Carbon Pricing and Green Finance in Clean Growth

Abstract

This paper develops a quantifiable dynamic general equilibrium model to uncover the roles of green finance on emissions reduction, production, innovation, and growth under scenarios with and without optimal carbon pricing. I model green finance as either green capital cost intervention that confers a cost advantage to green capital, or green innovation intervention that channels resources to foster green technology innovation. Quantitative results indicate that carbon pricing yields larger cuts in emission levels and intensities than green capital cost intervention, a finding consistent with reduced-form DiD event studies on a global sample of firms. To bridge the gap between the two in reducing emissions, the cost advantage for green capital needs to increase substantially. In optimal scenarios where carbon can be priced optimally, green finance is preferable to carbon pricing only in the presence of a strong innovation externality. Mapping model to data where carbon is underpriced, green finance complements carbon pricing in emissions reduction and achieving higher welfare gains. Additionally, advancements in green technology innovation can amplify the effectiveness of a carbon border adjustment mechanism.

Keywords: Growth and Environment; Carbon pricing; Green finance; Green technology innovation; Optimal policy; Carbon border adjustment mechanism

JEL: O44, Q54, F18, F42, G12

1 Introduction

Our world needs climate action on all fronts – everything, everywhere, all at once.
– UN secretary general, António Guterres

Addressing climate change demands multifaceted efforts on a global scale. Among the most widely adopted strategies are carbon pricing and green finance. Carbon pricing compels emitters to internalize environmental externalities through taxation or emission quotas. Green finance, broadly defined as sustainable interventions targeting environmental, social, and governance concerns, primarily operates through capital cost interventions or by channeling resources towards green technology innovation. However, its ability to achieve tangible environmental outcomes remains less understood. In light of these considerations, two questions arise: What roles does green finance play when carbon is not optimally priced, as of the implementation status of the current carbon pricing policy? Moreover, how would its roles change if carbon could be priced optimally?

I uncover the role of green finance in five steps. First, I provide empirical evidence on the environmental consequences of carbon pricing and green finance at the firm level. Second, I develop a structural model that incorporates carbon pricing, cost of capital intervention, and directed innovation, the primary mechanisms of these policies. Third, I analyze how these mechanisms translate into economic and environmental outcomes and quantify their contributions to emission levels, intensity, consumption, production, and welfare through the structural model. Fourth, I map the model into data to tackle the role of green finance when carbon is not priced optimally, as carbon is currently globally under-priced. Five, I discuss whether optimal environmental policy can be attained through carbon pricing without the involvement of green finance. To the best of my knowledge, this is the first paper that answers the role of green finance in influencing emissions reduction, innovation and growth, and welfare under scenarios with and without optimal carbon pricing.

Leveraging data on a global sample of firms, I begin with an empirical investigation of the increased prevalence of green finance and its distinct contribution to emissions compared to carbon pricing. Using DiD event study with multiple treatment periods method, I show that on the extensive margin, green finance has a smaller and sometimes ambiguous impact on reducing firm emission levels and emission intensities than carbon pricing.

I develop a dynamic general equilibrium model that incorporates carbon pricing and green finance. The model environment is a version of Eaton and Kortum (2002) (EK) Ricardian model, embedded with dynamic capital accumulation and directed innovation. My novelty lies in that, first, I design two types of capital: brown and green. Brown capital emits carbon and is thus subject to carbon pricing. Green capital, on the other hand, has no carbon impacts. Each type of capital has unique emission rates and follows an independent capital accumulation process. Second, I integrate directed innovation into the model, in that innovation happens in both the green technology sector and the brown technology sector, depending on the allocation of scientists. Scientists maximize profits in the current period rather than the discounted sum of future profits because of finite periods of monopoly rights. The laissez-faire allocation of research efforts is distorted because scientists do not capture the full social value of their innovation. This gives room to explore the second channel of green finance, the intervention in the allocation of research resources to advance green technology innovation, in optimal and non-optimal environments.

Exploring the mechanisms, the model implies that carbon pricing has a negative scale effect and a negative substitution effect on emission intensity, leading to an unambiguous reduction in emission intensity. The reduction in emission intensity, which captures emissions reduction per output unit, is termed the technique effect. On the contrary, the impact of green finance on the technique effect is ambiguous, due to its countervailing scale effect and substitution effect. The scale effect of green finance contributes to an increase in emission intensity because of the imperfect substitute between green capital and brown capital. The intuition lies in that a reduction in the cost of using green capital boosts not only the demand for green capital but also brown capital due to imperfect substitution between them in production. The substitution effect, where green capital replaces brown capital, always leads to a decline in emission intensity, though. Ultimately, which effect dominates hinges on the substitutability between green capital and brown capital. Unsurprisingly, more scientists working to advance green innovation always leads to lower emission intensity. Carbon pricing, as well as green finance capital cost intervention, has an indirect general equilibrium effect in triggering green technology innovation; only the green innovation intervention exerts a direct influence on green innovation.

Model quantification indicates that, when measuring both carbon pricing and green finance in the same policy extent of triggering a 1% cost difference between green capital and brown capital, carbon pricing is more effective in magnitude in reducing emission level and emission intensity than green capital cost intervention, yielding larger welfare gains. However, carbon pricing adversely affects real production in the short run when there is no advancement in green capital productivity. In the long run with green technology innovation, carbon pricing increases real production and yields substantial welfare gains.

I map a four-country version of my model to real data, to capture the current development status of carbon pricing and green capital cost intervention. The four countries represent the US, EU, China, and the rest of the world. I find that despite being less developed than carbon pricing, green finance complements carbon pricing in emissions reduction and achieving welfare gains, primarily due to the underpricing of carbon. However, for green finance to have an impact on par with carbon pricing, the cost advantage for green capital needs to rise substantially. The attainability of such a substantial gap in capital costs in real-life scenarios remains questionable.

The international context of my model allows me to speak to trade policy design to target emitters when global agreement on regulation stringency fails. Countries exhibit significant variation in their regulation efforts, as evidenced by the diverse carbon pricing rates depicted in Figure A.7. I design carbon border tariffs in light of the EU carbon border adjustment mechanism. Quantitative exercises show that, while carbon tariffs may reduce global emissions, they could potentially decrease welfare in the absence of changes in technology and innovation. However, when green technology innovation occurs, the results consistently demonstrate positive outcomes for both emissions and overall welfare under the carbon border adjustment mechanism.

In discussions of optimal environmental policy, green capital cost intervention can substitute for carbon pricing in addressing environmental externalities, provided that only the relative ratio, rather than the absolute levels, of green capital and brown capital is what matters for resolving environmental externalities in the economy. However, when innovation externality is present, green finance innovation interventions become indispensable to work in conjunction with optimal carbon pricing to reach optimal outcomes.

This paper connects to several strands of recent literature. First, I contribute to the line of literature that studies climate change and environmental policies through the lens of quantitative trade and spatial models, such as Arkolakis and Walsh (2023), Fan et al. (2023), Duan et al. (2021), Shapiro and Walker (2018), particularly studies focusing on the interaction between trade policy and climate policy, such as Weisbach et al. (2022), Farrokhi and Lashkaripour (2021), Hsiao (2022), Shapiro (2020). Building on this literature, I turn to examine the role of capital markets, and how heterogenous capital costs play a part in affecting the interaction between trade and environment.

Methodologically, this paper also relates to the literature applying directed technological change to study environment, such as Casey (2023), Acemoglu et al. (2023), Hemous (2016), Acemoglu et al. (2012). My contribution lies in investigating a fresh environmental tool, green finance, and its impact on advancing innovation in green technology for addressing climate change.

More broadly, this paper connects to the discussion on the merits of carbon taxes vs clean subsidies, including Behmer (2023), Borenstein and Kellogg (2023), Bistline et al. (2023).[1] I differ from these papers in that I focus not just on the environmental externality, but also on how carbon taxes and clean subsidies affect innovation and growth. By incorporating carbon taxes and clean subsidies in a richer context, I uncover their distinct roles more comprehensively and effectively.

Finally, this paper contributes to the burgeoning literature that studies environmental issues through asset pricing models or corporate governance, including Flammer (2021), Ľuboš Pástor et al. (2022), Caramichael and Rapp (2022), Daubanes et al. (2022). My distinct contribution is that I incorporate green finance in a quantifiable general equilibrium model, and provide the first evidence on how it compares to and can be of complement to carbon pricing in emissions reduction, production, technology innovation and growth.

2 Empirical Context and Motivating Facts

Carbon pricing, a climate change mitigation tool, has been in practice since the 1990s. This tool, implemented through carbon taxes or emission quotas (e.g., Emission Trading Schemes), compels the internalization of environmental externalities. Carbon taxes were first imposed in some Scandinavian countries in 1990, and the European emissions trading system, launched in 2005, has become the largest carbon market globally.

Despite being regarded as a highly effective tool to combat climate change (see a series of papers by Nordhaus, such as Nordhaus (2019), Nordhaus (2013)), less than a quarter of global greenhouse gas emissions are subject to any carbon pricing instrument as of 2023. Figure A.8 illustrates the progression in the adoption of carbon pricing instruments by jurisdictions from 1990 to 2023. Moreover, the highest carbon price in the globe remains below the estimated social cost of carbon by standard carbon models, as shown in Figure A.7, which plots the highest pricing instrument's cost in each jurisdiction. Table A.3 provides a detailed description of the adoption years of carbon taxes or emission trading schemes in various countries across the globe.

Green finance, a tool for both climate change mitigation and adaptation, has a relatively shorter development period than carbon pricing. Green finance is generally defined in terms of a sustainable regulation and investment policy that deploys regulatory tools and investment instruments to align with environmental, social, and governance considerations.

[1] Green finance encompasses a range of instruments, including but not limited to clean subsidies. In my structural modeling, however, I model green finance in the spirit of clean subsidies.

This includes green credit policies, green bonds, and all types of green initiatives and green subsidies (e.g., Inflation Reduction Act) that direct financial flows to support green technologies and sustainable resources.[2]

The working mechanisms of green finance are less clear than carbon pricing, but two mechanisms are well recognized: cost of capital intervention and channeling resources for green technology development. The cost of capital intervention mechanism is supported by studies such as Hong and Shore (2022), Hong et al. (2022), Caramichael and Rapp (2022), Pedersen (2023). On the discussion of channeling R & D resources to green technology, see Pless (2023), Huang and Kopytov (2023).[3]

In this section, I collect data from a global sample of firms spanning 57 countries, as in Table A.3. I then provide reduced-form evidence on the relative environmental effectiveness of these two environmental tools and examine the linkage between emission intensities and the capital market. To the best of my knowledge, there is no large sample empirical study relating carbon pricing to green finance, and comparing their distinct contributions to emissions reduction. This section fills this gap.

2.1 Data

Carbon is measured in CO2 equivalent tons that encompasses all greenhouse gas emissions in the whole paper unless noted otherwise. Firm-level emissions are measured as scope-1 carbon emissions provided by the Refinitiv LSEG database.[4] There are around 1600 firms from 50 countries spanning 2002 to 2022 in the sample. Among them, 477 firms are from the US, and 486 are located in Europe. The carbon pricing data for the 57 sample countries are from The-World-Bank (2022) carbon data Dashboard. It provides information on the year of implementing carbon tax or emission trading systems for each country,

[2]See a description of green financing at the United Nations Environment Program. Green finance tends to enjoy more public support than carbon pricing recently. Global sustainable investment reached $35.3 trillion in five major markets at the start of 2020, equating to 36% (larger than carbon pricing coverage of 15% in 2020) of all professionally managed assets across countries (Allianz (2021)), with an expectation to hit $53 trillion by 2025. The European Commission has established a Platform on Sustainable Finance, the G20 has formed a Sustainable Finance Working Group, and the OECD has launched a Centre on Green Finance and Investment. Green credit policies have been introduced and are rapidly evolving in China and India since 2012. The Inflation Reduction Act allocates two-thirds of the fiscal costs of the climate-related provisions towards investments in clean energy and energy efficiency by individuals.

[3]The allocation of proceeds detailed in Apple's annual green bond report, earmarked for renewable energy adoption and enhancing energy efficiency or low carbon design, exemplifies two key mechanisms of green finance, see Figure A.10.

[4]The Refinitiv LSEG database provides two sources of firm-level emissions data: one from the Carbon Disclosure Project and the other corrected by the LSEG carbon model. The main results in this section use the LSEG carbon model corrected measure, but they remain robust when using the other source of metrics.

year-specific carbon prices for each instrument measured in USD/ton, and associated carbon coverage of each instrument.

Firm-level ESG scores and other accounting data including total assets (TA), net property, plant, and equipment (PPE), market capitalization (Size), and total revenue (Rev) are from the Refinitiv database. I use the weighted average cost of capital to measure firm capital cost. It is constructed as: $WACC = C_E \times W_E + C_D \times W_D \times$ (1-tax) $+ C_{PS} \times W_{PS}$. The cost of equity C_E is measured by CAPM, the cost of debt C_D is measured by the yield to maturity, and the cost of preferred stock C_{PS} is computed by the dividend yield. The data for these variables are from StarMine, with observations from 2015 to 2022. Note that all of these measures, including the equity cost implied by the CAPM model, are ex-ante approaches. It is widely acknowledged in the finance literature that ex-ante measures are more effective in accurately capturing capital costs than ex-post methods.

For green finance policy instrument, I use green bond issuance as an instrument proxy due to a lack of access to other instrument data. Green bond issuance has become increasingly prevailing in the last decade, concurrent with the rise of green finance. In 2021, annual ESG bond issuance amounted to over $1 trillion, accounting for more than 10% of global new bond issuance, see Figure A.9. Hence it is pretty representative in revealing when a firm reacts to green finance.[5] I collect global green bonds' issuance data from Refinitiv. It has each green bond's characteristics, including issuers' information, year of issuance, domicile, and bond values. Note that the issuers are not just corporates, there are a lot of municipal/government-issued green bonds. For corporate issuers, I observe the year and the amount of their green bond issuance. I merge them with firm-level emissions and other accounting data using the issuer's ISIN numbers. When measuring a region's green investment, I include all types of green bonds issued in this region.

Summary statistics and more detailed data descriptions are in Appendix A.

2.2 Motivating facts

Fact 1: *Green finance has a smaller (and sometimes ambiguous) impact on reducing firm emission levels and emission intensities than carbon pricing.*

I use DiD with multiple-treatment-periods event study method proposed by Callaway

[5] A caveat lies in that this policy proxy may not be exogenous because of selection into being "green" issue. However, notice that the empirical analyses with carbon pricing and green finance focus only on extensive margins. Hence it's when a firm is treated that matters in such an empirical context, and the issuance of green bonds does reveal when a firm is being affected by green finance.

and Sant'Anna (2021) to document the extensive margin of these two policies on emission. Specifically, the average treatment effect of the treated (1) for Y (emission level or intensity) at e years after treatment is:

$$\mathrm{ATT}_e \equiv \mathbb{E}\left[Y_{i,g+e}(1) - Y_{i,g+e}(0) \mid G_i = g\right] \tag{2.1}$$

Where $G_i = g$ means group i is treated by either carbon pricing or green finance at year g.[6] I use the implementation of carbon taxes or ETS in a country as the treatment measure for carbon pricing. The treatment year is the year the country first implements a carbon tax or ETS.[7] For green finance, I use firm-level green bond issuance as the treatment measure. The treatment year is the year the firm first issues a green bond.[8]

On emission intensities, carbon pricing yields a more substantial decline in firm-level emission intensity compared to green finance, as in Figure 1. Shapiro and Walker (2018) takes a reduction in emission intensity as "the technique effect", carbon pricing can clearly stimulate a positive technique effect, while the influence of green finance on this effect remains ambiguous, even if it appears to be quicker in triggering this effect than carbon pricing. An intuitive reason supporting green finance triggers the technique effect more effectively is that companies that issue green bonds intend to use a large portion of the funds to develop green technology. For example, Apple uses most of its green bond proceeds to support innovative green technology.

