific disagreement cannot be distilled into disagreement between two blocks of investors, thereby mimicking factor disagreement.

Overall, as half of the agents in the economy are optimistic about the factor and half are pessimistic about the factor, we refer to high s_t times as periods of high *factor disagreement*. Similarly, when s_t is low, most disagreement surrounds the idiosyncratic component of dividends. As such, we label these times as periods of high *stock-specific disagreement*. Moreover, we assume that each agent starts with the same initial consumption shares, and the initial dividend shares of the stocks are the same.

Simulation results. Figure 2 shows the exposures of the agents in the economy to the stock-specific and factor shocks (as defined in equation (17)). The left (right) plot shows the exposures to the shocks when $s_t = 0.05$ ($s_t = 0.95$) and represents the exposures in a state of low (high) factor disagreement. As the economy transitions to a state of higher factor exposure, the share of factor disagreement increases. In the middle plot, we show the factor exposure as a continuous function of the share of factor disagreement (s_t). The figure shows that as the economy moves from low to high factor disagreement, the positions of the agents become more concentrated on the factor shock and less concentrated on the individual shocks (stocks). Put differently, agents take large speculative bets on the aggregate stock portfolio (ETF) instead of the individual stocks as we have more factor disagreement.

In Figure 2, we plot the standard deviation of the ETF and the average stock market correlation of the individual stocks in the ETF as we move from stock-specific (low s_t) to factor disagreement (high s_t). As one can see, both the volatility of the ETF and the average correlation increases in the share of factor disagreement. The reason for this is that, as we move from stock-specific to factor disagreement, the economy looks more and more like an economy with bets between two large groups of investors who have opposing views on the factor. Such "correlated" bets have larger impacts on the aggregate economy than stock-specific disagreement, as bets on individual stocks tend to diversify in the cross-section.[9]

Based on the figures and the intuition developed in Section 1.3 we have the following three model predictions:

Testable Predictions. *Increased (decreased) factor (stock-specific) disagreement drives*

(a) Larger flows into the ETF (i.e., more common factor exposure);

(b) Higher ETF-level return volatility;

(c) And higher average return correlations between stocks in the ETF.

In contrast, both increased factor and stock-specific disagreement drive

(d) Higher single-stock volatility at the single-stock level.

2 Empirical evidence

This section describes the data and empirical measures used to evaluate the four predictions of the model outlined in Section 1. Section 2.1 motivates the economic connection between ETFs and our theory. Section 2.2 provides an overview of the set of ETFs we use to test the relations among disagreement, fund flows, and risk-neutral volatility and correlation. Section 2.3 describes our empirical measures of factor and stock-specific disagreement. Sections 2.4 through 2.8 then use these measures to test our predictions.

2.1 Institutional details

The previous section introduced the key mechanism underlying our model: agents have differing subjective beliefs on the stock-specific and factor (common) component of returns. If, for example,

[9] One way to look at this is to consider a limiting version of our economy as the number of assets approaches infinity ($N \to \infty$). If agents are either optimists or pessimists for each individual stock, i.e., the perturbation of the belief is $+/-\Delta(1-s_t)$ on each individual stock with equal probability, then the consumption shares only depend on factor disagreement.

investors have differing beliefs about individual stocks, they will hold vastly disparate positions in individual firms, e.g., Apple, Microsoft, and Tesla. In contrast, if the agents have a strong desire to speculate on the technology sector, they will purchase or short a portfolio of technology stocks (e.g., the Nasdaq 100), acting as if there are far fewer agents in the economy.

The reality is that the transition from a state in which agents are only trading the individual technology stocks to one in which they are trading the Nasdaq 100 index can be executed in two ways. The first approach is that captured by our model, i.e., trading individual firms such that agents eventually hold a value-weighted portfolio of all the stocks underlying the Nasdaq 100 index. The second approach is to purchase derivative security such as a Nasdaq future or Nasdaq ETF (e.g., QQQ). The mere existence of this alternative reflects two facts: first, investors desire this kind of exposure and, second, investors are subject to frictions that these alternatives products mitigate (see, e.g., Ross (2015, 1976) on the issue of non-redundancy).

Most papers in the ETF literature assume that the primary friction is a participation cost for uninformed traders (see, e.g., Bond and Garcia, 2022). This ignores evidence, however, that informed traders also use ETFs (and, of course, futures) in their trading activity (see, e.g., Huang, O'Hara, and Zhong, 2021). Based on this evidence we propose an alternative friction that affects both retail and sophisticated traders alike: the cost of purchasing or shorting a basket of many securities versus purchasing or shorting a single security (ETF or future). Take, for example, a large fundamental hedge fund that would like to purchase $100m of the Nasdaq 100. Given that their value add to investors comes from understanding stocks' fundamental values, not from superior trade execution, trading via an index product (e.g., an ETF) would be more efficient than routing multiple stock orders through a relatively expensive program trading platform, such as one run by Goldman Sachs.

From this perspective, the creation and redemption activity of ETFs is a natural measure of changing factor demand that we link to changes in disagreement. We focus on ETF trading activity as our main measure of the demand for the underlying factors, since ETFs cover a wide variety of investment themes, styles, and factors, and span a relatively long time period. We also test our main hypothesis on the relationship between disagreement and futures open interest, but, importantly, note that the cross-section of futures contracts is relatively small versus ETFs.[10] This analysis lends additional credence to our proposed mechanism.

Finally, a natural question is why we are focused on ETFs and not passive, indexed mutual funds. First, and perhaps most importantly, mutual funds do not have timely pricing data, specifically on options. This inhibits our ability to test the predicted relations between disagreement, and forward-looking measures of volatility and correlations. Second, mutual fund holdings data are reported quarterly. In contrast, data on ETF constituents, ETF flows, and ETF-options prices are available at a much higher frequency, i.e., daily, which we exploit in our analysis.

2.2 Data and summary statistics

Our sample begins in January 2012, which is the first month in which ETF Global began providing granular and comprehensive data on ETF flows and constituents, and ends in December 2022. As the top panel of Figure 3 shows, ETFs are a relatively nascent security that only began trading in 1993 with the introduction of the SPDR S&P 500 Trust ETF (NYSE: SPY). While ETF trading volumes represented less than 5% of total dollar trading volume in the 1990s, the ETF market has come to represent approximately 25% to 30% of total dollar trading volume since 2010. This rapid increase in popularity reflects, in large part, the fact that ETFs provide investors with relatively cheap access to a wide variety of investment factors and styles. Beyond the fact that granular data on ETF holdings are only available beginning in 2012, there is an additional benefit of starting our sample period at this point in time: our theoretical analyses assumes that the dynamics of disagreement, and consequently flows into and out of an ETF, are stationary. It would appear that ETF trading activity has achieved a stable equilibrium in the time period underlying our analyses.

With these benefits of ETFs in mind, our analysis focuses on a small set of highly liquid US-focused equity ETFs for which options on both the ETF and its constituent stocks are actively traded.

[10] Futures are standardized products that have well known and transparent no-arbitrage relationships to their underlying cash securities. Program trading operations at investment banks have for decades imposed this future-cash relationship through their trading operations. ETF issuers have similarly clear rules about how to maintain the ETF versus underlying asset relationship. It should thus come as no surprise that it is these same investment banks that are usually those authorized to execute these rules, i.e., as authorized participants. See Evans et al. (2022) for a more detailed explanation of the role arbitrage and market makers play in maintaining this relationship.

This focus allows us to elicit accurate measures of the forward-looking volatility and correlation risk associated with the investment factor, theme, or style that the ETF tracks. The 13 ETFs in our sample are SPY (SPDR S&P 500 ETF Trust), DIA (SPDR Dow Jones Industrial Average ETF Trust), QQQ (Invesco QQQ Trust Series 1), XLK (Technology Select Sector SPDR Fund), XLB (Materials Select Sector SPDR), XLE (Energy Select Sector SPDR), XLI (Industrial Select Sector SPDR), XLP (Consumer Staples Select Sector SPDR), XLV (Health Care Select Sector SPDR), XLY (Consumer Discretionary Select Sector SPDR), XOP (SPDR S&P Oil & Gas Exploration & Production ETF), XBI (SPDR S&P Biotech ETF), and IBB (iShares Biotechnology ETF).

Although these 13 ETFs represent only a small number of the approximately 2,200 distinct ETFs that are now trading in U.S. markets, the bottom panel of Figure 3 demonstrates that these ETFs represent just under half of all dollar trading volume in US ETFs in the recent decade. Moreover, these ETFs represent a variety of investment styles. Three ETFs track broad market indices, while 10 track many of the various sectors underlying the U.S. economy. Thus, our sample represents an economically sizable portion of the US equity market.[11]

Table 1 reports several summary statistics related to the ETFs that comprise our sample. For instance, the table shows that the largest ETF in our sample is SPY, which has a net asset value (NAV) of $227.23b. The smallest ETF in our sample is XOP, the Oil & Gas Exploration & Production ETF, with a net asset value of $44.90b. Beyond showing relatively large differences in NAVs across ETFs, the table also shows large differences in the market capitalizations of the equities underlying these ETFs. For instance, the biotech (healthcare) firms underlying XBI (XLV) have a combined market value of $710.31b ($2795.04b).

ETF trading activity. We measure the trading activity associated with each ETF in one of

[11]While several other economically large ETFs exist, they are excluded from our sample because they feature very little option-trading activity at the index level. For instance, both VOO (Vanguard S&P 500 ETF) and IVV (iShares Core S&P 500 ETF) are two ETFs that have net asset values in excess of $200b, but typically have fewer than 1,000 options linked to either VOO or IVV traded each day. In contrast, hundreds of thousands of options linked to SPY are traded each day. Due to these differences in options-trading volume, we feel comfortable with our estimate risk-neutral moments for SPY, but are we are not for VOO and IVV.

two ways. First, we define the net dollar flows into an ETF m in month t as

$$\text{NDFlow}_{m,t} = \sum_{\tau=1}^{T_t} \text{NetFlow}_{m,t,\tau}. \tag{18}$$

Here, $\text{NDFlow}_{m,t,\tau}$ represents the net flow into ETF m on trading day τ of month t (expressed in dollars and from ETF Global), and T_t captures the total number of trading days in month t. This measure clearly has a positive time-series trend as market value of stocks tend to increase over time. To mitigate this trend all our regression specifications include the lagged dependent variable and time fixed effects.

Second, we complement this measure of flow with changes in shares outstanding in ETF m in month t:

$$\text{SFlow}_{m,t} = \sum_{\tau=1}^{T_t} \left(\text{ShrOut}_{m,t,\tau} - \text{ShrOut}_{m,t,\tau-1}\right). \tag{19}$$

Here, $\text{SFlow}_{m,t,\tau}$ represents the shares outstanding associated with associated with ETF m on trading day τ of month t. Beyond robustness, this specification mitigates the concern that our findings are being driven by non-stationary trends in prices. The economic intuition underlying equation (19) is that more shares of the index will be created when market participants have a greater demand for exposure to the common factor provided by the ETF rather than the idiosyncratic exposures of its constituent stocks.

Table 1 reports the average value of the absolute net flows into each ETF over the average month of the sample period, as well as the average dollar-trading volume in each ETF and its constituent stocks. The table shows that approximately \$30b flows into and out of SPY each month, and around \$1b moves into and out of XBI and XLB, the biotech and materials ETFs. While the nominal value of these latter flows is an order of magnitude smaller than the flow for SPY, they represent similar magnitudes relative to the aggregate values of the underlying equities held by each of these three ETFs. Similarly, the table shows that there is, on average, 10 to 20 times as much dollar-trading volume in individual stocks relative to an ETF.

Other summary statistics. Beyond the summary statistics outlined above, Table 1 also reports the average number of analysts following the average firm underlying each ETF, or $\mathbb{E}$ [Analysts]. The table shows that the ETFs are well balanced in terms of their analyst coverage, as most underlying stocks are followed by an average of 15 analysts. This fact is useful, as Section 2.3 uses data related to analyst forecasts to construct measures of ETF-level disagreement, which maps to the notion of disagreement in the model. Unreported summary statistics also indicate that almost all the individual stocks underlying these ETFs are optioned – a fact that we exploit later in this section when we use the options market to estimate forward-looking measures of volatility and correlation risk for each ETF in the sample.

2.3 Measuring systematic and idiosyncratic disagreement

Our empirical analysis requires measures of factor and stock-specific disagreement for each ETF. This allows us to empirically determine the extent to which s_t in equation (11) is closer to zero (more stock-specific disagreement) or one (more factor disagreement). Following Buraschi, Trojani, and Vedolin (2014), we estimate these measures using IBES data. IBES captures quarterly earnings estimates from *informed* Wall Street analysts; measures generated from these data are therefore particularly useful in highlighting how our results are driven by broader fundamentals (i.e., heterogenous beliefs) rather than dynamics from largely *uninformed* retail beliefs or flows.

For the stock-specific disagreement measure, we first calculate the mean absolute value of next period's earnings estimates (Est) for each stock across all analysts. These estimates are subject to the standard filters applied to the unadjusted IBES data file. For instance, we remove all analyst revisions reported after a firm's earnings announcement date and choose each analyst's most recent estimate within a month. Finally, we remove stale information by deleting forecasts that are outstanding for more than 105 days. The measure of stock-specific disagreement surrounding the stocks underlying ETF m at time t, denoted by $\text{StockDisagree}_{m,t}$, is then the equal-weighted sum of the individual security disagreement measures across all stocks in the ETF, or

$$\text{StockDisagree}_{m,t} = \frac{1}{J} \sum_{j=1}^{J} \left[\frac{\frac{1}{A} \sum_{a=1}^{A} |\text{Est}_{a,j,t} - \overline{\text{Est}}_{j,t}|}{\text{P}_{j,t-1}} \right]. \tag{20}$$

Here, $\text{Est}_{a,j,t}$ is the quarterly earnings estimate of analyst a for stock j at time t, $\overline{\text{Est}}_{j,t}$ is the average earnings estimate across all analysts for a given security at time t, and $\text{P}_{j,t-1}$ is the security's price at previous month end. $\text{P}_{j,t-1}$ normalizes the earnings dispersion to make the single-stock measure comparable across securities. We use an equal rather than market capitalization weighted measure to minimize the possibility of granularity in our stock-specific measure confounding the factor disagreement measure we define next (see, e.g., Gabaix, 2011).

