

In the Money? Low-Leverage Option Betting

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ABSTRACT

I examine the role of In-The-Money (ITM) options, a relatively underexplored yet economically significant segment of the options market, characterized by higher dollar investment. Using a unique options database, I find that ITM trading is concentrated in large-cap stocks, short-maturity options, and is strongly correlated with retail investor activity on social media. Despite their low leverage, ITM options attract unsophisticated investors who view them as bets with a higher probability of exercise and consistent, albeit smaller, profits compared to lottery-like Out-of-the-Money (OTM) options. However, these investors often underperform, as they tend to trade ITM options during periods of increase attention and high stock volatility. I propose a model suggesting an optimal short-term strategy for such investors investing in ITM options, providing new insights into their trading behavior and performance.

Keywords: options, leverage, retail investors, gambling, social networks.

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1. Introduction

What are the motives of investors to trade in equity options? Options appeal to investors primarily for their leverage benefits, offering the potential for higher returns compared to stocks. According to [Black \(1975\)](#), leverage is the key variable considered by informed investors when choosing to trade in the options market. High-leverage options not only offer higher expected returns [Coval and Shumway \(2001\)](#), but also provide opportunities for hedging [Goldstein, Li, and Yang \(2014\)](#) and exhibit lottery-like payoffs, which attract investors with gambling preferences [Boyer and Vorkink \(2014\)](#). Out-of-The-Money (OTM) options, which offer the highest leverage, dominate trading activity and thus have attracted significant attention in the literature. This paper, however, focuses on In-The-Money (ITM) options, which offer the lowest leverage. Despite being relatively underexplored, these low-leverage derivatives represent a larger share of investment in dollar terms in the overall options market than OTM options.

Studying ITM options is important not only because it addresses a significant gap in the literature, but also because it offers insights into the behavior of unsophisticated investors, especially retail investors, whose participation in the options market has increased significantly in recent years. While previous literature like [Han and Kumar \(2013\)](#) and [Filippou, Garcia-Ares, and Zapatero \(2018\)](#) have shown that retail traders with strong gambling propensity are attracted to OTM options because of their lottery-like payoffs, I present several stylized facts suggesting that ITM options are also appealing to retail traders and play a crucial role in their trading strategies. Despite their lower leverage, retail investors seem to favor ITM options due to their perceived higher probability of exercise and the potential for consistent, though smaller, profits.

While previous studies have explored retail trading in the options market and examined the reasons for choosing options over stocks, this research focuses on the behavior and motivations of individual investors trading different types of options based on their leverage.

By analyzing the economic motives behind trading ITM versus OTM options, this study enhances the understanding of retail investor performance in the options market. It contributes to recent studies of [Bryzgalova, Pavlova, and Sikorskaya \(2022\)](#), [Bogousslavsky and Muravyev \(2024\)](#), and [de Silva, Smith, and So \(2023\)](#), by highlighting how retail investor performance varies with leverage. My findings reveal that retail investors tend to underperform when trading ITM options during periods of increased stock volatility and heightened retail trading attention. Conversely, they perform better with these low-leverage options during periods of low stock volatility. To explain this, I introduce a simple model for investors with gambling preferences, which outlines an optimal strategy for trading ITM options. Despite retail investors trade ITM options of low-volatility stocks, as suggested by the model, they often overlook volatility dynamics, leading to underperformance in high volatile conditions driven by increases in retail option trading.

For this analysis, I constructed one of the most comprehensive open-close option databases. It covers approximately 70% of the entire equity options market, and it allows for precise identification of the direction of trading volumes and the type of investor involved in each option contract. To the best of my knowledge, this dataset stands as one of the most comprehensive and exhaustive options Open-Close datasets utilized in academic research focused on options markets. While previous studies typically relied on open-close option data from only one or two exchanges, my analysis integrates data from eight exchanges —CBOE, CBOE-C2, CBOE-BZX, CBOE-EDGX, ISE, PHLX, NOM, and GEMX. Specifically, [de Silva, Smith, and So \(2023\)](#) used for their analysis data from only two Nasdaq exchanges: PHOTO (PHLX) and NOTO (NOM) focusing on the “non-professional customers” category to examine retail investor trading. Similarly, I use the “non-professional customers” classification, but I further refine the analysis by conditioning on trade size, particularly focusing on smaller trades involving fewer than 100 contracts. To further identify the participation of retail investors in the options market, I exploit the data from one of the most popular social media plat-

forms among retail investors: Stocktwits. Indeed, social investment networks have become the primary source of investment advice for many retail investors ([Cookson, Mullins, and Niessner \(2024\)](#)) influencing their trading in both equity and options markets.

Among the stylized facts that suggest that retail investors are attracted to ITM options, first I find that ITM options generally exhibit a higher average dollar volume compared to other option types, particularly among small customers (small-size trades). On average, ITM options account for about 40% of the total dollar volume traded by small customers in equity options, compared to 35% for OTM options and 25% for ATM options. For professionals and firms, ITM options make up only 33% of the dollar volume. Second, the higher dollar volume in ITM options among small customers is predominantly driven by call options and is concentrated in those with short maturities of less than one week. This aligns with [Bryzgalova, Pavlova, and Sikorskaya \(2022\)](#), which found that retail investors favor call options over puts and that 50% of retail trades involve short-term options with less than a week until expiration. Similarly, [Bogousslavsky and Muravyev \(2024\)](#) observed a decrease in the median maturity of retail options from four days in 2020 to just one day in 2022.

As a third fact I find that investment in ITM options is largely concentrated in large stocks, especially technology companies. In contrast, the investment in OTM options is primarily in small-cap, high-risk stocks, including Gamestop (GME) and AMC meme stocks. This trend is consistent with [Bogousslavsky and Muravyev \(2024\)](#) and [Bryzgalova, Pavlova, and Sikorskaya \(2022\)](#), which also noted that retail trading is heavily focused on large-cap technology stocks.

The fourth fact is that there is a significant and robust relationship between the dollar trading volume of options by small customers and retail investor activity on Stocktwits. This correlation is similar in magnitude for both ITM and OTM options, particularly for call options, and intensifies when social media content specifically relates to options trading. In contrast, the connection between options trading by professionals and firms and social

media activity is substantially lower.

I then analyze the performance of ITM and OTM options traded by small customers using the aggregate net open interest across the eight exchanges. The results reveal a notable negative correlation between the performance of ITM call options and abnormal retail activity on StockTwits (both in dollar terms and percentage returns). This negative relationship is especially pronounced for ITM call options, indicating that these low-leverage options underperform more significantly when retail investor attention increases.

This raises a key question: why do unsophisticated investors trade ITM options? To answer this, I propose a model based on the Kelly Criterion, which suggests that for investors with gambling preferences, the optimal strategy involves investing in ITM options with short maturities and those of low-volatility underlying stocks. While existing literature often links gambling motives to trading options with positive skewness, such as deep OTM options with lottery-like payoffs, this paper examines the use of low-leverage ITM options as a form of betting. Evidence from StockTwits suggests that retail investors refer to the Kelly Criterion in their posts to guide their gambling strategies, viewing ITM options as offering consistent profits due to their higher probability of exercise compared to OTM options.

While these retail investors may follow the Kelly Criterion's guidance of trading ITM short maturity options of large-cap, low-volatility stocks, they fail to account for the dynamic of the stock's volatility, which can rise sharply with increased retail participation in options trading. Consequently, ITM options, which may seem like sound investments under stable market conditions, are prone to underperformance in volatile environments driven by surges in retail trading. Overall, this underscores the significant role ITM options play within the broader landscape of the options market.

My research contributes to several areas within the existing literature. First, it adds to the understanding of why investors are drawn to trading options and the distinctive features that options offer. [Sanghvi, Sharma, and Chandani \(2024\)](#) provide a comprehensive review

of literature elucidating the motives of individual investors to engage in equity derivatives trading, categorizing these motives as “hedging and speculation”, “returns versus risk”, and “gambling”. My paper aligns with the “gambling” motives. Specifically, it extends the literature that has shifted focus toward the asset pricing implications of models that depart from the conventional representative agent/expected utility framework to explain individual trading behavior in the options market. For instance, [Boyer and Vorkink \(2014\)](#) argue that the lottery-like features of options, implicit in their leverage and nonlinear payoff structures, appeal to investors with a preference for skewness. Additionally, recent research by [Filippou, Garcia-Ares, and Zapatero \(2018\)](#) suggests that OTM options serve as the primary securities with lottery characteristics for skewness-seeking investors, particularly among retail investors. However, my paper introduces another dimension to the motives driving derivative trading within the gambling framework. I argue that there are investors who are concerned about betting just the right amount to grow their money steadily without the risk of losing it all in one bet.

Second, it contributes to the literature that examines the various determinants of options based on their moneyness or leverage. [Pan and Poteshman \(2006\)](#) and [Ge, Lin, and Pearson \(2016\)](#) show that informed investors trade in option contracts with greater leverage. They found that signals derived from the trading volume of ITM options do not predict stock returns. In line with their findings, my paper shows that investors who are attracted to low leverage options are unsophisticated retail investors that have been attracted an unprecedented inflows to the options market using investing apps like Robinhood and other commission-free brokerages.

Third, it contributes to the recent literature demonstrating that retail investors generally lose money by trading options, as shown by [Bryzgalova, Pavlova, and Sikorskaya \(2022\)](#). Additionally, it provides an explanation for why these investors incur losses when trading these derivatives. It also aligns with literature that finds retail investors are generally unin-

formed and make systematic mistakes when selecting stocks, like [Barber and Odean \(2000\)](#), [Barber and Odean \(2000\)](#), and [Barber, Lin, and Odean \(2023\)](#).

