

The Market for ESG Ratings*

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Abstract

The boom in Environmental, Social, and Governance (ESG) investing has created a demand for ESG ratings. ESG ratings, unlike credit ratings, measure multiple unrelated categories. We provide a model of ESG ratings competition where raters provide information about these categories and set fees. Raters specializing in different categories maximizes the amount of information transmitted and total surplus, and is the competitive outcome when investors are less concerned about ESG performance. When investor concerns about ESG performance are large enough, the competitive outcome is for them to generalize – splitting their effort among the categories, resulting in less informative ESG ratings. In this case, generalizing increases the stand-alone value of the ratings, and, hence, the raters’ pricing power. The possibility of greenwashing by firms can make generalization the unique equilibrium. We also demonstrate that specialization maximizes ratings disagreement and, thus, empirical measures of disagreement may be poor measures of surplus.

JEL Classification: G24, G14, L13

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1 Introduction

Global sustainable investment has reached \$30.3 trillion (GSIA, 2022). Underlying this boom is the market to assess investments on environmental, social, and governance (ESG) criteria. A large number¹ of ESG rating providers have sprouted up to gather, analyze, and aggregate data for investors. Nevertheless, ESG ratings have come under intense scrutiny from the media, regulators, and academics for measurement and accuracy issues.²

Indeed, some studies use credit ratings as a benchmark for how inaccurate ESG ratings are. However, a comparison between ESG ratings and credit ratings is like comparing apples to oranges. Aside from having well established data and methodologies, credit ratings attempt to measure one variable: the probability a default will occur.³ ESG ratings measure a panoply of subcategories within E, S, and G that are very different from each other. For example, one of the largest raters, MSCI, states that within S, it includes the headings (with many subheadings): Human Capital, Product Liability, Stakeholder Opposition, and Social Opportunities, and within E, it includes the headings: Climate Change, Natural Capital, Pollution & Waste, and Environmental Opportunities.

Large investors understand that ESG ratings are not credit ratings. They purchase multiple ESG ratings and use the data from the categories rather than the aggregate score (SustainAbility, 2020; ESMA, 2022).

Given this, we analyze the market structure of the ESG ratings industry. In our model, there is a project that needs funding from an investor. In addition to the financial performance, the investor cares about two categories of ESG performance. We consider these may be two categories among E, S, and G, or two of their subcategories. This preference may represent a concern about externalities, long-term performance, or potential scandals. There are two ESG raters who choose how much effort to allocate to gathering information on the two categories. The two raters then compete over the fees they charge the investor for purchasing the gathered information.⁴ We assume that the ESG raters (i) provide the rating by category rather than just an aggregate, and (ii) communicate their information truthfully.

¹The European Securities and Markets Authority states that there are 59 ESG rating providers in Europe (ESMA, 2022). Still, the market is dominated by several larger providers.

²For examples, see “ESG ratings need regulation to fix inconsistencies and bias” by Hazel James Ilango, *energypost.eu*, October 31, 2022, “Greenwashing Unmasked: A critical examination of ESG ratings and actual environmental footprint” by Benjamin Laker, *Forbes*, August 4, 2023, and “ESG ratings: whose interests do they serve?” by Kenza Bryan, *Financial Times*, October 3, 2023.

³Note that Moody’s states that they also measure the amount of recovery upon default.

⁴We note that unlike corporate credit ratings, which are paid for by issuers, ESG ratings are bought by investors.

We find that, in equilibrium, the investor will purchase ratings from both raters. The investor may not be willing to, however, pay both raters their marginal value. This is because, depending on the investor’s ESG preferences and the raters’ ratings design, the combined value of the ratings, the maximum value the investor is willing to pay for ratings from both raters, could be less than the sum of the marginal value of each rater. In this case, the raters’ share from the combined value will depend on the stand-alone value of their ratings (the maximum amount they could charge the investor if they were the only rater).

The stand-alone value thus has important implications for the equilibrium design of the ratings. For instance, when the investor’s value for ESG performance is so high that she needs a positive update about the project’s performance in both categories to invest, the information about a single category has no value. In this case, a rater who specializes in one category has no stand-alone value, which pushes the raters away from specialization. In fact, in equilibrium, the raters will *generalize*: split their efforts between the rating categories. Generalization, of course, leads to a costly duplication of effort.

When the investor’s value for ESG performance is low, positive information about one category is sufficient for the investment. As such, the stand-alone values are large enough that they do not distort the raters’ incentives. The investor will pay each rater the marginal value of their ratings. In this case, raters will *specialize*: each rater will focus all of its effort on one category.

For intermediate values of the investor’s value for ESG performance, there are multiple equilibria – both specialization and generalization are equilibria. This is because, in this range, the raters’ decisions to specialize or generalize become strategic complements.

We find that having the raters specialize in different categories maximizes value, where we define value as the sum of the agents’ payoffs. This is because specialization maximizes the amount of information transmitted. When investors place a high value on ESG performance, this creates a wedge between the market solution and the value-maximizing solution as the market provides less information through generalization. This occurs exactly when investors value that information the most.

Many recent papers point to the divergence among ESG ratings as a metric for inaccuracy. For example, Berg et al. (2022) document that the correlations between ESG ratings range from 0.38 to 0.71; similar results are found in Chatterji et al. (2016), Christensen et al. (2022), and Gibson Brandon et al. (2021). We demonstrate that the correlation between the ratings of the two raters is minimized when the raters specialize in different categories. Given the result that specialization is value maximizing, this implies that divergence is not

a good measure of inaccuracy.⁵

There is much discussion of greenwashing in the ESG data provision: firms’ manipulation of their performance data so as to improve public perception. We show that greenwashing will impact the strategy of ESG raters - when greenwashing by firms is substantial, the unique equilibrium will be generalization. This arises due to investors’ rational suspicion of ratings due to greenwashing. Particularly, investors require a consistently high ESG performance across categories for their investment, which pushes the raters toward generalization. This implies that less information provision by firms makes ratings less informative (1) through a direct effect, and (2) by influencing the industrial organization of the ESG raters.

Our results are robust to alternative specifications of the model, including asymmetric costs of information acquisition for different categories, varying the allocation of market power in pricing, looking at more general information structures, allowing the social planner’s weight on ESG to differ from the investor’s, and allowing for divergence in measurement methods.

Berg et al. (2022) point to three sources of divergence in ESG ratings. The first is “scope divergence,” where ratings are based on different attributes. The second is “measurement divergence,” where raters measure the same attribute differently. The last is “weight divergence,” which captures raters weighting different attributes differently. Our model is able to capture scope divergence and measurement divergence, but not weight divergence, as we do not look at aggregate ratings. Berg et al. (2022) find little evidence of weight divergence; it contributes to only 6% of the divergence.

There is some evidence of specialization in the current marketplace. Institutional Shareholder Services (ISS), which has its roots as a proxy advisor, has been “praised most frequently for its governance scoring” (SustainAbility, 2020, p. 14). Of course, all of the major raters state that they have measurements for E, S, and G.

In the model, we are agnostic about why investors derive utility from the ratings. It could be because they care about the externalities the firms impose. Alternatively, it could be because it affects the firm’s payoffs through risk (physical, reputational, litigation, or regulatory) or proxies for a long-term approach. Amel-Zadeh and Serafeim (2018) provide survey evidence that institutional investors care about all of these (see their Table 3). In our model, the only important elements are that these categories affect the investor’s payoff

⁵This is in line with a statement by Jean Christophe Nicaise Chateau from the European Commission: “Users like the diversity of ratings. It gives them the ability to go to a number of different providers depending on the type of information they’re looking for. The more they know about what they’re buying, the more it will help them choose to go to the relevant data provider or ratings agency that may have a more specialized approach.” See: <https://www.sustainability.com/thinking/rating-the-raters-yet-again-increasing-esg-scrutiny-makes-current-rate-the-raters-study-even-more-crucial/>

from investing in the project and, therefore, the investor would like to learn more information about them.

In reality, the rating agencies themselves are unclear and varied in their objectives, discussing both risk and impact (for a summary, see Larcker et al., 2022). The literature also finds very mixed evidence on the relationship between ratings and social outcomes (e.g., Raghunandan and Rajgopal, 2022) and the relationship between ratings and financial performance (e.g., Hartzmark and Sussman, 2019). Berg et al. (2021) find that combining ratings leads to a stronger relationship between ESG and financial performance.

Our model is complementary to models of credit rating competition, as in Bolton et al. (2012), Bar-Isaac and Shapiro (2013), Bouvard and Levy (2018), and Piccolo (2021). However, we assume the raters tell the truth and do not depend on reputation. This permits us to broaden the analysis and allow for revealing information on multiple categories.

Our paper also contributes to the literature on information sales.⁶ Admati and Pfleiderer (1986) study how a monopolistic information provider should design signals about asset values when the value of its information is subject to dilution through equilibrium prices. Huang et al. (2018) extend Admati and Pfleiderer (1986) to the case with multiple information sellers. Bergemann et al. (2018) examine the optimal menu of signals sold by a monopolistic information provider when it is uncertain about the buyer’s valuation of the information. A key distinction of our model is that information providers decide how to gather information about multiple, possibly unrelated, variables, and then sell their assessments to investors. We show that raters may move away from specialization toward generalization, depending on the investor’s preferences, while generalization by both raters results in inefficient duplication of effort, and consequently, less overall information production.

The multidimensionality of relevant information in financial decisions has been analyzed in models of financial markets (Goldstein and Yang, 2015; Goldstein et al., 2022) and bank lending (Blickle et al., 2024). We contribute to this literature by studying the incentives of an information intermediary to gather, design, and sell multidimensional information.

A growing body of literature theoretically investigates the functioning of capital markets in financing socially responsible investments (Piccolo et al., 2022; Gupta et al., 2022; Piatti et al., 2023; Oehmke and Opp, 2024), and regulations to facilitate the transition toward a green economy (Oehmke and Opp, 2022; Hong et al., 2023; Huang and Kopytov, 2023; Döttling and Rola-Janicka, 2023; Inderst and Opp, 2024). Some studies have explored the asset pricing implications of ESG investing (e.g., see Pástor et al., 2021; Sauzet and Zerbib, 2022; Avramov et al., 2022). This paper contributes to this literature by exploring the

⁶For a review, see Bergemann and Bonatti, 2019.

production of ESG information, which is a key input for the functioning of capital markets.

The rest of this paper is organized as follows. Section 2 presents the setup. In Section 3, we characterize the equilibrium outcomes, and compare them with the value-maximizing outcomes. In Section 4, we explore the implications of disagreement between raters. In Section 5, we analyze the impact of greenwashing on the design of ESG ratings. In Section 6, we demonstrate the robustness of the key results under alternative assumptions about the information acquisition cost, the allocation of market power in pricing, and allowing for mixed strategies. We also analyze the equilibrium outcomes when raters choose between different measurement methods. Concluding remarks are in Section 7. Key proofs are in the appendix. The online appendix has additional proofs and (i) shows that our results remain robust when we allow for a more flexible information structure for the ratings, and (ii) examines the socially optimal production of ESG information when investors' ESG preferences are not aligned with the social planner.

2 The Model

We consider a model of competition between two ESG raters to sell information to an investor. There are three periods in the model: $t = 0, 0.5, 1$. There is a project that requires investment $I > 0$, which generates a certain financial output of $I + \Delta$, where $\Delta > 0$. A deep-pocketed investor considers investing in the project.⁷ The investor is concerned about the project's ESG performance, denoted by u . ESG performance may capture externalities of the project or long-term risk factors that contribute to or take away from the project's future cash flows. Therefore, ESG performance may be positive or negative. The investor assigns weight $\beta \geq 0$ to ESG performance. As such, from the investor's perspective, the total value created by the project is $\Delta + \beta u$.

The fundamental assumption in our model is that ESG performance has multiple dimensions, which can be unrelated. For instance, ESG rating agencies evaluate firms based on three major categories (environmental, social, and governance), with each category containing many sub-categories. The performance in one category (or subcategory) is not necessarily related to or informative about the performance in the others. This multidimensionality of non-financial performance differentiates our model from the equilibrium models of credit ratings, where the investors' objective is to learn about a default probability based on financial data.

⁷Note that as there is only one investor, ESG raters in our model do not have the incentive to differentiate in order to capture market share.

For simplicity, we focus on two categories: A and B . The categories could represent the major categories (i.e., E, S, and G), or subcategories of a major category. The project's performance in A and B is characterized by (w^A, w^B) , where the performance is binary in each category, i.e., $w^i \in \{L, H\}$, where $i = A, B$. A priori, the probability of the high state is $\eta \in (0, 1)$ for each category. The probabilities are independent across the categories. The project's type is publicly realized in period 1.

The equation below specifies the ESG performance (u) as a function of the project's type:

$$u = \begin{cases} V^{HH} & (w^A, w^B) = (H, H) \\ V^{HL} & (w^A, w^B) \in \{(H, L), (L, H)\} \\ V^{LL} & (w^A, w^B) = (L, L), \end{cases} \quad (1)$$

where:

$$V^{HH} > 0, \quad V^{LL} < 0, \quad V^{HL} \in (V^{LL}, V^{HH}). \quad (2)$$

We present our results under the assumption that the investor does not invest in the project if no information about the project's ESG performance is available. Assumption 1 formalizes this point. This assumption ensures that there is a demand for ESG information, as no investment would take place without the information.

Assumption 1.

$$\Delta + \beta \mathbb{E}[u] = \Delta + \beta \{\eta^2 V^{HH} + 2(1 - \eta)\eta V^{HL} + (1 - \eta)^2 V^{LL}\} < 0. \quad (3)$$

Furthermore, we assume that the investor is not indifferent between investing and not investing when the project's type is (H, L) or (L, H) , as formalized by Assumption 2. This assumption ensures that information about both categories is valuable for the investor, even when information about the other category is perfectly available.

Assumption 2.

$$\Delta + \beta V^{HL} \neq 0. \quad (4)$$

2.1 ESG Rating Agencies

Information about u is provided by two non-cooperative ESG rating agencies. The raters are indexed by $j = 1, 2$. In period zero, the raters design a rating technology that generates a rating for each of the two categories. The ratings are denoted by $S_j = (s_j^A, s_j^B)$. The ratings are binary, i.e., $s_j^i \in \{h, l\}$, for $j = 1, 2$, and $i = A, B$. The rating technology is characterized by the pair $\lambda_j = (\lambda_j^A, \lambda_j^B)$, where λ_j^i denotes the probability that the project receives a high

rating in a category for which it has a high type, which we label as precision. Putting it differently,

$$P(s_j^i = h|w^i = H) = \lambda_j^i, \quad P(s_j^i = h|w^i = L) = 0, \quad j = 1, 2, \quad i = A, B. \quad (5)$$

Note that under this signal structure, high-ratings perfectly reveal the underlying state.⁸ In Section OA.1, we consider a more general set of rating technologies that allow for both false-positive and false-negative errors in the ratings, and demonstrate the robustness of our results to our assumption about the signal structure. A few remarks about the ratings are in order. The raters report their ratings truthfully. Hence, we assume away any strategic behavior in the reporting of the ratings (as in, e.g., Bolton et al., 2012; Agrawal et al., 2023). Moreover, the raters report the rating in each category separately, rather than reporting an aggregated version. This is consistent with the fact that ESG rating agencies provide the break-down of their ratings to their subscribers, and this clearly dominates an arbitrary aggregation rule.

Since there is no false-positive error in the ratings, a high rating in a category by a rater is enough to verify that the project has a high performance in that category. Therefore, it is helpful to introduce the following notation for the combined ratings:

$$s^i = \begin{cases} h & \text{if } s_1^i = h \text{ or } s_2^i = h \\ l & \text{if } s_1^i = s_2^i = l \end{cases} \quad i = A, B. \quad (6)$$

The conditional probabilities for $s^i = h$, namely the probability of receiving a high rating in category i from at least one of the raters, is:

$$P(s^i = h|w^i = L) = 0, \quad (7)$$

$$\lambda^i \equiv P(s^i = h|w^i = H) = 1 - (1 - \lambda_1^i)(1 - \lambda_2^i) = \lambda_1^i + \lambda_2^i - \lambda_1^i \lambda_2^i, \quad i = A, B.$$

We assume that the raters choose their rating technology under the following technological constraint:

$$\lambda_j^A + \lambda_j^B \leq 1. \quad (8)$$

This constraint implies that each rater can perfectly disclose the project's performance in one of the categories, or provide a noisy rating for both categories. As a result, it is possible for the

⁸Note that (5) implies that in the extreme case of $\lambda_j^i = 0$, namely when rater j does not assess the project in category i , rater j assigns rating l for category i . However, this rating is uninformative.

raters to perfectly reveal the project’s type collectively.⁹ The ratings are independent across the raters conditional on the project’s performance, which can capture different measurement methodologies employed by ESG raters (Berg et al., 2022). All feasible rating technologies are equally costly, and the marginal cost is set to zero.

Two types of rating technologies are particularly important in our analysis:

Definition 1. We say rater $j \in \{1, 2\}$ *specializes* if $\lambda_j \in \{(1, 0), (0, 1)\}$. We denote the specialized rating technologies in categories A and B by λ^{SPA} and λ^{SPB} , respectively. Moreover, we say rater $j \in \{1, 2\}$ *generalizes* if $\lambda_j = \lambda^{GN} = (\frac{1}{2}, \frac{1}{2})$.

2.2 The Ratings Market

After designing the rating technologies simultaneously in period zero, the raters sequentially offer their set of ratings to the investor at fees ϕ_1 and ϕ_2 . Rater 1 sets its fee first. The sequential fee-setting enables us to refine the equilibrium outcomes since the equilibrium would not always be unique had we assumed a simultaneous fee-setting.¹⁰ In Section 6.2, we show the robustness of our results to the case that both raters set their fees first with a positive probability. After observing the fees, the investor decides whose ratings to purchase given the posted fees. The investor can purchase ratings from one rater, from both raters, or not purchase at all. The investor cannot see the ratings before purchasing them; the information for each set of ratings is revealed after purchasing them.

We define $V(\lambda_1, \lambda_2)$ as the expected value of the ratings to the investor when bought together. Moreover, let \mathbf{O} denote the uninformative rating technology, i.e., $\mathbf{O} = (0, 0)$. Therefore, $V(\lambda_1, \mathbf{O})$ and $V(\mathbf{O}, \lambda_2)$ are the “stand-alone” values of the ratings provided by raters 1 and 2, respectively. Equation 9 formally defines the value function:

$$V(\lambda_1, \lambda_2; \beta) = \sum_{s^A, s^B \in \{h, l\}} \max\{0, \Delta + \beta \mathbb{E}[u|s^A, s^B]\} P(s^A, s^B). \quad (9)$$

In equation 9, s^A and s^B denote the possible realizations of the combined ratings. $P(s^A, s^B)$ denotes the unconditional probability that $(s^A, s^B) \in \{h, l\}^2$ realizes. The maximum operator indicates that the investor may decide to invest or not given the realization of s^A and s^B . To avoid repetition, we omit the element β where possible, for simplicity.

⁹In Section 6.1, we demonstrate the robustness of our results in a case where the information acquisition costs differ across the categories so the raters cannot perfectly reveal the project’s type. This assumption helps us simplify the characterizations in the main model.

¹⁰To see this point, consider the following conceptually similar game: Suppose two players simultaneously request a fraction of a cake, analogous to the overall surplus created by the ratings. The requests are accepted as long as their sum does not exceed one. Then, any pair of fractions that sum up to one constitutes an equilibrium.

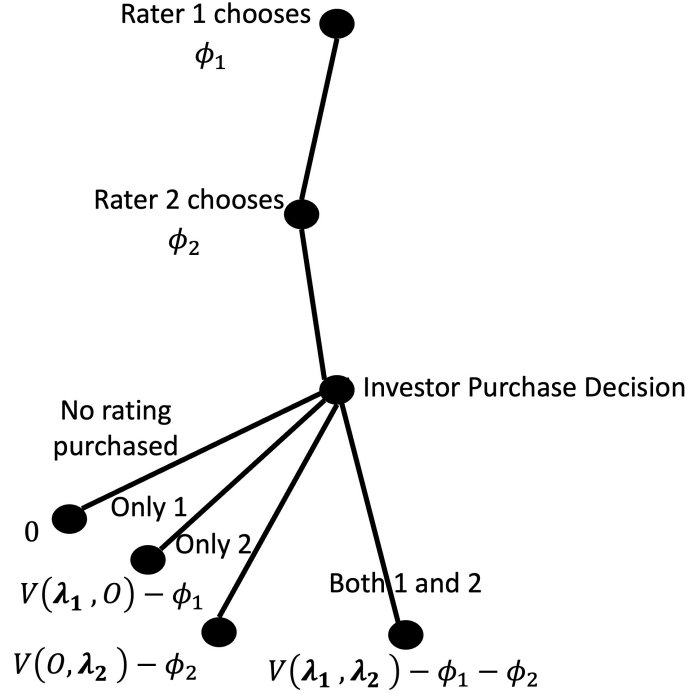


Figure 1. Sequence of actions and the investor's payoff in the ratings market stage

Figure 1 displays the moves and the investor's payoff. We assume that the investor breaks ties in her purchasing decision according to the following order, with the first being the most favored: (1) purchasing from both the raters, (2) purchasing only from Rater 1, (3) purchasing only from Rater 2, (4) no purchase. We analyze and discuss the equilibrium fees in detail in Section 3.1.

Timeline

Figure 2 presents the timeline of the model.

3 Equilibrium Choice of Rating Technology

This section analyzes what rating technologies the raters will choose. After solving for the market equilibrium, we contrast the results with the rating technologies that maximize surplus.

3.1 Equilibrium in the Ratings Market Stage

In this section, we analyze the raters' equilibrium fees given a pair of rating technologies (λ_1, λ_2) . Lemma 1 provides the equilibrium fees.

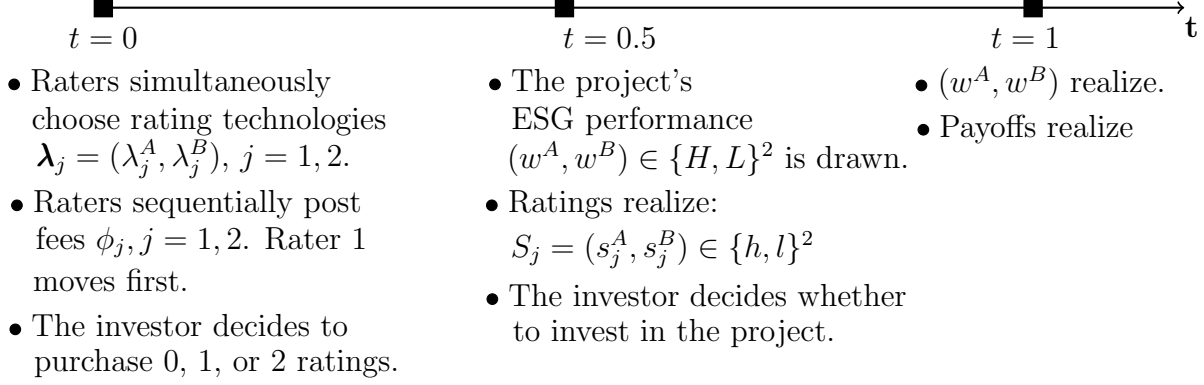


Figure 2. Timeline.

Lemma 1. *If the rating technologies chosen by the raters are (λ_1, λ_2) , then the following fees are set in equilibrium:*

$$\begin{aligned}\phi_1 &= V(\lambda_1, \lambda_2) - V(\mathbf{O}, \lambda_2) \\ \phi_2 &= \min\{V(\lambda_1, \lambda_2) - V(\lambda_1, \mathbf{O}), V(\mathbf{O}, \lambda_2) - V(\mathbf{O}, \mathbf{O})\}.\end{aligned}\tag{10}$$

Lemma 1 states that Rater 1 is always paid the marginal value of its ratings, while Rater 2 may be paid less. The reason is that the sum of the marginal contributions can exceed the total value created by the ratings, depending on the choice of rating technologies. In other words,

$$\underbrace{V(\lambda_1, \lambda_2) - V(\mathbf{O}, \lambda_2)}_{\text{Marginal contribution of Rater 1}} + \underbrace{V(\lambda_1, \lambda_2) - V(\lambda_1, \mathbf{O})}_{\text{Marginal contribution of Rater 2}} \geq \underbrace{V(\lambda_1, \lambda_2) - V(\mathbf{O}, \mathbf{O})}_{\text{Total value created by the ratings}}.\tag{11}$$

Consequently, the investor is not willing to pay both raters their marginal contribution.¹¹

In this case, Rater 2 charges the stand-alone value of its rating technology, as it is the maximum amount the rater can charge if the investor were only to buy Rater 2's ratings. Therefore, Rater 2's fee is bounded by the stand-alone value of its ratings, affecting the rater's incentives in the design of its rating technology.¹²

¹¹By rearranging equation 11, we obtain:

$$V(\lambda_1, \lambda_2) - V(\lambda_1, \mathbf{O}) \geq V(\mathbf{O}, \lambda_2) - V(\mathbf{O}, \mathbf{O}).\tag{11'}$$

This directly connects equation 11 to the Rater 2's equilibrium payoff. Note that we can see from equation 11' that the rating technologies are complements (the marginal value of the Rater 2 is larger than its stand-alone value) when the inequality holds, and substitutes otherwise. We use this formulation in the appendix.

¹²In Section 6.2, we demonstrate the robustness of our results when both raters move first with a positive probability, and consequently, both raters' payoffs will depend on their stand-alone value with a positive probability.

3.2 Market Outcomes

With the characterization of the fees, we can analyze the equilibrium design of the ratings. Note that (λ_1, λ_2) is an equilibrium outcome when λ_j , $j = 1, 2$, maximizes fee ϕ_j , given the choice of the other rater. Proposition 1 characterizes the equilibrium outcomes in pure strategies. In section 6.3, we analyze the set of mixed strategy equilibria.

Proposition 1. *Under Assumptions 1 and 2, the only possible pure strategy equilibrium outcomes are generalization by both raters and specialization by each rater in different categories. The following provides the characterization in detail (up to symmetries in the actions):*

a) *If $V^{HL} \geq 0$, the only equilibrium outcome is that the raters specialize in different categories.*

b) *If $V^{HL} < 0$, define:*

$$\beta^*(\lambda) = \sup\left\{\beta \mid \frac{(1-\eta)(\Delta + \beta V^{HL})}{\eta(\Delta + \beta V^{HH})} \geq \lambda - 1\right\}, \quad (12)$$

which is a decreasing function of λ .

b.1) *If $\beta^*(\frac{1}{4})$ is finite and $\beta > \beta^*(\frac{1}{4})$, then the unique equilibrium is generalization by both raters.*

b.2) *If $\beta \leq \beta^*(\frac{1}{4})$, specialization in different categories is an equilibrium. This is the only equilibrium outcome, except when $\beta \in [\beta^*(\frac{9}{16}), \beta^*(\frac{17}{32})] \cup \{\beta^*(\frac{1}{4})\}$. In this case, generalization by both raters is also an equilibrium.*

Proposition 1 states that if information about the project's ESG performance is essential for the investment (Assumption 1), and information about both categories always has a positive marginal value (Assumption 2), the only pure strategy equilibria are that the raters either both generalize or specialize in different categories. In Section 6.3, where we analyze the mixed strategy equilibria, we show that there is one additional equilibrium for some parameter values: one where the raters fail to coordinate on the category in which they specialize, and they randomize between specializing in categories A and B with equal probabilities.