The measure of factor disagreement for ETF m at time t, denoted by $\text{FactorDisagree}_{m,t}$, is constructed by first summing the forecasts of all component securities of an index across all analysts employed by a given broker. This "bottom up" approach mimics the methodology used by macroeconomic groups at brokerage firms when estimating earnings for the S&P 500 and other indexes (e.g., Darrough and Russell (2002)). We then compute the disagreement across brokers rather than analysts in the sample,

$$\text{FactorDisagree}_{m,t} = \frac{\frac{1}{B}\sum_{b=1}^{B} |\text{Est}_{b,m,t} - \overline{\text{Est}}_{m,t}|}{\text{P}_{m,t-1}}, \tag{21}$$

where B is the total number of brokers in the sample, $P_{m,t-1}$ is the price per share of ETF m at time $t-1$, $\text{Est}_{b,m,t}$ is the earnings estimate of broker b for ETF m, and $\overline{\text{Est}}_{m,t}$ is the equal-weighted average earnings estimate for ETF m across all brokers. This measure captures the extent to which brokerage firms disagree about the valuation of an index rather than the degree to which analysts disagree about the valuation of an individual constituent of the index. We elicit $\text{Est}_{b,m,t}$ directly from the analyst estimates of the constituent stocks,

$$\text{Est}_{b,m,t} = \sum_{j=1}^{J} \frac{w_{j,m,t} \cdot \text{P}_{m,t}}{\text{P}_{j,t}} \cdot \text{Est}_{b,j,t}, \tag{22}$$

where $w_{j,m,t}$ is the weights supplied on a daily basis by ETF Global, and captures the relative market capitalization of security j in ETF m. Equation (22) represents the earnings per share estimate of broker b for an ETF m.

In constructing this measure of factor disagreement, we find that individuals brokers do not necessarily cover all securities in a given ETF. For instance, while almost all brokers cover stocks with large market capitalizations, only some brokers cover stocks with smaller market capitalizations. The dropoff in coverage is not linear and falls precipitously from over 80% of market capitalization for the top 15 brokers to less than 30% for those outside of the top 15. We thus apply two filters to the data. First, we restrict our attention to the disagreement among the top 15 brokers in the sample. Second, if a given broker does not cover a particular stock at time t, we assume that the broker's estimate for the earnings-per-share of that stock is the consensus estimate. Since smaller stocks are less covered, but also have smaller weights in an index, this imputation has a minimal effect on our results, and, if anything, biases our findings towards insignificance.

To illustrate the relative importance of factor disagreement for a given ETF m in a given month t, we define the share of factor disagreement $s_{m,t}$ as

$$s_{m,t} = \frac{\text{FactorDisagree}_{m,t}}{\text{FactorDisagree}_{m,t} + \text{StockDisagree}_{m,t}}. \tag{23}$$

When this ratio $s_{m,t}$ approaches one, then the majority of the disagreement regarding the prospects of an ETF is related to the prospects of the underlying factor that the ETF tracks. In contrast, when $s_{m,t}$ approaches zero, then the majority of the disagreement surrounding an ETF is driven by the (idiosyncratic) prospects of the stocks that constitute the ETF. We plot the time-series dynamics of $s_{m,t}$ for SPY in Figure 4. While $s_{m,t}$ provides us with a plausible proxy for s_t from equation (11)—i.e., the main state variable governing the amount of disagreement about a stock's idiosyncratic shocks (when $s_t \to 0$) versus common factor (when $s_t \to 1$)—the measure is (i) highly persistent, complicating statistical inference, and (ii) confounded by time-varying volatility. For this reason, our primary empirical specification tests if changes in stock-specific and factor disagreement, separately, predict changes in our dependent variables of interest with different signs in the case of the ETF dynamics (i.e., predictions (a), (b), and (c) in Section 1.6) or with the same signs in the case of the single-stock dynamics (i.e., prediction (d) in Section 1.6).

Figure A.1.1 in the Online Appendix plots the values of stock-specific (or idiosyncratic) and factor disagreement for SPY, defined following equations (20) and (21). Most disagreement related to SPY is driven by the idiosyncratic prospects of the constituent stocks of that ETF. However, factor disagreement exceeds stock-specific disagreement in the period surrounding the 2013 debt-ceiling crisis, before the 2016 presidential election, and upon the onset of the economic effects of the COVID-19 pandemic that began in early 2020.[12]

2.4 Disagreement and flows

This section shows that increases (decreases) in disagreement about the common (stock-specific) factor underlying each ETF predict an increase in ETF inflows. That is, when investors face relatively more disagreement about the common component of an investment style rather than stock-specific disagreement about the constituent stocks, then they would rather trade the portfolio than trade the constituent stocks. This supports the first key prediction of the model in Section 1.6 that posits that higher levels of factor disagreement (i.e., higher values of s_t) predict an increased demand for factor exposure, as shown in Figure 1. Empirically this demand will manifest itself in both greater assets dedicated to an ETF, implying flows into (i.e., the creation of) the ETF, and greater ETF versus underlying trading activity.

We thus examine the relation between changes in factor versus stock-specific disagreement and the relative amount of trading activity in the ETF by estimating the following panel regression:

$$\begin{aligned}\text{TActivity}_{m,t} = &\alpha_m + \delta_t + \beta_{\text{Sys}} \cdot \Delta\text{Sys.Disagree}_{m,t-1} + \\ &\beta_{\text{Idio}} \cdot \Delta\text{Idio.Disagree}_{m,t-1} + \boldsymbol{\beta}\boldsymbol{X}_{m,t-1}^{T} + \varepsilon_{m,t}.\end{aligned} \tag{24}$$

Here, $\text{TActivity}_{m,t}$ represents either the net dollar flow into (see Equation (18)) or the net new shares created in ETF m at month t, $\Delta\text{Sys.Disagree}_{m,t-1}$ ($\Delta\text{Idio.Disagree}_{m,t-1}$) represents the change from month $t-2$ to $t-1$ in factor (stock-specific) disagreement, while $\boldsymbol{X}_{m,t-1}$ represents a vector of control variables, and includes the lagged flow variable, realized one-month volatility, one-month ETF returns,

[12] We provide summary statistics for this relative disagreement measure in Table A.1.1 of the Online Appendix, and report the correlation between these measures of relative disagreement across each pair of ETFs in Table A.1.2 of the Online Appendix.

and absolute returns. Lagged returns are an important control because flows in month t may simply arise from investors chasing high returns in month $t-1$ (Dannhauser and Pontiff, 2019). Similarly, this vector includes the average bid-ask spread of the ETF, as investors are less likely to invest in ETFs with larger transactions costs. As noted above, we test whether $\beta_{\text{Sys}} - \beta_{\text{Idio}} >> 0$.

To be consistent with our theory, we also include ETF and time fixed effects, denoted by δ_t and α_m, respectively. Time fixed effects absorb common shocks that affect all ETFs simultaneously (e.g., the effect of the Tax Cuts and Jobs Act of 2017 that triggered an inflow of funds into the equity market), while the ETF fixed effects absorb fixed differences in the level of flows and disagreement across ETFs, e.g., the fact that flows are unconditionally higher for SPY compared to XBI, the biotechnology sector ETF. We use robust estimated standard errors, all regressions are estimated at the monthly frequency (see, e.g., Buraschi, Trojani, and Vedolin (2014)), and both our independent and dependent variables are standardized. Finally, while our model is in continuous time — i.e., all relationships between variables of interest are instantaneous — our empirical setting is in discrete time. We therefore run predictive regressions that quantify what a shift in disagreement today tells us about factor demand tomorrow.

Table 2 reports the results of these panel regressions. Focusing on net flows in Panel A, column (1) shows that with only time fixed effects and no controls, relatively higher amounts of factor disagreement are associated with increases in trade flows (i.e., the creation of ETF units). A one-standard-deviation higher amount of factor disagreement leads to an 0.09-standard-deviation higher flow into the ETF the following month. Moreover, column (2) shows that the same result holds true if we control for ETF fixed effects. Finally, column (3) shows that the addition of lagged flows, realized volatility, past returns, and trading costs does not alter this result. Of particular importance is the inclusion of past returns, which is significant, in keeping with the findings of Dannhauser and Pontiff, but of smaller magnitude than our variables of interest. Neither the volatility-related controls (realized volatility and absolute returns) and bid-ask spread have little predictive power on future flows into the ETF.

Panel B (columns (4) through (6)) repeats the previous specifications, but now using monthly changes in shares outstanding as the dependent variable, and yield a similar results as Panel A (columns

(1) through (3)). Notably, the results show a strong positive association between the amount of factor versus stock-specific disagreement and the degree to which investors demand new shares in the ETF versus its underlying securities in month t. Columns (4) and (5) show that a one-standard-deviation increase in factor disagreement is associated with an approximately 0.07-standard-deviation increase in the amount of trading in the ETF relative to the underlying stocks. As column (6) shows, including all the controls from column (3), does little to decrease the impact of factor and stock-specific disagreement on the creation of new ETF shares. Combined, the tests in Panels A and B validate the intuition developed in our theoretical model. As disagreement increases in an individual firm's factor versus idiosyncratic risk, investors trade the factor exposure more aggressively. Additionally, dynamics in factor disagreement drive flow into the ETF and correspondingly out of the individual securities, consistent with our model's first prediction.

Finally, we note that the positive (negative) association between factor (stock-specific) disagreement and flows is the opposite to that predicted by Huang, O'Hara, and Zhong (2021) and Antoniou et al. (2022). In their frameworks, institutional investors use ETFs as a means to hedge the systematic risk exposures of firms, thereby isolating their investments to only the idiosyncratic components. If (i) the supply of ETFs to borrow were limited and (ii) speculative demand for shorts exogenously increased, additional units of the ETF would need to be created to satisfy the hedging motive. After all, at the margin, every short position in the ETF must be accompanied by a corresponding long position in the same security. This would result in flows into the ETF because of higher *idiosyncratic* disagreement — the opposite of what we find. To be clear, our results are not inconsistent with the notion that institutional investors deploy ETFs in the manner suggested by these papers. Rather, our contribution is to provide a coherent economic explanation for the relationship between factor and stock-specific disagreement and ETF flows in the data.

Index Futures. While we use ETFs as a laboratory to test for a positive association between the share of factor disagreement and the demand for factor exposure, this prediction is not confined to just this setting and should hold true in periods predating 2012. Testing the implications of our model in other asset classes is generally difficult due to a lack of available data; however, in Panel C (columns (7) through (9)) of Table 2 we show that the same relation holds true with respect to the open interest in the index futures market (see Bessembinder, Chan, and Seguin, 1996, for a similar interpretation of futures open interest as a measure of disagreement).

Following Hong and Yogo (2012), we compute the change in open interest in futures across all index contracts given on the last trading day for each month from the Commodity Futures Trading Commission's (CFTC) *Commitments of Traders in Commodity Futures* data and use that as our dependent variable ($\text{TActivity}_{m,t}$) in equation (24).[13] To our point of limited data, there are only three indices that have liquid futures over a long horizon – the Nasdaq, Dow Jones, and S&P 500. These correspond closely to the QQQ, DIA, and SPY ETFs, respectively.

Although we are restricted to this narrower cross-section, futures data has a much longer time series than ETF data: the S&P 500 runs from 1992, and the Dow Jones and Nasdaq samples run from 1997. There are two additional caveats to using the CFTC data that are important to mention. First, the CFTC computes separate open interest across different contract multipliers for a given index. For example, the original S&P 500 contract, with a multiplier of 250, and the E-Mini S&P 500 contract, with a multiplier of 50, are separate line items for a given month t. As the price level of the index has increased, trading has slowly shifted from the higher to lower contract multipliers; this is true of all three indices. We are careful to maintain a common multiplier across contracts when summing open interest. Second, the CFTC distinguishes open interest between two different classes of investors—specifically, between hedgers and speculators. From a theoretical perspective, this distinction is important; flow in our model is driven by shifting subjective expectations and relative wealth. In contrast to speculators, commercial users of stock index futures trade for a variety of other reasons, including shifting hedging demand induced by changes in indexation-related flow.

Table 2 present the results using futures interest in Panel C. As in columns (1) through (6), both the dependent and independent variables are standardized. A one-standard-deviation higher disagreement measure corresponds to a 0.13-standard-deviation higher speculator open interest. This is robust to the addition of index fixed effects and our controls in column (8). Given that our open

[13] See "Commitments of Traders" section at `https://www.cftc.gov/data`.

interest measures cross many expiry dates and contract multipliers, we do not include bid-ask spreads as a control. For our most stringent specification a one-standard-deviation increase in disagreement relates to a more than 0.12-standard-deviation higher speculator open interest. These results further validate the notion that increases in factor disagreement translate into greater demand for exposure to the underlying factor. Finally, in column (9) we repeat the specification, but focus on the open interest of hedgers as our dependent variable. Both factor and stock-specific disagreement load positively, with their difference being statistically insignificant. Importantly, this suggests the potential of a different mechanism from the one we have suggested underlying hedgers' motivations.

2.5 Index volatility and disagreement

Having shown that disagreement drives flows into ETFs, as predicted by Figure 1, this section documents that disagreement also drives an increase in ETF-level volatility. This confirms prediction (b) from Section 1.6, as outlined in the left panel of Figure 2. We establish this relation by showing that increases (decreases) in disagreement surrounding the factor (stock-specific) component of ETF earnings drive increases in ETF-level volatility.

Measuring volatility risk. Our primary specification utilizes at-the-money (ATM) implied volatility from OptionMetrics to obtain a *forward-looking* measure of ETF-level risk. Importantly, using options allow us to compute these forward-looking measures of risk across many different maturities (i.e., we consider options that expire 30 to 365 days in the future). This differentiates our work from other studies that examine the impact of ETF ownership on intra-ETF correlations and variance and that typically focus on higher frequency estimates of risk using realized returns (see, e.g., Ben-David, Franzoni, and Moussawi, 2018; Da and Shive, 2018). Longer-dated options allow us to measure *changes* in expectations and perceptions of the risk and diversification benefits (i.e., correlation) of an ETF. Existing studies also characterize the relationship between ETF flow and asset prices as being short-lived and quickly mean-reverting in nature. Any result we find would be expected to be long-dated due to the extended duration of the options we use.

Figure 5 shows the time series of risk-neutral volatility estimated using ATM volatility for the most prominent ETF in our sample: SPY. In particular, the figure plots the risk-neutral volatility of SPY in two ways. The first uses options written on SPY itself (the dashed blue line in the figure). The second uses the weighted sum of these risk-neutral volatilities across all pairs of firms in the index, where these volatilities are weighted by the importance of a given stock in each ETF on a given trading day t as reported by ETF Global.

This outlined procedure, which implicitly assumes that the returns of all pairs of stocks in each ETF are perfectly positively correlated, results in the dashed red line shown in the figure. While these two approaches for calculating the risk-neutral volatility of an ETF are highly correlated (for instance, each measure of volatility increases surrounding the 2016 presidential election and the onset of the recession induced by the COVID-19 virus), the wedge between the two lines suggests that investors' forward-looking perceptions of the intra-ETF correlation are less than one (i.e., investors do not believe the stocks within the S&P 500 are perfectly positively correlated) and vary over time.