Lastly, my paper also extends the literature on money management by exploring the Kelly Criterion ([Kelly, 1956](#)) as an alternative approach to Markowitz’s framework. Initially utilized by Edward Thorp for blackjack betting in Las Vegas casinos ([Thorp, 1966](#)), the Kelly Criterion was later adapted as a portfolio optimization method ([Thorp, 1975](#)). Subsequently, numerous researchers have scrutinized the Kelly Criterion, highlighting its benefits and drawbacks, and it has been adopted by hedge fund managers in their asset allocation strategies. Specifically, my research aligns with recent papers that have studied the application of the Kelly Criterion in option portfolios, like [Carta and Conversano \(2020\)](#), [Wu and Hung \(2018\)](#) and [Wu, Cao, Berger, Foote, Mahoney, Simonson, Anderson, Yab, Taylor, Boardman, et al. \(2017\)](#).

2. Data and Main Variables

2.1. Option data and variables

To construct my primary dataset, I aggregated daily record of option trading volume of all stocks spanning from January 2012 to December 2022. The dataset covers all stock’s option activity across the following exchanges:

1. CBOE: Open-Close Chicago Board Options Exchange C1 and C2 exchanges: CBOE, CBOE-C2, CBOE-BZX, CBOE-EDGX.
2. NOTO: Nasdaq Options Trade Outline.
3. PHOTO: PHLX Options Trade Outline.
4. ISE: International Securities Exchange Open/Close Trade Profile.
5. GEMX: GEMX Open/Close Trade Profile.

To the best of my knowledge, this dataset stands as one of the most comprehensive and exhaustive options Open-Close datasets utilized in academic research focused on options markets. It encapsulates approximately 70% of the total options trading volume reported by Optionmetrics. The details of every exchange and its coverage during the period of my analysis are depicted in Figure 1. It is important to mention that this analysis focuses on all the option contracts of stocks with share code 10 or 11 from the Center for Research in Security Prices (CRSP) from 8 different exchanges at the contract-day level. As a result, this is a big database, veraging 3,000 unique stocks, up to 4 million unique option contracts, and 200 million observations per year, as detailed in 1.

Each option contract is identified as a put or a call, by its strike price, by time of execution, and by time of expiration. Furthermore, each option is accompanied by its directional trading data, encompassing both its trading volume and the number of trades recorded at the close of each trading day, divided into four specific categories: opening buys, opening sells, closing buys, and closing sells. Opening buys refer to new trades that initiate a long position on the underlying, and closing buys to trades that close an existing short position. Conversely, opening sells refer to new trades that initiate a short position on the underlying, and closing sells to trades that close an existing long position.

The option volume is also categorized according to which investor classes initiate the trades: customers, professional customers, market makers, proprietary trading firms, and broker-dealers. These four types of investors collectively constitute the trading data for all non-market makers. Precisely, a “Professional Customer” is defined as an individual or entity that (i) is not a broker or dealer in securities, and (ii) places more than 390 orders in listed options per day on average during a calendar month for its own beneficial accounts. On the other hand, “Customers” also engage in trading on their own accounts, but their trading activity does not reach the threshold required to qualify them as “Professional Customers”. Furthermore, the trading activity of “Customers” is broken down into trade size buckets:

less than 100 contracts, 100-199 contracts, and greater than 199 contracts. This granular breakdown of trade size is an important feature for my analysis, as my primary variable of interest will be “Customers” trades with the smallest size, i.e., less than 100 contracts.

In my analysis, I merge each option contract recorded on OptionMetrics of all stocks with its corresponding open-close volume data from each exchange. The variables of Optionmetrics include the daily option price, forward price, implied volatility, and delta. This linkage is established by matching key parameters, including the ticker symbol, root, trade date, expiration date, option type (put or call), strike price, and settlement time (AM or PM). This matching process relies on the SecId-PERMNO crosswalk provided by WRDS. Additionally, I obtain the daily historical volatility of stocks from OptionMetrics for the preceding 10, 14, 30, and 60 days. Market volatility is represented using the daily CBOE Volatility Index (VIX).

I calculate the Trade Volume and Dollar Volume for every option contract by aggregating all opening buys, opening sells, closing buys, and closing sells. Unlike Trade Volume, which measures the number of contracts traded, Dollar Volume reflects the value of investor capital committed to the options market, denominated in US dollars. While Trade Volume is the simplest and most commonly used metric in the literature, Dollar Volume, which indirectly accounts for leverage using the price of the option contract, provides a more comprehensive representation of the wealth invested in the options market. Trade Volume $Volume(j, t)$ and Dollar Volume $DollarVolume(j, t)$ are calculated as follows:

$$\begin{aligned}
 Volume(j, t) &= OpenBuy_{j,t} + CloseBuy_{j,t} + OpenSell_{j,t} + CloseSell_{j,t} \\
 DollarVolume(j, t) &= OptionPrice_{j,t} \cdot Volume(j, t)
 \end{aligned}
 \tag{1}$$

Where $OpenBuy$, $CloseBuy$, $OpenSell$, $CloseSell$ represents the trading volume in number of contracts.

To account for the direction of each option trade, it is important to note that $OpenBuy$ and $CloseBuy$ account for buy volume, while $OpenSell$ and $CloseSell$ account for sell vol-

ume. Therefore to compute the buy-minus-sell volume, I calculate the Order Imbalance (OIB) in as follows:

$$OIB(j, t) = OpenBuy_{j,t} + CloseBuy_{j,t} - OpenSell_{j,t} - CloseSell_{j,t}$$

In dollar terms the Dollar Order Imbalance is calculated:

$$DollarOIB(j, t) = OptionPrice_{j,t} \cdot OIB(j, t) \quad (2)$$

Net Open Interest only measures the directional volume of options contracts that are traded, it doesn't measure the option positions of all contracts that have been traded but haven't been liquidated or exercised. Therefore, I calculate the Net Open Interest (NOI), which measures the current outstanding exposure for each contract, as follows:

$$NOI(j, t) = \sum_{s=0} OIB(j, t - s) \quad (3)$$

The calculation of NOI is complex as it requires accumulating the daily order imbalance since the inception of the option contract, considering all trades across all exchanges where the option contract is traded, and using balanced panel data. Since my database covers the option acti approximately 70% of all exchanges, it provides a reliable proxy for the net open interest of each option contract.

Using $NOI(j, t)$, I calculate the performance of every option contract. For my analysis I consider the performance in dollar termn and in percentage return. Specifically, the dollar performance of each option contract is calculated as follows:

$$\$PerfNOI_{j,t:t+1} = NOI_{j,t} \times 100 \times (Price_{j,t+1} - Price_{j,t})$$

While the performace in percentage of every option contract is computed:

$$\%PerfNOI_{j,t:t+1} = Direction_{NOI_{j,t}} \times \frac{Price_{j,t+1} - Price_{j,t}}{Price_{j,t}}$$

Where $Price_{j,t}$ and $Price_{j,t+1}$ are the prices of stock j on t and $t + 1$, respectively, and $Direction_{NOI_{j,t}}$ is the sign of the net open interest of stock j on day t . Using the net open interest to calculate the performance of the option contract, allows me to consider all contracts that are open at every time t and not only the contracts that are traded that day. As a result, both $PerfNOI_{j,t:t+1}$ and $\%PerfNOI_{j,t:t+1}$ are robust measures of performance.

To construct a database of option at a daily level, I aggregate the previously described option variables on a daily basis. To do this, I categorize the option contracts into different buckets based on their underlying stock, payoff type (Call or Put), time to maturity (τ), moneyness (F/K), and type of investor (Customers, Professionals and Firms). Regarding the maturity of the options, I consider four different buckets: 1 to 7 days, 8 to 30 days, 30 to 90 days, and over 91 days. To determine the level of moneyness of an option, I calculate the ratio (F/K) between the Forward Price of the Stock (F) and the Strike Price of the Option Contract (K). For call options, if $F/K < 0.975$, the contract is considered to be OTM, while if $F/K > 1.025$, it is ITM. Conversely, for put options, if $F/K < 0.975$, the contract is ITM, and if $F/K > 1.025$, it is OTM.

2.2. Stocks and Social Media data

For my analysis, I consider stocks with share codes 10 and 11 from the Center of Research in Security Prices (CRSP) from January 1, 2014 to December 31, 2022. I retrieve their corresponding ticker symbols, daily returns, and market capitalization. I also obtained data from one of the most popular social media platforms among retail investors called Stocktwits. Similar to Twitter, users can posts “tweets” or messages on the platform about stocks adding a \$ Cashtag symbol followed by the stock ticker symbol. Stocktwits data is obtained using an the api RapidAPI. I aggregate the number of posts related to each ticker in a daily basis. This data is then merged with the options data using the ticker symbol and date.

3. Stylized facts of ITM options

While OTM equity options dominate trading activity, a different picture emerges when considering dollar volume. Despite their lower leverage, ITM options represent a significant portion of the wealth invested in the options market. In dollar terms, ITM options frequently have comparable or even higher dollar volumes than OTM and ATM options. However, these low leverage derivatives remain underexplored in academic research. Motivated by this gap, I establish several stylized facts about ITM options, highlighting their crucial role for small investors.

According to [Black \(1975\)](#), informed investors are primarily attracted to equity options for their leverage potential. This idea was further supported by [Easley, O’hara, and Srinivas \(1998\)](#), who confirmed it under a natural set of assumptions. Additionally, [Pan and Poteshman \(2006\)](#) found that signals from OTM options exhibit the highest levels of predictability, while signals from lower-leverage contracts, such as ITM options, offer very limited, if any, predictive power. On the other hand, previous studies by [Barber and Odean \(2000\)](#), [Barber, Lee, Liu, and Odean \(2009\)](#), and [Barber and Odean \(2013\)](#) have shown that retail traders act as uninformed investors. Therefore, it is reasonable to think that investors trading low leverage options, like ITM options, are uninformed retail investors. To explore this hypothesis, in this section I document four stylized facts that connect retail investor activity to ITM options trading.