Part (a) of Proposition 1 states that specialization in different categories is the only form of equilibrium when $V^{HL} \geq 0$. The intuition is that high performance in a single category is sufficient for the investor to invest, which implies a positive stand-alone value for specialization. In fact, specialization has the largest stand-alone value in this case. Moreover, specialization in different categories identifies project types most efficiently, which means specialization also has the largest marginal value given the other rater specializes in the

other category. Therefore, in response to the other rater specializing, specialization in the other category is the best response for both raters. Moreover, both ratings are purchased in equilibrium since each rater identifies the high-performing projects in a single category; thus, both ratings have a positive marginal value.

When $V^{HL} < 0$, the equilibrium outcome depends on the investor's decision when the project has a mix of positive and negative ratings, i.e., $s^A = h$ and $s^B = l$, or vice versa. To this end, function $\beta^*(\lambda)$, when $\lambda \in [0, 1]$, represents the maximum value of β at which the investor is willing to invest when the project has a high rating in category i and both ratings are low in category $-i$ (i.e., the other category), with a combined precision of λ^{-i} (where λ^{-i} is defined in equation 7), and the argument of $\beta^*(\cdot)$ is $\lambda = \lambda^{-i}$. Note that for smaller values of λ , the investor with threshold type $\beta^*(\lambda)$ has less tolerance toward negative ratings, resulting in a negative relationship between $\beta^*(\lambda)$ and λ . For instance, when $\beta^*(0)$ is finite and $\beta > \beta^*(0)$, β is so large that a single high performance never justifies investment. Specifically, we have:

$$\begin{aligned} \beta > \beta^*(0) &\Rightarrow \eta(\Delta + \beta V^{HH}) + (1 - \eta)(\Delta + \beta V^{HL}) < 0 \\ &\Rightarrow \mathbb{E}[\Delta + \beta u | w^A = H] = \mathbb{E}[\Delta + \beta u | w^B = H] < 0. \end{aligned} \tag{13}$$

In this case, learning only w^A or w^B holds no value for the investor since she invests only when she receives a positive update about both categories. Therefore, Rater 2 cannot charge a positive fee when it specializes, pushing the rater toward generalization, which yields the highest stand-alone value. In response, Rater 1 moves away from specialization and chooses to generalize.

More broadly, stand-alone values play a role in the market equilibrium when β is sufficiently large. In our model, generalization has the highest stand-alone value when $\beta^*(\frac{1}{4})$ is finite and $\beta > \beta^*(\frac{1}{4})$. In other words, when this condition holds, the investor prefers obtaining a noisy signal about each category to a perfectly-revealing signal for a single category. Given this, Rater 2 generalizes, and consequently, the unique equilibrium is generalization by both raters. Here, the investor's strong preference for ESG investments leads, surprisingly, to less information being provided.

Another key force that shapes the market equilibrium is the strategic complementarity in the design of ratings. To see the strategic complementarity, note that the marginal value of specialization in a category is the highest when the other rater specializes in the other category. As the other rater moves from specialization towards generalization, the overlap between the two rating technologies increases, which reduces the marginal value of specialization. Conversely, the marginal value of generalization increases as the other rater moves

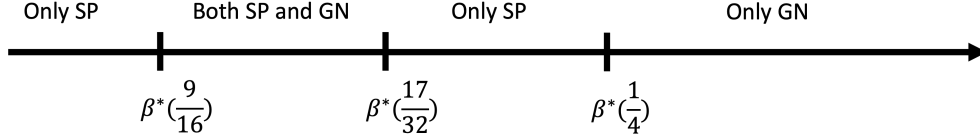


Figure 3. This diagram illustrates the set of market equilibria for different values of β when $V^{HL} < 0$ and $\beta^*(\frac{1}{4})$ is finite. “GN” and “SP” represent generalization by both raters and specialization in different categories, respectively.

from specialization to generalization. As such, both specialization in different categories and generalization by both raters can be equilibrium outcomes when the raters maximize their marginal value. Because of this strategic complementarity, there are multiple equilibria for some intermediate values of β . This occurs (1) when $\beta \geq \beta^*(\frac{9}{16})$, corresponding to when the marginal value of generalization is higher than the marginal value of specialization, given the other rater generalizes ($V(\lambda^{GN}, \lambda^{GN}) - V(\mathbf{O}, \lambda^{GN}) \geq V(\lambda^{SP_i}, \lambda^{GN}) - V(\mathbf{O}, \lambda^{GN})$), $i = A, B$; and (2) $\beta \leq \beta^*(\frac{17}{32})$, corresponding to when the marginal value of generalization (given the other rater generalizes) does not exceed the stand-alone value of generalization ($V(\lambda^{GN}, \mathbf{O}) \geq V(\lambda^{GN}, \lambda^{GN}) - V(\lambda^{GN}, \mathbf{O})$).

Figure 3 illustrates how the set of market equilibria varies with β when $V^{HL} < 0$. Figure 4 displays the equilibrium outcomes for different values of β and V^{HL} by depicting the raters’ best response behavior.

3.3 Value-maximizing Ratings

Next we analyze the choice of rating technologies that maximize the expected investment value. In particular, we call pair $(\lambda_1^*, \lambda_2^*)$ “value-maximizing” if it is a global maximizer of $V(\lambda_1, \lambda_2)$, as defined in equation 9, given the technological constraint in equation 8.

A value-maximizing pair can be interpreted in different ways. First, it can describe the solution to the social planner’s problem when the planner maximizes the total surplus of the investor and the raters. Second, it can describe the situation where the social planner’s objective is to maximize the expected social value of the investment and the social planner has the same preferences (i.e., β) as the investor. In Section OA.2, we examine the situation where the social planner and the investor may diverge in their valuations of ESG performance. Third, if the raters could collude and jointly decide about their rating technologies, they would choose a value-maximizing pair. As such, value-maximizing pairs provide us with a natural benchmark to study the impact of raters’ competition on the production of ESG information. Proposition 2 demonstrates that the only value-maximizing outcome is specialization in different categories.

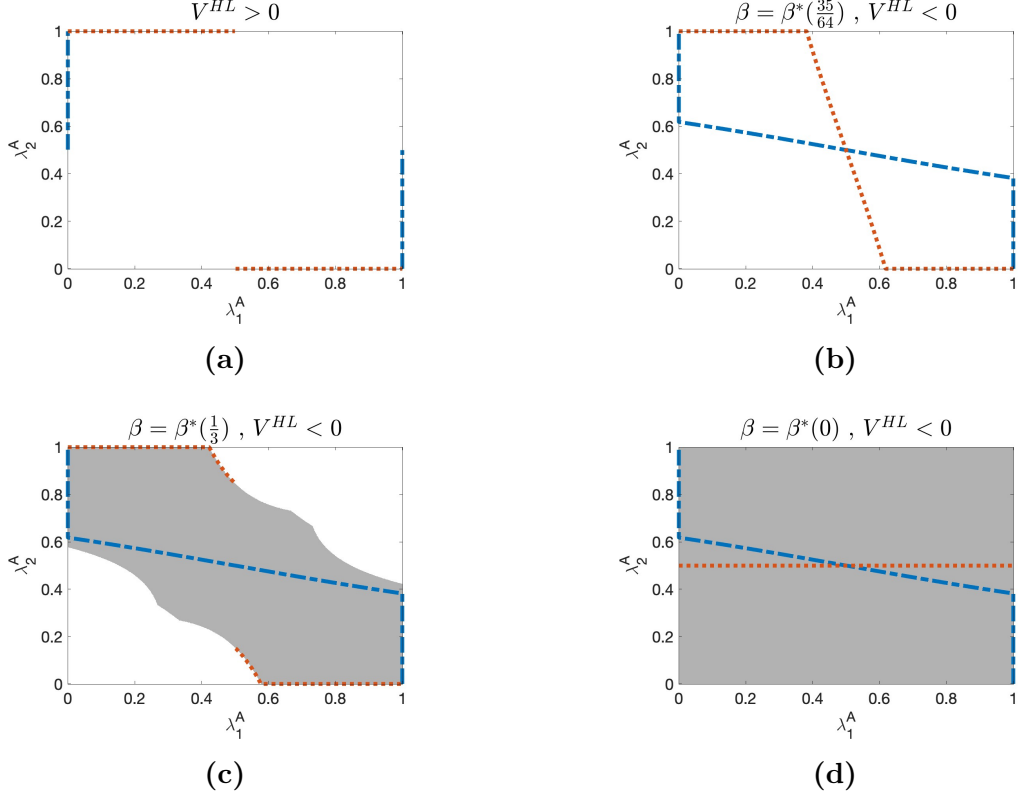


Figure 4. This figure displays the best response of each rater (blue (dashed) line for Rater 1 and red (dotted) line for Rater 2) for different values of β and V^{HL} . The intersection points designate the equilibrium outcomes. The white (gray) area represents the outcomes in which Rater 2 charges the marginal value (stand-alone value) of its ratings.

Proposition 2. *Under Assumptions 1 and 2, the only value-maximizing outcome is specialization in different categories, i.e., $(\lambda_1^*, \lambda_2^*) = (\lambda^{SP_A}, \lambda^{SP_B}), (\lambda^{SP_B}, \lambda^{SP_A})$.*

Note that when the raters specialize in different categories, the project's type is perfectly revealed to the investor, which results in perfect investment efficiency. In other words, the investment takes place iff it generates a positive value from the perspective of the investor (i.e., $\Delta + \beta u > 0$). No other pair of rating technologies implements this investment outcome since they result in some inefficient overlap in the information produced for each category. Figure 5 illustrates this point.

By juxtaposing Propositions 1 and 2, we learn that competition in the production of ESG information results in generalization when β is large and $V^{HL} < 0$, while the value-maximizing solution is specialization. Note that the deviation from the value-maximizing outcome happens when the investor is the most concerned about the project's ESG performance.

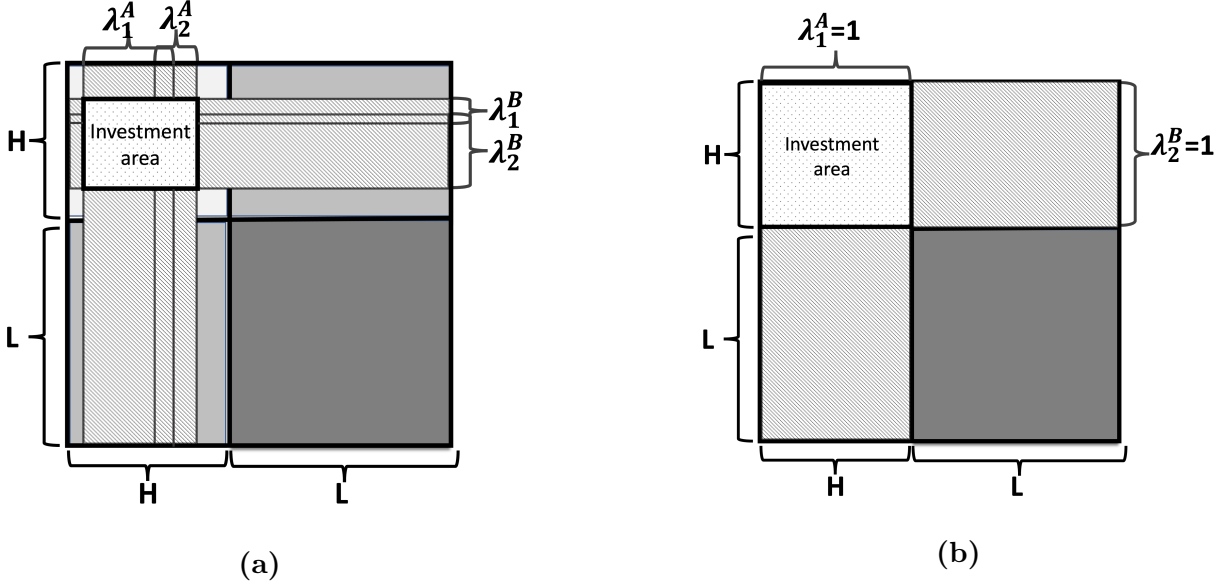


Figure 5. This figure displays the investment area for a (a) non-value-maximizing and (b) value-maximizing pair of rating technologies. In the figure, the parameter values are assumed to be such that the investor needs to receive a positive update in both categories to invest, i.e., when the condition in equation 13 is satisfied. The figure shows that when the raters specialize in different categories, a (H, H) -project always receives investment. No other pair can achieve this outcome.

The underlying insight here is that, while the specialization outcome maximizes the investment value, Rater 2 has incentives to generalize because its payoff is tied to the stand-alone value of its ratings. In other words, the strong demand for information about both categories A and B creates an incentive for raters to generalize in equilibrium, as specialized ratings hold no stand-alone value for the investor. Of course, as demonstrated earlier, strategic complementarities in the design of ratings can lead to generalization as well.

4 Disagreement in Ratings

Investors rely on ESG ratings and information to incorporate ESG considerations in their investments. Nevertheless, the available ESG ratings vastly disagree in their assessment of firms' ESG performance, potentially creating confusion for investors (Chatterji et al., 2016; Berg et al., 2022). In this section, we analyze the disagreement in the ratings implied by the model for the value-maximizing and market equilibrium pairs of rating technologies.

To analyze the disagreement, we examine the correlation between the expected investment payoff implied by each rater's ratings; that is,

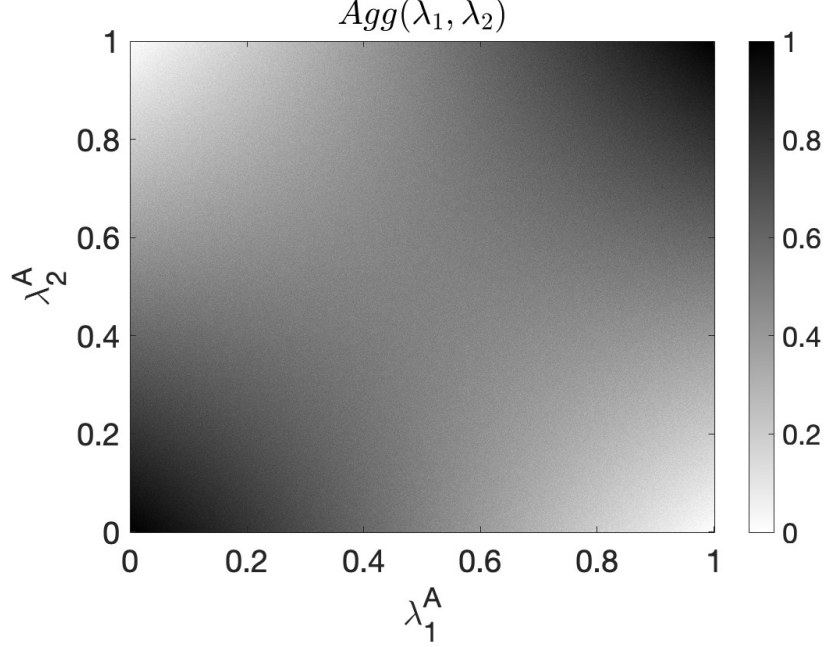


Figure 6. Agreement in the expected investment payoffs implied by the ratings ($Agg(\lambda_1, \lambda_2)$), as in equation 14. Darker values correspond to higher levels of agreement. Top-left and bottom-right points correspond to the outcomes in which the raters specialize in different categories. The middle point corresponds to the generalization outcome.

$$\begin{aligned} Agg(\lambda_1, \lambda_2) &= Corr(\mathbb{E}[\Delta + \beta u | s_1^A, s_1^B], \mathbb{E}[\Delta + \beta u | s_2^A, s_2^B]) \\ &= Corr(\mathbb{E}[u | s_1^A, s_1^B], \mathbb{E}[u | s_2^A, s_2^B]). \end{aligned} \quad (14)$$

We use this method of aggregation rather than being limited to an arbitrary aggregation rule. Figure 6 illustrates how the disagreement varies with the choice of rating technology. Darker points indicate more agreement (less disagreement). We see that specialization in different categories, which is value-maximizing, results in the highest level of disagreement. The agreement level is intermediate when both raters generalize. The highest level of agreement would be obtained if both raters specialize in the same category. We formalize this result in Proposition 3.

Proposition 3. *$Agg(\lambda_1, \lambda_2)$ is minimized when the raters specialize in different categories, and maximized when the raters specialize in the same category.*

To understand this result, note that disagreement arises from two sources: 1) providing inaccurate, but independent, ratings for a category, or 2) allocating resources differently among the categories. When both raters generalize, the former is the only source of discrepancy, and when the raters specialize in different categories, the latter is the primary

source. As a result, the disagreement observed in data can be attributed to a combination of these two sources, which stem from specialization and technological limitations. Therefore, disagreement in ESG ratings is efficient for investors when specialization is the driver.¹³

Berg et al. (2022) find that 38% of the discrepancy in the category ratings provided by major ESG rating agencies can be attributed to the differences in the subcategories examined (scope). This indicates that indeed a significant portion of the disagreement reflects specialization by the raters, and is thus beneficial from a social welfare perspective.¹⁴

5 Greenwashing and the Design of Ratings

In our model, we assume that the ESG raters can assess the true type of the project. However, in practice, ESG raters mostly utilize self-disclosed information provided by firms for their assessments. It means that there is room for manipulation, which is typically referred to as “greenwashing” in this context. For instance, a firm might announce a plan to reduce carbon emissions, which would help the firm receive a better rating in the environmental category. However, it likely would be difficult to assess the firm’s commitment to the plan. In this section, we analyze how greenwashing impacts the raters’ design of their ratings.

Specifically, suppose the raters assess a potentially manipulated type of the project, which we denote by (w_M^A, w_M^B) , where $w_M^A, w_M^B \in \{H, L\}$. In particular, with probability α , the project designer can successfully manipulate the type for any category with a low performance. The equations below describe the relationship between manipulated and actual types:

$$Prob(w_M^i = H | w^i = H) = 1, \quad Prob(w_M^i = H | w^i = L) = \alpha \in [0, 1), \quad i = A, B. \quad (15)$$

The probability α is lower when ESG disclosure requirements are tightened, or when greenwashing is costlier. The main model in the text corresponds to the case that greenwashing is not possible, i.e., $\alpha = 0$.

We maintain the mapping between types and ratings in equation 5 with the difference being that the types are the manipulated types:

$$P(s_j^i = h | w_M^i = H) = \lambda_j^i, \quad P(s_j^i = h | w_M^i = L) = 0, \quad j = 1, 2, \quad i = A, B. \quad (16)$$

¹³This intuition is robust to the way the ratings are aggregated. For instance, had the raters reported a precision-weighted average of their ratings, specialization in different (the same) categories would still induce the lowest (highest) correlation in the aggregated ratings.

¹⁴Berg et al. (2022) find that divergence in measurement methodologies also substantially contributes to the discrepancy in the ratings. In Section 6.4, we analyze the raters’ trade-offs in choosing among different measurement methods.

Note that the equations in (16) indicate that greenwashing results in the manipulation of the ratings. In our model, the investor forms beliefs rationally. Hence, she correctly accounts for the possibility of greenwashing in her evaluation of the project's ESG performance given a set of ESG ratings.

We assume that α is such that the ratings still induce investment when the project receives a high rating in both categories. In particular,

$$\begin{aligned} \Delta + \beta \mathbb{E}[u|s^A = s^B = h] &= \Delta + \beta \mathbb{E}[u|w_M^A = w_M^B = H] \\ &= \Delta + \frac{\beta}{(\eta + (1 - \eta)\alpha)^2} \left\{ \eta^2 V^{HH} + 2\eta(1 - \eta)\alpha V^{HL} + (1 - \eta)^2 \alpha^2 V^{LL} \right\} > 0. \end{aligned} \quad (17)$$

Note that greenwashing introduces additional noise to the ESG ratings. For instance, a high rating in both categories does not necessarily indicate high performance in both. Consequently, greenwashing causes investors to discount the expected ESG performance based on these ratings. For instance, if the investor requires a high performance only in one category when there is no greenwashing, she might require a high rating in two categories to account for the possibility of greenwashing. This can be thought of as a hedging mechanism against the greenwashing risk. In Proposition 4, we describe how greenwashing impacts the equilibrium design of ratings, through its effect on the investor's demand for ESG information.

Proposition 4. *For any values of β and V^{HL} , there exists an α^* such that the unique equilibrium is generalization by both raters when $\alpha > \alpha^*$ and α satisfies equation 17.*

Proposition 4 states that when the amount of greenwashing is sufficiently large, the unique equilibrium is generalization by both raters. The intuition is that when α is large enough, the investor requires a high rating in both categories to invest in the project. As a result, the stand-alone value of specialization is zero, which causes the raters to move away from the specialization outcome. Note that alpha must not be so large as to make the information in ESG ratings valueless (this is embodied in equation 17).

This result reveals a propagation mechanism through which greenwashing contaminates ESG ratings. Greenwashing does not only result in noisier ESG information, but also might push raters away from specialization since it is cumbersome to uncover the true performance in any category. Therefore, greenwashing results in more generalization, which leads to even less accurate ratings.

6 Robustness of the Model

This section demonstrates the insights delivered by our model are robust to the key assumptions and modeling choices made in the baseline model. Section 6.1 relaxes the symmetry between the two categories by allowing information production to be cheaper in one of the categories. Section 6.2 allows the second rater to move first with a positive probability, which enables us to examine the equilibrium outcomes for different allocations of surplus among the raters. In these two sections, we show that the set of equilibrium outcomes and value-maximizing outcomes remain qualitatively the same as the baseline model. Section 6.3 examines the equilibria in mixed strategies. Section 6.4 analyzes the raters' equilibrium behavior when they choose among different methods of measuring performance in a subcategory.

Additional robustness checks and discussions are provided in the Online Appendix. Section OA.1 demonstrates the robustness of our results to the assumption that the ratings have no false-positive error by considering a more general set of rating technologies that allow for flexible amounts of both false-positive and false-negative errors in the ratings. Section OA.2 examines the socially optimal information production when the investor and the social planner place different weights on the importance of ESG performance.

6.1 Unequal Information Acquisition Cost Across Categories

Thus far, we have assumed that information acquisition in the two categories is equally costly. In practice, performance is easier to measure in some categories than others. For instance, the Trucost database, offered by S&P Global, provides detailed information on firms' emissions of a variety of greenhouse gases, which helps investors and raters assess firms' performance in the environmental category.¹⁵ However, exposure to biodiversity risk is more complex to assess (Giglio et al., 2023). Therefore, the same amount of resources could result in different levels of precision in the ratings for different categories. In this section, we analyze how this heterogeneity impacts the equilibrium and value-maximizing design of the ratings.

Specifically, consider the following modification of the technological constraint in equation 8:

$$\lambda_j^A + \frac{\lambda_j^B}{b} \leq 1, \quad b \in (0, 1), \quad j = 1, 2. \quad (18)$$

In equation 18, b^{-1} captures the difficulty in acquiring information in category B , relative to that in category A . Note that specializing in category B , i.e., $\lambda_b^{SP_B} = (0, b)$, results in

¹⁵For more information, see <https://www.spglobal.com/esg/trucost>.

an imperfect rating about the project's performance in category B , while specialization in category A would perfectly disclose the project's performance in category A .

We assume that b is large enough that specialization in category B has a positive marginal value for the investor even when she is perfectly informed about the performance in category A . Assumption 2' formalizes this point. This Assumption boils down to Assumption 2 when $b = 1$.

Assumption 2'.

$$V(\lambda^{SP_A}, \lambda_b^{SP_B}) > V(\lambda^{SP_A}, \mathbf{O}). \quad (19)$$

In Propositions 5 and 6, we characterize the market equilibria and value-maximizing outcomes, respectively, in the presence of this heterogeneity in the information acquisition cost. Note that the raters cannot choose $\lambda^{GN} \equiv (\frac{1}{2}, \frac{1}{2})$ since it does not satisfy the technological constraint in equation 18. However, we see that raters choose interior rating technologies in equilibrium for some parameter values, mirroring the generalization outcome in our baseline model.

Proposition 5. *Under Assumptions 1 and 2', the only possible equilibria in pure strategies are specialization in different categories, and two more outcomes where both raters' rating technologies are interior (i.e., no rater specializes).¹⁶ The following provides the characterization of pure strategy equilibrium outcomes in detail:*

a) *If $V^{HL} \geq 0$, the only equilibrium outcome is that the raters specialize in different categories.*

b) *If $V^{HL} < 0$:*

b.1) *When $\beta > \beta^*(\frac{b}{4})$, the unique equilibrium outcome is that Rater 2 sets $\lambda_2 = (\frac{1}{2}, \frac{b}{2})$ and Rater 1 does not specialize.*

b.2) *When $\beta \leq \beta^*(\frac{b}{4})$, $(\lambda_b^{SP_B}, \lambda^{SP_A})$ is always an equilibrium, and $(\lambda^{SP_A}, \lambda_b^{SP_B})$ is also an equilibrium when $\beta \leq \beta^*(\frac{b}{(b+1)^2})$. These outcomes are the only equilibria except when:*

- $\beta = \beta^*(\frac{b}{4})$, where the interior outcome specified in Part (b.1) is also an equilibrium.
- b is above a threshold, and $\beta \in [\beta^*(x_1), \beta^*(x_2)]$, where $x_1 > x_2 > \frac{b}{(b+1)^2}$. In this case, there is an interior equilibrium in which both raters choose the same rating technologies with $\frac{1}{2} \geq \lambda_j^A > \lambda_j^B \geq \frac{b}{2}$, $j = 1, 2$.

Proposition 5 is similar to Proposition 1 for the baseline model. When $V^{HL} \geq 0$, specialization in different categories continues to be the only market equilibrium, even in the

¹⁶These outcomes are specified in the Appendix.