Following the intuition underlying our economic model, we implement a regression analysis to examine whether disagreement regarding the relative contribution of the systematic versus stock-specific risk of stock returns drives (part of) the cross-sectional differences in the forward-looking volatility of an ETF in each month. Section 2.6 then explores whether disagreement is associated with the intra-ETF correlation between stocks, as our model also predicts.

Regression analysis. We examine the relation between the relative importance of factor disagreement and ETF-level volatility risk by estimating the following panel regression:

$$\begin{aligned}\sigma^{Q}_{m,t,\tau} = \alpha_m + \delta_t + \beta_{\text{Sys}} \cdot \Delta\text{Sys.Disagree}_{m,t-1} + \\ \beta_{\text{Idio}} \cdot \Delta\text{Idio.Disagree}_{m,t-1} + \boldsymbol{\beta}\boldsymbol{X}^{T}_{m,t-1} + \varepsilon_{m,t}.\end{aligned} \tag{25}$$

As in regression (24), $\Delta\text{Sys.Disagree}_{m,t-1}$ and $\Delta\text{Idio.Disagree}_{m,t-1}$ correspond to changes in systematic and idiosyncratic disagreement regarding ETF m, as defined in equations (21) and (20), respectively. When estimating this regression, we measure the risk-neutral volatility of ETF m at time t by focusing on ATM options with $\tau \in \{30, 91, 182, 273, 365\}$ days to maturity. The controls we use are similar to those in regression (24), and include the lagged dependent variable, ETF returns, and bid-ask spreads. We also include ETF and time fixed effects, denoted by δ_t and α_m, respectively, and we standardized variables so that each point estimate can be interpreted as the effect of a one-standard-deviation change in the variable of interest. We then test whether $\beta_{\text{Sys}} - \beta_{\text{Idio}} >> 0$. Similar to Table 2, we expect this quantity to be positive, as increases (decreases) in factor (stock-specific) disagreement would lead to a higher variance of the ETF overall.

Table 3(a) reports the results of these panel regressions using 30-day options (the other maturities will be explored further in Section 2.7). Each column shows that increases in the systematic relative to idiosyncratic portion of disagreement result in higher ETF-level volatility. In column (1) we include date fixed effects and the lagged endogenous variable only. In column (2) we add ETF fixed effects. In columns (3) through (5) we add the bid-ask spread, past return and absolutely value of past returns as controls. The point estimates indicate that a one-standard-deviation increase in factor relatively to stock-specific disagreement raises forward-looking volatility by approximately 0.08-standard-deviations with none of the controls appreciably changing the statistical significance of $\beta_{\text{Sys}} - \beta_{\text{Idio}}$, which are all statistically significant at the 1% level. Interestingly, the lagged absolute past return loads statistically significantly even though the past implied volatility (lagged dependent) is also included in the regression.

For the sake of completeness, Table 3(b) reruns these regressions on a noisy proxy of expected volatility: the 22-day (intra-month) squared High-Low return (see Engle and Gallo, 2006)). Although $\beta_{\text{Sys}} - \beta_{\text{Idio}}$ in all specifications are now only marginally statistically significant, the signs on the coefficients of $\Delta\text{Sys.Disagree}_{m,t-1}$ and $\Delta\text{Idio.Disagree}_{m,t-1}$ in equation (25) are directionally consistent. Interestingly, the controls now load much more strongly in the regression, and with the expected signs, versus when using ATM volatility as the dependent variable. Higher bid-ask spreads, lower returns and higher absolute returns all strongly predict higher future realized volatility. This illustrates the complications involved in using *realized* stand-ins for *expected* measures. Altogether, however, these tests validate the intuition developed in our theoretical model; as disagreement in the factor (stock-specific) portion of returns increases (decreases) across agents, volatility increases by a statistically significant and economically large amount.

2.6 Intra-ETF correlation risk

The relationship between ETF flows and factor volatility is partially driven by changing correlations between the stocks composing the factor. In the context of our risk-neutral measure of implied volatility, one would therefore assume that correlation *risk* is related to disagreement. In this section we empirically test and confirm this prediction.

Measuring correlation risk. To construct a measure of the forward-looking diversification benefits of holding an ETF, we start by considering the definition of portfolio variance for ETF m on day t using risk-neutral measures of volatility constructed from options with τ days to maturity

$$\left(\sigma^Q_{m,t,\tau}\right)^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,m,t} w_{j,m,t} \sigma^Q_{i,t,\tau} \sigma^Q_{j,t,\tau} \rho^Q_{i,j,t,\tau}. \tag{26}$$

Here, $\sigma^Q_{m,t,\tau}$ denotes the risk-neutral volatility of ETF m, $\sigma^Q_{i,t,\tau}$ is the risk-neutral volatility of security i, $w_{i,m,t}$ denotes the weight of security i in ETF m, obtained from ETF Global, and $\rho^Q_{i,j,t,\tau}$ represents the risk-neutral correlation between stocks i and j on day t with τ days to maturity.

While exchange-traded options prices provide us with readily observable estimates of $\sigma^Q_{m,t,\tau}$ and $\sigma^Q_{i,t,\tau}$, the options market does not provide us with exchange-traded claims that deliver the correlation between a pair of securities at maturity. Consequently, we obtain the *average* risk-neutral correlation between the pairs of securities that constitute ETF m on day t by "inverting" equation (26) for the average value of $\rho^Q_{i,j,t,\tau}$. That is, we define

$$\overline{\rho}^Q_{m,t,\tau} = \frac{\left(\sigma^Q_{m,t,\tau}\right)^2}{\sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,m,t} w_{j,m,t} \sigma^Q_{i,t,\tau} \sigma^Q_{j,t,\tau}}. \tag{27}$$

as the risk-neutral correlation of stocks contained in ETF m at time t, calculated using options with τ days to maturity. This measure essentially reflects the time-varying wedge between the measures of index-level volatility computed using index-level and individual stock options in Figure 5. This procedure is similar to that done by Buss and Vilkov (2012) who show that correlation risk is priced using the S&P 500 index as a proxy for the market. In contrast, however, we are analyzing a much broader universe of securities, indices and underlying options, and, as such, utilize the simpler assumption that correlations (rather than correlation risk premia) are constant between constituents. This removes the necessity of estimating pairwise physical correlations.

Figure 6 displays the monthly time-series variation for the average $\tau = \{30, 91, 182, 273, 365\}$-day risk-neutral correlations ($\overline{\rho}^{Q}_{m,t,\tau}$) underlying two prominent ETFs in the sample: SPY and XOP, an ETF that tracks stocks involved in gas and oil exploration and production. There are four key takeaways from this figure. First, the estimated values of $\overline{\rho}^{Q}_{m,t,\tau}$ clearly satisfy the requirement that $\rho^{Q}_{m,t,} \in [0, 1]$. Second, the figure shows that these risk-neutral correlations vary substantially over time. While the average correlation between S&P 500 stocks was between 0.50 and 0.70 in 2012 depending on the maturity of the options used to compute $\overline{\rho}^{Q}_{m,t,\tau}$ in equation (27), these correlations dropped as low as 0.20 during the economic expansion of the 2010s before rising back to around 0.70 during the onset of the COVID-19 crisis. Third, there is also a large degree of cross-sectional heterogeneity between the risk-neutral correlations across ETFs at a given point in time. For instance, while stocks in the S&P 500 shared an average risk-neutral correlation of about 0.70 in 2020, the firms underlying XOP had an average risk-neutral correlation of about 0.95. Thus, ETF-level correlation risk can vary both over time and between different investment styles.

The final takeaway from Figure 6 is that there are stark differences in the *term structure* of risk-neutral correlations. Notably, while short-term measures of intra-ETF correlations (estimated using 30-day options) tend to be "fast-moving" and vary substantially over time, long-term measures of intra-ETF correlations (estimated using 365-day options) are relatively "slow-moving" and consequently vary less. However, these differences in the levels of correlations tend to diverge during good times and compress during bad times, such as the economic and financial shock induced by COVID-19. We will analyze the implications of these differences using different dated ATM options in Section 2.7.

Regression analysis. We now examine how disagreement regarding the systematic component of ETF returns may drive variation in the *level* of intra-ETF correlation risk by estimating the following panel regression:

$$\begin{aligned}\overline{\rho}^Q_{m,t} =&\delta_t + \alpha_m + \beta_{\text{Sys}} \cdot \Delta\text{Sys.Disagree}_{m,t-1} + \\ &\beta_{\text{Idio}} \cdot \Delta\text{Idio.Disagree}_{m,t-1} + \boldsymbol{\beta} \boldsymbol{X}^T_{m,t-1} + \varepsilon_{m,t}.\end{aligned} \tag{28}$$

Here, $\overline{\rho}^Q_{m,t,\tau}$ represents the average risk-neutral correlation for ETF m at time t, estimated using 30-day options; $\Delta\text{Sys.Disagree}_{m,t-1}$ and $\Delta\text{Idio.Disagree}_{m,t-1}$ correspond to changes in systematic and idiosyncratic disagreement regarding ETF m, as defined in equations (21) and (20), respectively; and the vector $X_{m,t}$ controls for lagged correlation, past returns, and bid-ask spreads. The regression includes both time fixed effects (δ_t) that account for common shocks that impact all ETFs at a given point in time, and ETF fixed effects (α_m) that account for unconditional differences in correlation risk across ETFs (e.g., the difference in the level of correlation between SPY and XOP in Figure 6). Finally, and similar to the tables above, both the independent and dependent variables are standardized for ease of interpretation.

The results of estimating equation (28) are reported in Table 4(a). The table highlights three key takeaways. First, as shown in Column (1), there is a strong predictive relationship between correlation and disagreement. A one-standard-deviation increase in factor relative to stock-specific disagreement results in a 0.083-standard-deviation increase in $\overline{\rho}^Q_{m,t,30}$, which empirically supports the intuition underlying model prediction (c) in Section 1.6. Specifically, these results support the prediction illustrated by the right panel of Figure 2. Through the lens of the model, this association arises because more concentrated trading in the ETF (i.e., a greater exposure to the systematic risk factor) increases the covariation between related securities and makes the market more susceptible to shocks that impact this systematic factor.

In Column (2) we add ETF fixed effects to the regression; the statistically significance of $\beta_{\text{Sys}} - \beta_{\text{Idio}}$ decreases only slightly. Now a one-standard-deviation increase in the relative factor disagreement predicts a 0.078-standard-deviation increase in $\overline{\rho}^Q_{m,t,30}$. This result indicates that the relationship holds true both dynamically and in the cross-section. In Columns (3) through (5) we add past bid-ask spreads, returns, and absolutely returns as controls. The statistical and economic significance of the relationship between lagged disagreement and correlation remains. In Table 4(b) we rerun these specifications utilizing the empirical measure of intra-index correlation, and High-Low return of Pollet and Wilson (2010) and Engle and Gallo (2006), respectively. The realized return-based correlation measure produces coefficients that are very similar to those generated using option-based correlations. Collectively, the evidence in this section further validates the economic mechanisms of our model.

2.7 Identifying the disagreement channel

One concern with regressing volatility and correlation directly onto proxies of disagreement is that these proxies may also be capturing other economic phenomenon and motives to trade. For example, uncertainty regarding the factor or stock-specific components of returns across agents could be the primary driver of the variation in our measures, more so than disagreement. Higher uncertainty in turn would drive higher volatility and correlation risk in the indices.

Fortunately, there is economic theory that can guide our empirical strategy. Dow and da Costa Werlang (1992) and Epstein and Schneider (2007), for example, show that uncertainty about the probability distribution of asset payoffs is *the* necessary ingredient required to match the low rates of aggregate stock participation found in the data. In their models Knightian Uncertainty dissuades agents from trading, i.e., taking either a short or long position in a particular security, driving a no-participation wedge that increases with uncertainty.[14] If $\Delta\text{Sys.Disagree}_{m,t}$ captures changes in factor *uncertainty* rather that *disagreement*, then it should predict an increase in the no-participation wedge, driving down, not up, the flows into index products. In contrast, if $\Delta\text{Idio.Disagree}_{m,t}$ is driven by stock-specific *uncertainty* rather than *disagreement*, then it should predict a larger no-participation wedge at the single-stock level, driving up flows into index products (see Cao, Wang, and Zhang, 2005).

This of course is the opposite of what we show in Section 2.4. To statistically verify that disagreement rather than uncertainty is driving our results, we further utilize this connection between our proxies and trading flows to conduct a 2SLS regression. First, we instrument flow by our measures of

[14] While empirical evidence of this mechanism in a dynamic environment is limited, Meyer and Uhr (2024) use brokerage-level data to show that investors re-invest exogenous increase in cash less aggressively during highly uncertain times.

factor and stock-specific disagreement. Specifically, our first stage regression is

$$\begin{aligned}\text{NDFlow}_{m,t} =& \beta_{\text{Lag}} \cdot \text{NDFlow}_{m,t-1} + \beta_{\text{Sys}} \cdot \Delta\text{Sys.Disagree}_{m,t-1} + \\ & \beta_{\text{Idio}} \cdot \Delta\text{Idio.Disagree}_{m,t-1} + \varepsilon_{m,t},\end{aligned} \tag{29}$$

where $\text{NDFlow}_{m,t}$ is the net dollar flows into ETF m during month t, and $\Delta\text{Sys.Disagree}_{m,t}$ and $\Delta\text{Idio.Disagree}_{m,t}$ are changes in factor and stock-specific disagreement as defined by equations (21) and (20), respectively. Equation (29) is closely related to regression (24). We then regress either risk-neutral volatility or correlation onto instrumented (i.e., $\widehat{\text{NDFlow}}_{m,t}$ or expected) flow

$$\begin{aligned}\text{RN-Moment}_{m,t,\tau} =& \delta_t + \alpha_m + \beta_{\text{Lag}} \cdot \text{RN-Moment}_{m,t-1,\tau} + \\ & \beta_{\text{NDFlow}} \cdot \widehat{\text{NDFlow}}_{m,t} + \boldsymbol{\beta} \boldsymbol{X}^T_{m,t-1} + \varepsilon_{m,t},\end{aligned} \tag{30}$$

where $\text{RN-Moment}_{m,t,\tau}$ is either risk-neutral volatility or correlation of ETF m at the end of month t using options that expire in τ days; β_{NDFlow} is our coefficient of interest, which we hypothesize to be greater than zero; and the vector $X_{m,t}$ controls for returns, the absolute value of returns and bid-ask spreads. As in our previous results, both the left and right-hand-side variables are standardized such that coefficients represent a one-standard-deviation change in the dependent variable, and t-statistics are estimated using robust standard errors.