3.1. Economic significance of ITM options

Fact 1: *The average dollar volume of ITM options exceeds that of OTM options for trades made by small customers. This trend is less pronounced for options traded by professionals and firms.*

I begin by calculating the trade volume and dollar volume for all option contracts, aggregating the data by stock, date, and moneyness. Moneyness is defined as the ratio F/K

rounded to two decimals, where F is the forward price of the underlying stock, and K is the option's strike price. In Figure 2, Panel A shows the average trade volume (number of trades), while Panel B displays the average dollar volume, both by different level of moneyness for options traded by small customers.

It is evident that OTM options dominate in terms of trade volume for both call and put options. However, this trend reverses when dollar volume is considered. On an average day, for an average stock, ITM options surpass other types, particularly OTM options, in dollar volume, reflecting a greater level of investment in ITM options. A similar, though less pronounced, trend is observed for options traded by professionals and firms, as shown in Figure 3.

I further aggregate the dollar volume, this time by type of moneyness instead, and report the summary statistics on Table 2 for call (Panel A) and put (Panel B) options by investor. For call options, if $F/K < 0.975$, the contract is considered to be OTM, while if $F/K > 1.025$, it is ITM. Conversely, for put options, if $F/K < 0.975$, the contract is ITM, and if $F/K > 1.025$, it is OTM. For an average day and for the average stock, the dollar volume of ITM options traded by small customers surpasses that of OTM and ATM options for both call and put options. Specifically, in Panel A for call options, aggregating the dollar volume across the entire sample period shows that ITM options account for 42% of the total, compared to 29% for OTM options and 29% for ATM options. This trend is reversed for professionals and firms, where the average dollar volume of ITM call options is lower than that of OTM and ATM call options, representing only 21% and 23% of the total dollar volume, respectively. A similar trend is observed for put options in Panel B, though the average dollar volume of ITM call options is significantly higher than that of ITM put options.

Overall, these results highlight the strong preference of small customers for investing in ITM options, particularly for call options, though to a lesser extent for puts. ITM options

account for a significant portion of the total dollar volume traded by small customers. In contrast, professionals and firms tend to favor OTM and ATM options, revealing distinct trading patterns between different type of investors.

3.2. ITM options trading distribution

Fact 2: *The dollar volume of ITM options traded by Small Customers is higher for maturities of less than 7 days.*

Next, I examine the distribution of dollar volume in equity options across different maturities. I calculate the daily average dollar volume for options within five maturity categories: 0 to 7 days, 7 to 30 days, 30 to 90 days, and over 90 days. The results are presented in Table 3, with Panel A showing data for call options and Panel B for put options. Panels C and D illustrate the percentage distribution of the aggregate dollar volume through the entire period of analysis.

For ITM options, the dollar volume is predominantly higher in options with maturities of less than 7 days. Notably, the daily average dollar volume for short-term call options is higher than that for put options when maturities are less than 7 days. Approximately 46% of the stock-daily average dollar volume for ITM call options and 48% for ITM put options is concentrated in contracts with less than a week to maturity. This pattern contrasts with the behavior of professionals and firms, who direct most of their dollar trading toward options with maturities longer than 90 days. These findings align with those of [Bryzgalova, Pavlova, and Sikorskaya \(2022\)](#) who found that 50% of retail trades in their sample are in ultra short-term options, that is, options with less than a week to expiration. Similarly, [Bogousslavsky and Muravyev \(2024\)](#) observed that the median option maturity for retail traders decreased from four days in 2020 to just one day in 2022.

Fact 3: *The dollar volume of ITM call options traded by Small Customers is predomi-*

nantly concentrated in large-cap technology stocks.

I then analyze the distribution of dollar volume in equity options based on the size of the underlying stocks. Table 4 presents a breakdown across NYSE market capitalization quantiles. The data, shown in Panel A for call options and Panel B for put options, indicate that the average dollar volume is higher for options on stocks with larger market capitalizations, particularly those in the highest quantile. Panels C and D further demonstrate that for large-cap stocks, the average dollar volume accounts for approximately 56% of call options and 49% of put options.

To deepen the analysis, I calculate for every ticker the daily average of the difference in dollar volume of ITM minus OTM options. Table 5 displays the top 25 underlying stocks with the highest daily average difference and the 25 stocks with the lowest. It stands out that for call options, in Panel A, the top 25 stocks where ITM options are most actively traded relative to OTM are predominantly technology companies. In contrast, the bottom 25, where OTM options dominate, are mostly small-cap high-risk investments including Gamestop (GME) and AMC meme stocks. This aligns with [Bogousslavsky and Muravyev \(2024\)](#), who found that retail trading in options is highly concentrated in large technology stocks and esoteric high-risk investments like Gamestop (GME). [Bryzgalova, Pavlova, and Sikorskaya \(2022\)](#) observed that retail investors tend to realize gains on options of large-cap stocks like Apple (AAPL), Nvidia (NVDA) and Google (GOOGL), while suffering losses from trading options in stocks like AMC, Gamestop (GME) and Tesla (TSLA).

For put options, this pattern is less pronounced. Notably, meme stocks like GME and AMC do not appear in either the top or bottom 25 lists of stocks with the largest ITM-OTM dollar volume differences, as they do for call options.

4. ITM options and Retail Attention

Fact 4: *There is a significant and robust relationship between the dollar trading volume of options by small customers and retail investor activity on Stocktwits.*

So far, I have demonstrated that ITM options are heavily traded by small customers. Similar to [de Silva, Smith, and So \(2023\)](#), I use the customer activity category from my open-close options database as a proxy for retail trading in the options market. However, while I focus on customers trading fewer than 100 contracts per transaction, this group cannot be definitively classified as retail traders. As [Bogousslavsky and Muravyev \(2024\)](#) pointed out, the “customer” category in daily signed volume from open-close data may also include other participants, such as professional hedge funds. To address this limitation, I explore the relationship between StockTwits activity and option trading, providing a more precise identification of retail investor behavior. Although several studies have used StockTwits to analyze retail trading, this paper is the first, to my knowledge, to examine its role in retail option trading.

I investigate the relationship between stocks heavily discussed on StockTwits by retail investors and increased trading activity in ITM options. Specifically, I analyze the abnormal dollar volume of options with different levels of moneyness around days with abnormal StockTwits activity. Days with abnormal activity are defined as those where the number of posts exceeds the stock’s historical average number of posts by more than two standard deviations. Figure 5 plots the abnormal dollar volume of ITM, OTM, and ATM options. The abnormal dollar volume is calculated as the difference in the log dollar volume (in millions) from the average during the benchmark period $t = [-20, -10]$, where $t = 0$ represents the day of abnormal StockTwits activity. The figure shows the event average as a solid line, with shaded areas representing the 95% confidence intervals. The figure reveals a significant increase in abnormal dollar volume across all types of moneyness, for both call and put options. However, the rise is more pronounced for ITM call options compared to others,

suggesting that there exists a correlation between the attention of retail investors on social media and higher dollar volume of ITM options, particularly in call options.

To further investigate the preference for ITM call options over ITM put options, I calculate the ratio of call option dollar volume to the total option dollar volume around days of abnormal StockTwits activity. For each stock j at time t during each event, the ratio is defined as follows:

$$Call(j, t)_{Ratio} = \frac{DollarVolume(j, t)^{CALL}}{DollarVolume(j, t)^{CALL} + DollarVolume(j, t)^{PUT}}$$

Figure 6 plots the average of this ratio for ITM options, comparing it to the corresponding ratios for OTM and ATM options, along with their respective confidence intervals. For all types of options, the ratio exceeds 0.50, indicating that the dollar volume of call options consistently surpasses that of put options on any given day t during the event window. However, for ITM options, there is a notable increase in the ratio leading up to day t , the day of abnormal StockTwits activity, a trend not observed for OTM or ATM options. This finding suggests that retail investors show a clear preference for trading ITM call options over ITM put options.

To ensure the economic significance of these results, I extend the analysis over the entire time series while controlling for various factors. Specifically, I calculate the number of abnormal posts as the difference between the average number of posts in the previous five days $[t - 5, t - 1]$ and the average during a benchmark period $[t - 60, t - 6]$ for every stock. I compute the abnormal dollar volume of options as the difference between the dollar volume on day t and the average dollar volume over the previous 60 days, for every stock. I then regress the abnormal dollar volume of options, categorized by different types of moneyness, on the abnormal number of StockTwits posts for each underlying stock in my database. Specifically, the regression model is as follows:

$$\begin{aligned}
AbnVolume(j, M)_t &= AbnPost(j, \tau)_{t-1} + AbnNews(j, \tau)_{t-1} + |Ret(j)_{[t-5, t-1]}| \\
&= + |Ret(j)_{[t-60, t-5]}| + Vol(j)_{[t-60, t-1]} + \alpha_j + \alpha_t + \varepsilon_{j,t}
\end{aligned} \tag{4}$$

Where $AbnVolume(j, M)_t$ represents the abnormal change in option dollar volume for stock j at time t compared to its average dollar volume over the period $[t - 60, t - 6]$, for different types of moneyness $M = ITM, OTM, ATM$, specifically for options traded by small customers. $AbnPost(j, \tau)_{t-1}$ is the abnormal number of StockTwits posts related to stock j , calculated as the difference between the average number of posts over the period $\tau = [t - 5, t - 1]$ and the average over $[t - 60, t - 6]$. A similar calculation is applied to $AbnNews(j, \tau)_{t-1}$, which represents the abnormal number of RavenPack news mentions for stock j during the period $\tau = [t - 5, t - 1]$, relative to the average over $[t - 60, t - 6]$. $Ret(j)_{[t-5, t-1]}$ and $Ret(j)_{[t-60, t-5]}$ capture the average stock returns of j over the periods $[t - 5, t - 1]$ and $[t - 60, t - 5]$, respectively. Lastly, $Vol(j)_{[t-60, t-1]}$ is the standard deviation of stock j returns over the period $[t - 60, t - 1]$. The model also incorporates stock-specific and time-specific fixed effects, α_j and α_t , respectively. The results, presented in Table 6, show a significant positive relationship between abnormal dollar volume and the abnormal number of StockTwits posts for both call options (columns 1 to 3) and put options (columns 4 to 6), across all types of moneyness. As expected for skewness-seeking retail investors there is a strong relationship for OTM options. But notably, there is also a strong relationship for ITM options. In fact for call options, the coefficient is greater for ITM than for OTM options, highlighting a greater responsiveness of ITM options to retail investor activity in dollar terms. This correlation remains robust even after controlling for variables such as abnormal news volume, past stock returns, and stock volatility. Importantly, these findings suggest that retail investors are not exclusively drawn to options with lottery-like payoffs, such as OTM options. Instead, a segment of retail investors demonstrates a preference for ITM options, rather than solely seeking skewed returns.