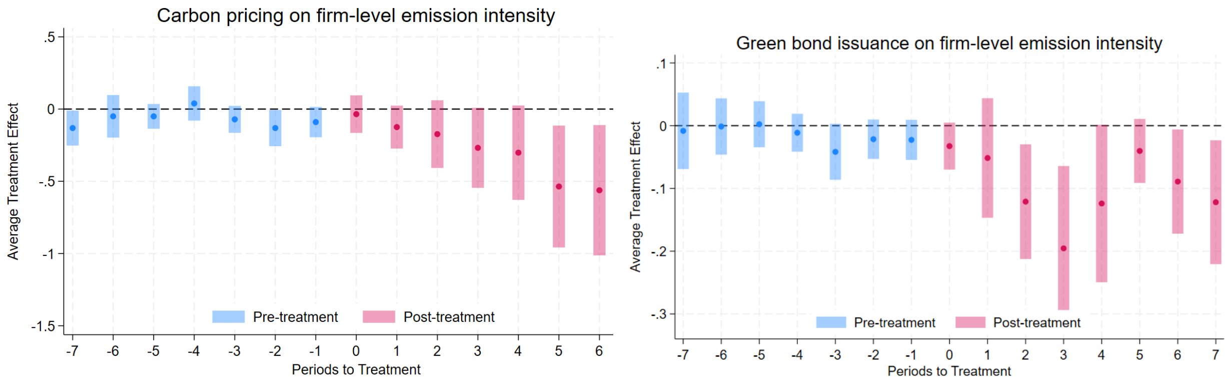
On emission levels, Figure A.11 shows that firms subject to carbon pricing policies exhibit a noticeable downward trend on emissions from the first year after implementation. In contrast, firms that have engaged in green finance activities exhibit a decline in emission levels only during the second and third years following their initiation, with no discernible long-term patterns of emissions reduction thereafter. Again, the reduction in emissions resulting from green finance is smaller in magnitude compared to the impact of carbon pricing. The patterns on emission intensities and levels are very robust, in that when I change to other measures of emission intensity or use scope-2(or -3) measures of emission

[6] In all results using this metric, I use the "Not-yet-treated" cohorts at time $g+e$ as the control group (0) to get the predicted change. The underlying parallel assumption using "not-yet-treated" as controls is $\mathbb{E}\left[Y_{i,t+e}(0) \mid G_i = g\right] - \mathbb{E}\left[Y_{i,g+b}(0) \mid G_i = g\right] = \mathbb{E}\left[Y_{i,g+e}(0) \mid G_i > g+e\right] - \mathbb{E}\left[Y_{i,g+b}(0) \mid G_i > g+e\right]$, where b is the b years after treatment.

[7] For example, in the US, the first ETS, the Regional Greenhouse Gas Initiative (RGGI), began in 2009, and there has never been a national carbon tax implemented. Therefore, the treatment year of carbon pricing for all firms in the US is 2009.

[8] For instance, during my observation years, Apple has issued three green bonds, with the first one issued in 2016, so Apple is treated in the year 2016.

Figure 1: Carbon pricing and green finance on firm-level emission intensity



Note: Emission intensity is measured as firm-level scope-1 emissions over firm-level total assets deflated by CPI, with the unit (kiloton CO2e)/(million USD).

level the results still hold. See Appendix A for more robustness tests.[9]

Fact 2: *Greener firms have lower costs of capital, this trend is robust controlling for industry, year, and country fixed effects.*

The descriptive scatters in Figure A.12 reveal a distinct negative association between greenness and capital costs. Firms ranking within the top quintile in environmental scores enjoy approximately a 4% - 5% cost of capital advantage over those in the bottom quintile in the US market.

I also regress capital costs on emissions using my sample:

$$WACC_{i,c,s,y} = \beta_0 + \beta_1 E_{i,c,s,y} + \gamma_c + \gamma_s + \gamma_y + \epsilon_{i,c,s,y} \tag{2.2}$$

Where $WACC_{i,c,s,y}$ is the weighted average cost of capital of firm i in country c industry s at year y. $E_{i,c,s,y}$ is an environmental performance measure, using environmental score or emission intensity, which is computed by firm-level CO2e emissions divided by (i) total asset, or (ii) gross property, plant, and equipment (PPE), or (iii) market capitalization, or (iv) total revenue (all in real value, deflated by CPI). The higher the environmental score or the lower the emission intensity, the greener the firm is.

Table 1 reinforces the link between greenness and costs of capital: firms with higher

[9] A caveat may arise that before being treated by carbon pricing (or green finance), firms might be treated by green finance (or carbon pricing). To address it, for carbon pricing, I keep those firms only treated by carbon pricing and never issued green bonds as my treatment groups; for green finance, I keep those firms that issued green bonds in regions never (or not yet in the year of green finance) implemented any carbon pricing tools as treatment groups. I get similar patterns.

emission intensity or lower environmental scores tend to exhibit higher weighted average costs of capital. This negative association remains robust across various measures of emission intensity and even after accounting for industry, country, and year fixed effects.

The observed negative correlation between greenness and capital costs suggests potential interventions that make greenness have come to influence capital use and internal decisions of firms. I will explore the linkage between carbon pricing, green finance interventions and firms' decisions on capital use through a structural point of view below.

Table 1: Weighted average cost of capital and greeness

	WACC				
Dependent Variable:	(1)	(2)	(3)	(4)	(5)
Environmental score	-0.007^{***}				
	(-4.38)				
$CO2e/totalasset$		0.089^{***}			
		(4.89)			
$CO2e/PPE$			0.055^{***}		
			(5.51)		
$CO2e/Size$				0.026^{**}	
				(1.97)	
$CO2e/Revenue$					0.032^{***}
					(3.58)
Cons	5.105***	6.905***	6.041***	6.214***	6.061***
	(5.97)	(5.58)	(5.54)	(5.78)	(5.65)
Country-FE	Yes	Yes	Yes	Yes	Yes
Industry-FE	Yes	Yes	Yes	Yes	Yes
Year-FE	Yes	Yes	Yes	Yes	Yes
N	8283	5601	6202	6481	6390
R^2	0.343	0.362	0.340	0.338	0.339

Note: WACC is the weighted average cost of capital. The units of the four emission intensity measures are (kiloton CO2e)/(million USD). Environmental scores are provided by Refinitiv. All regression results in the main text exclude financial firms (2-digit NAICS 52 or 53).

3 A Model with Carbon Pricing, Green Capital Cost Intervention, and Directed Innovation

I develop a dynamic general equilibrium model with carbon pricing, green capital cost intervention, and directed innovation. The model builds on an EK Ricardian multi-country framework, extended with two types of capital accumulation and directed technology change. There are two primary types of capital: green and brown, which are imperfect substitutes in production. Brown capital emits carbon, while green capital is carbon-neutral. Each period, finite researchers are allocated to work on improving green or brown capital productivity in the R & D sector. Environmental externality arises in the use of brown capital, and innovation externality emerges due to the effect of standing on the shoulders of giants.

3.1 Model setup

3.1.1 Production and directed innovation

In each region i, differentiated goods with variety ω is produced using technology $z_i(\omega)$, labor, and composite production capital:

$$y_i(\omega) = z_i(\omega) K_i^{\alpha_i} L_i^{1-\alpha_i} \tag{3.1}$$

Each variety draws an exogenous technology $z_i(\omega)$ from a Frechet distribution with location parameter T_i and shape parameter θ. The final good combines all varieties using a production function with constant elasticity of substitution σ. That is, $Y_i = \left[\int y_i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$.

The production capital K_i is produced by capital composite producer using raw green capital, raw brown capital purchased from households, and R & D resources (scientists) that can improve the productivity of the raw capital. The production capital is composed in a CES fashion:

$$K_i = \left[(E_{i,g})^{\frac{\epsilon-1}{\epsilon}} + (E_{i,b})^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{3.2}$$

with capital intermediates $E_{i,g}$, $E_{i,b}$ being produced by

$$E_{i,k} = A_{i,k,t} K_{i,k} \quad k \in \{g, b\} \tag{3.3}$$

where $K_{i,k}$ is the raw capital purchased from households. $A_{i,k,t}$ is the productivity for capital intermediate $E_{i,k}$; it is endogenous and decided by the equilibrium allocation of scientists.

Directed innovation. The productivity of capital intermediate $A_{i,k,t}$ evolves over time due to innovation. Scientists that innovate successfully over an intermediate raise that intermediate's productivity by a factor $\varphi > 1$, $A_{i,k,t} = \varphi A_{i,k,t-1}$. Scientists become the monopolist supplier of the intermediate. Assuming Bertrand competition, scientists are constrained by next-best technology, they set a limit price with a gross markup φ on the price of capital intermediate. The probability of success of innovation per scientist directed at capital type k is $\eta s_{i,k,t}^{-\psi}$, where $\psi > 0$ captures congestion effect and it rules out corner solution, η denotes research productivity.

The evolution of productivity of capital intermediate

$$A_{i,k,t} = \varphi^{\eta s_{i,k,t}^{1-\psi}} A_{i,k,t-1} \tag{3.4}$$

The balanced growth rate is $\varphi^{\eta s_{i,k,t}^{1-\psi}} - 1$. Notice from this expression, that if $\varphi = 1$ then the system shuts down innovation, hence shutting down growth. If $s = 0$, no innovation in technology k, the other technology type grows at $\varphi^{\eta} - 1$.

Scientists make the innovation decision by maximizing profits in the current period rather than the discounted sum of future profits, assuming the monopoly right lasts only 1 period. Expected return for innovation in capital intermediate $E_{k,t}$:

$$\Pi_{i,k,t} = \eta s_{i,k,t}^{-\psi} \left(1 - \frac{1}{\varphi}\right) P_{i,k,t} E_{i,k,t} \tag{3.5}$$

which is the probability of success times static profits. Hence, the price of capital intermediate: $P_{i,k,t} \equiv \frac{\varphi r_{i,t}}{A_{i,k,t}}$, where capital $K_{i,k,t}$ is provided by HH with the interest rate $r_{i,t}$, markup φ.

Each period, the total amount of R & D resources is fixed: $\sum_k s_{i,k,t} = 1 \quad \forall t$. The Laissez-faire allocation of scientists:

$$\frac{\Pi_{i,g,t}}{\Pi_{i,b,t}} = 1 \tag{3.6}$$

which is the innovation equilibrium, where scientists are driven by profits to the extent of being indifferent between investing in green and brown technology sectors.

The laissez-faire allocation of research efforts is distorted because scientists do not capture the full social value of their innovation, in that the spillover of current research on future innovation is uninternalized. This innovation externality exists when assuming the monopoly rights last more than one period, in that so long as monopoly rights last for finite periods and scientists are not able to capture the full future path of their innovation profits. Because the total amount of scientists is fixed and constant over time, only the relative allocation of scientists matters. Hence, one instrument is sufficient to address this externality even if it exists in both green and brown technologies.

Brown capital intermediate emits pollution, with emission rate ξ_b; green capital intermediate, on the other hand, does not emit carbon. The total emission: $D_{i,t} = \xi_{b,i} P_{i,b,t} E_{i,b,t}$. Hence, emissions intensity to output in a region is $\frac{D_{i,t}}{P_{i,t} Y_{i,t}}$, which will be an endogenous subject. Goulder et al. (2016) shows the equivalence between modeling carbon tax and emission trading quotas in a general equilibrium framework without policy uncertainty. Here I introduce carbon pricing in that when the government in region i implements carbon pricing, emissions will be priced at a carbon price $\tau^c_{i,t}$.

I model the first mechanism of green finance, the green capital cost intervention in that the use of green capital intermediate can be supported by a green capital cost advantage $g_{k,t}$. This way of modeling is in line with Joachim et al. (2023), though my paper yields very different quantitative results from this paper. Hence, the total cost for the use of brown capital intermediate is $(1+\tau^c_{i,t}\xi_{b,i})P_{i,b,t}E_{i,b,t}$, the total cost for the use of green capital intermediate is $(1-g_{i,k,t})P_{i,g,t}E_{i,g,t}$. Solving the problem of the capital intermediate producer under policy intervention, the final cost of capital composite $K_{i,t}$ is given by

$$P_{i,K,t} = \left[[(1+\tau^c_{i,t}\xi_{b,i})P_{i,b,t}]^{1-\epsilon} + [(1-g_{i,k,t})P_{i,g,t}]^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{3.7}$$

I introduce the second mechanism of green finance, green innovation intervention, by introducing subsidies to green technology innovation. The expected innovation profit of green innovation becomes $\Pi_{i,g,t}(1+g_{q,t})$, hence scientist allocation equilibrium changes to

$$(\frac{s_{g,t}}{s_{b,t}})^{\psi} = \frac{P_{g,t}E_{g,t}}{P_{b,t}E_{b,t}}(1+g_{q,t}) \tag{3.8}$$

in scenarios with green innovation intervention.

3.1.2 Trade

The unit price of an input bundle in region i is $x_i = (\frac{P_{i,K,t}}{\alpha_i})^{\alpha_i}(\frac{w_{i,t}}{1-\alpha_i})^{1-\alpha_i}$, with $P_{i,K,t} = \left[[(1+\tau^c_{i,t}\xi_{b,i})\frac{\varphi r_{i,t}}{A_{i,b,t}}]^{1-\epsilon} + [(1-g_{i,k,t})\frac{\varphi r_{i,t}}{A_{i,g,t}}]^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$ being where all policies kick in affecting trade.

Following Eaton and Kortum (2002), the Frechet structure implies that the share of goods purchased in location i from location n is given by:

$$\pi_{in} = \frac{T_n\left(x_n d_{in}\right)^{-\theta}}{\sum_k T_k\left(x_k d_{ik}\right)^{-\theta}} \tag{3.9}$$

The comparative advantage depends on Frechet location parameter T_n, which is exogenous; green and brown technology $A_{n,g,t}$ and $A_{n,b,t}$, which are endogenous and heterogenous across locations. The price index in a region i:

$$P_i^{-\theta} = \Gamma_i \sum_{n=1}^{N} T_n \left(x_n d_{in}\right)^{-\theta} \tag{3.10}$$

Before accounting for tariff, the market clearing is such that $X_i = w_{i,t}\bar{L}_i + P_{i,K}K_i$. For simplicity, trade is assumed to be balanced in each region i: $X_i = \sum_{n=1}^{N} \pi_{ni} X_n$. With tariffs, the total expenditure of a region becomes

$$X_i = w_{i,t}\bar{L}_i + P_{i,K}K_i + \sum_{n=1} \frac{\tau_{in}}{1+\tau_{i,n}} \pi_{i,n} X_i \tag{3.11}$$

Where the last term is the tariff revenue from importing from the world. Import for region i from region n is $M_{i,n} = X_i \frac{\pi_{i,n}}{1+\tau_{i,n}}$.

