How Carbon Is Priced in Cryptocurrencies

Abstract

This paper explores the impact of carbon intensity on cryptocurrency pricing through two channels: (1) the investment decisions of carbon-sensitive green investors, and (2) carbon emissions associated with cryptocurrency production. The CAPM-like pricing relation reveals two phenomena: first, ceteris paribus, the carbon premium in cryptocurrencies is lower compared to equities. Second, carbon intensity heightens cryptocurrencies' market portfolio exposure, which can be mitigated by utilizing green resources in production. Speculative behavior weakens carbon sensitivity, thereby lowering carbon premium in cryptocurrencies. Additionally, regulations targeting cryptocurrency carbon footprints may provoke negative market reactions as security concerns from lower energy use outweigh environmental benefits.

Keywords: Carbon emissions, Cryptocurrency, Proof-of-Work, Proof-of-Stake, Transactional benefits, Transition risk

1 Introduction

The carbon intensity of cryptocurrency mining has raised significant concerns in recent years, as the high energy consumption of the process contributes to a significant carbon footprint. For instance, as of February 2023, Bitcoin mining alone consumes an estimated 95.63 TWh per year, surpassing the electricity consumption of entire countries such as Ukraine (85.98 TWh) and the Czech Republic (55.95 TWh).[1] Such a high level of energy consumption results in a substantial carbon footprint since much of the world's electricity is still generated from fossil fuels.

How does this substantial carbon footprint influence the pricing of cryptocurrencies? From the perspective of cryptocurrency trading, on one hand, energy intensity is an integral part of ensuring the security of many established cryptocurrencies like Bitcoin. Thus, higher carbon intensity signals enhanced security and reduces the perceived risk of holding these cryptocurrencies. On the other hand, the presence of carbon-sensitive green investors in the market can affect the demand for, and consequently, the prices of cryptocurrencies with a high carbon footprint. As discussed in Pástor, Stambaugh, and Taylor (2020), the carbon intensity of a financial asset can negatively impact the utility of carbon-sensitive investors, prompting them to tilt their portfolios towards carbon-friendly assets and away from carbon-intensive ones. Therefore, investing in carbon-friendly assets may be a more attractive option for such investors, and this could negatively affect the pricing of cryptocurrencies with high carbon emissions.

The economic forces associated with carbon intensity of cryptocurrecnies extend beyond mere trading activities. From the production perspective, the process of cryptocurrency mining is notably carbon-intensive.[2] As economies worldwide transition from carbon-intensive activities to carbon-neutral alternatives, cryptocurrency miners face substantial transition risks. One of the primary risks miners face is increased regulation and compliance costs as governments implement carbon-reducing policies. Furthermore, investors' growing interest in carbon-neutral

[1] see https://ccaf.io/cbeci/index

[2] For instance, refer to the article at https://www.nytimes.com/2021/03/09/business/dealbook/bitcoin-climate-change.html

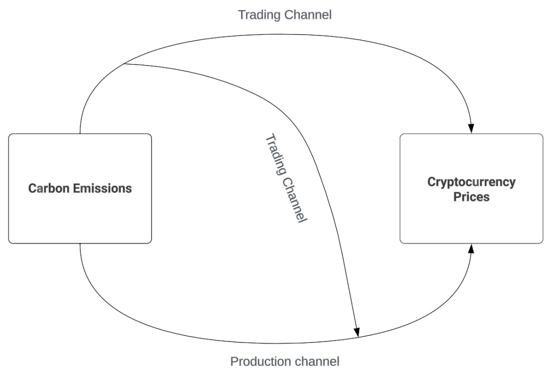

Figure 1. The trading and investment decisions of investors directly affect the price of carbon-intensive cryptocurrencies and, indirectly, miners' revenue. Additionally, miners are at risk from regulatory changes targeting carbon emissions, which can directly influence their revenue and the cryptocurrency's price accordingly.

cryptocurrencies and preference for sustainable investments, as noted in the trading channel, results in a diminished demand for established carbon-intensive cryptocurrencies. Consequently, this channel - also referred to as reputation risk - can result in a decline in both the price and transaction volume of these cryptocurrencies. Such a decline directly impacts miners' revenue, as their compensation is in the form of cryptocurrencies. Ultimately, both these factors collectively diminish miners' revenue potential.

The impact of reduced mining revenue on cryptocurrency pricing can be understood in two ways. First, to offset the decline in revenue, miners may attempt to collude to prioritize and validate transactions with higher fees, a behavior that can directly affect cryptocurrency pricing (Lehar and Parlour 2020). Second, diminished mining revenue renders operations infeasible for a portion of existing miners. Consequently, the number of active miners on the cryptocurrency platform decreases. This reduction in validators and entities maintaining the transaction ledger adversely impacts platform security, potentially discouraging investment and trading on the platform. Figure 1 illustrates the interaction between trading and production channels associated with carbon emissions in cryptocurrency pricing.

In this study, I explore the impact of carbon intensity on cryptocurrency pricing through a stylized general equilibrium model that integrates both the trading and production channels described previously. I model an ecosystem with overlapping generations of carbon-sensitive green investors who trade two types of cryptocurrencies, an equity, and a riskfree asset. One type of cryptocurrecy refers to the ones using Proof-of-Work (PoW) for consensus in the decentralized blockchain. PoW is a consensus mechanism in blockchain technology that requires computational work to be performed in order to validate transactions and create new blocks. The computational work in these platforms require energy consumption and produces a considerable amount of carbon. Thus, I refer to them as carbon-intensive cryptocurrencies. The computational work is conducted by entities called miners and in turn, they are rewarded with newly-generated cryptocurrencies. The other type of cryptocurrency uses Proof-of-Stake (PoS) for consensus in the decentralized blockchain. They are typically less-established platforms that reach consensus through a process known as staking. Staking is the process of holding a certain amount of cryptocurrency in a wallet or other designated accounts to support the operations of a blockchain network. Like miners, Stakers are typically rewarded with cryptocurrencies for contributing to the security and reliability of the network. Since staking does not require computational power, it is considered energy efficient and I refer to the associated cryptocurrencies as carbon-neutral.

Miners and stakers behave competitively and use the revenue from mining and staking (block reward + transaction fee) to cover their operating expenses. It is worth noting that miners use a portion of their mining revenue to cover the energy (electricity) cost of mining process, while stakers do not face such costs. However, stakers do face the opportunity cost of staking a certain amount of cryptocurrencies. For simplicity, the main text considers a single entity performs both mining and staking, and this entity can be viewed as a validator in the blockchain ecosystem. I relax this simplifying assumption in the Appendix by proposing that miners and stakers are independent entities and that a social planner determines the resource allocation between mining and staking.

Theoretically, assumptions within this ecosystem are made to present a pricing relation that resembles the Capital Asset Pricing Model (CAPM). This CAPM-like relationship suggests that the price disparities between PoW and PoS cryptocurrencies arise from both their anticipated transactional benefits and their distinct carbon footprints, resulting in three predominant pricing effects. First, the carbon intensity of cryptocurrency mining influences the level of exposure to the market portfolio. This finding reveals the presence of a transition risk in carbon-intensive cryptocurrencies, which is in line with previous studies on this concept (Bolton and Kacperczyk 2021b). As the entire economy is transitioning to carbon-neutral activities, carbon-intensive activities such as cryptocurrency mining face higher levels of risk, reflected in their beta exposure to the market portfolio. The theory demonstrates that the beta exposure of PoW cryptocurrency increases relative to its PoS counterpart as the carbon-intensity of PoW cryptocurrency rises. However, when miners use renewable resources in their energy portfolios, their beta exposure to the market portfolio will become relatively lower compared to the scenario where they use only fossil fuels for energy generation. This finding implies that the market demand for PoW cryptocurrencies is likely to decline in the future, unless miners take steps to substitute fossil fuels with renewable resources.

Second, investors' carbon sensitivity introduces a positive CAPM alpha for the carbon-intensive PoW cryptocurrency, while no alpha term associated with carbon emissions appears in the pricing relation of the carbon-neutral PoS cryptocurrency. This reflects the disutility experienced by carbon-sensitive investors from holding the carbon-intensive cryptocurrency, and the premium they require to compensate for their loss of utility. Third, a comparison of the carbon alpha between PoW cryptocurrency and equity shows that, ceteris paribus, PoW cryptocurrency exhibits a lower carbon alpha. This is attributed to the fact that the higher carbon intensity in PoW cryptocurrency signals stronger network security—a critical aspect of decentralized digital assets like cryptocurrencies. Stronger security reduces the perceived risk associated with holding PoW cryptocurrency in the portfolio, effectively leading to a negative premium.

Along with the carbon alpha, cryptocurrencies possess another alpha term associated with their distinctive transactional benefits. Established carbon-intensive cryptocurrencies like Bitcoin, or highly efficient ones such as Ethereum, are expected to exhibit a negative alpha related to transactional benefits. This, coupled with the positive carbon alpha, creates a complexity in interpreting the overall alpha in the cross-sectional analysis. It is likely that a negative alpha is observed for carbon-intensive cryptocurrencies since the negative alpha for transactional benefits dominates the positive carbon alpha. To address this confounding factor in the empirical verification of the theory, this paper proposes an Instrumental Variable (IV) analysis that demonstrates how exogenous variation in carbon sensitivity affects the prices of PoW cryptocurrencies. The results indicate that as investors' carbon sensitivity increases, the prices of carbon-intensive PoW cryptocurrencies decrease, supporting the existence of a carbon premium.

Considering the speculative nature of cryptocurrencies, I investigate the effect of speculative behavior on carbon sensitivity. I extend the baseline model to study an ecosystem consisting of two types of investors: rational investors, who optimize based on a subjective probability measure that aligns with the objective measure, and overconfident investors, who base their optimal decisions on a subjective probability measure that deviates from the objective measure. These overconfident investors are sometimes referred to as noise traders or speculators in the literature (Sockin and Xiong (2020) and De Long, Shleifer, Summers, and Waldmann (1990)). By endogenizing carbon sensitivity, I demonstrate that speculative trading weakens the aggregate market's carbon sensitivity and subsequently reduces the carbon premium. This occurs because the speculative motive tends to overshadow the environmental concerns regarding the carbon footprint of cryptocurrencies, leading investors to prioritize expected financial gains over climate considerations.

The main theory findings are based on the assumption that investors are sensitive to carbon emissions. The direct outcome of this preference is a carbon alpha in the pricing relation of PoW cryptocurrency. Additionally, exposure to transition risk is an indirect outcome of investors' carbon sensitivity. While previous studies have provided extensive evidence of carbon sensitivity in the equity market, it has not yet been documented in the crypto market. Hence, to support theory findings, providing evidence of carbon sensitivity in the crypto market is crucial. However, testing the implications of carbon sensitivity in the crypto market through an OLS regression in the cross-section is not straightforward due to confounding factors like unobservable transactional benefits. Moreover, due to fundamental differences between the cryptocurrency and equity markets, applying conventional equity market methods to the cryptocurrency market is not feasible and may lead to erroneous results. To overcome empirical limitations, I employ the Instrumental Variable (IV) method to investigate whether sensitivity to carbon emissions affects cryptocurrency prices. According to the theory, if investors are carbon-sensitive, higher carbon emissions should command a positive risk premium through two channels, a positive alpha and an increase in beta coefficient. As discussed in Pástor, Stambaugh, and Taylor (2020) and Pedersen, Fitzgibbons, and Pomorski (2021), higher expected returns correspond to lower price levels. Therefore, if carbon-sensitivity is present in the market, we anticipate that positive shocks to investors' carbon sensitivity lower the prices of carbon-intensive PoW cryptocurrencies.

In order to test the argument presented above, it is necessary to identify an appropriate proxy for carbon sensitivity, which is an unobservable preference parameter. Building upon prior research such as the work by Andrei and Hasler (2015) and Chen, Kumar, and Zhang (2020), I consider investors' attention to carbon-related topics, henceforth referred to as *carbon attention*, as a reflection of their carbon sensitivity. The rationale behind this is that as investors become more carbon-sensitive, they are likely to pay more attention to new information about carbon emissions. Thus, fluctuations in carbon attention can be utilized to infer the presence of carbon sensitivity. A higher level of sensitivity leads to stronger fluctuations in attention as investors react more strongly to new information. I employ Google Search Volume data to gauge the fluctuations in carbon attention, encompassing a range of subjects such as "carbon emission," "greenhouse gas emission," "environment," "ESG," and "climate." Subsequently, I investigate whether fluctuations in carbon attention are reflected in the pricing of carbon-intensive cryptocurrencies, which would suggest that carbon sensitivity is also priced.

To investigate whether carbon attention is priced in PoW cryptocurrencies, I focus exclusively on the four leading PoW cryptocurrencies: Bitcoin, Ethereum, Litecoin, and Dogecoin.[3]

[3] It's important to note that Ethereum underwent a significant transition to a PoS system with the Beacon Chain merge in September 2022.

This selection is driven by two primary considerations: firstly, cryptocurrencies represent a relatively new asset class, and many lesser-known cryptocurrencies have only a limited amount of time series data available. In contrast, these four cryptocurrencies provide at least five years of comprehensive data on various factors crucial for our analysis. This data richness allows for more robust and reliable results. Secondly, these cryptocurrencies are deeply established in the market and tend to exhibit relatively lower volatility compared to their smaller counterparts, which often have market capitalization smaller than 1 billion dollars. Consequently, they are less susceptible to speculative behavior and trading activities aimed at exploiting arbitrage opportunities.

To construct a valid instrument for carbon attention, I consider the interplay of economic forces associated with the network hash rate of PoW cryptocurrencies. The hash rate is a metric that quantifies the computational power of a PoW platform, shedding light on both the network's carbon intensity and its security level. To separate the portion of the hash rate that is indicative of carbon intensity from the aspect associated with perceived security, I regress the hash rate on the Google search volume for terms such as 'Bitcoin hack,' 'Ethereum hack,' 'Litecoin hack,' and 'Dogecoin hack.' These search terms are presumed to echo investors concerns regarding the security of these digital assets, with heightened search volumes possibly indicating increased concerns about network security breaches. By extracting the residuals from this regression, I can construct an instrument from hash rate that signals the network's carbon intensity, while ensuring it is uncorrelated with network security deduced from the hash rate.

Utilizing 2SLS regression and controlling for various factors, a significant negative relation between investors' carbon attention and the prices of BTC, ETH, LTC, and DOGE is discovered. The results indicate that carbon attention is indeed priced, which implies that investors' carbon sensitivity has a discernible impact on the value of carbon-intensive cryptocurrencies.

Moreover, I direct attention to the Ethereum Beacon Chain merge to present further empirical evidence that confirms the presence of carbon sensitivity in the cryptocurrency market. This noteworthy event occurred on September 15, 2022, signifying the transition of the Ethereum network from its existing PoW consensus algorithm to a PoS consensus algorithm. The merge resulted in a remarkable 99.95% reduction in Ethereum's energy consumption.[4] Following this event, I demonstrate a considerable increase in the proportional demand for Ethereum compared to Bitcoin. This surge in demand is evident through the flow of money deposited into the Ethereum platform via exchanges, leading to an upward push in the exchange rate of Ethereum to Bitcoin. Furthermore, there has been a noticeable rise in the number of new Ethereum addresses created subsequent to the announcement of the Beacon Chain merge, surpassing the growth seen in Bitcoin. This influx of new addresses further contributes to the increase in the exchange rate of Ethereum to Bitcoin.

1.1 Related literature

This paper bridges two distinct strands of literature. One strand explores the effect of carbon emissions, or more broadly, climate change, on asset prices (Albuquerque, Koskinen, and Zhang (2019), Avramov, Lioui, Liu, and Tarelli (2021), Bolton and Kacperczyk (2021a), Pástor, Stambaugh, and Taylor (2020), Barnett, Brock, and Hansen (2020) and Pedersen, Fitzgibbons, and Pomorski (2021)). Both Pástor, Stambaugh, and Taylor (2020) and Pedersen, Fitzgibbons, and Pomorski (2021) propose theoretical frameworks that incorporate carbon, or more broadly climate sensitivity into the utility function of investors. They show that climate sensitivity introduces a positive CAPM alpha term in carbon-intensive stocks. This study expands on existing research in two directions. First, it integrates the distinctive characteristics of cryptocurrencies, particularly the interplay between network security and carbon intensity, into the prevailing modeling framework. Second, it investigates the impact of carbon emissions from cryptocurrency production on the pricing of this digital asset. By introducing a new market participant that produces cryptocurrencies using carbon-intensive processes, I highlight the presence of transition risk in the cryptocurrency market. Furthermore, I endogenize the representative investor's aversion to carbon emissions and provide insights into the factors that shape this prefer-

[4] https://www.forbes.com/sites/qai/2022/09/27/proof-of-stake-will-the-ethereum-merge-really-lead-to-a-rally/?sh=4c498b65223d

ence. Additionally, I explore the impact of speculation in the cryptocurrency market on carbon sensitivity.

The theoretical findings of this paper are consistent with the empirical findings of Bolton and Kacperczyk (2021b) and Bolton and Kacperczyk (2021a). Specifically, this paper establishes that carbon intensity of PoW cryptocurrencies exposes them to a higher systematic risk, akin to the notion of transition risk discussed by Bolton and Kacperczyk (2021b). Also, in line with empirical findings of these two studies, this paper demonstrates that carbon sensitivity is more pronounced in wealthier economies and economies with stricter regulations on carbon emissions.

Another strand focuses on cryptocurrency pricing (Biais, Bisiere, Bouvard, Casamatta, and Menkveld (2020), Schilling and Uhlig (2019), Cong, Li, and Wang (2021), Sockin and Xiong (2020)). In this strand, researchers acknowledge the convenience yield from transacting on blockchain platforms as a fundamental source of value. Additionally, Pagnotta (2022) examine the impact of cryptocurrency infrastructure elements, such as security, on pricing. In this paper, I bridge the concepts from this strand with traditional asset pricing literature, proposing a CAPM-like pricing relation that accounts for both the convenience yield and the carbon intensity's impact on cryptocurrency pricing. Specifically, I concentrate on the microfoundation relating network security and cryptocurrency mining with carbon intensity. Utilizing this, I derive a set a findings that shed light on the sustainability aspect of cryptocurrencies.

2 Model

In this section, I first describe a stylized representative agent ecosystem to identify the impact of carbon emissions on cryptocurrency pricing. Then, I extend the model into a two-agent speculative ecosystem which comprises of a rational agent and an overconfident agent (i.e. a speculator).