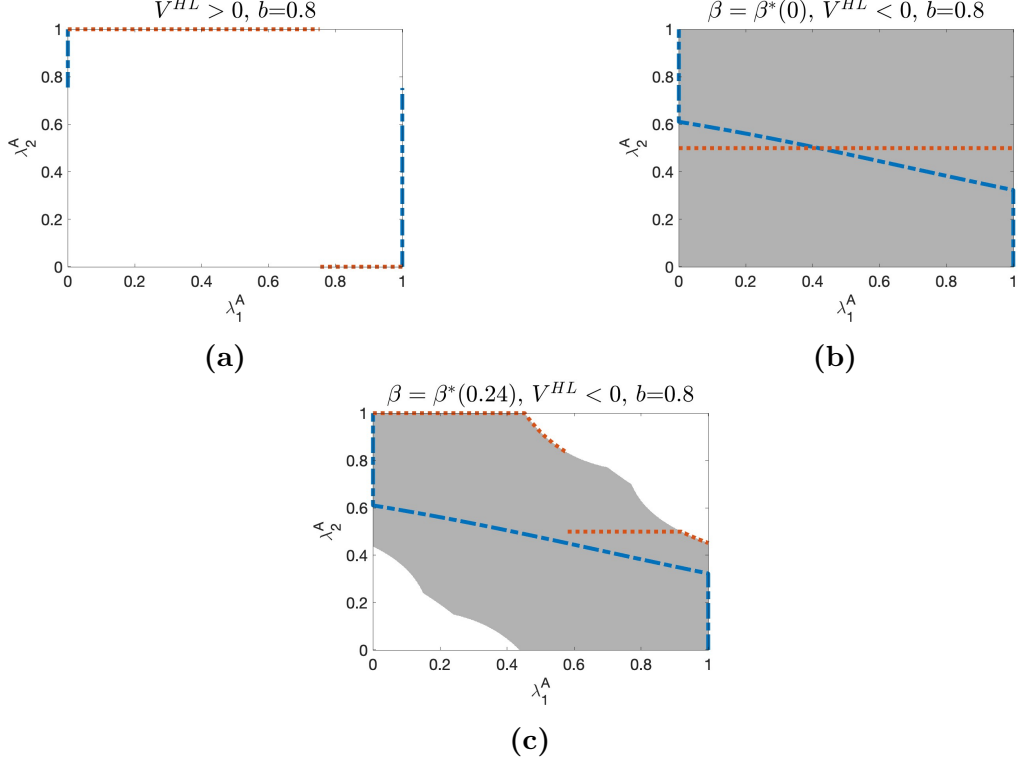


Figure 7. This figure displays the best response of each rater (blue (dashed) line for Rater 1 and red (dotted) line for Rater 2) for $b = 0.8$, and different values of β and V^{HL} . The intersection points designate the equilibrium outcomes. The white (gray) area represents the outcomes in which Rater 2 charges the marginal value (stand-alone value) of its ratings.

presence of heterogeneous information acquisition costs. Figure 7(a) illustrates this outcome. The equilibrium outcomes are similar when $V^{HL} < 0$ and β is small.

Part (b.1) of Proposition 5 states that when β is large, the raters choose interior rating technologies (see Figure 7(b)). Similar to the baseline model, the intuition is that specialization has no stand-alone value for high values of β , pushing Rater 2 away from specialization. Also, similar to the baseline model, Part (b.2) of the proposition indicates that for some intermediate values of β (when b is large), there is an equilibrium outcome in which the raters do not specialize, due to strategic complementarities.

Another interesting implication of Part (b.2) of Proposition 5 is that due to the heterogeneity in the information acquisition costs, the indeterminacy between the two specialization outcomes i.e., $(\lambda_b^{SP_B}, \lambda^{SP_A})$ and $(\lambda^{SP_A}, \lambda_b^{SP_B})$, might break for some values of β . This possibility is illustrated in Figure 7(c). In particular, for some values of β , the unique equilibrium is $(\lambda_b^{SP_B}, \lambda^{SP_A})$, in which Rater 2, who has a smaller bargaining power, specializes in category A , which has a lower information acquisition cost. The other specialization outcome

is not an equilibrium since $\lambda_b^{SP_B}$ has a low stand-alone value due to the higher information acquisition cost associated with category B , which would result in a low fee if set by Rater 2.

Now, we discuss the value-maximizing outcomes. Proposition 6 presents the value-maximizing pairs; that is, the set of pairs that maximize the investment value $V(\lambda_1, \lambda_2)$, given the technological constraint in equation 18.

Proposition 6. *Under Assumptions 1 and 2', specialization in different categories is the unique value-maximizing pair.*

Proposition 6 demonstrates the robustness of our earlier results that specialization in different categories maximizes the surplus, even with the presence of heterogeneity in the cost of information production. This result is intuitive since the overlap in information production is minimized when the raters specialize in different categories.

6.2 Stochastic Ordering of the Fee-setting

In our baseline model, we assume that the Rater 1 always sets its fee first, which gives Rater 1 stronger bargaining power in the allocation of the surplus created by the ratings. In this section, we relax this assumption by allowing Rater 2 to set its fee first with a positive probability. In particular, suppose Rater 1 sets its fee first with probability $p \in [0.5, 1]$. Figure 8 presents the moves and the investor's payoff considered in this section.

Note that this modification does not impact the investment value function (i.e., $V(\cdot, \cdot)$). As a result, specialization in different categories is the value-maximizing outcome.

Moreover, the expected payoff of each rater is a linear combination of the first-mover and second-mover's payoffs in equation 10:

$$\begin{aligned}\pi_1(\lambda_1, \lambda_2) &= p[V(\lambda_1, \lambda_2) - V(\mathbf{O}, \lambda_2)] + (1 - p) \min\{V(\lambda_1, \mathbf{O}) - V(\mathbf{O}, \mathbf{O}), V(\lambda_1, \lambda_2) - V(\mathbf{O}, \lambda_2)\} \\ \pi_2(\lambda_1, \lambda_2) &= (1 - p)[V(\lambda_1, \lambda_2) - V(\lambda_1, \mathbf{O})] + p \min\{V(\mathbf{O}, \lambda_2) - V(\mathbf{O}, \mathbf{O}), V(\lambda_1, \lambda_2) - V(\lambda_1, \mathbf{O})\}.\end{aligned}\tag{20}$$

Depending on whether the sum of the marginal value of the rating technologies exceeds their combined value or not (i.e., whether equation 11 holds or not), each rater's payoff is either their marginal value or a linear combination of their marginal value and stand-alone value. Specifically, when equation 11 holds, we have:

$$\begin{aligned}\pi_1(\lambda_1, \lambda_2) &= p[V(\lambda_1, \lambda_2) - V(\mathbf{O}, \lambda_2)] + (1 - p)(V(\lambda_1, \mathbf{O}) - V(\mathbf{O}, \mathbf{O})) \\ \pi_2(\lambda_1, \lambda_2) &= (1 - p)[V(\lambda_1, \lambda_2) - V(\lambda_1, \mathbf{O})] + p(V(\mathbf{O}, \lambda_2) - V(\mathbf{O}, \mathbf{O}))\end{aligned}\tag{21}$$

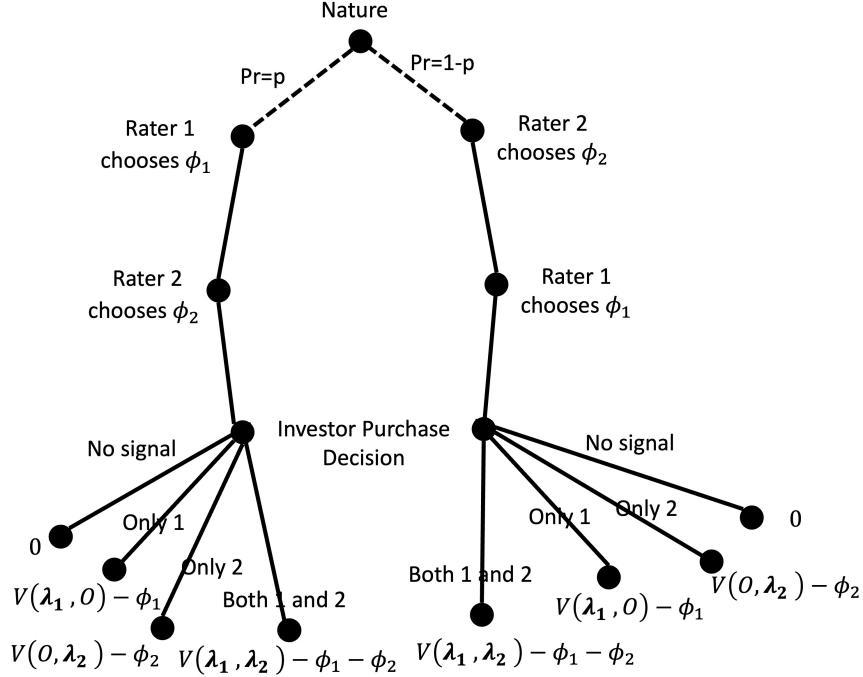


Figure 8. Sequence of actions and raters' payoff in the ratings market stage when the first mover is randomly determined.

We see that p determines the allocation of surplus between the raters in this case. Both raters receive their stand-alone value, plus a fraction of their marginal value. As such, p can be thought of as a parameter capturing the allocation of bargaining powers. In Proposition 7, we characterize the equilibrium outcomes. The diagrams in Figure 9 illustrate how the set of equilibrium outcomes varies with β .

Proposition 7. *Under Assumptions 1 and 2, the only possible equilibrium outcomes in pure strategies are generalization and specialization in different categories:*

a) *If $V^{HL} \geq 0$, the only equilibrium outcomes are specialization in different categories.*

b) *Suppose $V^{HL} < 0$:*

b.1) *If $p > 0.5$, generalization by both raters is the unique market equilibrium when $\beta > \beta^*((1 - \frac{1}{2p})^2)$.*

b.2) *If $p > 0.5$, specialization in different categories is always an equilibrium when $\beta \leq \beta^*((1 - \frac{1}{2p})^2)$. Moreover, generalization by both raters is an equilibrium when $\beta \in [\beta^*(\frac{9}{16}), \beta^*(\frac{17}{32})] \cup [\beta^*(\frac{1+3p}{16p}), \beta^*((1 - \frac{1}{2p})^2)]$.*

b.3) *If $p = 0.5$, specialization in different categories is always an equilibrium. Generalization by both raters is also an equilibrium when $\beta \in [\beta^*(\frac{9}{16}), \beta^*(\frac{17}{32})]$ and $\beta \geq \beta^*(\frac{5}{16})$.*

The main message of Proposition 7 is that the key insights of the baseline model are

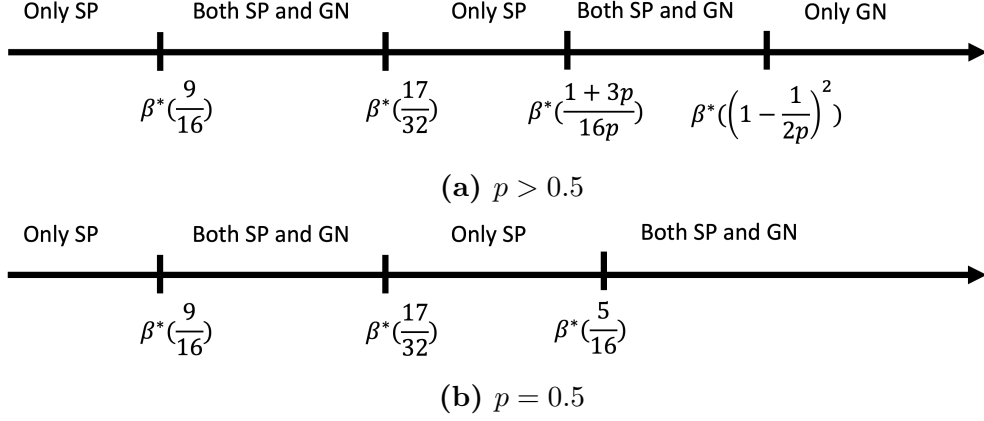


Figure 9. This diagram illustrates the set of market equilibria for different values of β when $V^{HL} < 0$ and the probability that Rater 1 sets its fee first $p \geq 0.5$. “GN” and “SP” represent generalization by both raters and specialization in different categories, respectively.

robust to the choice of the fee-setting mechanism. Particularly, generalization by both raters is the unique market equilibrium when β is sufficiently large (except for the knife-edge case of $p = 0.5$). The intuition is that the rater with a lower bargaining power (Rater 2) assigns a higher weight to the stand-alone value of its ratings, and specialization generates a small stand-alone value when β is large. Hence, this rater benefits from producing information in both categories to improve the stand-alone value of its ratings, and consequently, its pricing power.

The set of market equilibria remains the same for some parameter values, namely when $V^{HL} \geq 0$, and when $V^{HL} \leq 0$ and $\beta < \beta^*(\frac{17}{32})$; In these cases, both raters charge the marginal value of their ratings, thus their payoffs are the same as the baseline case. Both generalization by both raters and specialization in different categories can be equilibria for intermediate values of β due to the strategic complementarity motive discussed earlier.

6.3 Mixed Strategy Equilibria

In Section 3, we characterized the set of market equilibria in pure strategies. In this section, we characterize the mixed strategy equilibria.

In order to simplify the characterization of the set of mixed strategy equilibria, we focus on equilibria that are robust to small perturbations to β (the ESG preference parameter). We formalize this in Definition 2.

Definition 2. *[Robust Equilibrium] Let σ_1 and σ_2 be some probability density functions over the set of rating technologies. We say mixed strategies σ_1 and σ_2 constitute a “robust equilibrium” if for $i = 1, 2$, σ_i is a best response to σ_{-i} in a neighborhood of β .*

Intuitively, this refinement ensures that the raters' choices of rating technology are robust to some uncertainty about the investor's preference parameter. One can verify that the pure strategy equilibria characterized in Proposition 1 are *robust*, except at the threshold values (i.e., $\beta^*(\frac{9}{16})$, $\beta^*(\frac{17}{32})$, and $\beta^*(\frac{1}{4})$). In Proposition 8, we characterize the set of equilibria in mixed strategies.

Proposition 8. *Under Assumptions 1 and 2, the only outcome in mixed strategies that is a robust equilibrium for some values of β is that both raters randomize between λ^{SPA} and λ^{SPB} with equal probabilities. The following provides the details:*

a) *If $V^{HL} \geq 0$, the outcome above is an equilibrium. More generally, this is the only mixed strategy equilibrium outcome when not restricting to robust equilibria.*

b) *If $V^{HL} < 0$:*

b.1) *When $\beta \geq \beta^*(\frac{1}{3})$, there is no robust equilibrium in mixed strategies. If $\beta > \beta^*(\frac{1}{4})$, there is no equilibrium in mixed strategies even when not restricting to robust equilibria.*

b.2) *When $\beta < \beta^*(\frac{1}{3})$, the only robust equilibrium in mixed strategies is the outcome specified above.*

Proposition 8 demonstrates that both raters might mix between specialization in the two categories when $V^{HL} > 0$ or when $V^{HL} < 0$ and β is below a threshold value. This is intuitive since the pure specialization outcomes, i.e., $(\lambda^{SPA}, \lambda^{SPB})$ and $(\lambda^{SPB}, \lambda^{SPA})$, require coordination among the raters. In the absence of this coordination, the raters might randomly specialize in a category. Hence, with a positive probability, this leads to the inefficient outcome that both raters specialize in the same category. For this outcome to be an equilibrium, the raters should randomize with equal probabilities, as otherwise, the raters would specialize in the category that the other rater specializes in with a lower probability. Proposition 8 further states that this randomization between λ^{SPA} and λ^{SPB} is the only possible robust equilibrium in mixed strategies.

However, this outcome is not an equilibrium when $V^{HL} < 0$ and β is large. This is because as the stand-alone value of specialization decreases with β , so does Rater 2's payoff from specialization. In particular, when $\beta > \beta^*(\frac{1}{3})$ and Rater 1 randomizes between specialization in the two categories with equal probabilities, Rater 2 prefers to generalize instead of randomizing between λ^{SPA} and λ^{SPB} .

Furthermore, Part (b.1) of Proposition 8 verifies the robustness of the key insight that when β is sufficiently large, the unique equilibrium is generalization by both raters. The intuition is that, in this case, generalization is the unique best response of Rater 2 to any choice of rating technology by Rater 1. This arises from the fact that generalization achieves

the highest stand-alone value when $\beta \geq \beta^*(\frac{1}{4})$, and, according to Lemma 1, Rater 2's payoff is capped by the stand-alone value of its ratings. Because Rater 1's unique best response to generalization is also generalization, the generalization outcome is the only equilibrium outcome.

Overall, we see that allowing for mixed strategies does not affect the key insight developed by the baseline model: The raters generalize when the investor assigns a large weight to ESG performance, and may specialize otherwise.

6.4 Divergence in Measurement Methodologies

As discussed in Section 4, Berg et al. (2022) attribute 38% of ratings disagreement to raters examining different subcategories, which is along the lines of what we call specialization. They further attribute 56% of disagreement to differences in measurement. This observation indicates that raters might use different methodologies for subcategories that they have in common. In this section, we analyze the raters' measurement decisions when there are multiple methods available to measure performance in a single subcategory. We show that the key insights from the baseline model are applicable to this analysis.

Specifically, we depart from the baseline model by assuming that the investor is only concerned about the project's performance in a single subcategory, say subcategory X . X could represent a subcategory of the major categories E, S, or G. However, there are two noisy variables, m^a and m^b , that can be used to measure the performance. For instance, to measure a firm's performance in providing occupational safety, one can examine the number of employee injuries, as well as the number or frequency of safety training sessions. Both measures inform investors about the firm's commitment to creating a safe working environment for its employees. Nonetheless, neither of these measures is perfect.

The performance in subcategory X is binary and represented by $w^X \in \{H, L\}$, where $\eta_X \in (0, 1)$ is the probability of $w^X = H$. The investor's net payoff if she invests is $\Delta + \beta u$, where:

$$u = \begin{cases} \Omega^H > 0 & w^X = H \\ \Omega^L < 0 & w^X = L. \end{cases} \quad (22)$$

As in our main model, we assume that no investment takes place in the absence of ESG information. Thus, we modify Assumption 1 as follows:

Assumption 1'.

$$\Delta + \beta \mathbb{E}[u] = \eta_X(\Delta + \beta \Omega^H) + (1 - \eta_X)(\Delta + \beta \Omega^L) < 0. \quad (23)$$

Measurement variables m^a and m^b are independent and have the following conditional distributions:

$$Prob(m^i = H|w^X = H) = z_H, \quad Prob(m^i = H|w^X = L) = 1 - z_L, \quad i = a, b, \quad (24)$$

where $z_H, z_L \in (0, 1)$ capture the precision of these variables. We assume that the measurement variables are precise enough that the investor would invest if both variables indicate high performance:

$$\Delta + \beta \mathbb{E}[u|m^A = m^B = H] > 0. \quad (25)$$

Furthermore, as in Assumption 2, we rule out the possibility that the investor becomes indifferent between investing and not investing when the two measures contradict:

Assumption 2''.

$$\Delta + \beta \mathbb{E}[u|m^A = H, m^B = L] \neq 0. \quad (26)$$

Similar to the baseline model, the raters design rating technologies (λ_1, λ_2) that map the performance measures into ratings s_j^a and s_j^b , $j = 1, 2$:

$$P(s_j^i = h|m^i = H) = \lambda_j^i, \quad P(s_j^i = h|m^i = L) = 0, \quad j = 1, 2, \quad i = a, b. \quad (27)$$

Note that the raters can either perfectly measure the project's performance in one of the two methods, or generate some noisy ratings using both measurement methods. The raters are subject to the same technological constraint (equation 8) and follow the same fee-setting mechanism as in the baseline model. In Proposition 9, we describe the raters' equilibrium behavior:

Proposition 9. *Under Assumptions 1' and 2'', the only possible pure strategy equilibrium outcomes are that the raters specialize in different measurement methods, or generalize across the two methods. Specifically, define:*

$$\begin{aligned} P_m^{HH} &= \eta_X z_H^2 + (1 - \eta_X)(1 - z_L)^2, \\ P_m^{HL} &= \eta_X z_H(1 - z_H) + (1 - \eta_X)z_L(1 - z_L), \\ V_m^{HH} &= \mathbb{E}[u|m^A = m^B = H] = \frac{\eta_X z_H^2 m^H + (1 - \eta_X)(1 - z_L)^2 m^L}{\eta_X z_H^2 + (1 - \eta_X)(1 - z_L)^2}, \text{ and} \\ V_m^{HL} &= \mathbb{E}[u|m^A = H, m^B = L] = \frac{\eta_X z_H(1 - z_H)m^H + (1 - \eta_X)z_L(1 - z_L)m^L}{\eta_X z_H(1 - z_H) + (1 - \eta_X)z_L(1 - z_L)}. \end{aligned} \quad (28)$$

a) *If $V_m^{HL} \geq 0$, then the unique equilibrium is that the raters specialize in different measure-*

ment methods.

b) If $V_m^{HL} < 0$, define:

$$\beta_m^*(\lambda) = \sup\{\beta \mid \frac{P_m^{HL}(\Delta + \beta V_m^{HL})}{P_m^{HH}(\Delta + \beta V_m^{HH})} \geq \lambda - 1\}. \quad (29)$$

b.1) If $\beta_m^*(\frac{1}{4})$ is finite and $\beta > \beta_m^*(\frac{1}{4})$, then the unique equilibrium is generalization by both raters.

b.2) If $\beta \leq \beta_m^*(\frac{1}{4})$, specialization in different measurement methods is an equilibrium. If $\beta \in [\beta_m^*(\frac{9}{16}), \beta_m^*(\frac{17}{32})] \cup \{\beta_m^*(\frac{1}{4})\}$, then generalization in different measurement methods is also an equilibrium.

Proposition 9 states that both specializing in different measurement methods and generalizing in those are possible equilibria, depending on the parameter values. The specialization outcome is the unique equilibrium outcome when high performance in a single measurement variable is sufficient to indicate positive ESG performance in the corresponding subcategory ($V_m^{HL} \geq 0$). However, when β is sufficiently large and V_m^{HL} is sufficiently small, which happens when the measurements have a large false-positive error (small z_L), the raters generalize across the two methods.

With a logic similar to Proposition 2, one can show that the combined value is maximized when the raters specialize in different measurement methods. Therefore, the discrepancy documented in the measurements across raters is efficient to the extent that it captures specialization in independent methods of measurement.

7 Conclusion

ESG investing has become a key focus in financial markets. Investors need information on ESG factors in order to allocate their capital. ESG raters provide this information, but not without controversy. The media, regulators, and academics have attacked them for providing inaccurate ratings. We construct a model of the market for ESG ratings. In the model, raters provide information about multiple (unrelated) categories and set fees. Specializing in different categories can maximize surplus. However, the competitive outcome may be for raters to generalize among categories. We also demonstrate that specialization maximizes disagreement among raters, and, hence, disagreement may be a poor measure of welfare. Greenwashing by firms may push the raters towards generalizing. It would be interesting to expand the model to allow for dynamics and reputation, and examine the role of regulation.

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Appendix A: Proofs of Lemmas and Propositions

A.1 Proof of Lemma 1 (Equilibrium fees)

The following definition is helpful for analyzing the raters' fee-setting behavior.

Definition A.1. *The ratings generated by rating technologies $\lambda_1 = (\lambda_1^A, \lambda_1^B)$ and $\lambda_2 = (\lambda_2^A, \lambda_2^B)$ are called “**complements**” if*

$$V(\lambda_1, \lambda_2) - V(\lambda_1, \mathbf{O}) \geq V(\mathbf{O}, \lambda_2) - V(\mathbf{O}, \mathbf{O}). \quad (\text{A.1})$$

*Otherwise, we call the rating technologies “**substitutes**.”*

First, we consider the case that the rating technologies are complements. Suppose Rater 1 sets ϕ_1 above the stand-alone value of λ_1 , i.e., $\phi_1 > V(\lambda_1, \mathbf{O}) - V(\mathbf{O}, \mathbf{O})$. Therefore, the investor prefers purchasing no ratings to purchasing only Rater 1's ratings. To successfully sell its ratings, Rater 2 should set ϕ_2 such that the investor purchases its ratings with Rater 1's ratings, i.e., $\phi_2 = V(\lambda_1, \lambda_2) - V(\mathbf{O}, \mathbf{O}) - \phi_1$, or without Rater 1's ratings, i.e., $\phi_2 = V(\mathbf{O}, \lambda_2) - V(\mathbf{O}, \mathbf{O})$. Thus, Rater 2 sets $\phi_2 = \max\{V(\lambda_1, \lambda_2) - V(\mathbf{O}, \mathbf{O}) - \phi_1, V(\mathbf{O}, \lambda_2) - V(\mathbf{O}, \mathbf{O})\}$. Both raters' ratings are purchased when the first element is larger or when the elements are equal (according to the tie-breaking rule). When the second element is strictly larger, only Rater 2's ratings are purchased. Therefore, the maximum fee that Rater 1 can collect is:

$$\begin{aligned} \phi_1 &= \{V(\lambda_1, \lambda_2) - V(\mathbf{O}, \mathbf{O})\} - \{V(\mathbf{O}, \lambda_2) - V(\mathbf{O}, \mathbf{O})\} \\ &= V(\lambda_1, \lambda_2) - V(\mathbf{O}, \lambda_2) \geq V(\lambda_1, \mathbf{O}) - V(\mathbf{O}, \mathbf{O}). \end{aligned} \quad (\text{A.2})$$

As a result, $\phi_2 = V(\mathbf{O}, \lambda_2) - V(\mathbf{O}, \mathbf{O})$, and both sets of ratings are purchased. Furthermore, it is suboptimal for Rater 1 to set ϕ_1 below $V(\lambda_1, \mathbf{O}) - V(\mathbf{O}, \mathbf{O})$. It confirms the equilibrium fees for the case that the rating technologies are complements.

Now, we analyze the case that the rating technologies are substitutes. That is, according to Definition A.1, the following inequality holds:

$$V(\lambda_1, \lambda_2) - V(\mathbf{O}, \lambda_2) < V(\lambda_1, \mathbf{O}) - V(\mathbf{O}, \mathbf{O}). \quad (\text{A.3})$$

Similar to the previous case, suppose Rater 1 moves first and sets ϕ_1 . There are three cases depending on the value of ϕ_1 :

- If $\phi_1 \geq V(\lambda_1, \mathbf{O}) - V(\mathbf{O}, \mathbf{O})$, then the fee is larger than the stand-alone value of λ_1

and its marginal value given Rater 2's ratings. Therefore, Rater 1's ratings are not purchased regardless of ϕ_2 .

- If $V(\lambda_1, \lambda_2) - V(\mathbf{O}, \lambda_2) < \phi_1 < V(\lambda_1, \mathbf{O}) - V(\mathbf{O}, \mathbf{O})$, then Rater 1's fee is less than the stand-alone value of its ratings and more than their marginal value given Rater 2's ratings. Therefore, the ratings of both raters are not purchased together. To sell its ratings, Rater 2 sets ϕ_2 slightly below the value that makes the investor indifferent between λ_1 and λ_2 , i.e., $\phi_2 = V(\mathbf{O}, \lambda_2) + \phi_1 - V(\lambda_1, \mathbf{O}) - \varepsilon$ (for a sufficiently small value of $\varepsilon > 0$). With this choice, only λ_2 is purchased, and Rater 1's payoff is zero.
- If $\phi_1 \leq V(\lambda_1, \lambda_2) - V(\mathbf{O}, \lambda_2)$, then ϕ_1 is less than or equal to the marginal value of Rater 1's ratings given λ_2 . Rater 2 sets its fee equal to the marginal value of λ_2 given λ_1 , i.e., $\phi_2 = V(\lambda_1, \lambda_2) - V(\lambda_1, \mathbf{O})$, and both sets of ratings are purchased.