Tables 5(a) and 5(b) report the results of this procedure using risk-neutral volatility and correlation, respectively, as the dependent variable of interest. Columns (1) through (5), in sequence, represent the full regression specification using 30, 91, 182, 273 and 365 day ATM options, respectively. There are a few takeaways from these results. First, instrumented flow has strong positive predictive coefficients for most measures of risk-neutral volatility and correlation risk. For 30-day options, a one-standard-deviation increase in predicted flows drives ATM implied volatility (correlation) risk higher by nearly 0.25 (0.26) standard-deviations. The economic magnitudes of these coefficients are two to three times higher than those of the raw disagreement scores. It seems that the 2SLS approach better identifies our economic mechanism—i.e., time varying factor and idiosyncratic disagreement drive measures of trading activity that, in turn, drive higher volatility and correlation risk. Second, the economic magnitude and significance of instrumented flow almost monotonically decays as the maturity of the options used to estimate volatility and correlation risk increase. This is an intuitive result if one assumes, as our model does, that volatility, factor disagreement, and stock-specific disagreement mean revert to long-run averages. Finally, one concern is that equation (29), our first-stage regression, is weakly identified. At the bottom of both tables, we present F-statistic for this regression; all F-statistics are greater than 30, which typically indicates well-identified first-stage regressions.

2.8 Single-stock Volatility and Disagreement

Our model-derived hypotheses from Section 1.6 predict that, at the index-level, factor (stock-specific) disagreement should be positively (negatively) associated with index volatility, while, at the single-stock level, *both* factor and stock-specific disagreement should be positively associated with firm-level volatility. The intuition of this result is simple: at the aggregate (index) level, idiosyncratic volatility washes out while at the single-stock level it is total volatility that matters most for pricing volatility risk. In this section, we test for this contrast at the single-stock level by regressing firm-level ATM implied volatility onto our measures of disagreement,

$$\begin{aligned}\sigma^Q_{i,m,t,\tau} = \alpha_i + \delta_t + \beta_{\text{Sys}} \cdot \Delta\text{Sys.Disagree}_{m,t-1} + \\ \beta_{\text{Idio}} \cdot \Delta\text{Idio.Disagree}_{i,m,t-1} + \boldsymbol{\beta}\boldsymbol{X}^T_{i,m,t-1} + \varepsilon_{i,m,t}.\end{aligned} \tag{31}$$

Here, $\sigma^Q_{i,m,t,\tau}$ represents the risk-neutral volatility of firm i within for ETF m at time t, estimated using τ-day options; $\Delta\text{Idio.Disagree}_{i,m,t-1}$ and $\Delta\text{Sys.Disagree}_{m,t-1}$ correspond to changes in firm-level and associated ETF-level disagreement, respectively; and the vector $X_{i,m,t}$ controls for firm-level lagged volatility, past returns, absolute value of returns, and bid-ask spreads. A key distinction between our index-level panel regressions and equation (31) is that while factor disagreement is still defined as in equation (21), stock-specific disagreement is now defined as only the bracketed term in equation

(20)—i.e., this term is not averaged across all stock within the ETF. In addition to the controls in $X_{i,m,t}$, some specification use combinations of time (δ_t) and firm (α_i) fixed effects.

Tables 6(a) and 6(b) present the results of regression (31) using 30-day and 365-day ATM implied volatility as the dependent variables of interest, respectively. In column (1) we see that increases in both factor and stock-specific disagreement independently predict higher single-stock level volatility. A one-standard-deviation increase in factor (stock-specific) disagreement leads to a 0.023 (0.024) standard-deviation increase in volatility. In column (2) and (3) we add firm fixed effects and our firm-level controls; regression coefficients do not change by much. In columns (4) and (5) we add index by time fixed effects that subsumes the time fixed effect and factor disagreement as a predictive variable. Both regressions continue to show a predictive association between past stock-specific disagreement and single-stock volatility.

3 Conclusion

This paper studies how factor and stock-specific disagreement affect asset prices and risk. We start by building a theoretical model to examine the interplay between heterogeneity in subjective expectations about the stock-specific and common components of expected returns. We consider a pure exchange economy with multiple Lucas trees that are exposed to both factor risk and stock-specific shocks, and we allow agents to disagree about both dimensions of returns. As such, our model features periods of strong disagreement about the factor, and periods when disagreement about stocks dominates.

Our model predicts that (i) factor disagreement increases the exposure of investors in the economy to the assets that are most closely aligned with systematic risk; (ii) factor disagreement increases the return volatility of financial instruments aligned with the common factor (e.g., ETFs); and (iii) this increase in volatility is accompanied by the increased correlations between stocks that compose the factor (i.e., reduced diversification benefits).

We then use the return and flow dynamics of ETFs and their underlying securities to test these hypotheses. In keeping with the model's predictions, we find that disagreement is strongly related to

ETF flows—when factor versus stock-specific disagreement is one-standard-deviation higher, there is a 0.09-standard-deviation higher flow into the ETF. Second, disagreement also relates closely to forward-looking volatility and correlation risk—a one-standard-deviation increase in factor versus stock-specific disagreement measure leads to volatility and correlation expectations rising by approximately 0.08 standard deviations. Finally, we contrast these ETF-level findings with those at the single-stock level. While factor and stock-specific disagreement have opposite predictive signs at the index-level, the both have positive predictive coefficients at the single stock level, which is in keeping with our theoretical findings. Taken together, our theoretical and empirical results highlight the first-order effect of disagreement on trade flows and show how flows impact cross-sectional differences in volatility and correlation risk.

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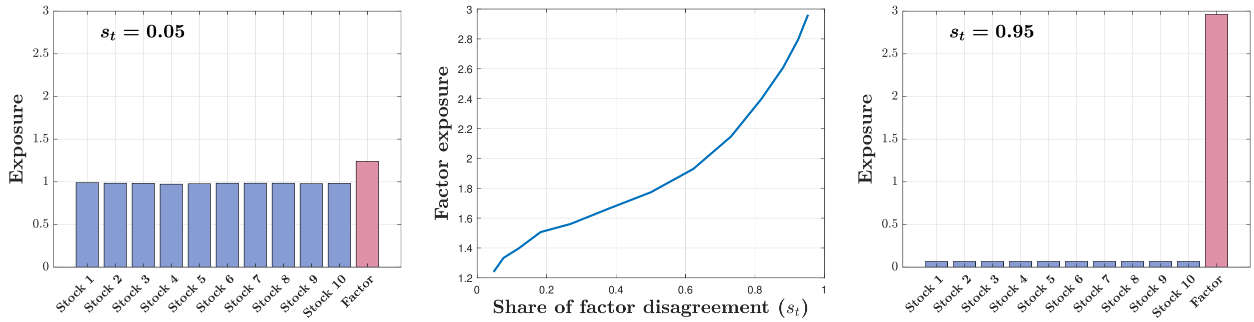

Figure 1: The figures show exposure when $s_t = 0.05$ for each of the 10 stocks and the factor (left), the factor exposure as function of the share of factor disagreement s (middle), and the exposure when $s_t = 0.95$ for each of the 10 stocks and the factor (right). The figures are based on a 10-stock economy. All agents have the same consumption shares, and each dividend share is the same for each stock. The total dividend share is 5%. The figures are based on the following parameters: $\gamma = 2$, $\rho = 0.02$, $\mu_E = 0.02$, $\sigma_E = 0.03$, $\sigma_Z = 0.03$, $\mu_Z = -0.5\sigma_Z^2$, $\sigma_n = 0.06$, $\mu_n = -0.5\sigma_n^2$ $\Delta = 0.2$, $\kappa_\delta = 0.1$, $\sigma_\delta = 0.2$, $\bar{\delta} = 0$. The results are generated by Monte-Carlo simulations based on $400,000$ paths of monthly observations of length of 100 years.

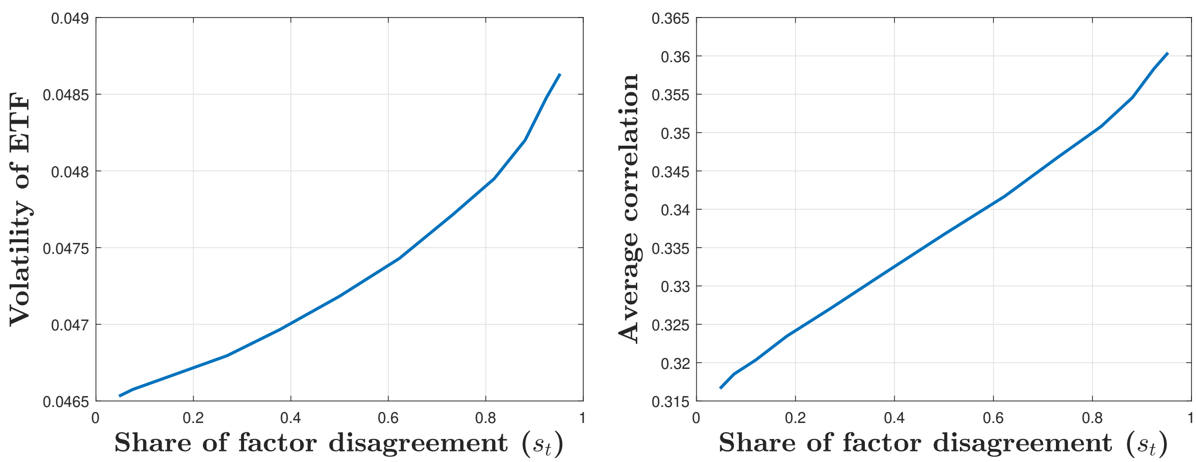

Figure 2: The figures show the standard deviation of the ETF (left) and the average stock return correlation (right) as a function of the share of factor disagreement s. The figures are based on a 10-stock economy. All agents have the same consumption shares, and each dividend share is the same for each stock. The total dividend share is 5%. The figures are based on the following parameters: $\gamma = 2$, $\rho = 0.02$, $\mu_E = 0.02$, $\sigma_E = 0.03$, $\sigma_Z = 0.03$, $\mu_Z = -0.5\sigma_Z^2$, $\sigma_n = 0.06$, $\mu_n = -0.5\sigma_n^2$ $\Delta = 0.2$, $\kappa_\delta = 0.1$, $\sigma_\delta = 0.2$, $\bar{\delta} = 0$. The results are generated by Monte-Carlo simulations based on $400,000$ paths of monthly observations of length of 100 years.

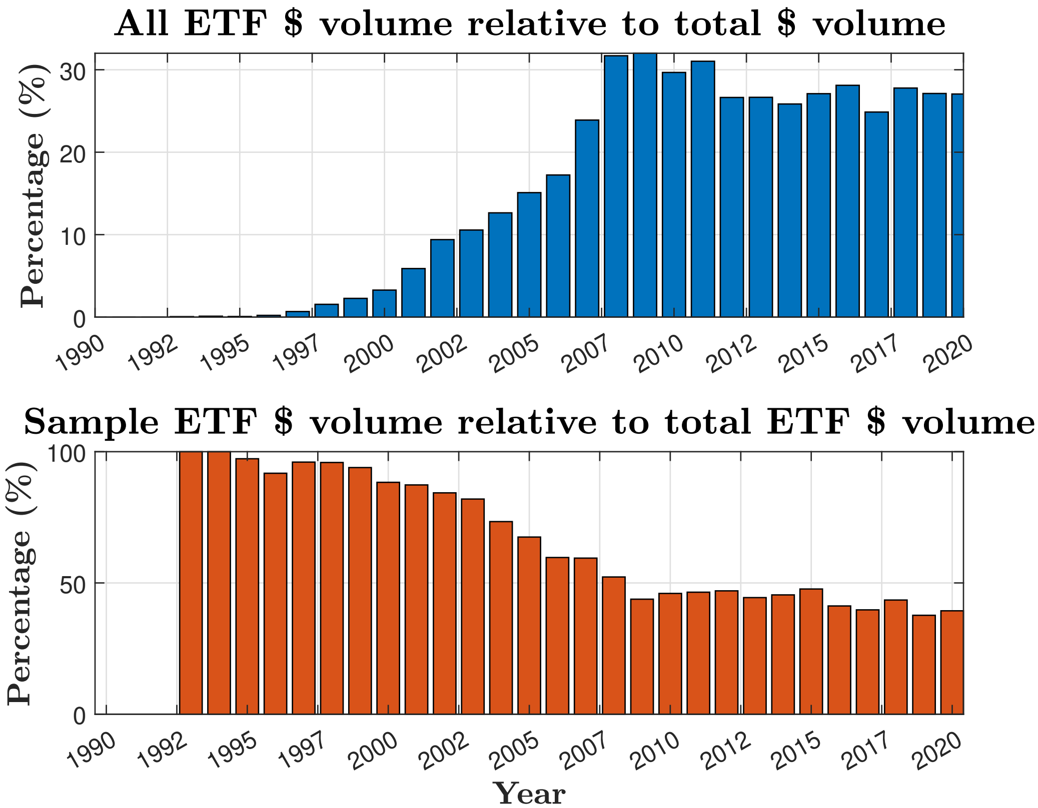

Figure 3: The figure displays the percentage of dollar-trading volume in ETFs relative to dollar-trading volume for the U.S. equity market for the period ranging from 1900 to 2020. ETFs are identified as securities in the CRSP Monthly dataset that have a share code (SHRCD) of 73. We then compute the monthly dollar trading volume of (i) all ETFs, and (ii) all securities in the CRSP Monthly universe, and aggregate these monthly trading volumes to the annual frequency within each trading year. The top panel of the figure then reports the percentage of ETF-related dollar trading volume, summed across all U.S. ETFs, relative to the aggregate amount of dollar-trading volume across all U.S. securities. The bottom panel of the figure reports the percentage of ETF-related dollar-trading volume, summed across the ETFs in our sample (see Table 1), relative to the aggregate amount of dollar-trading volume across all U.S. ETFs.