Trade balance condition becomes

$$\sum_{n=1} \frac{\pi_{i,n}}{1+\tau_{i,n}} X_i = \sum_{n=1}^{N} \frac{\pi_{n,i}}{1+\tau_{n,i}} X_n \tag{3.12}$$

This means for each region, the total import is equal to the total sales to the world.

3.1.3 Households

Households work and accumulate capital. Households are of measure zero.[10] I assume households are indifferent between green capital and brown capital and invest in them until the returns make them indifferent. Households solve the following problems:

$$\max_{\{C_{i,t}^H, K_{i,t+1}^g, K_{i,t+1}^b\}_{t=0}^{\infty}} U_{i,0} = \sum_{t=0}^{\infty} \beta^t \{\log\left(C_{i,t}^H\right) - \bar{\omega}\log\left(D_{i,t}\right)\}$$

$$\text{s.t. } P_{i,t}[C_{i,t}^H + K_{i,g,t+1} - (1-\delta)K_{i,g,t} + K_{i,b,t+1} - (1-\delta)K_{i,b,t}] = w_{i,t}\bar{L}_i + \varphi r_{i,t}(K_{i,g,t} + K_{i,b,t}) + TR$$

Static profits from the production side (owned by scientists or capital monopolist suppliers) are transferred to households. Household income includes wages, capital renting income, and profit. It is equivalent to $X_i = w_{i,t}\bar{L}_i + P_{i,g,t}E_{i,g,t} + P_{i,g,t}E_{i,g,t} = w_{i,t}\bar{L}_i + \varphi r_i(K_{i,g} + K_{i,b})$. I now assume government transfers TR change consumption by $P\Delta C = TR$, implied by government budget condition.

[10]Since households are of measure zero, they can not decide emission levels in the economy. Households take emission levels as given, that they do not internalize emissions into their consumption decision.

The solution for the above dynamic programming problem is:

$$C^H_{i,t} = (1-\beta)\left[\frac{\varphi}{\alpha_i} r_{i,t}/P_{i,t} + (1-\delta)\right](K_{i,g,t} + K_{i,b,t}) + \frac{\tau^c_{i,t}\xi_b P_{i,b,t}E_{i,b,t} - g_{i,k,t}P_{i,g,t}E_{i,g,t}}{P}$$
$$K_{i,g,t+1} = \beta\left[\frac{\varphi}{\alpha_i} r_{i,t+1}/P_{i,t+1} + (1-\delta)\right]K_{i,g,t} \tag{3.13}$$
$$K_{i,b,t+1} = \beta\left[\frac{\varphi}{\alpha_i} r_{i,t+1}/P_{i,t+1} + (1-\delta)\right]K_{i,b,t}$$

I provide the detailed derivation of these policy functions in Appendix B.

3.1.4 Government and welfare

The government in each region collects carbon revenues from firms. These tax revenues are reimbursed to firms in the direction of supporting their use of green capital, and to households in a lump-sum transfer.

Budget constraint of local government:

$$\tau^c_{i,t}\xi_b P_{i,b,t}E_{i,b,t} = g_{i,k,t}P_{i,g,t}E_{i,g,t} + TR \tag{3.14}$$

In decentralized equilibrium, TR has no impact on capital choice, but it will change household consumption C by $P\Delta C = TR$. Excluding tax rebates on household capital choice implies that tax rebate has no general equilibrium impact on the production side. In optimal cases, the planner considers the transfer's impact on capital choice.

Regional welfare in location i is also the welfare of households in this region:

$$U_{i,0} = \sum_{t=0}^{\infty}\beta^t\{\log\left(C^H_{i,t}\right) - \bar{\omega}\log\left(D_{i,t}\right)\} \tag{3.15}$$

Global welfare in the economy:

$$U_0 = \sum_{t=0}^{\infty}\beta^t\{\sum_i \log\left(C^H_{i,t}\right) - \bar{\omega}\sum_i \log\left(D_{i,t}\right)\} \tag{3.16}$$

Where I assume the elasticity of utility to consumption is 1, and the elasticity of utility to pollution (relative to consumption) is $\bar{\omega}$.

3.2 Equilibrium

I define the following equilibrium conditions.

Definition 1. **Sequential Equilibrium (decentralized).** A dynamic equilibrium is defined by a sequence of factor prices, goods prices, capital stocks, scientists allocation, and technology stocks $\{w_{i,t}, P_{i,K,t}, r_{i,t}, P_{i,t}, K_{i,k,t}, s_{i,k,t}, A_{i,k,t}\}_{i=1,t=0,k\in g,b}^{N,\infty}$, that solves (i) innovation equilibrium condition, scientists market clearing, the laws of motion for technologies; (ii) location price, trade flows, total expenditure, local market clearing; (iii) the capital accumulation condition; given initial technology levels, factor endowments, parameters and fundamentals (including τ^c, g_k).

Definition 2. **Balanced Growth Path.** Along the balanced growth path, all equilibrium variables grow at a constant rate. In particular, denote by g_y the growth rate of a generic variable y at the balanced growth path. At the balanced growth path the technology of type $k \in \{g, b\}$ grows at a rate $\varphi^{\eta s_k^{1-\psi}} - 1$.

Appendix B lays out the equilibrium system of all variables and equations along the balanced growth path. The appendix also shows how to detrend all the equilibrium variables and equilibrium conditions, namely, how to express them relative to their balanced long-run growth rates.

Definition 3. **Steady state without growth.** When there is no growth $\varphi = 1$, a steady state is characterized by time-invariant factors and goods prices, consumption, expenditure, capital stocks, scientist allocation, and technology stocks that satisfy the above sequential equilibrium conditions.

3.3 Dissecting mechanisms

To untangle the exact mechanisms linking carbon pricing and green finance to carbon emissions, technology innovation, and economic activities, I examine how carbon pricing, green capital cost intervention, and green intervention on R & D resource allocation affect emission intensity and green innovation.

Take a partial derivative of emission intensity to carbon pricing rate:

$$\frac{\partial(D_{i,t}/(P_{i,t}Y_{i,t}))}{\partial\tau^c} = \underbrace{\underbrace{\frac{\partial f}{\partial\tau^c}}_{<0}}_{\text{scale effect }-} + \underbrace{\underbrace{\frac{\partial f}{\partial K_g/K_b}}_{<0}\underbrace{\frac{\partial K_g/K_b}{\partial(\frac{1-g_k}{1+\tau^c\xi_{b,i}})}}_{<0}\underbrace{\frac{\partial(\frac{1-g_k}{1+\tau^c\xi_{b,i}})}{\partial\tau^c}}_{<0}}_{\text{substitution effect }-}$$

It indicates that carbon pricing has an unambiguous negative impact on emission intensity.

Take a partial derivative of emission intensity to green finance green capital cost shifter:

$$\frac{\partial(D_{i,t}/(P_{i,t}Y_{i,t}))}{\partial g_K} = \underbrace{\underbrace{\frac{\partial f}{\partial g_K}}_{>0}}_{\text{scale effect } +} + \underbrace{\underbrace{\frac{\partial f}{\partial K_g/K_b}}_{<0} \underbrace{\frac{\partial K_g/K_b}{\partial(\frac{1-g_k}{1+\tau^c \xi_{b,i}})}}_{<0} \underbrace{\frac{\partial(\frac{1-g_k}{1+\tau^c \xi_{b,i}})}{\partial g_K}}_{<0}}_{\text{substitution effect } -}$$

Green finance capital cost intervention yields an ambiguous impact on reducing emission intensity. This is due to its positive scale effect, in that falling user cost associated with the larger cost subsidies to green capital would increase demand in all inputs, including the use of brown capital, as green capital and brown capital are imperfect substitutes.

Green finance green intervention in technology innovation, on the other hand, can reduce emission intensity unambiguously:

$$\frac{\partial(D_{i,t}/(P_{i,t}Y_{i,t}))}{\partial(1+g_{q,t}))} = \underbrace{\underbrace{\frac{\partial f}{\partial K_g/K_b}}_{<0} \underbrace{\frac{\partial K_g/K_b}{\partial(s_{g,t}/s_{b,t})}}_{>0} \underbrace{\frac{\partial(s_{g,t}/s_{b,t})}{\partial(1+g_{q,t})}}_{>0}}_{-}$$

This is because advancements in green innovation associated with more scientists being allocated to the green technology sector always reduce emission intensity $\frac{\partial(D_{i,t}/(P_{i,t}Y_{i,t}))}{\partial(s_{g,t}/s_{b,t})} < 0$.

A crucial aspect is to understand how carbon pricing, green capital cost intervention, and green innovation subsidy impact green technology innovation. Notice:

$$d\log\left(\frac{s_{g,t}}{s_{b,t}}\right)^{\psi} = dlog(1+g_{q,t}) + d\log\left(K_{g,t}/K_{b,t}\right) = \underbrace{d\log(1+g_{q,t})}_{\text{Direct Effect}} + \underbrace{(-\epsilon)\times d\log\left((1-g_{k,t})/(1+\tau_t^c \xi_b)\right)}_{\text{General Equilibrium Effect}}$$

Carbon pricing and green capital cost intervention influence technology innovation through general equilibrium forces by altering producers' capital decisions, whereas green innovation subsidy directly impacts technology innovation. The above expression will further yield $\frac{\partial(s_{g,t}/s_{b,t})}{\partial g_{k,t}} > 0$, $\frac{\partial(s_{g,t}/s_{b,t})}{\partial \tau_t^c} > 0$, $\frac{\partial(s_{g,t}/s_{b,t})}{\partial g_{q,t}} > 0$. Therefore, each channel of these two policies has the potential to drive advancements in green technology innovation.

The detailed derivations for dissecting mechanisms are listed in Appendix B.

4 Optimal Environmental Policy

4.1 On the environmental externality

Focus only on addressing environmental externality, what's the role of green finance if carbon pricing is available and can be placed at any level? I derive the following proposition to tackle the role of green finance in the world with optimal carbon pricing under scenarios with only environmental externality.

Proposition 1. Ignoring innovation externality, addressing environmental externality requires carbon pricing and green capital cost intervention to satisfy *either*

1. Carbon price rate is at the optimal rate $\tau_t^c = \frac{-U'_{D,t}D'_{K_b,t}/U'_{C,t}}{\xi_b \partial Y_t/\partial K_{b,t}} = \frac{\varpi/K_{b,t}C_t}{\xi_b \varphi r_t/(P_t\alpha)}$ when there is no green capital intervention;
2. Green capital intervention is at the optimal rate $g_{k,t} = \frac{-U'_{D,t}D'_{K_b,t}/U'_{C,t}}{\partial Y_t/\partial K_{g,t}} = \frac{\varpi/K_{b,t}C_t}{\varphi r_t/(P_t\alpha)}$ when there is no carbon pricing;
3. Carbon price rate and green capital intervention satisfy $\tau_t^c \xi_b \frac{\partial Y_t}{\partial K_{b,t}} + g_{k,t}\frac{\partial Y_t}{\partial K_{g,t}} = \frac{-U'_{D,t}D'_{K_b,t}}{U'_{C,t}} = \frac{\varpi C_t}{K_{b,t}}$;

to achieve the optimal mix of green and brown capital in all countries.[11]

The detailed proof for Proposition 1 is in Appendix B. The intuition behind the proof is that the optimal intervention to address environmental externality is to make the shifter of carbon pricing or green finance to the marginal product of green or brown capital exactly offsets the environmental externality wedge in the social planner problem, taking into account capital decisions are predetermined.

Cases 1 and 2 demonstrate that optimal carbon pricing and optimal green finance capital cost intervention, with policy rates set to levels sufficient to address environmental externality independently, can act as substitutes for each other. The intuition is that carbon pricing and green finance capital cost intervention essentially in essence are cost shifters to the user costs of brown capital and green capital. They both influence the relative utilization of green capital to brown capital, affecting the relative ratio of green capital to brown capital.

In my model framework, the social planner problem hinges on the ratio of green to brown capital as the determinant of optimal outcomes in addressing environmental externalities.

[11] $\partial Y_t/\partial K_{b,t} = \frac{\varphi r_t}{\alpha P_t} = \partial Y_t/\partial K_{g,t}$ is the marginal product of green and brown capital under no intervention. $-U'_{D,t}D'_{K_b,t}/U'_C$ represents brown capital utility damage, measured as the marginal utility change resulting from the negative environmental externality by the use of brown capital, relative to the marginal utility of consumption.

The rationale for substitution lies in their alike ability to elevate the relative utilization of green capital over brown capital. However, if the separate levels of green capital and brown capital become critical in the social planner problem, this proposition may no longer hold.

Lemma 1. *Green capital cost intervention reduces the necessary optimal carbon pricing rates to address environmental externality.*

Proof for Lemma 1 can be seen easily from Proposition 1 case 3.[12]

4.2 Green technology innovation and innovation exnternality

To address only the environmental externality, the role of green finance capital cost intervention is not indispensable so long as carbon pricing is available and can be effectively placed at desired levels. However, when innovation becomes critical and market failures persist within the technology and innovation sector, how might that impact the role of green finance, especially the presence of green innovation intervention?

I introduce a new externality to capture market failures associated with innovation. The Laissez-faire allocation of scientists $\frac{\Pi_{g,t}}{\Pi_{b,t}} = 1$ is such that:

$$(\frac{s_{g,t}}{s_{b,t}})^{\psi} = \frac{P_{g,t}E_{g,t}}{P_{b,t}E_{b,t}} \tag{4.1}$$

This allocation is distorted, as scientists can not capture the full social value of their innovation, so the spillover of current research on future innovation is uninternalized.

Social planner desired allocation of scientists is

$$(\frac{s_{g,t}}{s_{b,t}})^{\psi} = \frac{\sum_{u=0}^{\infty} \frac{1}{1+r_{t+u}} P_{g,t+u}E_{g,t+u}}{\sum_{u=0}^{\infty} \frac{1}{1+r_{t+u}} P_{b,t+u}E_{b,t+u}} \tag{4.2}$$

Because of the total research resources are fixed, only the relative allocation of research resources matters. To address this externality, green finance green innovation intervention that works on channeling R & D can play a role, as in Proposition 2.

Proposition 2. Accounting for innovation externality, the optimal climate policy requires

[12] Notice in all 3 cases, the right-hand-side subjects are endogenous. Case 1 or 2 introduces one new unknown (τ^c with $g_k = 0$, or g_k with $\tau^c = 0$) and one new equation, while case 3 allows two new unknowns (τ^c and g_k) but with only one new equation to pin down them. In case 3 one can set g_k in the range of $[0, 1]$ and discretize the value then pin down each associated optimal τ^c sequentially.