In contrast, when I run the same regression model from Equation 4 to the abnormal dollar volume of options $AbnVolume(j, M)_t$ traded by professional customers, and firms, the results, presented in Table 7, show that while the coefficients are positive and statistically significant for all option types, they are much smaller and close to zero. This further highlights the strong reaction of retail investors towards StockTwits activity, distinct to the much weaker reaction of more sophisticated market participants.

The variable $AbnPost(j, \tau)_{t-1}$ is calculated taking into account all posts that are related to stock j . However, not all of these posts are necessarily related to option trading. Therefore as a robustness check, I further filter all posts that are specifically related to option trading. Using text analysis, I extract keywords from each post that are commonly associated with option trading, such as: “derivatives”, “calls”, “puts”, “call spread”, “put spread”, “itm”, “in the money”, “in-the-money”, “otm”, “out of the money”, “out-of-the-money”, “at-the-money”, “at the money”. The number of posts specifically related to option trading is significantly lower than the total number of posts on StockTwits. Figure 4 illustrates this distinction by showing the total number of posts aggregated by month. Panel A presents all posts, while Panel B shows the subset of posts containing at least one of the aforementioned option-trading-related keywords. The volume of option-related posts has surged since 2018, aligning with the introduction of commission-free options trading for retail investors by platforms like Robinhood.

I then estimate the following the regression:

$$\begin{aligned}
 AbnVolume(j, M)_t &= AbnPost(j, \tau)_{t-1} + \mathbb{1}_j^{\text{Option}} + AbnPost(j, \tau)_{t-1} \times \mathbb{1}_j^{\text{Option}} + AbnNews(j, \tau)_{t-1} \\
 &= + |Ret(j)_{[t-5, t-1]}| + |Ret(j)_{[t-60, t-5]}| + Vol(j)_{[t-60, t-1]} + \alpha_s + \alpha_t + \varepsilon_{j,t}
 \end{aligned}$$

where $AbnVolume(j, M)_t$, $AbnPost(j, \tau)_{t-1}$, $AbnNews(j, \tau)_{t-1}$, $|Ret(j)_{[t-5, t-1]}|$, $|Ret(j)_{[t-60, t-1]}|$ and $Vol(j)_{[t-60, t-1]}$ are defined in Equation 4. $\mathbb{1}_j^{\text{Option}}$ is a dummy variable set to one if a stock j has at least 60 posts related to option trading in the 60 preceding days. The results, displayed in Table 8, show that for both call and put options, the interaction term

between abnormal StockTwits posts and option-related content is positive and statistically significant. This effect is particularly pronounced for ITM and OTM options, and remains robust even after controlling for other variables. These results reveal that social media attention of retail investors, particularly posts related to option trading, has a significant impact on the options trading behavior.

Overall, all of the stylized facts shown in previous sections demonstrate that ITM options (specially call options) are of particular importance to retail investors, who are not solely attracted to options for their leverage. A segment of retail investors actively engages in ITM options trading, revealing more diverse option strategies than just seeking skewed return profiles associated with OTM options.

5. ITM options performance

Given that previous results indicate retail investors significantly trade ITM options, it is important to examine the consequences of this behavior, particularly regarding the performance of these low-leverage options. The literature on retail trading has highlighted concerns about the poor performance of unsophisticated investors in the options markets (Bryzgalova, Pavlova, and Sikorskaya, 2022, de Silva, Smith, and So (2023)). Therefore, in this section, I analyze the impact of trading ITM options on the wealth of small customers

For that, I calculate the cumulative performance, in millions of dollars and percentage return, for each option contract at different horizons h , as follows:

$$\$CumPerf(j, M)_{t,h} = \sum_{t-h}^t \$PerfNOI_j$$

$$\%CumPerf(j, M)_{t,h} = \sum_{t-h}^t \%PerfNOI_j$$

Then, I analyze the correlation of the cumulative performance aggregate by stock for options at different types of moneyness and the contemporaneous abnormal activity of that

stock in Stocktwits, by calculating the following regression:

$$\$CumPerf(j, M)_{t,h} = AbnPost(j, \tau)_{t,h} + \alpha_j + \alpha_t + \varepsilon_{j,t} \quad (5)$$

Where $\$CumPerf(j, M)_{t,h}$ is the sum of the performance of each option contract in millions of dollars from $t - h$ to t , $AbnPost(j, \tau)_{t,h}$ is the abnormal average of number of posts related to stock j over the horizon of h days $[t - h, t]$, minus the average on $[t - 60, t - h]$. α_s and α_t correspond to stock and day fixed effects, respectively. The results in Table 10 reveal a significant negative relationship between cumulative performance in dollar terms and abnormal retail investor activity on StockTwits for all option types, considering horizons of 5 days (Panel A), 10 days (Panel B), and 30 days (Panel C). Notably, this negative relationship is more pronounced for ITM call options compared to OTM call options, suggesting that ITM call options tend to underperform the most when retail investor attention increases.

The results in Table 10 consider the overall sample period of my analysis, from 2014 to 2022. To further link this poor performance in dollar terms to retail investors, I conduct a robustness check by dividing the sample period into two segments: before and after 2018. Robinhood was the first retail trading platform that introduce in January 2018 zero-comission trading for options to individual investors. Table 13 shows the results of the regression analysis based on equation 5 for these two periods. Notably, the coefficient for ITM and OTM call options more than doubles after 2018, across all horizons ($h = 5, 10, 30$ days). In contrast, the coefficient for put options remains relatively stable or decreases for both ITM and OTM options after 2018. Importantly, the relationship between cumulative performance of ITM options and abnormal StockTwits activity remains the most negative post-2018, confirming that these low leverage options underperform the most during periods of increased retail trading.

I further repeat this analysis for the cumulative performance in percentage return, by

regressing:

$$\%CumPerf(j, M)_{t,h} = AbnPost(j, \tau)_{t,h} + \alpha_j + \alpha_t + \varepsilon_{j,t}$$

Where $\%CumPerf(j, M)_{t,h}$ is the cumulative sum of percentage returns from $t - h$ to t for each option contract. The results on the entire sample, from 2014 to 2022 are depicted in Table 12. As with the dollar performance analysis, I find that the coefficient for ITM call options is the most negative for a 5-day horizon. However, as the horizon length increases, the returns for OTM call options become more negative and surpass those of ITM options for a 30-day horizon. When splitting the period into before and after 2018, as detailed in Table 13, the coefficient for both ITM and OTM call options becomes significantly more negative after 2018. This trend confirms that retail trading's impact is more pronounced post-2018, with ITM call options showing the most substantial negative performance.

Overall, these results reveal a significant negative correlation between cumulative performance (both in dollar terms and percentage returns) of ITM options and abnormal retail activity on StockTwits. This negative relationship is particularly pronounced for ITM call options, suggesting that these low-leverage options underperform more severely when retail investor attention increases. This highlights the impact of retail trading on the performance of ITM options, indicating that these options are particularly vulnerable to poor performance when retail trading volumes increase.

6. Why investors trade ITM options?

In the previous section, I demonstrated that ITM options underperform the most during periods of increased retail trading. This raises an important question: why do unsophisticated investors continue to trade ITM options? To address this, I propose a toy model based on the Kelly Criterion to calculate the optimal position size for investments in ITM options.

While much of the existing literature has linked the gambling motives of unsophisticated

investors to trading options with positive skewness, such as deep OTM options with lottery-like payoffs, this paper takes a different approach. It explores the possibility that these investors use low-leverage ITM options as a form of betting. The motivation for using the Kelly Criterion lies in its origins in gambling, particularly in sports betting and casino games like blackjack [Thorp \(1966\)](#). Over time, its application extended to financial markets, as noted by [Thorp \(1975\)](#).

Interestingly, there is evidence that investors on StockTwits mention the Kelly Criterion in their trading strategies. By scraping posts that include the terms "Kelly Criterion" or "Kelly Criteria," I identified 143 posts between 2014 and 2022, with a noticeable increase in recent years. Figure 7 highlights 10 examples of such posts. Many of these posts discuss how investors use the Kelly Criterion to determine position sizing, often viewing their trades as bets. Notably, most of the posts referencing the Kelly Criterion are connected to betting or gambling strategies. In fact, one of these posts, of User X6 in Figure 7 suggests the idea that some retail investors approach low-leverage options like ITM calls through a gambling perspective, using the Kelly Criterion.

In this section I provide the background and details of Kelly Criterion, along with its application in determining the optimal fraction of wealth to invest in both ITM and in an OTM options, and conduct comparative statistical analysis, taking into account the volatility of the stock at the option's maturity. Finally, I provide an explanation of why ITM options underperform when retail attention increases.

6.1. Kelly Criterion

The Kelly Criterion, introduced by Kelly in 1956 [Kelly \(1956\)](#), serves as a method to determine the optimal fraction or size of wealth to invest in a bet or a favorable investment opportunity, aiming to maximize the exponential growth rate.