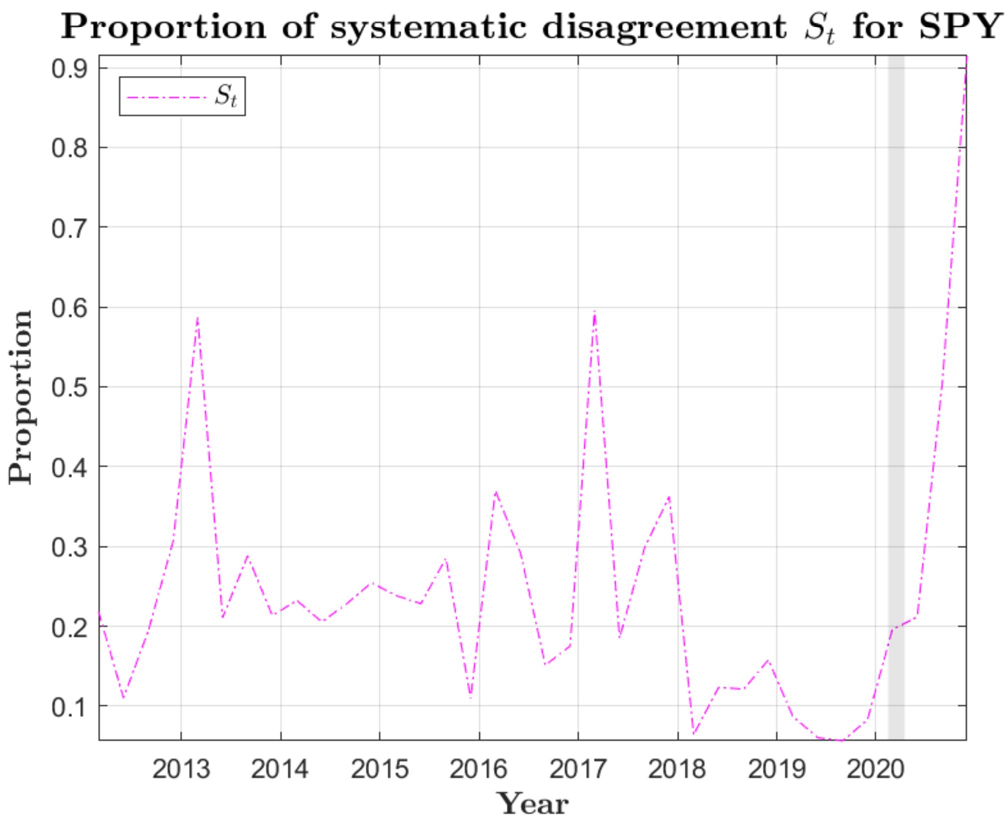

Figure 4: The figure reports the proportion of factor disagreement relative to total disagreement, as defined by equation (23) for SPY – an ETF that tracks the S&P 500. The individual components of disagreement are measured using equations (20) and (21), respectively. To visualize the data, we aggregate each measure of disagreement to the quarterly frequency by computing the mean value of each disagreement in a given quarter. The sample period ranges from 2012 to 2020.

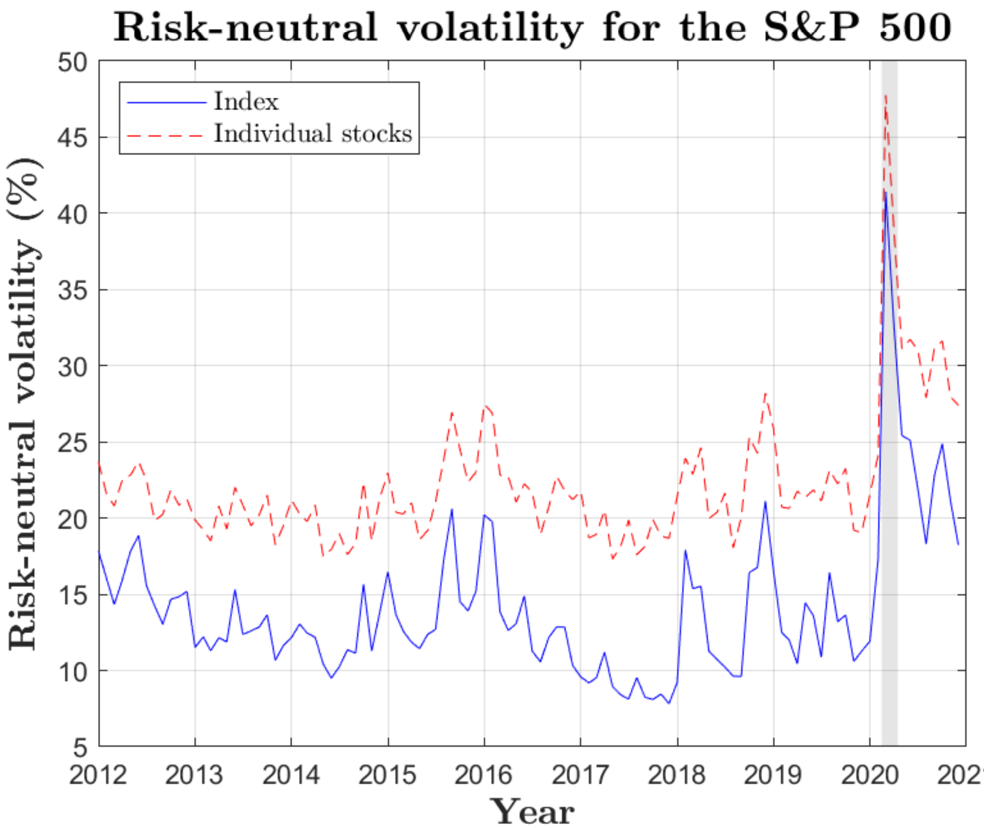

Figure 5: The figure displays the risk-neutral volatility for SPY, which tracks the S&P 500 stock market index. The risk-neutral volatility is computed in two ways. First, the solid blue line reports the risk-neutral volatility from using ETF-linked options. Second, the dashed red line reports the risk-neutral volatility for each stock i and j in ETF m, and then aggregated across all stocks in the ETF via $\sum_{i=1}^{N}\sum_{i=1}^{N} w_{i,m,t}w_{j,m,t}\sigma^{Q}_{i,t,\tau}\sigma^{Q}_{j,t,\tau}$, where $w_{i,m,t}$ and $w_{j,m,t}$ is the weight of firm i or j in ETF m at time t, respectively, and each risk-neutral volatility (i.e., $\sigma^{Q}_{i,t,\tau}$) is obtained by taking the square root of the risk-neutral variance. These risk-neutral volatilities are calculated daily, scaled to represent annualized volatilities, and aggregated to the monthly frequency by computing the average risk-neutral volatility within each month. In all the calculations above, we set $\tau = 30$ such that we estimate each risk-neutral volatility with 30-day options. Finally, the sample period spans January 2012 and December 2020.

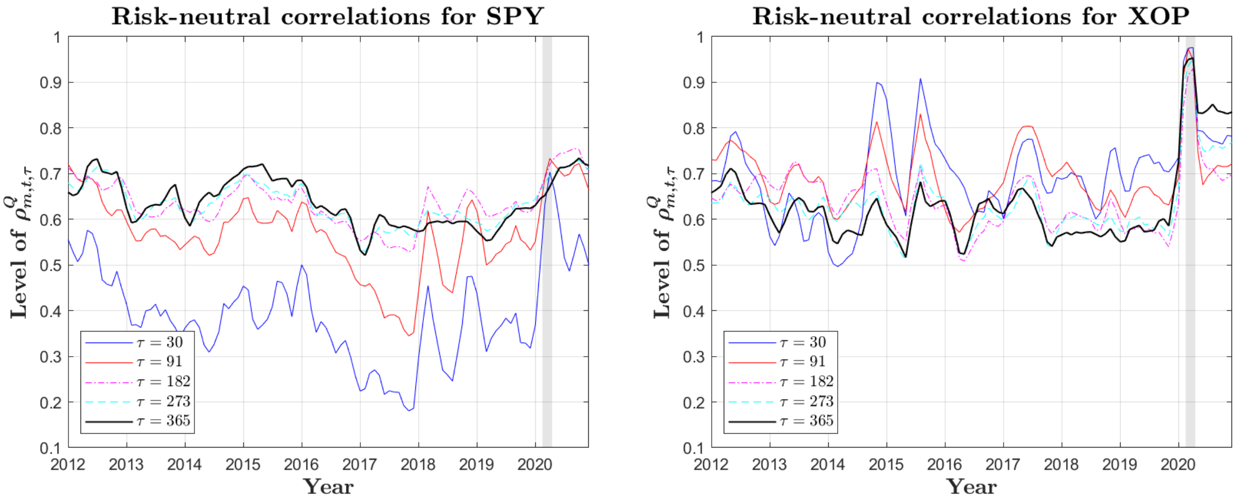

Figure 6: The figure displays the average risk-neutral correlation for the SPY ETF, which tracks the S&P 500 stock market index, and the XOP ETF, which tracks the returns of stocks involved in the exploration and production of oil and gas. The figure reports the $\tau = 30$-, 91-, 182-, 273-, and 365-day-ahead risk-neutral correlation of each index, obtained by solving equation (27) to obtain $\overline{\rho}^{Q}_{m,t,\tau}$ for each ETF m in each month t. To visualize the resulting risk-neutral correlations, we apply a moving-average filter to each monthly time-series of correlations, and report the average correlation over a window of $[-1, 1]$ months around each month t. The sample period spans January 2012 to December 2020.

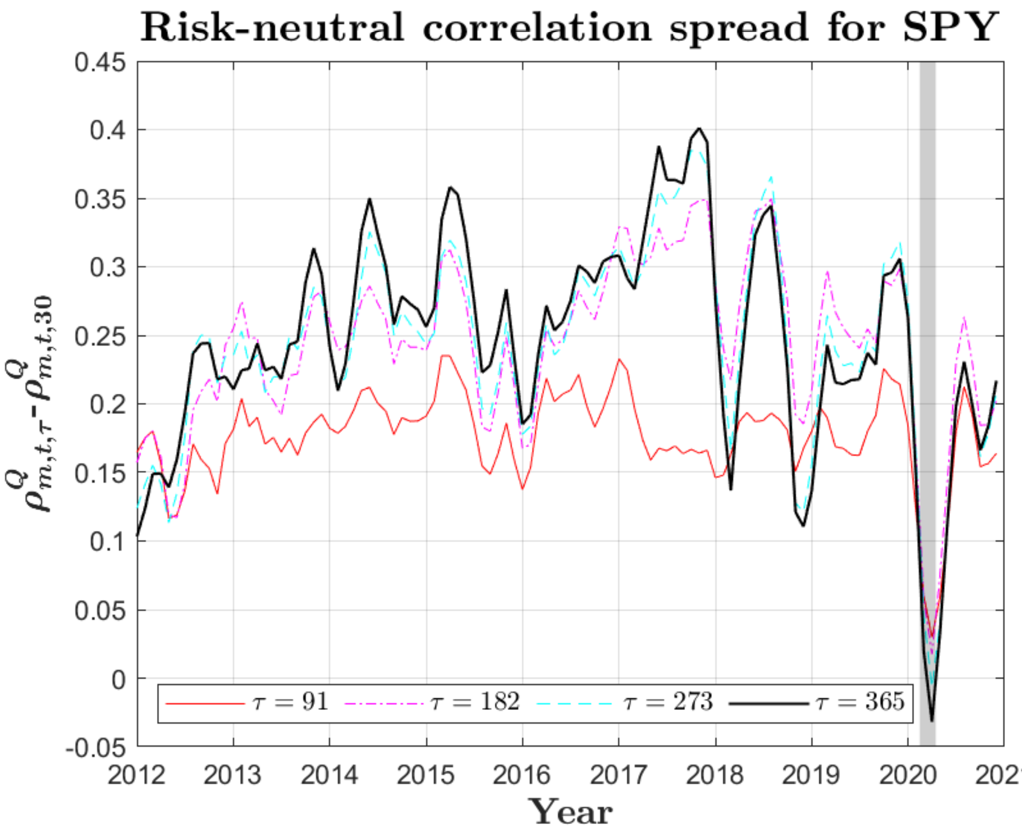

Figure 7: The figure reports the time-series of the spread in the average risk-neutral correlations among the constituents of the SPY ETF, an ETF that tracks the returns of the S&P 500 index. Specifically, the risk-neutral correlation $\overline{\rho}^Q_{m,t,\tau}$ represented by equation (27) is estimated using options with maturities of $\tau \in \{30, 91, 182, 273, 365\}$ days to maturity. With these risk-neutral correlations in hand, the figure then reports $\overline{\rho}^Q_{m,t,\tau} - \overline{\rho}^Q_{m,t,30}$ for $\tau \in \{91, 182, 273, 365\}$. As such, the figure displays the difference between various measures of the intra-ETF "long-run" correlations and the market's perception of the intra-ETF "short-run" correlation. To visualize the resulting risk-neutral correlation spreads, we apply a moving-average filter to each monthly time-series of correlation spreads, and report the average correlation spread over a window of $[-1, 1]$ months around each month t. The sample period spans January 2012 to December 2020.

Table 1: The table reports summary statistics for the ETFs in our sample. For each ETF, the table reports the ticker alongside the benchmark that the ETF tracks (denoted by "Style"). The net asset value of each fund is represented by "NAV (\$b)" and is reported by ETF Global. Similarly, "ME (\$b)" represents the average total market value represented by the stocks underlying each ETF over the sample period and is constructed from CRSP Monthly data. "|Flow|" reports the average amount of (absolute) flow into and out of each ETF, on average, over the sample period, and is also measured in billions of dollars from ETF Global. The columns "\$Vol(ETF)" and "\$Vol(Stocks)" report the average amount of dollar-trading volume associated with each ETF and its underlying stocks per month, respectively. $\mathbb{E}$ [Analysts] reports the average number of analysts following each firm in each ETF. The sample period ranges from 2012 to 2020.

ETF	Style	NAV (\$b)	ME (\$b)	|Flow| (\$b)	\$Vol(ETF) (\$b)	\$Vol(Stocks) (\$b)	E [Analyst]
SPY	Market	227.23	20227.72	29.36	473.85	2867.83	16.27
DIA	Dow Jones	200.56	5840.02	2.30	19.96	585.23	21.36
QQQ	Nasdaq	133.55	6219.90	7.01	102.88	1146.36	19.58
IBB	Biotech	187.63	764.66	0.83	7.52	163.19	5.92
XLK	Technology	55.02	4760.55	1.61	12.95	738.46	21.22
XLB	Materials	49.77	577.13	0.91	6.62	93.67	15.18
XLE	Energy	69.13	1346.49	2.17	22.64	197.01	24.22
XLI	Industrials	59.98	1979.36	2.02	14.65	281.70	15.12
XLP	Cons. Staples	50.01	1942.78	1.82	11.49	187.68	14.53
XLV	Health Care	72.35	2795.04	1.90	14.14	333.28	15.58
XLY	Cons. Disc.	85.73	2408.32	1.60	9.65	468.94	19.26
XOP	Oil & Gas	44.90	1163.23	1.17	10.47	180.15	17.04
XBI	Biotech	104.43	710.31	0.96	6.06	133.18	7.33

Table 2: The table documents how changes in factor versus stock-specific disagreement ($\beta_{Sys} - \beta_{Idio}$) are associated with future trading activity in the ETF or index futures. These results are obtained by estimating the panel regression outlined in equation (24). Net dollar flows into an ETF are constructed according to equation (18), net share flows into an ETF are constructed according to equation (19), open interests are computed according to the methodology of Hong and Yogo (2012), and the measures of disagreement are obtained via equations (20) and (21). Regressions control for one-month lagged trading activity as well as combinations of index and time (i.e., month) fixed effects and index-level returns. For ETF-associated regressions we also control for ETF-level bid-ask spreads, and t-statistics are estimated using robust standard errors. The sample period underlying this regression ranges from 2012 to 2022.

