Excess Volatility and Mispricing in the Presence of Sentiment and Institutional Investors*

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ABSTRACT

Against a prediction of standard models, an exacerbation of investor sentiment is often associated with lower stock return volatility in the data. We propose a model that can replicate this empirical pattern by interacting retail investor optimism with institutional benchmarking concerns. Both optimism and benchmarking separately increase the demand for a stock and decrease its risk premium. In contrast, their joint effect on return volatility is more ambiguous, as the transmission of fundamental news to prices combines benchmarking and relative-wealth channels. The latter channel, in particular, can induce a negative and asymmetric relation between investor sentiment and the stock return's excess volatility. It also explains how a greater institutionalization of financial markets can reduce excess volatility and mispricing in the presence of high sentiment. We show that these patterns are consistent with the data.

Keywords: Sentiment, Excess Volatility, Mispricing, Benchmarking, Institutional Investors.

JEL Classification: G11, G12, G18, G41.

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Abstract

Against a prediction of standard models, an exacerbation of investor sentiment is often associated with lower stock return volatility in the data. We propose a model that can replicate this empirical pattern by interacting retail investor optimism with institutional benchmarking concerns. Both optimism and benchmarking separately increase the demand for a stock and decrease its risk premium. In contrast, their joint effect on return volatility is more ambiguous, as the transmission of fundamental news to prices combines benchmarking and relative-wealth channels. The latter channel, in particular, can induce a negative and asymmetric relation between investor sentiment and the stock return's excess volatility. It also explains how a greater institutionalization of financial markets can reduce excess volatility and mispricing in the presence of high sentiment. We show that these patterns are consistent with the data.

1 Introduction

A vast body of literature documents how psychological biases and cognitive limits can shape the investment behavior of individuals, introducing an element of irrationality, or "sentiment," in financial markets.[1] In the presence of frictions that limit the activity of rational market participants, this sentiment can cause systematic deviations of prices from fundamentals and excess return volatility.[2]

Against this backdrop, the recent trend toward greater portfolio delegation to institutional investors ("institutionalization") raises new questions. On the one hand, economic intuition suggests that, to the extent that institutional investors are sophisticated and less prone to committing systematic mistakes ("smart money"), the greater institutionalization of markets should help correct sentiment-driven distortions. On the other hand, recent theoretical and empirical findings suggest a more nuanced description of their investment behavior and potential impact on markets. First, Basak and Pavlova (2013) show that institutions' performance concerns relative to benchmark indexes ("benchmarking concerns") can result in upward pressure on the stock index and amplify the volatility of index stocks and the aggregate stock market. Second, DeVault et al. (2019) find that at least part of the demand shocks captured by sentiment metrics are not necessarily due to irrational beliefs but rather reflect rational (e.g., risk management) institutions' decisions in response to their investment styles. A natural question is then: Can we expect institutional investors to correct or, on the contrary, worsen the financial distortions caused by sentiment? The interaction of financial institutions' features and investor beliefs underlying this question is at the core of a recent research agenda in asset pricing (Brunnermeier et al., 2021).

To help motivate our inquiry, we plot in Figure 1 the observed relationship between the U.S. stock market return volatility and Baker and Wurgler (2006)'s measure of sentiment for different levels of aggregate institutional stock ownership (IOR), over the period 1980–2021. If the participation of institutional investors in the stock market did not affect the relationship between sentiment and return volatility, the depicted patterns should be roughly similar across IOR levels. In contrast, we observe a marked difference in the levels and patterns of return volatility depending on the aggregate participation of institutional investors in the stock market. In particular, when institutional investors account for a larger share of the stock market ownership ("High IOR"), volatility can fall with sentiment even in times of overall optimism,[3] a pattern that is hard to rationalize within con-

[1] For surveys of this literature see, e.g., Barberis and Thaler (2003); Hirshleifer (2015).

[2] See, e.g., Gromb and Vayanos (2010).

[3] We take a more formal approach to examining the relationship between sentiment, institutions' stock ownership,

ventional models of sentiment trading.[4]

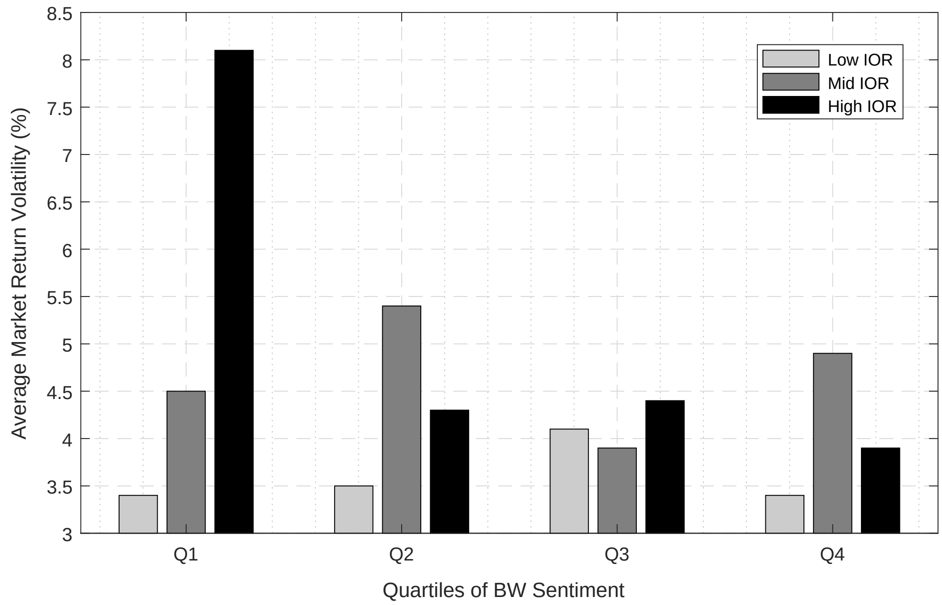

Figure 1: **Empirical relationship between market return volatility and sentiment**

This figure plots the average monthly stock market return volatility across quartiles of Baker and Wurgler (2006) (BW)'s measure of investor sentiment by terciles of institutional stock ownership (IOR). Quartiles of sentiment are created based on the overall sample from 1965/07 to 2022/06, available on Wurgler's website. Quartiles 1 and 2 correspond to strongly and moderately negative sentiment months, whereas quartiles 3 and 4 correspond to moderately and strongly positive sentiment months, respectively. This time series is normalized to have a mean value of zero. Data on quarterly institutional holdings is from Thomson/Refinitiv and covers the period from 1980 to 2021. Stock-level IOR is calculated as the ratio of shares held by 13F institutions to the number of shares outstanding. Stock-level values are then averaged across stocks each quarter using their market caps as weights. Volatility is the standard deviation of the daily market returns from Ken French's website, scaled to a monthly measure and reported in percentage points.

In this paper, we examine this and other effects of investor sentiment on asset prices within a dynamic general equilibrium model that accounts for the participation of institutional investors in financial markets. In the model, risk-averse investors trade continuously and frictionlessly in a risky asset (a "stock") and a riskless asset ("cash") over a finite investment period. Investors belong to either of two classes: "retail" or "institutional." Retail investors have standard preferences and can feature dogmatically optimistic or pessimistic beliefs about the stock's dividend growth rate. Thus, they can be subject to the type of positive ("bullish") or negative ("bearish") sentiment that is typically associated with retail trading in empirical studies.[5] Institutional investors have identical preferences to retail investors, except that their marginal utility of wealth is increasing in the level of

and both return volatility and prices in Section 6.

[4]See our literature review below.

[5]See, among others, Kumar and Lee (2006), Barber and Odean (2008), Greenwood and Nagel (2009), Barber et al. (2009), Da et al. (2015).

a benchmark index. This assumption follows Basak and Pavlova (2013)'s reduced-form approach to capturing the fact that, as agents for their delegating investors, institutional investors are typically evaluated (and compensated) in terms of both absolute and relative performance with respect to a benchmark portfolio. Unlike retail investors, they are rational in the sense of having the correct belief about the dividend growth rate.

We solve for the equilibrium in this economy and explicitly characterize asset prices and portfolio allocations. Based on this characterization, we first compare the separate implications of sentiment versus benchmarking concerns on the equilibrium levels and dynamics of prices. Besides providing relevant reference cases against which to compare their joint effect, this analysis can inform the extent to which the two features can be observationally equivalent. Because both introduce a wedge in the demand for the stock relative to an otherwise identical rational non-institutional investor, retail optimism and benchmarking concerns exert a similar upward pressure on the stock price, with the effect increasing in the severity of the feature. The similarity is such that, for any given intensity of benchmarking concerns, the "index effect" that these concerns induce on the stock's price-dividend ratio at a point in time is equivalent to the effect of a given ("threshold") level of retail optimism across all aggregate wealth distributions.

When considered in isolation, both benchmarking concerns and retail optimism exacerbate stock return volatility, although to different extents. The reason is that both features lead to portfolio heterogeneity across investors, amplifying the effect of fundamental shocks on stock returns via a relative-wealth channel. According to this channel, positive (respectively, negative) shocks to fundamentals transmit to prices not only via higher (lower) expectations of future payoffs but also via a greater (lower) demand pressure, as a result of a wealth effect, from the now relatively wealthier (poorer) traders whose portfolio is overexposed to the shock—i.e., the institutions in one case, and the optimistic retail investors in the other. The benchmark concerns of institutions introduce a second volatility amplification channel, as positive (negative) shocks to prices feed back into additional positive (negative) institutional demand for the stock to hedge relative performance risk. This effect exacerbates volatility beyond the levels induced by sentiment.

We then focus the analysis on our main case of interest, namely the pricing implications of sentiment-driven retail investors' trading with rational institutional investors. We are particularly interested in addressing two questions: (i) How does sentiment affect volatility in the presence of institutions? (ii) To what extent do rational but benchmark-concerned institutions correct sentiment-induced mispricing? While the mere addition of the effects of either type of feature on prices described above suggests an exacerbation of the associated distortions, the equilibrium analysis reveals several surprising patterns.

Concerning the first question, when the trading counterparts of the irrational investors are institutions instead of non-institutional investors, retail optimism can actually *dampen* volatility. This result contrasts with the one prevailing in a standard economy without institutions, where sentiment unambiguously creates excess stock market volatility (DeLong et al., 1990; Dumas et al., 2009). To understand this, assume, for example, that retail investors are slightly to moderately optimistic about the stock's prospects. Instead of meeting the extra demand for the stock of these investors (as rational non-institutional investors would in the standard economy), institutional investors demand more of the asset. They do so to the extent that, in equilibrium, none of the two investor types gets to lever up their portfolios as desired. Because both investor types end up having similar portfolios, shocks to fundamentals do not significantly alter the distribution of aggregate wealth, shutting down the relative-wealth amplification channel on volatility. The effect is such that, within this range of optimism, the stock excess volatility monotonically *falls* with investor sentiment. By comparison, this volatility increases monotonically within the same optimism range in the otherwise equivalent economy in which the rational counterparts of the sentiment investors are not institutions.

A similar intuition explains a related result, according to which the positive relationship between the strength of institutions' benchmarking concerns and volatility prevailing in a rational institutionalized economy switches signs for sufficiently high levels of optimism. Moreover, the range of optimism over which the relationship between sentiment and volatility is negative increases with the strength of these concerns. Thus, relative to retail rational investors, institutions attenuate excess volatility in the presence of retail optimism but exacerbate it under pessimism, creating an *asymmetric* sentiment-volatility pattern. The reason is that, due to their benchmarking concerns, institutions are willing to buy more of the stock that pessimistic retail investors want to sell than equivalent non-institutional counterparts. As a result, differences in portfolio composition across investor types and the consequent relative-wealth channel through which fundamental shocks translate into return volatility amplify under retail pessimism.

An analysis of the equilibrium dynamics of the model uncovers novel patterns, as the wealth distribution across investors responds endogenously to the arrival of cash-flow news to determine prices and allocations. In the reference cases where either optimistic retail or rational institutional investors trade with rational non-institutional investors, wealth effects give rise to a cyclical pattern in price-dividend ratios and a countercyclical pattern in Sharpe ratios. These patterns are exacerbated in the second reference case, as rising prices can increase institutions' stock demand, pushing prices even higher and Sharpe ratios even lower. The portfolio allocation of institutions additionally determines a cyclical pattern in stock return volatility, as benchmarking concerns amplify the transmission of fundamental shocks to returns.

More surprisingly, trading between the same rational institutions and optimistic retail investors can lead to the opposite, countercyclical pattern in return volatility. The switch in pattern follows from a switch in sign, in this case, of the relative-wealth effect on return volatility. Specifically, when retail sentiment is so high that its impact on the stock demand is stronger than the impact of institutions' benchmarking concerns, the retail investors are relatively overinvested in the stock. Positive fundamental news then makes them relatively wealthier. The news also increases the index risk-hedging demand of institutions for the stock, leading to a sharp decline in the market price of risk. However, as their relative wealth increases and the market price of risk plummets, retail investors reduce the fraction of their wealth invested in the stock, inducing an increasing relative risk aversion pattern absent in partial equilibrium. The greater wealth of the retail investors implies that their lower demand prevails over the greater demand of institutions in the aggregate, making the relative-wealth channel now *reduce* (rather than amplify) the sensitivity of prices to fundamental news. The stronger the wealth effect, the steeper the fall in volatility.

The same negative impact of the relative-wealth channel on volatility can push volatility levels below those prevailing under either sentiment or benchmarking concerns alone. This result has important implications for assessing the effect of institutions, on the one hand, and of sentiment, on the other hand, on financial markets. First, it implies that, in the presence of high sentiment, institutions can have a more substantial depressing effect on volatility than equivalently rational but non-institutional peers despite their benchmarking concerns. Second, in markets with a high presence of institutional investors (high institutionalization), sentiment need not create "excess volatility" but substantially reduce it. These implications highlight the importance of distinguishing the degree of institutionalization of markets in empirical analyses that associate excess return volatility, as inferred from, e.g., volatility-ratio tests, to irrational behavior and mispricing (e.g., Shiller, 1979, 1981; Giglio and Kelly, 2018).

Concerning the second question, whether greater institutionalization helps correct or distort sentiment-induced distortions on prices depends on the relative strength of retail sentiment vis-a-vis institutions' benchmarking concerns. For low sentiment levels, even optimistic retail investors choose to sell an increasing share of their stock holdings to the institutions as institutionalization grows. Because benchmark concerns increase their risk appetite, the institutions purchase these shares at increasingly higher prices, which worsens the stock overpricing as institutionalization rises. This result verifies the conjecture of DeVault et al. (2019) that the existence of sophisticated investors might push prices further away from their fundamental value than retail sentiment. However, the opposite happens for more severe levels of optimism, when retail sentiment leads to stronger demand for the stock than institutions' benchmarking concerns. In these situations, characterized by a low (potentially negative) stock risk premium, greater institutionalization can be accompanied by aggressive selling of the stock by the institutions, which helps to push the stock price closer to its fundamental value. Thus, institutions help correct severe sentiment-induced overpricing that would otherwise result in financial "bubbles"—a market with a negative risk premium.

In the last part of the paper, we summarize these results into two testable implications and take them to the data. The first testable implication is that the institutionalization of financial markets induces an *asymmetric* sentiment-excess volatility pattern, such that *(i)* an intensification of *pessimistic* sentiment always increases volatility while a similar intensification of *optimistic* sentiment can reduce it instead, and *(ii)* an intensification of benchmarking practices exacerbates volatility in markets with predominantly *pessimistic* sentiment but attenuates it in markets with predominantly *optimistic* sentiment. Our second testable implication is that, in the presence of strongly optimistic investor sentiment, a higher institutionalization of financial markets can lead to lower levels of stock overpricing, but not when investor sentiment is low or moderately optimistic.

We offer empirical support for both implications in the U.S. stock market. We form portfolios of stocks every month by sequentially sorting them based on their institutional ownership ratios (IOR) and their mispricing scores (MISP) from Stambaugh et al. (2015). Then, we examine the volatility and return of these portfolios in the month after portfolio formation. We find that, in periods with the lowest levels of sentiment, portfolios with high IOR have an average volatility that is 45% higher than portfolios with low IOR. This difference is large in magnitude and statistically significant. The result is consistent with part (ii) of our first implication above that, in negative-sentiment periods, the increased presence of institutions can lead to higher stock return volatility.

We also show that volatility decreases monotonically from periods of negative to positive sentiment for portfolios with either high or low institutional ownership. However, the rate at which volatility decreases across sentiment quartiles is stronger for high IOR portfolios. This is consistent with part (i) of our first theoretical implication that, in positive-sentiment periods, the larger the presence of institutions, the lower the stock return volatility. These results continue to hold after controlling for industrial production and NBER recessions, which account for most of the volatility variation in the extreme sentiment quartiles.

In addition to the portfolio-level analysis using marketwide sentiment, we also perform cross-sectional tests using sentiment and benchmarking intensity both measured at the stock level. We use order imbalances (OIB) in retail trades as a proxy for sentiment about a given stock, where positive (respectively, negative) imbalances represent positive (negative) sentiment. Retail trades are captured from the TAQ database following the methodology in Boehmer et al. (2021). We then use the stock-level benchmarking intensity measure proposed by Pavlova and Sikorskaya (2023). They compute benchmark intensity (BMI) as a stock's cumulative weight in all benchmarks, weighted by mutual funds' and ETFs' assets following each benchmark. Consistent with the portfolio-level analysis, we find that the positive effect of sentiment (retail OIB) on volatility decreases as BMI increases.

To test the overpricing implication, we examine how the predictive power of MISP on stock returns is affected by the presence of institutions in periods of moderate vs. strong positive sentiment. Again, we form portfolios by sequentially sorting on IOR and MISP to study the predictive power of MISP controlling for IOR. Portfolios with high (low) MISP are overpriced (underpriced) and should be shorted (purchased). We then create two subsamples, one that includes only months of moderately positive sentiment and one with only months of strongly positive sentiment. We empirically validate that the short leg of the MISP strategy consistently forecasts negative returns in the subsequent month across all subsamples. However, during periods characterized by moderately positive sentiment, such predictive ability remains comparable between portfolios with low vs. high IOR. This suggests that a higher presence of institutions does not help correct overpricing for moderate levels of positive sentiment. In contrast, for the subsample of periods with strongly positive sentiment, the predictability of the high MISP portfolio is significantly weaker in the high IOR portfolios, consistent with institutions significantly attenuating overpricing in such periods.