Therefore, the maximum fee that Rater 1 can set to successfully sell its ratings is $\phi_1 = V(\lambda_1, \lambda_2) - V(\mathbf{O}, \lambda_2)$. In this case, $\phi_2 = V(\lambda_1, \lambda_2) - V(\lambda_1, \mathbf{O})$, and both sets of ratings are purchased. We see that in both possibilities, Rater 1 sets ϕ_1 equal to the marginal value of its ratings, i.e., $V(\lambda_1, \lambda_2) - V(\mathbf{O}, \lambda_2)$. Rater 2's equilibrium fee is either $V(\mathbf{O}, \lambda_2) - V(\mathbf{O}, \mathbf{O})$, or $V(\lambda_1, \lambda_2) - V(\lambda_1, \mathbf{O})$, whichever is smaller.

A.2 Proof of Proposition 1 (Characterization of market equilibria)

To prove the proposition, we show that rating technologies λ_1 and λ_2 can constitute an equilibrium only if the raters specialize in different categories (i.e., $(\lambda_1, \lambda_2) \in \{(\lambda^{SPA}, \lambda^{SPB}), (\lambda^{SPB}, \lambda^{SPA})\}$), or both raters generalize (i.e., $(\lambda_1, \lambda_2) = (\lambda^{GN}, \lambda^{GN})$). After demonstrating that these outcomes are the only possible forms of equilibria in **Steps 1 and 2**, we complete the characterization in **Step 3**.

First, we prove two helpful Lemmas. Lemma A.1 simplifies the value function. Lemma A.2 provides a sufficient condition for a pair to form an equilibrium.

Lemma A.1. *Let λ^* be the solution to the following linear equation:*

$$\eta(1 - \lambda^*)(\Delta + \beta V^{HH}) + (1 - \eta)(\Delta + \beta V^{HL}) = 0. \quad (\text{A.4})$$

Then, the combined value is:

$$V(\lambda_1, \lambda_2) = \left\{ \lambda^A \lambda^B + \lambda^A [\lambda^* - \lambda^B]^+ + \lambda^B [\lambda^* - \lambda^A]^+ \right\} \eta^2 (\Delta + \beta V^{HH}) \quad (\text{A.5})$$

, where $[x]^+ = \max\{x, 0\}$, and

$$\lambda^i = \lambda_1^i + \lambda_2^i - \lambda_1^i \lambda_2^i, \quad i = A, B. \quad (\text{A.6})$$

Proof. Note that according to Assumption 1, the investment does not take place when all ratings are low, i.e., $(s^A, s^B) = (l, l)$. Therefore, we can expand equation 9 as below

$$\begin{aligned} V(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) &= (\Delta + \beta V^{HH}) \eta^2 \lambda^A \lambda^B \\ &+ \frac{[(\Delta + \beta V^{HH}) \eta^2 \lambda^A (1 - \lambda^B) + (\Delta + \beta V^{HL}) \eta (1 - \eta) \lambda^A]^+}{P(s^A = h, s^B = l)} P(s^A = h, s^B = l) \\ &+ \frac{[(\Delta + \beta V^{HH}) \eta^2 (1 - \lambda^A) \lambda^B + (\Delta + \beta V^{HL}) (1 - \eta) \eta \lambda^B]^+}{P(s^A = l, s^B = h)} P(s^A = l, s^B = h) \\ &= \eta^2 (\Delta + \beta V^{HH}) \lambda^A \lambda^B + \eta \lambda^A [\eta (1 - \lambda^B) (\Delta + \beta V^{HH}) + (1 - \eta) (\Delta + \beta V^{HL})]^+ \\ &\quad + \eta \lambda^B [\eta (1 - \lambda^A) (\Delta + \beta V^{HH}) + (1 - \eta) (\Delta + \beta V^{HL})]^+. \end{aligned} \quad (\text{A.7})$$

By employing (A.4), we can rewrite (A.7) as below:

$$\begin{aligned} V(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) &= \eta^2 (\Delta + \beta V^{HH}) \lambda^A \lambda^B + \eta \lambda^A [\eta (\Delta + \beta V^{HH}) (\lambda^* - \lambda^B)]^+ + \eta \lambda^B [\eta (\Delta + \beta V^{HH}) (\lambda^* - \lambda^A)]^+ \\ &= \left\{ \lambda^A \lambda^B + \lambda^A [\lambda^* - \lambda^B]^+ + \lambda^B [\lambda^* - \lambda^A]^+ \right\} \eta^2 (\Delta + \beta V^{HH}). \end{aligned} \quad (\text{A.8})$$

□

Lemma A.1 states that the value function is proportionate to $v(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2; \lambda^*)$, where

$$\begin{aligned} v(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2; \lambda^*) &= \lambda^A \lambda^B + \lambda^A [\lambda^* - \lambda^B]^+ + \lambda^B [\lambda^* - \lambda^A]^+ \\ &= \max\{\lambda^A \lambda^B, \lambda^A \lambda^*, \lambda^B \lambda^*, (\lambda^A + \lambda^B) \lambda^* - \lambda^A \lambda^B\}. \end{aligned} \quad (\text{A.9})$$

Therefore, according to Lemma 1, we can write the fees as below:

$$\begin{aligned} \phi_j(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) &= \eta^2 (\Delta + \beta V^{HH}) \hat{\phi}_j(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2), \quad j = 1, 2 \\ \hat{\phi}_1(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) &= v(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2; \lambda^*) - v(\mathbf{O}, \boldsymbol{\lambda}_2; \lambda^*) \\ \hat{\phi}_2(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) &= \min\{v(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2; \lambda^*) - v(\mathbf{O}, \boldsymbol{\lambda}_2; \lambda^*), v(\mathbf{O}, \boldsymbol{\lambda}_2; \lambda^*) - v(\mathbf{O}, \mathbf{O}; \lambda^*)\}. \end{aligned} \quad (\text{A.10})$$

Therefore, a key insight we obtain from Lemma A.1 is that λ^* is a sufficient variable to analyze the raters' best response behavior. Note that λ^* is bigger than one when $\Delta + \beta V^{HL} > 0$, and it is less than one otherwise. Moreover, note that the mapping between λ^* and β is the inverse of $\beta^*(\lambda)$, introduced in equation 12. That is, $\beta = \beta^*(\lambda^*)$. Based on this observation, we characterize the equilibria in terms of λ^* , and use the mapping to describe them in terms of β .

Lemma A.2. For rating technologies λ_1, λ_2 , and $\tilde{\lambda}_2$, suppose the following inequalities hold:

$$V(\lambda_1, \tilde{\lambda}_2) > V(\lambda_1, \lambda_2), \quad V(\mathbf{O}, \tilde{\lambda}_2) > V(\mathbf{O}, \lambda_2). \quad (\text{A.11})$$

Then, $\phi_2(\lambda_1, \tilde{\lambda}_2) > \phi_2(\lambda_1, \lambda_2)$. As a result, (λ_1, λ_2) is not an equilibrium.

Proof. The inequality below shows that Rater 2's payoff is larger with $\tilde{\lambda}_2$ than with λ_2 , given λ_1 . Therefore, (λ_1, λ_2) cannot be an equilibrium:

$$\begin{aligned} \phi_2(\lambda_1, \tilde{\lambda}_2) &= \min\{V(\lambda_1, \tilde{\lambda}_2) - V(\lambda_1, \mathbf{O}), V(\mathbf{O}, \tilde{\lambda}_2) - V(\mathbf{O}, \mathbf{O})\} \\ &> \min\{V(\lambda_1, \lambda_2) - V(\lambda_1, \mathbf{O}), V(\mathbf{O}, \lambda_2) - V(\mathbf{O}, \mathbf{O})\} = \phi_2(\lambda_1, \lambda_2). \end{aligned} \quad (\text{A.12})$$

□

An implication of Lemma A.2 is that to prove λ_2 is a best response to λ_1 for Rater 2, it is sufficient to show that λ_2 has the largest stand-alone value and combined value given λ_1 . Moreover, we can also employ Lemma A.2 to prove that the technological constraint should bind for both raters. Corollary A.1 formalizes this point.

Corollary A.1. If pair (λ_1, λ_2) forms an equilibrium, then the technological constraint 8 binds for both raters, i.e., $\lambda_j^A + \lambda_j^B = 1$, $j = 1, 2$.

Proof. Consider the contrary that the constraint does not bind for rater $j \in \{1, 2\}$. That is, $\lambda_j^A + \lambda_j^B < 1$. λ_j should have a positive marginal value, as otherwise, this would imply that λ_j earns a zero payoff for rater j (See Lemma 1). λ_j cannot be the best response since it is straightforward to show that when $\Delta + \beta V^{HL} \neq 0$ (Assumption 2), there is an action that obtains a positive payoff for any choice of the other rater.

Since λ_j has a positive marginal value, one can show that either $\frac{\partial v(\lambda_1, \lambda_2)}{\partial \lambda_j^A} > 0$ or $\frac{\partial v(\lambda_1, \lambda_2)}{\partial \lambda_j^B} > 0$. Moreover, if the rater increases the precision of its ratings in both categories, then the stand-alone value strictly increases. Therefore, rater j can increase both the marginal value and stand-alone value of its rating technology by switching to $\lambda'_j = (\lambda_j^{A'}, \lambda_j^{B'})$, where $\lambda_j^{i'} > \lambda_j^i$, $i = A, B$. According to Lemma A.2, λ'_j obtains a higher payoff, which is a contradiction. Therefore, the technological constraint should be binding for both raters in any equilibrium.

□

With this result, it is without loss to focus on pairs that the technological constraint 8 holds with equality for both raters. Now, we solve for the market equilibria in three steps.

Step 1: Possible equilibria when λ_1 and λ_2 are substitutes

Now, we examine which pairs of (λ_1, λ_2) can form an equilibrium if the rating technologies are substitutes. In this case, both raters charge their marginal contribution as their fee, according to Lemma 1. Particularly, we show that the only possible equilibrium outcomes are specialization in different categories and generalization by both raters. We divide cases based on the value of λ^* , as defined in equation A.4.

- **Case 1:** $\lambda^* \geq \max\{\lambda^A, \lambda^B\}$

Suppose Rater 2 does not specialize. We show that by specialization, the rater can increase both the stand-alone value and the marginal value of its ratings, which demonstrates the contradiction, according to Lemma A.2.

According to Lemma A.1, we have:

$$\begin{aligned} v(\mathbf{O}, \lambda_2) &= \lambda_2^A \lambda_2^B + \lambda_2^A [\lambda^* - \lambda_2^B]^+ + \lambda_2^B [\lambda^* - \lambda_2^A]^+ = \lambda^* - \lambda_2^A \lambda_2^B \\ v(\lambda_1, \lambda_2) &= \lambda^A \lambda^B + \lambda^A [\lambda^* - \lambda^B]^+ + \lambda^B [\lambda^* - \lambda^A]^+ = (\lambda^A + \lambda^B) \lambda^* - \lambda^A \lambda^B. \end{aligned} \quad (\text{A.13})$$

In the first line of (A.13), we use the fact that $\lambda_2^i \leq \lambda^i$, $i = A, B$. Since $\lambda_2^A, \lambda_2^B > 0$, the stand-alone value of λ_2 is less than that of specialization:

$$v(\mathbf{O}, \lambda^{SP_B}) = v(\mathbf{O}, \lambda^{SP_A}) = \lambda^*. \quad (\text{A.14})$$

Moreover, it is straightforward to show that $(\lambda^A + \lambda^B) \lambda^* - \lambda^A \lambda^B$ is convex in λ_2^A . Therefore, the expression obtains its maximum value in one of the extreme values, i.e., $\lambda_2^A = 0, 1$. Suppose the maximum value is obtained at $\lambda_2^A = 1$, implying $v(\lambda_1, \lambda^{SP_A}) > v(\lambda_1, \lambda_2)$.

Therefore, according to Lemma A.2, Rater 2 can increase its payoff by switching to λ^{SP_A} . The argument for the case that Rater 1 does not specialize is similar. As a result, the only possible equilibrium outcome in this case is specialization in different categories.

- **Case 2:** $\max\{\lambda^A, \lambda^B\} > \lambda^* \geq \min\{\lambda^A, \lambda^B\}$

Note that λ^* should be less than one since λ^A and λ^B are capped at one. Without loss, assume $\lambda^A \geq \lambda^B$. By applying equation A.9, we find $v(\lambda_1, \lambda_2) = \lambda^A \lambda^*$. If no rater specializes, Rater 1 can increase the combined value, and consequently, its

marginal value, by specializing in category A . If Rater 1 specializes in category A , then $v(\lambda^{SP_A}, \lambda_2) = \lambda^*$ since $\lambda_2^B \leq \lambda^B < \lambda^*$, implying $\phi_2 = 0$. Therefore, Rater 2 can increase its payoff by specializing in category B since it obtains a strictly positive payoff. With a similar argument, we can rule out the possibility that only Rater 2 specializes.

- **Case 3:** $\min\{\lambda^A, \lambda^B\} > \lambda^*$

Note that according to Definition A.1, if (λ_1, λ_2) are substitutes, the rating technologies are substitutes in an open neighborhood of the pair. Therefore, the first-order conditions are necessary for a pair to constitute an equilibrium.

Moreover, in this case, Lemma A.1 implies that $v(\lambda_1, \lambda_2) = \lambda^A \lambda^B$. Without loss, assume $\lambda_2^A \geq \lambda_2^B$. The first-order conditions imply:

$$\begin{aligned} \frac{\partial v(\lambda_1, \lambda_2)}{\partial \lambda_1^A} &\leq 0 \Rightarrow \lambda_2^A - \lambda_2^B \geq (\lambda_1^B - \lambda_1^A)(1 - \lambda_2^A)(1 - \lambda_2^B), \\ \frac{\partial v(\lambda_1, \lambda_2)}{\partial \lambda_2^A} &\geq 0 \Rightarrow \lambda_1^B - \lambda_1^A \geq (\lambda_2^A - \lambda_2^B)(1 - \lambda_1^A)(1 - \lambda_1^B), \end{aligned} \quad (\text{A.15})$$

where the equality holds for the first (second) inequality when the first (second) rater does not specialize. It is straightforward to show that the only pairs that can satisfy these conditions are specialization in different categories and generalization by both raters.

Step 2: Possible equilibria when λ_1 and λ_2 are complements

The goal of this step is to demonstrate when λ_1 and λ_2 are complements, then they cannot constitute an equilibrium unless the raters specialize in different categories, or they both generalize.

First, we prove that for two rating technologies to be complements, we need to have $\lambda^A, \lambda^B > \lambda^*$. To this end, we rule out the other possibilities:

- $\lambda^* \geq \lambda^A, \lambda^B$: In this case, $\lambda_j^i \leq \lambda^*$, for $i = A, B$ and $j = 1, 2$. Therefore, according to equation A.9, we have:

$$\begin{aligned} v(\lambda_1, \mathbf{O}) &= \lambda^* - \lambda_1^A \lambda_1^B, & v(\mathbf{O}, \lambda_2) &= \lambda^* - \lambda_2^A \lambda_2^B \\ v(\lambda_1, \lambda_2) &= (\lambda^A + \lambda^B) \lambda^* - \lambda^A \lambda^B = (2 - \lambda_1^A \lambda_2^A - \lambda_1^B \lambda_2^B) \lambda^* - \lambda^A \lambda^B. \end{aligned} \quad (\text{A.16})$$

To prove that λ_1 and λ_2 are substitutes, we need to show:¹⁷

$$\begin{aligned}
& v(\lambda_1, \mathbf{O}) + v(\mathbf{O}, \lambda_2) > v(\lambda_1, \lambda_2) \\
\iff & 2\lambda^* - \lambda_1^A \lambda_1^B - \lambda_2^A \lambda_2^B > (2 - \lambda_1^A \lambda_2^A - \lambda_1^B \lambda_2^B) \lambda^* - \lambda^A \lambda^B \quad (\text{A.17}) \\
\iff & \lambda^A \lambda^B - \lambda_1^A \lambda_1^B - \lambda_2^A \lambda_2^B > -\lambda^*(\lambda_1^A \lambda_2^A + \lambda_1^B \lambda_2^B).
\end{aligned}$$

Since $\lambda^*(\lambda_1^A \lambda_2^A + \lambda_1^B \lambda_2^B)$ is non-negative, we only need to show that $\lambda^A \lambda^B - \lambda_1^A \lambda_1^B - \lambda_2^A \lambda_2^B > 0$, which is demonstrated by the inequalities below:

$$\begin{aligned}
& \lambda^A \lambda^B - \lambda_1^A \lambda_1^B - \lambda_2^A \lambda_2^B \geq 0 \\
\iff & \{1 - \lambda^A - \lambda^B + \lambda^A \lambda^B\} - \{1 - \lambda^A - \lambda^B + \lambda_1^A \lambda_1^B + \lambda_2^A \lambda_2^B\} \geq 0 \quad (\text{A.18}) \\
\iff & (1 - \lambda_1^A)(1 - \lambda_1^B)(1 - \lambda_2^A)(1 - \lambda_2^B) + 1 - (\lambda_1^A + \lambda_2^B)(2 - \lambda_1^A - \lambda_2^B) \geq 0.
\end{aligned}$$

The inequality in (A.17) is strict since in (A.18), equality holds only when the raters specialize in the same category, which results in $\lambda_1^A \lambda_2^A + \lambda_1^B \lambda_2^B = 1 > 0$. It implies that any pair of rating technologies with $\lambda^* \geq \lambda^A, \lambda^B$ are substitutes.

- $\max\{\lambda^A, \lambda^B\} > \lambda^* \geq \min\{\lambda^A, \lambda^B\}$: Without loss, assume $\lambda^A > \lambda^B$. It implies that $\lambda_1^B, \lambda_2^B < \lambda^*$. Therefore:

$$v(\lambda_1, \lambda_2) = \lambda^A \lambda^* \leq \lambda_1^A \lambda^* + \lambda_2^A \lambda^* \leq v(\lambda_1, \mathbf{O}) + v(\mathbf{O}, \lambda_2). \quad (\text{A.19})$$

The first inequality is obtained from the definition of λ^A in (7). The second inequality is resulted from equation A.9. In (A.19), equality is never obtained. The reason is that equality is obtained in the first inequality only when either λ_1^A or λ_2^A is zero, which implies $\lambda^B \geq \max\{\lambda_1^B, \lambda_2^B\} = 1$. It is impossible since $1 \geq \lambda^A > \lambda^*$.

Therefore, $\lambda^A, \lambda^B > \lambda^*$ if λ_1 and λ_2 are complements, implying that $v(\lambda_1, \lambda_2) = \lambda^A \lambda^B$. Similar to the argument in Case 3 at Step 1, at least one rater can increase the combined value if the outcome is not specialization in different categories or generalization by both raters. Rater 1 charges its marginal contribution; thus λ_1 maximizes $v(\lambda_1, \lambda_2)$ if (λ_1, λ_2) is an equilibrium. Therefore, Rater 2 has to charge the stand-alone value of λ_2 . Now, we divide the cases based on whether $\min\{\lambda_2^A, \lambda_2^B\}$ is less than λ^* .

- **Case 1:** $\lambda^* \geq \min\{\lambda_2^A, \lambda_2^B\}$

Without loss, suppose $\lambda_2^A \geq \lambda_2^B$. As shown in (A.15), according to the first-order conditions, there are two possibilities: Either Rater 1 specializes in category B , or

¹⁷Note that $v(\mathbf{O}, \mathbf{O}) = 0$.

sets λ_1 such that $\lambda_1^B - \lambda_1^A = \frac{\lambda_2^A - \lambda_2^B}{(1-\lambda_2^A)(1-\lambda_2^B)} \geq \lambda_2^A - \lambda_2^B$. If Rater 1 specializes, Rater 2 can increase the combined value and the stand-alone value of its rating technology by specializing in category A , which means that the specialization would obtain a higher payoff, according to Lemma A.2. If Rater 1 generalizes, Rater 2 should also be generalizing, according to the first-order condition. We discuss the generalization outcome in Step 3.

If Rater 1 neither specializes nor generalizes, Rater 2 can increase its payoff by specializing more, namely by increasing λ_2^A :

$$\begin{aligned}
\lambda_1^B - \lambda_1^A &= \frac{\lambda_2^A - \lambda_2^B}{(1-\lambda_2^A)(1-\lambda_2^B)} > \lambda_2^A - \lambda_2^B \Rightarrow \lambda_1^B > \lambda_2^A \\
\Rightarrow \frac{\partial v}{\partial \lambda_2^A} &= \frac{\partial \lambda^A \lambda^B}{\partial \lambda_2^A} = (1-\lambda_1^A)\lambda^B - (1-\lambda_1^B)\lambda^A \\
&= \underbrace{\lambda_1^B}_{1-\lambda_1^A} (\lambda_1^B + \underbrace{\lambda_2^B \lambda_1^A}_{\lambda_2^B - \lambda_1^B \lambda_2^B}) - \underbrace{\lambda_1^A}_{1-\lambda_1^B} (\lambda_2^A + \underbrace{\lambda_2^B \lambda_1^A}_{\lambda_1^A - \lambda_1^A \lambda_2^A}) > 0 \\
\frac{\partial v(\mathbf{O}, \lambda_2)}{\partial \lambda_2^A} &= \frac{\partial \max\{\lambda_2^A \lambda^*, \lambda^* - \lambda_2^A \lambda_2^B\}}{\partial \lambda_2^A} > 0.
\end{aligned} \tag{A.20}$$

Therefore, according to Lemma A.2, λ_2 cannot be the best response to λ_1 .

• **Case 2:** $\min\{\lambda_2^A, \lambda_2^B\} > \lambda^*$

Note that λ^* should be less than $\frac{1}{2}$, implying that $v(\mathbf{O}, \lambda^{GN}) = \frac{1}{4}$. If Rater 2 generalizes, Rater 1's best response is also to generalize. Suppose $\lambda_2 = (\lambda_2^A, \lambda_2^B) \neq (\frac{1}{2}, \frac{1}{2})$. If λ_1 and λ^{GN} are complements, then Rater 2 can increase its payoff by generalizing as $\frac{1}{4} > \lambda_2^A \lambda_2^B$. If λ_1 and λ^{GN} are substitutes, then (A.18) implies that $\min\{\lambda_1^A, \lambda_1^B\} < \lambda^*$. Assume $\lambda_1^B \geq \lambda^* > \lambda_1^A$. Moreover, we show $\lambda^* > \frac{1}{4}$:

$$\begin{aligned}
(\frac{1}{2} + \lambda_1^B - \frac{1}{2}\lambda_1^B)(\frac{1}{2} + \lambda_1^A - \frac{1}{2}\lambda_1^A) &< \frac{1}{4} + \lambda_1^B \lambda^* \\
\Rightarrow \lambda_1^B < 1 + \lambda_1^B - \lambda_1^{B^2} < 4\lambda_1^B \lambda^* &\Rightarrow \frac{1}{4} < \lambda^*.
\end{aligned} \tag{A.21}$$

Note that

$$\hat{\phi}_2(\lambda_1, \lambda^{SPA}) = \min\{\lambda^*, \lambda_1^B(1-\lambda^*)\}. \tag{A.22}$$

Since $\hat{\phi}_2(\lambda_1, \lambda_2) = v(\mathbf{O}, \lambda_2) < \frac{1}{4}$, we only need to show $\hat{\phi}_2(\lambda_1, \lambda^{SPA}) > \frac{1}{4}$ to demonstrate that λ_2 is not the best response. It can be shown by noting that $\lambda^* \in (\frac{1}{4}, \frac{1}{2})$ and $\lambda_1^B \geq \frac{1}{2}$, implying $\lambda_1^B(1-\lambda^*) > \frac{1}{4}$.

Step 3: Completing the characterization of the equilibria

Thus far, we have found that the possible equilibrium outcomes, for any value of λ^* , are:

$$(\lambda^{SPA}, \lambda^{SPB}), (\lambda^{SPB}, \lambda^{SPA}), (\lambda^{GN}, \lambda^{GN}). \quad (\text{A.23})$$

We just need to examine which one of these outcomes forms an equilibrium for each value of λ^* .

- **Case 1:** $\lambda^* > \frac{9}{16}$

This case covers Part a of the Proposition since $\lambda^* > 1$ when $V^{HL} \geq 0$, as well as Part b.2 when $\beta \leq \beta^*(\frac{9}{16})$ and $V^{HL} < 0$.

We first show that $(\lambda^{GN}, \lambda^{GN})$ cannot be an equilibrium in this case. To see this, we compare Rater 1's payoff from generalization and specialization in response to generalization by Rater 2. According to Lemma 1, Rater 1 always obtains the marginal value of its ratings, i.e., $\phi_1 = V(\lambda_1, \lambda_2) - V(\mathbf{O}, \lambda_2)$. Therefore, λ_1 should maximize the combined value given λ_2 . The inequalities below show that $v(\lambda^{SPA}, \lambda^{GN}) > v(\lambda^{GN}, \lambda^{GN})$ when $\lambda^* > \frac{9}{16}$, which implies that the generalization outcome cannot be an equilibrium:

$$\begin{aligned} \lambda^* \geq 1 : \quad & v(\lambda^{SPA}, \lambda^{GN}) = \frac{3}{2}\lambda^* - \frac{1}{2}, \quad v(\lambda^{GN}, \lambda^{GN}) = \frac{3}{2}\lambda^* - \frac{9}{16} \\ \lambda^* \in (\frac{3}{4}, 1) : \quad & v(\lambda^{SPA}, \lambda^{GN}) = \lambda^*, \quad v(\lambda^{GN}, \lambda^{GN}) = \frac{3}{2}\lambda^* - \frac{9}{16} \\ \lambda^* \in (\frac{9}{16}, \frac{3}{4}) : \quad & v(\lambda^{SPA}, \lambda^{GN}) = \lambda^*, \quad v(\lambda^{GN}, \lambda^{GN}) = \frac{9}{16} \\ \Rightarrow & v(\lambda^{SPA}, \lambda^{GN}) > v(\lambda^{GN}, \lambda^{GN}) \Rightarrow V(\lambda^{SPA}, \lambda^{GN}) > V(\lambda^{GN}, \lambda^{GN}). \end{aligned} \quad (\text{A.24})$$

In the inequality above, we use the fact that the value function is proportional to $v(\cdot, \cdot)$, specified in (A.9).

It is straightforward to show that any rating technology is substitute with λ^{SPA} and λ^{SPB} when $\lambda^* > \frac{9}{16}$. Therefore, both raters charge their marginal contribution as their fees, according to Lemma 1. Note that maximizing the marginal value for a rater is equivalent to maximizing the combined value. In fact, specialization in different categories achieves the highest combined value, as the project's type would be perfectly revealed to the investor (we formally demonstrate this point in Proposition 2). Therefore, specialization in different categories is the only equilibrium outcome in pure strategies when $\lambda^* > \frac{9}{16}$.

• **Case 2:** $\lambda^* \in [\frac{17}{32}, \frac{9}{16}]$

This case corresponds to Part b.2 in the proposition when $\beta \in [\beta^*(\frac{9}{16}), \beta^*(\frac{17}{32})]$ and $V^{HL} < 0$.

In this case, one can show that any two rating technologies are substitutes.¹⁸ Therefore, both raters maximize the combined value of the ratings. We use this fact to prove that the best response to specialization (generalization) is specialization in the other category (generalization) for both raters.