In the representative agent ecosystem: (i) I solve the investor's optimization problem and establish a CAPM-like pricing relation for the equity and cryptocurrency pricing; (ii) I describe the optimal behavior of the competitive validator (miner and staker); (iii) By imposing the market-clearing conditions, I develop a proposition that explains how carbon intensity affects the beta exposure to the market portfolio; (iv) I endogenize carbon-sensitivity and identify the structure of investor's sensitivity to carbon emissions; and (v) I investigate the effects of policy interventions aimed at reducing the climate impact of cryptocurrency mining. This contains a study of how China's mining ban in May 2021 affects the price of Bitcoin.

In the two-agent speculative ecosystem: (i) I solve the optimization problem of each agent and establish a CAPM-like pricing relation for the equity and cryptocurrency pricing; (ii) I demonstrate how speculative behavior weakens carbon sensitivity.

2.1 The representative agent ecosystem:

The economy is populated with an infinitely-lived validator (miner and staker) and overlapping generations of investors who trades two types of cryptocurrencies, an equity, and a riskfree asset. Investors are carbon-sensitive, implying that the carbon emissions of their portfolio will adversely affect their utility.

I describe the two types of cryptocurrencies as follows: (1) a Proof-of-Work (PoW) cryptocurrency which requires computational power and is carbon-intensive, (2) a Proof-of-Stake (PoS) cryptocurrency which is energy-efficient and considered as carbon-neutral. The equity can be regarded as a major index such as S&P 500. The riskfree asset is in zero net supply and I normalize its return to 1. Also, one Dollar is set as the numeraire.

New cryptocurrencies (coins) are generated based on a predetermined rate. New coins are generated in the form of block rewards and paid out to the validator. Let θ_t^k be the number of coins of type k's platform at time t where $k = w, s$. $k = w$ represents a PoW platform and $k = s$ represents a PoS platform. Also φ^k is the pre-determined coin generation rate of type k's platform. We have:

$$\theta_{t+1}^k = \varphi^k \theta_t^k \quad \varphi^k > 1 \tag{1}$$

Overlapping generations of investors live for two dates t and $t+1$. In this section, I assume that there exists a representative agent in each generation. A young generation born at time t is endowed with wealth W_t. This young generation allocates its wealth among four available assets to maximize the utility of terminal payoff. Let r_{t+1}^k be the return on cryptocurrency of type k from t to $t+1$. Also, r_{t+1}^e describes the cum-dividend equity return from t to $t+1$. These returns are assumed to follow a normal distribution with the mean $E_t[r_{t+1}^k]$ and variance σ_{rk}^2 where $k = w, s, e$. Let f_t^k denote the transaction fee of trading one dollar value of type k cryptocurrency at time t. To keep the model as simple as possible, I assume that the transaction fee is exogenously given and is ex ante known by the investor. The budget constraint is expressed as below:

$$W_{t+1} = W_t(1 + X^s(r_{t+1}^s - f_t^s) + X^w(r_{t+1}^w - f_t^w) + X^e r_{t+1}^e) \tag{2}$$

where X^s and X^w denote the weight of PoS and PoW cryptocurrencies in the representative investor's portfolio. Also, X^e denotes the weight of equity in her portfolio.

Besides the financial gain, cryptocurrencies offer distinctive transactional benfits. These transactional benefits are unique to cryptocurrencies and cannot be provided by the equity or the riskfree asset. These benefits can be viewed as the "convenience yield" associated with holding cryptocurrencies. Inspired by previous studies (Cong, Li, and Wang (2021); Biais, Bisiere, Bouvard, Casamatta, and Menkveld (2020); Abadi and Brunnermeier (2018); and Sockin and Xiong (2020)), I define transactional benefits as the aggregation of platform productivity and platform security. Security in this context refers to promoting honest behavior within the decentralized network of cryptocurrencies. In PoW cryptocurrency, security is achieved through energy-intensive mining, making dishonesty economically unviable. Conversely, in PoS cryptocurrency, security is maintained through a more energy-efficient process known as staking, which involves validators committing their own cryptocurrency holdings to support network integrity. Platform productivity, on the other hand, refers to unique features specific to each cryptocurrency, such as Ethereum's smart contracts or Bitcoin's efficiency in cross-border transactions.

Let S_t^k denote the level of security that platform type k provides at time t to the representative investor per dollar value invested in the cryptocurrency. Additionally, let A_t^k denote the productivity that the platform type k offers per dollar value invested at time t where $k = w, s$.

To quantify the perceived transactional benefits per dollar value invested in cryptocurrency type k, denoted by λ_t^k, I employ the CES aggregator as follows:

$$\lambda_t^k = \gamma S_t^k + (1-\gamma) A_t^k \tag{3}$$

where $0 < \gamma < 1$ and determines the relative importance of security and productivity in the composition of the transactional benefits. I assume that S_t^k follows a normal distribution with the mean of $\bar{S}^k$ and variance of σ_{Sk}^2. I will present a parsimonious microfoundation which determines the structure of the mean and variance of network security. Also, I assume that A_t^k follows a normal distribution with the mean of $\bar{A}^k$ and variance of σ_{Ak}^2. The additive structure of Eq. 3 implies that λ_t^k is also normally distributed with the following mean and variance:

$$\bar{\lambda}^k = \gamma \bar{S}^k + (1-\gamma) \bar{A}^k$$

$$\sigma_{\lambda k}^2 = \gamma^2 \sigma_{Sk}^2 + (1-\gamma)^2 \sigma_{Ak}^2 + 2\gamma(1-\gamma)\mathrm{cov}(S_t^k, A_t^k)$$

In Eq. 3, security and productivity are assumed to be perfect substitutes. This assumption is particularly chosen to maintain the model's tractability and to enable the derivation of an analytical solution within the CARA-Normal framework. It's important to note, however, that this assumption is not restrictive in terms of the model's outcomes. As demonstrated in Appendix B, relaxing the assumption of perfect substitutability between security and productivity does not significantly alter the core results of the analysis.

In this model, both the productivity and platform security are treated as exogenous variables provided to the representative investor. In the context of a PoS platform, the security dimension is influenced by its inherent security mechanisms. Although PoS platforms have a complex security infrastructure, exploring these details falls outside the primary scope of this study, which concentrates on the implications of energy intensity and its associated carbon footprint. Consequently, the microfoundations of PoS security are not explored in depth.

PoW security and carbon emissions: In the context of PoW cryptocurrency, security can be characterized as the aggregation of the costs associated with generating network hash rate and other security features. Let's represent the per dollar value of the network hash rate and security features by h_t^w and F_t^w, respectively. Understanding the role of hash rate in provision of security necessitates an analysis of the interplay between energy consumption and hash generation. Cryptocurrency miners expend energy (electricity) to operate their mining rigs, which, in turn, generate hashes. This dynamic can be mathematically represented as follows:

$$h_t^w = g e_t^w \tag{4}$$

here, g symbolizes the efficiency of mining rigs (the number of hashes generated per unit of energy consumed) and e_t^w signifies the energy required for producing one dollar value of the PoW cryptocurrency, which is referred to as energy intensity.

In order to disrupt the PoW blockchain, an entity must acquire the majority of the network's hash rate. Therefore, as miners dedicate more energy to hash generation, the resource expenditure and associated costs required to acquire the majority of the network's hash rate and disrupt the blockchain increase, thereby enhancing the security of the PoW cryptocurrency. To characterize the process governing the hash rate, it requires to first introduce the process for e_t^w. The energy utilized for hash generation typically originates from two primary sources: fossil fuels and renewable (green) resources. Consequently, the following relationship can be established to represent this energy mix:

$$e_t^w = \alpha e_t^{wF} + (1-\alpha)e_t^{wG} \tag{5}$$

where $0 \leq \alpha \leq 1$, and e_t^{wF} and e_t^{wG} represent the per dollar value of resources available for hash generation from fossil fuels and green sources, respectively. Both e_t^{wF} and e_t^{wG} follow AR(1) process with the following structure:

$$e_t^{wF} = \eta^{wF} + \phi^w e_{t-1}^{wF} + \varepsilon_t^{wF} \tag{6}$$

$$e_t^{wG} = \eta^{wG} + \phi^w e_{t-1}^{wG} + \varepsilon_t^{wG} \tag{7}$$

where $Cov(\varepsilon_t^{wF}, \varepsilon_t^{wG}) = 0$. Therefore, the following process describes e_t^w:

$$e_t^w = \eta^w + \phi^w e_{t-1}^w + \varepsilon_t^w \tag{8}$$

and the mean and variance have the following structure:

$$E[e_t^w] = \frac{\alpha\eta^{wF} + (1-\alpha)\eta^{wG}}{1-\phi^w} \tag{9}$$

$$Var(e_t^w) = \alpha^2\sigma_{wF}^2 + (1-\alpha)^2\sigma_{wG}^2 \tag{10}$$

Considering above, the process for h_t^w yields the following structure:

$$h_t^w = \mu + \phi h_{t-1}^w + \varepsilon_t^w \tag{11}$$

where $\phi = g\phi^w$, $\mu = g\eta^w$, and $Var(h_t^w) = g^2 Var(e_t^w)$. Let p_{eng}^F and p_{eng}^G represent the ex ante known prices of fossil fuels and green energy, respectively. The assumption of known energy prices aids in model tractability and assists in deriving a closed-form solution. Although this represents a simplification, it aligns with some real-world practices, particularly the widespread usage of futures contracts in the energy market. However, it's important to note that relaxing this assumption would not significantly impact our results. For example, employing a log-normal distribution for both energy prices and energy intensity, and then applying log-linearization, could lead to similar findings. For a more detailed discussion on this topic, please refer to the Appendix B.

Employing the CES aggregator to characterize PoW platform security yields:

$$S_t^w = \pi \left(Hash\_Cost_t \right) + (1 - \pi) \left(F_t^w \right) \tag{12}$$

where $0 < \pi < 1$. The $Hash\_Cost_t$ has the following structure:

$$Hash\_Cost_t = p_{eng}^F \times \underbrace{\alpha e_t^{wF}}_{\substack{\text{First} \\ \text{term}}} + p_{eng}^G \times \underbrace{(1-\alpha) e_t^{wG}}_{\substack{\text{Second} \\ \text{term}}}$$

where the $First\ term$ represents the per dollar capacity for hash generation from fossil fuels and $Second\ term$ represents the per dollar capacity for hash generation from green energy.

F_t^w follows a normal distribution with the constant mean of $\bar{F}^w$ and variance of σ_F^2. F_t^w can be interpreted as the security provided by the blockchain's cryptographic architecture. In the PoW blockchain, each block contains a cryptographic hash, which is based on the data within that block as well as the hash of the preceding block. As the blockchain expands, the cryptographic links between the blocks strengthen, rendering it computationally impractical to alter past blocks, thereby solidifying the integrity and reliability of the entire PoW blockchain system.

The energy intensity of equity e_t^e, is defined as energy consumption per dollar value of the equity at time t. In the context of this discussion, there exists entities operating in the equity market that expend energy to produce output. The energy intensity of equity is assumed to follow an AR(1) process with the following structure:

$$e_t^e = \eta^e + \phi^e e_{t-1}^e + \varepsilon_t^e \tag{13}$$

where $\varepsilon_t^e \sim N(0, \sigma_e^2)$.

The carbon intensity is defined as the amount of carbon emissions per dollar value of asset type k at time t and is represented by C_t^k. As previously stated, PoS cryptocurrency is regarded as a carbon-neutral asset, leading to the conclusion that $C_t^s = 0$. The carbon emissions is the outcome of fossil fuel usage for energy generation. Let's define ψ as the conversion rate of fossil fuels usage to carbon emission, effectively representing the emission intensity of fossil fuels. It's straightforward to show that C_t^w follows an AR(1) process as delineated in the following equation:

$$C_t^w = \mu_C^w + \phi_C^w C_{t-1}^w + \varepsilon_t^w \tag{14}$$

where $\mu_C^w = \psi\alpha\eta^{wF}$ and $\phi_C^w = \psi\alpha\phi^w$. Consequently, the mean and the variance of C_t^w can be expressed as $\bar{C}^w = \frac{\mu_C^w}{1-\phi_C^w}$ and $Var(C_t^w) = \frac{\sigma_{Cw}^2}{1-(\phi_C^w)^2}$.

For equity, C_t^e denotes the carbon intensity of the equity at period t with a mean of $\bar{C}^e$ and a variance of σ_{Ce}^2. Given that the detailed analysis of the microfoundations underlying equity emissions falls outside the primary scope of this study, I will treat $\bar{C}^e$ and σ_{Ce}^2 as given.

In this ecosystem, a cap on carbon emissions is established, and the carbon market must clear at each time epoch. This emissions cap can be viewed as the initiative of a benevolent social planner who internalizes the externalities associated with carbon emissions. Let Cap_t denote the cap on carbon emissions at period t. It is important to note that the energy intensity and carbon intensity can be either perfectly correlated ($Corr(C_t^k, e_t^k) = 1$), which occurs when all energy is produced from fossil fuels, or imperfectly correlated ($Corr(C_t^k, e_t^k) = \alpha < 1$) when a portion of energy comes from green sources. To ensure positive values for energy intensity, carbon intensity, and hash rate, I assume that the mean of their respective processes is significantly larger than their volatility. In the Appendix B, I relax this assumption by considering a log-normal probability distribution for these three variables.

Utility function: Considering the Capital Asset Pricing Model (CAPM) as a benchmark, the utility function of the representative investor is structured in the following manner to derive a pricing relation that both resembles and is comparable to CAPM:

$$Utility = -e^{A_t \hat{W}_{t+1}} \tag{15}$$

where the terminal wealth ($\hat{W}_{t+1}$) is considered a bundle which aggregates the pecuniary and non-pecuniary benefits. The pecuniary benefits are derived from the capital gains of each asset, while non-pecuniary benefits stem from the utility of transactional benefits and the disutility associated with carbon emissions. A_t represents the coefficient of absolute risk aversion in generation t. The choice of overlapping generations model implies time-varying preference parameters across generations. However, for simplicity, I drop the time subscript unless necessary. Considering the above explanation, I identify the wealth bundle as below:

$$\hat{W}_{t+1} = W_t \left(\delta_1(1 + \mathbf{X'_t}(\mathbf{r_{t+1}} - \mathbf{f_t})) - \delta_2\mathbf{X'_t}\mathbf{C_t} + \delta_3\mathbf{X'}_t\lambda_\mathbf{t}\right) \tag{16}$$

where $\delta_1, \delta_2, \delta_3 > 0$, signifying the relative importance of different components within the bundle. Also, $\mathbf{X'_t} = (X_t^w, X_t^s, X_t^e)$ represents the vector of portfolio weights. The terms $\mathbf{r_{t+1}}$, $\mathbf{f_t}$, $\mathbf{C_t}$, and $\lambda_\mathbf{t}$ represent the vectorized formats of return, transaction fee, carbon intensity, and transactional benefits, respectively. Specifically, $\delta_1 W_t(1 + \mathbf{X'_t}(\mathbf{r_{t+1}} - \mathbf{f_t}))$ represents the pecuniary component of the wealth bundle, while $W_t\left(-\delta_2\mathbf{X'_t}\mathbf{C_t} + \delta_3\mathbf{X'}_t\lambda_\mathbf{t}\right)$ represents the non-pecuniary component. In this framework, every component, whether it is pecuniary or non-pecuniary in nature, is treated as a perfect substitute for the others. This simplifying assumption is deliberately adopted to enable the derivation of an analytical solution within the CARA-Normal framework. Such an approach facilitates comparative analysis with the CAPM. However, as detailed in Appendix B, the robustness of the results is maintained even when the assumption of perfect substitutability is relaxed.

Given this utility structure, the representative investor in generation t performs the following optimization over her expected utility function conditional on her information at time t:

$$\underset{X}{Max}\, E_t[-e^{-A\hat{W}_{t+1}}] \tag{17}$$

In the following subsection, I present the solution for the representative investor's problem. Then, I characterize the equilibrium pricing relation.

2.1.1 Portfolio allocation and the CAPM-like relation

In this subsection, I solve the optimization problem of the representative investor and find the equilibrium pricing relation within the context of this specific ecosystem.

Normal distribution of assets' returns, transactional benefits, and carbon intensity allows us to employ the normal characteristic function to rewrite the optimization problem of Eq. 17 as below:

$$\underset{X}{Max} -e^{-a[\delta_1(1+\mathbf{X}'(E_t[\mathbf{r_{t+1}}]-\mathbf{f_t}))-\delta_2\mathbf{X}'\bar{\mathbf{C}}+\delta_3\mathbf{X}'\bar{\lambda}_\mathbf{t}]+\frac{1}{2}a^2\mathbf{X}'\boldsymbol{\Sigma}_\mathbf{T}\mathbf{X}} \tag{18}$$

$$where \quad E_t[\mathbf{r_{t+1}}] = \begin{pmatrix} E_t[r^w_{t+1}] \\ E_t[r^s_{t+1}] \\ E_t[r^e_{t+1}] \end{pmatrix}$$

$$and \;\; \mathbf{f_t} = \begin{pmatrix} f^w_t \\ f^s_t \\ 0 \end{pmatrix} \quad \bar{\lambda} = \begin{pmatrix} \bar{\lambda}^w \\ \bar{\lambda}^s \\ 0 \end{pmatrix} \quad \bar{\mathbf{C}} = \begin{pmatrix} \bar{C}^w \\ 0 \\ \bar{C}^e \end{pmatrix}$$

We have $\Sigma_T = \delta_1^2\Sigma + \delta_2^2\Sigma_C + \delta_3^2\Sigma_\lambda - 2\delta_1\delta_2\Sigma_{rC} + 2\delta_1\delta_3\Sigma_{r\lambda} - 2\delta_2\delta_3\Sigma_{C\lambda}$ as the the variance-covariance matrix of this ecosystem. Also, $a = AW_t$ signifies the coefficient of relative risk aversion of the representative investor. Here, the effective risk aversion is $a\delta_1$, the effective preference for carbon emission, which I referred to as carbon sensitivity, is $a\delta_2$, and the effective preference for transactional benefits is $a\delta_3$. The signs of these effective preference parameters indicate that higher capital gain and transactional benefits enhance the investor's utility, whereas an increase in carbon emissions results in a utility loss.