Our paper is related to two main strands of the literature. First, it relates to the literature on equilibrium pricing implications of institutional investors' incentives. Cuoco and Kaniel (2011) find that symmetric benchmark-adjusted compensation has a significant and unambiguous positive effect on the price of benchmark assets and a negative impact on their Sharpe ratios. In contrast, asymmetric schemes have a more ambiguous effect. Using a highly tractable model, Basak and Pavlova (2013), explicitly characterize the institutions' portfolios in response to benchmarking incentives and their impact on the prices, Sharpe ratios, return volatilities, and correlations of benchmark versus non-benchmark assets. Several studies have built on this framework to rationalize observed asset pricing phenomena. Hong et al. (2014) use it to capture "status" (Keeping-Up-with-the-Joneses) concerns and explain the excessive trading of small local stocks and the trend-chasing behavior of individuals. Basak and Pavlova (2016) analyze the effect of the financialization of commodity futures markets on commodity futures prices, volatilities and correlations, and equity-commodity correlations. Buffa and Hodor (2022) study benchmark heterogeneity across asset managers to explain differences in the predictability of return comovement across cap-style and industry-sector portfolios. Hodor and Zapatero (2022) show that the interaction of institutions' short investment horizons and benchmarking concerns can rationalize a downward-sloping term structure of risk premia. Pavlova and Sikorskaya (2023) propose a theory-motivated measure ("benchmarking intensity," or BMI) of the benchmarking-induced stock demand of asset managers and present causal evidence of the effects of benchmarking concerns on fund portfolios and stock prices. While potentially accounting for wealth effects on portfolio allocations, these studies assume that all traders are rational and, thus, are not set up to assess the impact of sentiment on prices. Other studies in this literature do allow for the existence of irrational trading to explain how money managers subject to time-varying investors' flows (Vayanos and Woolley, 2013), or perceiving fees that depend on relative performance via an endogenous compensation contract (Buffa et al., 2019), can push prices away from fundamental value, as well as to study the effect of benchmarking concerns on information acquisition and market efficiency (Breugem and Buss, 2019; Sockin and Xiaolan, 2019). Because these studies assume constant absolute risk aversion (CARA) preferences for rational traders and leave the investment decisions of irrational traders unmodeled (i.e., irrational trading is likened to "noise"), they cannot account for the wealth effects of either institutions or sentiment investors that are key to our analysis.

Second, our paper is related to the literature that examines the impact of sentiment on prices in general equilibrium. Several studies show that different behavioral biases such as overconfidence (Daniel et al., 2001; Scheinkman and Xiong, 2003; Dumas et al., 2009), self-attribution bias (Daniel et al., 1998), extrapolative beliefs (Hong and Stein, 1999; Barberis and Shleifer, 2003; Barberis et al., 2015, 2018), among others, can lead to sentiment-like excess trading and have a significant impact on asset returns and volatility.[6] In modeling sentiment, we focus on the type of dogmatic

[6] For a comprehensive survey of asset pricing models based on psychological considerations, see Barberis (2018).

beliefs conducive to irrational optimism or pessimism considered by, e.g., Kogan et al. (2006). As Martin and Papadimitriou (2022) point out, this type of belief is consistent with the evidence documented by Giglio et al. (2021) and Meeuwis et al. (2021) in portfolio choice contexts. Similarly to both Kogan et al. (2006) and Martin and Papadimitriou (2022), we account for risk aversion and endogenous wealth effects on portfolio decisions and prices. Unlike these authors, we assume that the trading counterparts of the sentiment-driven investors are financial institutions rather than otherwise identical direct investors.[7] We show that due to benchmarking concerns, these institutions can either exacerbate or correct the distortions associated with sentiment depending on the relative strength of the sentiment- versus the benchmark-driven demands for the assets.

The rest of the paper proceeds as follows. Section 2 describes the financial markets and investors' problems. Section 3 characterizes explicitly the equilibrium prices and allocations in these markets. Section 4 compares the equilibrium in the general case in which sentiment investors trade with institutions to the reference cases in which only one of these features is present. Section 5 analyzes the equilibrium in these markets when the distribution of wealth is endogenously determined. Section 6 discusses the empirical implications of our findings. Section 7 offers concluding remarks. All proofs are presented in Appendix A.

2 Model

2.1 Economic Setting

We consider a pure exchange economy with a finite horizon T populated by two classes of traders, retail and institutional investors, each of which, in principle, can exhibit irrational sentiment (optimism or pessimism) about the economy's driving fundamentals.

Financial Market. The financial market consists of a single risky security (a stock market portfolio); one share of the stock is available for trading. The stock only pays a dividend at the final time T. Let S and D denote the stock and dividend (cash flow) processes, respectively. For simplicity, we assume that the process D follows a geometric Brownian motion, i.e.

$$dD_t = D_t(\mu dt + \sigma dB_t),$$

[7] Similarly, Krishnamurthy and Li (2021) study the effect of sentiment on financial crises in the presence of financial intermediaries.

where μ is the mean dividend growth rate, σ is the dividend volatility, and dB_t are the increments of the standard Wiener (cash flow "news") process under (the true) probability measure $\mathbb{P}$.

In addition, a zero-coupon bond is available in zero net supply. The zero-coupon bond delivers a sure payment of one at time T; following Kogan et al. (2006), we use the bond as the numeraire so its price is always equal to one.

Investor Preferences. Agents derive utility from their terminal wealth. Following Basak and Pavlova (2013), there are two classes of investors: retail (R) and institutional (I). Retail investors have standard logarithmic utility, i.e.

$$u_R(W_T^R) = \log W_T^R.$$

Institutional investors have otherwise identical preferences to retail investors except that their utility is scaled by the value of a benchmark index Y:

$$u_I(W_T^I) = (1 - \upsilon + \upsilon Y_T) \log W_T^I, \qquad \upsilon \in [0, 1), \tag{1}$$

where, without loss of generality, we let the benchmark index coincide with the stock market, i.e., we set $Y = S$. In the sequel, we show that a time-t measure of the strength of the institutional investor's benchmarking concern is:

$$q_t \triangleq \frac{\upsilon D_t e^{\mu(T-t)}}{1 - \upsilon + \upsilon D_t e^{\mu(T-t)}}, \qquad q_t \in [0, 1),$$

which depends positively on the benchmark weight υ in I's utility, the level of dividends D_t, and the remaining time horizon $T - t$.

Specification (1) follows Basak and Pavlova (2013)'s reduced-form approach to capturing the fact that, as agents for their delegating investors, institutional investors are typically evaluated (and compensated) in terms of both absolute and relative performance with respect to a benchmark portfolio, so their marginal utility is increasing in the level of this benchmark. The specification can also capture relative performance concerns facing, e.g., status-conscious investors (Hong et al., 2014).

Investor Beliefs. For $k \in \{R, I\}$, investor k believes that the mean growth rate of the dividend process D is constant and equal to μ^k. Investor k's beliefs are represented by an exponential martingale ξ^k whose evolution under $\mathbb{P}$ is given by

$$d\xi_t^k = \xi_t^k \delta^k \sigma dB_t,$$

where $\delta^k \triangleq (\mu^k - \mu)/\sigma^2$ is the "optimism" in investor's k beliefs, and $\xi_0^k = 1$. ξ_T^k is the Radon-Nikodym derivative of $\mathbb{P}^k$, the probability measure under which the dividend mean growth rate is equal to μ^k, with respect to $\mathbb{P}$. Under $\mathbb{P}^k$, the evolution of the dividend process D is given by

$$dD_t = D_t \left((\mu + \sigma^2 \delta^k) dt + \sigma dB_t^k \right),$$

where $dB_t^k = dB_t - \sigma\delta^k dt$ is the increment of a standard Wiener process under $\mathbb{P}^k$. Finally, we assume that investors agree to disagree, which reflects each class of investors' degree of overconfidence in its judgments. In the sequel, E_t^k denotes the conditional expectation at time t under investor k's beliefs.

Under $\mathbb{P}$, the dynamics of the stock price are given by

$$dS_t = S_t(\mu_{S,t} dt + \sigma_{S,t} dB_t).$$

At time t, investor k decides the fraction $\theta_t^k \in \mathbb{R}$ of her portfolio to allocate in the stock, with the remaining fraction $1 - \theta_t^k$ allocated in the bond. At time 0, and without loss of generality, investor k is endowed with a fraction λ^k of the stock share (with $\lambda^I + \lambda^R = 1$) and no bond. At time t, investor k's budget constraint is given by

$$dW_t^k = \theta_t^k W_t^k (\mu_{S,t}^k dt + \sigma_{S,t} dB_t^k), \tag{2}$$

where $\mu_{S,t}^k = \mu_{S,t} - \sigma_{S,t}\sigma\delta^k$, and $W_0^k = \lambda^k S_0$.

In most of the empirical and theoretical discussions on the topic (see references in Section 1), sentiment-driven trading is associated with retail investors. Thus, for comparability and without loss of generality, we assume that retail investors can display sentiment but institutional investors are fully rational ($\delta^I = 0, \xi_t^I = 1$ for all $t \in [0, T]$) throughout the rest of the analysis.

2.2 Portfolio Problem and Equilibrium Definition

At time t, investor k maximizes her lifetime utility of wealth

$$
\begin{aligned}
J_k(W_t^k) &= \max_{\theta^k} \; E_t^k[u_k(W_T^k)] \\
&= \max_{\theta^k} \; \frac{1}{\xi_t^k} E_t[\xi_T^k u_k(W_T^k)],
\end{aligned}
$$

subject to the budget constraint (2).

Clearly, markets are dynamically complete. This implies the existence of a unique state price density process π with $\mathbb{P}$-dynamics:

$$
d\pi_t = -\kappa_t \pi_t dB_t,
$$

where κ denotes the (endogenously determined) stock market price of risk.

We define equilibrium in these markets in the usual way, as consisting of a set of portfolio allocations and asset prices such that: (i) the individual portfolio allocations of the retail and institutional investors are optimal, and (ii) bond and stock markets clear.

3 Equilibrium Characterization

We start by characterizing the stock's equilibrium price-dividend ratio and price of risk:

Proposition 1. *The time-t equilibrium price-dividend ratio and the market price of risk are given by:*

$$
S_t/D_t = \overline{(S/D)}_t \frac{1}{\varpi_t^I\big(1-\gamma(T-t)q_t\big) + (1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big)}, \tag{3}
$$

$$
\kappa_t = \bar{\kappa}\left(1 - \frac{\varpi_t^I(1-\gamma(T-t))q_t + (1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big)\delta^R}{\varpi_t^I\big(1-\gamma(T-t)q_t\big) + (1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big)}\right), \tag{4}
$$

where $\gamma(x) \triangleq 1 - e^{-\sigma^2 x}$ $(\gamma(x) < 1, \gamma'(x) > 0)$, and $\overline{(S/D)}$ and $\bar{\kappa}$ are the equilibrium price-dividend ratio and the market price of risk in the standard ("STD") economy with no sentiment $(\delta^R = 0)$ or institutional investors $(\upsilon = 0)$, as given by:

$$
\begin{aligned}
\overline{(S/D)}_t &\triangleq (S_t/D_t)|_{\delta^R=0,\upsilon=0} = e^{(\mu-\sigma^2)(T-t)}, \\
\bar{\kappa} &\triangleq \kappa_t|_{\delta^R=0,\upsilon=0} = \sigma.
\end{aligned}
$$

Both greater optimism $\delta^R > 0$ and benchmarking concerns $q > 0$ lead to higher market valuations S_t/D_t in excess of fundamental values $\overline{(S/D)}_t$, with pessimism ($\delta^R < 0$) creating the opposite effect. The higher (respectively, lower) prices translate into lower (higher) market prices of risk κ_t, reducing (increasing) the appeal of the stock in the portfolio allocation problem of investors and restoring the market equilibrium between the increased (reduced) demand and supply. Thus, the introduction of either optimistic (pessimistic) or institutional investors to an otherwise standard economy induces asset "overvaluation" ("undervaluation") from the perspective of their rational non-institutional trading counterparts. The severity of this mispricing increases with sentiment or the intensity of benchmarking concerns.

To see how the pricing effects of sentiment and benchmarking relate to the demand for the stock of the retail and institutional investors, we next characterize their equilibrium portfolio allocations and the stock return volatility:

Proposition 2. *The time-t portfolio weights in the stock of the retail and institutional investors are:*

$$\theta_t^R = \frac{\kappa_t}{\sigma_{S,t}} + \frac{\sigma}{\sigma_{S,t}}\delta^R, \tag{5}$$

$$\theta_t^I = \frac{\kappa_t}{\sigma_{S,t}} + \frac{\sigma}{\sigma_{S,t}}q_t, \tag{6}$$

so that the leverage $(\theta_t^R - 1)\varpi_t^R$ of the retail investors is:

$$(\theta_t^R - 1)\varpi_t^R = \varpi_t^I(1 - \varpi_t^I)\frac{\sigma}{\sigma_{S,t}}(\delta^R - q_t). \tag{7}$$

The equilibrium stock return volatility is:

$$\sigma_{S,t} = \bar{\sigma}_S\left(1 + \varpi_t^I\frac{\gamma(T-t)q_t(1-q_t) + (1-\varpi_t^I)\big(\gamma(\delta^R(T-t)) - \gamma(T-t)q_t\big)(\delta^R - q_t)}{\varpi_t^I\big(1-\gamma(T-t)q_t\big) + (1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big)}\right) \geq \bar{\sigma}_S, \tag{8}$$

where $\bar{\sigma}_S = \sigma$ is the equilibrium price-dividend ratio and the market price of risk in the STD economy with no sentiment ($\delta^R = 0$) or institutional investors ($\upsilon = 0$).

Equation (7) shows that the strength of retail investors' stock demand relative to the stock demand of institutions, $\delta^R - q_t$, indicates whether the time-t stock allocation in R-investors' portfolio is levered ($\delta^R - q_t > 0$) or not ($\delta^R - q_t < 0$). In an all-rational investor economy with institutions, we have $\delta^R - q_t = -q_t < 0$, so the retail investors always lend money to the institutions. Proposition

2 shows that, because the strength q_t of their benchmarking concerns is always smaller than 1, institutional investors turn into lenders and retail investors into borrowers when the latter become sufficiently optimistic (i.e., $\delta^R > 1$) on the stock's prospects.

Whereas according to Eq. (8) both sentiment ($\delta^R \neq 0$) and benchmarking concerns ($q > 0$) create "excess volatility" with respect to the STD case (as $\sigma_{S,t} > \bar{\sigma}_S$ in both cases), the contribution of each of these features to this result is not entirely obvious from this expression. To assess these contributions, we decompose the stock return volatility into fundamental, benchmarking, and relative-wealth components. Specifically, let us formally write:

$$\begin{aligned} dq_t/q_t &= \mu_{q,t}dt + \sigma_{q,t}dB_t, \\ d\varpi_t^I/\varpi_t^I &= \mu_{\varpi^I,t}dt + \sigma_{\varpi^I,t}dB_t. \end{aligned}$$

Further letting $\varepsilon_{S,t}^x = \frac{\partial S_t}{\partial x_t} \times \frac{x_t}{S_t}$ denote the elasticity of the stock price with respect to x at time t, we have the following:

Lemma 1. *The equilibrium stock return volatility can be decomposed as:*

$$\sigma_{S,t} = \varepsilon_{S,t}^D \sigma_{D,t} + \varepsilon_{S,t}^q \sigma_{q,t} + \varepsilon_{S,t}^{\varpi^I} \sigma_{\varpi^I,t}, \tag{9}$$

where $\sigma_{q,t} = (1 - q_t)\sigma$, $\sigma_{\varpi^I,t} = -(1 - \varpi_t^I)(\delta^R - q_t)\sigma$, *and*

$$\begin{aligned} \varepsilon_{S,t}^D &= 1, \\ \varepsilon_{S,t}^q &= \frac{\varpi_t^I \gamma(T-t)q_t}{\varpi_t^I\big(1 - \gamma(T-t)q_t\big) + (1 - \varpi_t^I)\big(1 - \gamma(\delta^R(T-t))\big)} > 0, \qquad (10) \\ \varepsilon_{S,t}^{\varpi^I} &= \frac{\gamma(T-t)q_t - \gamma(\delta^R(T-t))}{\varpi_t^I\big(1 - \gamma(T-t)q_t\big) + (1 - \varpi_t^I)\big(1 - \gamma(\delta^R(T-t))\big)} \varpi_t^I. \qquad (11) \end{aligned}$$

Thus, the excess volatility ratio is:

$$EVR_t \triangleq \sigma_{S,t}/\bar{\sigma}_S - 1 = \Psi_{q,t} + \Psi_{\varpi^I,t} \geq 0, \tag{12}$$

where:

$$\begin{aligned} \Psi_{q,t} &= \varepsilon_{S,t}^q (1 - q_t) > 0, \qquad (13) \\ \Psi_{\varpi^I,t} &= -\varepsilon_{S,t}^{\varpi^I} (1 - \varpi_t^I)(\delta^R - q_t). \qquad (14) \end{aligned}$$

The first term in (9) is the direct effect of fundamental news on return volatility. It reflects the fact that positive (respectively, negative) cash flow news signals a greater (smaller) terminal dividend D_T, so the stock price S must adjust proportionally to reflect investors' updated expectations.

The second and third terms are the indirect impacts of these fundamental news on stock return volatility via the changes they induce in, respectively, the institutions' benchmarking intensity and relative-wealth share (i.e., the level of institutionalization), holding each other constant. Since these indirect impacts are the drivers of the excess volatility in this economy relative to the STD economy, we interpret them as the "benchmarking," Ψ_q, and "relative-wealth", Ψ_{ϖ^I}, propagation channels of fundamental shocks to excess volatility.

Benchmarking concerns create positive feedback from prices to the stock demand. In an economy with institutional investors, the higher (lower) price stemming from investors' updated expectations after positive (negative) cash flow news raises (depresses) the institution's benchmarking-related demand for the stock in order to keep up with the benchmark. Thus, the aggregate demand and the price for the stock change more than in the STD economy in response to the same cash flow news, amplifying the sensitivity of prices to dividend shocks. It is easy to see from Lemma 1 that, for a given benchmarking intensity q, the benchmarking channel is positive and increasing in the degree of optimism δ^R of the retail investors. Thus, the amplification of excess volatility induced by institutions is always greater when trading with optimist rather than rational retail counterparts.

The relative-wealth channel arises endogenously in equilibrium whenever $0 < \varpi_t^I < 1$, i.e., whenever no investor type absorbs the entire economy. In this case, differences in the portfolio compositions of institutional versus retail investors lead to differences in the dynamics of their relative wealth. To the extent that changes in wealth translate to changes in stock demands (as is the case with log preferences), the aggregate wealth distribution becomes a stochastic (state) variable whose volatility adds fundamental risk to the stock relative to the STD case.

One can check from Eq. (11) that the relative-wealth elasticity of stock prices $\varepsilon_S^{\varpi^I}$ decreases, while the relative demand strength $\delta^R - q_t$ increases, with this degree of optimism. Moreover, each of them can be positive or negative depending on how optimistic the retail investors are. Thus, in principle, it is possible that, in trading with institutions, optimistic retail investors attenuate the relative wealth-induced excess volatility of stock returns relative to their rational retail counterparts. In Section 4.2.1 below, we provide the conditions under which this possibility arises.

4 Analysis of Equilibrium

4.1 Reference Economies

To further isolate the channels through which the presence of both retail sentiment and institutions' benchmarking concerns affect financial markets equilibrium, in this section we examine two relevant reference economies: the Basak and Pavlova (2013)'s setting featuring institutional investors but no sentiment (BP), and an economy that features sentiment but no institutions (SENT). The proofs for all results in this section are given in Appendix A as special cases of the results in Section 3.