1. Carbon pricing and green capital cost intervention are at optimal levels as in Proposition 1.

2. A green innovation subsidy such that

$$\frac{P_{g,t}E_{g,t}}{P_{b,t}E_{b,t}}(1+g_{q,t}) = \frac{\sum_{u=0}^{\infty}\frac{1}{1+r_{t+u}}P_{g,t+u}E_{g,t+u}}{\sum_{u=0}^{\infty}\frac{1}{1+r_{t+u}}P_{b,t+u}E_{b,t+u}}.$$

It can be proved that so long as $\frac{K_{g,t+u}}{K_{b,t+u}} > \frac{K_{g,t}}{K_{b,t}}$ then $g_{q,t}$ is positive.

The intuition lies in that green innovation subsidy today crows out future undesirable brown innovations that would be made otherwise, and this crowd-out benefit is not internalized by the private sector due to finite periods of patent rights.

5 Quantification

5.1 Mapping model to data

I map the model to the most recently available data. Each time in my model corresponds to one year, I calibrate the discount rate $\beta = 0.98$, in line with DICE Nordhaus (2013). I calibrate the trade elasticity $\theta = 4.5$, within the range of trade literature, such as Caliendo and Parro (2015). The capital depreciation rate is $\delta = 0.025$, corresponding to 40 years of usage. Labor shares of different markets are taken from Our World in Data database. The elasticity of substitution between green capital and brown capital, ϵ, is assigned a value of 4 in main counterfactuals. This number is consistent with values assigned in Acemoglu et al. (2012), and a value greater than 1 also aligns with the substitution role between fossil fuels and clean energy in integrated assessment models, such as Cruz and Rossi-Hansberg (2021). The green capital share in each region's gross capital value, ζ, is calibrated as the share of renewable energy consumption over total final energy consumption, documented in the World Bank database. I map the country-specific brown capital emission rate to align with carbon intensity of oil equivalent energy use data in the World Bank. Innovation related parameters: $\varphi = 1.07$, $\eta = 1.4634$, from Acemoglu et al. (2023).

In Section 5.2 when quantifying the decentralized economy to understand the role of green finance in the context of the current carbon pricing policy, I map carbon price rates and green finance capital cost advantage rates to real data. When measuring the country-level effective carbon price rates. τ_i^c, I use a model implied relationship between carbon revenues over gross output, effective carbon price rates, output emission intensity e_n, and brown capital emission intensity $\xi_{b,i}$:

$$\frac{\text{carbon revenues}}{\text{gross output}} = \frac{\tau_i^c \xi_{b,i} r_i^b K_i^b}{P_i Y_i} = \tau_i^c IN_i$$

The left-hand side is the carbon revenue ratio and OECD-PINE documents carbon revenues over GDP in each region. The right-hand-side involves output emission intensity $e_n = \frac{e^b r_n^b K_n^b}{p_n y_n}$, and the World Bank database provides emission intensity at the country production level, measured in carbon quantity over the monetary value of output (e.g., ton/USD). This yields me values of effective carbon price rates within the price range of the various pricing instruments implemented in those countries; the order of calibrated carbon price rates of countries also aligns with their respective positions in The-World-Bank (2022).

When estimating the cost of capital differences, I employ asset pricing models to identify the cost of capital gap between brown and green capital in each market. I sort the firms in each country into 10 portfolios based on their ESG scores, and take the difference between the highest decile and lowest decile, that is, the average cost of capital of firms in the highest decile of ESG scores minus the average cost of capital of firms in the lowest decile of ESG scores. I test the robustness using other sorting strategies, including sorting on their emission intensity measured by emission per unit of real value total asset (or real value PPE, real value total sales). The advantage of using the portfolio sorting approach lies in that, portfolio characteristics are relatively robust, while individual firm-level capital price or individual capital-level price is extremely noisy. This approach yields costs of capital gaps similar to asset pricing studies on comparing asset returns between polluting firms and clean firms, such as Bolton and Kacperczyk (2021), Hsu et al. (2023).

I use the hat algebra developed in Dekle et al. (2008) to solve my model, assuming the economy is in steady state when innovation and growth are absent, or in a balanced growth path with growth. I quantify the effects of a policy by comparing an economy with it relative to the other one without it. This involves two scenarios. In the first scenario, I calibrate the policy rate in each market with a value of the counterfactual interest. In the second scenario, I assume this policy rate is removed, while keeping everything else identical to the first scenario. That is, what the economy would be if the policy is absent while keeping all other fundamentals exactly as they are? By comparing the economy with a policy versus a counterfactual economy in which it is removed, I measure the environmental as well as the overall economic consequences of this policy. The computation detail is in Appendix C.

5.2 Counterfactuals

I first perform counterfactual exercises to understand the mechanism difference between carbon pricing and green finance. Then, I conduct a series of counterfactual exercises to unravel the role of green finance in the current carbon pricing implementation status. Lastly, I discuss the carbon border adjustment mechanism using my structural model framework.

5.2.1 Quantifying model mechanisms

To make carbon pricing and green finance cost intervention comparable when assessing their distinct impacts on the economy, I design a scenario when carbon pricing and green finance cost intervention are implemented to the extent that they both trigger green capital to be 1% more affordable than brown capital[13]. In this scenario, the two policy channels result in an identical market equilibrium ratio of green capital and brown capital when eliminating technology differences and innovation. However, they will still affect capital levels differently, implying distinct impacts on economic activities. Structural parameters in this context are calibrated to the US economy.

Figure 2 shows that even if each of the policies is implemented to the extent of creating a 1% cost difference between green capital and brown capital, their impacts on emissions, production, and welfare vary substantially in magnitude and sometimes even direction. Carbon pricing brings a more substantial decline in emission levels and intensity, resulting in a greater welfare gain. However, the impact of green finance is comparatively smaller. It is important to note that carbon pricing decreases production when shutting down the technology channel, while green finance green capital cost subsidy increases it on the other hand. Leveraging green finance alongside carbon pricing can significantly enhance pollution reduction while minimizing declines in economic output, leading to greater welfare gains.

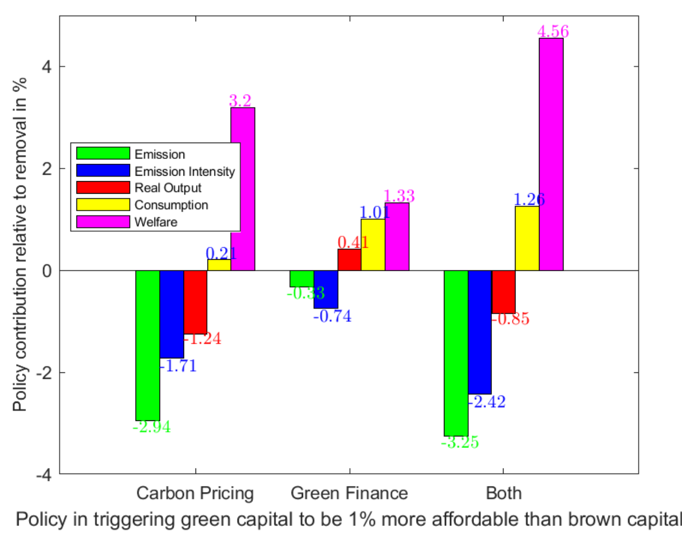
When accounting for technology differences and innovation, a scenario highly likely in the long run as innovation becomes consequential, the impacts of carbon pricing and green finance are altered compared to scenarios where technology differences and innovation are ignored. Figure 3 shows that, if innovation only occurs in the green technology sector, it amplifies the impacts of carbon pricing and green finance policies on emission levels and intensities. It also alters the effect of carbon pricing on real production; when green innovation is considered, the same level of carbon pricing implementation increases real

[13] That is, $\frac{(1-g_{k,i})}{(1+\tau_i^c \xi_{b,i})} = \frac{0.99}{1}$ by either carbon pricing τ_i^c or green finance capital cost shifter $g_{k,i}$, with i for the US. Consequently, the market equilibrium green capital and brown capital ratio will become $\frac{K_g}{K_b} = (\frac{A_g}{A_b})^{\epsilon-1}[\frac{(1-g_k)}{(1+\tau^c \xi_b)}]^{-\epsilon}$, as discussed in Section 3.3.

output instead of decreasing it. In the opposite case when innovation takes place only in the brown technology sector, it inadvertently reverses the effects of carbon pricing and green capital cost interventions on emissions, despite leading to an increase in real output that is more significant compared to when there is only green innovation.

In scenarios where both carbon pricing and green finance are jointly implemented at comparable levels, their combined gains initially appear to align almost linearly with the sum of their individual contributions, as depicted in Figure 2. However, this relationship cannot hold when technological innovation occurs disparately across the economy. Comparing column 6 and column 9 with column 3 in Figure 3, it becomes evident that the joint benefits derived from a 1% implementation of carbon pricing in conjunction with a 1% green capital cost intervention do not mirror the aggregated gains of their individual implementations. This $1 + 1 \neq 2$ scenario implies carbon pricing and green capital cost intervention may have nonlinear interaction when accounting for their contribution to technological change.

Figure 2: Quantifying mechanisms of carbon pricing and green finance(no innovation)

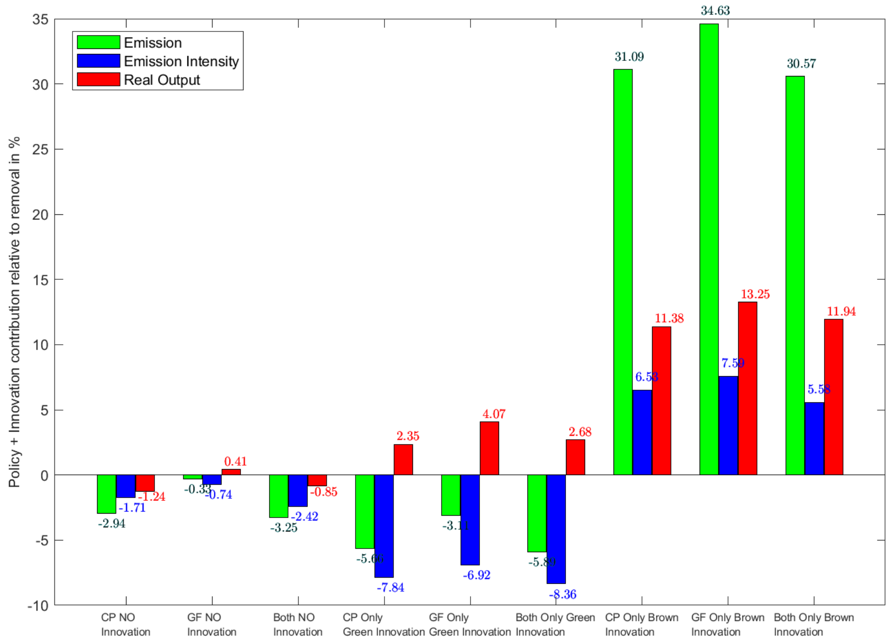


Note: Each policy is implemented to the extent of creating a 1% cost gap between green capital and brown capital. Numbers are comparisons from the model with policy being implemented to the above extent versus a counterfactual model where the policy is removed. This environment shuts down technology differences and innovation.

5.2.2 Green finance in current carbon pricing implementation status

What's the role of green finance when carbon is not priced optimally, as of the current implementation status of global carbon pricing policy?

Figure 3: Quantifying mechanisms of carbon pricing and green finance(with innovation)



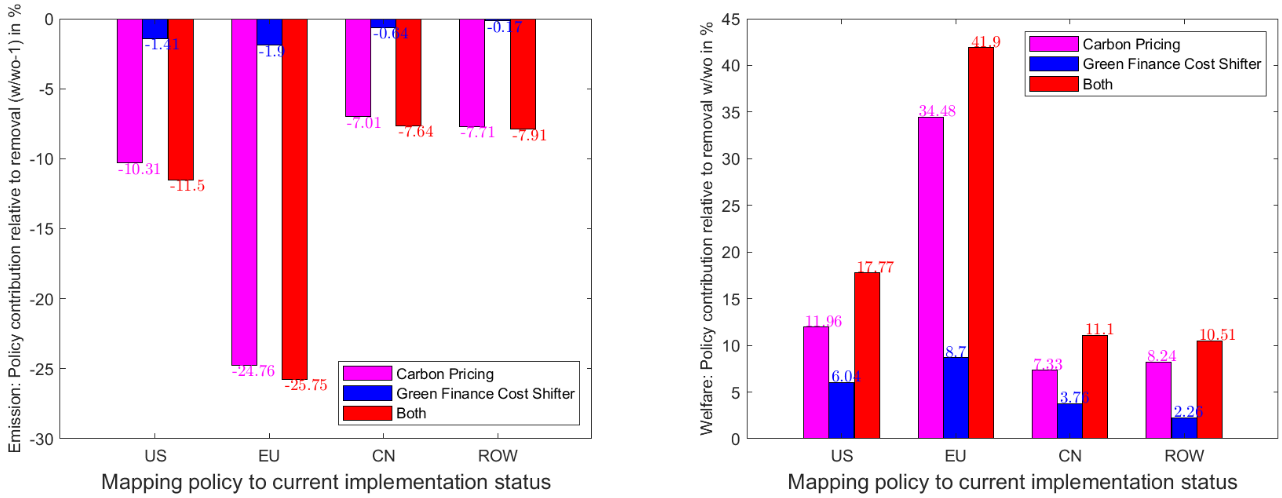
Note: Carbon pricing or green finance capital cost intervention is implemented to the extent of creating a 1% cost difference between green capital and brown capital. The numbers are comparisons from the model with policy being implemented to the above extent versus the counterfactual model where the policy is removed.

To answer this question, I map the carbon price rates and green finance cost intervention to current data, as calibrated in Section 5.1. I focus on carbon pricing and green finance capital cost intervention.[14] Specifically, I conduct counterfactual analyses by comparing an economy with carbon price rates (or green capital cost advantages) implemented to the levels observed in the data, against a counterfactual economy in which they are removed. I further perform another counterfactual in which both carbon price rates and green capital cost advantages are removed.

Figure 4 indicates that deploying green finance cost intervention along with carbon pricing does lead to larger reductions in emissions and greater welfare improvements compared to relying on carbon pricing alone. Green finance itself yields a smaller impact relative to carbon pricing. This is not only due to mechanism differences but also because the development status of current carbon pricing is more advanced compared to green

[14]Due to data availability and the difficulty in measuring the extent to which green finance intervention advances green technology innovation from data, quantifying the impacts of the current green innovation intervention is left to future work.

Figure 4: Emission and welfare change contributed by carbon pricing and green finance



Note: The numbers are comparisons from the model with policy as of data versus the counterfactual model where the policy is removed.

finance intervention. The impacts of carbon pricing and green finance differ across countries, given country-level heterogeneity in policy implementation status and the capital structures of production.

A novel question worthy of addressing is, given the implementation status of the current carbon pricing policy, how big should the green capital cost advantage be given to yield an environmental impact on par with it?

To address it, I vary the green capital cost intervention and resolve the model to get the corresponding emission change for the case of the US. The result is in Figure 5. To achieve an environmental impact equitable to carbon pricing, the green capital cost advantage needs to increase substantially. Specifically, my calibrated green capital cost advantage in the US is around 4%, and my model shows that it helps to reduce emissions by 1.41%. This green capital cost advantage must increase to around 50% to align with the 10.31% emissions reduction achieved through carbon pricing (with a calibrated around 6 USD/tCO2e country-level effective carbon price rate).