In response to specialization, the raters' best response is to specialize in the other category since specialization in different categories achieves the highest combined value. Next, we show Rater 1's best response to generalization is generalization (the same logic applies to Rater 2). Suppose $\lambda_1^A \geq \lambda_1^B$, then $\lambda^A = \frac{1}{2} + \frac{1}{2}\lambda_1^A \geq \frac{3}{4} > \lambda^*$. We consider two possibilities for λ^B . If $\lambda^B \leq \lambda^*$, then $v(\lambda_1, \lambda_2) = v(\lambda_1, \lambda^{GN}) = \lambda^A \lambda^* \leq \lambda^* \leq \frac{9}{16} = v(\lambda^{GN}, \lambda^{GN})$. If $\lambda^B > \lambda^*$, analyzing the first-order conditions reveals that $\lambda^A \lambda^B$ is maximized at $\lambda_1^A = \lambda_1^B = \frac{1}{2}$ (Note that $\lambda^A \lambda^B$ is concave in λ_1^A).

Case 3: $\lambda^* \in (\frac{1}{4}, \frac{17}{32})$

Similar to the previous case, one can show that Rater 1's best response to specialization is specialization in the other category. However, Rater 2's fee can be the stand-alone value or the marginal value its ratings depending on λ^* . It is straightforward to show that λ^{SPA} and λ^{SPB} are substitutes when $\lambda^* > \frac{1}{2}$ and complements when $\lambda^* \leq \frac{1}{2}$. When the two specialized rating technologies are substitutes, similar to Rater 1's case, we can verify that in response to specialization, specialization in the other category is Rater 2's best response. When the two specialized technologies are complements, then Rater 2 obtains the stand-alone value of its ratings, which is λ^* . When $\lambda^* \geq \frac{1}{4}$, specialization has the largest stand-alone value. Therefore, the optimality of specialization in this case can be verified as below:

$$\begin{aligned} \phi_2(\lambda^{SPA}, \lambda_2) &= \min\{V(\lambda^{SPA}, \lambda_2) - V(\lambda^{SPA}, \mathbf{O}), V(\mathbf{O}, \lambda_2)\} \leq V(\mathbf{O}, \lambda_2) \\ &\leq V(\mathbf{O}, \lambda^{SPB}) = \phi_2(\lambda^{SPA}, \lambda^{SPB}). \end{aligned} \tag{A.25}$$

One can observe that λ^{GN} is complement with itself when $\lambda^* < \frac{17}{32}$. We use this fact to demonstrate that Rater 2's best response to generalization is not generalization. Consider the following possibilities:

¹⁸The only exception is $(\lambda^{GN}, \lambda^{GN})$ that are at the borderline of being complements when $\lambda^* = \frac{17}{32}$, while being substitutes when $\lambda^* > \frac{17}{32}$.

- $\lambda^* \in (\frac{1}{2}, \frac{17}{32})$: The payoff of Rater 2 is the stand-alone value of its ratings in a sufficiently small neighborhood of λ_2^{GN} since $2V(\mathbf{O}, \lambda^{GN}) < V(\lambda^{GN}, \lambda^{GN})$. In this neighborhood $v(\mathbf{O}, \lambda_2) = \lambda^* - \lambda_2^A \lambda_2^B$. Therefore, generalization does not locally optimize Rater 2's payoff, implying that the generalization outcome cannot be an equilibrium.
- $\lambda^* \in (\frac{1}{4}, \frac{1}{2}]$: In this case, $\hat{\phi}_2(\lambda^{GN}, \lambda^{GN}) = \frac{1}{4}$ since the pair $(\lambda^{GN}, \lambda^{GN})$ are complements. Now, define $\lambda_2^A = \frac{1}{4\lambda^*} + \varepsilon < 1$, for a sufficiently small value of $\varepsilon > 0$. Note that:

$$\begin{aligned} v(\mathbf{O}, \lambda_2) &\geq \lambda_2^A \lambda^* > \frac{1}{4} \\ v(\lambda^{GN}, \lambda_2) - v(\lambda^{GN}, \mathbf{O}) &= (\frac{1}{2} + \frac{1}{2}\lambda_2^A)(\frac{1}{2} + \frac{1}{2}\lambda_2^B) - \frac{1}{4} = \frac{1}{4} + \frac{1}{4}\lambda_2^A \lambda_2^B > \frac{1}{4}. \end{aligned} \quad (\text{A.26})$$

Therefore, by applying Lemma A.2, we find that $\phi_2(\lambda^{GN}, \lambda_2) > \phi_2(\lambda^{GN}, \lambda^{GN})$.

• **Case 4:** $\lambda^* < \frac{1}{4}$

This case corresponds to Part b.1 of the proposition. In this case, one can show that any rating technology is complement with generalization. Moreover, λ^{SPA} and λ^{SPB} are complements. Therefore,

$$\hat{\phi}_2(\lambda^{SPA}, \lambda^{SPB}) = \lambda^* < \frac{1}{4} = \hat{\phi}_2(\lambda^{SPA}, \lambda^{GN}). \quad (\text{A.27})$$

As such, specialization in different categories is not an equilibrium. Generalization by both raters is an equilibrium since generalization has the highest stand-alone value, thus it maximizes Rater 2's payoff. Moreover, with a similar argument to Case 2, one can show that Rater 1's best response to generalization is generalization.

A.3 Proof of Proposition 2 (Value-maximizing ratings)

We show that the value function $V(\lambda_1, \lambda_2)$ attains its maximum only when

$$(\lambda_1, \lambda_2) \in \{(\lambda^{SPA}, \lambda^{SPB}), (\lambda^{SPB}, \lambda^{SPA})\}. \quad (\text{A.28})$$

Note that specialization in different categories achieves the highest possible investment value since the project's type is fully revealed, so the investment always takes place efficiently. We show that no other pair of rating technologies can achieve the investment value created by the specialization outcomes, i.e., $V(\lambda^{SPA}, \lambda^{SPB})$.

For this purpose, we use equation A.9. Recall that equation A.9 defines function $v(\lambda_1, \lambda_2)$ that is proportionate to the value function $V(\lambda_1, \lambda_2)$. We divide the cases into two:

$\Delta + \beta V^{HL} > 0$: In this case, $\lambda^* > 1$, where λ^* is defined in equation A.4. For the specialization outcome, we find the following from equation A.9:

$$v(\lambda^{SPA}, \lambda^{SPB}) = 2\lambda^* - 1. \quad (\text{A.29})$$

Note that all three elements in the right-hand side of (A.9) are positive or zero for any pair of (λ_1, λ_2) . Therefore:

$$v(\lambda_1, \lambda_2) = (\lambda^A + \lambda^B)\lambda^* - \lambda^A\lambda^B. \quad (\text{A.30})$$

Note that λ^A and λ^B (defined in (7)) are in $[0, 1]$. Therefore,

$$\begin{aligned} (1 - \lambda^A)(1 - \lambda^B) &\geq 0 \\ \Rightarrow 1 - \lambda^A\lambda^B &\leq 2 - \lambda^A - \lambda^B \leq (2 - \lambda^A - \lambda^B)\lambda^* \\ \Rightarrow v(\lambda_1, \lambda_2) &= (\lambda^A + \lambda^B)\lambda^* - \lambda^A\lambda^B \leq 2\lambda^* - 1 = v(\lambda^{SPA}, \lambda^{SPB}). \end{aligned} \quad (\text{A.31})$$

Equality holds only when $\lambda^A = \lambda^B = 1$, corresponding to the specialization outcome.

$\Delta + \beta V^{HL} < 0$: In this case, only projects with type (H, H) receive investment in the optimal investment outcome. Note that if the second or third term in (A.7) are positive, a project with type (H, L) or (L, H) receives investment with a positive probability, which is inefficient. Therefore, for a value-maximizing pair, we should have $V = \lambda^A\lambda^B\eta^2(\Delta + \beta V^{HH})$. The maximum value is obtained when $\lambda^A = \lambda^B = 1$, which corresponds to the specialization outcome.

A.4 Proof of Proposition 3 (Disagreement)

When the raters specialize in different categories, they are providing information about two independent categories. Therefore, their ratings are independent, and as a result, have a zero correlation, i.e., $Agg = 0$. However, when the raters specialize in the same category, they are perfectly revealing the project's type in that category. Therefore, their rating has a perfect correlation, i.e., $Agg = 1$. Since $Agg \in [0, 1]$, we observe that these outcomes obtain the extreme possible values of Agg .

A.5 Proof of Proposition 4 (Market equilibrium with greenwashing)

Define V_M^{HH} and V_M^{HL} as below:

$$V_M^{HH} = \mathbb{E}[u|w_M^A = w_M^B = H] = \frac{\eta^2 V^{HH} + 2\eta(1-\eta)\alpha V^{HL} + (1-\eta)^2 \alpha^2 V^{LL}}{\eta^2 + 2\eta(1-\eta)\alpha + (1-\eta)^2 \alpha^2} \quad (\text{A.32})$$

$$V_M^{HL} = \mathbb{E}[u|w_M^A = H, w_M^B = L] = \frac{\eta V^{HL} + (1-\eta)\alpha V^{LL}}{\eta + (1-\eta)\alpha}.$$

According to Condition 17, $V_M^{HH} > 0$, implying that $\Delta + \beta V_M^{HH} > 0$. Note that assumption 1 implies that $\Delta + \beta V_M^{HH}$ converges to a negative number as $\alpha \rightarrow 1$. Therefore, there exists $\bar{\alpha} \in (0, 1)$ such that $\Delta + \beta V_M^{HH} = 0$ when $\alpha = \bar{\alpha}$.¹⁹ Therefore, Condition 17 implies that $\alpha < \bar{\alpha}$. It is also clear that $V_M^{HL} > V^{LL}$.

Therefore, we can use Proposition 1 for the preference parameters $(V^{LL}, V_M^{HL}, V_M^{HH})$ to characterize the equilibrium outcomes. According to Proposition 1, the unique equilibrium is generalization by both raters when $\beta > \beta^*(\frac{1}{4})$, which corresponds to $\lambda^* < \frac{1}{4}$. We have (See equation A.4):

$$\lambda^* = 1 + \frac{(1-\eta)(\Delta + \beta V_M^{HL})}{\eta(\Delta + \beta V_M^{HH})}. \quad (\text{A.34})$$

Note that in the neighborhood of $\bar{\alpha}$, $V_M^{HL} < 0$. Therefore, λ^* converges to $-\infty$ as α goes to $\bar{\alpha}$ (the denominator goes to zero and the numerator converges to a negative number), implying the presence of α^* such that $\lambda^* < \frac{1}{4}$ when α is sufficiently close to $\bar{\alpha}$.

¹⁹There is a unique $\bar{\alpha} = 0$ for which $\Delta + \beta V_M^{HH} = 0$ since V_M^{HH} is decreasing in α . This can be verified by noting that:

$$\begin{aligned} V_M^{HH} &= V^{LL} + \frac{\eta^2(V^{HH} - V^{LL}) + 2\eta(1-\eta)\alpha(V^{HL} - V^{LL})}{\eta^2 + 2\eta(1-\eta)\alpha + (1-\eta)^2 \alpha^2} \\ &= V^{LL} + \frac{\eta^2(V^{HH} - V^{LL}) + 2\eta(1-\eta)\alpha(V^{HL} - V^{LL})}{\eta^2 + 2\eta(1-\eta)\alpha} \frac{\eta^2 + 2\eta(1-\eta)\alpha}{\eta^2 + 2\eta(1-\eta)\alpha + (1-\eta)^2 \alpha^2} \\ &= V^{LL} + (V^{HL} - V^{LL} + \frac{\eta^2(V^{HH} - V^{HL})}{\eta^2 + 2\eta(1-\eta)\alpha})(1 - \{\frac{(1-\eta)\alpha}{\eta + (1-\eta)\alpha}\}^2). \end{aligned} \quad (\text{A.33})$$

The expressions inside the parentheses in the last line of (A.33) are decreasing in α . Therefore, V_M^{HH} , as a function of α , crosses zero only once.

Online Appendix for “The Market for ESG Ratings”

Ehsan Azarmsa and Joel Shapiro

This Online Appendix contains supplementary results and proofs not included in the main manuscript. Section OA.1 demonstrates the robustness of our results to the assumption that the ratings have no false-positive error. Section OA.2 discusses the socially optimal communication when the social planner values ESG performance differently from the investor. Section OA.3 provides the omitted proofs from Appendix A.

OA.1 General Information Structure

In our main model, we assume that a high rating perfectly reveals the firm’s performance in the corresponding category. In other words, there is no false-positive error in the ratings. In this section, we relax this assumption by allowing the rating technologies to generate both false-positive and false-negative errors in the ratings.

Specifically, each rater chooses conditional probabilities $\lambda_j = (\lambda_j^{AH}, \lambda_j^{AL}, \lambda_j^{BH}, \lambda_j^{BL}) \in [0, 1]^4$, where:

$$\lambda_j^{iH} = \text{Prob}(s_j^i = h | w^i = H), \quad \lambda_j^{iL} = \text{Prob}(s_j^i = l | w^i = L), \quad j = 1, 2, \quad i = A, B, \quad (\text{OA.1})$$

and the technological constraint is:

$$\lambda_j^{AH} + \lambda_j^{AL} + \lambda_j^{BH} + \lambda_j^{BL} \leq 3. \quad (\text{OA.2})$$

Note that a higher value of λ_j^{iH} or λ_j^{iL} corresponds to a more precise rating in category i for rater j . Equation (OA.2) implies that the raters face two types of trade-offs. The first is the trade-off between the precision of the ratings in categories A and B . The second trade-off, which is absent in our baseline model, is between the level of false-negative and false-positive errors in the ratings for each category. For instance, they can make their ratings more tilted toward high ratings by equally increasing the conditional probability of a high rating for both performance levels, i.e., $\lambda_j^{iH}, \lambda_j^{iL} \rightarrow \lambda_j^{iH} + \varepsilon, \lambda_j^{iL} - \varepsilon$, for category $i \in \{A, B\}$ and rater $j \in \{1, 2\}$.

Without loss of generality, we impose the conditions below to ensure that a high-performing project is not less likely to receive a high rating in a category than a low-performing one:

$$\begin{aligned} \text{Prob}(s_j^i = h | w^i = H) &\geq \text{Prob}(s_j^i = h | w^i = L) \\ \Rightarrow \lambda_j^{AH} &\geq 1 - \lambda_j^{AL}, \quad \lambda_j^{BH} \geq 1 - \lambda_j^{BL}, \quad j = 1, 2. \end{aligned} \quad (\text{OA.3})$$

The baseline model corresponds to the case with $\lambda_j^{A_L} = \lambda_j^{B_L} = 1$, $j = 1, 2$, implying no false-positive errors in the ratings. The conditions above nest the technological constraint in the baseline model (equation (8)), implying that all feasible rating technologies in the baseline model remain available. For instance, the specialized rating technologies correspond to $\lambda^{SP_A} = (1, 1, 0, 1)$ and $\lambda^{SP_B} = (0, 1, 1, 1)$,²⁰ and generalization corresponds to $\lambda^{GN} = (\frac{1}{2}, 1, \frac{1}{2}, 1)$. To make the results comparable to those for the baseline setup, we set the right-hand side of Constraint OA.2 to three. This choice keeps these special rating technologies at the frontier of the rating technologies available to the raters.

Similar to the baseline model, the raters first simultaneously decide on their rating technologies, and then they sequentially set fees. The investor then decides which ratings to purchase and, lastly, decides whether or not to invest in the project given the realized ratings.

To simplify the characterization of the equilibria, we assume that the investor is so averse to investing in (L, L) -projects that she does not invest if, given her information, there is a positive probability that the project has low performance in both categories. This can be thought of as assigning a very low value to V^{LL} .

In Proposition OA.1, we describe the market equilibria in pure strategies. We once again employ the notion of robust equilibria, which is defined in Definition 3, to refine the set of equilibria.

Proposition OA.1. *Under Assumptions 1 and 2, generalization by both raters and specialization in different categories are the only outcomes that can form a robust equilibrium in pure strategies for some values of β . The following provides the characterization in detail:*

a) *If $V^{HL} \geq 0$, then the only robust equilibrium outcome is that the raters specialize in different categories. This equilibrium outcome remains unique even when not restricting to robust equilibria.*

b) *Suppose $V^{HL} < 0$:*

b.1) *If $\beta > \beta^*(0)$, the unique robust equilibrium is that both raters generalize. This uniqueness holds even when not restricting to robust equilibria.*

b.2) *If $\beta \in (\beta^*(\frac{1}{2}), \beta^*(0)]$, there is no robust equilibrium.*

b.3) *If $\beta \leq \beta^*(\frac{1}{2})$, the unique robust equilibrium is that the raters specialize in different categories.*

²⁰ λ^{SP_A} is informationally equivalent to any other rating technology that perfectly reveals the project's performance in category A and provides an uninformative rating for category B , i.e., $\lambda^{SP'_A} = (1, 1, x, 1 - x)$, for $x \in [0, 1]$. Likewise, λ^{SP_B} is equivalent to $\lambda^{SP'_B} = (y, 1 - y, 1, 1)$ for any $y \in [0, 1]$. For brevity, we do not report these equivalent choices in our characterizations.

The proof is provided in Section OA.3.6. Proposition OA.1 demonstrates that the key insights developed in the baseline model continue to hold if we allow for a more flexible set of rating technologies. Thus, they are robust to our earlier assumption about the information structure.

As stated in Part (a) of Proposition OA.1, the unique equilibrium is that the raters specialize in different categories when $V^{HL} \geq 0$. The intuition is similar to the baseline case: In this case, a single rating ensuring a high performance in a category is enough for the investor to invest. As a result, the stand-alone value of specialization is large enough that specialization in different categories, the value-maximizing outcome, is an equilibrium. We show this equilibrium outcome is unique as well.

When $V^{HL} < 0$, the unique equilibrium is generalization when the investor is highly concerned about the project's ESG performance (i.e., high β). To understand the intuition, consider the extreme case where $\frac{V^{HL}}{V^{HH}} < 0$ is very negative; that is, the loss from investing in a (L, H) - or (H, L) -project is substantially larger than the gain from investing in a (H, H) -project. In this case, the investor is more concerned about the false-positive error in the ratings than the false-negative error because the former might lead to investment in projects with low performance in a category, resulting in a very low payoff, while the latter just inefficiently screens out some good projects, which is not as costly. As a result, the raters have a strong incentive to eliminate the false-positive error in their ratings, reducing the model to the baseline setup, where we find that generalization is the unique equilibrium outcome when β exceeds a threshold value.

Furthermore, there is no robust equilibrium for some intermediate values of β , as stated in Part (b.2) of Proposition OA.1. This is because neither specialization nor generalization maximizes the stand-alone value of the ratings when $\beta \in (\beta^*(\frac{1}{2}), \beta^*(0))$, given that we consider a more flexible set of rating technologies. As a result, Rater 2 chooses a rating technology that depends on β , implying that the equilibrium is not robust to β .

Specialization achieves the largest stand-alone value when $\beta \leq \beta^*(\frac{1}{2})$. With this observation, we show that the best response to specialization is to specialize in the other category for both raters. Therefore, specialization in different categories forms an equilibrium, which is the only robust equilibrium outcome.

OA.2 When Social Surplus Does Not Equal Investor Surplus

The investor might underweight or overweight the importance of the ESG performance compared to the socially optimal benchmark. For instance, the investor might not be representative of the group benefiting or impacted by the investment, or may have incorrect beliefs

about the value of the ESG performance. However, as it is the investor who buys the ESG rating, this could lead to another source of inefficiency in the production of ESG information. In this section, we analyze how the misalignment between the social benefit and investor's preferences impacts the optimal design of rating technologies.

For this analysis, we assume that the social planner has a different β than the investor, which we denote by β^{SP} . Therefore, the social value created by the investment is $\Delta + \beta^{SP}u$. We assume that both β and β^{SP} also satisfy Assumption 1, implying that both the investor and the social planner need positive information about the project's ESG performance to approve the investment. Furthermore, to simplify our exposition, we assume that $\Delta + \beta^{SP}V^{HL} < 0$, meaning that the social planner only approves projects that have a high performance in both categories.

The goal is to find the socially optimal choices of (λ_1, λ_2) when the investor uses the realized ratings to guide her investment decision. To this end, we derive the expected social value generated by pair (λ_1, λ_2) , which we denote by $W(\lambda_1, \lambda_2)$, as below:

$$W(\lambda_1, \lambda_2) = \sum_{s^A, s^B \in \{h, l\}} \mathbb{E}[(\Delta + \beta^{SP}u) \mathbb{I}\{\mathbb{E}[\Delta + \beta u | s^A, s^B] \geq 0\} | s^A, s^B] P(s^A, s^B). \quad (\text{OA.4})$$

In equation (OA.4), $\Delta + \beta^{SP}u$ is the social welfare when the investment takes place. The indicator function $\mathbb{I}\{\mathbb{E}[\Delta + \beta u | s^A, s^B] \geq 0\}$ represents whether the investment takes place given the ratings realized. We call a pair of rating technologies “socially optimal” if it maximizes $W(\lambda_1, \lambda_2)$, given the technological constraint in equation (8). In Proposition OA.2, we characterize the socially optimal pairs of rating technologies.

Proposition OA.2.

a) If $\Delta + \beta V^{HL} < 0$, then it is socially optimal that the raters specialize in different categories to implement perfect disclosure. The investment takes place iff the project has a high performance in both categories.

b) If $\Delta + \beta V^{HL} > 0$:

b.1) When $\Delta + \beta^{SP}\{\eta V^{HH} + (1 - \eta)V^{HL}\} \geq 0$, it is socially optimal that the raters specialize in the same category. The investment takes place iff the project has a high performance in this category.

b.2) When $\Delta + \beta^{SP}\{\eta V^{HH} + (1 - \eta)V^{HL}\} < 0$, it is socially optimal that the raters provide no information. The investment does not take place in this case.

Section OA.3.7 provides the proof. Proposition OA.2(a) states that perfect disclosure

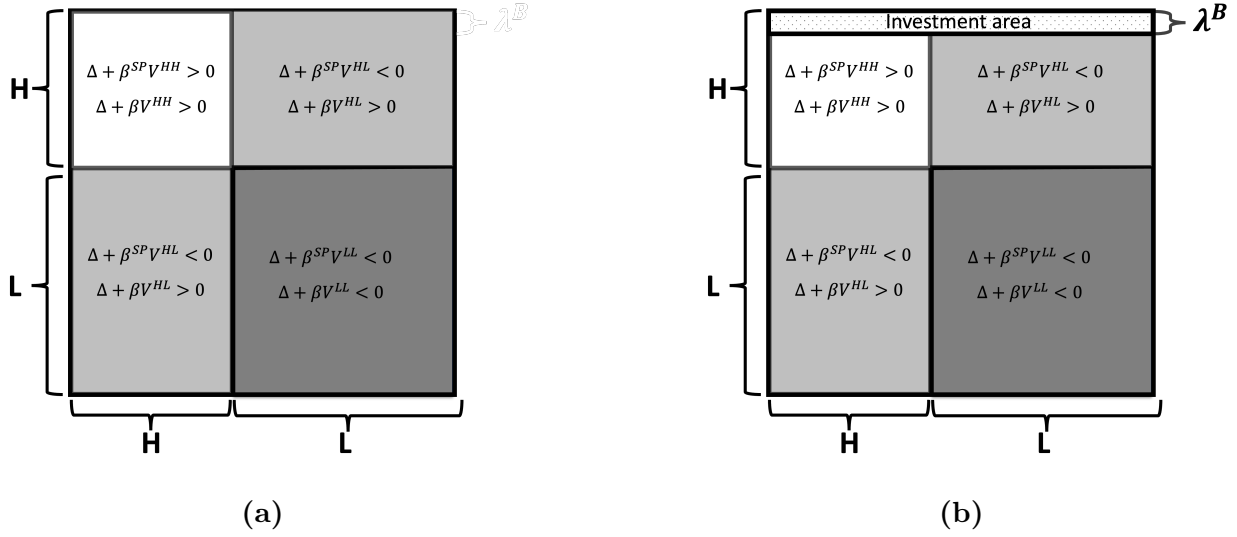


Figure OA.1. This figure displays the impact of an increase in the disclosure in a category on the investment outcome. Figure (a) corresponds to no disclosure. Figure (b) displays the effect of some disclosure in category B . The disclosure leads to some investment in the projects with a high performance in category B . The investment is socially efficient for the (H, H) -type and inefficient for the (L, H) -type. The increase in the disclosure might or might not be socially optimal depending on the composition of these two types.

of the ESG information is socially optimal, when $\Delta + \beta^{SP}V^{HL}$ and $\Delta + \beta V^{HL}$ are both negative. In this case, the investor and social planner agree on which type of projects should receive investment; thus, perfect disclosure is socially optimal.

When $\Delta + \beta^{SP}V^{HL}$ is negative and $\Delta + \beta V^{HL}$ is not, perfect disclosure is not optimal since some project types would inefficiently receive investment. In this case, depending on the relative importance of ESG performance for the social planner, he might decide not to disclose any ESG information or only disclose partially.

We make two observations here. First, in order for this parameter constellation to hold, it must be that V^{HL} is negative and, therefore, the social planner has a stronger preference for ESG performance than the investor ($\beta^{SP} > \beta$). Second, when $\Delta + \beta V^{HL} \geq 0$, more disclosure leads to more investment since the investor just wants to screen out (L, L) -type projects. The social planner prefers to also screen out (H, L) -type projects. Therefore, the social planner's decision depends on the risk that (H, L) -type projects may arise. When this risk is high enough, the social planner prefers no information to be provided by the raters, so as to induce the investor not to invest.

Figure OA.1 illustrates this point. The figure shows the impact of slightly increasing disclosure when the social planner and investor disagree on the optimal investment decision.

Figure OA.1a corresponds to the case with no disclosure, which leads to no investment due to Assumption 1. In Figure OA.1b, there is some disclosure in category B . The disclosure identifies some projects with $w^B = H$, which the investor finds attractive to invest. Whether the increase in investment is socially optimal depends on the composition of the project types that receive investment and the social investment payoff associated with each type. If the additional investment mostly goes to (H, L) -type projects, or if their resulting social investment payoff $(\Delta + \beta^{SP} V^{HL})$ is sufficiently low, then the social planner prefers not to increase the disclosure. Overall, a key insight provided in this section is that more ESG disclosure is not necessarily desirable when the investor's preferences are not aligned with those of the social planner.

OA.3 Omitted Proofs

This section provides the proof of Propositions in Section 6, OA.1, and OA.2.

OA.3.1 Proof of Proposition 5 (Market equilibria with heterogeneous information acquisition costs)

Recall from the proof of Proposition 1 that the raters' preferences across outcomes depend solely on λ^* , as defined in equation A.4. The details are provided in equations A.9 and A.10. We establish the proof through three steps. First, we determine the values of λ^* for which the specialization outcomes—i.e., $(\lambda^{SPA}, \lambda^{SPb-B}), (\lambda^{SPb-B}, \lambda^{SPA})$ —constitute an equilibrium. Second, employing a method akin to the proof of Proposition 1, we demonstrate that there is only one possible interior equilibrium outcome in which the rating technologies are substitutes, and find that this interior outcome is an equilibrium only when b exceeds a threshold value. Lastly, we establish that there is only one possible interior equilibrium outcome in which the ratings are complements. This interior outcome is the unique equilibrium when $\lambda^* < \frac{b}{4}$.