4.1.1 Benchmarking Concerns and No Sentiment (BP)

Basak and Pavlova (2013) introduce heterogeneity across investor types in an STD economy ($\delta^R = 0, \upsilon = 0$) by including positive benchmarking concerns ($0 < \upsilon < 1$) in the objective function (1) of the institutions.[8] The authors show that these benchmarking concerns induce an extra demand for the stock that raises the price-dividend ratio above and depresses the stock market price of risk below the levels prevailing in the STD economy, such that:

$$(S/D)_t^{BP} \triangleq (S_t/D_t)|_{\delta^R=0} = \overline{(S/D)}_t \frac{1}{1-\gamma(T-t)\varpi_t^I q_t} \geq \overline{(S/D)}_t, \tag{15}$$

$$\kappa_t^{BP} \triangleq \kappa_t|_{\delta^R=0} = \bar{\kappa}\left(1 - \frac{(1-\gamma(T-t))\varpi_t^I q_t}{1-\gamma(T-t)\varpi_t^I q_t}\right) \leq \bar{\kappa}. \tag{16}$$

For both $(S/D)_t^{BP}$ and κ_t^{BP}, the differences from their equilibrium values in the STD economy $\overline{(S/D)}_t$ and $\bar{\kappa}$ increase with the "benchmarked wealth" $\varpi_t^I q_t$, which we identify with the product of the fraction of aggregate wealth in I's hands, ϖ_t^I, and the intensity of their benchmarking concerns, q_t. Basak and Pavlova (2013) refer to the upward pressure on prices and depressing effect on market price of risk resulting from benchmarking concerns as an "index effect."

These authors show that the presence of institutions further increases the stock return volatility relative to the STD economy. The effect is an increasing function of the benchmarked wealth $\varpi_t^I q_t$:

$$\sigma_{S,t}^{BP} \triangleq \sigma_{S,t}|_{\delta^R=0} = \bar{\sigma}_S \left(1 + \gamma(T-t)\frac{\varpi_t^I q_t(1-\varpi_t^I q_t)}{1-\gamma(T-t)\varpi_t^I q_t}\right) \geq \bar{\sigma}_S. \tag{17}$$

Using our results from Section 3, we can decompose the associated excess volatility ratio, EVR^{BP}, into its benchmarking and relative-wealth shock propagation channels as $EVR_t^{BP} = \Psi_{q,t}^{BP} + \Psi_{\varpi^I,t}^{BP}$,

[8] In this section we only highlight the aspects of these authors' analysis that are most relevant for our purposes.

with:

$$\begin{aligned}\Psi_{q,t}^{BP} &= \tfrac{\gamma(T-t)\varpi_t^I q_t}{1-\gamma(T-t)\varpi_t^I q_t}(1-q_t) > 0,\\ \Psi_{\varpi^I,t}^{BP} &= \tfrac{\gamma(T-t)\varpi_t^I(1-\varpi_t^I)q_t^2}{1-\gamma(T-t)\varpi_t^I q_t} > 0.\end{aligned}$$

The positive sign of Ψ_q^{BP} is expected from our more general result (13), and its expression implies that the amplifying effect on return volatility of the institutions' benchmarking concerns rises with the extent of institutionalization ϖ^I of the economy, the more so, the more intense benchmarking concerns q are.

The positive sign of $\Psi_{\varpi^I}^{BP}$ implies that in the presence of institutions but no sentiment, relative-wealth effects also exacerbate the response of stock return to fundamental shocks. Intuitively, a positive (negative) dividend shock makes institutions, which overweight the stock in their portfolios, relatively wealthier (poorer). Confronted with a higher (lower) wealth, the positive sensitivity of their demand to wealth leads institutions to demand more (less) of the stock, pushing its price even higher (lower).

4.1.2 Sentiment and No Benchmarking Concerns (SENT)

In this economy, sentiment-driven (either optimistic or pessimistic) retail investors trade in the stock and the bond alongside identical but rational investors. The specialization of our framework to this case ($\delta^R \neq 0$ and $\upsilon = 0$) resembles the setup of Kogan et al. (2006) with log preferences and is formally equivalent to a model of differences of opinion (e.g., Panageas, 2020) in which one of the two investors classes has the correct prior about the dividend growth rate μ.

Sentiment introduces a wedge between the demands for the stock of irrational and rational investors, with optimistic investors ($\delta^R > 0$) overweighting and pessimistic investors ($\delta^R < 0$) underweighting the stock in their portfolios. The following result summarizes the impact of the ensuing pressure on prices:

Lemma 2. *In the presence of sentiment and absence of institutional investors, the stock market price-dividend ratio and price of risk are*

$$(S/D)_t^{SE} \triangleq (S_t/D_t)|_{\upsilon=0} = \overline{(S/D)}_t \frac{1}{1-\varpi_t^R \gamma(\delta^R(T-t))}, \tag{18}$$

$$\kappa_t^{SE} \triangleq \kappa_t|_{\upsilon=0} = \bar{\kappa}\left(1 - \frac{\varpi_t^R\big(1-\gamma(\delta^R(T-t))\big)\delta^R}{1-\varpi_t^R\gamma(\delta^R(T-t))}\right). \tag{19}$$

Thus, the price-dividend ratio rises above (respectively, falls below) the corresponding ratio $\overline{(S/D)}$

in the STD economy when sentiment investors are optimistic (pessimistic), with the difference $|(S/D)_t^{SE} - \overline{(S/D)}|$ *increasing in sentiment* $|\delta^R|$*. Similarly, the market price of risk under optimistic (respectively, pessimistic) sentiment falls below (rises above) its equilibrium value* $\bar{\kappa}$ *in the STD economy, with the difference* $|\kappa_t^{SE} - \bar{\kappa}|$ *increasing in sentiment.*

Importantly, sentiment also creates "excess volatility" in stock returns:

Lemma 3. *In the presence of sentiment and absence of institutional investors, the stock return volatility and excess volatility ratio are:*

$$\sigma_{S,t}^{SE} \triangleq \sigma_{S,t}|_{\upsilon=0} = \bar{\sigma}_S \left(1 + \frac{\varpi_t^R(1-\varpi_t^R)\gamma(\delta^R(T-t))}{1-\varpi_t^R\gamma(\delta^R(T-t))}\delta^R\right) \geq \bar{\sigma}_S, \tag{20}$$

$$EVR_t^{SE} \triangleq EVR_t|_{\upsilon=0} = \Psi_{\varpi^I,t}^{SE} = \frac{\varpi_t^R(1-\varpi_t^R)\gamma(\delta^R(T-t))}{1-\varpi_t^R\gamma(\delta^R(T-t))}\delta^R \geq 0. \tag{21}$$

Thus, under heterogeneity in investor types ($0 < \varpi_t^R < 1$)*, both positive (optimism) and negative (pessimism) sentiment exacerbate the stock return volatility relative to the STD economy* ($EVR_t^{SE} > 0$)*. Moreover,* $\sigma_{S,t}^{SE}$ (EVR_t^{SE}) *is increasing in sentiment for fixed wealth distribution and reverts to the STD equilibrium value* $\bar{\sigma}_S$ (0) *when all investors are sentiment driven* ($\varpi_t^R = 1$)*.*

Whenever sentiment-driven investors trade in the stock with rational investors, the positive impact of sentiment on volatility arises under *both* optimism and pessimism. Moreover, it increases monotonically and symmetrically with the level of (positive or negative) sentiment.[9]

The excess volatility of stock returns in the SENT economy is purely a relative-wealth effect. Similarly to the institutional investor in the BP economy, a sentiment-driven *optimistic* investor (a rational investor in the case of *pessimistic* sentiment) is overexposed to the stock compared to the other investor type. A positive dividend shock makes the investor type that is overexposed to the stock relatively wealthier. The wealthier investor then demands more stock, pushing its price even higher. Conversely, a negative dividend shock makes the same investor relatively poorer, negatively impacting her demand for the stock and, as a result, the stock price. In this sense, the presence of sentiment in an all-retail economy *always* exacerbates return volatility with respect to the STD case, as does the presence of institutions in the BP setting. This positive relationship between sentiment and excess volatility is consistent with the prediction of earlier models of noise trading and sentiment risk (DeLong et al., 1990; Dumas et al., 2009).

[9]This effect is robust to more general constant relative risk aversion (CRRA) preferences, with its intensity decreasing in the coefficient of relative risk aversion.

4.1.3 Comparison of the Pure Effects of Benchmarking vs. Sentiment

The similarities in effects on prices and return dynamics of benchmarking concerns (section 4.1.1) and sentiment (Section 4.1.2) raise the question of how the asset pricing implications of these features compare. One approach to address this question is to make the two features quantitatively comparable and examine the resulting similarities and differences in equilibrium values. We have shown that, in the SENT economy, the stock price-dividend ratio, the market price of risk, and the return volatility change monotonically with the degree of optimism δ^R. Thus, keeping all other parameters and the horizon $T-t$ fixed, it is possible to find the value $\delta^R = \check{\delta}_t^R$ such that the equilibrium values of any one of these variables in the SENT and BP economies coincide. We can then compare the equilibrium effect of benchmarking concerns and sentiment in the other endogenous variables.[10] The following result shows that the level of optimism that equates price-dividend ratios across the two economies is independent of the distribution of wealth ϖ_t^R across the agents:

Lemma 4. *The degree of optimism $\check{\delta}_t^R$ that leads to identical price-dividend ratios for the stock in the BP and SENT economies at any given horizon $T-t$ equals:*

$$0 < \check{\delta}_t^R = \frac{\log(1-\gamma(T-t)q_t)}{\log(1-\gamma(T-t))} < q_t. \tag{22}$$

At this level of optimism, $\kappa_t^{BP} \geq \kappa_t^{SE}$, and $\sigma_{S,t}^{BP} \geq \sigma_{S,t}^{SE}$.

Lemma 4 shows that, for any parameterization of the BP economy, there is a level of optimism in the SENT economy that creates the same upward shift in price-dividend ratios across *all* distributions of aggregate wealth as the "index effect" identified by Basak and Pavlova (2013). Moreover, at this level of optimism, the index effect on the market price of risk and the stock volatility is always greater than the effect of sentiment.

Fig. 2 illustrates the equilibrium initial price-dividend ratio, the market price of risk, and the return volatility, under the degree of optimism $\check{\delta}_t^R$. Equilibrium values are plotted as a function of the share of aggregate wealth $1-\varpi_t^R$ of the institutional or the sentiment-prone (optimistic) retail investors, respectively, in the BP and SENT economies.[11] The stock overvaluation relative to the

[10] In this exercise, we assign the distribution weights ϖ^R and $1-\varpi^R$ in the SENT economy to, respectively, the rational and irrational investors. In this way, the irrational investor has the same weight as the institutional investor of the BP economy ($\varpi^I = 1-\varpi^R$).

[11] For comparability, the rest of the model parameters follow the baseline parameterization of Basak and Pavlova (2013)'s single-stock economy.

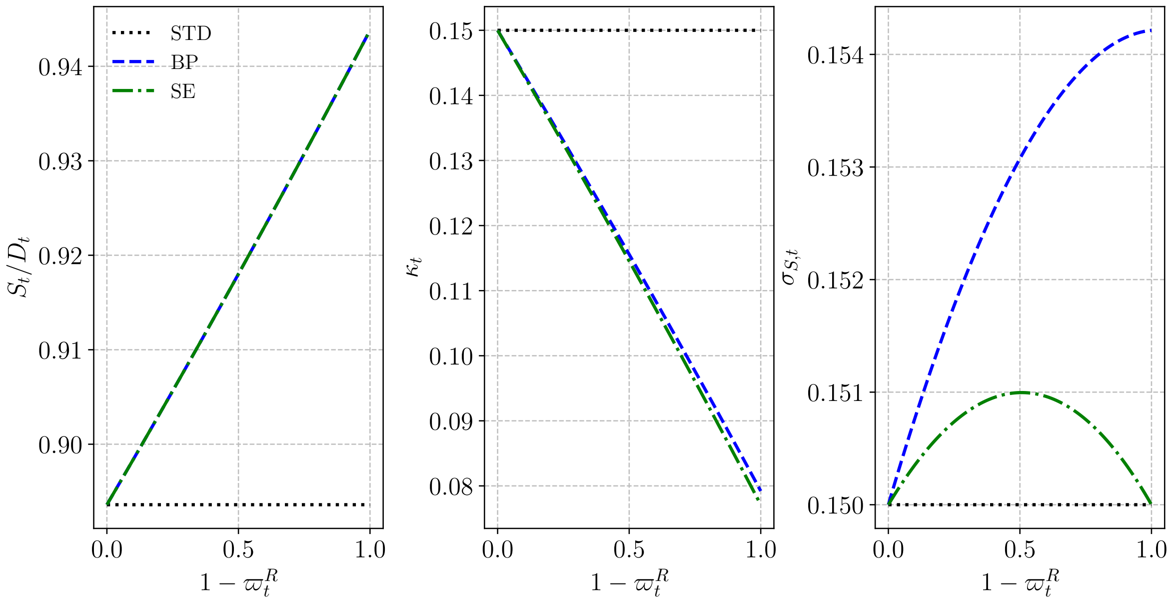

Figure 2: **Equilibrium under the reference economies**

This figure plots the equilibrium price-dividend ratio (leftmost panel), the market price of risk (center panel), and the stock return volatility (rightmost panel), under the STD (dotted black line), BP (dashed blue line), and SENT (dash-and-dot green line) economies. Equilibrium values are depicted as a function of the wealth shares of either I-investors (BP case) or sentiment R-investors (SENT case). Across all graphs, $\delta^R = \check{\delta}_0^R = 0.486$. The rest of the model parameters follow the parameterization in Basak and Pavlova (2013): $\mu = 0, \sigma = 0.15, t = 0, T = 5, D_0 = 1, \upsilon = 0.5$.

STD economy rises with the wealth share of the institutions (BP case) or the sentiment investors (SENT case) in each of the economies. Accordingly, equilibrium stock market prices of risk follow very similar decreasing patterns across the two economies, with values being (slightly) lower in the SENT economy in line with Lemma 4.

The right-most panel quantitatively illustrates the difference, following Lemma 4, in the amplification effects of benchmarking concerns and sentiment on stock return volatility. For low institutional or sentiment investors' shares of aggregate wealth, excess volatility increases with either share. Still, it does so more rapidly in the BP case—as expected from the presence of a benchmarking channel on excess volatility only in this case. As these shares become large enough, the pattern remains positive in the BP case but turns negative in the SENT case, where the excess volatility disappears as sentiment investors become the only investor type.[12]

[12]It can be additionally shown that whenever the wealth share of the retail rational investors ϖ_t^R is small enough (with $\varpi_t^R < 0.5$ being sufficient), the magnitude of the relative-wealth channel is larger, i.e., changes in relative wealth

4.2 General Case: Interaction of Benchmarking and Sentiment

We have shown that introducing either optimistic sentiment or benchmarking concerns to an otherwise standard economy has similar boosting effects on stock prices and return volatilities, with possibly (significantly) different magnitudes in the case of volatility. In this section, we analyze the equilibrium under the general ("GE") case in which sentiment retail investors trade alongside institutional investors.

4.2.1 Excess volatility

The simple addition of the effects of benchmarking concerns (Section 4.1.1) and sentiment (Section 4.1.2) on excess volatility may suggest that in the presence of both optimistic R-investors and institutional I-investors, the stock return volatility must rise beyond the BP and SENT levels. The following result indicates that this intuition need not hold:

Proposition 3. *In the presence of institutional ($\upsilon > 0$) and irrational retail ($\delta^R \neq 0$) investors:*

(a) *There exists a unique degree of optimism $\hat{\delta}^R(D_t, \varpi_t^I, T-t) > \check{\delta}_t^R > 0$ such that:*

$$\frac{\partial \sigma_{S,t}}{\partial \delta^R} \begin{cases} > 0, & \delta^R > \hat{\delta}^R(D_t, \varpi_t^I, T-t) \\ = 0, & \delta^R = \hat{\delta}^R(D_t, \varpi_t^I, T-t) \\ < 0, & \delta^R < \hat{\delta}^R(D_t, \varpi_t^I, T-t) \end{cases}.$$

This implies, in particular, that for $0 < \delta^R < \check{\delta}_t^R$, the effect of optimistic sentiment is to reduce *the stock return volatility across all wealth distributions ϖ_t^I relative to the BP case.*

(b) *As long as aggregate wealth is not concentrated in institutional investors' hands, the stock return volatility $\sigma_{S,t}$ monotonically increases with the intensity of benchmarking concerns q_t under no or low sentiment, but monotonically* decreases *with q_t when sentiment is sufficiently* optimistic. *Otherwise, $\sigma_{S,t}$ first increases and then decreases (i.e., is hump-shaped) with q_t.*

According to part (a) of Proposition 3, within a range of no-to-moderate optimism $0 < \delta^R < \check{\delta}_t^R$ the stock excess volatility *monotonically falls* with investor sentiment across all aggregate wealth

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lead to greater volatility of stock returns in the BP economy than in the SENT economy. More precisely,

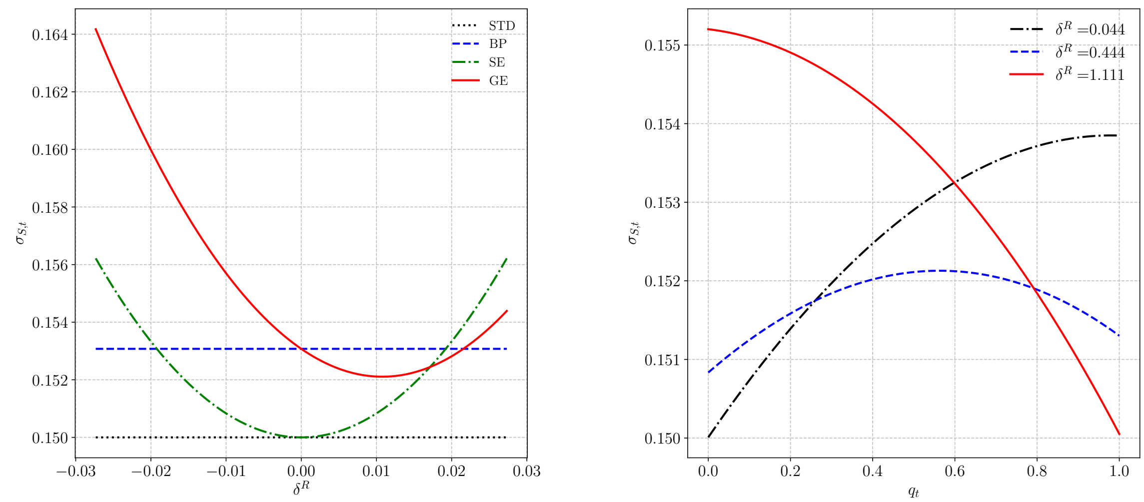
$$\Psi^{BP}_{\varpi^I,t} > \Psi^{SE}_{\varpi^I,t} \Leftrightarrow \varpi_t^R < \frac{\check{\delta}_t^R - (1 - \gamma(T-t)q_t)}{\check{\delta}_t^R + 1} < 0.5.$$

distributions. Thus, the presence of sentiment in an institutionalized economy can *dampen* the excess volatility of the stock market. This can be seen by comparing the equilibrium stock return volatility at different levels of optimism of the R-investors in Panel (a) of Figure 3 in the GE (red solid line) and BP (dashed blue line) economies. The negative relationship between optimism and volatility in the GE case contrasts with the pattern arising in the otherwise equivalent SENT economy (dash-and-dot green line), whereby greater optimism always translates into *greater* excess volatility—in line with the commonly held view.