In contrast with conventional portfolio optimization methods like mean-variance analysis,

which seek to maximize a portfolio's expected returns, the Kelly Criterion pursues the maximization of the expected value of the logarithm of wealth, essentially optimizing expected logarithmic utility. The key idea behind the Kelly Criterion is to allocate more capital to opportunities with higher expected returns and favorable odds, while also considering the possibility of losses.

The Kelly criterion corresponds to the following Bayesian decision problem under binary uncertainty that optimizes the bidding fraction of total assets. Consider a sequence of i.i.d. bets where the probabilities of events are known and independent, where p is the probability of a win, $q = 1 - p$ is the probability of a loss, and f ($0 < f < 1$) is the bidding fraction of the total assets at each turn.

6.2. Kelly Criterion with Asymmetric Payoffs

Given the initial capital X_0 and after W number of wins and L number of losses ($W + L = n$), the capital X_n at the n -th trial is:

$$X_T = X_0 ((1 + bf)^W (1 - f)^L)$$

The quantity that measures the exponential rate of increase per trial is the growth rate of wealth:

$$G_n(f) = \log \left[\frac{X_n}{X_0} \right]^{1/n} = \frac{W}{n} \log(1 + bf) + \frac{L}{n} \log(1 - f)$$

Kelly chose to maximize the expected value of growth rate coefficient as follows:

$$g_n(f) = E \left(\log \left[\frac{X_n}{X_0} \right]^{1/n} \right) = p \log(1 + bf) + (1 - p) \log(1 - f)$$

$$g'(f) = \frac{pb}{1 + bf} - \frac{1 - p}{1 - f} = 0$$

The optimal betting fraction, f^* , is:

$$f^* = \frac{p(1+b) - 1}{b}$$

Assuming Black and Scholes, we can implement these framework in the context of options. The bet size will be determined by the price paid for the option. In the case of call options, this will be $C(S_0, T)$, where S_0 is the stock price at time $t = 0$ and T is the maturity of the option. The gain per unit bet is the profit earned when the option is exercised, for Call options is $F_T - K$, where $F_T = S_0 \exp^{rT}$ is forward stock price maturing at $t = T$ assuming a risk free r , and K is the strike price of the option. Therefore for Call options b will be:

$$b = \frac{F_T - K}{C(S_0, T)}$$

Furthermore, the probability of winning the bet p , for options can be interpreted as the probability of exercising the option, which is captured by the Delta of the option Δ and it is defined as $\Delta = N(d_1)$. Thus, plugging all numbers in f^* :

$$f^* = \frac{\Delta \left(1 + \frac{F_T - K}{C(S_0, T)}\right) - 1}{\frac{F_T - K}{C(S_0, T)}} = \frac{N(d_1) \left(1 + \frac{F_T - K}{C(S_0, T)}\right) - 1}{\frac{F_T - K}{C(S_0, T)}} = N(d_1) - (1 - N(d_1)) \frac{C(S_0, T)}{F_T - K}$$

Where:

$$F_T = S_0 \exp^{rT}$$

$$d_1 = \frac{\log\left(\frac{S_0}{K}\right) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$C(S_0, T) = S_0 N(d_1) - \exp^{-rT} K N(d_2)$$

Note that since $F_T = S_0 \exp^{rT}$, then $b = \frac{S_0 \exp^{rT} - K}{S_0 N(d_1) - \exp^{-rT} K N(d_2)}$. Therefore for every level of moneyness α such that $K = \alpha S_0$, then b does not depend of the stock price S_0 at time

$t = 0$ since $b = \frac{S_0 \exp^{rT} - \alpha S_0}{S_0 N(d_1) - \exp^{-rT} \alpha S_0 N(d_2)} = \frac{\exp^{rT} - \alpha}{N(d_1) - \exp^{-rT} \alpha N(d_2)}$. In other words, changes in the stock price will not change the value of b on every period of investment.

6.3. Comparative Statics

This section offers comparative statics results on the changes in the optimal investment fraction, denoted as f^* according to the Kelly Criterion, in response to variations in the maturity (T) and volatility (σ) of options, considering both ITM and OTM options. The findings unveil two crucial observations: 1) a negative correlation between the optimal fraction invested in ITM options and the option's maturity, and 2) a negative correlation between the optimal fraction invested in ITM options and the historical volatility of the underlying stock. The following four propositions offer a more detailed exploration of these relationships.

6.3.1. Maturity

Proposition 1 *For ITM options, when $F_T - K > 0$, then $\frac{\partial f^*}{\partial T} < 0$ for all strike prices $K < S_0 \exp^{-(r+\sigma^2/2)T}$. That is, there is a negative relationship between the optimal fraction of investment and the maturity of ITM options.*

Proof: See Appendix

6.3.2. Volatility

Proposition 2 *For ITM options, when $F_T - K > 0$, then $\frac{\partial f^*}{\partial \sigma} > 0$, for all strike prices $K < S \exp^{(r-\sigma^2/2)T}$ and $\sigma^2/2 > r$. That is, there is a positive relationship between the optimal fraction of investment and the volatility of the stock for ITM options.*

Proof: See Appendix

Proposition 3 *For OTM options, when $F_T - K > 0$, then $\frac{\partial f^*}{\partial \sigma} > 0$, for all strike prices $K > C(S_0, T) + F_T$ and $\sigma^2/2 > r$. That is, there is a positive relationship between the optimal fraction of investment and the volatility of OTM options.*

Proof: See Appendix

In Figure 8, I illustrate these findings. Panel A depicts the optimal investment fraction f^* and its variations across different maturity levels. Specifically, subplots (a) and (b) respectively depict this relation for OTM and ITM options, considering different moneyness levels. Similarly, Panel B presents the optimal investment fraction f^* across varying levels of historical stock volatility. Subplots (c) and (d) represent this relationship for OTM and ITM options, accounting for different moneyness levels as well.

6.4. ITM performance and Retail Attention

Finally, in this section, I provide an explanation for why ITM options underperform when retail attention surges. According to the Kelly Criterion, investing in ITM options is optimal for options with underlying stocks with low volatility. However, increased retail attention often drives up the volatility of the underlying stocks, creating suboptimal conditions for ITM options. In contrast, during periods of low volatility, when there is no abnormal retail attention, ITM options perform better, highlighting the sensitivity of their performance to volatility driven by higher retail trading activity.

To show this, I conduct the following regression:

$$\$CumPerf(j, M)_{[t, t-h]} = \mathbb{1}_{HighVol} + \mathbb{1}_{LowVol} + \alpha_j + \alpha_t + \varepsilon_{j,t}$$

Where $\$CumPerf(j, M)_{[t, t-h]}$ corresponds to the cumulative performance in millions of dollars, on the period $[t, t-h]$ days of stock j at time t , for different horizons $h = 5, 10, 30$ days, and different type of moneyness $M = \{ITM, OTM, ATM\}$ traded by small customers. $\mathbb{1}_{HighVol}$ ($\mathbb{1}_{LowVol}$) is a dummy that takes the value of 1 if the stock's volatility during the period $[t, t-h]$ falls within the top (bottom) quintile of its time-series. Otherwise, it is set to 0. The volatility of stock j is defined as the standard deviation of its returns over the period $[t, t-h]$.

The results, presented in Table 14, show that ITM call options underperform during periods of high volatility but outperform during periods of low volatility. This pattern holds across all horizons—5 days (Panel A), 10 days (Panel B), and 30 days (Panel C). When splitting the sample by market capitalization, the effect is more pronounced for large-cap stocks. As discussed in the previous section, small customers tend to focus their ITM option trading on large-cap stocks, which typically exhibit lower volatility than small-cap stocks. This aligns with the Kelly Criterion’s guidance of investing in low-volatility stocks. However, these unsophisticated investors fail to account for the fact that volatility is dynamic and increases with heightened retail trading activity, leading to suboptimal performance during volatile periods.

Similarly I run the same regression, but this time with the performance in percentage returns as well, as follows:

$$\%CumPerf(j, M)_{[t, t-h]} = \mathbb{1}_{HighVol} + \mathbb{1}_{LowVol} + \alpha_j + \alpha_t + \varepsilon_{j,t}$$

Where $\%CumPerf(j, M)_{[t, t-h]}$ corresponds to the cumulative performance in percentage, on the period $[t - h, t]$ days of stock j at time t , for different horizons $h = 5, 10, 30$ days, and different type of moneyness $M = \{ITM, OTM, ATM\}$. The results in Table15 show a similar story, like the ones with the performance in millions of dollars. ITM call options underperform (overperform) on periods of high (low) stock volatility.

Overall, these findings explain why small customers underperform when trading ITM options during periods of heightened retail activity. While these retail investors may follow the Kelly Criterion’s guidance of focusing on low-volatility stocks, they fail to account for the dynamic of the stock’s volatility, which can rise sharply with increased retail participation in options trading. Consequently, ITM options, which may seem like sound investments under stable market conditions, are prone to underperformance in volatile environments driven by surges in retail trading. This underscores the significant role ITM options play within the broader landscape of the options market and the need for investors to consider volatility

dynamics when trading these low leverage options as a form of betting or gambling.

7. Conclusion

In conclusion, this paper highlights the growing interest in In-The-Money (ITM) options among unsophisticated investors, a segment often overlooked in the literature. It provides new insights into the trading behavior of retail investors, particularly trading ITM options, and the broader implications of retail trading activity in the options market. The analysis reveals that ITM options are particularly appealing to retail investors with gambling preferences, driven by perceived higher probabilities of exercise and consistent profits. By analyzing the performance of ITM options in relation to volatility, this research highlights that retail investors tend to trade ITM options on large-cap, low-volatility stocks, aligning with the Kelly Criterion's principles. However, these investors often overlook the dynamics of stock volatility, which increases during periods of heightened retail activity. As a result, ITM options, which typically perform well in stable, low-volatility environments, tend to underperform during more volatile periods when retail attention surges.