5.2.3 Carbon border adjustment mechanism

Carbon emissions have global consequences, yet countries exhibit significant variation in the stringency of their regulation, as evidenced by the diverse carbon pricing rates depicted in Figure A.7. When some countries free-ride and put minimal effort into domestic regulation, the international community can intervene by targeting emitters with import tariffs, such as carbon border adjustments tariffs. The European Union adopted the Carbon Border Adjustment Mechanism (CBAM) in 2023, with plans to fully implement it in 2026.

Figure 5: Green finance cost of capital intervention and corresponding emission change

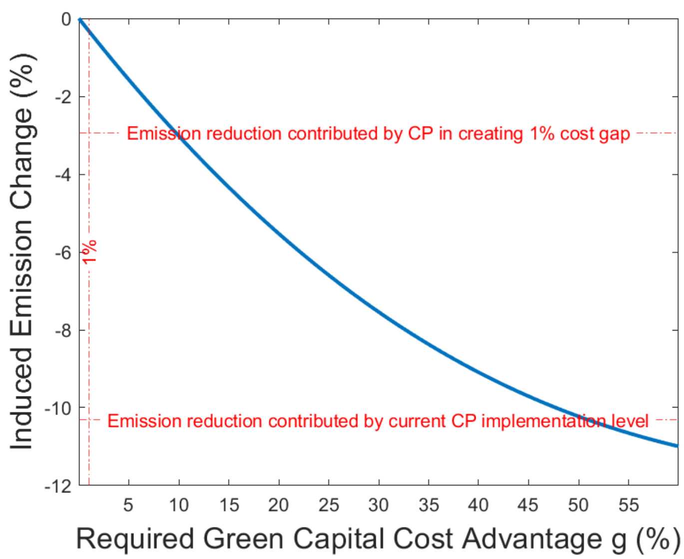
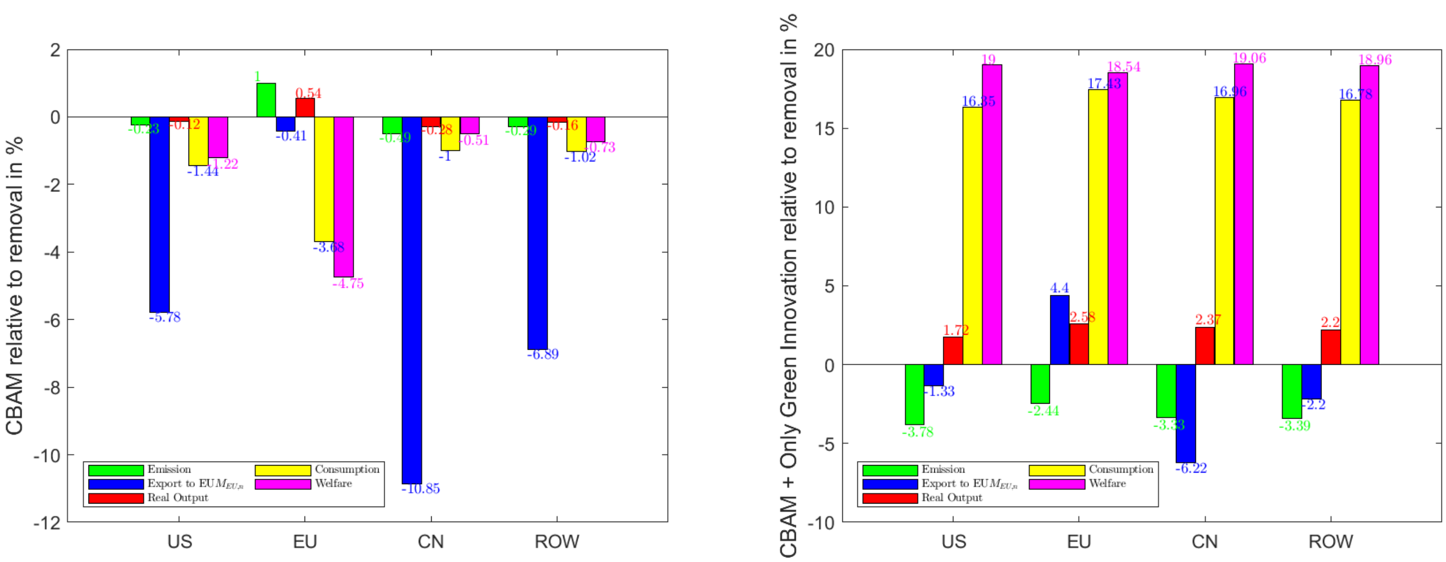


Figure 6: Carbon border adjustment mechanism simulation



Note: The numbers on the left figure are comparisons from the model with CBAM adjusting for the carbon price difference between the source country and the EU versus the counterfactual model where CBAM is removed. The numbers on the left figure are comparisons from the model with CBAM adjusting for the carbon price difference between the source country and the EU, as well as green innovation happening globally versus the counterfactual model where CBAM is removed.

My model can provide insights into the potential impacts of the carbon border mechanism on the global economy. I design a simple CBAM in which the EU sets carbon tariff to adjust for the carbon price difference between the source country and the EU.[15] I compare a counterfactual economy with CBAM working on imports entering the EU and a baseline economy in which it is removed, in a scenario shutting down technology and innovation, and a scenario where innovation happens only in the green technology sector.

Figure 6 shows that, when coupled with green technology innovation, the carbon border adjustment mechanism lowers emissions and intensities, delivering significant welfare gains. However, without innovation, the mechanism lowers emissions for all countries except the EU, although global emissions are decreasing. The rise in emission levels of the EU is due to its rise in production, and the model's oversight of trade in intermediate goods, and the lack of input-output linkage. In such a parsimonious international structure, tariffs decrease EU imports, diminish consumer consumption, and lead to welfare decline. These numbers undoubtedly overstate the consumption and welfare losses while underestimating the benefits of emissions reductions. Nevertheless, comparing the outcomes with and without green innovation underscores the significance of considering innovation in assessing the impacts of global climate agreements.

6 Conclusion

This paper asks a novel question on the role of green finance in a world without and with optimal carbon pricing, motivated by the increased prevalence of green finance. I develop a structural model with carbon pricing, green finance capital cost intervention, and green innovation intervention to capture the primary working mechanisms of carbon pricing and green finance. I analytically examine and quantify these mechanisms to understand how they convert into concrete environmental and economic outcomes, in impacting emissions, innovation, output, and welfare.

I highlight that the role of green finance in relation to carbon pricing—whether it is complementary, substitutable, or redundant—depends on the specific context they are in. When carbon is under-priced, green finance plays a role in complementing carbon pricing to further reduce emissions, increase production, and raise welfare. When carbon can be otherwise priced optimally, green finance capital cost intervention can lower the optimal

[15]Specifically, the source country carbon price is τ_i^c, the carbon component incorporated in the goods entering the EU is $IN_i X_{EU,i}$, with IN_i being emission intensity to output of region i, $X_{EU,i}$ trade flow. CBAM imposes carbon tariffs with rates equal to $(\tau_{EU}^c - \tau_i^c)IN_i$ (unit $\frac{usd}{t}\frac{t}{usd}$) on these goods.

carbon pricing rate to address environmental externality. If the carbon pricing rate is at the optimal level that it suffices to address environmental externality, there is no room for green finance capital cost intervention; but when innovation externality is present, green finance intervention in innovation becomes indispensable in complementing optimal carbon pricing to achieve optimal policy outcomes.

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Appendix

A Empirical Details

Descriptive data, figures, and tables

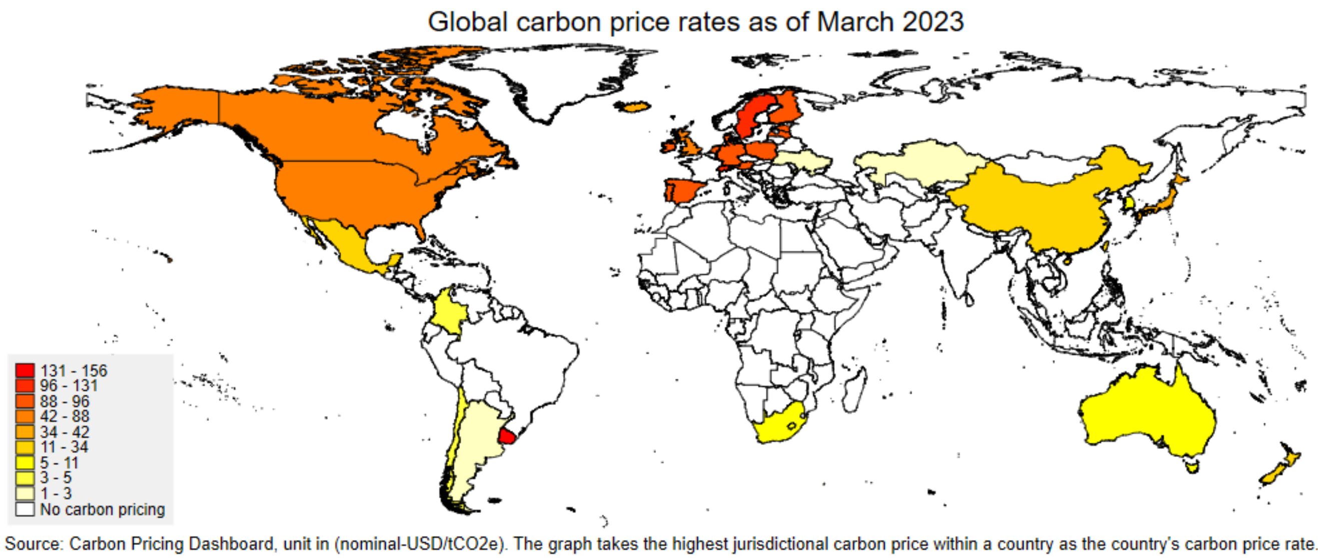


Figure A.7: Carbon pricing in practice

Note: Data is from The-World-Bank (2022) Carbon Pricing Dashboard and the author's calculation. The "No carbon pricing" column indicates jurisdictions for which data is not collected in the World Bank's carbon pricing dashboard as of March 2023. The World Bank Carbon Pricing Dashboard provides yearly data for each instrument (carbon tax or emission trading scheme) implemented in each jurisdiction. It calculates ETS prices largely based on auction prices. Numbers in this plot are the highest pricing rate among all instruments in a jurisdiction in 2023. The Social cost of carbon is $170/tCO2$ (discount rate 2%) for 2020 period (in 2019 international \$), according to Barrage and Nordhaus (2023).

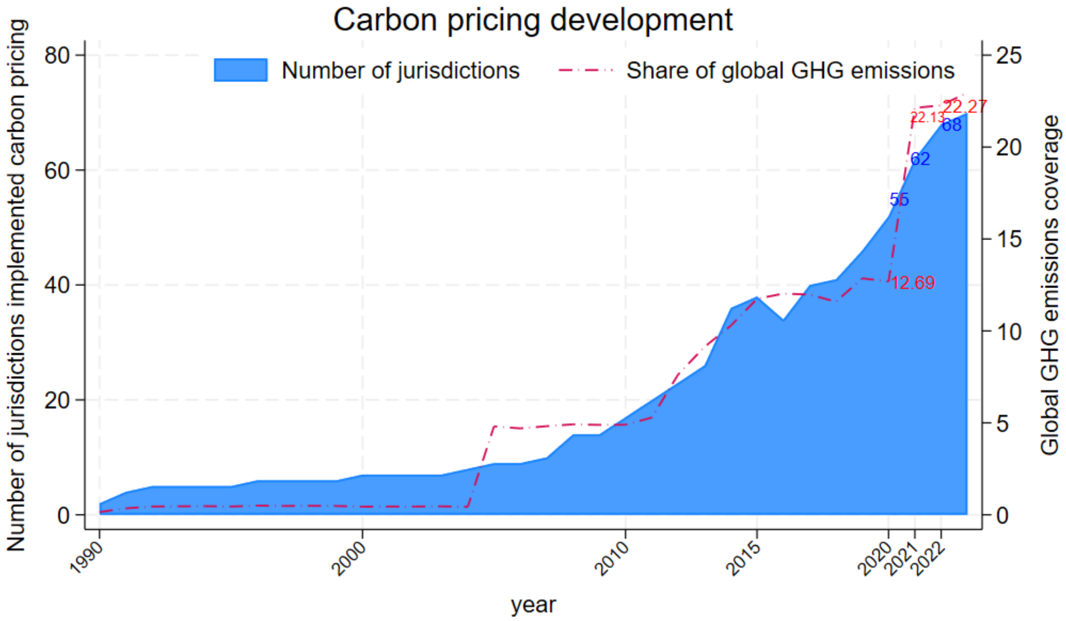


Figure A.8: Carbon pricing implementation and coverage

Note: Data is from The-World-Bank (2022) Carbon Pricing Dashboard and author's calculation.

Table A.2: Firm-level Variable Summary Statistics

Variable	N	Mean	SD	Min	Max
CO2(s1)	11380	3407.056	14051.35	.0001	240369.2
CO2e/TA	10318	.3757989	1.786879	.0001	24.22439
CO2e/PPE	10210	.5845261	3.509912	.0001	71.62435
CO2e/Size	10963	1.097949	9.04798	.0001	239.0919
CO2e/Rev	11272	.730509	3.55699	.0001	56.89043
ESGscore	17705	53.22515	19.77304	.4023902	95.73563
WACC	11953	6.522627	3.237547	.226126	29.9197

CO2(s1) in Kiloton CO2e. Unit of the latter 4 vars: Kiloton CO2e/Million USD (CPI deflated)

Table A.3: Countries of the firms in the empirical sample

	country	ETS	CarbonTax
1	Poland	2005	1990
2	Finland	2005	1990
3	Sweden	2005	1991
4	Norway	2008	1991
5	Denmark	2005	1992
6	Slovenia	2005	1996
7	Estonia	2005	2000
8	Latvia	2005	2004
9	Switzerland	2008	2008
10	Ireland	2005	2010
11	Iceland	2013	2010
12	Ukraine	*N/A*	2011
13	Japan	2010	2012
14	Australia	*N/A*	2012
15	United Kingdom	2005	2013
16	France	2005	2014
17	Spain	2005	2014
18	Mexico	*N/A*	2014
19	Portugal	2005	2015
20	Colombia	*N/A*	2017
21	Chile	*N/A*	2017
22	Argentina	*N/A*	2018
23	Germany	2005	2019
24	Canada	2013	2019
25	Nigeria	N/A	2019
26	South Africa	*N/A*	2019
27	Singapore	N/A	2019
28	Uruguay	*N/A*	2022
29	Italy	2005	*N/A*
30	Netherlands	2005	*N/A*
31	Cyprus	2005	*N/A*
32	Greece	2005	*N/A*
33	Malta	2005	*N/A*
34	Slovakia	2005	*N/A*
35	Czechia	2005	*N/A*
36	Belgium	2005	*N/A*
37	Austria	2005	*N/A*
38	Lithuania	2005	*N/A*
39	Luxembourg	2005	*N/A*
40	Hungary	2005	N/A
41	Bulgaria	2007	*N/A*
42	Romania	2007	*N/A*
43	New Zealand	2008	*N/A*
44	United States	2009	*N/A*
45	Croatia	2013	N/A
46	Liechtenstein	2013	*N/A*
47	China	2013	*N/A*
48	Kazakhstan	2013	N/A
49	Korea,Rep.	2015	*N/A*
50	Isle of Man	*N/A*	*N/A*
51	Russia	*N/A*	*N/A*
52	Turkey	*N/A*	*N/A*
53	Guemsey	*N/A*	*N/A*
54	Jersey	*N/A*	N/A
55	India	*N/A*	*N/A*
56	Brazil	*N/A*	*N/A*
57	Israel	*N/A*	*N/A*

Note: Data is from The-World-Bank (2022) Carbon Pricing Dashboard and the author's calculation. The adoption year of a country is the year the country first implements a carbon tax or ETS or cap and trade. I double-check to confirm each country's adoption year through online searching.