According to part (b), the positive relationship between benchmarking intensity and volatility prevailing in a rational institutionalized (BP) economy switches signs for sufficiently high levels of optimism. This is illustrated in Panel (b) of Figure 3, which plots the stock return volatility as a function of institutions' benchmarking intensity at different levels of retail sentiment. For very low (pessimistic) and null levels of sentiment, $\sigma_{S,t}$ increases monotonically with q_t. The relationship is nonmonotonic for moderate levels of optimism, and becomes monotonically negative for high optimism. The effect is such that, following Proposition 3(a), under strong benchmarking concerns the stock return volatility is smaller at high than at low levels of optimism.

The economic intuition behind these effects can be traced back to the decomposition of excess volatility into the benchmarking and relative-wealth shock propagation channels. Starting with part (a) of Proposition 3, for $0 < \delta^R < \check{\delta}_t^R$ the benchmarking effect on the stock demand is stronger than the sentiment effect ($\delta^R < q_t$) and creates heterogeneity in the portfolio holdings across the two investor types. However, as δ^R rises, the gap between the two stock demands shrinks until disappearing at $\delta^R \approx q_t$. At this point, none of the two investor types gets to lever up their portfolios, as they would if trading with rational retail counterparts, and the relative-wealth channel shuts down. For even higher levels of optimism, the demand strength of the sentiment-driven investors exceeds that of the institutional investors ($\delta^R > q_t$), the difference $\delta^R - \check{\delta}_t^R$ turns positive, and the ensuing differences in portfolios activate the relative-wealth channel's positive effect on return volatility.

The same changes in relative wealth as a function of the distance between δ^R and q_t explain why, according to part (b) of Proposition 3, return volatility can fall as q_t increases toward δ^R for sufficiently optimistic sentiment. Importantly, however, the institutions' benchmarking intensity is bounded above by $q_t = 1$, as institutions can favor the stock over the riskfree asset, regardless of fundamentals, only as much as their benchmark does. This implies that, unlike the comparative statics of $\sigma_{S,t}$ with respect to δ^R, the negative relationship between q_t and $\sigma_{S,t}$ need not turn positive



(a) Effect of sentiment

(b) Effect of benchmarking

Figure 3: **Equilibrium volatility**

The figure plots the equilibrium stock return volatility as a function of either sentiment (Panel (a)) or institutions' benchmarking intensity (Panel (b)). In Panel (a), equilibrium relationships are illustrated for the GE (red solid line), BP (dashed blue line), SENT (dash-and-dot green line), and STD (dotted black line) economies. In Panel (b), equilibrium relationships are illustrated for the GE economy under high (red solid line), moderate (dashed blue line) and low (dash-and-dot black line) levels of optimistic sentiment. In both Panels, we set $\varpi_t^I = 0.5$. The rest of the model parameters (other than δ^R in Panel (a), and υ in Panel (b)) are as in Fig. 2.

for sufficiently high q_t when $\delta^R > 1$.

Following this argument, one would expect that the range of optimism over which the relationship between sentiment and volatility is negative increases with the intensity q_t of institutions' benchmarking concerns. Figure 4, which illustrates this pattern across different levels of q_t, confirms this. It further highlights an *asymmetry* in this relationship that is caused only by the presence of institutions: relative to retail rational investors, institutions attenuate excess volatility in the presence of optimistic sentiment but exacerbate it under widespread pessimism. To understand the latter effect, notice that institutions are willing to buy more of the stock shares sold by pessimistic retail investors than equivalently rational but retail counterparts. This creates greater differences in the composition of R- and I-investors' portfolios, making the relative-wealth channel amplify return volatility.

4.2.2 Effect of institutionalization

DeVault et al. (2019) conjecture that the existence of sophisticated investors need not help prices

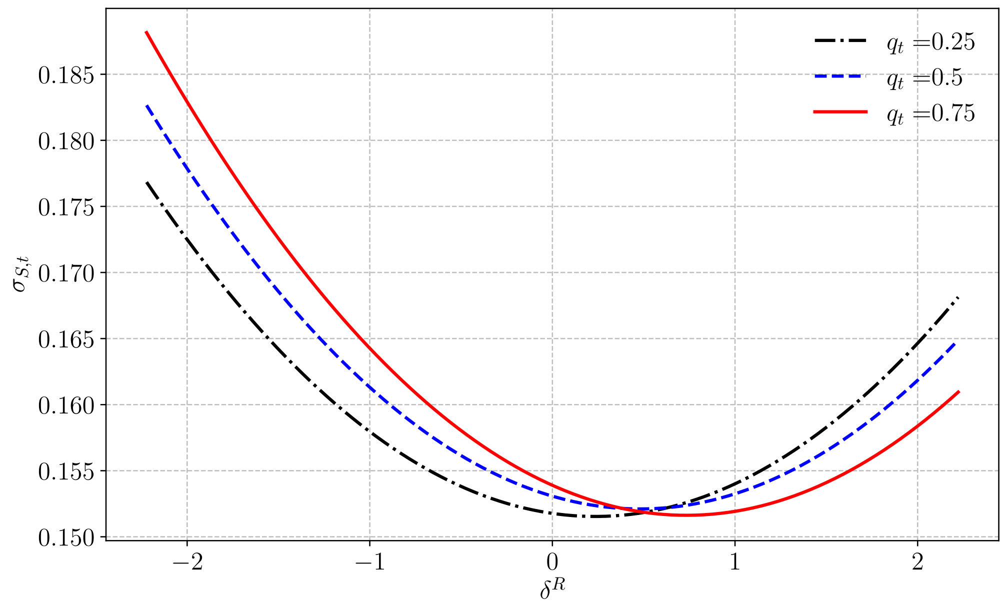


Figure 4: **Sentiment and equilibrium volatility**

The figure plots the equilibrium relationship between sentiment and stock return volatility under the GE economies for different levels of institutions' benchmarking intensity q_t. We set $\varpi_t^I = 0.5$. The rest of the model parameters are as in Fig. 2.

converge to, and might make them deviate even more from, fundamental value. We examine this conjecture within our setup by studying whether the introduction of institutions to (i.e., the institutionalization of) a market populated by optimistic retail investors exacerbates or, on the contrary, helps correct overpricing.

To this aim, we analyze how the price-dividend ratio S_t/D_t changes in Eq. (3) as the share ϖ_t^I of aggregate wealth in I's hands increases, for different levels of sentiment of the R investors. We find that for high enough levels of sentiment, a greater level of institutionalization of markets always helps *correct* overpricing:

Proposition 4. *In an economy populated by irrational retail and rational institutional investors, whether a higher level of institutionalization decreases, does not change, or increases the stock price-dividend ratio depends on whether the level of retail optimism δ^R exceeds, equals, or falls below the threshold $\check{\delta}_t^R$ that equalizes price-dividend ratios in the BP and SENT economies.*

Whether greater institutionalization helps correct or exacerbates sentiment-induced price distortions depends on the relative strength of retail sentiment vis-a-vis institutions' benchmarking concerns. Figure 5, which plots the equilibrium stock price-dividend ratios and market prices of risk as

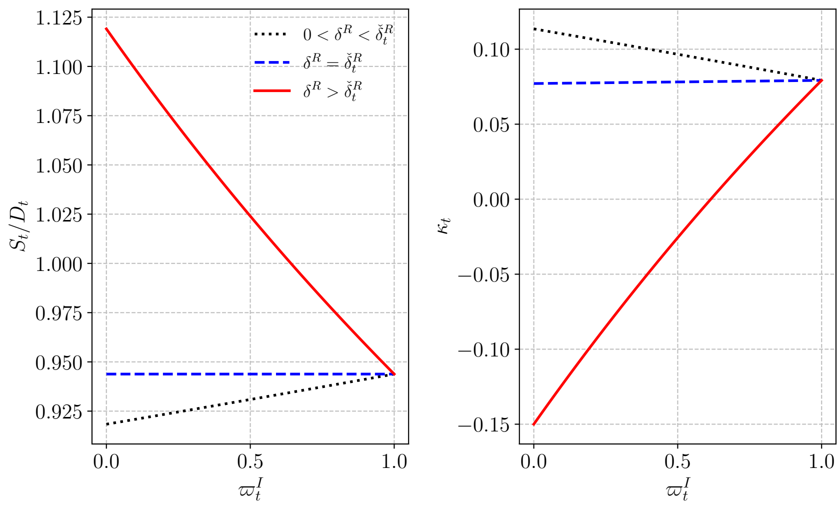


Figure 5: **Equilibrium prices in the GE case under mild to high levels of optimism**
This figure plots the equilibrium price-dividend ratio (left panel) and market price of risk (right panel) under the GE economy for three levels of optimism δ^R of the R-investors: mild optimism ($0 < \delta^R < \check{\delta}_t^R$, dotted black line), middle-ranged optimism ($\delta^R = \check{\delta}_t^R$, dashed blue line), and high optimism ($\delta^R > \check{\delta}_t^R$, solid red line). Equilibrium values are depicted as a function of the share of aggregate wealth of Iinvestors. Model parameters are as in Fig. 2.

a function of the share of institutional investors in aggregate wealth, and Figure 6, which illustrates the associated optimal asset allocations, offer intuition for this result. For low levels of sentiment ($\delta^R < \check{\delta}_t^R$), even optimistic retail investors choose to sell an increasing fraction of their stock holdings to the institutions as the level of institutionalization rises. Because benchmark concerns increase their risk appetite, the institutions purchase these shares at increasingly higher price-dividend ratios, determining a pattern of stock overpricing that worsens with institutionalization. This result verifies the conjecture of DeVault et al. (2019) that the existence of sophisticated investors might push prices further away from their fundamental value than the presence of sentiment-driven retail investors.

However, the opposite holds for higher levels of optimism ($\delta^R > \check{\delta}_t^R$), when retail sentiment leads to stronger demand for the stock than institutions' benchmarking concerns. Such strong retail optimism can lead to severe levels of overvaluation and a negative market risk premium, akin to a financial "bubble," under low levels of institutionalization. When the risk premium is low enough, however, rational institutions, no matter how concerned about their benchmark, will find it optimal to reduce their portfolio allocation in the stock. As institutionalization grows, aggressive selling by the institutions pushes the stock price closer to fundamental value (see the STD case in Fig. 2)

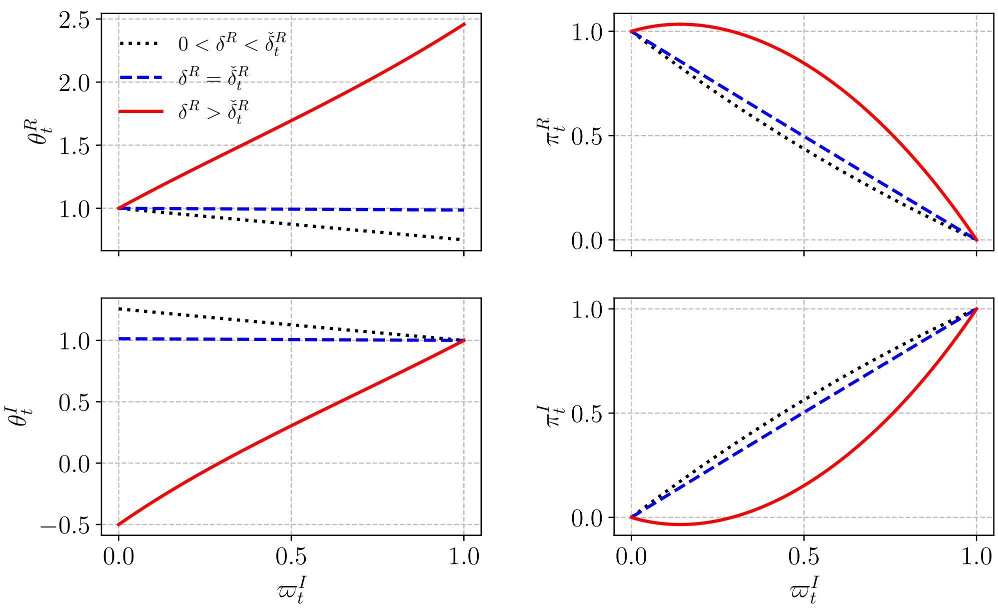


Figure 6: **Equilibrium allocations in the GE case under mild to high levels of optimism** This figure plots the equilibrium weights (left panels) and number of shares $\pi_t^k = \theta_t^k W_t^k / S_t, k \in \{I, R\}$ of the stock (right panels) in the portfolios of optimistic retail (top panels) and rational institutional (bottom panels) investors under the equilibrium cases illustrated in Fig. 5: mild optimism ($0 < \delta^R < \check{\delta}_t^R$, dotted black line), middle-ranged optimism ($\delta^R = \check{\delta}_t^R$, dashed blue line), and high optimism ($\delta^R > \check{\delta}_t^R$, solid red line). Equilibrium values are depicted as a function of the share of aggregate wealth of Iinvestors. Model parameters are as in Fig. 2.

and eventually reverses it to levels consistent with a positive risk premium. Notably, the threshold that separates "low" from "high" sentiment, $\check{\delta}_t^R$, is invariant to the level of institutionalization and coincides with the threshold level of retail optimism in which the "pure" effects of sentiment on the stock's price dividend ratio is identical to the impact of benchmarking in an all-rational-investors economy (BP).

5 Dynamic effects

Over time, aggregate wealth distribution across investor types changes endogenously in response to the arrival of positive and negative cash flow news, affecting the dynamics of equilibrium prices and allocations. To characterize these dynamics, we fix the initial stock share endowments λ^I and λ^R of the I and R investors to λ and $1 - \lambda$, respectively, and solve for the corresponding time-t aggregate wealth shares to obtain the following:

Lemma 5. *Given initial wealth distribution* $\varpi_0^I = \lambda$ *and* $\varpi_0^R = 1 - \lambda$ *for, respectively, the* I *and* R *investors, the equilibrium market price of risk, price-dividend ratio, and return volatility are given by Eqs.* (4), (3), *and* (8), *while the equilibrium portfolio allocations to the stock and the optimal borrowing are given by Eqs.* (5) *and* (7), *for:*

$$\varpi_t^I = \frac{\lambda}{\lambda + \left(1 - \gamma(\frac{1}{2}\delta^R(\delta^R - 1)t)\right)\left(\frac{q_t}{q_0}\right)^{\delta^R}\left(\frac{1-q_t}{1-q_0}\right)^{1-\delta^R}(1-\lambda)}, \tag{23}$$

$$\varpi_t^R = 1 - \varpi_t^I. \tag{24}$$

Moreover, whether positive cash flow news decrease *the institutional investors' share of aggregate wealth depends on whether the sentiment retail investors are sufficiently optimistic, as given by:*

$$\frac{\partial(\varpi_t^I)}{\partial D_t}\begin{cases} < 0, & \delta^R > q_t \\ = 0, & \delta^R = q_t \\ > 0, & \delta^R < q_t \end{cases}. \tag{25}$$

To provide intuition for this result, we first proceed as in Section 4.1.3 and compare the equilibria under the BP and SENT economies where the only non-standard features are, respectively, the benchmark concerns of I investors or the sentiment of R investors. We do so by referring to Fig. 7, which illustrates the interim equilibrium as of $t = 1$ (for $T = 5$) under the parameterization of Fig. 2, conditional on different realizations of the cash flow news D_t.

Based on the analysis above, one would expect similarities and differences between the effects on sentiment, on the one hand, and benchmark concerns, on the other, on equilibrium configurations. Lemma 5 indicates that good (bad) cash flow news, as represented by values of $dD_t > 0$ (< 0), make the institutional investors in the BP case and the optimistic investors in the SENT case wealthier (poorer), thus increasing (decreasing) the aggregate stock demand and, accordingly, the stock's price-dividend ratio. These wealth effects lead to a countercyclical behavior for the stock's Sharpe ratio (market price of risk). The SENT case shows that the presence of wealth effects is enough to create this pattern, even with a fixed level of optimism. The BP case shows that benchmarking concerns further exacerbate it through their interaction with wealth effects.

Where the BP and SENT cases differ, once again, is in the impact of cash flow news on return volatility. Indeed, the stock's excess volatility rises substantially in the BP economy but stays approximately constant in the SENT economy as cash flow news and the relative wealth of either

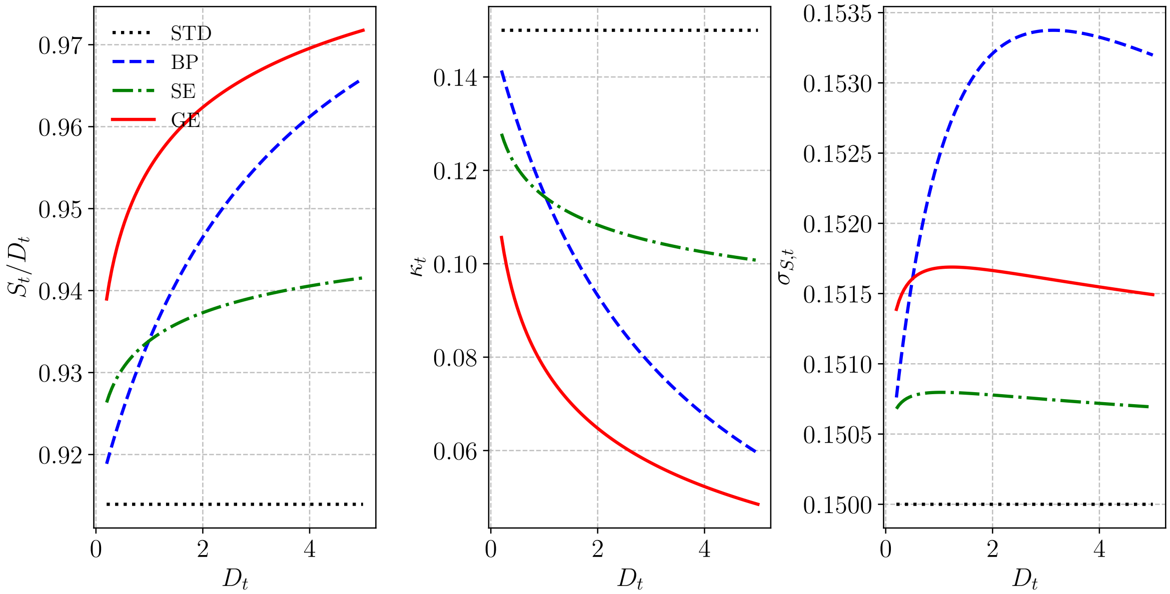


Figure 7: **Interim equilibrium in the GE and reference economies**

This figure plots the equilibrium price-dividend ratio (leftmost panel), the market price of risk (center panel), and the stock return volatility (rightmost panel), under the STD (dotted black line), BP (dashed blue line), SENT (dash-and-dot green line), and GE (red solid line) economies. Equilibrium values are depicted as a function of cash flows D_t as of $t = 1$, for a fixed initial share of aggregate wealth $\varpi_0^I = 0.5$. The rest of the model parameters are as in Fig.2.

the I investor or the optimistic R investor, respectively, rises. As we saw before, this occurs because positive news increases the stock market price, which, along with the associated wealth effects, leads I investors to demand more of the stock. To attenuate this demand, volatility must necessarily rise. Note that this implies a *cyclical* pattern of volatility over a wide range of positive cash flow news in the BP economy.