Overall, this paper contributes to a deeper understanding of the behavioral patterns of retail investors in the options market. While much of the existing literature has linked the gambling motives of unsophisticated investors to trading options with positive skewness, such as deep OTM options with lottery-like payoffs, this paper shows evidence that these investors use low-leverage ITM options as a form of betting. This research not only fills a critical gap in options literature but also provides valuable insights into retail trading dynamics and potential policy implications.

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Figure 1. Exchange Volume Coverage

This figure shows the monthly aggregated volume of options of stocks listed in the S&P 500 Index, as percentage of their total volume reported on Optionmetrics. The sample period is from January 1, 2012, to December 31, 2021.

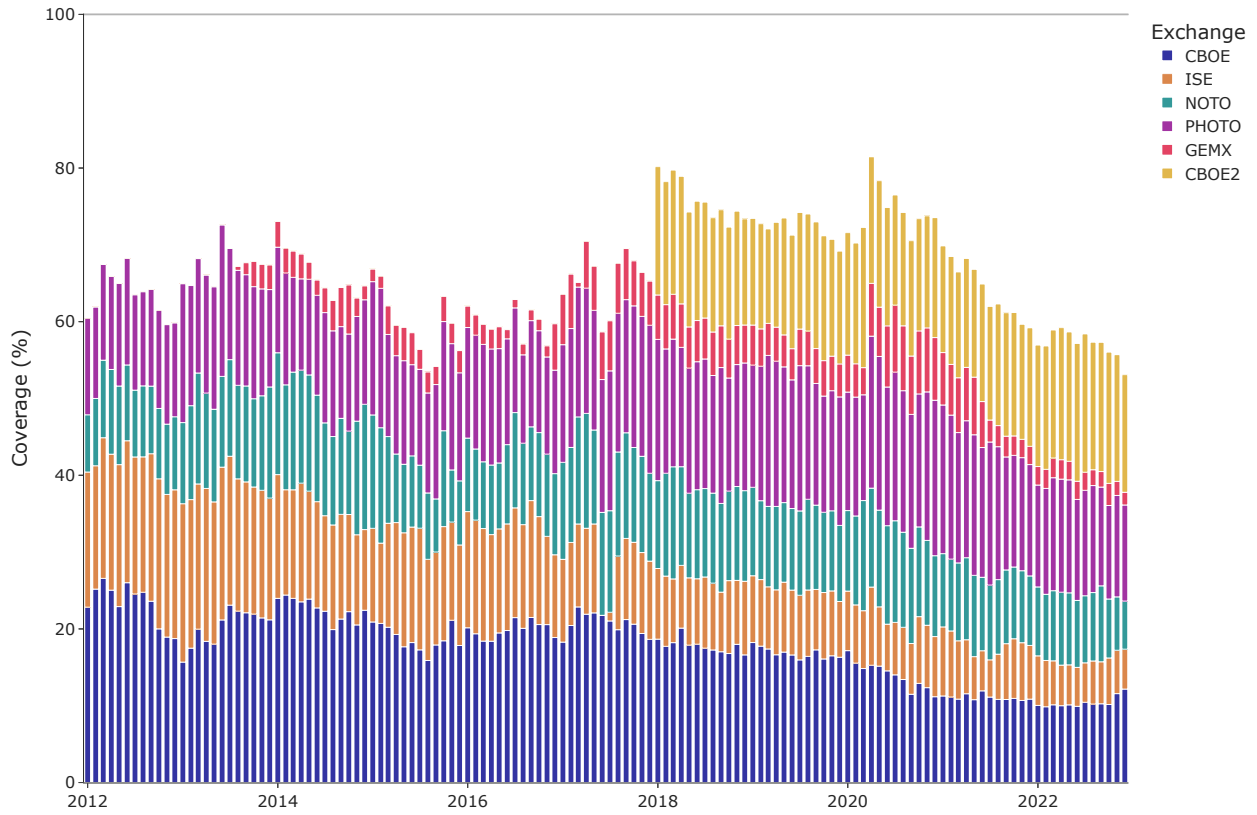


Figure 2. Average Trade and Dollar Volume of options traded by Small Customers

This figure displays the average stock-daily trade volume in Panel A and the average stock-daily dollar volume in Panel B, segmented by different levels of moneyness for call and put options traded by small customers. The level of moneyness F/K is calculated as the ratio between the Forward Price of the Stock (F) and the Strike Price of the Option Contract (K). The sample period January 2014 to December 2022 for options of all stocks considered in the analysis.

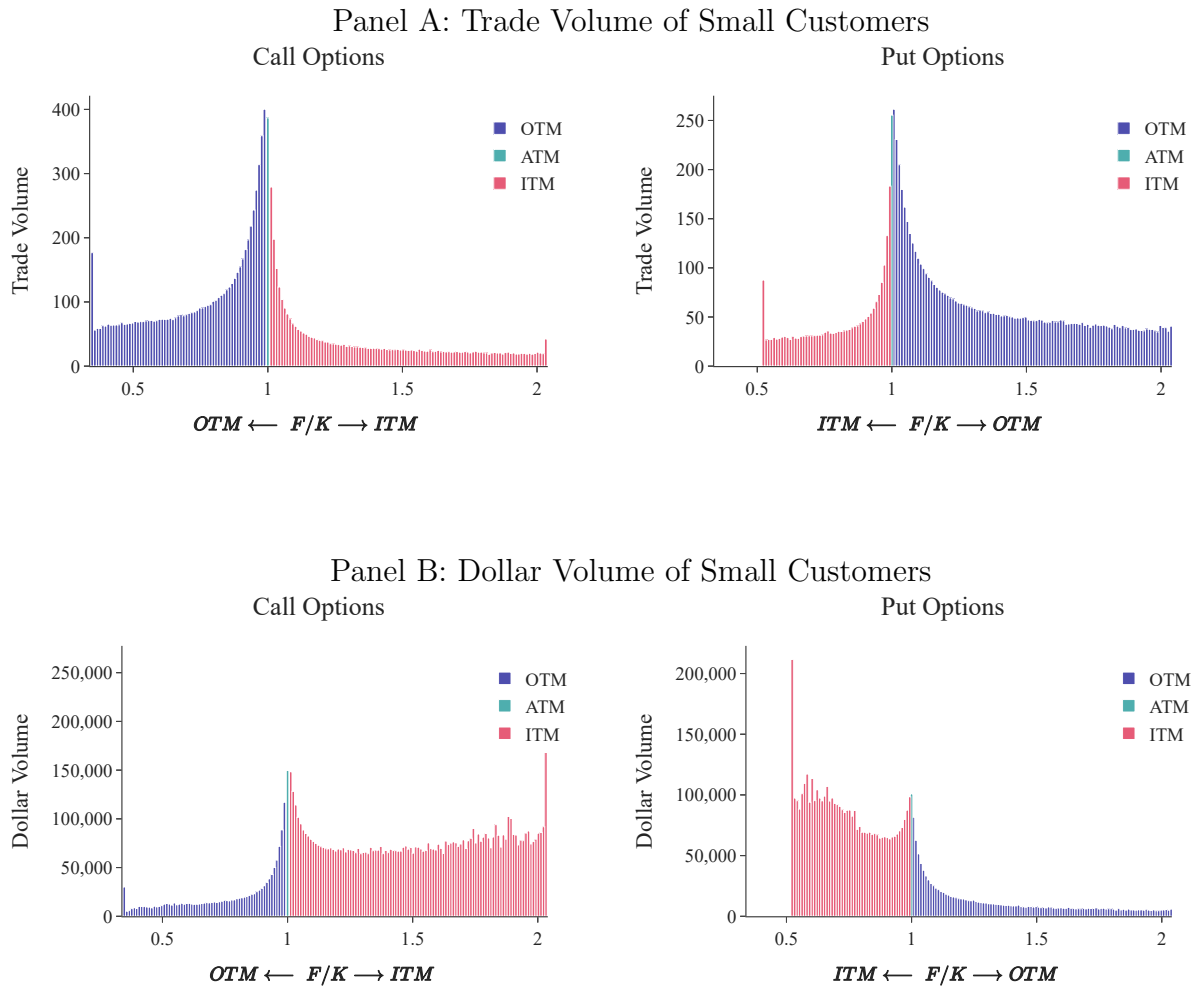


Figure 3. Average Trade and Dollar Volume of options traded by Professionals and Firms

This figure displays in Panel A the average stock-daily trade volume and the average stock-daily dollar volume for call and put options traded by professionals and segmented by different levels of moneyness. Panel B shows the average stock-daily trade volume and the average stock-daily dollar volume for call and put options traded by firms and segmented by different levels of moneyness. The level of moneyness F/K is calculated as the ratio between the Forward Price of the Stock (F) and the Strike Price of the Option Contract (K). The sample period January 2014 to December 2022 for options of all stocks considered in the analysis.