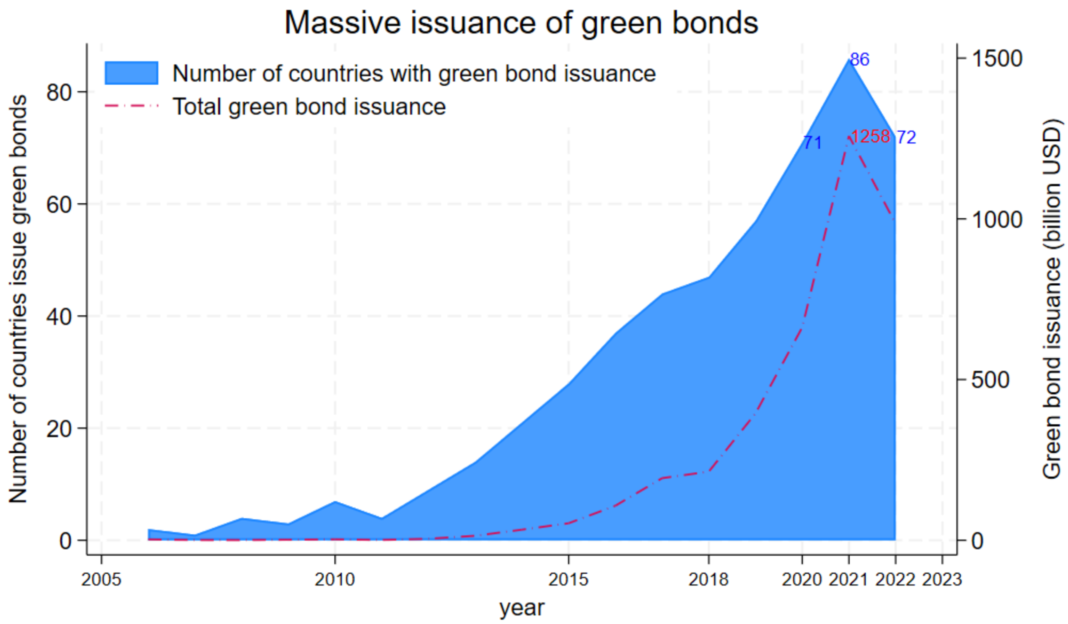


Figure A.9: Green green bond issuance.

Note: The data is from the Refinitiv database.

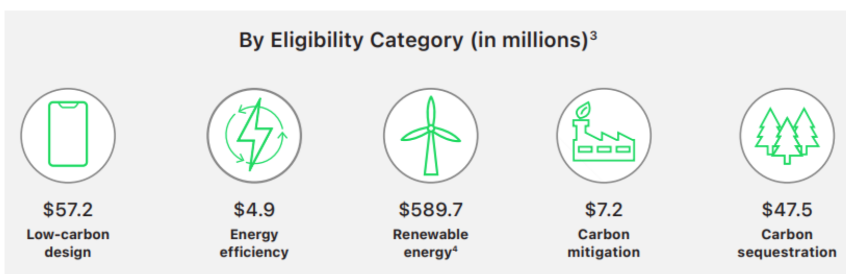


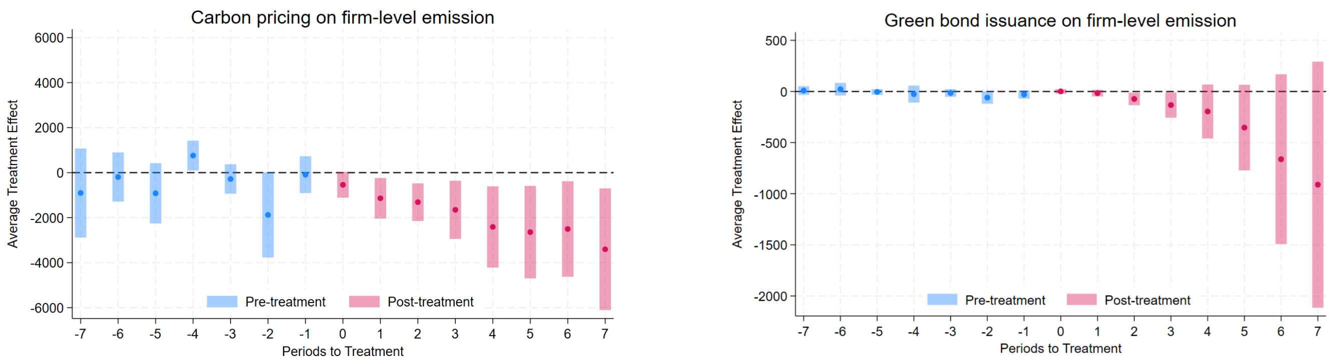
Figure A.10: The use of apple green bond proceeds in 2022.

Note: The graph is from Apple's Annual Green Bond Impact Report (Fiscal Year 2022 Updated).

1. Green capital cost intervention on green input: solar panel
2. Green technology innovation: 'more efficient' solar panel → green input more 'productive'.

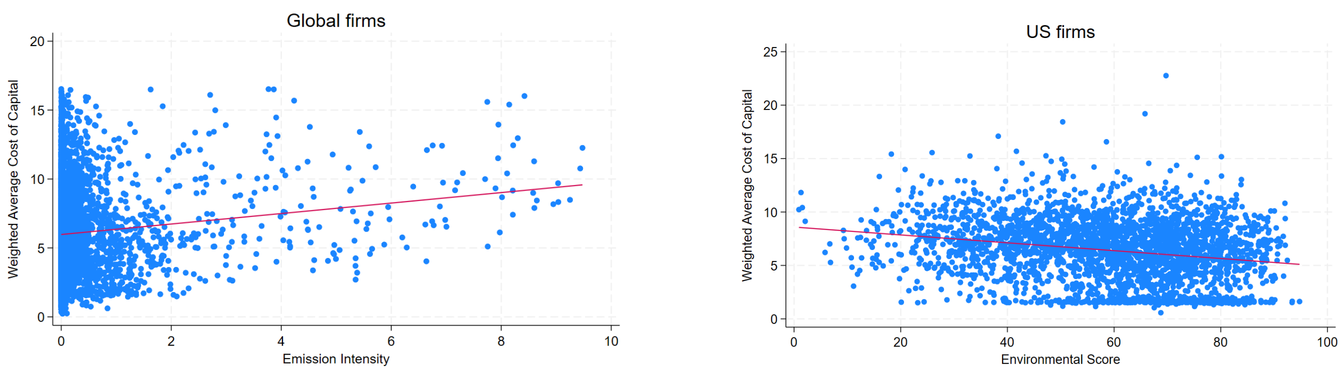
Reduced-form evidence

Figure A.11: Carbon pricing and green finance on firm-level emission



Note: Carbon pricing is the treatment on a firm subject to the first carbon pricing tool. Green finance is the treatment of a firm issuing the first green bond. Firm-level emissions are scope-1 CO2e emissions (kiloton).

Figure A.12: Greeness and cost of capital



Note: Firm-level emissions are scope-1 CO2e emissions. The unit is in kiloton. Emission intensity is measured as firm-level scope-1 emissions over firm-level total assets deflated by CPI, with the unit (kiloton CO2e)/(million USD). Environmental scores are provided by Refinitiv.

B Model Derivations and Proofs

Dissecting Mechanisms Derivations

Emission intensity: $\frac{D_{i,t}}{P_{i,t}Y_{i,t}} = \frac{\xi_{b,i}P_{i,b,t}E_{i,b,t}}{\frac{1}{\alpha_i}[(1+\tau^c_{i,t}\xi_{b,i})P_{i,b,t}E_{i,b,t}+(1-g_{i,k,t})P_{i,g,t}E_{i,g,t}]} = \frac{\alpha_i\xi_{b,i}}{(1+\tau^c_{i,t}\xi_{b,i})+(1-g_{i,k,t})K_{i,g,t}/K_{i,b,t}} = f(\tau^c, g_K, K_g/K_b(\tau^c, g_K))$

$$\frac{\partial(D_{i,t}/(P_{i,t}Y_{i,t}))}{\partial g_K} = \underbrace{\underbrace{\frac{\partial f}{\partial g_K}}_{>0}}_{\text{scale effect }+} + \underbrace{\underbrace{\frac{\partial f}{\partial K_g/K_b}}_{<0}\underbrace{\frac{\partial K_g/K_b}{\partial(\frac{1-g_k}{1+\tau^c\xi_b})}}_{<0}\underbrace{\frac{\partial(\frac{1-g_k}{1+\tau^c\xi_b})}{\partial g_K}}_{<0}}_{\text{substitution effect }-}$$

i) Carbon pricing: negative

$$\frac{\partial(D_{i,t}/(P_{i,t}Y_{i,t}))}{\partial \tau^c} = \underbrace{\underbrace{\frac{\partial f}{\partial \tau^c}}_{<0}}_{\text{scale effect }-} + \underbrace{\underbrace{\frac{\partial f}{\partial K_g/K_b}}_{<0}\underbrace{\frac{\partial K_g/K_b}{\partial(\frac{1-g_k}{1+\tau^c\xi_b})}}_{<0}\underbrace{\frac{\partial(\frac{1-g_k}{1+\tau^c\xi_b})}{\partial \tau^c}}_{<0}}_{\text{substitution effect }-}$$

ii) Green capital cost intervention: ambiguous Scale effect +: falling user cost increases demand in all input, 'imperfect substitute' Notice $P_{i,k,t}E_{i,k,t} \equiv \varphi r_{i,t}K_{i,k,t}$, so $\frac{P_gE_g}{P_bE_b} = \frac{K_g}{K_b} = (\frac{A_g}{A_g})^{\epsilon-1}[\frac{(1-g_k)}{(1+\tau^c\xi_{b,i})}]^{-\epsilon}$, the second = from FOC. If $\epsilon > 1$, the substitution effect dominates.

The FOC is such that

$$\frac{E_g}{E_b} = \frac{A_gK_g}{A_bK_b} = [\frac{(1-g)P_g}{(1+\tau^c\xi_b)P_b}]^{-\epsilon} = [\frac{(1-g_k)\varphi r/A_g}{(1+\tau^c\xi_b)\varphi r/A_b}]^{-\epsilon} \tag{B.1}$$

This is given by the problem of choosing each capital intermediate type, notice scientists become monopolist supplier on capital intermediates

$$\begin{aligned} &Max_{\{K_b,K_g\}}P_KK{-}(1+\tau^c\xi_b)\varphi rK_b - (1-g_k)\varphi rK_g \\ \rightarrow\ &Max_{\{K_b,K_g\}}P_KK{-}(1+\tau^c\xi_b)P_bE_b - (1-g_k)P_gE_g \end{aligned}$$

Recall the problem of deciding how many scientists on each technology type:

$$\begin{aligned} \frac{\Pi_{i,g,t}}{\Pi_{i,b,t}} &= 1 \text{ innovation equilibrium, scientist indifference} \\ \Pi_{i,k,t} &= \eta s_{i,k,t}^{-\psi}\left(1-\frac{1}{\varphi}\right)P_{i,k,t}E_{i,k,t} \end{aligned}$$

iii) If introducing green innovation subsidy, expected innovation profit of green innovation becomes

$\Pi_{i,g,t}(1+g_{q,t})$; scientist allocation equilibrium becomes $(\frac{s_{g,t}}{s_{b,t}})^{\psi} = \frac{P_{g,t}E_{g,t}}{P_{b,t}E_{b,t}}(1+g_{q,t}) = \frac{K_{g,t}}{K_{b,t}}(1+g_{q,t})$

$$\frac{\partial(D_{i,t}/(P_{i,t}Y_{i,t}))}{\partial(1+g_{q,t}))} = \underbrace{\underbrace{\frac{\partial f}{\partial K_g/K_b}}_{<0}\underbrace{\frac{\partial K_g/K_b}{\partial(s_{g,t}/s_{b,t})}}_{>0}\underbrace{\frac{\partial(s_{g,t}/s_{b,t})}{\partial(1+g_{q,t})}}_{>0}}_{-}$$

iii) green innovation subsidy: negative. This is because advancements in green technology innovation associated with more scientists being allocated to the green technology sector will always reduce emission intensity

$$\frac{\partial(D_{i,t}/(P_{i,t}Y_{i,t}))}{\partial(s_{g,t}/s_{b,t})} = \underbrace{\underbrace{\frac{\partial f}{\partial K_g/K_b}}_{<0}\underbrace{\frac{\partial K_g/K_b}{\partial(s_{g,t}/s_{b,t})}}_{>0}}_{-}$$

[3)] How does i) carbon pricing, ii) green capital cost intervention, iii) green innovation subsidy affect green technology innovation?

Equilibrium scientist allocation $(\frac{s_{g,t}}{s_{b,t}})^{\psi} = \frac{P_{g,t}E_{g,t}}{P_{b,t}E_{b,t}}(1+g_{q,t}) = \frac{K_{g,t}}{K_{b,t}}(1+g_{q,t})$ From capital composite producer problem, $-\epsilon = \frac{d\log\left(K_{g,t}/K_{b,t}\right)}{d\log\left((1-g_{k,t})/(1+\tau_t^c\xi_b)\right)} < 0$

$$d\log\left(\frac{s_{g,t}}{s_{b,t}}\right)^{\psi} = dlog(1+g_{q,t}) + d\log\left(K_{g,t}/K_{b,t}\right) = \underbrace{d\log(1+g_{q,t})}_{\text{Direct Effect}} + \underbrace{(-\epsilon)\times d\log\left((1-g_{k,t})/(1+\tau_t^c\xi_b)\right)}_{\text{General Equilibrium Effect}}$$

Hence, $\frac{\partial(s_{g,t}/s_{b,t})}{\partial g_{k,t}} > 0$, $\frac{\partial(s_{g,t}/s_{b,t})}{\partial \tau_t^c} > 0$, $\frac{\partial(s_{g,t}/s_{b,t})}{\partial g_{q,t}} > 0$.

Carbon pricing, green capital cost intervention affect technology innovation through GE Green innovation subsidy affects technology innovation directly.