The different effects of wealth on the stock demand or, equivalently, on the demand for leverage of institutional vs. sentiment-driven investors can be understood by reference to the differences in portfolio allocations across the BP and SENT economies, as illustrated in Fig. 8. For low levels of D_t (e.g., for $D_t < 2$ in the figure), the fraction of wealth allocated to the stock (leftmost bottom panel) monotonically increases among I investors (dashed blue line) but decreases instead among R investors, with D_t. The allocations determine opposite leverage patterns across the two investor types—increasing for I investors, decreasing for R investors.[13]

[13]To facilitate comparison with the BP case, the portfolio of the sentiment-driven retail investors in the SENT economy is identified with the superscript "I" and plotted in the bottom panels in Fig. 8. In contrast, the superscript

These trading patterns highlight an interesting contrast: even though their intrinsic (log) preferences display constant relative risk aversion (CRRA), the optimistic investors behave as if their relative risk aversion *increased* with wealth instead (IRRA preferences) by decreasing the fraction of their portfolio allocated to the stock in response to positive wealth shocks. This apparent contradiction is purely an equilibrium outcome. As their wealth increases, the increased demand of optimistic investors pushes the stock price higher and the Sharpe ratio lower, worsening the risk-return tradeoff that the stock offers. This effect is absent in partial equilibrium. In the current context, it implies that the optimists' demand for the stock grows less than proportionally with their wealth, leading to the observed pattern in portfolio weights.

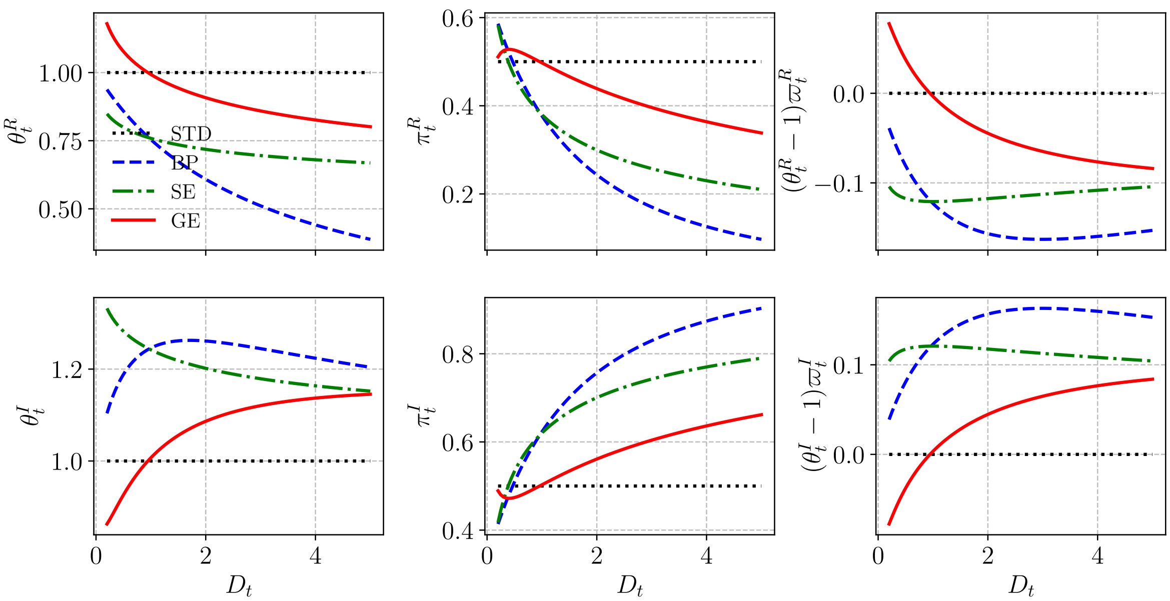


Figure 8: **Interim equilibrium portfolio allocations under the GE and reference economies**

This figure plots the equilibrium weights (leftmost panels) and number of shares $\pi_t^k = \theta_t^k W_t^k / S_t, k \in \{I, R\}$ of the stock (center panels) in the portfolios, and the leverage (rightmost panels), of optimistic retail (top panels) and rational institutional (bottom panels) investors. Depicted cases correspond to the STD (dotted black line), BP (dashed blue line), SENT (dash-and-dot green line), and GE (red solid line) economies. Across panels, equilibrium values are plotted against cash flows D_t as of $t = 1$, for a fixed initial share of aggregate wealth $\varpi_0^I = 0.5$. Model parameters are as in Fig.7.

What happens when both the sentiment-driven and the institutional investors trade with each other? As in our characterization of equilibrium of Section 3, the GE case of Fig. 7 shows that

"R" (and the top row of panels) is reserved for their rational retail counterparts.

the effect on prices and Sharpe ratios is as anticipated: At all levels of cash flows D_t, the former increase, while the latter falls, beyond the levels in the benchmark BP and SENT economies.

The effect on volatility, once again, is less obvious: In markets with predominantly good news ($D_t > D_0$), the introduction of optimistic investors to a BP economy (GE case) not only reduces excess volatility substantially at all D_t levels, but it also turns the cyclical pattern into a flat or decreasing pattern. A comparison of the top and bottom rows of Fig. 8 reveals that under these volatility and Sharpe ratios patterns, the institutional investors purchase increasingly more shares of the stock from the optimistic investors, which they finance by borrowing increasing amounts of money. The result represents a divorce in the patterns of volatility and institutions' leverage, both of which are tightly related in the BP economy, where the trading counterparts of the institutions are not rational but sentiment-driven retail investors.

The economic intuition for this result is as follows. The benchmarking channel pushes the level of volatility around neutral cumulative news ($D_t = D_0$) above that prevailing in the SENT economy but below that prevailing in the BP economy. The fact that volatility (slightly) *falls* as cumulative cash flow news becomes either positive ($D_t > D_0$) or negative ($D_t < D_0$) then implies that changes in the opposite direction in the relative-wealth channel are offsetting the associated changes in this amplification channel. Given that aggregate wealth is evenly split at $t = 0$ in this illustration, the changes in interim relative wealth associated with this cash flow news must be small or nil. Then it must be that the higher demand for the stock of I-investors in response to, e.g., a positive cash flow news, is being met by a lower demand of the R-investors. A comparison of the solid red and dash-and-dot green lines in the top left panel of Fig. 8 shows that this is indeed the case, as the weight of the stock in the portfolio of the optimistic R-investor falls more steeply when trading with a rational I investor (red line) than with a rational R investor (green line).

Why does this happen? Looking at Fig. 7, we see that as D_t rises, the market price of risk in the GE case falls more steeply (because of the effect of benchmarking) than in the SENT case, justifying the greater fall in R's stock demand in the former economy. Something similar happens with negative cash flow news: the demand of the optimistic R-investors increases more when trading with rational I investors than when trading with otherwise equivalent R investors because the benchmarking concerns induce the former to reduce their position in the stock very aggressively, leading to a steeper increase in the market price of risk.

How different can the patterns in volatility be in the GE case compared to the SENT and BP reference cases? In particular, can volatility be countercyclical over a "reasonable" range of the

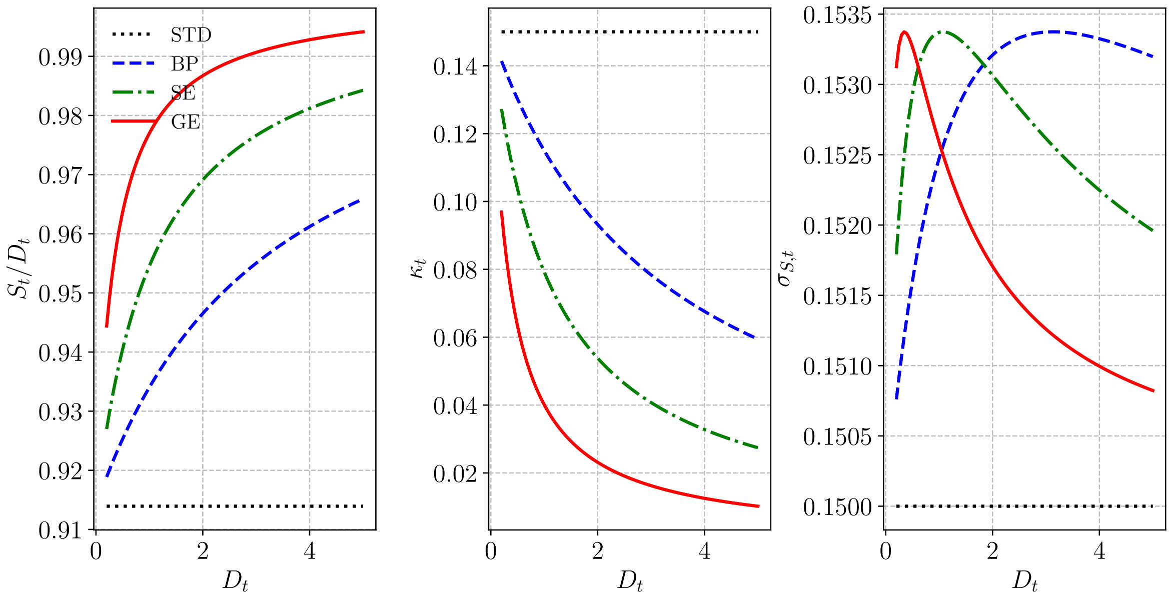


Figure 9: **Interim equilibrium: higher optimism**

This figure plots the equilibrium price-dividend ratio (leftmost panel), the market price of risk (center panel), and the stock return volatility (rightmost panel), under the STD (dotted black line), BP (dashed blue line), and SENT (dash-and-dot green line) economies, for a relatively high level of optimism $\delta^R = 1$. Equilibrium values are depicted as a function of cash flows D_t as of $t = 1$, for a fixed initial share of aggregate wealth $\varpi_0^I = 0.5$. The rest of the model parameters are as in Fig.2.

cash flows D_t? This feature would be desirably from an empirical point of view, given the typically countercyclical pattern of volatility that the literature (see, e.g., Mele, 2007) has documented.

Fig. 9 illustrates the interim equilibrium at $t = 1$ for high levels of optimism of the sentiment-driven investor, under an otherwise identical parameterization as in Fig. 7.[14] Two features of the volatility pattern in the GE case, in comparison with either the SENT or BP cases, are noticeable:

(i) Volatility is highly countercyclical over almost the entire range of D_t, even when it is cyclical for low-D_t states in the reference SENT economy and for both low- and high-D_t states in the BP economy.

(ii) Volatility can be lower not only than in the case of an economy with all rational investors in which some are institutional (BP), but also than in an economy where there are only retail investors but some are optimistic (SENT).

[14]More precisely, $\delta^R = 1$. At this level of optimism, the demand for the stock of the I investors cannot dominate the demand of optimistic R investors ($\delta^R \geq q_t$). According to Lemma 5, positive cash flow news then increases the latter's share of wealth.

The intuition for this result is as follows. With high optimism ($\delta^R > q_t$), according to Proposition 2, the R investors are relatively overinvested in the stock compared to the I investors. Following Lemma 5, positive cash flow news makes R investors relatively wealthier and I investors relatively poorer. However, as R investors become wealthier, their "equilibrium-induced" increasing relative risk aversion leads them to reduce the fraction of wealth invested in the stock. The positive cash flow news still leads I investors to increase the weight of their portfolio allocated in the stock. However, because R investors become relatively wealthier, their reduced demand prevails in the aggregate, inducing the relative-wealth channel to *reduce* the sensitivity of prices to CF news. Remember that, by contrast, the relative-wealth channel always increases this sensitivity in the BP and SENT economies. The negative impact of the relative-wealth channel on volatility is specific to the GE case and explains both the *level* effect (volatility is lower in the GE case than in either the BP or SENT cases) and the *slope* effect (volatility is countercyclical). Because stronger wealth effects lead to steeper falls in volatility, both effects are exacerbated at higher levels of optimism.

6 Empirical Analysis

Our model results suggest potentially large gains in explanatory power from additionally accounting for either sentiment or institutions' benchmarking concerns in setups where only one of these features is present. As illustrated by Fig. 3 and 4, the additional explanatory power could help rationalize the aggregate evidence on the changing relationship between optimism and volatility depending on the level of institutionalization of markets presented in Fig. 1. In this section, we summarize our model predictions with regard to this relationship, formulate other novel testable implications, and contrast them all with the data.

6.1 Testable Implications

The results in Section 4.2.1 highlight an attenuating impact of the relative-wealth channel on the stock's excess volatility. They imply, in particular, that in markets with a high presence of institutional investors, greater optimism need not exacerbate volatility but substantially reduce it, and that whether a greater incidence of benchmarking concerns in financial markets increases or decreases volatility depends on the prevailing level of investor sentiment. These results, which are summarized by Fig. 4, lead to the following:

Testable Implication 1 (TI1): *The institutionalization of financial markets induces an asym-*

metric sentiment-excess volatility pattern, such that (i) *an intensification of* pessimistic *sentiment always increases volatility while a similar intensification of* optimistic *sentiment can reduce it instead, and* (ii) *an intensification of benchmarking practices exacerbates volatility in markets with predominantly* pessimistic *sentiment but attenuates it in markets with predominantly* optimistic *sentiment.*

Second, according to Section 4.2.2, benchmarking concerns have a positive but limited influence on the demand of rational institutional investors relative to the demand of optimistic retail investors. This result implies that institutions are unlikely to help correct situations of low to moderate asset overpricing, but can exert a significant correcting force for more severe overpricing levels, leading to the following:

Testable Implication 2 (TI2): *A higher institutionalization of financial markets exacerbates stock overpricing when sentiment is low or moderately optimistic but* attenuates *it instead in the presence of strongly optimistic investor sentiment.*

In the following section we test these implications using data on the U.S. stock market.

6.2 Evidence

Our empirical tests require three main measures: (i) sentiment, (ii) institutional ownership, and (iii) mispricing. Our main proxy for (i) is Baker and Wurgler (2006)'s sentiment metric, which we denote as SentBW. SentBW is an aggregate monthly time-series from 1965/07 to 2022/06 and is available from Wurgler's webpage[15]. This time-series is normalized to have a mean value of zero. We create quartiles of SentBW over the whole sample from 1965 to 2022, and as a result quartiles 1 and 2 correspond to negative ("bearish") sentiment months, whereas quartiles 3 and 4 correspond to moderately and strongly positive ("bullish") months, respectively.

We extract quarterly institutional holdings data covering the period from 1980 to 2021 from Thomson/Refinitiv[16]. Our stock-level proxy for (ii), IOR, is the ratio of shares held by institutions to the number of shares outstanding.

We use the mispricing scores (MISP) provided by Stambaugh et al. (2015) as our proxy for (iii).

[15] https://pages.stern.nyu.edu/~jwurgler/

[16] This data is extracted from the 13F form that investment companies and professional money managers are required to file with the SEC.

MISP is a combination of 11 well-known asset pricing anomalies.[17] They argue that the reason to focus on this subset of anomalies is that these are more likely driven by market mispricing, as opposed to rational risk premia. They show that the predictability of these anomalies is higher after periods of high sentiment, and is mostly driven by the overpriced stocks (High MISP), as short selling is more costly compared to going long.[18] We collect the monthly stock-level MISP from Stambaugh's webpage,[19] which covers the period from 1965 to 2016.

To sum up, after combining the three datasets above, our final sample covers the period from 1980/04 to 2016/12,[20] and includes three main variables of interest with different frequencies and scopes: IOR (quarterly, stock-level), MISP (monthly, stock-level), and SentBW (monthly, market-level).

6.2.1 On the Relation Between Sentiment and Volatility

Our testable implication TI1 implies that, in markets with a significant presence of institutions, the observation of *low* return volatility will be more indicative than *high* volatility of positive sentiment. We test this implication using both market-level and stock-level sentiment proxies.

Portfolio Analysis Using Market-Level Sentiment To test the effect of institutions on the relationship between sentiment and volatility, we form stock portfolios monthly by sequentially sorting them first into terciles based on the previous quarter's IOR first and then into deciles based on MISP. This results in 30 monthly rebalanced portfolios. The average number of stocks in these portfolios ranges from 85 in 2009 to 150 in 1997. Of these 30 portfolios, we keep only the portfolios with MISP deciles 6 to 10, i.e., portfolios with above-median MISP, and portfolios in IOR terciles 1 and 3. Thus, for this analysis, we exclude the Mid-IOR portfolios (tercile 2) and below-median MISP portfolios (deciles 1 to 5). The reason to focus the analysis on this subset of 10 portfolios is that prior literature has shown that most of the effects of MISP are concentrated in the above-

[17]The individual anomalies and the studies uncovering them (in parenthesis) are: momentum (Jegadeesh and Titman, 1993), gross profitability (Novy-Marx, 2013), asset growth (Cooper et al., 2008), investment to assets (Titman et al., 2004), return on assets (Fama and French, 2006), net operating assets (Hirshleifer et al., 2004), accruals (Sloan, 1996), net stock issues (Loughran and Ritter, 1995), composite equity issues (Daniel and Titmans, 2006), failure probability (Campbell et al., 2008), and O-score (Ohlson, 1980 and Dichev, 1998).

[18]Chu et al. (2020) show that the relation between short selling costs and the predictability of these anomalies is causal, providing further support for the argument that this predictability is driven by mispricing.

[19]https://finance.wharton.upenn.edu/∼stambaug/

[20]This is 64% of the period covered by the SentBW dataset. Of the 441 months in our final sample, 42 months (9.5%) are strongly negative-sentiment months (i.e., fall in SentBW quartile 1). This is about 25% of months in SentBW Q1 from 1965 to 2022. This is the least populated quartile of SentBW in our final sample, which works against us finding any significant results in those months.

median group (see, e.g., Stambaugh et al., 2012, and Chu et al., 2020). The reason to use only terciles 1 and 3 of IOR is to compare and contrast the High IOR group with the Low IOR group, which are our main groups of interest.

For each of these 10 portfolios, we compute next month's daily returns by weighting the stock-level daily returns with their gross returns in the formation month. Equal-weighted portfolio returns can lead to various statistical and microstructure biases, as discussed in Asparouhova et al. (2013). Still, for testing purposes, it is useful to make use of the information in small firms, because small firms are especially informative in understanding the effects of volatility and mispricing. Our main tests, therefore, use gross-return-weighted portfolio returns.[21]

After calculating the gross-return-weighted daily returns the month after portfolio formation for each of our 10 portfolios, we compute the standard deviation of those daily portfolio returns and multiply them by the square root of 21 to scale them to a monthly measure. This gives us monthly volatilities for each portfolio in the month after portfolio formation, and these portfolios are rebalanced monthly. We use these volatilities as our dependent variable in the regression results reported in Table 1 below.