Taking first order condition with respect to the vector of portfolio weights $X' = (X^w, X^s, X^e)$ from Eq. 18 gives us the optimal portfolio weight in generation t:[5]

$$X^* = \frac{1}{a}\Sigma_T^{-1}(\delta_1(E_t[\mathbf{r_{t+1}}] - \mathbf{f_t}) - \delta_2\bar{\mathbf{C}} + \delta_3\bar{\lambda}) \tag{19}$$

Appendix A provides the proof. The optimal portfolio can be conceptually broken down into three components. The first component ($\Sigma_T^{-1}(E_t[\mathbf{r_{t+1}}] - \mathbf{f_t})$) represents the standard portfolio with the highest Sharpe ratio, indicating the most efficient allocation of assets based on their expected financial gain. The second component ($\Sigma_T^{-1}\bar{\mathbf{C}}$) represents the negative impact of carbon emissions on the portfolio allocation and investment decision, reflecting the investor's carbon sensitivity. Lastly, the third component ($\Sigma_T^{-1}\bar{\lambda}$) illustrates the adjustment in portfolio allocation influenced by transactional benefits, particularly in the context of cryptocurrency investments.

Utilizing the optimal portfolio allocation of the investor, I can characterize the equilibrium return. Building on the conclusions of the preceding section, the mean transactional benefits for both PoW ($\bar{\lambda}^w$) and PoS ($\bar{\lambda}^s$) cryptocurrencies are characterized as follows:

$$\bar{\lambda}^w = a_1\bar{C}^w + a_2$$

$$\bar{\lambda}^s = \gamma\bar{S}^s + (1-\gamma)\bar{A}^s$$

$$where\, a_1 = \gamma\pi\frac{p_{eng}^F}{\psi}\cdot\frac{1-\phi_C^w}{1-\phi^w}$$

$$and\;\; a_2 = \gamma\pi(1-\alpha)p_{eng}^G\frac{\eta^{wG}}{1-\phi^w} + \gamma(1-\pi)\bar{F}^w + (1-\gamma)\bar{A}^w$$

In this context, "a_1" represents the perceived impact of carbon intensity on network security.

[5] The portfolio weight must have a time subscript since it represents the optimal decision of generation t. However, for the sake of brevity, I drop the time subscript.

Let's define X_M as the market portfolio which consists of the 3 risky assets: PoW cryptocurrency, PoS cryptocurrency, and the equity. Therefore, in the market equilibrium, the market weights represent the portfolio weight of each risky asset, i.e., $X'_M = (X_M^w, X_M^s, X_M^e)$. In light of this equilibrium condition, the return of each risky asset at generation t can be derived as follows:

$$E_t[r_{t+1}^w] = \frac{a}{\delta_1} e'_1 \Sigma_T X_M + (f_t^w - \frac{\delta_3}{\delta_1} a_2) + (\frac{\delta_2 - \delta_3 a_1}{\delta_1}) \bar{C}^w \tag{20}$$

$$E_t[r_{t+1}^s] = \frac{a}{\delta_1} e'_2 \Sigma_T X_M + (f_t^s - \frac{\delta_3}{\delta_1} \bar{\lambda}^s) \tag{21}$$

$$E_t[r_{t+1}^e] = \frac{a}{\delta_1} e'_3 \Sigma_T X_M + \frac{\delta_2}{\delta_1} \bar{C}^e \tag{22}$$

where e_i denotes the i^{th} basis vector of $\mathbf{R}^n$ (having 1 in the i^{th} place and 0 elsewhere).

Appendix A provides the proof. In Eq. 20, likewise the optimal portfolio allocation, the expected return has 3 components. $\frac{a}{\delta_1} e'_1 \Sigma_T X_M$ represents the compensation for the volatility of PoW cryptocurrency and its covariance with other risky assets in the market. $\frac{\delta_2 - \delta_3 a_1}{\delta_1} \bar{C}^w$ represents a premium for carbon emissions. This premium may be positive or negative, contingent on how carbon intensity influences network security. Finally, $f_t^w - \frac{\delta_3}{\delta_1} a_2$ represents the premium for the net of carbon-independent transactional benefits that the PoW cryptocurrency is expected to offer.

In Eq. 21 and 22, there are 2 components that explain the expected return. For both equations, Similar to the PoW cryptocurrency, $\frac{a}{\delta_1} e'_2 \Sigma_T X_M$ (for PoS) and $\frac{a}{\delta_1} e'_3 \Sigma_T X_M$ (for equity) represent the risk compensation. In PoS, $f_t^s - \frac{\delta_3}{\delta_1} \bar{\lambda}^s$ represents the negative premium linked to the net transactional benefits. For equity, $\frac{\delta_2}{\delta_1} \bar{C}^e$ represents the positive premium associated with equity carbon emissions. Within the framework of CAPM, these premiums in excess of the risk compensation are often referred to as "alpha." Here, the model finds a transactional benefits alpha for PoS and PoW cryptocurrencies and a carbon alpha for PoW cryptocurrency and equity.

Comparing the expected return of PoW cryptocurrency with the equity, it becomes apparent that the carbon alpha for each unit of emission is lower in PoW cryptocurrency than in equity. Mathematically speaking, we have:

$$\frac{\delta_2}{\delta_1} > \frac{\delta_2 - \delta_3 a_1}{\delta_1}$$

This holds since $a_1 > 0$. Interestingly, when the emission intensity (ψ) increases, the difference between equity carbon alpha and the PoW carbon alpha decreases as indicated by $\frac{\partial(\frac{\delta_3 a_1}{\delta_1})}{\partial \psi} < 0$. Intuitively, when fossil fuels become more polluting, $\delta_3 a_1$ gets smaller which suggests that although higher carbon emissions might be perceived as an indicator of stronger network security, lower emission efficiency actually weakens the positive effect of this enhanced security.

Considering the expected returns of the three risky assets, I can develop a CAPM-like pricing relation, as stated in the following Proposition.

Proposition 1: The expected return of each risky asset in this ecosystem follows a CAPM-like relation as described below:

$$E_t[r^w_{t+1}] = \beta^w E_t[r^M_{t+1}] + \frac{\delta_2 - \delta_3 a_1}{\delta_1}\bar{C}^w + (f^w_t - \frac{\delta_3}{\delta_1}a_2) \tag{23}$$

$$E_t[r^s_{t+1}] = \beta^s E_t[r^M_{t+1}] + (f^s_t - \frac{\delta_3}{\delta_1}\bar{\lambda}^s) \tag{24}$$

$$E_t[r^e_{t+1}] = \beta^e E_t[r^M_{t+1}] + \frac{\delta_2}{\delta_1}\bar{C}^e \tag{25}$$

Proof: see Appendix A.

where $\beta = \frac{\Sigma_T X_M}{\hat{\sigma}^2_M}$ and $\hat{\sigma}^2_M = \sigma^2_M + \bar{C}_M - \bar{T}_M$ represents the adjusted variance. Here, $\bar{C}_M = \frac{1}{a}X'_M(\delta_2\bar{C})$ represents the average market premium for carbon emissions adjusted by risk aversion and $\bar{T}_M = \frac{1}{a}X'_M(\delta_3\overline{\lambda} - \delta_1\mathbf{f}_t)$ represents the average market premium for the net of transactional benefits adjusted by risk aversion. Similar to the approach of Pástor, Stambaugh, and Taylor (2020), one can normalize $\bar{C}_M - \bar{T}_M = 0$ and adjusts the alpha terms accordingly.

However, here I retain the current structure and and the adjustment is made to the variance term. From an econometrics perspective, the inclusion of additional preference parameters causes the CAPM beta to deviate from its conventional model value, which typically includes only risk aversion as a preference parameter.

In Proposition 1, presence of carbon sensitive investors ($\delta_2 > 0$) introduces a positive carbon alpha to the pricing relation of equity, as documented in previous studies such as Pástor, Stambaugh, and Taylor (2020) and Zerbib (2022). On the other hand, the sign of carbon alpha for PoW cryptocurrency is unclear. It can even be negative if investors perceive higher emissions as a sufficiently strong signal of network security. Furthermore, the carbon alpha in PoW cryptocurrency tends to be lower or equivalent per unit of carbon emission, suggesting that cryptocurrency investors are less sensitive to the carbon emissions of this asset class when compared to equity (stock) investors.

2.1.2 Competitive validator

A single entity called validator undertakes both mining and staking activities. This entity is competitive, meaning that it uses the revenue from mining and staking (transaction fee + block reward) to offset operating expenses. Specifically, mining expenses comprise equipment maintenance costs and the expenditure on energy (electricity) necessary for the mining operations. Expenses related to staking are primarily the opportunity costs associated with staked tokens. Moreover, the validator operates honestly and gains no private benefit from altering the blockchain.[6] In the latter part of this subsection and further explored in Appendix B, I discuss about a more comprehensive scenario where miners and stakers are treated as separate, independent entities.

As our primary focus is on the impact of energy costs and associated carbon emissions, I aggregate all other expenses, such as equipment maintenance, into a single term denoted as M_t.

[6] The concept of a representative validator simplifies the model for this analysis. However, introducing competition among miners in the setup does not significantly change the outcomes.

Therefore, M_t encapsulates the sum of mining and staking expenses, excluding energy cost.

In the baseline model, I assume that the energy utilized for mining exclusively originates from fossil fuels, thus $e_t^w = e_t^{wF}$. However, in Section 2.1.5, I will extend the model by considering a scenario where miners also use green energy for mining. Earlier, I discussed the application of a linear conversion method to translate energy intensity into carbon intensity (Barnett, Brock, and Hansen 2020). Hence, the relationship between energy intensity and carbon intensity can be expressed as follows:

$$C_t^w = \psi e_t^w \tag{26}$$

Now defining p_t^w and p_t^s as the price of one unit of PoW and PoS cryptocurrencies at time t respectively, I characterize the budget constraint of the validator as below:

$$\underbrace{(\theta_{t+1}^s - \theta_t^s)p_t^s}_{Block\ reward\ from\ staking} + \underbrace{(\theta_{t+1}^w - \theta_t^w)p_t^w}_{Block\ reward\ from\ mining} + \underbrace{f_t^s X^s W_t}_{TX\ fee\ of\ PoS} + \underbrace{f_t^w X^w W_t}_{TX\ fee\ of\ PoW} = \underbrace{p_{eng}^F e_t^w p_t^w (\theta_{t+1}^w - \theta_t^w)}_{Energy\ cost\ of\ mining} + \underbrace{M_t}_{Other\ expenses} \tag{27}$$

Considering Eq. 1, I can express block reward as $(\varphi^k - 1)\theta_t^k p_t^k$ for $k = w, s$. Note that $\theta_t^k p_t^k$ represents the market cap of type k's cryptocurrency. Thus, we have:

$$(\varphi^k - 1)\theta_t^k p_t^k = (\varphi^k - 1)W_t X^k \tag{28}$$

for $k = w, s$. Replacing 28 into 27, and simplifying gives us:

$$(\varphi^w - 1 + f_t^w)X^w + (\varphi^s - 1 + f_t^s)X^s = p_{eng}^F e_t^w (\varphi^w - 1)X^w + \frac{M_t}{W_t} \tag{29}$$

I use Eq. 29 along with the market-clearing conditions described in the next section to derive relationship between systematic risk exposure and carbon intensity in cryptocurrency pricing.

Assuming a single validator carries an important implication: the validator can cross-subsidize resources between mining and staking. While this assumption reflects certain real-world scenarios, a more common perspective views miners and stakers as separate, independent entities. In Appendix B, I explore this perspective by considering a representative miner and a representative staker. Subsequently, I introduce a social planner to determine the optimal resource allocation between mining and staking. I then demonstrate that the findings that emerge remain consistent with the current ecosystem.

2.1.3 Market-clearing conditions

This ecosystem has two market-clearing conditions. First, the market for risky assets must clear. Since the riskfree asset is in zero net supply, the market-clearing condition for risky assets is:

$$X^w + X^s + X^e = 1 \tag{30}$$

Also, there is a cap for carbon emissions. So, the carbon market must clear which implies:

$$C_t^w W_t X^w + C_t^e W_t X^e = Cap_t \tag{31}$$

where Cap_t represents the cap on carbon emissions at time t. As elaborated earlier, this emissions cap may result from the actions of a benevolent social planner who internalizes the externalities associated with carbon emissions.

2.1.4 Systematic impact of carbon emissions on cryptocurrency pricing

Gathering together the equations 29, 30, and 31, we form a linear system with three equations and three unknowns, outlined as follows:

$$\begin{cases} b_1 X^w + b_2 X^s = b_3 \\ X^w + X^s + X^e = 1 \\ C_t^w X^w + C_t^e X^e = \frac{Cap_t}{W_t} \end{cases} \tag{32}$$

$$where \quad b_1 = (\varphi^w - 1)(1 - p_{eng}^F e_t^w) + f_t^w$$

$$b_2 = (\varphi^s - 1 + f_t^s)$$

$$b_3 = \frac{M_t}{W_t}$$

Here, b_1 represents the marginal revenue net of electricity cost of mining per dollar value of PoW cryptocurrency, while b_2 represents the marginal revenue of staking per dollar value of PoS cryptocurrency. Additionaly, b_3 represents the marginal operating cost. The solution to this linear system gives the structure of optimal portfolio weights in equilibrium. The following equations characterizes the solution.

$$X_M^w = \frac{C_t^e(b_2 - b_3) - \frac{Cap_t}{W_t} b_2}{C_t^e(b_2 - 1) - C_t^w b_2} \tag{33}$$

$$X_M^s = \frac{C_t^e(b_3 - b_1) + \frac{Cap_t}{W_t} b_1 - C_t^w b_3}{C_t^e(b_2 - 1) - C_t^w b_2} \tag{34}$$

$$X_M^e = \frac{C_t^w(b_3 - b_2) + \frac{Cap_t}{W_t}(b_2 - b_1)}{C_t^e(b_2 - 1) - C_t^w b_2} \tag{35}$$

For technical reason, $\frac{C_t^w}{C_t^e} \neq \frac{b_2 - 1}{b_2}$ must hold. The weights described in Eq. 33, 34, and 35 must be equivalent to the equilibrium weights described in Eq. 19. I utilize this equivalence to develop Proposition 2 which captures the impact of carbon intensity on systematic exposure to the market portfolio. Prior to that, I provide the intuition for Eq. 33, 34, and 35 through a baseline simulation. I consider each period as a year. I take the coin generation rate for PoW cryptocurrency as $\varphi^w - 1 = 0.002$. This is taken based on BTC generation rate. Currently, generation per block is 6.25 and each 10 minutes, a block is mined. Over a year, at this generation rate, almost 52500 new BTC will be generated. Considering the total number of BTC which is almost 19000000, the 0.002 rate is obtained. For PoS, I take the coin generation rate as $\varphi^s - 1 = 0.046$. This is obtained based on the annual inflation rate of ADA as one of the leading PoS cryptocurrencies.

The average transaction fee per dollar value of transaction for PoW platform is $f_t^w = 0.075$. This is also obtained from Bitcoin. The average transaction fee is currencly around 50 satoshi/BTC. Each Satoshi is 0.00000001 BTC. Considering the average price of 15000$ for Bitcoin, the average transaction fee of 0.075 is obtained. The average transaction fee per dollar value of transaction for PoW platform is $f_t^s = 0.22$. This is also taken from Cardano (ADA). The average transaction fee in Cardano platform is 0.17 ADA per transaction. Mutliply it by ADA's avrage price over the sample, we obtain $f_t^s = 0.22$.

I take the wealth of representative investor as $W_t = 35000\$$. I consider 3 kWh for mining one unit of PoW cryptocurrency. Also, I take the average energy price as 13.33 cent/kWh. I take other operating expenses $M_t = 10000\$$. To calculate per capita emission cap, I took the total carbon emission in the U.S. as 4 Billion ton. Considering a population of 300000000 people, the per capita emission would be $Cap_t = 13.33$.

As shown in Figure 2, the maximum weight in PoW cryptocurrency occurs when the carbon footprint (intensity) of equity is high and the carbon footprint (intensity) of PoW cryptocurrency is low. In this scenario, the optimal weight falls within the range of 20% to 25%. Additionally, as the carbon footprint (intensity) of both equity and PoW approaches zero, the weight of PoW in the investor's portfolio significantly increases. This is due to the expectation that PoW cryptocurrency offers transactional benefits and financial gains, unlike equity which provides only

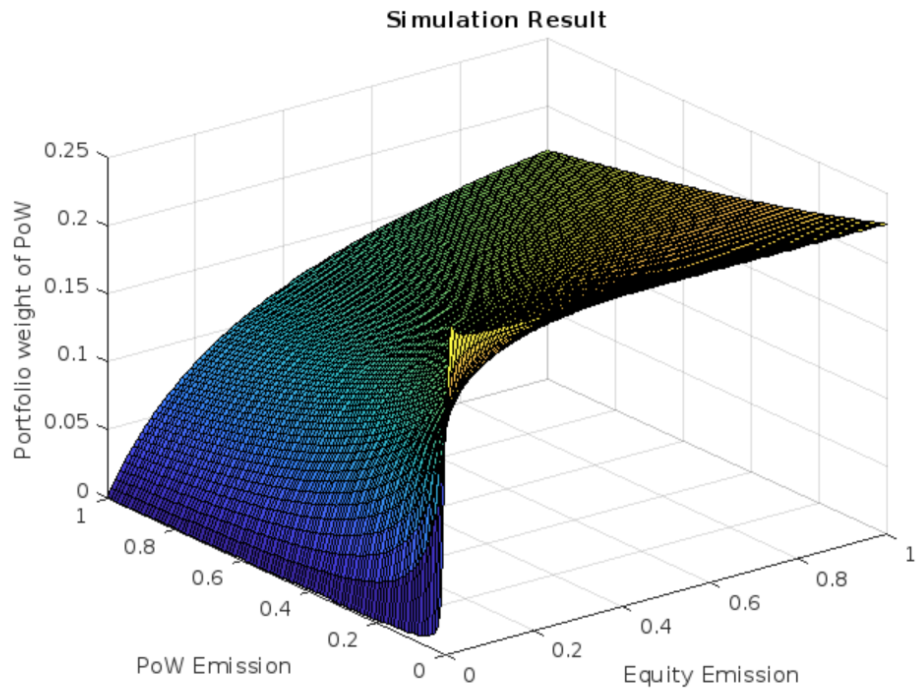

Figure 2. Simulation results This figure demonstrates the simulated weight of PoW cryptocurrency for the given parameters.

financial gain.

To develop Proposition 2, I define the inverse of emission intensity ($\frac{1}{\psi}$) as carbon efficiency. Intuitively, when fossil fuels emit a lower level of carbon per unit of energy consumption (a low ψ), they can be considered carbon-efficient. Thus, we have: $Carbon\_Eff = \frac{1}{\psi}$. I also define the ratio of PoW carbon intensity to equity carbon intensity as $I_t = \frac{C_t^w}{C_t^e}$.