Specialization outcomes

Let $BR_j(\lambda_{-j})$ denote the set of best response rating technologies for rater $j \in \{1, 2\}$, when the other rater chooses λ_{-j} . Since the first rater effectively optimizes the combined value of the ratings by maximizing the marginal value of its ratings, it is straightforward to show $\lambda^{SPA} = BR_1(\lambda_b^{SPB})$ and $\lambda_b^{SPB} = BR_1(\lambda_b^{SPA})$. Therefore, we only need to find values of λ^* for which specialization in a category is the best response for the second rater when the first rater specializes in the other category.

First, we prove that $\lambda^{SPA} = BR_2(\lambda_b^{SPB})$ iff $\lambda^* \geq \frac{b}{4}$, implying that $(\lambda_b^{SPB}, \lambda^{SPA})$ is an equilibrium outcome for this set of values of λ^* . When $\lambda^* \geq \frac{b}{4}$, λ^{SPA} yields the highest stand-alone value. The argument as follows: The stand-alone value of $\lambda_2 = (\lambda_2^A, \lambda_2^B)$ is $\max\{\lambda^* - \lambda_2^A \lambda_2^B, \lambda_2^A \lambda^*, \lambda_2^B \lambda^*, \lambda_2^A \lambda_2^B\}$. The first three terms are maximized at $(1, 0)$, and attain a value of λ^* . $\lambda_2^A \lambda_2^B \leq \frac{b}{4}$ when $\lambda_2^A + b^{-1} \lambda_2^B \leq 1$. Therefore, $\lambda^{SPA} = (1, 0)$ obtains the highest stand-alone value when $\lambda^* \in [\frac{b}{4}, 1]$.

Moreover: $v(\lambda_b^{SPB}, \lambda_2) = \max\{(\lambda_2^A + \lambda^B) \lambda^* - \lambda_2^A \lambda^B, \lambda_2^A \lambda^B, \lambda^* \lambda^B\}$ (See equation A.9). The first two terms are maximized at $\lambda_2 = \lambda^{SPA}$, and attain a value of $(1 + b) \lambda^* - b$ and b , respectively. These two values exceed λ^* , which is the maximum value that the third term can obtain. Therefore, λ^{SPA} is Rater 2's best response to λ_b^{SPB} , and consequently, $(\lambda_b^{SPB}, \lambda^{SPA})$ is an equilibrium outcome when $\lambda^* \geq \frac{b}{4}$.

To see that this outcome is not an equilibrium outcome when $\lambda^* < \frac{b}{4}$, note that $\lambda_{b_1}^{GN} \equiv (\frac{1}{2}, \frac{b}{2})$ has a larger stand-alone value, and $\lambda_{b_1}^{GN}$ and λ_b^{SPB} are complements for these values of λ^* :

$$\begin{aligned} v(\lambda_b^{SPB}, \lambda_{b_1}^{GN}) &= \frac{3}{4}b - \frac{1}{4}b^2, \quad v(\lambda_b^{SPB}, \mathbf{O}) = b\lambda^*, \quad v(\mathbf{O}, \lambda_{b_1}^{GN}) = \frac{b}{4} \\ \Rightarrow v(\mathbf{O}, \lambda_{b_1}^{GN}) + v(\lambda_b^{SPB}, \mathbf{O}) &= \frac{b}{4} + b\lambda^* < \frac{b}{4} + \frac{b^2}{4} < \frac{3}{4}b - \frac{1}{4}b^2 = v(\lambda^{SPB-B}, \lambda_{b_1}^{GN}) \quad (\text{OA.5}) \\ &\Rightarrow \hat{\phi}_2(\lambda_b^{SPB}, \lambda^{SPA}) \leq \lambda^* < \frac{b}{4} = \hat{\phi}_2(\lambda_b^{SPB}, \lambda_{b_1}^{GN}). \end{aligned}$$

Now, we show that $\lambda_b^{SPB} = BR_2(\lambda^{SPA})$, and consequently $(\lambda^{SPA}, \lambda_b^{SPB})$ is an equilibrium, iff $\lambda^* \geq \frac{b}{(1+b)^2}$. To see this, note that $\lambda_2 = \lambda_b^{SPB}$ has the highest marginal value when $\lambda_1 = \lambda^{SPA}$. Therefore, the only possibility for $\lambda_b^{SPB} \neq BR_2(\lambda^{SPA})$ is the existence of λ_2 with a higher stand-alone value such that λ_2 and λ^{SPA} are complements. More specifically, these two conditions should jointly hold for some λ_2 for $(\lambda^{SPA}, \lambda_b^{SPB})$ not to be an equilibrium:

$$\begin{aligned} v(\mathbf{O}, \lambda_2) &> v(\mathbf{O}, \lambda_b^{SPB}) \\ v(\lambda^{SPA}, \lambda_2) &\geq v(\lambda^{SPA}, \mathbf{O}) + v(\mathbf{O}, \lambda_2). \end{aligned} \quad (\text{OA.6})$$

These inequalities imply $\lambda_2^A, \lambda_2^B \geq \lambda^*$. The second inequality further implies:

$$\lambda_2^B \geq \lambda^* + \lambda_2^A \lambda_2^B \iff \lambda_2^B (1 - \lambda_2^A) \geq \lambda^* \iff b(1 - \lambda_2^A)^2 \geq \lambda^*. \quad (\text{OA.7})$$

Moreover, the first inequality in (OA.6) implies:

$$\lambda_2^A \lambda_2^B > b\lambda^* \iff \lambda_2^A (1 - \lambda_2^A) > \lambda^*. \quad (\text{OA.8})$$

By combining the inequalities above, we find that λ_2^A exists that satisfies both inequalities iff $\lambda^* < \frac{b}{(1+b)^2}$, which proves our claim.

Interior equilibrium outcomes with substitute rating technologies

Now, we analyze interior equilibrium outcomes in which the rating technologies are substitutes. By interior, we mean outcomes in which at least one rater does not specialize. Suppose (λ_1, λ_2) is an equilibrium pair of rating technologies where λ_1 and λ_2 are substitutes. Similar to the proof of Proposition 1, if $\min\{\lambda^A, \lambda^B\} \leq \lambda^*$, we can show that the pair cannot constitute an equilibrium unless $(\lambda_1, \lambda_2) \in \{(\lambda^{SP_A}, \lambda_b^{SP_B}), (\lambda_b^{SP_B}, \lambda^{SP_A})\}$ since at least a rater can increase the combined value and stand-alone value (if necessary) by switching to specialization. If $\lambda^A, \lambda^B \geq \lambda^*$, then $v(\lambda_1, \lambda_2) = \lambda^A \lambda^B$. Note that both raters effectively maximize the combined value by optimizing the marginal value of their rating technologies. Since the set of pairs that the rating technologies are substitutes is open (according to Definition A.1), the following first-order conditions should hold (by taking the derivative of $v(\lambda_1, \lambda_2) = \lambda^A \lambda^B$ with respect to λ_1^A and λ_2^A , and considering that technological constraint 18 binds for both raters):

$$\begin{aligned} [\lambda_1^A] : \quad & (1 - \lambda_2^A) \lambda^B = b(1 - \lambda_2^B) \lambda^A \\ [\lambda_2^A] : \quad & (1 - \lambda_1^A) \lambda^B = b(1 - \lambda_1^B) \lambda^A. \end{aligned} \tag{OA.9}$$

Note that these conditions result in a symmetric pair, i.e., $\lambda_1^A = \lambda_2^A$. Let us denote this rating technology by $\lambda_{b_2}^{GN} = (x, y)$. The first-order conditions imply the following equation for x and y (in addition to $x + b^{-1}y = 1$):

$$(1 - x)(2y - y^2) = b(1 - y)(2x - x^2). \tag{OA.10}$$

An implication of equation OA.10 is that $x \geq y$:

$$\frac{2y - y^2}{1 - y} = b \frac{2x - x^2}{1 - x} \Rightarrow (1 - y)^{-1} - (1 - y) < (1 - x)^{-1} - (1 - x) \Rightarrow y < x, \tag{OA.11}$$

where we used the fact that $(1 - z)^{-1} - (1 - z)$ is an increasing function in $[0, 1]$. Moreover, equation OA.10 implies that $x \leq \frac{1}{2}$. To see this, consider the contrary:

$$\begin{aligned} (1 - x)(2y - y^2) &= b(1 - y)(2x - x^2) \xrightarrow{y=b(1-x)} (1 - x)^2(2 - y) = (2x - x^2)(1 - y) \\ &\xrightarrow{\text{if } x \geq \frac{1}{2}} \frac{1}{4} > (1 - x)^2 = \frac{1 - y}{2 - y}(2x - x^2) = \underbrace{\frac{1 - b(1 - x)}{2 - b(1 - x)}}_{> \frac{1}{3}} \underbrace{(2x - x^2)}_{> \frac{3}{4}} > \frac{1}{4}, \end{aligned} \quad (\text{OA.12})$$

which is a contradiction. Hence, $x \leq \frac{1}{2}$.

$(\lambda_{b_2}^{GN}, \lambda_{b_2}^{GN})$ constitutes an equilibrium when $\lambda^* \leq \lambda^A \lambda^B = (2x - x^2)(2y - y^2)$ (since Rater 1 should not benefit from deviating to λ^{SPA}) and $\lambda_{b_2}^{GN}$ is substitute with itself. In other words, the following inequalities should hold:

$$\begin{aligned} v(\lambda_{b_2}^{GN}, \lambda_{b_2}^{GN}) &\geq \lambda^* \iff (2x - x^2)(2y - y^2) \geq \lambda^* \\ 2V(\lambda_{b_2}^{GN}, \mathbf{O}) &> V(\lambda_{b_2}^{GN}, \lambda_{b_2}^{GN}) \iff 2 \max\{xy, x\lambda^*, y\lambda^*, (x + y)\lambda^* - xy\} > (2x - x^2)(2y - y^2) \end{aligned} \quad (\text{OA.13})$$

If $x, y \geq \lambda^*$, then the left-hand side in the second inequality of (OA.13) is $2xy$, which implies:

$$2xy > (2x - x^2)(2y - y^2) \Rightarrow 2 > (2 - x)(2 - y) \geq (2 - x)^2 \geq \frac{9}{4} > 2, \quad (\text{OA.14})$$

which is not possible. Additionally, we cannot have $x \geq \lambda^* > y$ since that would imply:

$$\underbrace{2x\lambda^*}_{< \lambda^*} > (2x - x^2)(2y - y^2) \geq \lambda^*. \quad (\text{OA.15})$$

Therefore, $x, y < \lambda^*$, which implies that the left-hand side in the second inequality of (OA.13) is $(x + y)\lambda^* - xy$. This implies that the following inequality for x and y :

$$2(x + y)(2x - x^2)(2y - y^2) - 2xy \geq 2(x + y)\lambda^* - 2xy > (2x - x^2)(2y - y^2). \quad (\text{OA.16})$$

Note that x and y are functions of b . One can show that there exists threshold value $b^* \in (0, 1)$ such that this inequality holds when $b > b^*$. When $b < b^*$, $(\lambda_{b_2}^{GN}, \lambda_{b_2}^{GN})$ is never an equilibrium. Conversely, when $b \geq b^*$, there exists an interior range of λ^* for which $(\lambda_{b_2}^{GN}, \lambda_{b_2}^{GN})$ is an equilibrium.

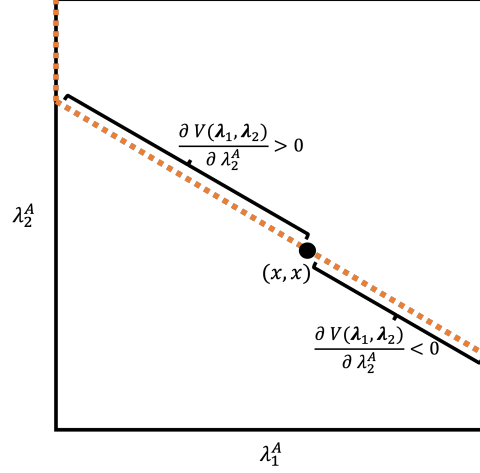


Figure OA.2. The dashed line depicts the choices of λ_1^A that maximize $\lambda^A \lambda^B$ given λ_2 . The dark circle represents $(\lambda_{b_2}^{GN}, \lambda_{b_2}^{GN})$, where $\lambda_{b_2}^{GN} = (x, y)$.

Interior equilibrium outcomes with complement rating technologies

As shown in the proof of Proposition 1, $\lambda^A, \lambda^B \geq \lambda^*$ when two ratings are complements (See Step 2 in the proof). Therefore, λ_1 is set to maximize $\lambda^A \lambda^B$, given λ_2 . We show that the only possible interior equilibrium outcome with complement rating technologies is the one with $\lambda_2 = \lambda_{b_1}^{GN} = (\frac{1}{2}, \frac{b}{2})$. Note that this is the only interior outcome for which $\frac{\partial V(\mathbf{O}, \lambda_2)}{\partial \lambda_2^A} = 0$; specifically, the stand-alone is maximized locally.

Figure OA.2 depicts the pairs of (λ_1, λ_2) for which λ_1 has the highest marginal value given λ_2 . Note that $(\lambda_{b_2}^{GN}, \lambda_{b_2}^{GN})$ is also among these pairs, as marked by a dark circle in the Figure.

Consider pair (λ_1, λ_2) that corresponds to a point on the dashed line, where $\lambda_2 \neq \lambda_{b_1}^{GN}$ and $\lambda_2^A \geq x$, i.e., $(\lambda_1^A, \lambda_2^A)$ is left to (x, x) on the dashed line in Figure OA.2. This means that $\frac{\partial V(\lambda_1, \lambda_2)}{\partial \lambda_2^A} > 0$. The proof for the other case is similar. There are two possibilities for λ_2^A :

- $\lambda_2^A \in [x, \frac{1}{2}]$: We show that $\frac{\partial v(\mathbf{O}, \lambda_2)}{\partial \lambda_2^A} > 0$ in this case, which means that Rater 2 can increase its payoff by increasing λ_2^A . Since $\lambda_2^A \geq x > y \geq \lambda_2^B$, there are three possibilities.

– If $\lambda_2^A > \lambda_2^B > \lambda^*$, then

$$\frac{\partial v(\mathbf{O}, \lambda_2)}{\partial \lambda_2^A} = \frac{\partial \lambda_2^A \lambda_2^B}{\partial \lambda_2^A} = b(1 - 2\lambda_2^A) > 0. \quad (\text{OA.17})$$

– If $\lambda_2^A \geq \lambda^* \geq \lambda_2^B$, then

$$\frac{\partial v(\mathbf{O}, \boldsymbol{\lambda}_2)}{\partial \lambda_2^A} = \frac{\partial \lambda_2^A \lambda^*}{\partial \lambda_2^A} = \lambda^* > 0. \quad (\text{OA.18})$$

– If $\lambda^* > \lambda_2^A > \lambda_2^B$. Note that since $\lambda_2^A > \frac{b}{1+b}$, λ^* should also exceed $\frac{b}{1+b}$. Therefore,

$$\begin{aligned} \frac{\partial v(\mathbf{O}, \boldsymbol{\lambda}_2)}{\partial \lambda_2^A} &= \frac{\partial [\lambda_2^A \lambda^* + \lambda_2^B \lambda^* - \lambda_2^A \lambda_2^B]}{\partial \lambda_2^A} = \\ (1-b)\lambda^* - b + 2b \underbrace{\lambda_2^A}_{> \frac{b}{1+b}} &> (1-b)\lambda^* - b + 2\frac{b^2}{1+b} = (1-b)(\lambda^* - \frac{b}{1+b}) > 0. \end{aligned} \quad (\text{OA.19})$$

- $\lambda_2^A > \frac{1}{2}$: Similar to the previous case, one can show that if $\min\{\lambda_2^A, \lambda_2^B\} \leq \lambda^*$, then $\frac{\partial v(\mathbf{O}, \boldsymbol{\lambda}_2)}{\partial \lambda_2^A} > 0$; It means that Rater 2 can increase both the stand-alone value and marginal value of its ratings by increasing λ_2^A .

Now, suppose $\min\{\lambda_2^A, \lambda_2^B\} > \lambda^*$. Since $\lambda_2^B = b(1 - \lambda_2^A) < \frac{b}{2}$, this implies that $\lambda^* < \frac{b}{2}$. Since $\boldsymbol{\lambda}_1$ maximizes $\lambda^A \lambda^B$ given $\boldsymbol{\lambda}_2$, we have the following first-order condition:

$$\lambda_1^B - b\lambda_1^A \leq \frac{b\lambda_2^A - \lambda_2^B + (1-b)\lambda_2^A \lambda_2^B}{(1 - \lambda_2^A)(1 - \lambda_2^B)}, \quad (\text{OA.20})$$

where the inequality is strict only for the extreme values of λ_1^A . Given that $\lambda_2^A > \frac{1}{2}$, the expression above implies that:

$$\lambda_1^B - b\lambda_1^A > 0 \Rightarrow \lambda_1^B > \frac{b}{1+b} > \frac{b}{2} > \lambda^*. \quad (\text{OA.21})$$

The inequalities in (A.18) imply that if $\lambda_1^A, \lambda_1^B > \lambda^*$, then $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_{b_1}^{GN}$ are also complements (Note that $\boldsymbol{\lambda}_{b_1}^{GN} = (\frac{1}{2}, \frac{b}{2})$ and $\frac{1}{2} > \frac{b}{2} > \lambda^*$). Therefore, Rater 2 can increase its payoff by switching to $\boldsymbol{\lambda}_{b_1}^{GN}$:

$$\hat{\phi}_2(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = \lambda_2^A \lambda_2^B = b\lambda_2^A(1 - \lambda_2^A) < \frac{b}{4} = \hat{\phi}_2(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_{b_1}^{GN}). \quad (\text{OA.22})$$

If $\lambda_1^A < \lambda^* < \lambda_1^B$, then $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ cannot be an equilibrium if $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_{b_1}^{GN}$ are also complements, with a logic similar to what is used in equation OA.22. If $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_{b_1}^{GN}$

are substitutes, then we have:

$$\begin{aligned} v(\boldsymbol{\lambda}_1, \mathbf{O}) + v(\mathbf{O}, \boldsymbol{\lambda}_{b_1}^{GN}) &> v(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_{b_1}^{GN}) \\ \Rightarrow \lambda_1^B \lambda^* + \frac{b}{4} &> \left(\frac{1}{2} + \frac{\lambda_1^A}{2}\right) \left(\frac{b}{2} + \left(1 - \frac{b}{2} \lambda_1^B\right)\right) \end{aligned} \quad (\text{OA.23})$$

By expanding the right-hand side and making some rearrangements, we obtain:

$$\begin{aligned} \lambda^* \lambda_1^B &> \frac{b}{4} + \left(\frac{3}{4} - \frac{b}{2}\right) \lambda_1^B - \left(\frac{b^{-1}}{2} - \frac{1}{4}\right) \lambda_1^{B^2} \\ &= \frac{b}{4} \lambda_1^B + \left\{ \frac{b}{4} + \frac{3}{4}(1-b) \lambda_1^B - \left(\frac{b^{-1}}{2} - \frac{1}{4}\right) \lambda_1^{B^2} \right\}. \end{aligned} \quad (\text{OA.24})$$

The expression inside the curly brackets is positive. To verify this, note that the expression is concave in λ_1^B , implying that the minimum values are obtained in one of the extreme values, i.e., $\lambda_1^B = 0, b$. It is straightforward to verify that the expression is non-negative for both values. Therefore, $\lambda^* > \frac{b}{4}$. Now, we show that Rater 2 can increase its payoff by switching to $\boldsymbol{\lambda}^{SPA}$, which implies that $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ cannot form an equilibrium:

$$\hat{\phi}_2(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}^{SPA}) = \min\{\lambda_1^B - \lambda_1^B \lambda^*, \lambda^*\} > \min\left\{\frac{b}{2}\left(1 - \frac{b}{4}\right), \frac{b}{4}\right\} = \frac{b}{4} > \hat{\phi}_2(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2). \quad (\text{OA.25})$$

Therefore, the only possible interior pair with complement rating technologies is $(BR_1(\boldsymbol{\lambda}_{b_1}^{GN}), \boldsymbol{\lambda}_{b_1}^{GN})$. Suppose $BR_1(\boldsymbol{\lambda}_{b_1}^{GN}) = (z^A, z^B)$. Since $\boldsymbol{\lambda}_{b_1}^{GN}$ should optimize the stand-alone value locally, than $\lambda^* \leq \frac{b}{2}$. Moreover, by examining the first-order condition in (OA.20), we find:

$$z^B - bz^A = \frac{(1-b)b}{2(1-\frac{b}{2})} > 0 \Rightarrow z^A < \frac{1}{2}. \quad (\text{OA.26})$$

When $\lambda^* \leq \frac{b}{4}$, $\boldsymbol{\lambda}_{b_1}^{GN}$ has the highest stand-alone value and is complement with any other rating technology. Therefore, it is an equilibrium. However, it is not an equilibrium when $\lambda^* > \frac{b}{4}$, as Rater 2 can increase its payoff by specializing in category A:

$$\begin{aligned} \hat{\phi}_2((z^A, z^B), \boldsymbol{\lambda}^{SPA}) &= \min\{v((z^A, z^B), \boldsymbol{\lambda}^{SPA}) - v((z^A, z^B), \mathbf{O})\} = \min\{z^B(1 - z^A), \lambda^*\} \\ &= \min\{b(1 - z_A)^2, \lambda^*\} > \frac{b}{4} = v(\mathbf{O}, \boldsymbol{\lambda}_{b_1}^{GN}) = \hat{\phi}_2((z^A, z^B), \boldsymbol{\lambda}_{b_1}^{GN}). \end{aligned} \quad (\text{OA.27})$$

OA.3.2 Proof of Proposition 6 (Value-maximizing pairs with heterogeneous information acquisition costs)

From Lemma A.1, recall that the value created by pair (λ_1, λ_2) is:

$$\begin{aligned} V(\lambda_1, \lambda_2) &= (\Delta + \beta V^{HH})v(\lambda_1, \lambda_2) \\ v(\lambda_1, \lambda_2) &= \lambda^A \lambda^B + \lambda^A [\lambda^* - \lambda^B]^+ + \lambda^B [\lambda^* - \lambda^A]^+. \end{aligned} \quad (\text{OA.28})$$

We show that for any pair such as $(\hat{\lambda}_1, \hat{\lambda}_2)$, we have $v(\hat{\lambda}_1, \hat{\lambda}_2) \leq v(\lambda^{SPA}, \lambda_b^{SPB})$, and equality is only obtained when raters specialize in different categories. Define,

$$\hat{\lambda}^i = \hat{\lambda}_1^i + \hat{\lambda}_2^i - \hat{\lambda}_1^i \hat{\lambda}_2^i, \quad i = A, B. \quad (\text{OA.29})$$

There are three possibilities:

- $\lambda^* > \hat{\lambda}^A, \hat{\lambda}^B$: In this case, $v(\hat{\lambda}_1, \hat{\lambda}_2) = (\hat{\lambda}^A + \hat{\lambda}^B)\lambda^* - \hat{\lambda}^A \hat{\lambda}^B$. Since the expression is convex in λ_1^A and λ_2^A , the maximum value should be attained when both raters specialize. It is straightforward to show that specialization in different categories dominates specialization in the same category.
- $\max\{\hat{\lambda}^A, \hat{\lambda}^B\} \geq \lambda^* \geq \min\{\hat{\lambda}^A, \hat{\lambda}^B\}$: In this case, we have:

$$v(\hat{\lambda}_1, \hat{\lambda}_2) = \max\{\hat{\lambda}^A, \hat{\lambda}^B\}\lambda^* \leq \lambda^* < b = v(\lambda^{SPA}, \lambda_b^{SPB}), \quad (\text{OA.30})$$

where $b > \lambda^*$ is implied from Assumption 2'.

- $\hat{\lambda}^A, \hat{\lambda}^B > \lambda^*$: In this case, $v(\hat{\lambda}_1, \hat{\lambda}_2) = \hat{\lambda}^A \hat{\lambda}^B$. We show that $\hat{\lambda}^A \hat{\lambda}^B \leq b$, and equality is only achieved when the raters specialize in different categories:

$$\hat{\lambda}^A \hat{\lambda}^B \leq (\hat{\lambda}_1^A + \hat{\lambda}_2^A)(\hat{\lambda}_1^B + \hat{\lambda}_2^B) \leq b(\hat{\lambda}_1^A + \hat{\lambda}_2^A)(2 - \hat{\lambda}_1^A - \hat{\lambda}_2^A) \leq b. \quad (\text{OA.31})$$

Note that equality is achieved when $\hat{\lambda}_1^A \hat{\lambda}_2^A = \hat{\lambda}_1^B \hat{\lambda}_2^B = 0$ and $\hat{\lambda}_1^A + \hat{\lambda}_2^A = 1$, which occur only when raters specialize in different categories.

OA.3.3 Proof of Proposition 7 (Random ordering of the fee-setting)

With steps similar to those in the proof of Proposition 1, one can show that the only equilibria are generalization and specialization in different categories. With this fact, we only need to examine for what values of β these outcomes form an equilibrium.

In the proof of Proposition 1, we show that any two ratings are substitutes when $\lambda^* > \frac{17}{32}$, where λ^* is defined in equation A.4. In this case, since the raters receive the marginal value of their ratings, the payoffs are similar to the baseline case ($p = 1$), and consequently, so are the equilibrium outcomes. This corresponds to Parts a and b.2.

Now, we analyze for what values of λ^* , specialization in different categories is an equilibrium, when $\lambda^* < \frac{17}{32}$. Note that the specialization outcome is value-maximizing, and specialization in a category obtains the largest stand-alone value when $\lambda^* \geq 0.25$. Therefore, one can show that no other action yields a higher expected payoff for either of the raters. As such, specialization in different categories remains an equilibrium when $\lambda^* \geq 0.25$ for all values of $p \in [0.5, 1]$.

When $\lambda^* < 0.25$, the specialized rating technologies are complements. In this case, the first rater's expected payoffs is:

$$\pi_2(\lambda^{SPA}, \lambda^{SPB}) = (1-p)(1-\lambda^*) + p\lambda^* = 1-p + (2p-1)\lambda^*. \quad (\text{OA.32})$$

In response to $\lambda_1 = \lambda^{SPA}$, consider Rater 2 alternatively chooses $\lambda_2 = (\lambda_2^A, \lambda_2^B)$. It is straightforward to show that λ^{SPB} dominates λ_2 if $\lambda_2^A \leq \lambda^*$, and it is suboptimal to set $\lambda_2^A > \lambda_2^B$. Thus, suppose $\lambda_2^B \geq \lambda_2^A > \lambda^*$. Rater 2's expected payoff is:

$$\pi_2(\lambda^{SPA}, \lambda_2) = (1-p)(\lambda_2^B - \lambda^*) + p\lambda_2^A \lambda_2^B \Rightarrow \frac{d\pi_2}{d\lambda_2^B} = 1 - 2p\lambda_2^B. \quad (\text{OA.33})$$

Note that if $p = 0.5$, the profit function is increasing in λ_2^B , which implies that λ^{SPB} is indeed Rater 2's best response to λ^{SPA} . If $p > 0.5$, then the highest expected payoff Rater 2 can obtain from choices with $\lambda_2^B \geq \lambda_2^A > \lambda^*$ is when $\lambda_2^B = \frac{1}{2p}$. One can show that the expected payoff from this choice exceeds that from specializing in category B when $\lambda^* < (1 - \frac{1}{2p})^2$. In this case, specialization in different categories is not an equilibrium.