	Dollar Flows into ETF			Δ ETF Shares Outstanding			Δ Futures Open Interest		
							Speculators		Hedgers
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\beta_{\text{Sys}} - \beta_{\text{Idio}}$	0.091***	0.092***	0.094***	0.069**	0.069**	0.072**	0.134**	0.123**	−0.060
	[3.29]	[3.24]	[3.29]	[2.05]	[2.08]	[2.14]	[2.29]	[2.12]	[−1.04]
Components:									
ΔSys.Disagree$_{m,t-1}$	0.041***	0.041***	0.044***	0.028	0.028	0.034*	0.064	0.058	0.027
	[2.67]	[2.65]	[2.84]	[1.40]	[1.42]	[1.68]	[1.57]	[1.45]	[0.65]
ΔIdio.Disagree$_{m,t-1}$	−0.051***	−0.051***	−0.050***	−0.041*	−0.041**	−0.038*	−0.071*	−0.065	0.086**
	[−2.86]	[−2.83]	[−2.72]	[−1.94]	[−1.97]	[−1.82]	[−1.72]	[−1.58]	[2.12]
Controls:									
ΔVolatility$_{m,t-1}$			−0.003			0.003		0.037	0.050
			[−0.14]			[0.14]		[0.92]	[1.28]
Bid-Ask$_{m,t-1}$			0.000			−0.007			
			[0.02]			[−0.85]			
Return$_{m,t-1}$			0.040**			0.063***		0.096**	0.076*
			[2.34]			[2.90]		[2.25]	[1.89]
$\lvert$Return$\rvert_{m,t-1}$			−0.003			0.011		0.022	0.060
			[−0.17]			[0.49]		[0.46]	[1.45]
Date FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Index FE	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes
Lag Dependent	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2,210	2,210	2,210	2,210	2,210	2,210	929	929	929
Within R^2	0.0074	0.0095	0.0111	0.0028	0.0026	0.0068	0.0086	0.0195	0.0195

Table 3: The table documents how changes in factor versus stock-specific ($\beta_{Sys} - \beta_{Idio}$) disagreement about an ETF are associated with the *forward-looking* risk of an ETF, measured using option-implied volatility. These results are obtained by estimating the panel regression outlined in equation (25), where the forward-looking risk of an ETF is obtained via 30-day at-the-money implied volatility in table 3(a) and monthly realized volatility in table 3(b). The measures of disagreement are obtained via equations (20) and (21). Each regression also controls for ETF-level returns and absolute value of returns, bid-ask spreads, as well as combinations of ETF and time (i.e., month) fixed effects. t-statistics are estimated using robust standard errors. The sample period underlying this regression ranges from 2012 to 2022.

(a) 30-day Risk Neutral Volatility

	(1)	(2)	(3)	(4)	(5)
$\beta_{\text{Sys}} - \beta_{\text{Idio}}$	0.083**	0.081**	0.081**	0.079**	0.081**
	[2.49]	[2.51]	[2.50]	[2.41]	[2.44]
Components:					
ΔSys.Disagree$_{m,t-1}$	0.079***	0.080***	0.080***	0.080***	0.082***
	[3.40]	[3.57]	[3.56]	[3.51]	[3.57]
ΔIdio.Disagree$_{m,t-1}$	−0.004	−0.001	−0.001	0.001	0.001
	[−0.17]	[−0.05]	[−0.04]	[0.04]	[0.07]
Controls:					
Bid-Ask$_{m,t-1}$			0.008	0.008	0.008
			[0.36]	[0.36]	[0.33]
Return$_{m,t-1}$				0.046	0.026
				[1.22]	[0.62]
$\lvert$Return$\rvert_{m,t-1}$					0.081*
					[1.86]
Date FE	Yes	Yes	Yes	Yes	Yes
Index FE	No	Yes	Yes	Yes	Yes
Lag Dependent	Yes	Yes	Yes	Yes	Yes
Observations	2,227	2,227	2,227	2,227	2,227
Within R^2	0.0062	0.0064	0.0065	0.0086	0.0142

Table 3: Volatility Regressions—(Continued)

(b) 30-day Realized Volatility

	(1)	(2)	(3)	(4)	(5)
$\beta_{\text{Sys}} - \beta_{\text{Idio}}$	0.055	0.062*	0.060*	0.056	0.059*
	[1.55]	[1.77]	[1.72]	[1.61]	[1.73]
Components:					
ΔSys.Disagree$_{m,t-1}$	0.037	0.043*	0.042*	0.034	0.035
	[1.45]	[1.75]	[1.72]	[1.39]	[1.49]
ΔIdio.Disagree$_{m,t-1}$	−0.018	−0.019	−0.018	−0.022	−0.023
	[−0.96]	[−1.01]	[−0.97]	[−1.21]	[−1.28]
Controls:					
Bid-Ask$_{m,t-1}$			0.067***	0.068***	0.067***
			[3.20]	[3.34]	[3.45]
Return$_{m,t-1}$				−0.100***	−0.143***
				[−3.08]	[−3.91]
$\lvert\text{Return}\rvert_{m,t-1}$					0.181***
					[4.79]
Date FE	Yes	Yes	Yes	Yes	Yes
Index FE	No	Yes	Yes	Yes	Yes
Lag Dependent	Yes	Yes	Yes	Yes	Yes
Observations	2,210	2,210	2,210	2,210	2,210
Within R^2	0.0028	0.0022	0.0067	0.0163	0.0438

Table 4: The table documents how changes in factor versus stock-specific ($\beta_{Sys} - \beta_{Idio}$) disagreement about an ETF are associated with the *forward-looking* risk of an ETF, measured using option-implied correlation. These results are obtained by estimating the panel regression outlined in equation (28), the forward-looking correlation risk of an ETF is obtained via equation (27), and the measures of disagreement are obtained via equations (20) and (21). Each regression also controls for ETF-level returns and absolute value of returns, bid-ask spreads, as well as combinations of ETF and time (i.e., month) fixed effects. t-statistics are estimated using robust standard errors. The sample period underlying this regression ranges from 2012 to 2022.

(a) 30-day Risk Neutral Correlation

	(1)	(2)	(3)	(4)	(5)
$\beta_{\text{Sys}} - \beta_{\text{Idio}}$	0.083**	0.078**	0.077**	0.075**	0.077**
	[2.56]	[2.35]	[2.32]	[2.26]	[2.29]
Components:					
ΔSys.Disagree$_{m,t-1}$	0.049**	0.047**	0.046**	0.046**	0.048**
	[2.21]	[2.18]	[2.15]	[2.13]	[2.21]
ΔIdio.Disagree$_{m,t-1}$	−0.034	−0.031	−0.031	−0.029	−0.029
	[−1.63]	[−1.43]	[−1.41]	[−1.35]	[−1.32]
Controls:					
Bid-Ask$_{m,t-1}$			0.035**	0.036**	0.033**
			[2.20]	[2.18]	[2.06]
Return$_{m,t-1}$				0.036	0.022
				[1.38]	[0.79]
$\lvert$Return$\rvert_{m,t-1}$					0.065**
					[2.40]
Date FE	Yes	Yes	Yes	Yes	Yes
Index FE	No	Yes	Yes	Yes	Yes
Lag Dependent	Yes	Yes	Yes	Yes	Yes
Observations	2,227	2,227	2,227	2,227	2,227
Within R^2	0.0028	0.0025	0.0038	0.0051	0.0091

Table 4: Correlation Regressions—(Continued)

(b) 30-day Realized Correlation

	(1)	(2)	(3)	(4)	(5)
$\beta_{\text{Sys}} - \beta_{\text{Idio}}$	0.079**	0.081**	0.080**	0.080**	0.081**
	[2.17]	[2.23]	[2.21]	[2.20]	[2.23]
Components:					
ΔSys.Disagree$_{m,t-1}$	0.041	0.044*	0.044*	0.043	0.044*
	[1.56]	[1.68]	[1.67]	[1.64]	[1.67]
ΔIdio.Disagree$_{m,t-1}$	−0.038**	−0.037**	−0.037**	−0.037**	−0.037**
	[−2.13]	[−2.11]	[−2.10]	[−2.10]	[−2.11]
Controls:					
Bid-Ask$_{m,t-1}$			0.016	0.016	0.014
			[0.89]	[0.89]	[0.75]
Return$_{m,t-1}$				−0.001	−0.015
				[−0.05]	[−0.50]
$\lvert$Return$\rvert_{m,t-1}$					0.068**
					[2.16]
Date FE	Yes	Yes	Yes	Yes	Yes
Index FE	No	Yes	Yes	Yes	Yes
Lag Dependent	Yes	Yes	Yes	Yes	Yes
Observations	2,210	2,210	2,210	2,210	2,210
Within R^2	0.0028	0.0031	0.0033	0.0033	0.0076

Table 5: The table documents how changes in factor and stock-specific disagreement are associated with ETF-related flow, which in turn impact the *forward-looking* risk of an ETF, measured using option-implied volatility and correlation. The identification of this channel is done through a 2SLS framework. These results are obtained by procedure outlined in Section 2.7, the forward-looking risk of an ETF is obtained via at-the-money implied volatility for options that expire in 30, 91, 182, 273 and 365-days for Table 5(a) and applying equation (27) to each of those options for Table 5(b). The measures of disagreement are obtained via equations (20) and (21). Each regression also controls for ETF-level returns and absolute value of returns, bid-ask spreads, as well as combinations of ETF and time (i.e., month) fixed effects. t-statistics are estimated using robust standard errors. The sample period underlying this regression ranges from 2012 to 2022.

(a) Volatility

	30 days (1)	91 days (2)	182 days (3)	273 days (4)	365 days (5)
$\widehat{\text{NDFlow}}_t$	0.249**	0.227**	0.210**	0.200**	0.169*
	[2.38]	[2.22]	[2.20]	[2.09]	[1.91]
Controls:					
Bid-Ask$_{m,t-1}$	0.010	0.031	0.041	0.058**	0.038*
	[0.44]	[1.23]	[1.61]	[2.43]	[1.69]
$\lvert\text{Return}\rvert_{m,t-1}$	0.077*	0.111**	0.109**	0.102**	0.119**
	[1.74]	[2.30]	[2.38]	[2.16]	[2.27]
Return$_{m,t-1}$	0.023	0.028	0.024	0.020	0.028
	[0.56]	[0.62]	[0.57]	[0.43]	[0.55]
Date FE	Yes	Yes	Yes	Yes	Yes
ETF FE	Yes	Yes	Yes	Yes	Yes
Lag Dependent	Yes	Yes	Yes	Yes	Yes
Observations	2,210	2,210	2,210	2,210	2,210
F-statistic	37.4196	37.4184	37.4478	37.5100	37.4371

Table 5: 2SLS Regressions—(Continued)

(b) Correlation

	30 days (1)	91 days (2)	182 days (3)	273 days (4)	365 days (5)
$\widehat{\text{NDFlow}}_t$	0.263**	0.222**	0.151*	0.205**	0.104
	[2.50]	[2.32]	[1.85]	[2.26]	[1.38]
Controls:					
$\text{Bid-Ask}_{m,t-1}$	0.034**	0.040**	0.044**	0.051**	0.027
	[2.09]	[2.30]	[2.39]	[2.53]	[1.64]
$\lvert\text{Return}\rvert_{m,t-1}$	0.062**	0.060**	0.023	0.014	0.019
	[2.25]	[1.98]	[0.83]	[0.45]	[0.61]
$\text{Return}_{m,t-1}$	0.020	0.045	0.063**	0.042	0.053*
	[0.69]	[1.49]	[2.20]	[1.33]	[1.68]
Date FE	Yes	Yes	Yes	Yes	Yes
ETF FE	Yes	Yes	Yes	Yes	Yes
Lag Dependent	Yes	Yes	Yes	Yes	Yes
Observations	2,210	2,210	2,210	2,210	2,210
F-statistic	37.6056	37.5176	37.5416	37.7815	37.4374

Table 6: The table documents how changes in factor and stock-specific disagreement are associated with the *forward-looking* risk of the single-stocks constituents in our ETFs of interest. These results are obtained by estimating the panel regression outlined in equation (31), the forward-looking risk of an ETF is obtained via 30-day at-the-money implied volatility for Table 6(a) and 365-day at-the-money volatility for Table 6(b). The measures of disagreement are obtained via equations (20) and (21). Each regression also controls for stock-level returns and absolute value of returns, bid-ask spreads, as well as combinations of stock, ETF and time (i.e., month) fixed effects. t-statistics are estimated using robust standard errors. The sample period underlying this regression ranges from 2012 to 2022.

(a) 30-day Volatility

	(1)	(2)	(3)	(4)	(5)
ΔSys.Disagree$_{m,t-1}$	0.023***	0.019***	0.019***		
	[9.08]	[7.28]	[7.46]		
ΔIdio.Disagree$_{i,m,t-1}$	0.024***	0.024***	0.024***	0.018***	0.018***
	[6.08]	[6.54]	[6.58]	[5.34]	[5.41]
Controls:					
Bid-Ask$_{i,m,t-1}$			−0.042***		−0.033***
			[−12.22]		[−10.29]
Return$_{i,m,t-1}$			−0.050***		−0.047***
			[−10.35]		[−9.60]
$\lvert$Return$\rvert_{i,m,t-1}$			0.076***		0.066***
			[14.11]		[12.44]
Date FE	Yes	Yes	Yes	No	No
Firm FE	No	Yes	Yes	Yes	Yes
Date × Index FE	No	No	No	Yes	Yes
Lag Dependent	Yes	Yes	Yes	Yes	Yes
Observations	189,659	189,578	189,524	189,578	189,524
Within R^2	0.0129	0.0143	0.0196	0.0133	0.0174

Table 6: Single Stock Regressions—(Continued)

(b) 365-day Volatility

	(1)	(2)	(3)	(4)	(5)
ΔSys.Disagree$_{m,t-1}$	0.012***	0.006**	0.007**		
	[3.89]	[2.12]	[2.27]		
ΔIdio.Disagree$_{i,m,t-1}$	0.015***	0.011***	0.011***	0.009***	0.009***
	[4.66]	[6.02]	[6.01]	[5.29]	[5.36]
Controls:					
Bid-Ask$_{i,m,t-1}$			−0.027***		−0.021***
			[−6.26]		[−6.70]
Return$_{i,m,t-1}$			−0.044***		−0.041***
			[−6.32]		[−6.53]
$\lvert$Return$\rvert_{i,m,t-1}$			0.073***		0.061***
			[6.71]		[6.60]
Date FE	Yes	Yes	Yes	No	No
Firm FE	No	Yes	Yes	Yes	Yes
Date × Index FE	No	No	No	Yes	Yes
Lag Dependent	Yes	Yes	Yes	Yes	Yes
Observations	189,659	189,578	189,524	189,578	189,524
Within R^2	0.0080	0.0081	0.0127	0.0077	0.0110

A Internet Appendix

A.1 Additional tables and figures

Table A.1.1: The table presents summary statistics for the relative importance of factor disagreement ($s_{m,t}$ from equation (23)) for each ETF m in our sample. In particular, the table reports the mean, median, and standard deviation of the relative disagreement measure for each ETF, as well as the 25^{th} and 75^{th} percentile of this measure. The sample period ranges from January 2012 to December 2020.