We control for industrial production and an indicator for NBER recessions, which we extract from Baker and Wurgler (2006). These variables aim to account for countercyclical variation in volatility.[22]

We incrementally add dummy variables for 3 out of the 4 quartiles of SentBW and one dummy for the High IOR (IOR tercile 3) portfolios. In the full model (column (7)), we only leave out quartile 1 of SentBW and tercile 1 of IOR, which means that our baseline group for comparison in that regression specification is the group of portfolios with low IOR in strongly negative-sentiment months (SentBW Q1=1).

Overall, this table shows that the coefficient on SentBW Q3 is consistently negative and statistically significant across the different specifications. The baseline group in column (2) includes only the negative-sentiment months (SentBW Q1=1 and SentBW Q2=1). Compared to these months, the volatility in moderately positive-sentiment months (SentBW Q3=1) is, on average, 0.74% lower

[21] As discussed in Asparouhova et al. (2013), gross-return-weighting places similar weight in drawing inferences on the information provided by each stock in the sample while mitigating the statistical and microstructure biases associated with equal-weighted portfolio returns.

[22] In unreported analysis we have found that without such controls our sentiment dummies for strongly negative- and strongly positive-sentiment months are consistently significant. The business cycle variable and the recession dummy capture most of the volatility changes in the extreme sentiment months. The results without business cycle controls are available from the authors upon request.

Table 1: **Effects of Sentiment and Institutions on Volatility**

This table reports the results of regressing next month's volatility of gross-return-weighted portfolio returns on sentiment quartile (SentBW Q) and institutional ownership (IOR) tercile dummies and their interactions. This analysis includes only the 10 portfolios with MISP deciles between 6 and 10 and IOR terciles 1 and 3. We include controls for industrial production and NBER recessions. Newey and West (1987) standard errors with 12 lags are reported in parentheses, and ∗, ∗∗, ∗ ∗ ∗ represent significance at the 10%, 5%, and 1% levels, respectively.

Dependent Var.: Next Month Volatility of Gross-Return-Weighted Portfolio Returns							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
SentBW Q4	-0.1161	-0.4953	-1.1356**		0.0411	-0.2703	-0.6795
	(-0.47)	(-1.62)	(-2.43)		(0.16)	(-0.92)	(-1.63)
SentBW Q3		-0.7431***	-1.4017***			-0.5979***	-1.0254***
		(-3.16)	(-3.25)			(-2.88)	(-2.77)
SentBW Q2			-0.8774*				-0.5697
			(-1.88)				(-1.43)
High IOR				1.4268***	1.5152***	1.6507***	2.1131***
				(14.05)	(12.87)	(9.09)	(6.02)
High IOR × SentBW Q4					-0.3144	-0.4499*	-0.9123**
					(-1.35)	(-1.67)	(-2.26)
High IOR × SentBW Q3						-0.2903	-0.7527**
						(-1.26)	(-1.99)
High IOR × SentBW Q2							-0.6153
							(-1.52)
Indpro	0.0520***	0.0499***	0.0506***	0.0533***	0.0520***	0.0499***	0.0506***
	(8.57)	(8.30)	(8.41)	(8.39)	(8.60)	(8.34)	(8.45)
Recess	3.0184***	2.9619***	2.9030***	3.0056***	3.0184***	2.9619***	2.9030***
	(4.85)	(4.92)	(4.93)	(4.91)	(4.86)	(4.92)	(4.93)
Intercept	-0.1221	0.4067	1.0115*	-0.9623*	-0.8797*	-0.4187	-0.045
	(-0.25)	(0.82)	(1.65)	(-1.85)	(-1.77)	(-0.83)	(-0.08)

and statistically significant at 1% level. However, for strongly positive-sentiment months (SentBW Q4=1) the portfolio volatility is indistinguishable from that in negative-sentiment months. This is consistent with the theoretical U-shape pattern in excess volatility that we document in Fig. 4.

In column (3), we only leave out the SentBW Q1 dummy, which means that the baseline group is now the set of months with strongly negative sentiment. The coefficients on dummy variables SentBW Q2 to Q4 are all negative and significant, implying that portfolio volatilities decrease as we move away from strongly negative sentiment months. This is evidence consistent with part (i) of implication TI1 described above. In terms of the magnitude of the coefficients, the least negative is that of SentBW Q2 and the most negative is that of SentBW Q3. These empirical estimates are consistent with the left-skewed U-shaped curve of Fig. 4, a pattern that is independent of the intensity of benchmarking concerns. It shows that the level of volatility is highest on the left side of the curve compared to the right side, and dips in the moderately positive-sentiment section of the curve.

In column (4), we leave out the sentiment dummies and introduce the high-ownership dummy (High IOR). The coefficient on High IOR is positive and significant, implying that portfolios with high institutional ownership exhibit higher volatility compared to portfolios with low institutional ownership. This is consistent with the theoretical amplifying effect of institutions on stock volatility in Basak and Pavlova (2013), also present in our model (see, e.g., the rightmost panel of Fig. 2).

In columns (5)-(7), we study the effect of institutional ownership on the sentiment-volatility relationship. The full model in column (7) shows that the interactions of High IOR with SentBW Q3 and Q4 are negative and significant at the 5% level, but the interaction with SentBW Q2 is not significant. This implies that volatility is lower in positive-sentiment months for portfolios with high IOR compared to portfolios with low IOR. The coefficient on High IOR captures the average volatility in the portfolios with high IOR in months with strongly negative sentiment. If we set High IOR equal to zero, the volatility-sentiment relationship for portfolios with low institutional ownership is captured by the coefficients on the sentiment dummies. The results show that volatility also dips in the moderately positive-sentiment months for low IOR portfolios. This is consistent with the pattern exhibited by all the curves in Fig. 4, but in particular the dash-dotted line, which corresponds to the case with low benchmarking concerns.

When we set High IOR equal to one, the interactions of High IOR with the sentiment dummies capture the differences in volatilities between high and low ownership portfolios for different sentiment months. This would be similar to comparing the dash-dotted line with the solid line in Fig. 4. In the positive sentiment section of the graph, the solid line lies below the dash-dotted line, which means that for the same level of positive sentiment, higher institutional presence is associated with lower volatility. Consistent with these theoretical results, column (7) of Table 1 reports negative and significant coefficients for the interactions between High IOR and SentBW Q3 and Q4. This is evidence consistent with part (ii) of implication TI1 described above.

Cross-Sectional Analysis Using Stock-Level Measures of Sentiment and Benchmarking Intensity Proxy We next extend our analysis of the volatility-sentiment relationship using a stock-level measure of sentiment instead of the market-level proxy proposed by Baker and Wurgler (2006). The measure that we use follows Boehmer et al. (2021), which proposes a method to identify retail trades in the TAQ database. We follow the same methodology to identify stock purchases and sales most likely submitted by retail traders. We then use order imbalances (OIB) in retail trades as a proxy for sentiment about a given stock, where positive (respectively, negative) imbalances represent positive (negative) retail sentiment. We average daily OIB to create monthly values. In addition to using this continuous variable, we also create dummy variables to proxy for different levels of sentiment. Specifically, we split positive and negative retail OIB by their respective median values and create four dummy variables for high and low values of positive and negative sentiment, respectively.

One goal of our cross-sectional analysis is to independently examine the impact of changes in the benchmarking intensity of institutions, following part (b) of Proposition 3 and part (ii) of TI1, on stock volatility for different levels of sentiment. To this aim, we introduce the benchmarking intensity measure proposed by Pavlova and Sikorskaya (2023), BMI, in our analysis. These authors compute benchmark intensity (BMI) as a stock's cumulative weight in all benchmarks, weighted by mutual funds' and ETFs' assets following each benchmark. Their database covers the period from 1998 to 2018, over which they provide the values of BMI for May and June each year. We identify BMI with the intensity of benchmark concerns as captured by the parameter q_t in our theoretical model.

Following this reasoning, we proxy for change in benchmarking concerns using the change in BMI from May to June of each year. We then interact these changes with the stock-level sentiment proxy and examine their joint impact on stock volatility in June. Table 2 reports the results of Fama-MacBeth cross-sectional regressions of stock volatility in June on sentiment and changes in BMI and their interactions.

In column (1), we include three dummies for sentiment, leaving out the one that identifies the highly negative sentiment stocks to serve as baseline group. The main coefficient of interest in column (1) is on the interaction term $\Delta BMI \times HighPosSent$. Following part (ii) of our testable implication TI1, we expect the coefficient on this interaction to be negative, consistent with an intensification of benchmarking intensity reducing volatility in strongly positive-sentiment markets. In line with this prediction, the coefficient is indeed negative and significant at the 1% level. This result is also broadly consistent with the portfolio-level results described in the previous section.

The specification in column (2) uses sentiment as a continuous variable but discretizes ΔBMI. Again, we split negative and positive changes in BMI in two groups using their respective median values, resulting in four dummies. We leave out the dummy for the most negative changes in BMI. The interactions of these dummies with the continuous sentiment proxy are however not significant. This suggests that we find no significant slope differences in the sentiment-volatility curve for different groups of changes in BMI.

In column (3), we use both variables of interest in their continuous form, without discretization. The main coefficient of interest is the one for the interaction term $\Delta \text{BMI} \times Sent(Continuous)$, which we expect to be negative from either part of TI1. Consistent with this prediction of our model, the estimated coefficient is negative and significant at 5% level, implying that the positive effect of

Table 2: **Stock-Level Effects of Sentiment and Benchmarking Intensity on Volatility**

This table reports the results of Fama-MacBeth cross-sectional regressions of stock volatility in the month of June each year on changes in benchmarking intensity (ΔBMI) from May to June, following Pavlova and Sikorskaya (2023), and stock-level sentiment in May, as proxied by order imbalances in retail trading, following Boehmer et al. (2021). These regressions control for the log of market capitalization and the number of shares on the float as a percent of shares outstanding. Newey and West (1987) standard errors are reported in parentheses, and $*$, $**$, $***$ represent significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)
ΔBMI	0.033		-0.002
	(0.038)		(0.031)
High Pos Sent	0.004**		
	(0.002)		
Low Pos Sent	0.021***		
	(0.003)		
Low Neg Sent	0.016***		
	(0.003)		
ΔBMI × High Pos Sent	-0.119***		
	(0.034)		
ΔBMI × Low Pos Sent	-0.001		
	(0.034)		
ΔBMI × Low Neg Sent	-0.029		
	(0.034)		
Sent (Continuous)		0.019***	0.015**
		(0.007)	(0.006)
High Pos ΔBMI		0.005	
		(0.004)	
Low Pos ΔBMI		-0.009**	
		(0.003)	
Low Neg ΔBMI		-0.010***	
		(0.002)	
High Pos ΔBMI × Sent (Continuous)		0.000	
		(0.008)	
Low Pos ΔBMI × Sent (Continuous)		-0.005	
		(0.005)	
Low Neg ΔBMI × Sent (Continuous)		-0.009	
		(0.006)	
ΔBMI × Sent (Continuous)			-0.082**
			(0.031)
Log(Mcap) in May	-0.013***	-0.013***	-0.013***
	(0.001)	(0.001)	(0.001)
Shares on Float in May	-0.029***	-0.029***	-0.029***
	(0.007)	(0.008)	(0.008)
Intercept	0.221***	0.233***	0.228***
	(0.017)	(0.018)	(0.018)
Adj R2	0.172***	0.165***	0.152***
	(0.009)	(0.008)	(0.008)
Obs	28,500	28,500	28,500

sentiment on volatility (as captured by the coefficient on *Sent*) falls with the changes in institutions' benchmarking intensity (ΔBMI).

6.2.2 On the Relation Between Sentiment and Mispricing

Our theoretical results imply that while arbitrarily high levels of overpricing can be consistent with sufficiently high levels of irrational sentiment, they are not easily rationalized by reference solely to the benchmarking concerns of rational institutional investors.[23] To test this implication on overpricing, we examine how the predictive power of the mispricing (MISP) measure of Stambaugh et al. (2015) is affected by the presence of institutions in periods of moderately vs. strongly positive sentiment. We again form 30 portfolios by sequentially sorting on IOR terciles and MISP deciles. Portfolios with high (low) MISP are overpriced (underpriced) and should be shorted (purchased). We then create two subsamples, one that includes only months with moderately positive sentiment (SentBW Q3), and one that includes only months with strongly positive sentiment (SentBW Q4). Table 3 reports the results of this portfolio sorting exercise.

We confirm that the overpriced stocks (decile 10 of MISP, i.e., High MISP) consistently exhibit negative returns in the following month.[24] However, in periods with moderately positive sentiment, there is no difference in predictability between portfolios with low vs. high IOR (see column (2)-(1) and row (2) for average returns, and row (6) for Fama and French (2015) 5-factor (FF5) alphas). This suggests that a larger presence of institutions does not help correct overpricing for moderate levels of positive sentiment. In contrast, in the subsample of periods with strongly positive sentiment (columns (4), (5), and (5)-(4)), the predictability of the High MISP portfolio is significantly lower for the High IOR portfolios, consistent with institutions reducing overpricing when sentiment is strongly positive. Specifically, in the SentBW Q4 subsample, the FF5 alpha for the overpriced portfolio (High MISP) with a low presence of institutions (Low IOR) is -1.9% (t-stat=-8.07), while for the High IOR counterpart, it is only -57 bps (t-stat=-1.94). The difference between the two portfolios is 1.33% and is statistically significant at the 1% level (t-stat=3.96).

Table 3 separately examines moderate and strongly positive-sentiment subsamples. This does not allow us to compare the predictability of overpricing scores across the two subsamples. To

[23] Sotes-Paladino and Zapatero (2019) show that option-like, benchmark-linked incentives can induce rational institutional investors to overinvest in overpriced, "bubble" securities, but mostly when these securities do not belong to the benchmark.

[24] For brevity, we only report gross-return-weighted portfolio results, but the results are qualitatively similar using value-weighted returns, which are available from the authors upon request.

Table 3: **Sequential Portfolio Sorts on Institutional Ownership and Mispricing**

This table reports the portfolio sorting results using gross-return-weighted returns. In the first 3 columns, we report the results for the subsample of months with moderately positive sentiment (SentBW Q3=1), and in the remaining columns, we use the strongly positive-sentiment months (SentBW Q4=1). We use 30 portfolios for each subsample, but we only report the results for the extreme portfolios, i.e., terciles 1 and 3 for IOR and deciles 1 and 10 for MISP. Newey and West (1987) standard errors with 12 lags are reported in parentheses, and $*$, $**$, $***$ represent significance at the 10%, 5%, and 1% levels, respectively.

			SentBW Q3			SentBW Q4		
			Low IOR	High IOR	High-Low	Low IOR	High IOR	High-Low
			(1)	(2)	(2)-(1)	(4)	(5)	(5)-(4)
Average	Low MISP	(1)	1.5313***	1.0663***	-0.4650***	2.3177***	1.9847***	-0.333
			(3.74)	(2.80)	(-2.84)	(6.51)	(6.33)	(-1.45)
	High MISP	(2)	-0.5945	-0.1889	0.4057	-1.0458**	0.3352	1.3810***
			(-1.38)	(-0.41)	(1.50)	(-2.27)	(0.87)	(4.76)
	High-Low	(2)-(1)	-2.1259***	-1.2552***	0.8706***	-3.3635***	-1.6495***	1.7140***
			(-7.60)	(-4.05)	(3.54)	(-14.51)	(-6.71)	(6.02)
CAPM	Low MISP	(3)	0.8448***	0.3075	-0.5372***	1.3041***	0.9339***	-0.3702
			(2.93)	(1.42)	(-3.18)	(3.74)	(3.40)	(-1.58)
	High MISP	(4)	-1.3274***	-1.0809***	0.2465	-2.2924***	-0.9732***	1.3192***
			(-3.87)	(-3.40)	(0.90)	(-8.26)	(-2.81)	(3.96)
	High-Low	(4)-(3)	-2.1722***	-1.3885***	0.7837***	-3.5965***	-1.9071***	1.6894***
			(-7.49)	(-4.30)	(3.09)	(-21.89)	(-6.57)	(5.90)
FF5	Low MISP	(5)	0.7377***	0.153	-0.5847***	1.0448***	0.3819**	-0.6629***
			(3.69)	(1.03)	(-2.98)	(6.62)	(2.53)	(-3.33)
	High MISP	(6)	-1.3582***	-1.1830***	0.1752	-1.8961***	-0.5698*	1.3263***
			(-5.96)	(-5.10)	(0.69)	(-8.07)	(-1.94)	(3.96)
	High-Low	(6)-(5)	-2.0959***	-1.3361***	0.7599***	-2.9410***	-0.9518***	1.9892***
			(-7.58)	(-4.33)	(2.83)	(-16.92)	(-2.62)	(5.69)

accomplish this we use a regression setting, and we report the results of that analysis in Table 4.

Table 4: **Effect of Institutional Ownership on Mispricing in Positive-Sentiment Periods**

This table reports the regression results of next month's gross-return-weighted portfolio returns on dummies for high sentiment and high institutional ownership. We also include the market factor in columns (4) to (6) and the Fama and French (2015) five factors in columns (7) to (9). Newey and West (1987) standard errors with 12 lags are reported in parentheses, and $*$, $**$, $***$ represent significance at the 10%, 5%, and 1% levels, respectively.

	Dep Var: Next-Month Gross-Return-Weighted Portfolio Returns in Excess of Riskless Rate								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
SentBW Q4	-0.1483		-0.6271	-0.3493		-0.8281*	-0.1631		-0.6418**
	(-0.20)		(-0.81)	(-0.90)		(-1.81)	(-0.66)		(-2.03)
High IOR		0.8508***	0.4143		0.8508***	0.4143		0.8508***	0.4143
		(3.13)	(1.19)		(3.63)	(1.33)		(3.79)	(1.38)
High IOR × SentBW Q4			0.9575*			0.9575**			0.9575**
			(1.94)			(2.31)			(2.44)
MKTRF				1.2165***	1.2157***	1.2165***	1.0886***	1.0854***	1.0886***
				(22.70)	(22.57)	(22.76)	(32.20)	(34.01)	(32.51)
SMB							0.7967***	0.7948***	0.7967***
							(11.54)	(11.31)	(11.55)
HML							0.2244***	0.2220***	0.2244***
							(3.24)	(3.20)	(3.25)
RMW							-0.1395	-0.1429	-0.1395
							(-1.17)	(-1.23)	(-1.17)
CMA							-0.4379***	-0.4435***	-0.4379***
							(-3.23)	(-3.37)	(-3.24)
Intercept	-0.751	-1.2440***	-0.9582**	-1.2360***	-1.8203***	-1.4432***	-1.2152***	-1.7078***	-1.4224***
	(-1.59)	(-3.29)	(-1.97)	(-4.13)	(-7.74)	(-4.15)	(-7.43)	(-9.01)	(-6.59)

Our dependent variable is the next month's return of the short leg of the MISP strategy, i.e., MISP decile 10, in excess of the risk-free return. We only keep months with positive sentiment, i.e., SentBW Q3 and Q4, and the extreme terciles of IOR (terciles 1 and 3). The regression model includes a sentiment dummy (SentBW Q4) and an IOR dummy (High IOR). When both dummies are set to zero, the baseline case (intercept) is the group of portfolios with low institutional ownership (High IOR=0) in months with moderately positive sentiment (SentBW Q3).