Generalization is complement with itself when $\lambda^* \leq \frac{17}{32}$. Therefore, the second rater's expected payoff from generalization is:

$$\pi_2(\lambda^{GN}, \lambda^{GN}) = (1-p)\left(\frac{9}{16} - \frac{1}{4}\right) + p\frac{1}{4} = \frac{1}{4} + \frac{1-p}{16}. \quad (\text{OA.34})$$

Again, assume $\lambda_2^B \geq \lambda_2^A$. It is straightforward to show that $\lambda_2 = \lambda^{GN}$ yields a higher expected payoff compared to all choices with $\lambda_2^A, \lambda_2^B \geq \lambda^*$ since $\lambda_2^A \lambda_2^B \leq \frac{1}{4}$ and generalization has the highest marginal value given the other rater generalizes when $\lambda^* < \frac{9}{16}$. Moreover, when $\lambda^* \leq \frac{1}{4}$, generalization has the highest stand-alone value among all rating technologies, implying that generalization is always an equilibrium when $\lambda^* \leq \frac{1}{4}$.

When $\lambda^* > \frac{1}{4}$, among choices with $\lambda_2^A < \lambda^*$, specialization yields the highest expected payoff. Therefore, we only need to find for what values of λ^* , $\pi_2(\boldsymbol{\lambda}^{GN}, \boldsymbol{\lambda}^{SP_B}) > \pi_2(\boldsymbol{\lambda}^{GN}, \boldsymbol{\lambda}^{GN})$. This translates into:

$$\frac{1-p}{4} + p\lambda^* > \frac{1}{4} + \frac{1-p}{16} \Rightarrow \lambda^* > \frac{1+3p}{16p}. \quad (\text{OA.35})$$

OA.3.4 Proof of Proposition 8 (Mixed strategy equilibria)

a) According to equation A.4, $\lambda^* \geq 1$ when $V^{HL} > 0$. Therefore, Lemma A.1 implies that the value functions are proportional to:

$$v(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) = (\lambda^A + \lambda^B)\lambda^* - \lambda^A\lambda^B. \quad (\text{OA.36})$$

Moreover, any two rating technologies are substitutes when $\lambda^* \geq 1$ (shown in the proof of Proposition 1). Therefore, both raters receive their marginal contribution as their payoff. To prove the statement in Part a of the proposition, we show that the best response to any mixed strategy is specialization or randomizing between specializing in the two categories.

The expression above is convex in λ_1^A and λ_2^A . Since a linear combination of convex functions is also convex, the expected payoffs are also convex when the other rater follows a mixed strategy. Therefore, the best response to any mixed strategy is to choose among the extreme values, i.e., $\lambda_i^A \in \{0, 1\}$, corresponding to $\boldsymbol{\lambda}^{SP_B}$ and $\boldsymbol{\lambda}^{SP_A}$.

It is straightforward to show that if rater $-i$ specializes in a category with a higher probability, the unique best response for rater i is to specialize in the other category. Therefore, the only possible mixed strategy equilibrium is that both raters randomize between specializing in the two categories with equal probabilities.

Part b.1) When $\lambda^* < \frac{1}{4}$, Rater 2's best response is always generalization, and it is unique:

$$\phi_2(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2) \leq V(\mathbf{O}, \boldsymbol{\lambda}_2) \leq V(\mathbf{O}, \boldsymbol{\lambda}^{GN}) = \phi_2(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}^{GN}). \quad (\text{OA.37})$$

In the inequalities above, the first inequality is obtained from Lemma 1. The second inequality reflects that generalization has the highest stand-alone value when $\lambda^* \leq \frac{1}{4}$, and the equality in the end is resulted from the fact that generalization is complement with any other rating technology when $\lambda^* \leq \frac{1}{4}$. Therefore, the only equilibrium is that both raters generalize, even when considering mixed strategy equilibria.

b.2) Suppose (σ_1, σ_2) is a pair of mixed strategies that constitute a robust equilibrium. Let Λ_i , $i = 1, 2$, be the support of σ_i ; that is, the set of rating technologies that are selected with a positive probability by rater i . Since Rater 1's payoff is always the marginal value

of its ratings, any $\lambda_1 \in \Lambda_1$ should maximize the expected combined value given σ_2 in a neighborhood of β . Therefore, any rating technology in Λ_1 should maximize the expression below in a neighborhood of λ^*

$$\lambda_1 \in \operatorname{argmax} v(\hat{\lambda}, \sigma_2; \tilde{\lambda}^*), \quad \forall \lambda_1 \in \Lambda_1, \tilde{\lambda}^* \in (\lambda^* - \varepsilon, \lambda^* + \varepsilon), \quad (\text{OA.38})$$

where

$$v(\hat{\lambda}, \sigma_2; \tilde{\lambda}^*) = \int_0^1 v(\hat{\lambda}, \lambda_2; \tilde{\lambda}^*) \sigma_2(\lambda_2) d\lambda_2^A. \quad (\text{OA.39})$$

Now, we show that the robustness of (σ_1, σ_2) implies that if there is an interior solution, then the solution is the unique best response to σ_2 .

If λ_1 is an interior solution, it implies that the right derivative of $v(\hat{\lambda}, \sigma_2; \tilde{\lambda}^*)$ is zero at $\hat{\lambda}^A = \lambda_1^A$ in a neighborhood of λ^* .²¹ Therefore,

$$\frac{\partial^2 v(\lambda_1, \sigma_2; \lambda^*)}{\partial_+ \lambda_1^A \partial \lambda^*} = 0. \quad (\text{OA.40})$$

Note that $\frac{\partial}{\partial_+ \lambda_1^A} v(\cdot, \sigma_2; \lambda^*)$ is linear in λ^* :

$$\begin{aligned} \frac{\partial v(\lambda_1, \sigma_2; \lambda^*)}{\partial_+ \lambda_1^A} &= \sum_{\lambda_2 \in \Lambda_2} \frac{\partial}{\partial \lambda_1^A} \lambda^A \lambda^B \mathbb{I}_{\{\lambda^* \leq \lambda^A, \lambda^B\}} \sigma_2(\lambda_2) \\ &+ \lambda^* \left[\sum_{\lambda_2 \in \Lambda_2} (1 - \lambda_2^A) \mathbb{I}_{\{\lambda^* \geq \lambda^B\}} \sigma_2(\lambda_2) - \sum_{\lambda_2 \in \Lambda_2} (1 - \lambda_2^B) \mathbb{I}_{\{\lambda^* \geq \lambda^A\}} \sigma_2(\lambda_2) \right]. \end{aligned} \quad (\text{OA.41})$$

Equation OA.40 implies that the second line in (OA.41) is zero. Moreover, the expression in the bracket is weakly increasing in λ_1^A . Consider the contrary that there is another interior solution, say λ'_1 . Therefore, the signs of $\lambda^A - \lambda^*$ and $\lambda^B - \lambda^*$ should be the same for any pair of λ_1 and λ'_1 with any rating technology in Λ_2 . Therefore, the first term should be zero for both interior solutions. However, it would contradict with the concavity of $\lambda^A \lambda^B$ since the derivative cannot be zero for both λ_1 and λ'_1 .

Now, we show that λ^{SPA} or λ^{SPB} cannot be optimum. To see this, note that expression OA.41 implies that $V(\lambda^{SPA}, \sigma_2) - V(\lambda_1, \sigma_2)$ increases in λ^* or stays unchanged since the second term in (OA.41) is weakly increasing in λ_1^A . If it increases, it means that λ_1 and λ^{SPA} cannot be jointly optimal in a neighborhood of λ^* . Even if the second term in (OA.41) is zero for both λ_1 and λ^{SPA} , the concavity of the first term implies that λ_1 and λ^{SPA} cannot

²¹Since function v is the maximum of multiple differentiable functions, the right derivative is not smaller than the left derivative at any point. Therefore, if the right derivative is negative and there is a kink at λ_1 for some $\lambda_2 \in \Lambda_2$, then the left derivative is also negative, meaning that λ_1 could not be a local optimum.

be jointly optimal. A similar argument also applies to λ^{SP_B} . Therefore, if σ_2 has an interior best response, the best response is unique. One can show that the only possibility that two actions are among the robust best responses of Rater 2 to a pure strategy of Rater 1 is that those two actions are λ^{SP_A} and λ^{SP_B} .

Therefore, the only possibility for a robust mixed strategy equilibrium is that both raters randomize between specializing in the two categories. With a logic similar to part (a), the raters should randomize with equal probabilities. One can show that this mixed strategies is dominated by generalization when $\lambda^* < \frac{1}{3}$.

OA.3.5 Proof of Proposition 9 (Divergence in measurement)

Define $V_m(\lambda_1, \lambda_2)$ as the combined value of the ratings. By applying equation 9, we find:

$$\begin{aligned} V_m(\lambda_1, \lambda_2) &= P_m^{HH} \lambda^A \lambda^B (\Delta + \beta V_m^{HH}) + [P_m^{HH} \lambda^A (1 - \lambda^B) (\Delta + \beta V_m^{HH}) + P_m^{HL} \lambda^A (\Delta + \beta V_m^{HL})]^+ \\ &\quad + [P_m^{HH} \lambda^B (1 - \lambda^A) (\Delta + \beta V_m^{HH}) + P_m^{HL} \lambda^B (\Delta + \beta V_m^{HL})]^+ \\ &= P_m^{HH} (\Delta + \beta V_m^{HH}) \left\{ \lambda^A \lambda^B + \lambda^A [\lambda_m^* - \lambda^B]^+ + \lambda^B [\lambda_m^* - \lambda^A]^+ \right\}, \end{aligned} \tag{OA.42}$$

where λ_m^* is such that

$$P_m^{HH} (1 - \lambda_m^*) (\Delta + \beta V_m^{HH}) + P_m^{HL} (\Delta + \beta V_m^{HL}) = 0. \tag{OA.43}$$

Therefore, the payoffs, and consequently the equilibrium outcomes, depend on λ_m^* in a manner similar to how they depend on λ^* in Proposition 1.

OA.3.6 Proof of Proposition OA.1 (Market equilibria with the general structure of rating technologies)

By expanding equation 9, one can show that the combined value of any two rating technologies can be written as below:

$$\begin{aligned} V(\lambda_1, \lambda_2) &= \eta^2 (\Delta + \beta V^{HH}) v(\lambda_1, \lambda_2) \\ v(\lambda_1, \lambda_2) &= l^A M^B + l^B M^A - l^A l^B, \end{aligned} \tag{OA.44}$$

where:

$$\begin{aligned}
l^i &= 1 - (1 - \lambda_1^{i_H} \mathbb{I}\{\lambda_1^{i_L} = 1\})(1 - \lambda_2^{i_H} \mathbb{I}\{\lambda_2^{i_L} = 1\}), \quad i = A, B \\
M^i &= [\lambda_1^{i_H} \lambda_2^{i_H} + (\lambda^* - 1)(1 - \lambda_1^{i_L})(1 - \lambda_2^{i_L})]^+ + [(1 - \lambda_1^{i_H})\lambda_2^{i_H} + (\lambda^* - 1)\lambda_1^{i_L}(1 - \lambda_2^{i_L})]^+ \\
&+ [\lambda_1^{i_H}(1 - \lambda_2^{i_H}) + (\lambda^* - 1)(1 - \lambda_1^{i_L})\lambda_2^{i_L}]^+ + [(1 - \lambda_1^{i_H})(1 - \lambda_2^{i_H}) + (\lambda^* - 1)\lambda_1^{i_L}\lambda_2^{i_L}]^+, \quad i = A, B,
\end{aligned} \tag{OA.45}$$

and λ^* is defined in equation A.4. The intuition for equation OA.44 is as follows: The investment takes place only if the possibility of type (L, L) is ruled out by the realized ratings. For instance, if $\lambda_1^{A_L} = 1$, s_1^A has no false-positive error, meaning that $\text{Prob}(w^A = L | s_1^A = h) = 0$. If $w^A = H$, with probability l^A , the project receives a high rating in category A that is free of a false-positive error. Given this high rating is in category A , the investor might invest depending on the realizations of s_1^B and s_2^B , which lead to an expected payoff of $\eta V^{HH} M^B$, explaining the first term in equation OA.44. The intuition for the second term is similar. The last term corrects for double-counting. Now, we analyze the equilibria for different values of λ^* :

$\lambda^* \geq 1$

This case corresponds to $V^{HL} \geq 0$ (See equation A.4). Since $\lambda^* \geq 1$, all terms in M^i in equation OA.45 are positive. Therefore, $M^A = M^B = \lambda^*$, and consequently

$$v(\lambda_1, \lambda_2) = (l^A + l^B)\lambda^* - l^A l^B. \tag{OA.46}$$

Since $\lambda^* \geq 1 \geq l^A, l^B$, the combined value is increasing in l^A and l^B . As a result, it is suboptimal to set $\lambda_j^{i_L} < 1$ for any $i = A, B$. The same logic applies to stand-alone values. In other words, it is suboptimal for the raters to introduce false-positive errors in their ratings since it would reduce l^i . Therefore, $\lambda_j^{A_L} = \lambda_j^{B_L} = 1$, $j = 1, 2$, which simplifies the game to the baseline case. According to Proposition 1, the only equilibrium in pure strategies are specialization in different categories.

$\lambda^* < 0$

This case corresponds to Part b.1. We prove the following statements:

Statement 1: $(\lambda^{GN}, \lambda^{GN})$ is an equilibrium for any $\lambda^* < 0$.

Statement 2: There is no other equilibrium for these values of λ^* .

Proof of Statement 1: According to Lemma 1, Rater 2 receives the minimum of the stand-alone value of its ratings and their marginal value. With this observation, we only need to show: (1) λ^{GN} continues to uniquely maximize the stand-alone value when $\lambda^* < 0$. In the generalization outcome, Rater 2 obtains $V(\mathbf{O}, \lambda^{GN})$, which exceeds its payoff from other choices of rating technology, as they have a lower stand-alone value²². (2) Rater 1's best response to λ^{GN} is also λ^{GN} .

The stand-alone value of rating technology $\lambda_2 = (\lambda_2^{A_H}, \lambda_2^{A_L}, \lambda_2^{B_H}, \lambda_2^{B_L})$ can be obtained from equation OA.44:

$$v(\mathbf{O}, \lambda_2) = \lambda_2^{A_H} \mathbb{I}\{\lambda_2^{A_L} = 1\} [\lambda_2^{B_H} + (\lambda^* - 1)(1 - \lambda_2^{B_L})]^+ + \lambda_2^{B_H} \mathbb{I}\{\lambda_2^{B_L} = 1\} [\lambda_2^{A_H} + (\lambda^* - 1)(1 - \lambda_2^{A_L})]^+ - \lambda_2^{A_H} \lambda_2^{B_H} \mathbb{I}\{\lambda_2^{A_L} = 1\} \mathbb{I}\{\lambda_2^{B_L} = 1\}. \quad (\text{OA.47})$$

According to equation OA.47, the stand-alone value would be zero if both $\lambda_2^{A_L}$ and $\lambda_2^{B_L}$ are less than one. This is intuitive since if $\lambda_2^{A_L}, \lambda_2^{B_L} < 1$ and the investor observes a high rating in both categories from Rater 2 (i.e., $s_2^A = s_2^B = h$), there is still a positive probability that the project has low performance in both categories. Hence, the investor never invests if she only purchases Rater 2's rating technology, resulting in a payoff of zero. Therefore, it is suboptimal for Rater 2 to choose a rating technology with $\lambda_2^{A_L}, \lambda_2^{B_L} < 1$, as it would lead to a payoff of zero.

If $\lambda_2^{A_L} = 1$ and $\lambda_2^{B_L} < 1$, it is straightforward to employ equation OA.47 to show that the stand-alone value of $\lambda_2 = (\lambda_2^{A_H}, 1, \lambda_2^{B_H}, \lambda_2^{B_L})$ is strictly less than that of $\lambda'_2 = (\lambda_2^{A_H}, 1, \lambda_2^{B_H} + \lambda_2^{B_L} - 1, 1)$. We earlier proved that generalization has the highest stand-alone value among rating technologies with $\lambda_2^{A_L} = \lambda_2^{B_L} = 1$ (i.e., the set of rating technologies with no false-positive error). Therefore, generalization maximizes the stand-alone value under the more general information structure when $\lambda^* < 0$, which completes step (1).

Step (2) is to prove that Rater 1's best response to generalization is generalization. To this end, we only need to show that it is suboptimal to set $\lambda_1^{A_L} < 1$ or $\lambda_1^{B_L} < 1$ when $\lambda_2 = \lambda^{GN}$ and $\lambda^* < 0$; because, Proposition 1 implies that generalization is the best response in the set of rating technologies with no false-positive error, i.e., $\lambda_1^{A_L} = \lambda_1^{B_L} = 1$.

Specifically, define $\lambda'_1 = (\lambda_1^{A_H} + \lambda_1^{A_L} - 1, 1, \lambda_1^{B_H} + \lambda_1^{B_L} - 1, 1)$. We prove that $V(\lambda'_1, \lambda^{GN}) > V(\lambda_1, \lambda^{GN})$ if $\lambda_1^{A_L} < 1$ or $\lambda_1^{B_L} < 1$. Consider the case that both $\lambda_1^{A_L} < 1$ and $\lambda_1^{B_L} < 1$ hold.

²²Note that we earlier showed that generalization is complement with itself when $\lambda^* \leq \frac{17}{32}$.

The proof strategy for the other possibilities is similar:

$$\begin{aligned}
v(\lambda_1, \lambda^{GN}) &= \frac{1}{2} \left[\frac{1}{2} + \underbrace{\left\{ \frac{1}{2} \lambda_1^{B_H} + (\lambda^* - 1)(1 - \lambda_1^{B_L}) \right\}^+}_{< \frac{1}{2}(\lambda_1^{B_H} + \lambda_1^{B_L} - 1)} + \frac{1}{2} + \underbrace{\left\{ \frac{1}{2} \lambda_1^{A_H} + (\lambda^* - 1)(1 - \lambda_1^{A_L}) \right\}^+}_{< \frac{1}{2}(\lambda_1^{A_H} + \lambda_1^{A_L} - 1)} \right] - \frac{1}{4} \\
&< \left(\frac{1}{2} + \frac{1}{2}(\lambda_1^{B_H} + \lambda_1^{B_L} - 1) \right) \left(\frac{1}{2} + \frac{1}{2}(\lambda_1^{A_H} + \lambda_1^{A_L} - 1) \right) = v(\lambda'_1, \lambda^{GN})
\end{aligned} \tag{OA.48}$$

As such, any rating technology with $\lambda_1^{A_L} < 1$ or $\lambda_1^{B_L} < 1$ is dominated by a rating technology with $\lambda_1^{A_L} = \lambda_1^{B_L} = 1$, which is the set of available rating technologies in the baseline case. Therefore, generalization is the unique best response for Rater 1 in response to $\lambda_2 = \lambda^{GN}$.

Proof of Statement 2:

Recall that in the baseline model, where we consider the restricted case with $\lambda_j^{A_L} = \lambda_j^{B_L} = 1$, $j = 1, 2$, Rater 2's best response to any rating technology is to generalize. The intuition is that any two rating technologies with no false-positive error are complements, and generalization yields the highest stand-alone value for Rater 2. However, this argument does not work in the more general case considered here since λ^{GN} is substitute with some rating technologies. To prove that $(\lambda^{GN}, \lambda^{GN})$ remains the unique equilibrium in pure strategies, we rule out other possibilities of (λ_1, λ_2) in several steps. In particular, we divide the cases based on the values of $\lambda_1^{A_L}, \lambda_1^{B_L}, \lambda_2^{A_L}, \lambda_2^{B_L}$.

$\lambda_1^{A_L}, \lambda_1^{B_L} < 1$: The combined value in this case is,

$$v(\lambda_1, \lambda_2) = \lambda_2^{A_H} \mathbb{I}\{\lambda_2^{A_L} = 1\} M^B + \lambda_2^{B_H} \mathbb{I}\{\lambda_2^{B_L} = 1\} M^A - \lambda_2^{A_H} \lambda_2^{B_H} \mathbb{I}\{\lambda_2^{A_L} = 1\} \mathbb{I}\{\lambda_2^{B_L} = 1\}, \tag{OA.49}$$

where M^A and M^B are defined in (OA.45). If $\lambda_2^{A_L} < 1$, then Rater 1's rating for category B has no impact on the investor's decision, and consequently, the combined value. Therefore, specializing in category A would increase the combined value, which generates no false-positive error in category A . A similar argument applies when $\lambda_2^{B_L} < 1$. If $\lambda_2^{A_L} = \lambda_2^{B_L} = 1$, then one can show that the combined value increases if Rater 1 switches to $\lambda'_1 = (\lambda_1^{A_H} + \lambda_1^{A_L} - 1, 1, \lambda_1^{B_H} + \lambda_1^{B_L} - 1, 1)$. Therefore, there is no equilibrium in which $\lambda_1^{A_L}, \lambda_1^{B_L} < 1$.

$\lambda_1^{A_L}, \lambda_1^{B_L} = 1$: In this case, λ_1 is complement with λ^{GN} . Therefore, Rater 2 can obtain $\phi_2 = V(\mathcal{O}, \lambda^{GN})$ by generalizing, which is the largest possible payoff. As such, λ^{GN} is Rater 2's best response, meaning that the only possible pure strategy equilibrium in this case is $(\lambda^{GN}, \lambda^{GN})$.

$\lambda_1^{A_L} = 1, \lambda_1^{B_L} < 1$: First, we show that we should have $\lambda_2^{A_L} = 1$. If $\lambda_2^{A_L}, \lambda_2^{B_L} < 1$, then the stand-alone value of λ_2 is zero, which means that the rater can increase its payoff by

switching to another rating technology with a positive marginal value and the stand-alone value (such as generalization).

If $\lambda_2^{A_L} < 1, \lambda_2^{B_L} = 1$, then the combined value is:

$$\begin{aligned} v(\lambda_1, \lambda_2) = & \lambda_1^{A_H} \lambda_2^{B_H} + \lambda_1^{A_H} [\lambda_1^{B_H} (1 - \lambda_2^{B_H}) + (\lambda^* - 1)(1 - \lambda_1^{B_L})]^+ \\ & + \lambda_2^{B_H} [\lambda_2^{A_H} (1 - \lambda_1^{A_H}) + (\lambda^* - 1)(1 - \lambda_2^{A_L})]^+. \end{aligned} \quad (\text{OA.50})$$

Rater 2 can increase both the combined value and stand-alone value of its ratings by switching to $\lambda'_2 = (\lambda_2^{A_H} + \lambda_2^{A_L} - 1, 1, \lambda_2^{B_H}, 1)$. To see this, take the derivative of (OA.50) with respect to $\lambda_2^{A_H}$ and $\lambda_2^{A_L}$, and note that the latter is larger than the former since $1 - \lambda^* > 1 \geq 1 - \lambda_1^{A_H}$.

The only remaining possibility is $\lambda_2^{A_L} = 1$. It implies that a high rating in category A , either from Rater 1 or 2, is sufficient to ensure that $w^A = H$. In this case, the combined value is:

$$v(\lambda_1, \lambda_2) = (\lambda_1^{A_H} + \lambda_2^{A_H} - \lambda_1^{A_H} \lambda_2^{A_H}) M^B, \quad (\text{OA.51})$$

where

$$\begin{aligned} M^B = \max\{ & \lambda_2^{B_H} \lambda_1^{B_H} + (\lambda^* - 1)(1 - \lambda_2^{B_L})(1 - \lambda_1^{B_L}), \lambda_2^{B_H} + (\lambda^* - 1)(1 - \lambda_2^{B_L}), \\ & \lambda_1^{B_H} + (\lambda^* - 1)(1 - \lambda_1^{B_L}), \lambda_2^{B_H} + \lambda_1^{B_H} - \lambda_2^{B_H} \lambda_1^{B_H} + (1 - \lambda^*)(1 - \lambda_2^{B_L} \lambda_1^{B_L}) \} \end{aligned} \quad (\text{OA.52})$$

The four items above reflect the four possibilities in the investment rule: The first item corresponds to the case that the investment requires a high rating in category B from both raters. The second (third) item represents the possibility that only the rating of Rater 2 (1) in category B is used for the investment. The last item corresponds to the possibility that a single high rating in category B is enough for the investment, in addition to receiving a high rating in category A . If M^B is equal to the second, third, or last item, then Rater 2 can increase the stand-alone and the combined value, by switching to $\lambda_2 = (\lambda_2^{A_H}, 1, \lambda_2^{B_H} + \lambda_2^{B_L} - 1, 1)$. Therefore, if (λ_1, λ_2) forms an equilibrium and $\lambda_1^{A_L} = \lambda_2^{A_L} = 1$, then M^B should be equal to the first item.²³

With this observation, we can write the combined value as below:

$$v(\lambda_1, \lambda_2) = (\lambda_1^{A_H} + \lambda_2^{A_H} - \lambda_1^{A_H} \lambda_2^{A_H})(\lambda_2^{B_H} \lambda_1^{B_H} + (\lambda^* - 1)(1 - \lambda_2^{B_L})(1 - \lambda_1^{B_L})). \quad (\text{OA.53})$$

Note that Rater 1 chooses λ_1 to maximize the combined value. The first-order conditions with respect to $\lambda_1^{B_H}$ and $\lambda_1^{B_L}$ reveal that we should have $\lambda_1^{B_H} = 1$. This is because λ_2 has a positive stand-alone value, which implies that $\lambda_2^{B_H} > (1 - \lambda^*)(1 - \lambda_2^{B_L})$. Therefore,

²³Here, we use the result from Lemma A.2.

$\lambda_1 = (a, 1, 1, 1 - a)$, for some $a \in [0, 1]$.

Furthermore, $a < \frac{1}{1-\lambda^*}$. This is because the first-order conditions for Rater 2 imply that we should have $\lambda_2^{B_L} = 1$ if $a \geq \frac{1}{1-\lambda^*}$. This implies that Rater 1's best response is to specialize in category A , i.e., $a = 1$,²⁴ which is ruled out earlier.