Model Derivations

Consumption Households work and accumulate capital:

$$\max_{\{C_{i,t}^H, K_{i,t+1}^g, K_{i,t+1}^b\}_{t=0}^{\infty}} U_{i,0} = \sum_{t=0}^{\infty} \beta^t \{\log\left(C_{i,t}^H\right) - \bar{\omega}\log\left(D_{i,t}\right)\}$$

s.t. $P_{i,t}[C_{i,t}^H + K_{i,g,t+1} - (1-\delta)K_{i,g,t} + K_{i,b,t+1} - (1-\delta)K_{i,b,t}] = w_{i,t}\bar{L}_i + \varphi r_{i,t}(K_{i,g,t} + K_{i,b,t}) + \Delta C$

The model assumes households are indifferent between the two types of capital. Notice there are static profits transfer to households, which is why the right hand side above equals wage, capital, and profit, equivalent to $X_i = w_{i,t}\bar{L}_i + P_{i,g,t}E_{i,g,t} + P_{i,g,t}E_{i,g,t} = w_{i,t}\bar{L}_i + \varphi r_i(K_{i,g} + K_{i,b})$.

Before adding transfer change, The solution for consumption is such

$$C_{i,t}^H = (1-\beta)\left[\frac{\varphi}{\alpha_i} r_{i,t}/P_{i,t} + (1-\delta)\right](K_{i,g,t} + K_{i,b,t})$$
$$K_{i,g,t+1} = \beta\left[\frac{\varphi}{\alpha_i} r_{i,t+1}/P_{i,t+1} + (1-\delta)\right] K_{i,g,t}$$
$$K_{i,b,t+1} = \beta\left[\frac{\varphi}{\alpha_i} r_{i,t+1}/P_{i,t+1} + (1-\delta)\right] K_{i,b,t}$$

It is derived in the way that in equilibrium, $\frac{w_{i,t}\bar{L}_i}{1-\alpha_i} = \frac{P_{K,t}K_{i,t}}{\alpha_i} = \frac{\varphi r(K_{i,g,t}+K_{i,b,t})}{\alpha_i}$; so $w_{i,t}\bar{L}_i + \varphi r_{i,t}(K_{i,g,t}+K_{i,b,t}) = \frac{\varphi}{\alpha_i} r_{i,t}(K_{i,g,t}+K_{i,b,t})$. The budget constraint can be written as $C_{i,t}^H + (K_{i,g,t+1} + K_{i,b,t+1}) - (1-\delta)(K_{i,g,t} + K_{i,b,t}) = \frac{\varphi}{\alpha_i}\frac{r_{i,t}}{P_{i,t}}(K_{i,g,t} + K_{i,b,t})$. The new Euler equation is:

$$\frac{1}{C_{i,t}^H} = \beta\frac{1}{C_{i,t+1}^H}\left[\frac{\varphi}{\alpha_i} r_{i,t+1}/P_{i,t+1} + (1-\delta)\right] = \beta\frac{1}{C_{i,t+1}^H} R_{i,t+1}$$

Guess $C_{i,t}^H = \zeta R_{i,t}(K_{i,g,t} + K_{i,b,t})$, by a) and b) get $\zeta = 1-\beta$

If now assume TR changes consumption by $P\Delta C = TR$.

$$C_{i,t}^H = (1-\beta)\left[\frac{\varphi}{\alpha} r_{i,t}/P_{i,t} + (1-\delta)\right](K_{i,g,t} + K_{i,b,t}) + \Delta C \qquad \text{(Consumption)}$$

which is

$$C_{i,0}^H = \frac{1-\beta}{\beta}(K_{i,g,0} + K_{i,b,0}) + (\tau_{i,t}^c \xi_b P_{i,b,t} E_{i,b,t} - g_{i,k,t} P_{i,g,t} E_{i,g,t})/P$$

Balanced Growth Path

Define $\tilde{y}_t \equiv y_t/(1+g_y)^t$, and $g_y \equiv \frac{d\log y}{dt} = \frac{dy/dt}{y} \approx \frac{(y_t - y_{t-1})/1}{y_{t-1}}$ is the growth rate of variable y_t at BGP. The whole model system, without accounting for trade tariffs, in the decentralized economy can be expressed as:

- $A_{i,g,t} = \tilde{A}_{i,g,t}(1+g_{Ag})^t$, and $g_{Ag} = \varphi^{\eta s_{i,k,t}^{1-\psi}} - 1$, and $s_{i,g,t} + s_{i,b,t} = 1$.
- The final cost of capital composite $K_{i,t}$: $\tilde{P}_{i,K,t}^{1-\epsilon} = [(1+\tau_{i,t}^c\xi_b)\frac{\varphi\tilde{r}_{i,t}}{\tilde{A}_{i,b,t}}]^{1-\epsilon} + [(1-g_{i,k,t})\frac{\varphi\tilde{r}_{i,t}}{\tilde{A}_{i,g,t}}]^{1-\epsilon}$

- $g_{P_K} = g_{r/A_b} = g_{r/A_g}$

- Price index: $\tilde{P}_{i,t}^{-\theta} = \Gamma_i \sum_{n=1}^N T_n(\tilde{x}_{n,t} d_{in})^{-\theta}$

- the growth rate of unit price equals to the growth rate of price index $g_x = g_P$;

- Trade share: $\tilde{\pi}_{i,n,t} = \frac{T_n(\tilde{x}_{n,t}d_{in})^{-\theta}}{\sum_k T_k(\tilde{x}_{k,t}d_{ik})^{-\theta}}$

- The growth rate of trade share is 0, $g_\pi = 0$;

- the growth rate of wage has such a connection from unit price equation $(1+g_x) = (1+g_{P_K})^{\alpha}(1+g_w)^{1-\alpha}$.

- Trade balance : $\sum_{n=1} \frac{\tilde{\pi}_{i,n}}{1+\tau_{i,n}} \tilde{X}_i = \sum_{n=1}^{N} \frac{\tilde{\pi}_{n,i}}{1+\tau_{n,i}} \tilde{X}_n$

- capital accumulation

$$\tilde{K}_{i,g,t+1} = \tilde{K}_{i,g} = \frac{\beta}{(1+g_{Kg})} \left(\frac{\varphi}{\alpha_i} \frac{\tilde{r}_{i,t}}{\tilde{P}_{i,t}} + (1-\delta) \right) \tilde{K}_{i,g,t}$$

$$\tilde{K}_{i,b,t+1} = \tilde{K}_{i,b} = \frac{\beta}{(1+g_{Kb})} \left(\frac{\varphi}{\alpha_i} \frac{\tilde{r}_{i,t}}{\tilde{P}_{i,t}} + (1-\delta) \right) \tilde{K}_{i,b,t}$$

$$\tilde{C}^H_{i,t} = (1-\beta) \left[\frac{\varphi}{\alpha_i} \tilde{r}_{i,t}/\tilde{P}_{i,t} + (1-\delta) \right] (\tilde{K}_{i,g,t} + \tilde{K}_{i,b,t}) + \tilde{\Delta C}$$

- $g_{Kg} = g_{Kb} = g_{C^H}$ and $g_{r/P} = 0$, so $g_r = g_P$.

Proof for propositions

Proposition 1: Ignoring innovation externality, addressing environmental externality requires carbon pricing and green capital cost intervention to satisfy *either*

1. Carbon price rate is at the optimal rate $\tau_t^c = \frac{-U'_{D,t}D'_{K_b,t}/U'_{C,t}}{\xi_b \partial Y_t/\partial K_{b,t}} = \frac{\varpi/K_{b,t}C_t}{\xi_b \varphi r_t/(P_t\alpha)}$ when there is no green capital intervention;
2. Green capital intervention is at the optimal rate $g_{k,t} = \frac{-U'_{D,t}D'_{K_b,t}/U'_{C,t}}{\partial Y_t/\partial K_{g,t}} = \frac{\varpi/K_{b,t}C_t}{\varphi r_t/(P_t\alpha)}$ when there is no carbon pricing;
3. Carbon price rate and green capital intervention satisfy $\tau_t^c \xi_b \frac{\partial Y_t}{\partial K_{b,t}} + g_{k,t}\frac{\partial Y_t}{\partial K_{g,t}} = \frac{-U'_{D,t}D'_{K_b,t}}{U'_{C,t}} = \frac{\varpi C_t}{K_{b,t}}$;

to achieve the optimal mix of green and brown capital in all countries.

Proof: General form. Countries are symmetric in the choice of consumption and capital, hence I can take the whole world as one country.

A local planner chooses C_i^H, $K_{i,g,t+1}$, $K_{i,b,t+1}$ to max U_i, subject to resource constraint in region i, $C_{i,t}^H + K_{i,g,t+1} - (1-\delta)K_{i,g,t} + K_{i,b,t+1} - (1-\delta)K_{i,b,t} = Y_i$. This is isomorphic to a global planner choose C^H, $K_{g,t+1}$, $K_{b,t+1}$ to maximize

$$U_0 = \sum_{t=0}^{\infty}\beta^t\{U_t\} = \sum_{t=0}^{\infty}\beta^t\{\sum_i U_{i,t}\}$$

Subject to resource constraint:$\sum_i [C_{i,t}^H + K_{i,g,t+1} - (1-\delta)K_{i,g,t} + K_{i,b,t+1} - (1-\delta)K_{i,b,t}] = \sum_i Y_{i,t}$

$$C_t^H + K_{g,t+1} - (1-\delta)K_{g,t} + K_{b,t+1} - (1-\delta)K_{b,t} = Y_t$$

From the production side, without intervention, $PY = WL + P_K K = \frac{P_K K}{\alpha} = \frac{\varphi r K_g + \varphi r K_b}{\alpha}$ so $Y = \frac{\varphi r K_g + \varphi r K_b}{\alpha P}$, which holds for all time period t. The marginal product of green capital and brown capital under no intervention equal to $\partial Y_t/\partial K_{b,t} = \frac{\varphi r_t}{\alpha P_t} = \partial Y_t/\partial K_{g,t}$, which is markup φ times the real interest rate r/P divided by capital share, and now with no intervention $\frac{\partial K}{\partial K_g} = \frac{\partial K}{\partial K_b}$.

With intervention, $PY = WL + P_K K = \frac{P_K K}{\alpha} = \frac{\varphi r K_g + \varphi r K_b}{\alpha}$ so $Y = \frac{(1+\tau^c\xi_b)\varphi r K_g + (1-g_k)\varphi r K_b}{\alpha P}$ for all time period t. Carbon pricing shifts the marginal product of brown capital by scale $\xi_b\tau^c$, and green capital cost interventions shift the marginal product of green capital by $-g_k$.

Take first order condition to consumption, green capital, and brown capital respectively, and denote the Lagrangian multiplier as λ^{lag}

$$\begin{aligned} & U'_{C,t} = \lambda^{lag}_t \\ & -\lambda^{lag}_t + \beta[-U'_{D,t+1}D'_{K_b,t+1} + \lambda^{lag}_{t+1}(\frac{\partial Y_{t+1}}{\partial K_{t+1,b}} + 1 - \delta)] = 0 \\ & -\lambda^{lag}_t + \beta[\lambda^{lag}_{t+1}(\frac{\partial Y_{t+1}}{\partial K_{t+1,g}} + 1 - \delta)] = 0 \end{aligned} \tag{B.2}$$

The intervention to address environmental externality is to *make the shifter of carbon pricing or green finance to the marginal product of green or brown capital exactly offsets the environmental externality wedge* $-U'_{D,t+1}D'_{K_b,t+1}$ in the social planner problem, taking into account capital decisions are predetermined.

Hence the optimal carbon price rate at time $t+1$ to address environmental externality alone is,

$$\tau^c_{t+1}\xi_b\frac{\partial Y_{t+1}}{\partial K_{t+1,g}} = \frac{-U'_{D,t+1}D'_{K_b,t+1}}{U'_{C,t+1}}$$

where $\frac{\partial Y_{t+1}}{\partial K_{t+1,b}}$ is the marginal product of brown capital under no intervention, which is $\partial Y_t/\partial K_{b,t} = \frac{\varphi r_t}{\alpha P_t}$.

Notice that first-order conditions to green capital and brown capital can be connected by λ^{lag}_t. Hence, $\beta[-U'_{D,t+1}D'_{K_b,t+1}+\lambda^{lag}_{t+1}(\frac{\partial Y_{t+1}}{\partial K_{t+1,b}}+1-\delta)] = \beta[\lambda^{lag}_{t+1}(\frac{\partial Y_{t+1}}{\partial K_{t+1,g}}+1-\delta)]$. In the absence of carbon pricing, the marginal cost of brown capital can not be shifted to offset environmental externality, green capital cost intervention can be leveraged to the level to offset it alone:

$$g_{k,t+1}\frac{\partial Y_{t+1}}{\partial K_{t+1,g}} = \frac{-U'_{D,t+1}D'_{K_b,t+1}}{U'_{C,t+1}}$$

where $\frac{\partial Y_{t+1}}{\partial K_{t+1,g}}$ is the marginal product of brown capital under no intervention, which is $\partial Y_t/\partial K_{g,t} = \frac{\varphi r_t}{\alpha P_t}$.

If exploiting both carbon pricing and green capital cost intervention to offset environmental externality, then the optimal policy implementation levels should satisfy:

$$\tau^c_{t+1}\xi_b\frac{\partial Y_{t+1}}{\partial K_{t+1,b}} + g_{k,t+1}\frac{\partial Y_{t+1}}{\partial K_{t+1,g}} = \frac{-U'_{D,t+1}D'_{K_b,t+1}}{U'_{C,t+1}}$$

Carbon price is measured in dollars per ton of carbon, green capital cost intervention is measured in percentage. The global optimal carbon price rate and green capital cost intervention are also the local optimal ones.

Proof: Plugging in specific functional forms of utility and production.

The planner chooses C_i^H, $K_{i,g,t+1}$, $K_{i,b,t+1}$ to max

$$U_0 = \sum_{t=0}^{\infty} \beta^t \{ \sum_i \log\left(C_{i,t}^H\right) - \bar{\omega} \sum_i \log\left(D_{i,t}\right) \}$$

Resource constraint in any region i:

$$C_{i,t}^H + K_{i,g,t+1} - (1-\delta)K_{i,g,t} + K_{i,b,t+1} - (1-\delta)K_{i,b,t} = Y_i$$

- Note that $P_{i,t}Y_{i,t} = \frac{1}{\alpha_i}[(1+\tau_{i,t}^c\xi_b)P_{i,b,t}E_{i,b,t} + (1-g_{i,k,t})P_{i,g,t}E_{i,g,t}]$ when there is CP/GF

FOC1 $\lambda_{i,t}P_{i,t} = \beta\{-\frac{\varpi}{K_{i,b,t+1}} + \lambda_{i,t+1}[\frac{(1+\tau_{t+1}^c\xi_b)\varphi r_{i,t+1}}{\alpha_i} + P_{i,t+1}(1-\delta)]\}$

FOC2 $\lambda_{i,t}P_{i,t} = \beta\{\lambda_{i,t+1}[\frac{(1-g_{i,k,t})\varphi r_{i,t+1}}{\alpha_i} + P_{i,t+1}(1-\delta)]\}$

This delivers an equivalence btw the 2 cost shifters, $\frac{\varpi}{K_{i,b,t+1}} = \lambda_{t+1}\frac{\varphi r_{t+1}}{\alpha_i}[(1+\tau_{i,t}^c\xi_b)-(1-g_{i,k,t})]$ which also decides the optimal level or combination of them

Proposition 2: Prove that so long as $\frac{K_{g,t+u}}{K_{b,t+u}} > \frac{K_{g,t}}{K_{b,t}}$ then $g_{q,t}$ is positive. Intuitively, if future periods demand more green capital than brown capital, then a strictly positive subsidy for green innovation is required.