We expect to find negative coefficients on the intercept because we only use the short-leg returns as the dependent variable. This is indeed what we find, implying the correct classification of these portfolios of stocks as being overpriced. In column (3), the interaction of High IOR with SentBW Q4 is positive and significant at the 10% level. The fact that a high MISP is not as predictive of negative returns in strongly positive-sentiment months suggests that when the presence of institutions is high, the average return of the overpriced portfolio is larger in strongly positive-sentiment months than in moderately positive-sentiment months.

The results remain qualitatively similar after accounting for the market excess returns in columns (4) to (6) and the five factors of Fama and French (2015) in columns (7) to (9). Together, these results provide evidence consistent with the testable implication TI2 of our theoretical model that institutions help correct overpricing to a larger extent in strongly positive periods, compared to periods of moderately positive sentiment.

7 Conclusion

Despite the significant trend toward the portfolio delegation of households to institutional managers in recent years, most studies on the effect of sentiment-driven trading on asset prices assume that the other side of the trade is taken by direct investors. Similarly, against the abundant evidence of irrational trading by individual investors, most of the literature on the role of institutions in asset pricing assumes that the trading counterparts of institutions are rational. In this paper, we account for the simultaneous presence of institutions and sentiment-driven retail trading in financial markets to find several novel equilibrium patterns.

First, the joint effect of sentiment and benchmarking concerns on stock volatility can be radically different from the addition of the effects stemming from either feature in isolation. When the trading counterparts are standard rational investors, both sentiment and benchmarking concerns introduce portfolio heterogeneity across investor types and create a relative-wealth channel that unambiguously amplifies the transmission of fundamental shocks to asset prices. The benchmarking concerns of institutions induce a positive feedback effect from prices to demand that further amplifies the impact of fundamental shocks on stock returns. When optimistic retail and institutional investors trade with each other, the latter channel can only amplify the stock return variation in response to fundamental shocks. By contrast, the relative-wealth channel can instead attenuate this volatility.

This attenuation effect has rich implications for the level of volatility in financial markets and its dynamics over the business cycle. It can push volatility levels below those prevailing under either sentiment or benchmarking concerns. This result implies that rational institutions can have a stronger depressing effect on volatility in the presence of high sentiment than similar non-institutional peers. It also implies that in markets with a high presence of institutional investors, sentiment need not create "excess volatility." Finally, it can lead to a countercyclical return volatility pattern broadly consistent with the existing empirical evidence.

Second, for low to moderate levels of optimism, institutions can exacerbate, while for greater optimism, they help correct the overpricing of the stock market that sentiment induces. The result highlights the often overlooked fact that the benchmarking-related demand for a benchmark stock, thus the pressure on its price, is positive but bounded. By contrast, the non-benchmark-related demand for the same stock is unbounded and, in the presence of sentiment-induced overpricing, can have the opposite sign, potentially leading to an overall negative—and large—price pressure.

Our results have several implications for the ongoing debate around the role of institutional investors in financial markets, as well as for the empirical inference of sentiment from market-determined variables such as prices and volatility. Importantly, it implies that neither the impact of the trend towards a greater institutionalization of markets in the correction of sentiment-driven distortions nor the degree to which sentiment distorts prices and volatility in the first place is linear but results from a complex interaction between sentiment, benchmarking, and wealth effects.

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Appendix

A Proofs

Proof of Proposition 1. Market completeness allows us to rewrite the investors' optimization problems as:

$$\begin{aligned} \max_{W_T^k} \quad & E_t[\xi_T^k U_k(W_T^k)] \\ \text{s.t.} \quad & E_t[\pi_T W_T^k] \leq W_0^k. \end{aligned}$$

The first order conditions are given by

$$\begin{aligned} \frac{\xi_T^R}{W_T^R} &= \psi_R \pi_T \\ \frac{\xi_T^I Y_T}{W_T^I} &= \psi_I \pi_T, \end{aligned}$$

where ψ_R and ψ_I are the Lagrange multipliers associated with the retail and institutional investors' budget constraints respectively. Using the fact that $W_T^R + W_T^I = S_T = D_T$, we find that

$$\begin{aligned} \pi_t &= E_t[\pi_T] \\ &= \frac{1}{\psi_R} E_t\left[\frac{\xi_T^R}{D_T}\right] + \frac{1}{\psi_I} E_t\left[\frac{\xi_T^I Y_T}{D_T}\right]. \end{aligned}$$

It follows that

$$\begin{aligned} W_t^R &= \frac{1}{\pi_t} E_t\left[\pi_T W_T^R\right] \\ &= \frac{\xi_t^R}{\psi_R \pi_t}. \end{aligned} \tag{26}$$

and similarly

$$\begin{aligned} W_t^I &= \frac{1}{\psi_I \pi_t} E_t\left[\xi_T^I Y_T\right] \\ &= \frac{\xi_t^I}{\psi_I \pi_t}\left(1 - \upsilon + \upsilon D_t e^{\mu(T-t)}\right). \end{aligned} \tag{27}$$

Finally let ϖ_t^R and ϖ_t^I denote the shares of wealth of the retail and institutional investors, respectively, so that $\varpi_t^R + \varpi_t^I = 1$. Note that

$$\frac{\xi_t^R \psi_I}{\xi_t^I \psi_R} = \frac{\varpi_t^R}{\varpi_t^I}\left(1 - \upsilon + \upsilon D_t e^{\mu(T-t)}\right).$$

The state price density is given by

$$\pi_t = \frac{\xi_t^I}{\psi_I D_t}\left[\frac{\xi_t^R \psi_I}{\xi_t^I \psi_R} e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \upsilon D_t + (1-\upsilon) e^{-(\mu-\sigma^2)(T-t)}\right].$$

Writing $d\pi_t = -\pi_t \kappa_t dB_t$ and applying Ito's lemma, we find that the market price of risk κ is given by:

$$\begin{aligned}
\kappa_t &= \sigma\left(1 - \frac{\delta^R \varpi_t^R \big(1-\upsilon+\upsilon D_t e^{\mu(T-t)}\big) e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I \upsilon D_t}{\varpi_t^R\big(1-\upsilon+\upsilon D_t e^{\mu(T-t)}\big)e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I\big(\upsilon D_t + (1-\upsilon)e^{-(\mu-\sigma^2)(T-t)}\big)}\right) \\
&= \sigma\left(1 - \frac{\delta^R \varpi_t^R e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I q_t e^{-\mu(T-t)}}{\varpi_t^R e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I \frac{\upsilon D_t + (1-\upsilon)e^{-(\mu-\sigma^2)(T-t)}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}}\right) \\
&= \sigma\left(1 - \frac{\delta^R \varpi_t^R e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I q_t e^{-\mu(T-t)}}{\varpi_t^R e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I e^{-(\mu-\sigma^2)(T-t)}\big(1 - q_t(1-e^{-\sigma^2(T-t)})\big)}\right) \\
&= \sigma\left(1 - \frac{\delta^R \varpi_t^R e^{-\sigma^2\delta^R(T-t)} + \varpi_t^I q_t e^{-\sigma^2(T-t)}}{\varpi_t^R e^{-\sigma^2\delta^R(T-t)} + \varpi_t^I\big(1 - q_t(1-e^{-\sigma^2(T-t)})\big)}\right) \\
&= \sigma\left(1 - \frac{\delta^R(1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big) + \big(1-\gamma(T-t)\big)\varpi_t^I q_t}{(1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big) + \varpi_t^I\big(1-q_t\gamma(T-t)\big)}\right),
\end{aligned}$$

where we used to fact that $q_t = \frac{\upsilon D_t e^{\mu(T-t)}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}$ and $\gamma(\tau) = 1 - e^{-\sigma^2\tau}$. Observe that in the absence of institutional investors, i.e., $\upsilon = 0$, we simply have

$$\kappa_t \triangleq \kappa_t^{SE} = \sigma(1-\delta^R).$$

The case $\delta^R = 0$ defines the BP economy of Section 4.1.1, and we have

$$\kappa_t^{BP} = \sigma\left(1 - \frac{\big(1-\gamma(T-t)\big)\varpi_t^I q_t}{1-\varpi_t^I + \varpi_t^I\big(1-q_t\gamma(T-t)\big)}\right)$$

Comparing the market price of risk given in relation (4) with its equilibrium value in an economy where there is no institutional investor, i.e., $\upsilon = 0$ given in relation (A), it is easy to verify that $\kappa_t < \kappa_t^{SE}$ whenever

$$\delta^R - q_t < q_t \frac{1-(1-q_t)\gamma(T-t)}{1-q_t\gamma(T-t)}.$$

The equilibrium stock price over dividend ratio is given by

$$\begin{aligned}
S_t/D_t &= (W_t^R + W_t^I)/D_t \\
&= \frac{1}{\varpi_t^R e^{-(\mu+\sigma^2(\delta^R-1))(T-t)} + \varpi_t^I \frac{\upsilon D_t + (1-\upsilon)e^{-(\mu-\sigma^2)(T-t)}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}} \\
&= \frac{e^{(\mu-\sigma^2)(T-t)}}{\varpi_t^R + \varpi_t^I \frac{\upsilon D_t e^{(\mu-\sigma^2)(T-t)} + (1-\upsilon)}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}} \\
&= \overline{(S/D)}_t \frac{1}{\varpi_t^R\big(1-\gamma(\delta^R(T-t))\big) + \varpi_t^I\big(1-\gamma(T-t)q_t\big)},
\end{aligned}$$

where we used the definition of q_t, the fact that $\varpi_t^R = 1 - \varpi_t^I$ as well as $1 - \upsilon = \frac{1-q_t}{q_t}\upsilon D_t e^{\mu(T-t)}$ and $\gamma(T-t) = 1 - e^{-\sigma^2(T-t)}$. □

Proof of Proposition 2. We use relations (26) and (27) to apply Ito's lemma and we identify the diffusion terms with those given by budget constraints (2). This leads to

$$\begin{aligned}\theta_t^R \sigma_{S,t} &= \kappa_t + \sigma\delta^R \\ \theta_t^I \sigma_{S,t} &= \kappa_t + \sigma q_t.\end{aligned}$$

Then, since $\varpi_t^R \theta_t^R + \varpi_t^I \theta_t^I = 1$, we observe that leverage $(\theta_t^R - 1)\varpi_t^R$ is given by:

$$(\theta_t^R - 1)\varpi_t^R = \varpi_t^R \varpi_t^I \frac{\sigma}{\sigma_{S,t}}(\delta^R - q_t).$$

From $\varpi_t^R \theta_t^R + \varpi_t^I \theta_t^I = 1$, we also obtain that

$$\begin{aligned}\sigma_{S,t} &= \kappa_t + (1-\varpi_t^I)\sigma\delta^R + \varpi_t^I \sigma q_t \\ &= \sigma\left(1 - \frac{\delta^R(1-\varpi_t^I)e^{-\sigma^2\delta^R(T-t)} + \big(1-\gamma(T-t)\big)\varpi_t^I q_t}{(1-\varpi_t^I)e^{-\sigma^2\delta^R(T-t)} + \varpi_t^I\big(1-q_t\gamma(T-t)\big)} + \varpi_t^R\delta^R + \varpi_t^I q_t\right) \\ &= \bar{\sigma}_S\left(1 + \varpi_t^I \frac{\gamma(T-t)q_t(1-q_t) + (1-\varpi_t^I)\big(1-\gamma(T-t)q_t - \big(1-\gamma(\delta^R(T-t))\big)\big)(\delta^R - q_t)}{(1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big) + \varpi_t^I\big(1-\gamma(T-t)q_t\big)}\right).\end{aligned}$$

If $\upsilon = 0$, then $q_t = 0, \delta^R - q_t = \delta^R$, so we have

$$\sigma_{S,t} = \bar{\sigma}_S\left(1 + \delta^R\varpi_t^I(1-\varpi_t^I)\frac{\gamma(\delta^R(T-t))}{(1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big) + \varpi_t^I}\right)$$

It is easy to verify that in this case the volatility is increasing in the degree of optimism δ^R.

Finally, we show that $\sigma_{S,t} \geq \overline{\sigma}_S$: Given relation (8) in the paper, it is enough to show that

$$\gamma(T-t)q_t(1-q_t) + \big(\gamma(\delta^R(T-t)) - \gamma(T-t)q_t\big)(\delta^R - q_t) \geq 0$$

The first term is always positive and the second term is negative iff $\check{\delta}^R \leq \delta^R \leq q_t$, where $\check{\delta}^R$ is defined in (22). For $0 < a \leq x \leq 1$, define auxiliary function φ_a with

$$\varphi_a(x) = \gamma(T-t)x + \gamma(a(T-t))(a-x) - \gamma(T-t)ax.$$

φ_a is linear in x with $\varphi_a(a) = \gamma(T-t)a(1-a)$ and $\varphi_a(1) = (1-a)\big(\gamma(T-t) - \gamma(a(T-t))\big) > 0$ as $a \leq 1$ and function γ is increasing. We conclude that φ_a is non-negative. □

Proof of Lemma 1. Recall that

$$S_t/D_t = \overline{(S/D)}_t \frac{1}{\varpi_t^I\big(1-\gamma(T-t)q_t\big) + (1-\varpi_t^I)\big(1-\gamma(\delta^R(T-t))\big)},$$

and $q_t = \frac{\upsilon D_t e^{\mu(T-t)}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}$. Let us formally write

$$dq_t/q_t = \mu_{q_t}dt + \sigma_{q_t}dB_t,$$

and observe that by Ito's lemma $\sigma_{q_t} = \frac{\frac{\partial q_t}{\partial D_t}}{q_t}D_t\sigma = (1-q_t)\sigma$. Then, we have the following stock volatility decomposition

$$\sigma_{S,t} = \varepsilon_{S,t}^D\sigma + \varepsilon_{S,t}^q\sigma_{q_t} + \varepsilon_{S,t}^{\varpi^I}\sigma_{\varpi_t^I}, \tag{28}$$

where $\varepsilon^x_{S,t} = \frac{\partial S_t}{\partial x_t} \times \frac{x_t}{S_t}$ denotes the elasticity of the stock price with respect to variable x at time t, and

$$\begin{aligned}
\varepsilon^D_{S,t} &= 1, \\
\varepsilon^q_{S,t} &= \frac{\varpi^I_t \gamma(T-t)q_t}{\varpi^I_t\big(1-\gamma(T-t)q_t\big) + (1-\varpi^I_t)\big(1-\gamma(\delta^R(T-t))\big)} > 0, \\
\varepsilon^{\varpi^I}_{S,t} &= \frac{\gamma(T-t)q_t - \gamma(\delta^R(T-t))}{\varpi^I_t\big(1-\gamma(T-t)q_t\big) + (1-\varpi^I_t)\big(1-\gamma(\delta^R(T-t))\big)}\varpi^I_t.
\end{aligned}$$

Given definition (12), expressions (13) and (14) then follow in a straightforward way from decomposition (9). □

Proof of Lemma 2. This is a special case of the proof of Proposition 1 when $\upsilon = 0$. □

Proof of Lemma 3. This is a special case of the proof of Proposition 2 when $\upsilon = 0$. □

Proof of Lemma 4. Replacing variable ϖ^R_t by variable ϖ^I_t in relation (18), the price-dividend ratios in the BP and in the SENT economies are equal if and only if

$$\frac{1}{1-\varpi^I_t(1-e^{-\sigma^2\delta^R(T-t)})} = \frac{1}{1-\gamma(T-t)\varpi^I_t q_t},$$

or, equivalently, $\gamma(T-t)q_t = 1 - e^{-\sigma^2\delta^R(T-t)}$, i.e., $\delta^R_t = \frac{\log(1-\gamma(T-t)q_t)}{\log(1-\gamma(T-t))} \triangleq \check{\delta}^R_t$.

To show that $\check{\delta}^R < q_t$, i.e., that $\log[1-\gamma(T-t)q_t] < q_t \log[1-\gamma(T-t)]$ define, for $(a,x) \in (0,1)^2$, the auxiliary function φ_a, with $\varphi_a(x) = \log[1-ax] - x\log[1-a]$. Observe that $\varphi''_a(x) = -a^2/(1-ax)^2 < 0$, so that φ_a is concave with $\varphi_a(0) = \varphi_a(1) = 0$, so φ_a must be positive on $[0,1]$. We conclude that $\check{\delta}^R < q_t$.

Next, we show that $\kappa^{BP}_t > \kappa^{SE}_t|_{\delta^R=\check{\delta}^R}$. When $\delta^R = \check{\delta}^R$, we have $\gamma(T-t)q_t = \gamma(\delta^R(T-t))$, and replacing ϖ^R_t by ϖ^I_t in relation (19), we find that

$$\kappa^{SE}_t = \overline{\kappa}\left(1 - \frac{\varpi^I_t(1-\gamma(T-t)q_t)\delta^R}{1-\varpi^I_t\gamma(T-t)q_t}\right).$$

Thus $\kappa^{BP}_t \geq \kappa^{SE}_t$ iff

$$(1-\gamma(T-t)q_t)\delta^R \geq (1-\gamma(T-t))q_t,$$

or equivalently

$$\frac{\log[1-\gamma(T-t)q_t]}{\log[1-\gamma(T-t)]} \geq \frac{(1-\gamma(T-t))q_t}{1-\gamma(T-t)q_t}.$$

For $a \in (0,1)$, define auxiliary function φ_a with

$$\varphi_a(x) = (1-ax)\log(1-ax) - (1-a)x\log(1-a),$$

for $x \in [0,1]$. Observe that $\varphi''_a(x) = \frac{a^2}{1-ax} > 0$. Since $\varphi_a(0) = \varphi_a(1) = 0$ and φ_a is convex, it must be the case that φ_a is negative on $[0,1]$. Since $\log[1-\gamma(T-t)] < 0$, we conclude that we always have $\kappa^{BP}_t \geq \kappa^{SE}_t|_{\delta^R=\check{\delta}^R}$.