Assume, without loss of generality, that in equilibrium, $X^w + X^s = n$, $X^w = m$ and consequently $X^e = 1 - n$ where $n = \frac{(b_2 - b_1)(C_t^e - \frac{Cap_t}{W_t}) - C_t^w b_3}{C_t^e (b_2 - 1) - C_t^w b_2}$. Considering this assumption, the following Proposition describes the effect of carbon intensity on exposure to systematic risk.

Proposition 2: When the staking revenue is sufficiently high, meaning that $b_2 > \frac{b_1}{I_t + 1}$ and carbon efficiency of PoW cryptocurrency is sufficiently low, denoted by $Carbon\_Eff < T$ (where T is detailed in the Appendix), the beta of PoW cryptocurrency, which represents its exposure to market portfolio fluctuations, increases in comparison to PoS cryptocurrency as the carbon intensity of the PoW cryptocurrency rises.

$$\beta^w - \beta^s = U \tag{36}$$
$$\frac{\partial U}{\partial C_t^w} > 0$$
$$where \quad U = \frac{a(\Delta\sigma)^2}{\delta_1 E_t[r_{t+1}^M]} \left(\frac{C_t^e(b_2 - b_3) - \frac{Cap_t}{W_t} b_2}{C_t^e(b_2 - 1) - C_t^w b_2} + \frac{l}{a(\Delta\sigma)^2} \right)$$
$$and \quad l = a((1-n)(\sigma_{ew} - \sigma_{es}) + n(\sigma_{sw} - \sigma_s^2))$$

Proof: see Appendix A.

Intuitively, when the available resources for mining have low carbon efficiency and the revenue from staking is sufficiently high, validators have a strong financial incentive to shift from mining to staking. This shift is driven by two factors: a cap on emissions that restricts PoW cryptocurrency adoption, thereby reducing potential mining revenues, and a growing preference for carbon-neutral investments, which promotes the adoption of PoS cryptocurrency and increases potential staking revenue. As a result of this shift, the security of PoS cryptocurrency improves, making it a more appealing investment instrument. In this scenario, Ceteris Paribus, the market tends to favor and invest more in the more energy-efficient PoS over PoW. Proposition 2 underlines this dynamic, suggesting that increased carbon emissions from PoW cryptocurrency mining contribute to heightened systematic risk in PoW cryptocurrency. This perspective is in line with the notion of transition risks explored in the works of Bolton and Kacperczyk (2021b) and Bolton and Kacperczyk (2021a).

2.1.5 When miners use renewable energies

Could utilizing green energy in cryptocurrency mining reduce systematic risk exposure? If we interpret higher systematic risk exposure as a form of transition risk, then integrating green energy sources for mining should mitigate this risk. To explore this, and in line with the recent trend in the Bitcoin mining industry, I will consider a scenario in which miners utilize renewable green energy sources to mine PoW cryptocurrencies.[7]

Consider a scenario where the energy needed to mine one dollar's worth of PoW cryptocurrency stems from a mix of fossil fuels and green energy, as detailed below:

$$e_t^w = \alpha e_t^{wF} + (1-\alpha)e_t^{wG} \tag{37}$$

$$where \quad 0 < \alpha < 1$$

As elaborated earlier, I assume a zero carbon footprint for green energy sources. Thus, it becomes apparent that the carbon intensity of PoW cryptocurrency adjusts accordingly to reflect the utilization of green energy sources in the mining process.

$$C_t^w = \hat{\psi} e_t^w \tag{38}$$

$$where \ \hat{\psi} = \alpha\psi$$

It's straightforward that $\hat{\psi} < \psi$ which implies that the use of green energy sources leads to a reduction in the emission intensity of PoW cryptocurrency. Now, the following proposition captures the mitigating effect of using green energy sources on exposure to the systematic risk.

Proposition 3: As miners use more green energy ($(1-\alpha)$ increases), the relative beta coefficient of PoW cryptocurrency experiences a decline in comparison to PoS cryptocurrency:

$$\beta^w - \beta^s = U \tag{39}$$

[7]Several large mining pools have publicized their business plans to utilize green energy for Bitcoin mining. Notable examples include Greenidge Generation and Hut 8, which use renewable energies for mining, as stated in their business plans.

$$\frac{\partial U}{\partial(1-\alpha)} < 0$$

$$where \quad U = \frac{a(\Delta\sigma)^2}{\delta_1 E_t[r^M_{t+1}]}\left(\frac{C^e_t(b_2-b_3) - \frac{Cap_t}{W_t}b_2}{C^e_t(b_2-1) - C^w_t b_2} + \frac{l}{a(\Delta\sigma)^2}\right)$$

$$and \quad l = a((1-n)(\sigma_{ew} - \sigma_{es}) + n(\sigma_{sw} - \sigma^2_s))$$

Proof: see Appendix A.

Proposition 3 indicates that as miners transition from utilizing fossil fuels to renewable resources (i.e., a decrease in α, and subsequently an increase in $(1-\alpha)$), the systematic exposure to the market portfolio decreases.

2.1.6 Endogenous carbon sensitivity

In this subsection, I endogenize carbon-sensitivity ($a\delta_2$) in the representative investor's problem described in Eq. 18 which results in the following problem:

$$\underset{X,\delta_2}{Max}\{-a[\delta_1(1 + X'(E_t[\mathbf{r_{t+1}}] - \mathbf{f_t})) - \delta_2(X'\bar{\mathbf{C}}) + \delta_3(X'\bar{\lambda})] + \frac{1}{2}a^2 X'\Sigma_T X\} \tag{40}$$

$$s.t. \quad W_{t+1} = W_t(1 + X_s(r^s_{t+1} - f^s_t) + X_w(r^w_{t+1} - f^w_t) + X_e r^e_{t+1})$$

The structure of optimal portfolio weight is similar to Eq. 19. To find the optimal carbon sensitivity, one should take the F.O.C with respect to δ_2. The structure of optimal carbon sensitivity in generation t is described in the following equation:[8]

$$a{\delta_2}^* = \frac{a\delta_1\sigma_{rC} + a\delta_3\sigma_{C\lambda} - \frac{E_t[Cap_t]}{W_t}}{\sigma^2_C} \tag{41}$$

[8]Recall that I dropped time subscript for the sake of brevity.

$$
where \quad \begin{cases} X'\Sigma_C X = \sigma_C^2 \\ X'\Sigma_{rC} X = \sigma_{rC} \\ X'\Sigma_{C\lambda} X = \sigma_{C\lambda} \end{cases}
$$

For proof, see Appendix A. Eq. 41 outlines that the structure of carbon sensitivity can be explained by four components. Firstly, when the correlation between carbon intensity and the return (σ_{rC}) increases, the degree of carbon sensitivity increases. Intuitively, a strong correlation suggests that carbon intensity poses a significant financial risk to the economy, necessitating a high premium. Consequently, the elevated risk exposure resulting from higher carbon intensity makes the representative investor more sensitive to carbon emissions. Also, the effect of the correlation between carbon intensity and the return is determined by the effective risk aversion $a\delta_1$, emphasizing the importance of financial risk protection in this component.

Second, the correlation between carbon intensity and transactional benefits provided by cryptocurrencies ($\sigma_{C\lambda}$). We can interpret this component in two ways. Firstly, if we regard carbon efficiency (defined as lower emissions intensity) as a key aspect of cryptocurrency productivity, we would anticipate a negative correlation. Hence, the more productivity which stems from higher carbon efficiency make the representative investor less carbon sensitive since activity on such cryptocurrency platforms won't harm the environment. Conversely, in scenarios where increased productivity or enhanced security aligns with elevated carbon emissions, the investor's carbon sensitivity is heightened. In these instances, the appeal of transactional benefits offered by cryptocurrencies might be overshadowed by their environmental cost. For example, while Bitcoin's reliability has cemented its status for international money transfers, the realization of its significant carbon footprint could lead investors to seek more eco-friendly alternatives.

Third, a higher expected cap on carbon emissions decreases the degree of carbon sensitivity. Forth, higher wealth level increases carbon sensitivity. The third and forth components are empirically documented in various studies such as Bolton and Kacperczyk (2021b). In wealthier societies, investors tend to show a deeper concern for climate change and the ramifications of carbon emissions. Similarly, in economies with stricter carbon emissions regulation, a higher level of environmental consciousness and carbon sensitivity among investors is often observed, as detailed in Bolton and Kacperczyk (2021b).

2.1.7 China's Bitcoin mining ban: A policy intervention to mitigate climate impact

In this subsection, I explore the potential market reactions to policy measures aimed at either reducing the carbon footprint of cryptocurrency mining or at restricting mining activities for other reasons. To provide better insight into this analysis, I focus on a notable event: In May 2021, China's State Council issued a statement emphasizing the need to control financial risks associated with cryptocurrencies, including Bitcoin mining. Subsequently, a ban on Bitcoin mining was implemented in various provinces and regions within China. Shortly after the ban, the hash rate on Bitcoin's network experienced a substantial reduction of 50%. This mining ban had a significant impact on the price of Bitcoin, which underwent a noteworthy decline from nearly \$55,000 to approximately \$30,000 over the subsequent months.[9]

Given the pronounced negative shock to the hash rate, it is expected that both the energy intensity and carbon intensity of Bitcoin decrease accordingly. Under the assumption that investors are carbon-sensitive, one might anticipate an appreciation in Bitcoin's price as lower carbon intensity attenuates the carbon premium. However, why do we observe such a substantial price decline? Is this situation puzzling?

In the following, I will explain how the combined effects of carbon intensity on network security and the carbon sensitivity of investors can account for this sharply negative market reaction. The decision made by the China State Council was an exogenous and destructive shock to Bitcoin miners who engage in hash production. One can interpret this exogenous shock as a destructive jump that follows a Poisson distribution. To capture this, I re-define the

[9] https://worldcoin.org/articles/china-crypto-ban

hash rate process as follows:

$$h_t^{wJ} = \underbrace{\mu + \phi h_{t-1}^w + \varepsilon_t^w}_{Normal\ \ Path} - \underbrace{\eta\mu Y}_{Poisson\ Jump} \tag{42}$$

here, η represents the jump size and Y follows a Poisson distribution with intensity θ given by:

$$P(Y = k) = \frac{e^{-\theta}\theta^k}{k!}$$

Assuming that the jump component and the normal error terms are uncorrelated, the hash rate process described by Eq. 42 has the following mean and variance:

$$\bar{h}^J = \frac{\mu - \eta\theta}{1 - \phi}$$

$$Var(h_t^{wJ}) = \frac{\sigma_h^2 + \eta^2\theta}{1 - \phi^2}$$

The impact of the Poisson shock on the hash rate is reflected in the energy intensity, as shown below:

$$e_t^w - \bar{e}^w = \frac{-\eta\mu Y + (h_t^w - \bar{h})}{g}$$

Assuming that all energy comes from fossil fuels ($e_t^w = \psi C_t^w$), the impact of the Poisson jump on carbon intensity can be interpreted as:

$$C_t^w - \bar{C}^w = \frac{-\psi\eta\mu Y + \psi(h_t^w - \bar{h})}{g}$$

Therefore, the impact of jump on carbon intensity would be (1) a reduction in mean carbon intensity and (2) an increase in the volatility of carbon intensity. The mean and variance of this impact are given by:

$$Mean = \frac{-\psi\eta\mu\theta}{g}$$

$$Variance = (\frac{\psi\eta\mu}{g})^2\theta$$

The above characterization demonstrates that the jump component influences the volatility of PoW cryptocurrency, its security (through affecting the hash rate), and its carbon intensity. Assuming a large sample size, the Central Limit Theorem allows us to approximate the Poisson distribution with a normal distribution (Ross 2014). Leveraging this property and the dynamics described above, I quantify the net effect of the Poisson jump on the expected return of PoW cryptocurrencies. Specifically, I evaluate the impact of the jump intensity (θ) and jump size (η) on the expected return.

$$\frac{\partial E_t[r^w_{t+1}]}{\partial \theta} = \underbrace{aX_w \left(\delta_3^2 (\beta\pi)^2 \frac{\eta^2}{1-\phi^2} + \delta_2^2 (\frac{\psi\eta\mu}{g})^2 - 2\delta_2\delta_3 \frac{\beta\pi\psi}{g} (\eta\mu)^2 \right)}_{\text{volatility effect}} + \underbrace{\delta_3\beta\pi\frac{\eta}{1-\phi}}_{\text{security effect}} - \underbrace{\delta_2\frac{\psi\eta\mu}{g}}_{\text{carbon effect}} \tag{43}$$

$$\frac{\partial E_t[r^w_{t+1}]}{\partial \eta} = \underbrace{aX_w \left(\delta_3^2 (\beta\pi)^2 \frac{2\eta\theta}{1-\phi^2} + 2\eta\theta (\frac{\delta_2\psi\mu}{g})^2 - 4\delta_2\delta_3 \frac{\beta\pi\psi}{g} \eta\theta\mu^2 \right)}_{\text{volatility effect}} + \underbrace{\delta_3\beta\pi\frac{\theta}{1-\phi}}_{\text{security effect}} - \underbrace{\delta_2\frac{\psi\theta\mu}{g}}_{\text{carbon effect}} \tag{44}$$

The proofs for Equations 43 and 44 are located in Appendix A. Both equations uncover three simultaneous effects of a jump on the expected return: the volatility effect, the security effect, and the carbon intensity effect. The volatility effect can be either positive or negative, contingent on the model parameters. The security effect is positive, signaling an increase in the expected return (i.e., price depreciation).[10] The carbon intensity effect is negative, resulting in a decrease in the expected return (i.e., price appreciation). Therefore, capturing the net effect is not straightforward. In the China case, the positive effect (price depreciation) dominates the negative effect (price appreciation). Hence, while the carbon intensity of Bitcoin declines after this event, the adverse impact on the Bitcoin platform's security outweighs the effect of the decline in carbon intensity.

[10] This is consistent with the finding of Pagnotta (2022)

2.2 The two-agent speculative ecosystem

In this section, I investigate the impact of speculative behavior on cryptocurrency pricing and carbon sensitivity. The environment considered is similar to the representative agent ecosystem, with one key difference: each generation has two agents with different beliefs about the expected returns. One agent is rational and optimizes her expected utility based on a subjective probability measure that aligns with the objective measure. The other agent is overconfident and optimizes her expected utility based on a subjective probability measure that deviates from the objective measure. As in prior literature, I refer to the overconfident agent as a speculator (Sockin and Xiong (2020); De Long, Shleifer, Summers, and Waldmann (1990)). In this environment, both agents receive the same set of information, but process it differently. Additionally, agents are non-cooperative and bet on the relative accuracy of their beliefs (David 2008). To simplify the analysis, it is assumed that investors hold heterogeneous beliefs only regarding the expected returns of cryptocurrencies.

Let W_t^i denote the endowment of investor type i in generation t where $i = R, O$. Here, R represents rational and O stands for overconfident. The aggregate wealth in the economy is given by $W_t = W_t^R + W_t^O$.

The rational agent uses the objective probability measure to solve the following optimization problem:

$$\underset{X,\,\delta_2}{Max}\, E_t[-e^{-A\left(\delta_1 W_{t+1}^R - \delta_2 W_t^R X' \mathbf{C_t} + \delta_3 W_t^R X' \lambda_\mathbf{t}\right)}]$$

$$s.t. \quad W_{t+1}^R = W_t^R(1 + X_s(r_{t+1}^s - f_t^s) + X_w(r_{t+1}^w - f_t^w) + X_e r_{t+1}^e)$$

Similar to the representative agent case, I can use the normal characteristic function to rewrite the above problem as follows:

$$\underset{X,\delta_2}{Max}\{-a_R[\delta_1(1 + X'_R(E_t[\mathbf{r_{t+1}}] - \mathbf{f_t})) - \delta_2 X'_R \bar{\mathbf{C}} + \delta_3 X'_R \bar{\lambda}] + \frac{1}{2} a_R^2 X'_R \Sigma_T X_R\}$$

where $E_t[\cdot]$ represents the expectation function under the objective probability measure conditional on information available at time t. The structure of optimal portfolio weight for the rational agent is identical to the optimal portfolio of the representative agent derived in the previous section:

$$X_R = \frac{1}{a_R}\Sigma_T^{-1}(\delta_1(E_t[\mathbf{r_{t+1}}] - \mathbf{f_t}) - \delta_2\bar{\mathbf{C}} + \delta_3\bar{\lambda})$$

where $a_R = AW_t^R$ represents the relative risk aversion of the rational investor. Also, the optimal carbon sensitivity of the rational investor would have the same structure as Eq. 41.

The overconfident investor solves the following optimization problem:

$$\underset{X,\hat{\delta}_2}{Max}\, E_t^O[-e^{-A\left(\delta_1 W_{t+1}^O - \hat{\delta}_2 W_t^O X' \, \mathbf{C_t} + \delta_3 W_t^O X' \, \lambda_\mathbf{t}\right)}] \quad (45)$$

$$s.t. \quad W_{t+1}^O = W_t^O(1 + X_s(r_{t+1}^s - f_t^s) + X_w(r_{t+1}^w - f_t^w) + X_e r_{t+1}^e)$$

where $\hat{\delta}_2$ represents the endogenous carbon sensitivity of the overconfident investor. Also, $E_t^O[\cdot]$ represents the expectation function under the subjective probability measure of the overconfident investor, conditional on the available information at time t.

A standard approach to solve a heterogeneous belief problem is to rewrite the expectation function of the overconfident agent using the objective probability measure. To proceed with this approach, I employ the Radon-Nikodym derivative, as detailed in Dumas, Kurshev, and Uppal (2009):

$$E_t^O[\mathbf{r_{t+1}}] = E_t[\xi \mathbf{r_{t+1}}] \quad (46)$$

where ξ represents the Radon-Nikodym derivative and can be interpreted as overconfident investor's (speculator's) sentiment (see Sockin and Xiong (2020)). Also, one can write relation between the perceived variance-covariance matrix in the view of overconfident and the variance-covariance under the objective probability measure as below:

$$\Sigma_T^O = \Lambda \Sigma_T \quad (47)$$

where Λ is a 3×3 matrix that captures the deviation of the perceived variance-covariance matrix from its objective value in the view of the overconfident investor.