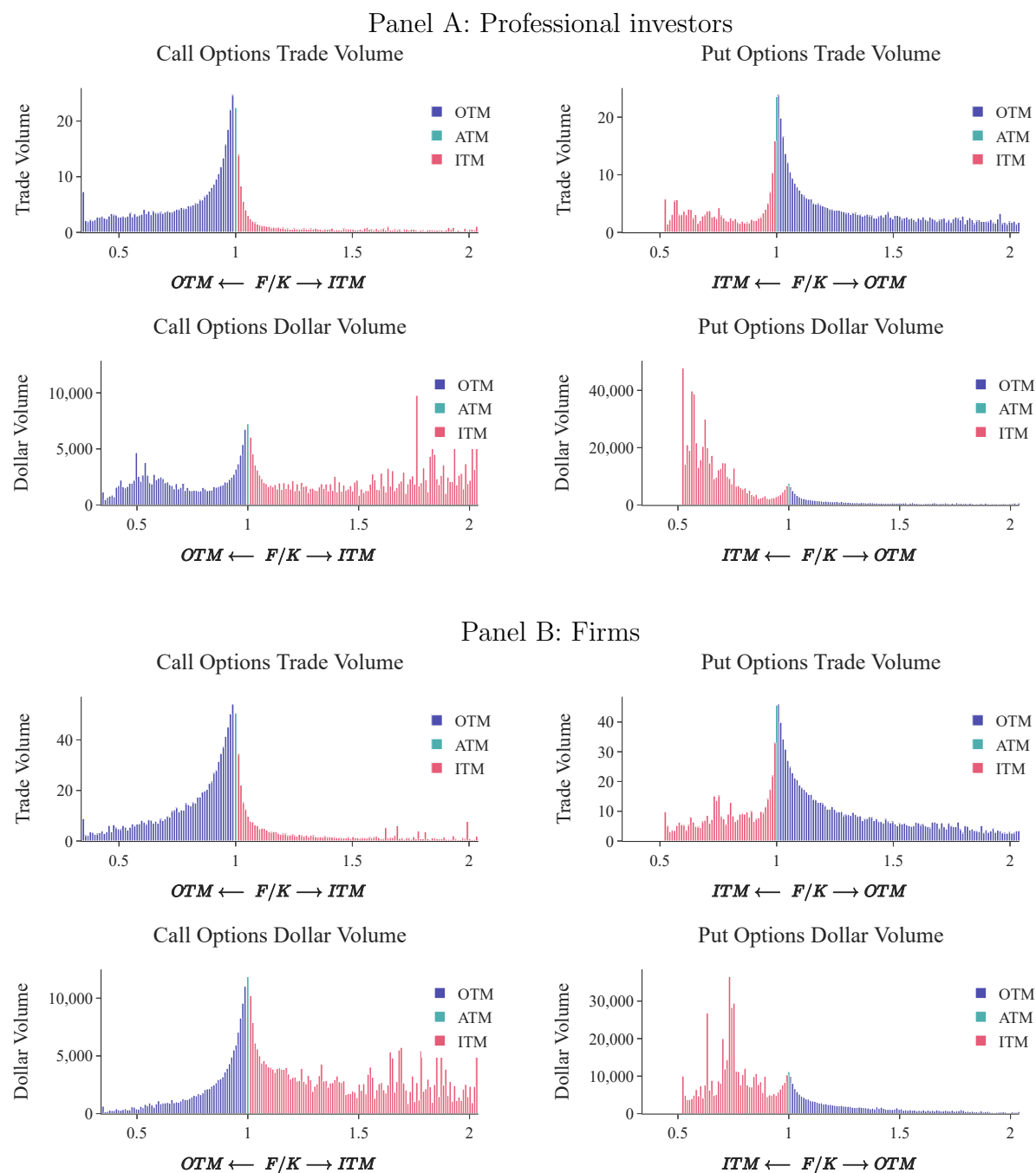
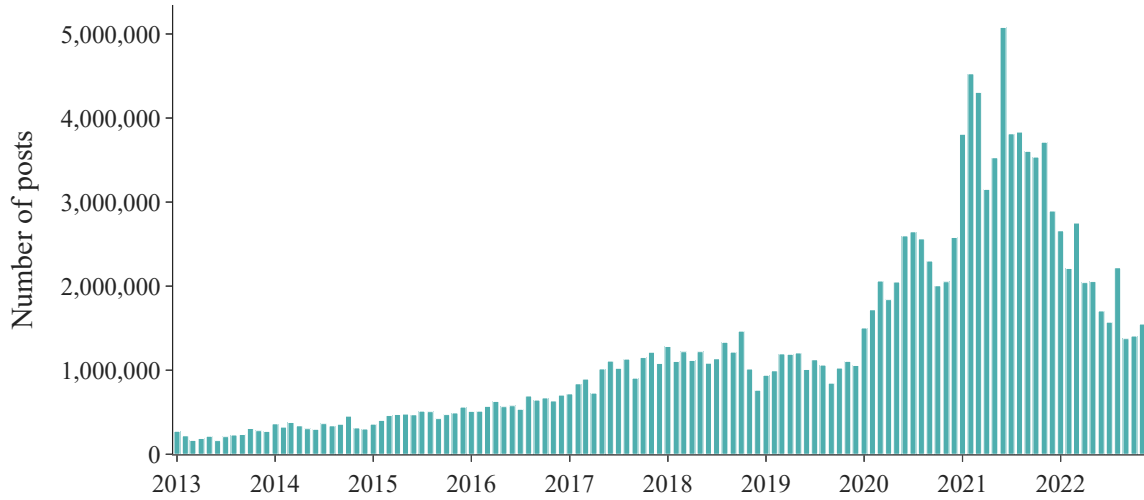


Figure 4. Information production on Stocktwits

This figure shows the monthly number of stock-specific posts on StockTwits on Panel A. The monthly number of stock-specific posts related to option trading on Stocktwits on Panel B. The sample period is from January 1, 2013, to December 31, 2022.

Panel A: Number of stock-specific posts on StockTwits



Panel B: Number of stock-specific posts on StockTwits related to option trading

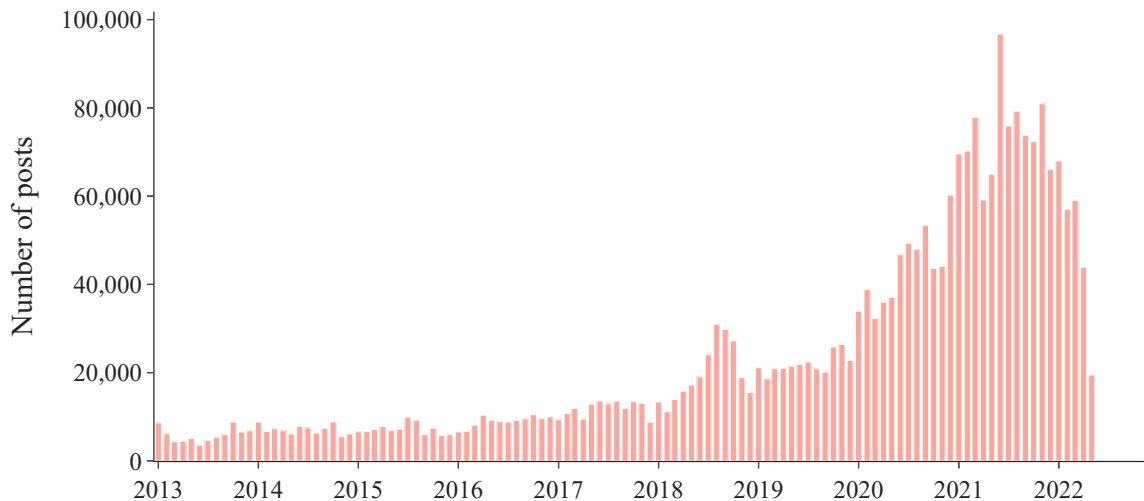


Figure 5. Dollar option volume around abnormal Stocktwits activity of Small Customers

This figure illustrates the dollar volume of options traded by small customers on days with abnormal post activity on StockTwits. Abnormal activity is defined as a daily change in the number of posts for a specific stock that exceeds three standard deviations. The solid line represents the average, while the shaded area indicates the 95% confidence intervals. Panel A and Panel B depicts the results for call options and put options, respectively. The events considered occurred from January 1, 2013, to December 31, 2022.

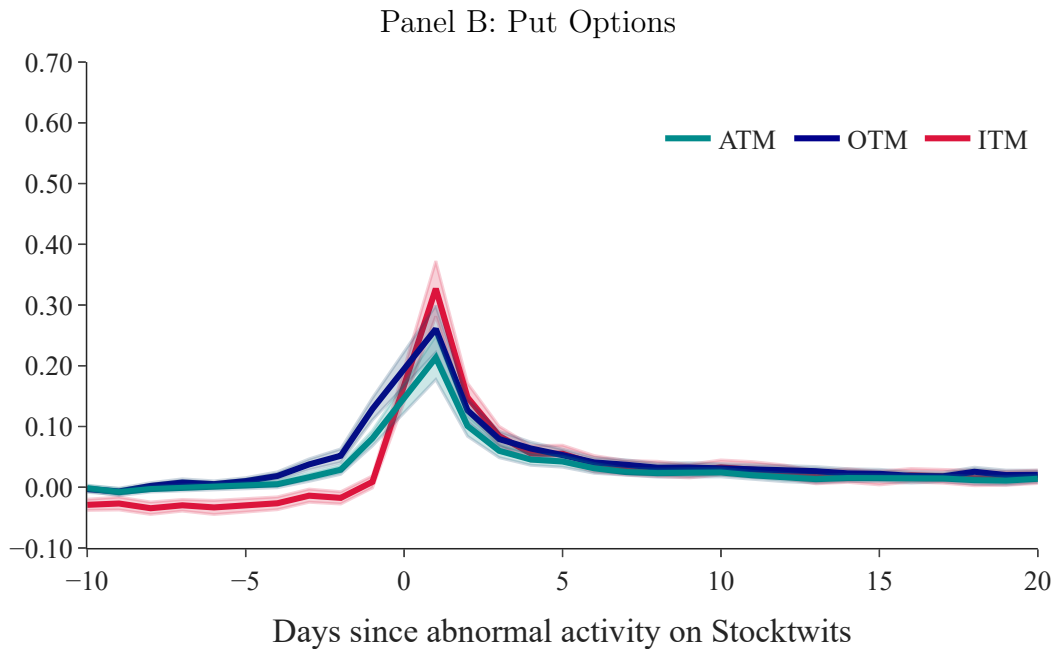
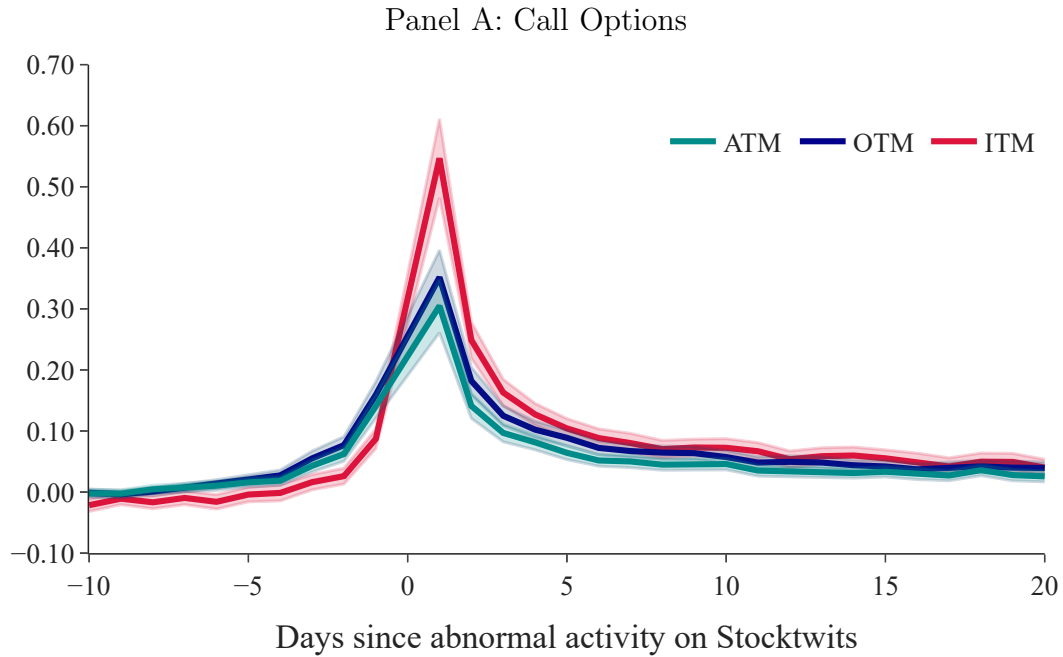