Finally, we show that $\sigma^{BP}_{S,t} \geq \sigma^{SE}_{S,t}|_{\delta^R=\check{\delta}^R}$. When $\delta^R = \check{\delta}^R$ we have, replacing ϖ^R_t by ϖ^I_t in relation (20),

$$\sigma^{SE}_{S,t}|_{\delta^R=\check{\delta}^R} = \overline{\sigma}\left(1 + \frac{\varpi^I_t(1-\varpi^I_t)\gamma(T-t)q_t}{1-\varpi^I_t\gamma(T-t)q_t}\frac{\log[1-\gamma(T-t)q_t]}{\log[1-\gamma(T-t)]}\right)$$

Thus $\sigma^{BP}_{S,t} \geq \sigma^{SE}_{S,t}$ iff

$$(1-\varpi^I_t)\frac{\log[1-\gamma(T-t)q_t]}{\log[1-\gamma(T-t)]} < 1 - \varpi^I_t q_t.$$

It is easy to verify that auxiliary function φ, with $\varphi(x) = -1 - \varpi x + (1-\varpi)\frac{\log[1-ax]}{\log[1-a]}$ and $(a, \varpi) \in (0,1)^2$, is increasing on $[0,1]$ and that $\varphi(1) = 0$. Thus, φ is negative on $[0,1]$. We conclude that when $\delta^R = \check{\delta}^R$, we always have $\sigma^{BP}_{S,t} \geq \sigma^{SE}_{S,t}$. □

Proof of Proposition 3. We prove part (a) first. To this end, recall that

$$\begin{aligned}\sigma_{S,t} &= \sigma\left(1 - \frac{\delta^R \varpi_t^R e^{-\sigma^2\delta^R(T-t)} + \varpi_t^I q_t\big(1-\gamma(T-t)\big)}{\varpi_t^R e^{-\sigma^2\delta^R(T-t)} + \varpi_t^I\big(1-q_t\gamma(T-t)\big)} + \varpi_t^R(\delta^R - q_t) + q_t\right)\\ &= \sigma\left(1 + \varpi_t^I q_t + \varpi_t^I\Big(-x - \frac{q_t\big(1-\gamma(T-t)\big) - x\big(1-q_t\gamma(T-t)\big)}{\varpi_t^R e^{-\sigma^2\delta^R(T-t)} + \varpi_t^I\big(1-q_t\gamma(T-t)\big)}\Big)\right)\\ &= \sigma\left(1 + \varpi_t^I q_t + \varpi_t^I\Big(-x + \frac{x-a}{\varpi_t^I + b\varpi_t^R e^{-\theta x}}\Big)\right)\end{aligned}$$

with

$$\begin{aligned} x &= \delta^R\\ a &= \frac{q_t\big(1-\gamma(T-t)\big)}{1-q_t\gamma(T-t)}\\ b &= \frac{1}{1-q_t\gamma(T-t)}\\ \theta &= \sigma^2(T-t).\end{aligned}$$

Then, define

$$\varphi(x) = -x + \frac{x-a}{\varpi_t^I + b\varpi_t^R e^{-\theta x}}.$$

We have .

$$\varphi'(x) = -1 + \frac{(\varpi_t^I + b\varpi_t^R e^{-\theta x}) + \theta(x-a)b\varpi_t^R e^{-\theta x}}{(\varpi_t^I + b\varpi_t^R e^{-\theta x})^2}.$$

Set $z = \varpi_t^I + b\varpi_t^R e^{-\theta x}$ so that

$$\varphi'(x) = \frac{z-\varpi_t^I}{z^2}\Big(\frac{(1-z)z}{z-\varpi_t^I} - \log(z-\varpi_t^I) - a\theta + \log(b\varpi_t^R)\Big).$$

Finally, consider $\psi(z) = \frac{(1-z)z}{z-\varpi_t^I} - \log(z-\varpi_t^I) - a\theta + \log(b\varpi_t^R)$ with $z > \varpi_t^I$. We have

$$\psi'(z) = -1 - \frac{\varpi_t^I\varpi_t^R}{(z-\varpi_t^I)^2} - \frac{1}{z-\varpi_t^I} < 0.$$

Then, note that $\lim_{z\to(\varpi_t^I)^+}\psi(z) = \infty$ and $\lim_{z\to\infty}\psi(z) = -\infty$. It follows that there is a unique z^* such that $\psi(z^*) = 0$ and $\psi > 0$ (respectively, < 0) on (ϖ_t^I, z^*) (resp., (z^*, ∞)). Then, define

$$x^* = -\frac{1}{\theta}\log\frac{z^*-\varpi_t^I}{b\varpi_t^R}.$$

Since variable z is decreasing in variable x, we deduce that $\varphi'(x) > 0$ (resp., < 0) iff $x > x^*$ (resp., $x < x^*$).

Finally observe that when $\delta^R = \check{\delta}^R$, we have $x = \check{x}$ and $\check{x}$ is such that $b = e^{\theta\check{x}}$, which implies that the corresponding value of z denotes $\check{z}$ is such that $\check{z} = 1$. Then, $\psi(1) = -a\theta + \log b$. Then, set $s = 1 - e^{-\theta} \in (0,1)$

and observe that $a = \frac{q_t(1-s)}{1-q_t s}$ and $b = \frac{1}{1-q_t s}$, so that

$$\psi(1) = q_t(1-s)\log(1-s) - (1-q_t s)\log(1-q_t s).$$

Then recall that $q_t \in [0,1]$ and notice that $g(y) = y\log y$ is a convex function so that

$$g(q_t(1-s) + 1 - q_t) < q_t g(1-s) + (1-q_t)g(1).$$

As $g(1) = 0$, we obtain that $\psi(1) > 0$. Thus, we must have $\check{z} < z^*$, which implies that for all $x < \check{x}$, i.e., $\delta^R < \check{\delta}_t^R$, as δ^R increases, the stock volatility decreases for any wealth distribution. This concludes the proof of part (a).

To prove part (b), note that for the case of no sentiment (BP economy) we have:

$$\sigma_{S_t} = \overline{\sigma}\left[1 + \gamma(T-t)\frac{\varpi_t^I q_t(1-\varpi_t^I q_t)}{1-\gamma(T-t)\varpi_t^I q_t}\right].$$

Define the auxiliary function $\varphi(x)$ as:

$$\varphi(x) = \frac{x(1-ax)}{1-\gamma ax},$$

for $x \in [0,1]$ with $(a,\gamma) \in [0,1]^2$. The derivative of $\varphi(x)$ is:

$$\varphi'(x) = \frac{\gamma a^2 x^2 - 2ax + 1}{(1-\gamma ax)^2}.$$

Since $\gamma \in [0,1]$, the function $z \mapsto \gamma z^2 - 2z + 1$ is decreasing in z, with:

$$\varphi'(0) = 1 > 0 \quad \text{and} \quad \varphi'(1) = \gamma a^2 - 2a + 1.$$

The expression $\varphi'(1)$ is decreasing in a. Moreover, when $a = 0$, $\varphi'(1) = 1 > 0$, and when $a = 1$, $\varphi'(1) = \gamma - 1 < 0$. Let $0 < \overline{q}_{1t} < \overline{q}_{2t}$ be the roots of the quadratic equation:

$$\gamma(T-t)(\varpi_t^I)^2 x^2 - 2\varpi_t^I x + 1 = 0,$$

i.e.:

$$\overline{q}_{1t} = \frac{1-\sqrt{1-\gamma(T-t)}}{\gamma(T-t)\varpi_t^I}, \quad \overline{q}_{2t} = \frac{1+\sqrt{1-\gamma(T-t)}}{\gamma(T-t)\varpi_t^I}.$$

Note that $\frac{1-\sqrt{1-\gamma(T-t)}}{\gamma(T-t)} < 1$, and $\overline{q}_{1t} \leq 1$ (resp. $\overline{q}_{1t} > 1$) if $\varpi_t^I \geq \overline{\varpi}_t^I = \frac{1-\sqrt{1-\gamma(T-t)}}{\gamma(T-t)} \in [1/2, 1]$. It is easy to check that $\overline{q}_{2t} > 1$.

Next, for $y \in (0,1]$, define the auxiliary function $g(y)$ as:

$$g(y) = \frac{1-\sqrt{1-y}}{y} = \frac{1}{1+\sqrt{1-y}}.$$

Clearly, $g(y)$ is increasing from $\lim_{y\to 0^+} g(y) = \frac{1}{2}$ up to $g(1) = 1$. Since $\gamma(T-t)$ is increasing, we conclude that as $T-t$ decreases, $\overline{\varpi}_t^I$ decreases from 1 to $\frac{1}{2}$. Summarizing the case of no sentiment, we have that: (i) When $\varpi_t^I \leq \overline{\varpi}_t^I$, σ_{S_t} is always increasing in q_t; (ii) when $\varpi_t^I \geq \overline{\varpi}_t^I$, σ_{S_t} is increasing (resp. decreasing) in q_t on $[0, \overline{q}_{1t}]$ (resp. $[\overline{q}_{1t}, 1]$).

In the presence of sentiment (GE economy), recall that

$$\sigma_{S_t} = \overline{\sigma}\left[1 + \varpi_t^I \frac{\gamma(T-t)q_t(1-q_t) + (1-\varpi_t^I)[\gamma(\delta^R(T-t)) - \gamma(T-t)q_t](\delta^R - q_t)}{\varpi_t^I[1-\gamma(T-t)q_t] + (1-\varpi_t^I)[1-\gamma(\delta^R(T-t))]}\right],$$

which can be rewritten as:

$$\sigma_{S_t} = \overline{\sigma}\left[1 + \varpi_t^I \frac{\gamma(T-t)q_t(1-\varpi_t^I q_t) - (1-\varpi_t^I)[\gamma(\delta^R(T-t)) + \gamma(T-t)\delta^R]q_t + \gamma(\delta^R(T-t))(1-\varpi_t^I)\delta^R}{\varpi_t^I[1-\gamma(T-t)q_t] + (1-\varpi_t^I)[1-\gamma(\delta^R(T-t))]}\right]$$

Define the auxiliary function φ as:

$$\varphi(x) = \frac{-a_2x^2 + a_1x + a_0}{b_0 - a_2x},$$

for $x \in [0,1]$ where:

$$\begin{aligned}
a_2 &= a\gamma \in [0,1], \\
a_1 &= \gamma - (1-a)[\gamma^R + \gamma\delta], \\
a_0 &= (1-a)\gamma^R\delta > 0, \\
b_0 &= 1 - (1-a)\gamma^R \in [0,1],
\end{aligned}$$

where we set $a = \varpi_t^I$, $\gamma = \gamma(T-t)$, $\delta = \delta^R$, and $\gamma^R = \gamma(\delta^R(T-t)$. It is easy to verify that $b_0 > a_2$. Moreover,

$$\begin{aligned}
b_0^2 - (a_2a_0 + a_1b_0) =& b_0[1-\gamma+(1-a)\gamma\delta] - a\gamma(1-b_0)\delta \\
=& b_0(1-\gamma) + \gamma\delta(b_0 - a) \\
=& b_0(1-\gamma) + \gamma\delta(1-a)(1-\gamma^R) > 0.
\end{aligned}$$

where (a, γ, γ^R) in $[0,1]^3$ and $\delta > 0$. We have:

$$\varphi'(x) = \frac{a_2^2x^2 - 2a_2b_0x + a_2a_0 + a_1b_0}{(b_0 - a_2x)^2}.$$

The discriminant of the quadratic (numerator) is

$$\begin{aligned}
\Delta =& 4a_2^2[b_0^2 - (a_2a_0 + a_1b_0)] \\
=& 4a_2^2[b_0(1-\gamma) + \gamma\delta(1-a)(1-\gamma^R)] > 0.
\end{aligned}$$

This implies that there are two roots:

$$\begin{aligned}
\overline{q}_{1t} &= \frac{b_0 - \sqrt{b_0(1-\gamma) + \gamma\delta(1-a)(1-\gamma^R)}}{a_2}, \\
\overline{q}_{2t} &= \frac{b_0 + \sqrt{b_0(1-\gamma) + \gamma\delta(1-a)(1-\gamma^R)}}{a_2}.
\end{aligned}$$

Since $b_0 > a_2$, we have $\overline{q}_{2t} > 1$. This leaves us with two possible cases:

Case 1: If $a_2a_0 + a_1b_0 \geq 0$, both roots $\overline{q}_{1t}$ and $\overline{q}_{2t}$ are positive. This is (always) satisfied when δ^R is small enough, including the case of $\delta^R = 0$, as in this case $a_2a_0 + a_1b_0 = \gamma > 0$. If $a = 1$, we have

$$a_2a_0 + a_1b_0 = \gamma > 0,$$

so the condition is met, while when $a = 0$, we have

$$a_2a_0 + a_1b_0 = \gamma - (\gamma^R + \gamma\delta), \tag{29}$$

which is nonnegative if and only if δ is sufficiently small.

In this case, $\varphi'(x)$ is positive on $[0, \overline{q}_{1t}]$ and negative on $[\overline{q}_{1t}, \overline{q}_{2t}]$. Then, there are two cases:

- If $\overline{q}_{1t} < 1$, then $\sigma_{S,t}$ is increasing in q_t on $[0, \overline{q}_{1t}]$ and decreasing in q_t on $[\overline{q}_{1t}, 1]$.
- If $\overline{q}_{1t} \geq 1$, $\sigma_{S,t}$ is always increasing in q_t. The condition $\overline{q}_{1t} < 1$ is

$$(b_0 - a\gamma)^2 < b_0(1-\gamma) + \gamma\delta(1-a)(1-\gamma^R).$$

If $a = 1$ this last condition is always satisfied. If $a = 0$, we must have

$$0 < \gamma^R - \gamma + \gamma\delta,$$

which is incompatible with condition (29). We conclude that if ϖ_t^I is close enough to 0 we have $\overline{q}_{1t} > 1$.

Case 2: $a_2 a_0 + a_1 b_0 < 0$. We have $\overline{q}_{1t} < 0$. Since $\overline{q}_{2t} > 1$, we conclude that φ' is negative on $[0,1]$ so $\sigma_{S,t}$ is decreasing in q_t:

$$\begin{aligned}
&\varpi_t^I \gamma(T-t)\left(1-\varpi_t^I\right)\gamma\left(\delta^R(T-t)\right)\delta^R \\
&\quad + \left[\gamma(T-t) - (1-\varpi_t^I)\left[\gamma\left(\delta^R(T-t)\right) + \gamma(T-t)\delta^R\right]\right] \\
&\quad \times \left[1 - (1-\varpi_t^I)\gamma\left(\delta^R(T-t)\right)\right] \\
&< 0.
\end{aligned}$$

Note that when δ^R is large, the left-hand side of the inequality is equivalent to

$$\varpi_t^I\left[\gamma(T-t) - (1-\varpi_t^I)\right],$$

which is negative (positive) if $\varpi_t^I \leq (\geq) 1 - \gamma(T-t)$.

Summing up:

1. If δ^R is small enough, σ_S is increasing in q_t as long as ϖ_t^I is sufficiently small; otherwise, it is hump-shaped in q_t.

2. If δ^R is large enough, σ_S is decreasing in q_t as long as ϖ_t^I is sufficiently small; otherwise, it is hump-shaped in q_t. □

Proof of Proposition 4. To account for the presence of rational institutions, we let $\upsilon > 0$. Replacing ϖ_t^R by $1 - \varpi_t^I$ in Eq. (3) and taking the partial derivative of S_t/D_t with respect to ϖ_t^I, we obtain the condition

$$\frac{\partial(S_t/D_t)}{\partial \varpi_t^I} < 0 \text{ iff } e^{-\sigma^2\delta^R(T-t)} < 1 - \gamma(T-t)q_t,$$

or, equivalently, iff $\delta^R > \log\left(1-\gamma(T-t)q_t\right)/\log\left(1-\gamma(T-t)\right) = \check{\delta}_t^R$. To sum up, we have:

$$\frac{\partial(S_t/D_t)}{\partial \varpi_t^I}\begin{cases} > 0, & \delta^R < \check{\delta}_t^R \\ = 0, & \delta^R = \check{\delta}_t^R \\ < 0, & \delta^R > \check{\delta}_t^R \end{cases}.$$

□

Proof of Lemma 5. Recall that

$$\frac{\xi_t^R \psi_I}{\xi_t^I \psi_R} = \frac{\varpi_t^R}{\varpi_t^I}\left(1 - \upsilon + \upsilon D_t e^{\mu(T-t)}\right).$$

It follows that

$$\frac{\varpi_0^R}{\varpi_0^I} e^{-\frac{1}{2}\sigma^2(\delta^R)^2 t + \sigma\delta^R B_t} = \frac{\varpi_t^R}{\varpi_t^I} \frac{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}{1-\upsilon+\upsilon D_0 e^{\mu T}}.$$

Then, as $D_t = D_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma B_t}$, given $D_0 > 0$, we find that

$$\frac{\varpi_0^R}{\varpi_0^I} e^{-\frac{1}{2}\sigma^2(\delta^R)^2 t - (\mu - \frac{\sigma^2}{2})\delta^R t} \left(D_t/D_0\right)^{\delta^R} = \frac{\varpi_t^R}{\varpi_t^I} \frac{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}{1-\upsilon+\upsilon D_0 e^{\mu T}},$$

i.e.,

$$\frac{\varpi_0^R}{\varpi_0^I} e^{-\frac{1}{2}\sigma^2(\delta^R)^2 t - (\mu - \frac{\sigma^2}{2})\delta^R t} \left(D_t/D_0\right)^{\delta^R} = \frac{\varpi_t^R}{\varpi_t^I} \frac{1-\upsilon+\upsilon D_t e^{\mu(T-t)}}{1-\upsilon+\upsilon D_0 e^{\mu T}},$$

i.e.,

$$\varpi_t^I = \frac{\varpi_0^I}{(1-\varpi_0^I)\frac{1-\upsilon+\upsilon D_0 e^{\mu T}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}} \left(D_t/D_0\right)^{\delta^R} e^{-\frac{1}{2}\sigma^2(\delta^R)^2 t - (\mu - \frac{\sigma^2}{2})\delta^R t} + \varpi_0^I}.$$

Then, from the definition of q_t, one can check that

$$\begin{aligned} D_t/D_0 &= \frac{q_t}{1-q_t}\frac{1-q_0}{q_0} e^{\mu t}, \\ \frac{1-\upsilon+\upsilon D_0 e^{\mu T}}{1-\upsilon+\upsilon D_t e^{\mu(T-t)}} &= \frac{q_t}{q_0}\frac{D_0}{D_t} e^{\mu t} = \frac{1-q_t}{1-q_0}. \end{aligned}$$

It follows that

$$\begin{aligned} \varpi_t^I &= \frac{\varpi_0^I}{\varpi_0^I + (1-\varpi_0^I)\frac{1-q_t}{1-q_0}\left(\frac{q_t}{1-q_t}\frac{1-q_0}{q_0} e^{\mu t}\right)^{\delta^R} e^{-\frac{1}{2}\sigma^2(\delta^R)^2 t - (\mu - \frac{\sigma^2}{2})\delta^R t}} \\ &= \frac{\varpi_0^I}{\varpi_0^I + (1-\varpi_0^I)\left(\frac{q_t}{q_0}\right)^{\delta^R}\left(\frac{1-q_t}{1-q_0}\right)^{1-\delta^R} e^{-\frac{1}{2}\sigma^2\delta^R(\delta^R-1)t}}. \end{aligned}$$

Note that when $\delta^R = 0$ (BP), the expression is much simpler as we get

$$\varpi_t^I = \frac{\varpi_0^I}{\varpi_0^I + (1-\varpi_0^I)\left(\frac{1-q_t}{1-q_0}\right)}.$$

Finally, since $\frac{\partial q_t}{\partial D_t} > 0$, we find that ϖ_t^I is increasing (decreasing) in cash flows D iff auxiliary function φ is decreasing (increasing) where $\varphi(q) = q^{\delta^R}(1-q)^{1-\delta^R}$. φ is a smooth function and

$$\begin{aligned} \varphi'(q) &= q^{\delta^R - 1}(1-q)^{-\delta^R}\left(\delta^R(1-q) - (1-\delta^R)q\right) \\ &= q^{\delta^R - 1}(1-q)^{-\delta^R}(\delta^R - q). \end{aligned}$$

To sum up, we have:

$$\frac{\partial(\varpi_t^I)}{\partial D_t} \begin{cases} > 0, & \delta^R < q_t \\ = 0, & \delta^R = q_t \\ < 0, & \delta^R > q_t \end{cases}.$$

□