If λ_1 and λ_2 are substitutes, then by examining the first-order conditions with respect to $\lambda_2^{B_H}$ and $\lambda_2^{B_L}$, we find that we should have $\lambda_2^{B_H} = 1$ since we just showed that $1 > (1 - \lambda^*)a$. This would imply $\lambda_2 = (b, 1, 1, 1 - b)$ for some $b \in [0, 1]$. However, in the inequalities below, we show that rating technologies of this form are all complements:

$$\begin{aligned}
& v((a, 1, 1, 1 - a), (b, 1, 1, 1 - b)) - v((a, 1, 1, 1 - a), \mathbf{O}) - v(\mathbf{O}, (b, 1, 1, 1 - b)) \\
&= (a + b - ab)(1 + (\lambda^* - 1)ab) - a(1 + (\lambda^* - 1)a) - b(1 + (\lambda^* - 1)b) \\
&= -ab + (1 - \lambda^*)(a^2 + b^2 - ab(a + b - ab)) \\
&\geq a^2 + b^2 - ab(1 + a + b - ab) = a^2 + b^2 - ab(2 - (1 - a)(1 - b)) \geq a^2 + b^2 - ab \geq 0.
\end{aligned} \tag{OA.54}$$

Therefore, $\lambda_1 = (a, 1, 1, 1 - a)$ and $\lambda_2 = (\lambda_2^{A_H}, 1, \lambda_2^{B_H}, \lambda_2^{B_L})$ should be complements. The combined value is:

$$v(\lambda_1, \lambda_2) = (a + \lambda_2^{A_H} - a\lambda_2^{A_H})(\lambda_2^{B_H} + (\lambda^* - 1)a(1 - \lambda_2^{B_L})). \tag{OA.55}$$

Furthermore, we should have $\lambda_2^{B_L} < 1$, as otherwise, $a = 1$, namely specialization in category A would be optimal, to which generalization is Rater 2's best response. However, one can show that Rater 2 can increase the stand-alone value of its ratings by changing λ_2^H and λ_2^L to $\lambda_2^H - \varepsilon$ and $\lambda_2^L + \varepsilon$, for some sufficiently small $\varepsilon > 0$.²⁵ This perturbation is only feasible when λ_1 and λ_2 are on the border between the complements and substitutes regions. In other words, the sum of the stand-alone values should be equal to the combined value:

$$\begin{aligned}
v(\lambda_1, \lambda_2) &= v(\lambda_1, \mathbf{O}) + v(\mathbf{O}, \lambda_2) \Rightarrow (a + \lambda_2^{A_H} - a\lambda_2^{A_H})(\lambda_2^{B_H} + (\lambda^* - 1)a(1 - \lambda_2^{B_L})) \\
&= a(1 + (\lambda^* - 1)a) + \lambda_2^{A_H}(\lambda_2^{B_H} + (\lambda^* - 1)(1 - \lambda_2^{B_L})).
\end{aligned} \tag{OA.56}$$

Note that Rater 1 chooses λ_1 to maximize the combined value given λ_2 . Therefore, the

²⁴Note that both rating technologies $(1, 1, 1, 0)$ and $(1, 1, 0, 1)$ represent specialization in category A .

²⁵Note that $\lambda_2^{B_H} > 0$, as otherwise, the stand-alone value of λ_2 would be zero.

combined value should not increase by switching to λ^{SP_B} :

$$\begin{aligned} V(\lambda^{SP_B}, \lambda_2) &\leq V(\lambda_1, \lambda_2) \Rightarrow \lambda_2^{A_H} \leq a(1 + (\lambda^* - 1)a) + \lambda_2^{A_H}(\lambda_2^{B_H} + (\lambda^* - 1)(1 - \lambda_2^{B_L})) \\ &\Rightarrow \lambda_2^{A_H}(1 - \lambda_2^{B_H} + (1 - \lambda^*)(1 - \lambda_2^{B_L})) \leq a(1 + (\lambda^* - 1)a). \end{aligned} \quad (\text{OA.57})$$

Note that Constraint (OA.2), along with $\lambda_2^{A_L} = 1$, implies that:

$$\lambda_2^{A_H} \leq 2 - \lambda_2^{B_H} - \lambda_2^{B_L} \leq 1 - \lambda_2^{B_H} + (1 - \lambda^*)(1 - \lambda_2^{B_L}). \quad (\text{OA.58})$$

Moreover, note that

$$a(1 + (\lambda^* - 1)a) \leq \frac{1}{4(1 - \lambda^*)}, \quad (\text{OA.59})$$

where the upper bound is attained at $a = \frac{1}{2(1 - \lambda^*)}$. Therefore, by combining inequalities OA.57-OA.59, we find:

$$\lambda_2^{A_H} \leq \sqrt{\frac{1}{4(1 - \lambda^*)}} < \frac{1}{2}. \quad (\text{OA.60})$$

Likewise, the combined value should not increase if Rater 1 switches to λ^{SP_A} :

$$\begin{aligned} V(\lambda^{SP_A}, \lambda_2) &\leq V(\lambda_1, \lambda_2) \\ \Rightarrow \lambda_2^{B_H} + (\lambda^* - 1)(1 - \lambda_2^{B_L}) &\leq a(1 + (\lambda^* - 1)a) + \lambda_2^{A_H}(\lambda_2^{B_H} + (\lambda^* - 1)(1 - \lambda_2^{B_L})) \quad (\text{OA.61}) \\ \Rightarrow \lambda_2^{B_H} + (\lambda^* - 1)(1 - \lambda_2^{B_L}) &\leq \frac{1}{1 - \lambda_2^{A_H}} a(1 + (\lambda^* - 1)a) \leq \frac{1}{2(1 - \lambda^*)}. \end{aligned}$$

Therefore, by combining the last two inequalities, we find:

$$\hat{\phi}_2 \leq v(\mathbf{O}, \lambda_2) = \lambda_2^{A_H}(\lambda_2^{B_H} + (\lambda^* - 1)(1 - \lambda_2^{B_L})) < \frac{1}{4(1 - \lambda^*)}. \quad (\text{OA.62})$$

However, it is a contradiction since Rater 2 can increase its payoff by switching to $\lambda'_2 = (\frac{1}{2(1 - \lambda^*)}, 1, 1, 1 - \frac{1}{2(1 - \lambda^*)})$. Because, it achieves a stand-alone value of $\frac{1}{4(1 - \lambda^*)}$, and it is complement with $\lambda_1 = (a, 1, 1, 1 - a)$, as demonstrated in (OA.54).

As such, there is no equilibrium in which $\lambda_1^{A_L} = 1, \lambda_1^{B_L} < 1$. Likewise, we can rule out the possibility that $\lambda_1^{A_L} < 1, \lambda_1^{B_L} = 1$. It completes the proof of the second statement.

$\lambda^* \in [0, 1)$

This case corresponds to Parts b.2 and b.3 of the proposition. We examine the robust equilibrium outcomes by dividing the cases based on $\lambda_2^{A_L}, \lambda_2^{B_L}, \lambda_1^{A_L}, \lambda_1^{B_L}$. Specifically, we

examine which pairs of (λ_1, λ_2) form a robust equilibrium under, namely the pair is an equilibrium in a neighborhood of λ^* .

Note that the stand-alone value of λ_2 should be positive so Rater 2 obtains a positive fee. Therefore, $\lambda_2^{A_L} \geq 1$ or $\lambda_2^{B_L} \geq 1$.

$\lambda_2^{A_L} = \lambda_2^{B_L} = 1$: In this case, λ_2 generates no false-positive error, and belongs to the baseline set of feasible rating technologies. However, λ_1 can be chosen from the general set. By analyzing the first-order conditions, one can show that if $\lambda_2^{A_H} > \lambda^*$, then $\lambda_1^{A_L} = 1$. Moreover, if $\lambda_2^{A_H} < \lambda^*$, then $\lambda_1^{A_H} = 1$.²⁶ Likewise, either $\lambda_1^{B_H}$ or $\lambda_1^{B_L}$ is equal to one depending on whether $\lambda_2^{B_H}$ is smaller or bigger than λ^* .

Therefore, λ_1 has to take one of the following three forms:

- $\lambda_1 = (1, x, 1, 1 - x)$ for some $x \in (0, 1)$: The combined value in this case is

$$v(\lambda_1, \lambda_2) = \lambda_2^{A_H} \lambda_2^{B_H} + \lambda_2^{A_H} [1 - \lambda_2^{B_H} + (\lambda^* - 1)x]^+ + \lambda_2^{B_H} [1 - \lambda_2^{A_H} + (\lambda^* - 1)x]^+. \quad (\text{OA.63})$$

If $1 - \lambda_2^{B_H} + (\lambda^* - 1)x \leq 0$, then the realization of s_1^B has no impact on the investor's decision. As a result, Rater 1 could increase its payoff by specializing in category A , which is against the assumption about λ_1 . Therefore, $1 - \lambda_2^{B_H} + (\lambda^* - 1)x > 0$. With the same logic, the last term in equation OA.63 should also be positive.

By analyzing the first-order conditions, we find that the only possibility is that $x = \lambda_2^{A_H} = \lambda_2^{B_H} = 0.5$. Therefore, $v(\lambda_1, \lambda^{GN}) = 0.25 + 0.5\lambda^*$. However, it is less than $v(\lambda^{SP}, \lambda^{GN})$:

$$v(\lambda^{SP}, \lambda^{GN}) = \max\{0.5, \lambda^*\} \geq 0.5(0.5 + \lambda^*) = v(\lambda_1, \lambda^{GN}). \quad (\text{OA.64})$$

Equality is achieved only for $\lambda^* = 0.5$. Therefore, there cannot be a robust equilibrium in this scenario.

- $\lambda_1 = (x, 1, 1, 1 - x)$ for some $x \in (0, 1)$:²⁷ The combined value is

$$v(\lambda_1, \lambda_2) = (x + \lambda_2^{A_H} - x\lambda_2^{A_H})(\lambda_2^{B_H} + [1 - \lambda_2^{B_H} + (\lambda^* - 1)x]^+). \quad (\text{OA.65})$$

If $1 - \lambda_2^{B_H} + (\lambda^* - 1)x \leq 0$, then s_1^B has no impact on the investor's decision since the investor would invest only when $s_2^B = h$ and either s_1^A or s_2^A are also h . This implies

²⁶We do not analyze the knife-edge case that $\lambda_2^{A_H}$ or $\lambda_2^{B_H}$ is λ^* since the outcome cannot be a robust equilibrium.

²⁷The extreme cases (i.e., $x = 0, 1$) correspond to specialization in one of the categories, which are analyzed in the next case.

that λ_1 is a suboptimal choice. If $1 - \lambda_2^{B_H} + (\lambda^* - 1)x > 0$, then s_2^B is redundant since $M^B = 1 + (\lambda^* - 1)x$, meaning that the investor invests when $s_1^B = h$ and either s_1^A or s_2^A is h . It is straightforward to show that Rater 2 can increase both stand-alone value and combined value by switching to λ^{SPA} . As such, there is no equilibrium in this scenario.

- $\lambda_1 = (x, 1, 1 - x, 1)$ for some $x \in [0, 1]$: In this case, λ_1 also belongs to the set of rating technologies in the baseline model. In Proposition 1, we show that the only possible equilibria in pure strategies are generalization by both raters and specialization in different categories. Therefore, we only need to examine if those outcomes remain in equilibrium when we expand the set of rating technologies available to the raters.

- The generalization outcome $(\lambda^{GN}, \lambda^{GN})$: We show that the generalization outcome is not an equilibrium for any value of $\lambda^* \in (0, 1)$. To demonstrate this point, we show that (1) λ^{GN} is not Rater 1's best response to λ^{GN} when $\lambda^* > 0.5$. And, (2) λ^{GN} is not Rater 2's best response to λ^{GN} when $\lambda^* \leq 0.5$.

Define $\lambda^{(x)} = (x, 1, 1, 1 - x)$. We show that when $\lambda^* \in (0.5, 1)$, $v(\lambda^{(x)}, \lambda^{GN}) > v(\lambda^{GN}, \lambda^{GN})$ for some $x \in (0, 0.5)$, which proves (1) since Rater 1 effectively maximizes the combined value.

$$v(\lambda^{(x)}, \lambda^{GN}) = 0.5(1 + x)(1 + (\lambda^* - 1)x), \quad x < 0.5. \quad (\text{OA.66})$$

For $\hat{x} = \frac{\lambda^*}{2(1-\lambda^*)}$, we have:

$$v(\lambda^{(\hat{x})}, \lambda^{GN}) = 0.5 + \frac{\lambda^{*2}}{8(1-\lambda^*)} > \frac{9}{16} = v(\lambda^{GN}, \lambda^{GN}) \quad \lambda^* > 0.5. \quad (\text{OA.67})$$

It proves (1).

To prove (2), we note that λ^{GN} is complement with itself when $\lambda^* \leq \frac{17}{32}$. In fact, any two rating technologies are complements in a neighborhood of $(\lambda^{GN}, \lambda^{GN})$ when $\lambda^* \leq 0.5 < \frac{17}{32}$. Therefore, λ^{GN} is complement with $\lambda'_2 = (\frac{1}{2}, 1, \frac{1}{2} + \varepsilon, 1 - \varepsilon)$ for a sufficiently small value of $\varepsilon > 0$. λ'_2 generates a larger stand-alone value:

$$v(\mathbf{O}, \lambda'_2) = \frac{1}{2}(\frac{1}{2} + \varepsilon + (\lambda^* - 1)\varepsilon) = 0.25 + 0.5\lambda^* = 0.25 > v(\mathbf{O}, \lambda^{GN}). \quad (\text{OA.68})$$

As such, the generalization outcome is not an equilibrium when $\lambda^* \in (0, 1)$, whereas it was an equilibrium in the baseline case for some values of λ^* in this

range.

- Specialization in different categories $(\lambda^{SP_A}, \lambda^{SP_B}), (\lambda^{SP_B}, \lambda^{SP_A})$: We show that **(1)** this outcome is not an equilibrium when $\lambda^* \in [0, 0.5)$, and **(2)** this outcome remains an equilibrium when $\lambda^* \in [0.5, 1]$.

To prove (1), note that λ^{SP_A} and λ^{SP_B} are complements when $\lambda^* \in [0, 0.5)$:

$$v(\lambda^{SP_A}, \mathbf{O}) + v(\mathbf{O}, \lambda^{SP_B}) = 2\lambda^* < 1 = v(\lambda^{SP_A}, \lambda^{SP_B}). \quad (\text{OA.69})$$

Therefore, rating technologies are complements in a neighborhood of $(\lambda^{SP_A}, \lambda^{SP_B})$. Let $\varepsilon > 0$ be sufficiently small such that $\lambda^{(1-\varepsilon)} = (1 - \varepsilon, 1, 1, \varepsilon)$ and λ^{SP_B} are complements. The inequality below demonstrates that the stand-alone value of $\lambda^{(1-\varepsilon)}$ exceeds that of specialization:

$$\begin{aligned} v(\mathbf{O}, \lambda^{(1-\varepsilon)}) &= (1 - \varepsilon)(1 + (\lambda^* - 1)(1 - \varepsilon)) \\ &= \lambda^* + (1 - 2\lambda^*)\varepsilon(1 - \frac{1 - \lambda^*}{1 - 2\lambda^*}\varepsilon) > \lambda^* = v(\mathbf{O}, \lambda^{SP_A}). \end{aligned} \quad (\text{OA.70})$$

Therefore, λ^{SP_A} is not Rater 2's best response to λ^{SP_B} .

To prove (2), we show that specialization in a category maximizes the stand-alone value. From Proposition 2, we know that for any value of λ^* , specializing in category A (B) obtains the highest combined value when the other rater specializes in category B (A). Therefore, according to lemma A.2. Each rater's best response to specialization is to specialize in the other category.

To prove that specialization obtains the largest stand-alone value, we need to show this choice maximizes the following objective function (assuming $\lambda_2^{A_L} = 1$ to ensure a positive stand-alone value):

$$\max_{\lambda_2^{A_H} + \lambda_2^{B_H} + \lambda_2^{B_L} \leq 2} v(\mathbf{O}, \lambda_2) = \lambda_2^{A_H} \max\{\lambda^*, \lambda_2^{B_H} + (\lambda^* - 1)(1 - \lambda_2^{B_L})\}. \quad (\text{OA.71})$$

Since $\lambda^* \in (0, 1]$, $\lambda_2^{B_H} = 1$, as otherwise, the stand-alone value could be increased by slightly increasing $\lambda_2^{B_H}$ and decreasing $\lambda_2^{B_L}$ by the same amount. Therefore, the rating technology with the largest stand-alone value is $\lambda^{(x)} = (x, 1, 1, 1 - x)$ for some $x \in [0, 1]$. In fact, x should maximize $x(1 + (\lambda^* - 1)x)$. The maximizing value of x is one for $\lambda^* \in [0, 1]$ and it is $x^* = \frac{1}{2(1-\lambda^*)}$ for $\lambda^* \in (0, 0.5)$. Since $x = 1$ corresponds to λ^{SP_A} , we see that specialization obtains the highest stand-alone value. Hence, specialization in different categories is an equilibrium when

$$\lambda^* \in [0.5, 1].$$

$\lambda_2^{A_L} = 1$ and $\lambda_2^{B_L} < 1$.²⁸ First, we show that it is not possible to have $\lambda_1^{A_L} < 1$ in equilibrium. If $\lambda_1^{B_L} < 1$ too, then s_1^A has no impact on the investor's decision, implying that it cannot happen in equilibrium since Rater 1's payoff would increase by switching to λ^{SP_B} . If $\lambda_1^{B_L} = 1$, the combined value is:

$$\begin{aligned} v(\lambda_1, \lambda_2) = & \lambda_2^{A_H} \lambda_1^{B_H} + \lambda_2^{A_H} [\lambda_2^{B_H} (1 - \lambda_1^{B_H}) + (\lambda^* - 1)(1 - \lambda_2^{B_L})]^+ \\ & + \lambda_2^{A_H} [(1 - \lambda_2^{B_H})(1 - \lambda_1^{B_H}) + (\lambda^* - 1)\lambda_2^{B_L}]^+ + \lambda_1^{B_H} [\lambda_1^{A_H} (1 - \lambda_2^{A_H}) + (\lambda^* - 1)(1 - \lambda_1^{A_L})]^+ \\ & + \lambda_1^{B_H} [(1 - \lambda_1^{A_H})(1 - \lambda_2^{A_H}) + (\lambda^* - 1)\lambda_1^{A_L}]^+. \end{aligned} \quad (\text{OA.72})$$

In equation OA.72, the terms correspond to the investor's value from signal realizations $(s_2^A = h, s_1^B = h), (s_2^A = h, s_1^B = l, s_2^B = h), (s_2^A = h, s_1^B = l, s_2^B = l), (s_2^A = l, s_1^A = h, s_1^B = h)$, and $(s_2^A = l, s_1^A = l, s_1^B = h)$, respectively. If the third or fifth term is positive, then signal realizations in categories B and A , respectively, have no impact on the investor's decision, which cannot happen in equilibrium since Rater 1 could increase the combined value by specializing in one of the categories. Moreover, the fourth term should be positive, as otherwise, the realization of s_1^A has no impact on the investor's decision. Moreover, if $\lambda_2^{A_H} > \lambda^*$, Rater 1 can increase its payoff by switching to $\lambda_1' = (\lambda_1^{A_H} + \lambda_1^{A_L} - 1, 1, \lambda_1^{B_H}, 1)$, which would violate the assumption that $\lambda_1^{A_L} < 1$. Therefore, we should have $\lambda_2^{A_H} < \lambda^*$, which implies that $\lambda_1^{A_H} = 1$. That is, $\lambda_1 = (1, 1 - y, y, 1)$ for some $y \in [0, 1]$. Thus, the combined value can be rewritten as below:

$$v(\lambda_1, \lambda_2) = y[1 + (\lambda^* - 1)y] + \lambda_2^{A_H} [\lambda_2^{B_H} (1 - y) + (\lambda^* - 1)(1 - \lambda_2^{B_L})]^+. \quad (\text{OA.73})$$

The first term in equation OA.73 represents the stand-alone value of λ_1 , and the second term is less than the stand-alone value of λ_2 . Therefore, two rating technologies are substitutes. As a result, λ_2 should maximize the combined value given λ_1 . The first-order conditions imply that if $y > \lambda^*$, then $\lambda_2^{B_L} = 1$, which contradicts our original assumption. Therefore, $y < \lambda^*$, which further implies $\lambda_2^{B_H} = 1$. As a result, the only possibility is that $\lambda_2 = (x, 1, 1, 1 - x)$ for some $x \in [0, 1]$. The combined value is:

$$x + y - xy + (\lambda^* - 1)(x^2 + y^2). \quad (\text{OA.74})$$

The first-order conditions with respect to x and y imply that $x = y = \frac{1}{1+2(1-\lambda^*)}$. This

²⁸The argument is similar for $\lambda_2^{A_L} < 1$ and $\lambda_2^{B_L} = 1$

implies a combined value of $v(\lambda_1, \lambda_2) = \frac{1}{1+2(1-\lambda^*)} = y$, which is less than λ^* , as shown earlier. Therefore, Rater 1 can increase the combined value by specializing in a category, meaning that λ_1 and λ_2 cannot form an equilibrium. Therefore, there is no equilibrium in which $\lambda_2^{A_L} = \lambda_1^{B_L} = 1$ and $\lambda_2^{B_L}, \lambda_1^{A_L} < 1$.

Lastly, we analyze the possibility that $\lambda_1^{A_L} = 1$ and $\lambda_1^{B_L} \leq 1$. The combined value is:

$$v(\lambda_1, \lambda_2) = \lambda^A M^B, \quad (\text{OA.75})$$

where $\lambda^A = \lambda_1^{A_H} + \lambda_2^{A_H} - \lambda_1^{A_H} \lambda_1^{B_H}$, and M^B is defined in equation OA.45. By examining the first-order conditions, we find that we should have either $\lambda_1^{B_H} = 1$ or $\lambda_1^{B_L} = 1$ for a robust equilibrium. Moreover, since a robust equilibrium cannot be on the borderline of the set of complement rating technology pairs for all values in a neighborhood of λ^* , λ_2 should maximize the combined value or stand-alone value in a neighborhood of (λ_1, λ_2) , depending on whether λ_1 and λ_2 are substitutes or complements.

First, we rule out the possibility that the rating technologies are complements: For $\lambda^* \in [0.5, 1]$, λ^{SP_A} and λ^{SP_B} maximize the stand-alone value. For $\lambda^* < 0.5$, the rating technology that maximizes the stand-alone value depends on λ^* , so it cannot be part of a robust equilibrium. Therefore, λ_1 and λ_2 should be substitutes.

By analyzing the first-order conditions, we can show that there are two possibilities: either $\lambda_2^{B_H} = 1$ or $\lambda_2^{B_L} = 1$, where the latter is ruled out by the case assumption, as analyzed earlier. Similarly, one can show either $\lambda_1^{B_H} = 1$ or $\lambda_1^{B_L} = 1$. Therefore, we only need to examine the following possibilities:

- $\lambda_1 = (y, 1, 1 - y, 1)$, $\lambda_2 = (x, 1, 1, 1 - x)$, and λ_1 and λ_2 are substitutes: x and y should maximize the combined value, which is:

$$v(\lambda_1, \lambda_2) = (x + y - xy)(1 - y + [y + (\lambda^* - 1)x]^+). \quad (\text{OA.76})$$

If the term inside the bracket is positive, we should have $y = 1$, and consequently $\lambda_1 = \lambda^{SP_A}$, which is analyzed earlier.

- $\lambda_1 = (y, 1, 1, 1 - y)$, $\lambda_2 = (x, 1, 1, 1 - x)$, and λ_1 and λ_2 are substitutes: The combined value is

$$v(\lambda_1, \lambda_2) = (x + y - xy)(1 + (\lambda^* - 1)xy). \quad (\text{OA.77})$$

By examining the first-order conditions, we find that $x = y$ and they depend on λ^* . Thus, there is no robust equilibrium under this possibility.

OA.3.7 Proof of Proposition OA.2 (Socially optimal information design)

Similar to Lemma A.1, one can show that the social value function can be written as below:

$$\begin{aligned} W(\lambda_1, \lambda_2) &= \eta^2(\Delta + \beta^{SP}V^{HH})w(\lambda^A, \lambda^B), \\ w(\lambda_1, \lambda_2) &= \lambda^A\lambda^B + \lambda^A[\lambda_{SP}^* - \lambda^B]\mathbb{I}\{\lambda^* \geq \lambda^B\} + \lambda^B[\lambda_{SP}^* - \lambda^A]\mathbb{I}\{\lambda^* \geq \lambda^A\}, \end{aligned} \quad (\text{OA.78})$$

where λ^* is defined in (A.4), and λ_{SP}^* is defined similarly for β^{SP} :

$$\eta(1 - \lambda_{SP}^*)(\Delta + \beta^{SP}V^{HH}) + (1 - \eta)(\Delta + \beta^{SP}V^{HL}) = 0. \quad (\text{OA.79})$$

From (OA.78), we can see that for a socially optimal design, (λ^A, λ^B) should maximize $w(\lambda^A, \lambda^B)$.

a) In this case, $\Delta + \beta^{SP}V^{HL}$ and $\Delta + \beta V^{HL}$ have the same signs; that is, they both agree that only projects with type (H, H) should receive investment. Therefore, it is trivial that the social planner can implement the socially optimal investment decision by perfectly disclosing the project's type, which is achieved when the raters specialize in different categories.

b) From (A.4) and (OA.79), we can see that $\lambda^* > 1 > \lambda_{SP}^*$. In particular, $\lambda_{SP}^* \in [0, 1)$ when $\Delta + \beta^{SP}\{\eta V^{HH} + (1 - \eta)V^{HL}\} \geq 0$ (Part b.1), and $\lambda_{SP}^* < 0$ otherwise (Part b.2). Since $\lambda^A, \lambda^B \leq 1$, we can write $w(\lambda^A, \lambda^B)$ in (OA.78) as below:

$$w(\lambda^A, \lambda^B) = \lambda_{SP}^*(\lambda^A + \lambda^B) - \lambda^A\lambda^B. \quad (\text{OA.80})$$

When $\lambda_{SP}^* < 0$, it is clear that $w(\lambda^A, \lambda^B) \leq 0$ and the equality is attained only when $\lambda^A = \lambda^B = 0$. It proves the statement in Part (b.2).

Now, we consider the case that $\lambda_{SP}^* \in [0, 1)$, corresponding to Part (b.1). Note that when the raters specialize in the same category, we have $\lambda^A = 1, \lambda^B = 0$ or $\lambda^A = 0, \lambda^B = 1$, which results in $w = \lambda_{SP}^*$. The inequalities below show that $\lambda_{SP}^* \geq w(\lambda^A, \lambda^B)$ for any λ^A and λ^B in this case:

$$\begin{aligned} \lambda^A, \lambda^B \leq 1 &\Rightarrow (1 - \lambda^A)(1 - \lambda^B) \geq 0 \\ \Rightarrow \lambda^A\lambda^B &\geq \lambda^A + \lambda^B - 1 \Rightarrow \lambda^A\lambda^B \geq \lambda_{SP}^*(\lambda^A + \lambda^B - 1) \\ \Rightarrow \lambda_{SP}^* &\geq \lambda_{SP}^*(\lambda^A + \lambda^B) - \lambda^A\lambda^B = w(\lambda^A, \lambda^B). \end{aligned} \quad (\text{OA.81})$$

Equality is obtained when $\lambda^A = 1, \lambda^B = 0$ or $\lambda^A = 0, \lambda^B = 1$. These outcomes correspond to specialization in the same category.