The normal characteristic function implies that the overconfident agent solves the following optimization problem:

$$\underset{X,\hat{\delta}_2}{Max}\{-a_O[\delta_1(1+X'_O(E_t[\xi\mathbf{r_{t+1}}]-\mathbf{f_t}))-\hat{\delta}_2X'_O\bar{\mathbf{C}}+\delta_3(X'_O\bar{\lambda}_\mathbf{t})]+\frac{1}{2}a_O^2X'_O(\Lambda\Sigma_T)X_O\} \quad (48)$$

The first order condition with respect to the portfolio weight gives the optimal portfolio of the overconfident agent:

$$X_O=\frac{1}{a_O}\Sigma_T^{-1}\Lambda^{-1}(\delta_1(E_t[\xi\mathbf{r_{t+1}}]-\mathbf{f_t})-\hat{\delta}_2\bar{\mathbf{C}}+\delta_3\bar{\lambda}) \quad (49)$$

In the following, I elaborate on two points. Firstly, the carbon sensitivity of the overconfident investor depends on her sentiment (i.e., speculative motive). Second, as the overconfident investor becomes more optimistic and speculates more aggressively, her carbon sensitivity weakens.

To characterize the equilibrium expected return and the optimal endogenous carbon sensitivity, I start by defining the following:

$$E_t[\xi\mathbf{r_{t+1}}]=E_t[\mathbf{r_{t+1}}]-\mathbf{B_t} \quad (50)$$

$$where \quad \mathbf{B_t}=E_t[(\xi-\mathbf{1})\mathbf{r_{t+1}}]$$

$\mathbf{B_t}$ reflects the impact of speculation on the expected return in the market, also regarded as the ”bubble” component. Next, I derive the optimal carbon sensitivity in the following proposition.

Proposition 4: In generation t, the optimal aggregate carbon sensitivity has the following structure:

$$\delta_2^{agg}=\frac{\delta_1(\sigma_{rC}-\frac{W_t^O}{W_t}\sigma_{BC})+\delta_3\sigma_{C\lambda}+\frac{E_t[Cap_t]}{aW_t}}{\sigma_C^2} \quad (51)$$

$$where \quad \sigma_{BC} = X'E[(\tilde{\mathbf{B}}_\mathbf{t} - \mathbf{B}_\mathbf{t})'(\tilde{\mathbf{C}}_\mathbf{t} - \bar{\mathbf{C}})]X$$

Proof: See Appendix A

Let's define $\kappa = \frac{\sigma_{BC}}{\sigma_C^2}$ represent the disparity in carbon sensitivity from the perspective of an overconfident investor, as opposed to when the same investor adopts a rational viewpoint. As evident, this disparity is induced by speculation, underscoring that "the carbon sensitivity of the overconfident investor depends on her sentiment (i.e., speculative motive)." In the following, I derive the equilibrium expected return by imposing the market-clearing condition:

$$W_t X_M = W_t^O X_O + W_t^R X_R \tag{52}$$

Proposition 5: In generation t, the equilibrium expected return in the speculative environment follows:

$$E_t[\mathbf{r}_{\mathbf{t+1}}] = \frac{a}{\delta_1}\hat{\Sigma}_T X_M - (\mathbf{I} + \Lambda)^{-1}(\mathbf{B}_\mathbf{t} + \frac{\kappa}{\delta_1}\bar{\mathbf{C}}) - (\frac{\delta_3}{\delta_1}\bar{\lambda} - \mathbf{f}_\mathbf{t}) + \frac{\delta_2}{\delta_1}\bar{\mathbf{C}} \tag{53}$$

where $\hat{\Sigma}_T = \Phi\Sigma_T$ represents the adjusted matrix of Σ_T. The adjustment factor is $\Phi = \frac{1}{\det(\mathbf{I}+\Lambda^{-1})} adj(\mathbf{I} + \Lambda^{-1})$, and $adj(.)$ represents the adjugate matrix.

The CAPM-like relation has the following structure:

$$E_t[\mathbf{r}_{\mathbf{t+1}}] = \beta E_t[\mathbf{r}_{t+1}^M] - (\mathbf{I} + \Lambda)^{-1}(\mathbf{B}_\mathbf{t} + \frac{\kappa}{\delta_1}\bar{\mathbf{C}}) - (\frac{\delta_3}{\delta_1}\bar{\lambda} - \mathbf{f}_\mathbf{t}) + \frac{\delta_2}{\delta_1}\bar{\mathbf{C}} \tag{54}$$

where $\beta = \frac{\hat{\Sigma}_T X_M}{\widehat{\sigma}^2}$.

Proof: See Appendix A.

An immediate implication of Proposition 5 is that the disparity between the carbon sensitivity of overconfident and rational investors is priced in the market. Building upon the findings of Proposition 4, I can now prove the argument I presented earlier in this section: "as the overconfident investor becomes more optimistic and speculates more aggressively, her carbon sensitivity weakens." When an overconfident investor is relatively more optimistic, she assigns a higher weight to desirable returns and a lower weight to undesirable returns compared to a rational investor. To characterize this behavior, I will approximate the normal distribution of returns with a binomial path. Thus, assuming that we observe two states: an "Up" state with positive return denoted by $r^U > 0$, and a "Down" state with negative return denoted by $r^D < 0$. Both states are equally likely. This approximation implies that:

$$E_t[r_{t+1}] = \frac{1}{2}r^U_{t+1} + \frac{1}{2}r^D_{t+1} \tag{55}$$

To ensure that no arbitrage opportunity exists, The expected return must be positive which implies that $r^U_{t+1} > \left|r^D_{t+1}\right|$. In the view of overconfident investor, the expected return has the following structure:

$$E^O_t[r_{t+1}] = \frac{1}{2}\xi_1 r^U_{t+1} + \frac{1}{2}\xi_2 r^D_{t+1} \tag{56}$$

where $\{\xi_1, \xi_2\}$ are discrete states counterpart of Radon-Nikodym derivative ξ which was defined earlier. Optimism in overconfident view implies that $\xi_1 > 1$ and $\xi_2 < 1$. The properties of Radon-Nikodym derivative and some further derivations are provided in the Appendix A. Intuitively, an optimistic overconfident assigns higher probability to "Up" state which is desirable and lower probability to "Down" state which is undesirable. Now, by considering this characterization, and the definition of $\kappa = \frac{\sigma_{BC}}{\sigma^2_C}$, I develop the following proposition.

Proposition 6: When the overconfident investor is relatively more optimistic and speculating in the market (i.e. $\xi_1 > 1$ and $\xi_2 < 1$), we observe that $\kappa > 0$, which indicates a weakening of carbon sensitivity.

Proof: See Appendix A.

Since the aggregate carbon sensitivity in the market is the wealth-weighted average of the carbon sensitivity of each type, i.e., $\delta^{agg}_2 = \frac{W^R_t}{W_t}\delta_2 + \frac{W^O_t}{W_t}\hat{\delta}_2$, a weakening in the carbon sensitivity of the overconfident investor would lead to a weakening of the aggregate carbon sensitivity in the market as well.

3 Some empirical evidence

The theoretical findings of the paper are based on the assumption that investors are carbon-sensitive. While this hypothesis is empirically well-supported in the equity market, its applicability to the cryptocurrency market has not yet been tested. Additionally, from a theoretical perspective, introducing a new preference parameter into a model influences price behavior. Thus, providing empirical evidence for the actual existence of this preference is imperative to substantiate the credibility of the theoretical findings. To this end, I present empirical evidence to support the notion of carbon sensitivity within the cryptocurrency market. Firstly, I employ the Instrumental Variable approach to demonstrate that carbon sensitivity is factored into cryptocurrency pricing. Secondly, I offer further supporting evidence for carbon sensitivity by analyzing the Ethereum Beacon Chain merge event.

3.1 Impact of carbon sensitivity on cryptocurrency prices

This section provides empirical evidence supporting the existence of carbon sensitivity in the cryptocurrency market. There are three important considerations in this analysis. Firstly, carbon sensitivity is not directly observable, necessitating the use of a proxy to capture this preference parameter.

Secondly, employing a simple OLS regression analysis could yield misleading results under certain conditions. For instance, in times of increased carbon sensitivity, a carbon-intensive cryptocurrency might offer significant non-pecuniary transactional benefits. However, since an econometrician cannot directly measure or observe these benefits, and given that the carbon alpha and the alpha for these non-pecuniary benefits have opposing signs, an OLS analysis may face endogeneity issues.

Third, the structure of the cryptocurrency market is significantly different from that of the equity market. In the equity market, there are a large set of stocks, and no single stock has the majority of market share. Thus, by constructing portfolios and calculating returns, we can reduce the noise and obtain reliable results. Unlike the equity market, the cryptocurrency market is highly volatile and has a dominant player, which is Bitcoin. Since its emergence in 2009, Bitcoin has always held more than 50% of the cryptocurrency market share. Combined with Ethereum, their market share has sometimes exceeded 85% of the total. In addition, smaller cryptocurrencies are highly correlated with Bitcoin and Ethereum, and even if one constructs an equally weighted portfolio, the noise component would still remain pronounced. Furthermore, since PoW cryptocurrencies are mined in different areas of the world using various energy sources and mining rigs, unlike equities, estimating their carbon emissions is not straightforward.

In light of these considerations, I adopt an empirical approach, which is detailed below, to test whether cryptocurrency investors are carbon sensitive. To address the first consideration, I draw on previous research on attention, such as the studies conducted by Andrei and Hasler (2015). They argue that when investors are sensitive to a particular topic, new information causes fluctuations in their attention toward that topic. Conversely, if there is no sensitivity to an issue, new information is unlikely to shift investors' attention. Therefore, following the footsteps of previous studies,[11] I use Google search volume for carbon-related topics as a proxy for investors' carbon attention. Fluctuations in carbon-related search volume can indicate investors' sensitivity to the impact of carbon emissions. My goal is to demonstrate whether fluctuations in carbon attention, induced by carbon sensitivity, are reflected in the pricing of PoW cryptocurrencies. To achieve this, I collect data on investors' attention towards topics such as "carbon emission," "greenhouse gas emission," "climate," "environment," and "ESG" to capture the impact of investors' carbon sensitivity.

[11] See, for example, Andrei and Hasler (2015), Chen, Kumar, and Zhang (2020), and Choi, Gao, and Jiang (2020), among others

To mitigate potential endogeneity concerns and obtain a reliable estimate of the relationship between carbon-intensive PoW cryptocurrency prices and investors' carbon sensitivity, I employ an instrumental variable approach. The estimation results reveal that an increase in investors' carbon attention leads to a decrease in the price of these cryptocurrencies. This decline in price suggests that investors demand a higher expected return for investing in PoW cryptocurrencies, reflecting their carbon sensitivity.

I focus exclusively on four well-established PoW cryptocurrencies: Bitcoin, Ethereum, Litecoin, and Dogecoin.[12] One of the reasons for selecting these cryptocurrencies is the availability of extensive data, with at least five years of historical information (equivalent to around 60 monthly observations) for each. This data richness allows for more robust and reliable results. Additionally, these cryptocurrencies are deeply established in the market and exhibit relatively lower volatility compared to smaller counterparts, which often have market caps less than 1 billion dollars. Consequently, they are less susceptible to speculative behavior, market manipulation, and trading activities aimed at exploiting arbitrage opportunities.

3.1.1 Data

I merge two data sources to conduct the empirical analysis. Data for the prices and hash rates of Bitcoin, Ethereum, Litecoin, and Dogecoin are obtained from coinmetrics.io. Additionally, I collect data on market capitalization, realized volatility (RV), and the number of active wallet addresses for these four cryptocurrencies to control for various factors in the analysis. This data is available on a daily frequency. To align with the Google search data and ensure consistency, I aggregate this daily data into a monthly dataset for a comprehensive analysis.

Additionally, to analyze the impact of the Beacon Chain merge, I gather data on the number of new wallet addresses and the exchange deposits for Bitcoin and Ethereum from coinmetrics.io. These exchange deposits represent the inflow of funds into these cryptocurrencies

[12] It's important to note that Ethereum underwent a significant transition to a PoS system with the Beacon Chain merge in September 2022.

through exchanges. I retain the daily frequency for these two features.

The data collection spans five years, from January 2018 to the January 2023. For Bitcoin, Litecoin, and Dogecoin, the dataset includes 60 observations, representing monthly data. However, for Ethereum, there is a gap in the data following the Beacon Chain merge in September 2022. Consequently, I exclude observations from September 2022 onward for Ethereum. For the specific analysis of the Beacon Chain merge, I collect daily data from July 2022 to November 2022, which captures approximately two and a half months before and after the merge. Its crucial to note that extending this data range might lead to unreliable results, as a longer range could include other events that might influence Ethereum or the broader crypto market. A summary of the statistics for the cryptocurrency features is presented in Table 1.

Variables	**Mean**	**Std. dev.**	**Min**	**Max**
BTC price (USD)	20509.08	16820.82	3659.201	60684.55
ETH price (USD)	1161.375	1190.12	107.5334	4442.732
LTC price (USD)	97.3452	56.98269	28.58892	263.289
DOGE price (USD)	0.0639779	0.0958728	0.0019219	0.4418706
BTC cap (USD)	3.83e+11	3.20e+11	6.39e+10	1.15e+12
ETH cap (USD)	1.34e+11	1.41e+11	1.12e+10	5.22e+11
LTC cap (USD)	6.37e+09	3.83e+09	1.70e+09	1.78e+10
DOGE cap (USD)	8.43e+09	1.26e+10	2.27e+08	5.72e+10
BTC hash (MH/s)	1.24e+08	7.16e+07	2.21e+07	2.97e+08
ETH hash (MH/s)	391.808	296.9157	137.6272	1027.384
LTC hash (MH/s)	310.8362	121.8788	136.6777	681.0606
DOGE hash (MH/s)	293.1968	129.1495	102.8585	685.1851
BTC 30-day RV	0.0410665	0.0243679	0.0098636	0.1438445
ETH 30-day RV	0.0508009	0.0219743	0.0149183	0.1168316
LTC 30-day RV	0.052693	0.0300589	0.0080375	0.2177028
DOGE 30-day RV	0.0573091	0.0381475	0.015616	0.3018972
BTC new addresses	407175.2	35324.86	311779	523145
ETH new addresses	95098.56	38228.53	58897	228738
BTC exchange deposit (USD)	4.94e+08	5.28e+08	1.15e+08	5.76e+09
ETH exchange deposit (USD)	4.06e+08	2.99e+08	3.81e+07	2.39e+09

Table 1. Summary of statistics for cryptocurrencies The table provides monthly statistics for price, market cap, hash rate, and realized volatility for four cryptocurrencies: Bitcoin, Dogecoin, Ethereum, and Litecoin. Additionally, the table includes daily statistics for the number of new wallet addresses and the exchange deposits into BTC and ETH, presented in the bottom four rows.

I collect data on Google search volume to construct a proxy for investors' attention toward a topic. This methodology aligns with previous studies such as Andrei and Hasler (2015), Chen, Kumar, and Zhang (2020), and Choi, Gao, and Jiang (2020), among others. I then aggregate the weekly data to create a monthly dataset for the following topics: carbon emissions, greenhouse gas emissions, ESG, climate, and environment. This approach allows to infer investor sensitivity towards carbon emission-related issues. To represent the level of interest in these topics, Google normalizes the data relative to the highest point within a specific time interval, where a value of 100 represents the peak interest, and a value of 0 represents the least interest in a given topic.

In addition to the above-mentioned topics, inspired by the approach of Liu and Tsyvinski (2021), I also consider investors' negative attention to infer about their perception toward security. I collect data on Google search volume for topics such as ”Bitcoin hack”, ”Ethereum hack”, ”Litecoin hack”, and ”Dogecoin hack.” Attention toward these topics reflects investors' perception of the network security in these PoW cryptocurrencies. High search volumes for these terms suggest increased investor concerns about the security of these PoW cryptocurrencies. By aggregating weekly data, I produce a monthly average of relative interest in each topic. Table 2 represents a summary statistics of the variables and parameters that have been used in this paper.

Variables	**Mean**	**Std. dev.**
carbon Emission	44.479	21.146
Environment	44.799	9.514
Climate	55.668	17.849
ESG	36.758	25.07
Greenhouse Gas Emission	53.098	15.536
Bitcoin Hack	16.99098	7.746626
Ethereum Hack	19.93607	11.59983
Dogecoin Hack	5.580328	5.974714
Litecoin Hack	12.06844	10.67873

Table 2. Summary of statistics for attention The statistics for attention toward the five topics of carbon emission, climate, ESG, greenhouse gas emission and environment are presented. Also, attention toward hack related topics are presented

3.1.2 2SLS regression

Hash rates and prices are non-stationary; therefore, I transform them into their natural logarithms to achieve stationarity. My aim is to construct an instrument that impacts PoW cryptocurrency prices solely through the carbon sensitivity channel. To construct this instrumental variable, I draw on the microfoundations of PoW cryptocurrencies outlined in the theory section. As discussed in Eq. 4 and 12, the network hash rate is linked to both network security and energy intensity (carbon intensity). Considering this, first I run the regression of log hash rate on attention toward the topic of hack in different cryptocurrencies. As discussed earlier, attention toward the topic of hacks reflects investors' perception of network security in a given cryptocurrency. Let $Hash_t^i$ denote the log of hash rate for cryptocurrency i at time t where $i = BTC, ETH, LTC, DOGE$. Then, I run the the following regression for each cryptocurrency.

$$Hash_t^i = \alpha + \beta Hack\_Attention_t^i + \varepsilon_t^i \tag{57}$$

Taking the residuals from regression 57 isolates the component of the hash rate that is uncorrelated with perceptions toward network security. Consequently, this component is solely reflective of the energy intensity and carbon intensity in PoW cryptocurrencies. Let's call this residual component $Residual_t^i$.

Now, I run 2SLS regression using $Residual_t^i$ as the instrument. The first stage regression is:

$$Attention_t = \alpha_{0i} + \alpha_{1i} Residual_t^i + \alpha_{2i} Controls_t^i + \varepsilon_t^i \tag{58}$$

The second stage is as follows:

$$Price_t^i = \beta_{0i} + \beta_{1i} \widehat{Attention}_t + \beta_{2i} Controls_t^i + e_t^i \tag{59}$$

The coefficient of interest is β_{1i}. In both stages of the analysis, I control for the size of the cryptocurrency, which is represented by the natural logarithm of market capitalization. Additionally, I account for the 30-day realized volatility of returns, serving as an indicative measure of speculative activities. A rise in speculative activity often leads to heightened volatility and vice versa, as elaborated by Dumas, Kurshev, and Uppal (2009). In Appendix B, I repeat the analysis with different sets of control variables and demonstrate that the results reported in the main text remain robust.