ETF	Style	Mean	Std	$p25$	Median	$p75$
SPY	Market	0.27	0.20	0.12	0.21	0.36
DIA	Dow Jones	0.04	0.04	0.01	0.02	0.05
QQQ	Nasdaq	0.12	0.12	0.04	0.08	0.15
IBB	Biotech	0.24	0.17	0.11	0.18	0.31
XLK	Technology	0.09	0.11	0.03	0.06	0.10
XLB	Materials	0.03	0.04	0.01	0.02	0.04
XLE	Energy	0.21	0.19	0.09	0.16	0.28
XLI	Industrials	0.03	0.05	0.01	0.01	0.03
XLP	Cons. Staples	0.01	0.02	0.01	0.01	0.01
XLV	Health care	0.03	0.03	0.01	0.02	0.03
XLY	Cons. Discretionary	0.07	0.06	0.02	0.04	0.08
XOP	Oil & gas	0.31	0.19	0.19	0.25	0.40
XBI	Biotech	0.21	0.19	0.07	0.15	0.27

Table A.1.2: The table presents the correlation between the measures of the relative importance of factor disagreement ($s_{m,t}$ from equation (23)) for each pair of ETFs in our sample. The sample period ranges from January 2012 to December 2020.

	SPY	DIA	QQQ	XLK	XLB	XLE	XLI	XLP	XLV	XLY	XOP	XBI	IBB
SPY	1.00	0.23	0.09	-0.01	0.07	0.17	-0.01	0.21	0.42	0.18	0.13	0.03	0.14
DIA	-	1.00	-0.03	0.14	0.14	0.01	0.02	0.18	0.11	0.28	-0.05	-0.09	-0.03
QQQ	-	-	1.00	0.13	0.22	0.18	0.29	0.26	0.28	0.15	0.08	-0.11	0.12
XLK	-	-	-	1.00	-0.11	-0.01	0.07	-0.10	0.04	-0.01	0.04	0.12	0.01
XLB	-	-	-	-	1.00	-0.00	0.22	0.05	-0.03	0.17	-0.02	-0.08	0.15
XLE	-	-	-	-	-	1.00	0.19	0.46	0.04	0.33	0.15	0.04	0.31
XLI	-	-	-	-	-	-	1.00	0.06	0.05	0.01	0.05	0.03	0.01
XLP	-	-	-	-	-	-	-	1.00	0.30	0.48	0.10	-0.12	0.30
XLV	-	-	-	-	-	-	-	-	1.00	0.16	0.09	-0.12	0.35
XLY	-	-	-	-	-	-	-	-	-	1.00	0.05	-0.11	0.32
XOP	-	-	-	-	-	-	-	-	-	-	1.00	0.18	0.06
XBI	-	-	-	-	-	-	-	-	-	-	-	1.00	-0.11
IBB	-	-	-	-	-	-	-	-	-	-	-	-	1.00

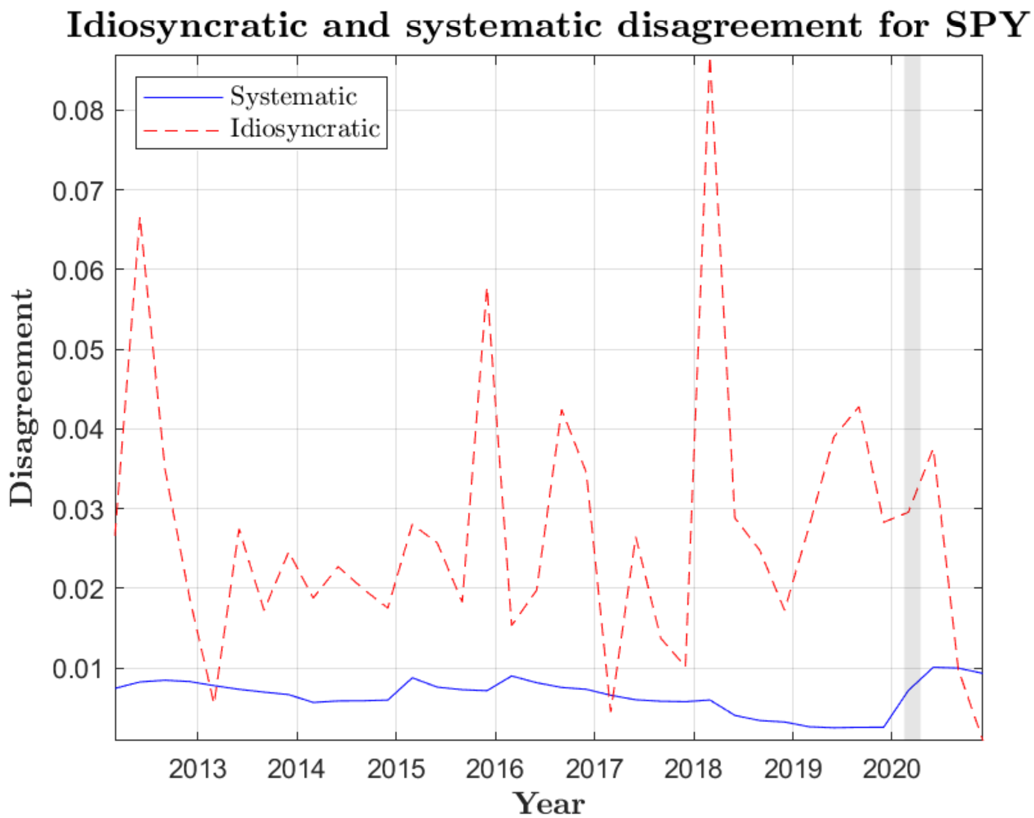

Figure A.1.1: The figure reports the levels of systematic (factor) and idiosyncratic disagreement for SPY – an ETF that tracks the S&P 500 – as measured using equations (20) and (21), respectively. To visualize the data, we aggregate each measure of disagreement to the quarterly frequency by computing the mean value of each disagreement in a given quarter. The sample period ranges from 2012 to 2020.

A.2 Complete Model and Simulation Details

In this section we provide the additional details of the model in Section 1. Details of the cash flow dynamics and the information structure can be found in the main body of the paper, so we do not repeat it here. To simplify some of the exposition we define the dividend diffusion coefficients for stock n as σ_{D_n}. Note that from an application of Ito's lemma on the dividend dynamics we have

$$\sigma_{D_n} = \sigma_E + \sigma_z + \sigma_n \tag{A.2.1}$$

In the reminder of this section, we stack all the Brownian shocks into a vector w_t, with $w_t = (w_{E,t}, w_{\delta,t}, w_{z,t}, w_{1,t}, \ldots, w_{N,t}) \in \mathbb{R}^{N+3}$. Similarly, define agent j's disagreement vector at time t as $\Delta_t^j = \left(0, 0, \Delta_z^j s_t, \Delta_1^j (1 - s_t), \ldots, \Delta_N^j (1 - s_t)\right)$. The disagreement vector of agent j captures how distorted the belief is about each shock in the Brownian vector w_t at time t.

A.2.1 Security markets

Agents can trade a locally risk-free asset in zero net supply, $N+1$ stocks, and two additional zero net supply derivatives. Since the market has $N+3$ shocks, the market is potentially complete. The locally risk-free asset follows

$$dB_t = r_t B_t dt, \tag{A.2.2}$$

where r_t is real short rate determined in equilibrium. In addition, since stock $n = 1, \ldots, N$ is a claim to $D_{n,t}$, the return dynamics of stock n is

$$dR_{n,t} = \frac{dP_{n,t} + D_{n,t}dt}{P_{n,t}} = \mu_{R_n,t}dt + \sigma'_{R_n,t}dw_t, \tag{A.2.3}$$

where $\sigma_{R_n,t} \in \mathbb{R}^{N+3}$. We solve for $\sigma_{R_n,t}$ and $\mu_{R_n,t}$ in equilibrium. We also assume that agents can trade a claim on the first Lucas tree, E_t, with return dynamics given by

$$dR_{E,t} = \frac{dP_{E,t} + E_t dt}{P_{E,t}} = \mu_{R_E,t}dt + \sigma'_{R_E,t}dw_t. \tag{A.2.4}$$

Besides the risk-free asset and the $N+1$ stocks, we assume that there are two zero net supply claims (derivatives) that agents can trade. The first derivative is linked to the shock to δ_t

$$dR_{w_\delta,t} = \mu_{w_\delta,t}dt + dw_{\delta,t}, \tag{A.2.5}$$

and the second derivative is linked to the shock to E_t,

$$dR_{w_E,t} = \mu_{w_E,t}dt + dw_{E,t}, \tag{A.2.6}$$

where $\mu_{w_\delta,t}$ and $\mu_{w_E,t}$ are determined in equilibrium. It is convenient to summarize the price system in terms of the stochastic discount factor. In our economy, agents have different beliefs, and therefore perceive different market prices of risk. Consequently, each agent perceives the stochastic discount factor differently. The dynamics of the stochastic discount factor as perceived by agent j is

$$dM_t^j = -r_t M_t^j dt - \theta_{E,t} M_t^j dw_{E,t} - \theta_{z,t}^j M_t^j dw_{z,t}^j - \sum_{n=1}^{N} \theta_{n,t}^j M_t^j dw_{n,t}^j. \tag{A.2.7}$$

Under the true measure the stochastic discount factor has the dynamics

$$dM_t = -r_t M_t^j dt - \theta_{E,t} M_t dw_{E,t} - \theta_{z,t}^j M_t dw_{z,t} - \sum_{n=1}^{N} \theta_{n,t} M_t dw_{n,t}, \tag{A.2.8}$$

where $\theta_{z,t}^j = \theta_{z,t} + \Delta_z^j s_t$ and $\theta_{n,t}^j = \theta_{n,t} + \Delta_n^j (1 - s_t)$. In equilibrium, we have $\mu_{i,t} = r_t + \theta_t' \sigma_{i,t}$ for $i = E, w_E, w_\delta, R_1, \ldots, R_N$. Hence, the expected return perceived by agent j is related to the expected return under the true measure by

$$\mu_{i,t}^j = \mu_{i,t} + \Delta_{z,t}^j s_t \sigma_{i,z,t} + \sum_{n=1}^{N} \Delta_{n,t}^j (1 - s_t) \sigma_{i,n,t}. \tag{A.2.9}$$

As noted above, if Δ_z^j (or Δ_n^j) is positive it implies that agent j is optimistic about z (or ϵ_n). In that case, we see from Equation (A.2.9) that the agent will also perceive a higher expected return provided that the loading of the asset $\sigma_{i,z,t}$ (or $\sigma_{i,n,t}$) is positive. Note that we can define the disagreement process of agent j by η_t^j that links the perceived stochastic discount factor of agent j, M_t^j, to the stochastic discount factor under the true measure, M_t, by $M_t^j = M_t/\eta_t^j$. The disagreement process is formally a Radon-Nikodym derivative with dynamics

$$d\eta_t^j = \Delta_z^j s_t \eta_t^j dw_{z,t} + \sum_{n=1}^{N} \Delta_n^j (1 - s_t) \eta_t^j dw_{n,t}. \tag{A.2.10}$$

A.2.2 Preferences

Agents maximize lifetime utility given by

$$\mathbb{E}^j \left[\int_0^\infty e^{-\rho t} \frac{c_{j,t}^{1-\gamma}}{1-\gamma} dt \right], \tag{A.2.11}$$

subject to the dynamic budget constraint

$$\begin{aligned} dW_t^j &= \left(r_t W_t^j + \pi_{E,t}^j \left(\mu_{E,t}^j - r_t \right) + \pi_{w_E,t}^j \left(\mu_{w_E,t} - r_t \right) \right) dt \\ &+ \left(\pi_{w_\delta,t}^j \left(\mu_{w_\delta,t} - r_t \right) + \sum_{n=1}^{N} \pi_{n,t}^j \left(\mu_{R_n,t}^j - r_t \right) - c_{j,t} \right) dt \\ &+ \pi_{w_E,t}^j dw_{E,t} + \pi_{w_\delta,t}^j dw_{\delta,t} \\ &+ \pi_{E,t}^j \sigma_{E,t}' dw_t + \sum_{n=1}^{N} \pi_{n,t}^j \sigma_{R_n,t}' dw_t, \end{aligned} \tag{A.2.12}$$

with $W_0^j = w^j$ and where $\pi_{i,t}^j$ for $i = E, w_E, w_\delta, 1, \ldots, N$ is the dollar amount invested in asset i by agent j. Note the expectation in Equation (A.2.11) and the dynamics of the wealth in Equation (A.2.12) are under the belief of agent j.

A.2.3 Equilibrium

We start by defining the equilibrium.

Definition 1. *Given preferences, endowments, and beliefs, an equilibrium is a collection of allocations*

$\left(c_{j,t}, \pi_{i,t}^{j}\right)$ *and a price system* $\left(r_t, \mu_{E,t}, \{\mu_{R_n,t}\}_{n=1}^{N}, \mu_{w_E,t}, \mu_{w_\delta,t}, \sigma_{E,t}, \{\sigma_{R_n,t}\}_{n=1}^{N}\right)$ *for* $j = 1, \ldots, J$ *and such that the processes* $\left(c_{j,t}, \pi_{i,t}^{j}\right)$ *maximize lifetime utility in Equation* (A.2.11) *subject to the dynamic budget condition in* (A.2.12) *and all market clear.*

Since the market is complete, we can solve the individual problem using Martingale methods as in Karatzas, Lehoczky, and Shreve (1987) and Cox and Huang (1989). The first-order conditions yield

$$c_{j,t} = \left(\kappa_j M_t / \eta_t^j e^{\rho t}\right)^{-1/\gamma}, \tag{A.2.13}$$

where κ_j is the Lagrange multiplier from the static optimization problem. The Lagrange multiplier is linked to the initial wealth of agent j. It is convenient to define $\lambda_{j,t} = \eta_t^j / \kappa_j$. By using the optimal consumption in (A.2.13) and the market clearing in the commodity market we have the following Proposition.