Notice $P_{g,t}E_{g,t} = \varphi r_t K_{g,t}$, so $\frac{P_{g,t}E_{g,t}}{P_{b,t}E_{b,t}}(1+g_{q,t}) = \frac{K_{g,t}}{K_{b,t}}(1+g_{q,t}) = \frac{\sum_{u=0}^{\infty}\frac{1}{1+r_{t+u}}K_{g,t+u}}{\sum_{u=0}^{\infty}\frac{1}{1+r_{t+u}}K_{b,t+u}}$. Manage terms,

$$(1+g_{q,t}) = \frac{\sum_{u=0}^{\infty}\frac{1}{1+r_{t+u}}K_{g,t+u}/K_{g,t}}{\sum_{u=0}^{\infty}\frac{1}{1+r_{t+u}}K_{b,t+u}/K_{b,t}}$$

If $\frac{K_{g,t+u}}{K_{b,t+u}} > \frac{K_{g,t}}{K_{b,t}}$, which is equivalent to $\frac{K_{g,t+u}}{K_{g,t}} > \frac{K_{b,t+u}}{K_{b,t}}$, then the right-hand side above will always be larger than 1. Hence the subsidy to green innovation must be strictly positive, that $g_{q,t} > 0$.

C Quantification and Simulation Details

The whole model system

0. Endogenous law of motion for capital productivity:

$$A_{i,k,t} = \varphi^{\eta s_{i,k,t}^{1-\psi}} A_{i,k,t-1} \tag{A.0}$$

with $k \in \{g, b\}$, and $s_{i,g,t} + s_{i,b,t} = 1$.

- Capital intermediate price: $P_{i,k,t} \equiv \frac{\varphi r_{i,t}}{A_{i,k,t}}$

1. The final cost of capital composite $K_{i,t}$: $P_{i,K,t} = \left[[(1+\tau_{i,t}^c \xi_b) P_{i,b,t}]^{1-\epsilon} + [(1-g_{i,k,t}) P_{i,g,t}]^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$

$$P_{i,K,t} = \left[[(1+\tau_{i,t}^c \xi_b) \frac{\varphi r_{i,t}}{A_{i,b,t}}]^{1-\epsilon} + [(1-g_{i,k,t}) \frac{\varphi r_{i,t}}{A_{i,g,t}}]^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{A.1}$$

2. Unit price of an input bundle:

$$x_i = (\frac{P_{i,K,t}}{\alpha})^{\alpha} (\frac{w_{i,t}}{1-\alpha})^{1-\alpha} \tag{A.2}$$

3. Price index:

$$P_i^{-\theta} = \Gamma_i \sum_{n=1}^{N} T_n \left(x_n d_{in} \right)^{-\theta} \tag{A.3}$$

4. Trade share:

$$\pi_{in} = \frac{T_n \left(x_n d_{in} \right)^{-\theta}}{\sum_k T_k \left(x_k d_{ik} \right)^{-\theta}} \tag{A.4}$$

- Market clearing: $X_i = w_{i,t} \bar{L}_i + P_{i,K} K_i + \sum_{n=1} \frac{\tau_{in}}{1+\tau_{i,n}} \pi_{i,n} X_i$
  - Import for region i from region n is $M_{i,n} = X_i \frac{\pi_{i,n}}{1+\tau_{i,n}}$
  - Shut down tariff until the discussion of CBAM

5. Trade balance accounting tariff:

$$\sum_{n=1} \frac{\pi_{i,n}}{1+\tau_{i,n}} X_i = \sum_{n=1}^{N} \frac{\pi_{n,i}}{1+\tau_{n,i}} X_n \tag{A.5}$$

which can be managed terms into

$$\frac{P_{iK} K_i}{\alpha} \frac{\sum_{n=1} \frac{\pi_{i,n}}{1+\tau_{i \leftarrow n}}}{1 - \sum_{n=1} \frac{\tau_{i,n} \pi_{i,n}}{1+\tau_{i,n}}} = \sum_{n=1}^{N} \frac{\pi_{n,i}}{1+\tau_{n \leftarrow i}} \frac{P_{in} K_n}{\alpha} \frac{1}{1 - \sum_{n=1} \frac{\tau_{n,i} \pi_{n,i}}{1+\tau_{n,i}}}$$

- HH

$$K_{i,g,t+1} = \beta \left[\frac{\varphi}{\alpha} r_{i,t+1}/P_{i,t+1} + (1-\delta)\right] K_{i,g,t}$$
$$K_{i,b,t+1} = \beta \left[\frac{\varphi}{\alpha} r_{i,t+1}/P_{i,t+1} + (1-\delta)\right] K_{i,b,t} \tag{A.6}$$

7. Consumption accounting for transfer:

$$C^H_{i,t} = (1-\beta)\left[\frac{\varphi}{\alpha} r_{i,t}/P_{i,t} + (1-\delta)\right](K_{i,g,t} + K_{i,b,t}) + (\tau^c_{i,t}\xi_b P_{i,b,t}E_{i,b,t} - g_{i,k,t}P_{i,g,t}E_{i,g,t})/P_{i,t} \tag{A.7}$$

8. Emission level:

$$D_{i,t} = \xi_b P_{i,b,t}E_{i,b,t} = \xi_b \varphi r_{i,t}/A_b K_{b,i,t} \tag{A.8}$$

- Notice $K_b = \frac{1}{A_b}[\frac{(1+\tau^c_{i,t}\xi_b)\frac{\varphi r_{i,t}}{A_{i,b,t}}}{P_K}]^{-\epsilon} K$

9. Emission intensity:

$$IN_{i,t} = \frac{\xi_b P_{i,b,t}E_{i,b,t}}{P_i Y_i} = \frac{\xi_b P_{i,b,t}E_{i,b,t}}{X_i} = \frac{\alpha \xi_b}{(1+\tau^c_{i,t}\xi_b) + (1-g_{i,k,t})K_{i,g,t}/K_{i,b,t}} \tag{A.9}$$

10. Regional welfare:

$$U_{0,i} = \sum_{t=0}^{\infty} \beta^t \{\log\left(C^H_{i,t}\right) - \bar{\omega}\log\left(D_{i,t}\right)\} \tag{A.10}$$

- A dynamic equilibrium is defined by a sequence of factor prices, goods prices, capital stocks, scientists allocation, and technology stocks $\{w_{i,t}, P_{i,K,t}, r_{i,t}, P_{i,t}, K_{i,k,t}, s_{i,k,t}, A_{i,k,t}\}^{N,\infty}_{i=1,t=0,k\in g,b}$, that solves (i) innovation equilibrium condition, scientists market clearing, the laws of motion for technologies; (ii) location price, trade flows, total expenditure, local market clearing; (iii) the capital accumulation condition; given initial technology levels, factor endowments, parameters and fundamentals (including policy variables).

Model system without endogenous growth

Without endogenous innovation, hence without endogenous growth (either $s_{i,k,t} = 0$ or $\varphi = 1$), the whole system can reach a steady state. That is, $X_t = X_{t+1} = X$ for every variable X. Express steady state in change:

- Define $\Xi_{n,t,GF} = (1 - g_{k,n})$, $\Xi_{n,t,CP} = (1 + \tau^c_n \xi_{b,n})$

1. Price for production capital

$$\hat{P}^{1-\epsilon}_{i,K,t} = \zeta_i(\hat{\Xi}_{CP}\frac{\hat{r}_i}{\hat{A}_b})^{1-\epsilon} + (1-\zeta_i)(\hat{\Xi}_{GF}\frac{\hat{r}_i}{\hat{A}_g})^{1-\epsilon} \tag{S.1}$$

- The ratio $\zeta_i \equiv \frac{r_i K_{i,g}}{P_{i,K} K_i}$ the share of green capital in a region's gross capital value

2. Unit price equation, notice $wL/(1-\alpha) = KP_K/\alpha$ and L fixed, so $x = (\frac{K}{L})^{1-\alpha}(\frac{1}{\alpha})^{\alpha} P_K$

$$\hat{x}_i = \hat{P}_{i,K}(\hat{K}_i)^{1-\alpha} \tag{S.2}$$

3. Price index: $P_i^{-\theta} = \Gamma_i \sum_{n=1}^{N} T_n (x_n d_{in})^{-\theta}$, in change

$$\hat{P}_i^{-\theta} = \sum_{n=1}^{N} \pi_{i,n} \left(\hat{P}_{n,K}(\hat{K}_n)^{1-\alpha}\hat{d}_{in}\right)^{-\theta}. \tag{S.3}$$

4. Trade share in change

$$\pi'_{in} = \pi_{in} \frac{\left(\hat{P}_{n,K}(\hat{K}_n)^{1-\alpha}\hat{d}_{in}\right)^{-\theta}}{\hat{P}_i^{-\theta}} \tag{S.4}$$

5. Trade balance

$$\sum_{n=1} \frac{\pi'_{i,n}}{1+\tau'_{i,n}} X'_i = \sum_{n=1}^{N} \frac{\pi'_{n,i}}{1+\tau'_{n,i}} X'_n \tag{S.5}$$

6. From HH dynamic capital accumulation $K_{i,t+1} = K_{i,t}$, get $\frac{\varphi}{\alpha} r/P + (1-\delta) = \frac{1}{\beta}$, so

$$\hat{r}_i = \hat{P}_i \tag{S.6}$$

7. Consumption accounting for transfer:

$$\hat{C}_i^H = \frac{[1 - g' \times \frac{\alpha_i(1-\beta+\delta\beta)}{1-\beta}]\frac{K_g \times \hat{K}_g}{K_b \times \hat{K}_b} + 1 + \tau^{c'}\xi_b \times \frac{\alpha_i(1-\beta+\delta\beta)}{1-\beta}}{[1 - g \times \frac{\alpha_i(1-\beta+\delta\beta)}{1-\beta}]\frac{K_g}{K_b} + 1 + \tau^{c}\xi_b \times \frac{\alpha_i(1-\beta+\delta\beta)}{1-\beta}} \times \hat{K}_b \tag{S.7}$$

8. Emission level:

$$\hat{D}_i = \frac{\hat{r}_i}{\hat{A}_b} \hat{K}_{b,i} \tag{S.8}$$

9. Emission intensity:

$$I\hat{N}_{i,t} = \frac{\hat{D}_i}{\hat{X}_i} \tag{S.9}$$

10. Regional welfare:

$$\hat{U}_{0,i} = \log\left(\frac{\hat{C}_{i,t}^H}{(\hat{D}_{i,t})^{\bar{\omega}}}\right) \tag{S.10}$$

Computation of steady state in change

With data on value-added and trade share, with exogenous parameters known, with changes in fundamentals $\hat{\Xi}_{CP}$, $\hat{\Xi}_{GF}$, $\hat{d}_{in}$ known, with $\hat{A}_g, \hat{A}_b$ known, then the following steps solve this system

- Step 1: Given an initial guess on $\hat{P}_{K,i}$ for all i, then solve the corresponding $\hat{r}_i$ using production capital price equation S.1, then solve the price $\hat{P}_i$ from HH dynamic capital accumulation S.6.

- Step 2: Use fsolve to solve the regional price index equation S.3, with LHS $\hat{P}_i$ and RHS $\hat{P}_{K,i}$ given by Step 1, and trade share π from data. This delivers $\hat{K}_i$ for all i.

- Step 3: Solve the trade share in counterfactual using S.4. Then solve trade balance equation S.5, which delivers a new $\hat{P}_{K,i}^{New}$ to update

- Notice $X_i'[1-\sum_{n=1}\frac{\tau_{i,n}'\pi_{i,n}'}{1+\tau_{i,n}'}] = \frac{P_{iK}'K_i'}{\alpha}$, and $\frac{P_{iK}'K_i'}{\alpha}\frac{\sum_{n=1}\frac{\pi_{i,n}'}{1+\tau_{i,n}}}{1-\sum_{n=1}\frac{\tau_{i,n}'\pi_{i,n}'}{1+\tau_{i,n}'}} = \sum_{n=1}^{N}\frac{\pi_{n,i}'}{1+\tau_{n\leftarrow i}'}\frac{P_{in}'K_n'}{\alpha}\frac{1}{1-\sum_{n=1}\frac{\tau_{n,i}'\pi_{n,i}'}{1+\tau_{n,i}'}}$, because $X_i = \frac{1}{\alpha}P_{iK}K_i + X_i\sum_{n=1}\frac{\tau_{in}}{1+\tau_{i\leftarrow n}}\pi_{i,n}$, and $\frac{P_{iK}'K_i'}{\alpha} = VA_{baseline}\hat{P}_K\hat{K}$

- Step 4: Check if $\hat{P}_{K,i}^{New}$ equals the initial guess $\hat{P}_{K,i}$, if not, Update $\hat{P}_{K,i}^{New}$ following $\hat{P}_{K,i}^{update} = \hat{P}_{K,i} * \left(1 + \frac{(\hat{P}_{K,i}^{New} - \hat{P}_{K,i})*(-0.1)}{\hat{P}_{K,i}}\right)$, until it converges.

- Step 5: With $\hat{P}_{K,i}$ and $\hat{K}_i$ solved by Step 1 to Step 4, one can recover changes in consumption, emission level, intensity, and welfare using equations S.7, S.8, S.9 and S.10.

The innovation simulation in this version takes an extreme case that innovation only happens in the green (or brown) sector, all scientists are working in the green (or brown) sector. There is a change in the green technology level, the change level is $\hat{A}_g = \frac{A_g'}{A_g} = \varphi^{\eta}$, implying from $A_{i,k,t} = \varphi^{\eta s_{i,k,t}^{1-\psi}} A_{i,k,t-1}$.

Table of parameters for simulation. In this version, the values for the parameters are

Table C.4: Table of parameters for current simulation

Parameter	Description	Value	Source
ϵ	elasticity of substitution	4	Acemoglu et al. (2012)
θ	trade elasticity	4.5	Caliendo and Parro (2015)
β	discount rate	0.98	2% as in Nordhaus (2015)
δ	depreciation	0.025	40 years capital life
$1-\alpha_i$	labor share	$[0.604, 0.58, 0.507, 0.538]$	Our World in Data
ζ	green capital share	0.248	OECD World average renewable energy use in energy consumption
$\xi_{b,i}$	brown capital emission rate	$[4.6, 4.1, 6.4, 4.6]$	CO2 intensity of oil equivalent energy use, $\times 10^{-3}$ to t/USD
$\bar{\omega}$	The elasticity of utility to pollution	1	equal weight to C and D
φ	gross markup	1.07	Acemoglu et al. (2023)
η	research productivity	1.4634	Acemoglu et al. (2023)

Note:i represents US, EU, CN ROW respectively.