If β_{1i} becomes negative, it implies that stronger attention to carbon emissions has a negative impact on the price (depreciates the price) of PoW cryptocurrencies. Such a finding lends support to the assumption that investors are carbon-sensitive.

Table 3. BTC 2SLS estimation

	(1) BTC Price	(2) BTC Price	(3) BTC Price	(4) BTC Price	(5) BTC Price	(6) BTC Price
Carbon Emission	-0.1913672*** (-3.81)					
Environment		-0.4886356** (-2.36)				
Climate			-0.121581*** (-4.38)			
GHG Emission				-0.1427119*** (-4.32)		
ESG					-0.0837787*** (-12.55)	
PC(1)						-0.0263804 *** (-6.01)
Constant	-16.22733*** (-84.36)	-14.54274*** (-24.29)	-15.63318*** (-107.42)	-16.12167*** (-102.33)	-16.86588*** (-174.86)	-16.72143*** (-92.38)
Controls	YES	YES	YES	YES	YES	YES
Observations	61	61	61	61	61	61
$F-stat$	10.8704	5.54038	18.6033	14.533	57.9809	27.1401

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.1.3 Effect of carbon sensitivity on Bitcoin price

In this section, I present the results of 2SLS estimation for Bitcoin. The results, as shown in Table 3, reveal a strong negative relation between investors' attention to carbon emission, greenhouse gas emission, ESG, climate, and environment and the price of Bitcoin. I also construct a composite measure for carbon attention by taking the first principal component of attention to these five topics. Column (6) presents the regression results using this composite measure. These findings support the assumption that cryptocurrency investors are carbon-sensitive. All coefficients are statistically significant, and the Wald test confirms that the F-stat is greater than 10 in all regressions with the exception of "Environment", indicating that $Residual_t^{BTC}$ is a valid instrument for carbon attention. This exception suggests that $Residual_t^{BTC}$ is not as effective an instrument for gauging attention to "Environment" as it is for the other categories. This discrepancy may be attributed to the fact that environmental topics cover a broad spectrum, many aspects of which are not directly associated with carbon emissions or climate change.

3.1.4 Effect of carbon sensitivity on Ethereum price

In this section, I presents the results of 2SLS estimation for Ethereum. The results, as shown in Table 4, also reveal a strong negative relationship between the attention to carbon emission, greenhouse gas emission, ESG, climate, and environment and the price of Ethereum. Column (6) presents the regression results using the composite measure. These findings further support the assumption that investors are sensitive to carbon emissions, similar to Bitcoin. It is important to note that I exclude data from September 2022 onwards due to the Beacon Chain merge, which has transitioned Ethereum into an environmentally friendly cryptocurrency. The Wald test reveals a relatively high F-stat for all attention variables, with the exception of "Environment". This suggests that a segment of climate attention may pertain to topics not directly linked to the topic of Environment.

Table 4. ETH 2SLS estimation

	(1) ETH Price	(2) ETH Price	(3) ETH Price	(4) ETH Price	(5) ETH Price	(6) ETH Price
Carbon Emission	-0.2433865***					
	(-5.19)					
Environment		-1.431186				
		(-1.39)				
Climate			-0.1845539***			
			(-4.83)			
GHG Emission				-0.2042883***		
				(-5.16)		
ESG					-0.1228259***	
					(-18.85)	
PC(1)						-0.0404422***
						(-6.72)
Constant	-17.67146***	-13.62421***	-17.11804***	-17.79246***	-18.50761***	-18.59822***
	(-126.76)	(-4.52)	(-87.04)	(-127.15)	(-331.43)	(-111.52)
Controls	YES	YES	YES	YES	YES	YES
Observations	58	58	58	58	58	58
$F-stat$	23.4755	1.79966	18.6319	26.5833	63.2226	35.8544

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.1.5 Effect of carbon sensitivity on Litecoin price

In this section, I present the results of 2SLS estimation for Litecoin. The results, as shown in Table 5, reveal a a strong negative relation between price and and carbon attention similar to Bitcoin and Ethreum. Here, we also observe an insignificant coefficient for attention to climate. In other cases, coefficients are significant. Also, the Wald test represents a relatively high F-stat, implying that the hash rate is a valid instrument for carbon attention.

Table 5. LTC 2SLS estimation

	(1) LTC Price	(2) LTC Price	(3) LTC Price	(4) LTC Price	(5) LTC Price	(6) LTC Price
Carbon Emission	-0.2588251***					
	(-5.79)					
Environment		-0.5612355***				
		(-3.74)				
Climate			1.125417			
			(0.74)			
GHG Emission				-0.1771417***		
				(-6.24)		
ESG					-0.1264595***	
					(-15.10)	
PC(1)						-0.0415502***
						(-7.45)
Constant	-17.14847***	-15.45295***	-24.74867**	-17.10713***	-18.07457***	-17.94431***
	(-60.88)	(-24.18)	(-2.37)	(-65.38)	(-147.52)	(-74.62)
Controls	YES	YES	YES	YES	YES	YES
Observations	61	61	61	61	61	61
$F - stat$	20.7027	19.8869	0.637196	44.3142	27.905	29.7491

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.1.6 Effect of carbon sensitivity on Dogecoin price

In this section, I present the results of 2SLS estimation for Dogecoin. In Table 6, we also observe a strong negative relationship between carbon attention and DOGE price. However, similar to ETH and LTC, the coefficient of climate attention is insignificant. The Wald test indicates weak correlation between DOGE hash rate and attention toward carbon emission and greenhouse gas emission.

Table 6. DOGE 2SLS estimation

	(1) DOGE Price	(2) DOGE Price	(3) DOGE Price	(4) DOGE Price	(5) DOGE Price	(6) DOGE Price
Carbon Emission	-0.2430837***					
	(-2.97)					
Environment		-0.4071554**				
		(-2.33)				
Climate			0.3945112			
			(0.80)			
GHG Emission				-0.13016***		
				(-3.89)		
ESG					-0.1156866***	
					(-11.66)	
PC(1)						-0.0349909**
						(-4.40)
Constant	-24.76878***	-23.79735***	-26.25048***	-24.92615***	-25.33594***	-25.57466***
	(-235.10)	(-45.20)	(-16.65)	(-443.79)	(-762.82)	(-186.68)
Controls	YES	YES	YES	YES	YES	YES
Observations	61	61	61	61	61	61
$F-stat$	6.49274	7.26493	0.808558	19.2462	13.5639	12.1386

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.1.7 Instrument validity: exclusion restriction

In this section, my aim is to demonstrate that the proposed instrument satisfies the exclusion restriction condition, meaning it is uncorrelated with other factors influencing cryptocurrency prices.

For this purpose, I regress the proposed instrument on proxies indicative of speculative activities and fundamental valuation in PoW cryptocurrencies. Following a large body of literature, including Dumas, Kurshev, and Uppal (2009), I use 30-days realized volatility as a reflective measure of speculative activities in the cryptocurrency market. Higher realized volatility is indicative of increased speculative activities, which in turn influence prices. Also, in line with Liu and Tsyvinski (2021) and Lashkaripour (2023), I consider the number of active wallet addresses as a component that captures the fundamental value of a given cryptocurrency. Users engaged in on-chain transactions, as opposed to merely trading on exchanges, directly benefit from transactional advantages like swift cross-border money transfers. Hence, the number of active wallet addresses can serve as an indicator of the fundamental value in cryptocurrencies. For some cryptocurrencies in our sample, data on the number of active wallet addresses is unavailable. Therefore, I use the market value to realized value (MVRV) ratio as an alternative measure. The realized value in cryptocurrencies refers to the value of the circulating supply. A low MVRV ratio suggests that a significant portion of the current cryptocurrency supply is engaged in on-chain activities, reflecting fundamental market activities. Considering this explanation, I run the following regression:

$$Residual_t^i = \phi_0 + \phi_1 RV_t^i + \phi_2 Fundametal_t^i + \varepsilon_t^i \tag{60}$$

Here, RV_t^i refers to the 30-days realized volatility of cryptocurrency i at time t where $i = BTC, ETH, LTC, DOGE$. Also, $Fundametal_t^i$ denotes either the number of active wallet addresses or the MVRV ratio of cryptocurrency i at time t. The result of the regression are presented in Table 7.

The findings presented in Table 7 reveal that all coefficients are statistically insignificant, except for Litecoin. This general lack of significance suggests that the instrument does not correlate with the proxies for speculative activities or the fundamental value of cryptocurrencies, thereby reinforcing its validity as a reliable instrument. The significant coefficient of 30-day realized volatility for Litecoin should not affect the 2SLS results, as this variable has been used as a control variable in both stages.

Table 7. Instrument validity: Exclusion restriction

	(1)	(2)	(3)	(4)
	Bitcoin Residual	Ethereum Residual	Litecoin Residual	Dogecoin Residual
30-days RV	-9.28288	-6.013041	-11.35569***	2.243628
	(-1.57)	(-1.12)	(-3.38)	(1.30)
# New Wallets	-3.35e-06	-1.36e-06		
	(-0.49)	(-0.49)		
# Active wallets	8.74e-06			
	(1.45)			
MVRV	-0.3126974	0.0866549	0.0661064	-0.2338657
	(-1.48)	(0.45)	(0.64)	(-1.25)
Constant	-1.760304***	0.2951925	0.5309032***	0.1399178
	(-2.95)	(0.76)	(2.78)	(0.92)
Observations	61	58	59	58
$R-Square$	0.3889	0.0314	0.2019	0.0488

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3.2 Evidence from Beacon Chain merge

The Beacon Chain merge refers to the process of integrating the Beacon Chain, a PoS blockchain, with the Ethereum mainnet, thereby replacing the existing Ethereum PoW mechanism. On August 24, 2022, the Ethereum Foundation Blog announced that the transition to proof-of-stake would be activated on September 15, 2022.[13] As a result of this merge, the energy consumption of Ethereum dropped by almost 99.95%.[14]

In this section, I aim to examine the response of investors to this significant event. If cryptocurrency investors are indeed carbon-sensitive, a notable reaction to this event should be evident. In order to provide insight, I undertake a comparative analysis between Ethereum (which transitioned from being carbon-intensive to carbon-neutral) and Bitcoin (the leading carbon-intensive cryptocurrency), both before and after the announcement of the Beacon Chain merge.

[13] https://blog.ethereum.org/2022/08/24/mainnet-merge-announcement
[14] https://ethereum.org/en/energy-consumption/

Specifically, the primary focus is to determine whether investors exhibit a noticeable increase in interest and engagement with Ethereum compared to Bitcoin after the merge. I provide the following evidence using a subset of the data described in Section 3.1.1.

Initially, I present a first-hand evidence by regressing the exchange rate of Ethereum to Bitcoin on a time dummy variable. This dummy variable takes the value of zero for pre-announcement dates and one for post-announcement dates of the Beacon Chain merge. I control for size (log of market capitalization) along with time trend by including the the 3-day Moving Average of the exchange rate as a control variable. Table 8 represents the result of the regressions.

Table 8. Evidence from Beacon Chain merge

	(1) (ETH price/BTC price)
Time Dummy	0.0014382***
	(3.01)
MA(ETH price/BTC price)	0.6950107***
	(16.14)
Size	0.0119667***
	(6.42)
Constant	-0.2884263***
	(-6.26)
Observations	142
R^2	0.8957

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The positive coefficient of the time dummy underscores an increase in the ETH to BTC exchange rate, signifying a surge in demand for Ethereum compared to Bitcoin following the Beacon Chain merge. The rise in the exchange rate suggests that Ethereum becomes relatively more valuable compared to Bitcoin, potentially due to a reduction in the carbon premium associated with Ethereum since carbon-sensitive investors now regard Ethereum as a cryptocurrency with low carbon intensity.

In what follows, I present evidence that highlights the increase in investors' interest and demand for Ethereum following the Beacon Chain merge. To substantiate this, I use two proxies for demand. Firstly, I consider the number of new wallet addresses as an indicator of investors' interest in investing and transacting on the Ethereum network. Secondly, acknowledging that some investors trade Ethereum on exchanges like Binance and Kraken, I use new deposits into Ethereum via these exchanges as an alternative measure of investor demand. I apply z-score normalization to both the number of new wallet addresses and new deposits. Then, I run the following two regressions.

$$
\begin{aligned}
log\left(\frac{ETH\_price_t}{BTC\_price_t}\right) &= a_0 + a_1 Time\_Dummy_t + a_2 New\_Addr_t \\
&+ a_3\left(New\_Addr_t \times Time\_Dummy_t\right) + a_4 ETH\_Size_t + \varepsilon_t
\end{aligned}
$$

$$
\begin{aligned}
log\left(\frac{ETH\_price_t}{BTC\_price_t}\right) &= b_0 + b_1 Time\_Dummy_t + b_2 Excg\_Dep_t \\
&+ b_3\left(Excg\_Dep_t \times Time\_Dummy_t\right) + b_4 ETH\_Size_t + \varepsilon_t
\end{aligned}
$$

The regression results are presented in Table 9. The first regression shows that the demand for Ethereum, triggered by the Beacon Chain merge, has had a significant positive impact on the Ethereum-to-Bitcoin exchange rate. Specifically, the coefficient of the interaction term reveals a noticeable influence of the demand on the post-merge exchange rate. When compared to the entire sample, we observe a change in sign and an increase in magnitude, indicating a shift in investors' behavior and a stronger inclination to invest in Ethereum after the event. In the second regression, we observe a similar pattern. The Beacon Chain merge has led to an increase in the exchange rate, and the coefficient of the interaction term confirms that investors' demand for Ethereum after the event has played a significant role in driving up the exchange rate. These results provide evidence that investors have responded to the event, and their interest in Ethereum has significantly increased. Therefore, we can conclude that the energy efficiency of Ethereum has become a major consideration for investors, and the improvement in energy efficiency has stimulated them to reallocate their investments toward Ethereum.

Table 9. Demand for Ethereum

	(1)	(2)
	Log excg rate	Log excg rate
Time Dummy	0.0082413***	0.0063679***
	(7.36)	(10.76)
New addr	-0.0029583*	
	(-1.67)	
Excg dept		-0.0013314 *
		(-1.90)
Time Dummy*New addr	0.0041912 **	
	(2.31)	
Time Dummy*Excg dept		0.0021722***
		(2.89)
Size	0.0394412***	0.0343309 ***
	(19.13)	(17.78)
Constant	-0.9546452***	-0.8204789***
	(-17.77)	(-16.36)
Observations	142	142
R^2	0.7407	0.7260

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

4 Conclusion

Cryptocurrency production, commonly referred to as mining, requires significant energy use, leading to a substantial carbon footprint. In this study, I introduce a stylized general equilibrium model to examine the impact of carbon intensity on cryptocurrency pricing. I identify two main pricing effects. First, ceteris paribus, the carbon premium in cryptocurrencies is lower compared to that in equities. Second, the beta exposure to the market portfolio intensifies as the carbon intensity of cryptocurrencies increases. However, this level of beta exposure diminishes when miners use renewable resources for energy generation. The first effect arises as higher carbon intensity indicates stronger network security, thus reducing the security risk in the decentralized network of PoW cryptocurrencies, resulting in a negative risk premium. The second effect, which can be interpreted as transition risk, is an indirect outcome of a cap on carbon emissions and investors' carbon sensitivity.

Given the presence of unobserved confounding factors, such as unobservable transactional benefits, testing the implications of the theory is not straightforward. Hence, I employ Instrumental Variable approach to assess the influence of investors' carbon sensitivity. I consider investors' attention to carbon and climate-related topics as an indicator of their carbon sensitivity. Empirically, I uncover a strong negative relationship between investors' attention to carbon or climate-related topics, such as carbon emissions, ESG, greenhouse gas emissions, environment, and climate, and the prices of major carbon-intensive cryptocurrencies. This empirical finding supports the theoretical prediction of a negative premium associated with carbon sensitivity. I also provide further evidence from the Ethereum Beacon Chain merge to confirm the existence of carbon sensitivity in the cryptocurrency market. I also demonstrate that speculative behavior in the cryptocurrency market weakens investors' carbon sensitivity.

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A Appendix A

A.1 Proof of Equation 19:

Optimizing the Eq. 18 is equivalent to optimizing the following equation:

$$\underset{X}{Max}\{-a[\delta_1(1+X'(E_t[\mathbf{r_{t+1}}]-\mathbf{f_t}))-\delta_2 X'\,\bar{\mathbf{C}}+\delta_3(X'\,\bar{\lambda})]+\frac{1}{2}a^2 X'\Sigma_T X\} \tag{61}$$

Now, taking first order condition with respect to vector X and restructuring completes the proof of Equation 19.

A.2 Proof of Equations 20, 21, and 22:

Using Eq. 19, one can write the following:

$$a\Sigma_T X_M = \delta_1(E_t[\mathbf{r_{t+1}}]-\mathbf{f_t})-\delta_2\bar{\mathbf{C}}+\delta_3\bar{\lambda} \tag{62}$$

A simple restructuring gives us:

$$E_t[\mathbf{r_{t+1}}] = \frac{a}{\delta_1}\Sigma_T X_M + \mathbf{f_t} + \frac{\delta_2}{\delta_1}\bar{\mathbf{C}} - \frac{\delta_3}{\delta_1}\bar{\lambda} \tag{63}$$

Now, using the basis vector, one can easily prove the findings of Equations 20, 21, and 22.

A.3 Proof of Proposition 1:

Let's multiply the vector of market portfolio to both sides of Eq. 62. We have:

$$aX'_M\Sigma_T X_M = \delta_1(X'_M E_t[\mathbf{r_{t+1}}]-X'_M\mathbf{f_t})-\delta_2 X'_M\bar{\mathbf{C}}+\delta_3 X'_M\,\bar{\lambda} \tag{64}$$

Considering $X'_M \Sigma_T X_M = \sigma_M^2$ as the variance of the market portfolio on the left-hand-side, one can write the following:

$$\frac{a}{\delta_1}\sigma_M^2 = E_t[r_{t+1}^M] - X'_M \mathbf{f_t} - \frac{\delta_2}{\delta_1} X'_M \bar{\mathbf{C}} + \frac{\delta_3}{\delta_1} X'_M \bar{\lambda} \tag{65}$$

where $E_t[r_{t+1}^M]$ is the expected return of the market portfolio. Let's define $\bar{C}_M = \frac{1}{a} X'_M (\delta_2 \bar{C})$ which represents the average carbon emissions premium of the market portfolio ($X'_M C_t$) adjusted by the relative risk aversion. Also, let's define $\bar{T}_M = \frac{1}{a} X'_M (\delta_3 \bar{\lambda} - \delta_1 \mathbf{f}_t)$ which represents the average of net transactional benefits premium of the market portfolio normalized by relative risk aversion. Now, we can define $\hat{\sigma}_M^2 = \sigma_M^2 + \bar{C}_M - \bar{T}_M$ as an adjusted variance of the market portfolio. In some works, like the one of (Pástor, Stambaugh, and Taylor 2020), they normalize the market average values of various terms to zero. Here, I keep the average value and adjust the variance of the market portfolio accordingly. Thus, the CAPM-like pricing relation in this economy becomes:

$$E_t[\mathbf{r_{t+1}}] = \beta E_t[r_{t+1}^M] + \frac{\delta_2}{\delta_1} \bar{\mathbf{C}} + (\mathbf{f_t} - \frac{\delta_3}{\delta_1} \bar{\lambda}) \tag{66}$$

where $\beta = \frac{\Sigma_T X_M}{\hat{\sigma}_M^2}$.