Proposition 1. *In equilibrium the optimal consumption of agent* $j = 1, \ldots, J$ *is*

$$c_{j,t} = f_{j,t} C_t, \tag{A.2.14}$$

where the consumption share, $f_{j,t}$*, is*

$$f_{j,t} = \frac{\lambda_{j,t}^{\frac{1}{\gamma}}}{\sum_k^J \lambda_{k,t}^{\frac{1}{\gamma}}}. \tag{A.2.15}$$

Moreover, the stochastic discount factor, M_t*, is*

$$M_t = e^{-\rho t} \left(\sum_j^J \lambda_{j,t}^{\frac{1}{\gamma}} \right)^{\gamma} C_t^{-\gamma}. \tag{A.2.16}$$

Proof. From the first order conditions we have

$$\lambda_{j,t} c_{j,t}^{-\gamma} = \lambda_{l,t} c_{l,t}^{-\gamma} \tag{A.2.17}$$

for any two agent j, l. Rearranging the above and using the clearing of the commodity market we get the result. $\square$

The next proposition shows the equilibrium real short rate and the market prices of risk.

Proposition 2. *The equilibrium real rate is*

$$r_t = \rho + \gamma\left(\mu_{C,t} + \sigma'_{C,t}\mathcal{E}_t(\Delta)\right) - \frac{1}{2}\gamma(1+\gamma)\sigma'_{C,t}\sigma_{C,t} + \frac{1}{2}\left(1 - \frac{1}{\gamma}\right)\mathcal{V}_t(\Delta) \tag{A.2.18}$$

and the market prices of risks are

$$\theta_t = \gamma\sigma_{C,t} - \mathcal{E}_t(\Delta) \tag{A.2.19}$$

where

$$\mathcal{E}_t(\Delta) = \sum_{j=1}^{J} f_{j,t}\Delta_t^j \tag{A.2.20}$$

is the consumption weighted average and

$$\mathcal{V}_t(\Delta) = \sum_{j=1}^{J} f_{j,t}\left(\Delta_t^j - \mathcal{E}_t(\Delta)\right)'\left(\Delta_t^j - \mathcal{E}_t(\Delta)\right) \tag{A.2.21}$$

is the consumption weighted average total variance of disagreement vector Δ_t^j

Proof. The results follow from an application of Ito's lemma on the stochastic discount factor, M_t, in (A.2.16) and matching the terms with the dynamics in Equation (A.2.8). $\square$

The risk-free rate is similar to the standard risk-free rate in a homogeneous beliefs economy with constant relative risk aversion (CRRA) preference with two exceptions. First, the true expected growth rate, $\mu_{C,t}$ is replaced with $\mu_{C,t} + \sigma'_{C,t}\mathcal{E}_t(\Delta)$. This turns out to be equivalent to the consumption share weighed average belief about the consumption growth. This is what Heyerdahl-Larsen and Illeditsch (2020b) refer to as the market view. Second, there is an additional term that depends on the consumption share weighted total variance of the disagreement vector, $\mathcal{V}_t(\Delta)$. This term is due to the speculative trade between the agents in the economy and the sign is linked to the value of γ. For $\gamma > 1$, the risk-free rate is higher when there is more disagreement in the economy. This is the channel explored in Ehling et al. (2018). We calculate stock price dynamic of stock n using Malliavin calculus. Specifically, it can be shown that the stock price loadings on the shocks are

$$\sigma_{R_n,t} = \theta_t + \frac{\mathbb{E}_t\left(\int_t^\infty \mathcal{D}_t\left(M_u D_{n,u}\right) du\right)}{\mathbb{E}_t\left(\int_t^\infty M_u D_{n,u} du\right)} \tag{A.2.22}$$

where $\mathcal{D}_t x_u = \left(\mathcal{D}_{E,t} x_u, \mathcal{D}_{\delta,t} x_u, \mathcal{D}_{z,t} x_u, \mathcal{D}_{1,t} x_u, \ldots, \mathcal{D}_{N,t} x_u\right)$ denotes the Malliavin derivative of x_u at time t. The next return dynamics is given by the next proposition.

Proposition 3. *The dynamics of stock $n = 1 \ldots, N$ is*

$$dR_{n,t} = \mu_{R_n,t} dt + \sigma'_{R_n,t} dw_t \tag{A.2.23}$$

where

$$\mu_{R_n,t} = r_t + \theta'_t \sigma_{R_n,t}, \tag{A.2.24}$$

and where

$$\sigma_{R_n,t} = \sigma_{D_n} + \theta_t + \frac{\mathbb{E}_t\left(\int_t^\infty M_u\left(\mathcal{E}_u\left(\mathcal{D}_t log\left(\lambda_u\right)\right) - \gamma \mathcal{D}_t log\left(C_u\right)\right) du\right)}{\mathbb{E}_t\left(\int_t^\infty M_u D_{n,u} du\right)}, \tag{A.2.25}$$

and where $\mathcal{E}_u\left(\mathcal{D}_t log\left(\lambda_u\right)\right)$ is the consumption share weighted average of the Malliavin derivative of the log disagreement process η_u.

Proof. To derive the result note that we have

$$M_t P_{n,t} + \int_0^t M_u D_{n,u} du = \mathbb{E}_t\left[\int_0^\infty M_u D_{n,u} du\right] \tag{A.2.26}$$

From equation (A.2.26) we have that the right hand side is a local martingale. An application of Ito's lemma to the left hand side of equation (A.2.26) yields

$$d\left(M_t P_{n,t} + \int_0^t M_u D_{n,u} du\right) = \ldots dt + M_t P_{n,t}\left(\sigma_{R_n,t} - \theta_t\right)' dw_t \tag{A.2.27}$$

For the right hand side of equation (A.2.26), we apply the Clark-Ocone theorem, implying

$$d\mathbb{E}_t\left[\int_0^\infty M_u D_{n,u} du\right] = \mathbb{E}_t\left[\int_0^\infty \mathcal{D}_t\left(M_u D_{n,u}\right) du\right]' dw_t \tag{A.2.28}$$

Next we calculate the Malliavin derivative in equation (A.2.28)

$$\begin{aligned}
\mathcal{D}_t\left(M_u D_{n,u}\right) &= D_{n,u}\mathcal{D}_t M_u + M_u \mathcal{D}_t D_{n,u} \\
&= D_{n,u}\mathcal{D}_t\left(e^{-\rho u}\left(\sum_j^J \lambda_{j,u}^{\frac{1}{\gamma}}\right)^{\gamma} C_u^{-\gamma}\right) + M_u D_{n,u}\sigma_{D_n} \\
&= D_{n,u}e^{-\rho u}\gamma\left(\sum_j^J \lambda_{j,u}^{\frac{1}{\gamma}}\right)^{\gamma-1} \frac{1}{\gamma}\sum_{j=1}^J \lambda_u^{\frac{1}{\gamma}}\mathcal{D}_t log\left(\lambda_u\right) + M_u D_{n,u}\sigma_{D_n} \\
&= M_u D_{n,u}\left(\sum_{j=1}^J f_u^j \mathcal{D}_t log\left(\lambda_u\right) + \sigma_{D_n}\right) \\
&= M_u D_{n,u}\left(\mathcal{E}_u\left(\mathcal{D}_t log\left(\lambda_u\right)\right) + \sigma_{D_n}\right)
\end{aligned} \tag{A.2.29}$$

Inserting equation (A.2.29) into equation (A.2.28), equating it with the diffusion coefficients in equation (A.2.27) and solving for the stock price diffusion coefficients $\sigma_{R_n,t}$ yields the result. □

As we are interested in the exposure of the agents' wealth to the shocks in the economy, we also need to find the wealth dynamics. The next proposition characterizes the wealth dynamics of the agents in the economy.

Proposition 4. *Let W_t^j be the wealth of agent j at time t with dynamics*

$$dW_t^j = \mu_{W^j,t} W_t^j dt + \sigma'_{W^j,t} W_t^j dw_t, \tag{A.2.30}$$

Then the exposures of agent j's wealth to the shocks, $\sigma_{W^j,t}$, are

$$\sigma_{W^j,t} = \theta_t + \frac{\mathbb{E}_t\left(\int_t^\infty \mathcal{D}_t\left(M_u C_{j,u}\right) du\right)}{\mathbb{E}_t\left(\int_t^\infty M_u C_{j,u} du\right)} \tag{A.2.31}$$

Proof. Note that we have

$$M_t W_t^j + \int_0^t M_u C_{j,u} du = \mathbb{E}_t \left(\int_0^\infty M_u C_{j,u} du \right) \tag{A.2.32}$$

Following a similar approach as for the stock price diffusions coefficients, i.e., apply Ito's lemma on the left hand side and Clark-Ocone theorem on the right hand side then equating the diffusion coefficients yields the result. □

From Proposition 4 together with the wealth dynamics in Equation (A.2.12) one can calculate the optimal portfolios. As we are interested in the economy wide loading on the factor shock $w_{z,t}$, we instead directly examine the total absolute exposure to the factor shock:

$$TE_{ETF,t} = \sum_{j=1}^{J} \mid \sigma_{W^j,z,t} \mid \tag{A.2.33}$$

A.3 Simulation details

In this section we provide complete details on the simulation of the model economy from Section A.2 that underlies the testable predictions in Figures 1 and 2 in Section 1.6.

Specifying beliefs. Our main numerical illustration of the model assumes that there are $J = 2N$ agents in the economy. We set $\Delta_z^j = \Delta > 0$ for agent $j = 1, \ldots, N$. This implies that half of the agents are *optimistic* about the growth rate of the factor (recall Equation (6)). We set $\Delta_z^j = -\Delta$ for the remaining agents, implying that half of the population is *pessimistic* about the factor's growth rate. While this assumption is not necessary, and is both discussed and relaxed below, it ensures that our baseline results are not simply driven by an ex ante imbalance between optimists and pessimists. Specifically, in terms of the stock-specific beliefs underlying Equation (9) we make the following assumptions. First, we assume that all investors are optimistic about half and pessimistic about the other half of the stock-specific components. Specifically, for investor j's belief about the stock-specific component of stock n we assume that

$$\Delta_n^j = \begin{cases} \frac{\Delta}{\sqrt{N}}, & \text{if optimistic} \\ -\frac{\Delta}{\sqrt{N}}, & \text{if pessimistic} \end{cases} \tag{A.3.34}$$

We normalize the stock-specific component by $\sqrt{N}$ to keep the total disagreement about the stock-specific component equal to that of the factor disagreement for $s_t = 0.5$. Second, we assume that for each stock there are just as many optimists as pessimists. Finally, we assume that for each optimist about the factor component, there is an investor with the same beliefs about the stock-specific components that is pessimistic about the factor component. In this way, we turn off any correlation between the structure of the beliefs in the economy about the factor and stock-specific disagreement.

Overall, as half of the agents in the economy are optimistic about the factor and half of the agents in the economy are pessimistic about the factor, we refer to high s_t times as periods of high *factor disagreement*. Similarly, when s_t is low, most disagreement surrounds the idiosyncratic component of dividends. As such, we label these times as periods of high *stock-specific disagreement*.

Other parameters. Each simulation considers an economy with ten stocks ($N = 10$) and a disagreement parameter $\Delta = 0.2$. Thus, there are $J = 2N = 20$ agents in the economy. Moreover, we assume that each agent starts with the same initial consumption shares. We solve the model using a social planner problem with Pareto weight $\frac{1}{\kappa_j}$ for agent $j = 1, \ldots, N$. Starting with the same initial consumption shares is equivalent to assuming the homogeneous Pareto weights. There is a mapping between the Pareto weights and initial wealth. We also assume that the dividend shares of each stock are the same, and that the total dividends are initially 5% of total consumption.

We define the ETF as the value-weighted portfolio of all the ten stocks. As we consider a frictionless market, there is no intrinsic demand for the ETF in the model since investors could, in principle, trade the individual stocks underlying the ETF. Hence, we assume that agents prefer to trade the ETF instead of the underlying stocks in the ETF if their agent's goal is to take on factor exposure.[15] We average all model-implied results across 400,000 simulated paths of monthly data that each span 100 years.

A.4 Different trading measures

In the main body of the paper, we consider the total factor exposure as our measure of flows and trading activity. In this section we examine two alternative measures. First, in the main body of the paper the exposure is the total dollar exposure. We can instead measure the exposure as the weighted average exposures of each of the investors in the economy. Specifically, we define the weighted exposure as WE_t where

$$WE_t = \sum_{j=1}^{J} f_{j,t}^{w} \left| \frac{\sigma_{W_j,z,t}}{W_{j,t}} \right| \tag{A.4.35}$$

In addition, we consider a measure based on the excess exposure as defined in Heyerdahl-Larsen and Illeditsch (2020a). Specifically, the excess exposure is defined as

$$XE_t = WE_t - |\sigma_{ETF,z,t}|. \tag{A.4.36}$$

The excess exposure captures the variation in wealth that is over and beyond that of a passive position. Finally, all the measure of exposures are monotonically increasing, so if we instead look at changes with respect to the factor disagreement share process, the measures also imply that the total exposure increases in the factor share process.

[15] For instance, if the ETF were an equal-weighted portfolio of the ten stocks in the economy, and agent j was holding one unit of each of stocks one to nine, and two units of stock ten, then we assume that the agent is holding one unit of the ETF and one additional unit of stock ten.

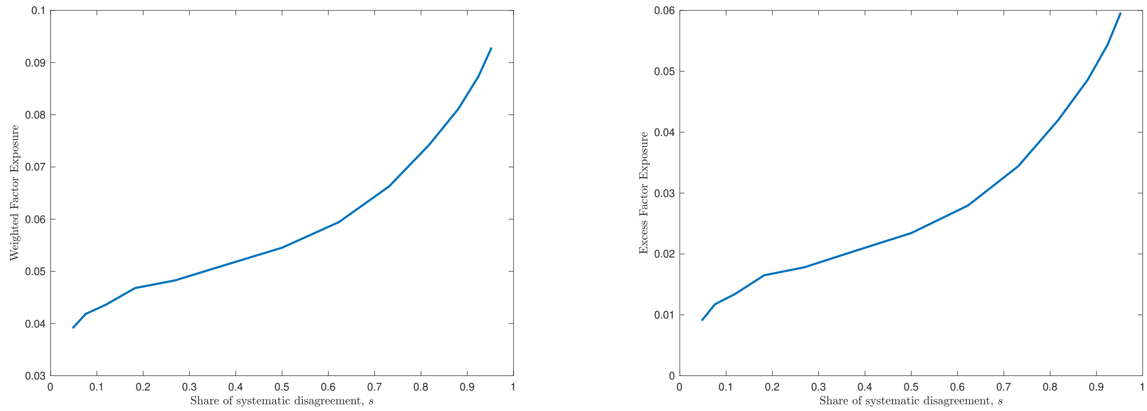

Figure A.4.2: The figure plots two measures of exposure for the baseline calibration. The left hand plot is the weighted exposures and the right hand plot is the excess exposure. Parameters are as in the baseline case.