A.4 Proof of Proposition 2:

Assuming, without loss of generality, that $X^w + X^s = n$, $X^w = m$, and $X^e = 1 - n$ holds in equilibrium, where $n > 0$ and $m > 0$. In equilibrium, the following first order conditions hold for PoW and PoS cryptocurrencies respectively:

$$-a(\delta_1(E_t[r_{t+1}^w] - f_t^w) + \delta_3 \bar{\lambda}^w - \delta_2 \bar{C}^w) + \frac{1}{2} a^2 (2X^w \sigma_w^2 + 2(n - X^w)\sigma_{sw} + 2(1-n)\sigma_{ew}) = 0 \tag{67}$$

$$-a(\delta_1(E_t[r_{t+1}^s] - f_t^s) + \delta_3 \bar{\lambda}^s) + \frac{1}{2} a^2 (2(n - X^w)\sigma_s^2 + 2X^w \sigma_{sw} + 2(1-n)\sigma_{es}) = 0 \tag{68}$$

Subtracting Eq. 68 from Eq. 67 gives:

$$-a(\delta_1(\Delta E_t[r_{t+1}]-\Delta f_t)+\delta_3(\Delta\bar{\lambda})-\delta_2\bar{C}^w)+a^2(X^w\sigma_w^2-(n-X^w)\sigma_s^2+(n-2X^w)\sigma_{sw}+(1-n)\sigma_{ew}-(1-n)\sigma_{es})=0 \tag{69}$$

Restructuring Eq. 69 gives:

$$a((X_w\sigma_w^2+X_w\sigma_s^2-2X_w\sigma_{sw})+(1-n)\sigma_{ew}-(1-n)\sigma_{es}-n\sigma_s^2+n\sigma_{sw})=(\delta_1(\Delta E_t[r_{t+1}]-\Delta f_t)+\delta_3(\Delta\bar{\lambda})-\delta_2\bar{C}^w) \tag{70}$$

It's clear that $\sigma_w^2+\sigma_s^2-2\sigma_{sw}=(\sigma_w-\sigma_s)^2$. Also, we can define the following to ease notational complexity:

$$l=a((1-n)(\sigma_{ew}-\sigma_{es})+n(\sigma_{sw}-\sigma_s^2))$$

Thus, we have:

$$aX^w(\sigma_w-\sigma_s)^2=\delta_1(\Delta E_t[r_{t+1}]-\Delta f_t)+\delta_3(\Delta\bar{\lambda})-\delta_2\bar{C}^w-l \tag{71}$$

which results is:

$$X^w=\frac{\delta_1(\Delta E_t[r_{t+1}]-\Delta f_t)+\delta_3(\Delta\bar{\lambda})-\delta_2\bar{C}^w-l}{a(\Delta\sigma)^2} \tag{72}$$

Considering Eq. 33, we have:

$$\frac{C_t^e(b_2-b_3)-\frac{Cap_t}{W_t}b_2}{C_t^e(b_2-1)-C_t^w b_2}+\frac{l}{a(\Delta\sigma)^2}=\frac{\delta_1(\beta^w-\beta^s)E_t[r_{t+1}^M]}{a(\Delta\sigma)^2} \tag{73}$$

Restructuring the above equation gives:

$$\beta^w-\beta^s=U \tag{74}$$

$$where \quad U=\frac{a(\Delta\sigma)^2}{\delta_1 E_t[r_{t+1}^M]}(\frac{C_t^e(b_2-b_3)-\frac{Cap_t}{W_t}b_2}{C_t^e(b_2-1)-C_t^w b_2}+\frac{l}{a(\Delta\sigma)^2})$$

Now, I want to derive the sign of the first order derivative of U with respect to C_t^w. To proceed, first I take the first order derivative with respect to C_t^w from $\frac{C_t^e(b_2-b_3)-\frac{Cap_t}{W_t}b_2}{C_t^e(b_2-1)-C_t^w b_2}$.

$$\frac{\partial\left(\frac{C_t^e(b_2-b_3)-\frac{Cap_t}{W_t}b_2}{C_t^e(b_2-1)-C_t^w b_2}\right)}{\partial C_t^w}=\frac{C_t^e(b_2-b_3)b_2-\frac{Cap_t}{W_t}b_2^2}{\left(C_t^e(b_2-1)-C_t^w b_2\right)^2} \tag{75}$$

For the first order derivative to be positive, we require:

$$C_t^e(b_2-b_3)b_2-\frac{Cap_t}{W_t}b_2^2>0$$

After some simplification, we have:

$$C_t^e(b_2-b_3)>\frac{Cap_t}{W_t}b_2$$

In equilibrium, we have $\frac{Cap_t}{W_t}=mC_t^w+(1-n)C_t^e$. Also, considering the definition of carbon efficiency, we have $C_t^w=Carbon\_Eff\times e_t^w$. Replacing these into the above equation gives us:

$$\frac{b_2-b_3}{b_2}>m\frac{e_t^w}{C_t^e}\times Carbon\_Eff+(1-n) \tag{76}$$

Further simplification implies that the first order derivative is positive if the following inequality holds:

$$Carbon\_Eff<\frac{(nb_2-b_3)C_t^e}{mb_2e_t^w} \tag{77}$$

Second, I take the first order derivative with respect to C_t^w from $\frac{l}{a(\Delta\sigma)^2}$. Prior to taking the first order derivative, let's analyze the sign of $(\sigma_{sw}-\sigma_s^2)$. Intuitively, we expect that investors increase their investment in the PoW cryptocurrency when the PoS cryptocurrency becomes riskier. Mathematically speaking, we have:

$$\sigma_s^2 \nearrow \Rightarrow X^w \nearrow \Rightarrow \ l \searrow$$

This implies that $\sigma_{sw} - \sigma_s^2 > 0$. Furthermore, let's assume that the correlation of PoW and PoS cryptocurrencies with equity are close to each other. This is also supported by real-world data as cryptocurrencies are highly correlated with each other. Thus, we have $\sigma_{ew} - \sigma_{es} \simeq 0$. Therefore, the sign of first order derivative of $\frac{l}{a(\Delta\sigma)^2}$ with respect to C_t^w is equivalent to the sign of first order derivative of $n = \frac{(b_2-b_1)(C_t^e - \frac{Cap_t}{W_t}) - C_t^w b_3}{C_t^e(b_2-1) - C_t^w b_2}$ with respect to C_t^w since the former is a positive affine transformation of the latter. So, to evaluate the sign, we only need to calculate the first order derivative of n with respect to C_t^w. For notational simplicity, let's write n as follows:

$$n = \frac{A - BC_t^w}{C - DC_t^w} \tag{78}$$

$$A = (b_2 - (\varphi^w - 1) - f_t^w)(C_t^e - \frac{Cap_t}{W_t})$$

$$B = b_3 - (\varphi^w - 1)(C_t^e - \frac{Cap_t}{W_t})p_{eng}^F \frac{1}{\psi}$$

$$C = C_t^e(b_2 - 1)$$

$$D = b_2$$

The first order derivative of Eq. 78 with respect to C_t^w has the following structure:

$$\frac{\partial(\frac{A-BC_t^w}{C-DC_t^w})}{\partial C_t^w} = \frac{AD - BC}{(C - DC_t^w)^2} \tag{79}$$

In order to have a positive sign, we require that $AD > BC$. This means that:

$$(C_t^e - \frac{Cap_t}{W_t})(b_2 - (\varphi^w - 1) - f_t^w)b_2 > (b_3 - (\varphi^w - 1)(C_t^e - \frac{Cap_t}{W_t})p_{eng}^F \frac{1}{\psi})C_t^e(b_2 - 1) \tag{80}$$

For ease of understanding, let's define the followings:

$$f = (C_t^e - \frac{Cap_t}{W_t})(b_2 - (\varphi^w - 1) - f_t^w)b_2$$

$$g = b_3 C_t^e (b_2 - 1)$$

$$q = (\varphi^w - 1)(C_t^e - \frac{Cap_t}{W_t})p_{eng}^F C_t^e (1 - b_2)$$

To complete the proof, I proceed with the following lemma:

Lemma 1: $q > 0$ if the staking revenue is sufficiently high, meaning that $b_2 > \frac{b_1}{I_t+1}$.

Proof: By definition, $\varphi^w - 1 > 0$. Also p_{eng}^F and C_t^e are positive. [15] It's also straightforward to show that $(1 - b_2) > 0$, since the revenue per dollar value invested in PoS cryptocurrncy is lower than one dollar; i.e., the revenue is less that 100% of the actual investment. Now, to complete the proof, we only have to show that $C_t^e - \frac{Cap_t}{W_t} > 0$. From the equilibrium condition, we can rewrite it as $C_t^e - \frac{Cap_t}{W_t} = nC_t^e - mC_t^w > 0$. Hence we have:

$$\frac{n}{m} > \frac{C_t^w}{C_t^e} \tag{81}$$

Considering Equations 34 and 33, we can replace the following for $\frac{n}{m}$:

$$\frac{n}{m} = \frac{(b_2 - b_1)Q_t - b_3 C_t^w}{b_2 Q_t - b_3 C_t^e}$$

$$where \ \ Q_t = C_t^e - \frac{Cap_t}{W_t}$$

A simple use of algebra gives:

$$(b_2 - b_1)Q_t < b_2 Q_t I_t$$

[15] From the assumption of the main text, we assume that mean of C_t^e is sufficiently higher than its variance to ensure that carbon intensity is positive.

Note that since $m > 0$, we have $b_2Q_t - b_3C_t^e < 0$ and by using algebra, the direction of inequality changes.

A simple restructuring of

$$(b_2I_t + b_2 - b_1)Q_t > 0$$

For $Q_t = C_t^e - \frac{Cap_t}{W_t}$ to be positive, we need:

$$(b_2I_t + b_2 - b_1) > 0 \tag{82}$$

Thus, we have:

$$b_2 > \frac{b_1}{1 + I_t} \tag{83}$$

The above inequality indicates that the revenue from staking must reach a certain threshold to incentivize validators to engage in staking. The proof is complete.

Now, I continue with the proof of Proposition 2. Considering that the condition described in inequality 83 holds, then the first order derivative of n with respect to C_t^w will be positive if the following relation holds:

$$Carbon\_Eff < \frac{f - g}{q} \tag{84}$$

Combining equations 77 and 84, it's straightforward to show that $\frac{\partial U}{\partial C_t^w} > 0$ when the following condition holds:

$$Carbon\_Eff < T$$

$$where\ T = Min\left\{\frac{(nb_2 - b_3)C_t^e}{mb_2e_t^w}, \frac{f - g}{q}\right\}$$

A.5 Proof of Proposition 3:

To prove this propsition, I use the chain rule as below:

$$\frac{\partial U}{\partial(1-\alpha)} = \frac{\partial U}{\partial C_t^w} \cdot \frac{\partial C_t^w}{\partial(1-\alpha)} \tag{85}$$

In Proposition 2, I showed that $\frac{\partial U}{\partial C_t^w} > 0$ under the condition $b_2 > \frac{b_1}{I_t+1}$ and $Carbon\_Eff < T$. Now, I have to calculate $\frac{\partial C_t^w}{\partial(1-\alpha)}$ as below:

$$\frac{\partial C_t^w}{\partial(1-\alpha)} = -\psi e_t^w \tag{86}$$

Since the energy intensity (e_t^w) and emission intensity (ψ) are strictly positive, thus we have $\frac{\partial C_t^w}{\partial(1-\alpha)} = -\psi e_t^w < 0$. So, the multiplication of a positive and a negative value will be negative $\frac{\partial U}{\partial(1-\alpha)} = \frac{\partial U}{\partial C_t^w} \cdot \frac{\partial C_t^w}{\partial(1-\alpha)} < 0$ and the proof is complete.

A.6 Proof of Equation 41:

Taking F.O.C from Eq. 40 with respect to δ_2 gives us the following:

$$aX'\bar{\mathbf{C}} + \frac{1}{2}a^2 X'(2\delta_2\Sigma_C - 2\delta_1\Sigma_{rC} - 2\delta_3\Sigma_{C\lambda})X = 0 \tag{87}$$

Further simplification gives:

$$X'\bar{\mathbf{C}} + a\delta_2\sigma_C^2 - a\delta_1\sigma_{rC} - a\delta_3\sigma_{C\lambda} = 0 \tag{88}$$

Where the market level variance of carbon emissions and its covariances with expected return and cryptocurrency platform productivity are presented below:

$$X'\Sigma_C X = \sigma_C^2$$
$$X'\Sigma_{rC} X = \sigma_{rC}$$
$$X'\Sigma_{C\lambda} X = \sigma_{C\lambda}$$

By considering above equations and restructuring Eq. 88, we have:

$$a\delta_2 = \frac{a\delta_1\sigma_{rC} + a\delta_3\sigma_{C\lambda} - \frac{E_t[Cap_t]}{W_t}}{\sigma_C^2}$$

where $X'\bar{\mathbf{C}} = \frac{E_t[Cap_t]}{W_t}$ reflects the emission cap per dollar value of the ecosystem.

A.7 Proof of Proposition 4:

I can rewrite the optimization problem of the overconfident investor under the objective probability measure as below:

$$\underset{X_O,\hat{\delta}_2}{Max}\{-a_O[1+\delta_1 X'_O(E_t[\xi \mathbf{r_{t+1}}]-\mathbf{f_t})+\delta_3(X'_O\bar{\lambda})-\hat{\delta}_2 X'_O\bar{\mathbf{C}}]+\frac{1}{2}a_O^2 X'_O(\hat{\Sigma}_T^O+\hat{\delta}_2^2\Sigma_C-2\delta_1\hat{\delta}_2\mathbf{\Psi}\Sigma_{rC}-2\delta_3\hat{\delta}_2\Sigma_{C\lambda})X_O\} \tag{89}$$

$$where\ \ \hat{\Sigma}_T^O = \Sigma_T^O - \hat{\delta}_2^2\Sigma_C + 2\delta_1\hat{\delta}_2\mathbf{\Psi}\Sigma_{rC} + 2\delta_3\hat{\delta}_2\Sigma_{C\lambda}$$

here $\hat{\Sigma}_T^O$ is the part of variance-covariance matrix that is independent of $\hat{\delta}_2$. Also $\mathbf{\Psi}$ refers to a 3×3 matrix used to adjust the covariance between carbon emissions and expected return calculated under the objective probability measure, to reflect the perception of the overconfident investor regarding the covariance between these two variables in the ecosystem. I characterize this matrix later

Taking F.O.C with respect to carbon sensitivity of overconfident investor ($\hat{\delta}_2$) gives:

$$a_O X'_O\bar{\mathbf{C}} - a_O^2(\hat{\delta}_2 X'_O\Sigma_C X_O - \delta_1 X'_O(\mathbf{\Psi}\Sigma_{rC})X_O - \delta_3 X'_O\Sigma_{C\lambda}X_O) = 0 \tag{90}$$

The covariance between expected return and carbon emissions in the view of overconfident investor can be written as below:

$$\mathbf{\Psi}\Sigma_{rC} = E[(\xi r_{t+1} - E_t[\xi r_{t+1}])'(\mathbf{C_t} - \bar{\mathbf{C}})] \tag{91}$$

Using the definition of $\mathbf{B_t}$, I can write the right-hand-side of above equation as below:

$$E((r_{t+1} - E_t[r_{t+1}] - (\tilde{\mathbf{B}}_\mathbf{t} - \mathbf{B_t}))'(\mathbf{C_t} - \bar{\mathbf{C}})) = \Sigma_{rC} - \Sigma_{BC} \tag{92}$$

Thus, the matrix $\mathbf{\Psi}$ has the following structure:

$$\mathbf{\Psi} = \mathbf{I} - \Sigma_{BC}\Sigma_{rC}^{-1} \tag{93}$$

Now consider the following:

$$X'E((r_{t+1} - E_t[r_{t+1}] - (\tilde{\mathbf{B}}_\mathbf{t} - \mathbf{B_t}))'(\mathbf{C_t} - \bar{\mathbf{C}}))X = \sigma_{rC} - \sigma_{BC} \tag{94}$$

$$where \;\; \sigma_{rC} = X'\Sigma_{rC}X$$
$$and \quad \sigma_{BC} = X'\Sigma_{BC}X$$

Replacing the above argument into the F.O.C and restructuring completes the proof as described below:

$$\hat{\delta}_2 = \frac{\delta_1(\sigma_{rC} - \sigma_{BC}) + \delta_3\sigma_{C\lambda} + X'_O\bar{\mathbf{C}}}{a_O\sigma_C^2} \tag{95}$$

It's also straightforward to show that the optimal carbon sensitivity of the rational investor (δ_2) has the following structure:

$$\delta_2 = \frac{\delta_1\sigma_{rC} + \delta_3\sigma_{C\lambda} + X'_R\bar{\mathbf{C}}}{a_R\sigma_C^2}$$

The aggregate carbon sensitivity in this economy can be described as below:

$$\delta_2^{agg} = \left(\frac{W_t^R}{W_t}\right)\delta_2 + \left(\frac{W_t^O}{W_t}\right)\hat{\delta}_2$$

By further simplification, we can derive the following structure for the aggregate carbon sensitivity in this economy:

$$\delta_2^{agg} = \frac{\delta_1(\sigma_{rC} - \frac{W_t^O}{W_t}\sigma_{BC}) + \delta_3\sigma_{C\lambda} + \frac{E_t[Cap_t]}{aW_t}}{\sigma_C^2}$$

A.8 Proof of Proposition 5:

Plugging the optimal portfolio weights into the market clearing condition gives us:

$$W_t X_M = W_t^O \frac{1}{AW_t^O}\Sigma_T^{-1}\Lambda^{-1}(\delta_1(E_t[\xi\mathbf{r_{t+1}}]-\mathbf{f}_t)+\delta_3\bar{\lambda}-\hat{\delta}_2\bar{\mathbf{C}})+W_t^R\frac{1}{AW_t^R}\Sigma_T^{-1}(\delta_1(E_t[\mathbf{r_{t+1}}]-\mathbf{f}_t)+\delta_3\bar{\lambda}-\delta_2\bar{\mathbf{C}}) \tag{96}$$

Multiply both sides of the above equation with Σ_T and restructuring gives us:

$$a\Sigma_T X_M = \delta_1(E_t[\mathbf{r_{t+1}}] - \mathbf{f}_t) + \delta_3\bar{\lambda} - \delta_2\bar{\mathbf{C}} + \Lambda^{-1}(\delta_1(E_t[\xi\mathbf{r_{t+1}}] - \mathbf{f}_t) + \delta_3\bar{\lambda} - \hat{\delta}_2\bar{\mathbf{C}}) \tag{97}$$

As shown in Proposition 4, I can write $\hat{\delta}_2 = \delta_2 - \kappa$ where $\kappa = a\delta_1\frac{\sigma_{BC}}{\sigma_C^2}$. One can restructure the above equation as below:

$$a\Sigma_T X_M = (\mathbf{I}+\Lambda^{-1})(\delta_3\bar{\lambda} - \delta_1\mathbf{f_t} - \delta_2\bar{\mathbf{C}}) + \delta_1(\mathbf{I}+\Lambda^{-1})E_t[\mathbf{r_{t+1}}] + \Lambda^{-1}(\delta_1 E_t[(\xi - \mathbf{1})\mathbf{r_{t+1}}] + \kappa\bar{\mathbf{C}}) \tag{98}$$

Consider the following and plug-in into the above equation:

$$E_t[\xi \mathbf{r_{t+1}}] = E_t[\mathbf{r_{t+1}}] - \mathbf{B_t}$$
$$where \quad \mathbf{B_t} = E_t[(\xi - \mathbf{1})\mathbf{r_{t+1}}]$$

Then, restructuring and pre-multiplying by $(\mathbf{I} + \Lambda^{-1})^{-1}$ gives us:

$$E_t[\mathbf{r_{t+1}}] = \frac{a}{\delta_1}(\mathbf{I} + \Lambda^{-1})^{-1}\Sigma_T X_M - (\mathbf{I} + \Lambda^{-1})^{-1}\Lambda^{-1}(\mathbf{B_t} + \frac{\kappa}{\delta_1}\bar{\mathbf{C}}) - (\frac{\delta_3}{\delta_1}\bar{\lambda} - \mathbf{f_t}) + \frac{\delta_2}{\delta_1}\bar{\mathbf{C}} \quad (99)$$

Using algebra, one can write $(\mathbf{I} + \Lambda^{-1})^{-1}\Lambda^{-1}$ as below:

$$(\mathbf{I} + \Lambda^{-1})^{-1}\Lambda^{-1} = (\mathbf{I} + \Lambda)^{-1} \quad (100)$$

Also, using standard algebra gives us:

$$(\mathbf{I} + \Lambda^{-1})^{-1} = \frac{1}{\det(\mathbf{I} + \Lambda^{-1})} adj(\mathbf{I} + \Lambda^{-1}) \quad (101)$$

Let's define the above matrix $\Phi = \frac{1}{\det(\mathbf{I}+\Lambda^{-1})} adj(\mathbf{I} + \Lambda^{-1})$ which can be interpreted as an adjustment factor. Indeed, Φ adjusts the variance in the market by incorporating the speculativ behavior of overconfident investors.

By defining the adjusted variance as $\hat{\Sigma} = \Phi\Sigma$, we can complete the first part of the proof as below:

$$E_t[\mathbf{r_{t+1}}] = \frac{a}{\delta_1}\hat{\Sigma}_T X_M - (\mathbf{I} + \Lambda)^{-1}(\mathbf{B_t} + \frac{\kappa}{\delta_1}\bar{\mathbf{C}}) - (\frac{\delta_3}{\delta_1}\bar{\lambda} - \mathbf{f_t}) + \frac{\delta_2}{\delta_1}\bar{\mathbf{C}}$$

To derive the CAPM-like relation, we follow the similar procedure as the representative agent case. The return on the market portfolio in this environment can be calculated as below:

$$E_t[\mathbf{r}^M_{t+1}] = \frac{a}{\delta_1}X'_M\hat{\Sigma}_T X_M - X'_M(\mathbf{I} + \Lambda)^{-1}(\mathbf{B_t} + \frac{\kappa}{\delta_1}\bar{\mathbf{C}}) - X'_M(\frac{\delta_3}{\delta_1}\bar{\lambda} - \mathbf{f_t}) + \frac{\delta_2}{\delta_1}X'_M\bar{\mathbf{C}} \quad (102)$$

Now, defining $\widehat{\sigma}_T{}^2 = X'_M \hat{\Sigma} X_M + \bar{C}_M - \bar{T}_M - \bar{B}$ where $\bar{B}_M = X'_M(\mathbf{1}+\Lambda)^{-1}(\mathbf{B}+\frac{\kappa}{\delta_1}\mathbf{C_t})$ let us to write a CAPM-like relation as below:

$$E_t[\mathbf{r_{t+1}}] = \beta E_t[\mathbf{r}^M_{t+1}] - (\mathbf{I}+\Lambda)^{-1}(\mathbf{B_t} + \frac{\kappa}{\delta_1}\bar{\mathbf{C}}) - (\frac{\delta_3}{\delta_1}\overline{\lambda} - \mathbf{f}_t) + \frac{\delta_2}{\delta_1}\bar{\mathbf{C}}$$

$$where \quad \beta = \frac{\hat{\Sigma} X_M}{\widehat{\sigma}_T{}^2}$$

A.9 Proof of Proposition 6:

We want to prove that if $\xi_1 > 1$ and $\xi_2 < 1$, then $\kappa > 0$. To show that $\kappa > 0$, I need to prove that $\sigma_{BC} > 0$.

It's necessary to point out two things. First, the speculation is only in the cryptocurrency market. Second, the carbon emissions of PoS cryptocurrency is zero. Thus, we can write σ_{BC} as below:

$$\sigma_{BC} = X_w^2 E[(\tilde{B}^w_t - B^w_t)(C^w_t - \bar{C}^w)] \tag{103}$$

Therefore, we only need to show that if $\xi_1 > 1$ and $\xi_2 < 1$ then we have $E[(\tilde{B}^w_t - B^w_t)(C^w_t - \bar{C}^w)] > 0$.

Let's consider C^U and C^D as the carbon emissions in Up and Down states respectively where $C^U > 0$ and $C^D > 0$ and $C^U > C^D$. Now, using the structure of covariance function $E[(\tilde{B}^w_t - B^w_t)(C^w_t - \bar{C}^w)] = E[\tilde{B}^w_t C^w_t] - \bar{C}^w B^w_t$, we can write the following:

$$E_t[\tilde{B}^w_t C^w_t] = \frac{1}{2}(\xi_1 - 1) r^U C^U + \frac{1}{2}(\xi_2 - 1) r^D C^D \tag{104}$$

Using the property of Radon-Nikodym derivative implies (Shreve et al. 2004):

$$E[\xi] = \frac{1}{2}\xi_1 + \frac{1}{2}\xi_2 = 1 \tag{105}$$

Considering this property, we can write Eq. 104 as below:

$$E_t[\tilde{B}_t^w C_t^w] = \frac{1}{2}(\xi_1 - 1)(r^U C^U - r^D C^D) \tag{106}$$

Also, we can calculate B_t^w as below:

$$B_t^w = \frac{1}{2}(\xi_1 - 1)(r^U - r^D) \tag{107}$$

Considering Eq. 106 and 107, we can write the covariance function as below:

$$E[(\tilde{B}_t^w - B_t^w)(C_t^w - \bar{C}^w)] = \frac{1}{4}(\xi_1 - 1)(C^U - C^D)(r^U + r^D) \tag{108}$$

We know that $(C^U - C^D) > 0$ since the carbon emissions of PoW cryptocurrency is positive. Also, PoW cryptocurrency must offer positive risk premium which implies that $r^U > \left|r^D\right|$. Thus, $r^U + r^D > 0$.

Therefore, if $\xi_1 > 1$, then $\sigma_{BC} > 0$ which completes the proof. Note that the propetry of Radon-Nikodym derivative described in Eq. 105 implies that when $\xi_1 > 1$, we should observe $\xi_2 < 1$.

A.10 Proof of Equations 43 and 44:

With the possibility of a Poisson jump, the following holds for carbon intensity:

$$C_t^{wJ} - \bar{C}^w = -\frac{\psi\eta\mu Y}{g} + \frac{\psi}{g}(h_t^w - \bar{h}) \tag{109}$$

Thus, the impact of Poisson jump on the mean and variance of carbon intensity can be charavterized as below:

$$\begin{cases} Var(C_t^{wJ} - \bar{C}^w) = (\frac{\psi\eta\mu}{g})^2\theta + some\_term \\ Mean(C_t^{wJ} - \bar{C}^w) = -\frac{\psi\eta\mu\theta}{g} \end{cases} \tag{110}$$

The effect of jump on the mean and variance of transactional benefits is as follows:

$$\begin{cases} Mean(\lambda_t^{wJ}) = \beta\pi\frac{\mu-\eta\theta}{1-\phi} + some\_term \\ Var(\lambda_t^{wJ}) = (\beta\pi)^2\frac{\sigma_h^2+\eta^2\theta}{1-\phi^2} + some\_term \end{cases} \tag{111}$$

Now, we can use the following expression to derive the first order derivative:

$$\frac{\partial E_t[r_{t+1}^w]}{\partial\theta} = \frac{\partial ae'_1\Sigma_T X_M}{\partial\theta} + \frac{\partial(-\delta_3\bar{\lambda}^{wJ})}{\partial\theta} + \frac{\delta_2\bar{C}^{wJ}}{\partial\theta} \tag{112}$$

A simple use of algebra give us the following:

$$\frac{\partial E_t[r_{t+1}^w]}{\partial\theta} = aX_w\left(\delta_3^2(\beta\pi)^2\frac{\eta^2}{1-\phi^2} + \delta_2^2(\frac{\psi\eta\mu}{g})^2 - 2\delta_2\delta_3\frac{\beta\pi\psi}{g}(\eta\mu)^2\right) + \delta_3\beta\pi\frac{\eta}{1-\phi} - \delta_2\frac{\psi\eta\mu}{g}$$

A similar path would give us the expression for $\frac{\partial E_t[r_{t+1}^w]}{\partial\eta}$.

B Appendix B

B.1 Model extension by relaxing the simplifying assumptions:

Let's assume that the representative investor has a quadratic utility function, described as below. Note that it is also possible to consider that the representative investor has a power utility, with log-normal variables determining her wealth bundle. Such an approach yields the same trade-off between the mean and variance of the wealth bundle, characterized by risk aversion. For further details, see Chapter 2 of Campbell and Viceira (2002).

$$E_t\left[U(\hat{W}_{t+1})\right] = E_t[\hat{W}_{t+1}] - \frac{A}{2}Var(\hat{W}_{t+1}) \tag{113}$$

where $\hat{W}_{t+1}$ represents the wealth bundle and is the aggregation of pecuniary wealth and non-pecuniary benefits. The dynamics of $\hat{W}_t$ has the following structure:

$$\hat{W}_{t+1} = W_t X' J_{t+1} \tag{114}$$

where $J_{t+1} = \begin{pmatrix} J^w_{t+1} \\ J^s_{t+1} \\ J^e_{t+1} \end{pmatrix}$ represents the vector of aggregated pecuniary and non-pecuniary benefits in each asset. The Cobb-Douglas aggregator is used to describe J_{t+1}.

$$J^w_{t+1} = (e^{R^w_{t+1}})^{\alpha}(e^{\lambda^w_t})^{\beta}(e^{-C^w_t})^{1-\alpha-\beta} \tag{115}$$

$$J^s_{t+1} = (e^{R^s_{t+1}})^{\alpha}(e^{\lambda^s_t})^{\beta} \tag{116}$$

$$J^e_{t+1} = (e^{R^e_{t+1}})^{\alpha}(e^{-C^s_t})^{1-\alpha-\beta} \tag{117}$$

where $R^k_{t+1} = 1 + r^k_{t+1}$. Here, J_t is log-normally distributed.

Taking F.O.C with respect to X from Eq. 113 gives the optimal portfolio weights:

$$X = \frac{1}{a}\Sigma_J^{-1}\left(E_t[J_{t+1}]\right) \tag{118}$$

To find the expected return relation, I restructure the the above equation:

$$a\Sigma_J X = E_t[J_{t+1}] \tag{119}$$

The normal characteristic function allows us to write the following:

$$E_t[J_{t+1}] = e^{\alpha E[R^w_{t+1}] + \beta\bar{\lambda}^w - (1-\alpha-\beta)\bar{C}^w + \frac{1}{2}\mathbf{1}'\Sigma_J\mathbf{1}} \tag{120}$$

Thus, we have:

$$e^{\alpha E[R^w_{t+1}] + \beta\bar{\lambda}^w - (1-\alpha-\beta)\bar{C}^w + \frac{1}{2}\mathbf{1}'\Sigma_J\mathbf{1}} = a\Sigma_J X \tag{121}$$

Now, let's define $\mathbf{1}'\Sigma_J\mathbf{1} = \sigma_T^2$. Then, taking natural logarithm from both sides gives the following expression for PoW cryptocurrency:

$$\alpha E[R_{t+1}^w] + \beta\bar{\lambda}^w - (1-\alpha-\beta)\bar{C}^w + \frac{1}{2}\sigma_T^2 = \ln(ae'_1\Sigma_J X) \tag{122}$$

For PoS cryptocurrency and Equity, the procedure is the same. By, restructuring the Eq. 122, we have the following:

$$E[R_{t+1}^w] + \frac{1}{2}\hat{\sigma}_T^2 = \ln(ae'_1\Sigma_J X) - \frac{\beta}{\alpha}\bar{\lambda}^w + \frac{1-\alpha-\beta}{\alpha}\bar{C}^w \tag{123}$$

where $\sigma_T^2 = \alpha\hat{\sigma}_T^2$ and $\frac{1}{2}\hat{\sigma}_T^2$ is the log-normal adjustment. The above equation identifies 3 priced components for PoW cryptocurrency similar to Theorem 2 in the main text. For PoS cryptocurrency and equity, we can use a similar approach to derive the results of Theorem 2.

Now, let's assume that variables described in the theory section are log-normally distributed. In other words, let's assume that λ_t^k, e_t^k, C_t^k, and R_t^k are log-normally distributed for $k = w, s, e$. The rationale behind this assumption is that the variables such as carbon emissions do not admit negative values. Now, in order to solve the model, we must define a new aggregator to bundle together the financial gain from the expected return, the non-pecuniary transactional benefits of crypturrencies, and the disutility of carbon intensity. To do so, I retain the structure I proposed earlier in this section and define the following Cobb-Douglas aggregator:

$$\begin{aligned}
J_{t+1}^w &= (e^{\ln(R_{t+1}^w)})^\alpha (e^{\ln(\lambda_t^w)})^\beta (e^{-\ln(C_t^w)})^{1-\alpha-\beta} \\
J_{t+1}^s &= (e^{\ln(R_{t+1}^s)})^\alpha (e^{\ln(\lambda_t^s)})^\beta \\
J_{t+1}^e &= (e^{\ln(R_{t+1}^e)})^\alpha (e^{-\ln(C_t^s)})^{1-\alpha-\beta}
\end{aligned}$$

Here, $\ln(R_{t+1}^k)$, $n(\lambda_t^w)$, and $\ln(C_t^w)$ are normally distributed. Hence, utilizing this structure for the aggregator along with appropriate parameters give similar results to what we have derived in Eq. 122, 123, and Theorem 2.

B.2 Relaxing the assumption of a single competitive validator

In this section, I move away from the simplifying assumption of a single representative entity producing both PoW and PoS cryptocurrencies. Instead, I posit that one representative entity produces PoW cryptocurrencies and is called ”miner,” while another representative entity produces PoS cryptocurrencies and is referred to as a ”staker.” The miner expends energy to produce hashes and ensure the blockchain security. In compensation, the miner receives transaction fees and block reward. To characterize this, I formulate the miner's profit as below:

$$Miner'sprofit = (\theta^w_{t+1} - \theta^w_t)p^w_t + f^w_t X_w W_t - p^F_{eng} e^w_t p^w_t(\theta^w_{t+1} - \theta^w_t) - M^w_t \tag{124}$$

where M^w_t denotes the maintenance costs associated with mining, such as the maintenance cost of mining device.

The staker holds a certain value of PoS cryptocurrencies in a wallet and, in return, receives cryptocurrency rewards in the form of transaction fees and block rewards. It's important to note that there are no maintenance costs associated with staking. The staker only incurs the opportunity cost of the staked tokens. To depict this, I formulate the staker's profit as follows:

$$Staker's\ profit = (\theta^s_{t+1} - \theta^s_t)p^s_t + f^s_t X_s W - M^s_t \tag{125}$$

where M^s_t denotes the time-varying opportunity cost of staked tokens. For brevity, let's define $M_t = M^w_t + M^s_t$. In this ecosystem, I introduce a social planner who determines the optimal resource allocation between mining and staking as characterized below:

$$\underset{\omega}{Max} \left\{ \begin{array}{l} \omega\left((\theta^w_{t+1} - \theta^w_t)p^w_t + f^w_t X_w W_t - p^F_{eng} e^w_t p^w_t(\theta^w_{t+1} - \theta^w_t) - M^w_t\right) \\ +(1-\omega)\left((\theta^s_{t+1} - \theta^s_t)p^s_t + f^s_t X_s W - M^s_t\right) \end{array} \right\}$$

where $0 \leq \omega \leq 1$. Taking first order condition with respect to ω yields the optimal resource allocation as outlined below:

$$(\theta^w_{t+1} - \theta^w_t)p^w_t + f^w_t X_w W_t + (\theta^s_{t+1} - \theta^s_t)p^s_t + f^s_t X_s W = p^F_{eng} e^w_t p^w_t (\theta^w_{t+1} - \theta^w_t) + (M^s_t + M^w_t)$$

This corresponds to Eq. 27. Thus, the conclusions drawn from the single validator assumption remain valid when considering separate and independent miners and stakers.