Figure 6. Call volume of options around abnormal Stocktwits activity of Small Customers

This figure illustrates the ratio of call option dollar volume to the total option dollar volume around days of abnormal StockTwits activity. For each stock j at time t during each event,. Abnormal activity is defined as a daily change in the number of posts for a specific stock that exceeds three standard deviations.

$$Call(j, t)_{Ratio} = \frac{DollarVolume(j, t)^{CALL}}{DollarVolume(j, t)^{CALL} + DollarVolume(j, t)^{PUT}}$$

The solid line represents the average, while the shaded area indicates the 95% confidence intervals. Panel A and Panel B show the results for call options and put options, respectively. The events considered occurred from January 1, 2013, to December 31, 2022.

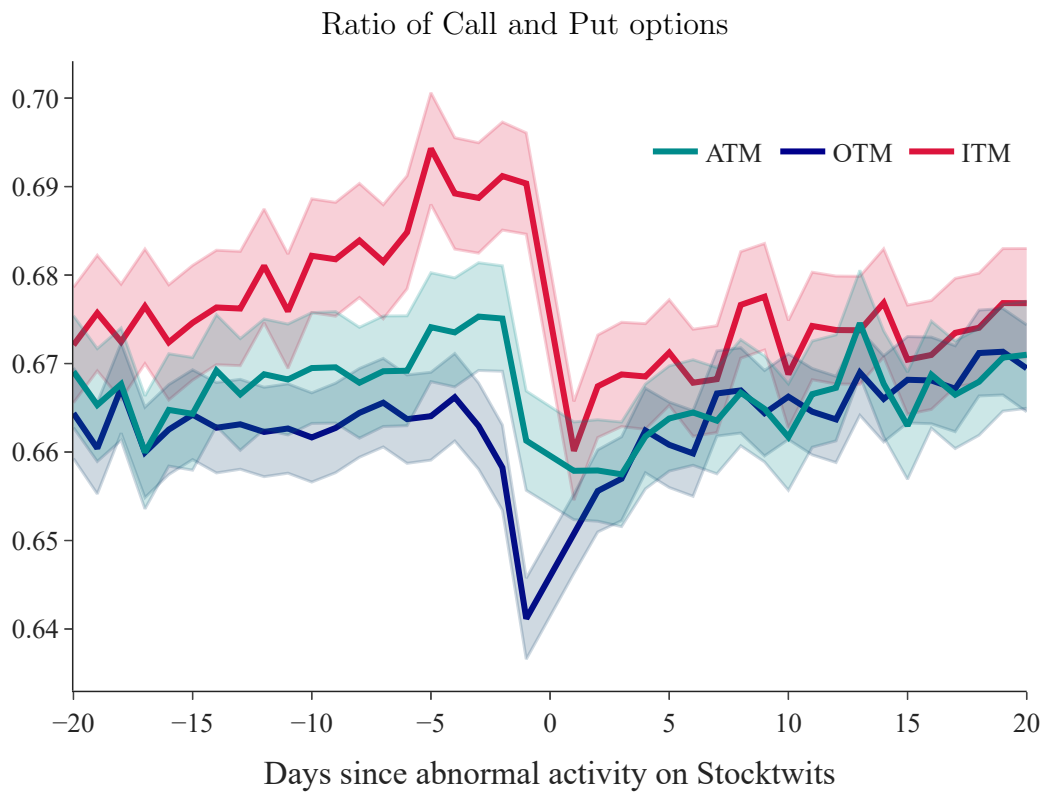


Figure 7. Stocktwits and Kelly Criterion

This figure presents examples of Stocktwits posts that reference the Kelly Criterion in their content.

User X1 2014-04-25 8:09:17 PM
@deepockettrader Sinclair talks abt using **Kelly Criterion** for trade sizing
<http://stks.co/a0TtG> bottom line expected return/volatility

User X2 2017-05-04 9:27:33 PM
@zQuant@MrE_PlusME Exactly. The **Kelly criterion** is the way.

User X3 2017-09-20 1:16:10 PM
@SD_Trader sorry million things on the go here. I don't know too much about him but a lot of these guys rely on **Kelly Criterion**

User X4 2018-01-10 9:53:39 AM
@andrewc0324 Nah, if you manage your **Kelly criterion** well in re: volatility the total loss potential is pretty low. Critical for L/S strat!

User X5 2020-07-15 11:15:05 PM
@valuationgman money management method for gamblers. good old **kelly criterion**, even buffet uses it

User X6 2020-07-17 3:42:15 PM
@RobbHaney@dixieseoul yeah dixie is right using dollar cost averaging, **itm options** and the **kelly criterion** of asset allocation it reaps pretty big returns.

User X7 2020-08-02 11:39:40 AM
@fergusonandrew631 @croftmemphis Congrats!
I myself might not be in a great position to give advice since I too am somewhat of a gambler (8-9% of my portfolio in \$MFAC warrants) but it may be a good idea to use the **Kelly criterion** when sizing your positions to at least feel somewhat sound about the rationality of your gambles

User X8 2021-04-09 1:55:50 PM
@godsposterchild My only advice is to research the **Kelly Criterion** and understand it in its entirety. Apply it to your portfolio. Its very important to understand the maximum size bet you should be making so you don't cause yourself irreparable harm.

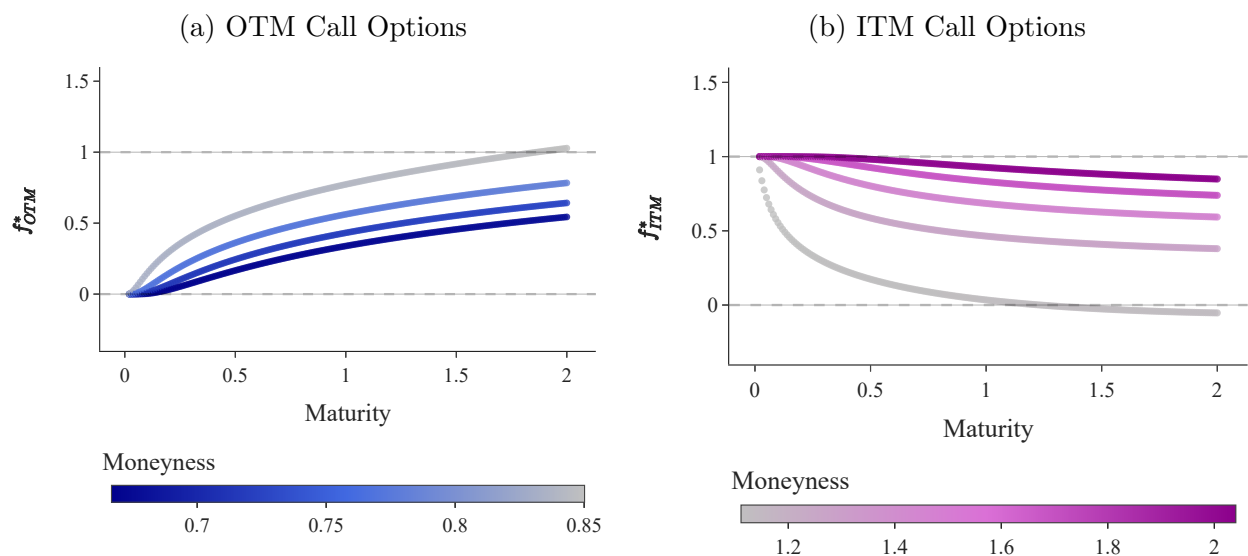
User X9 2021-10-17 4:14:12 PM
@Ironbro don't bet your whole account. Use the **Kelly criterion** to determine position sizing

User X10 2022-03-17 10:52:07 AM
@_777_ it is gambling. calculated gambling. there's some good articles out there about using **Kelly criterion** to size your bets. it's challenging because it's not human to cut your losses, but that's what you have to do sometimes - throw away a bad hand like in poker.

Figure 8. The optimal fraction investment f^* according to Kelly Criterion
 This figure shows the optimal fraction of investment f^* according to Kelly Criterion to different levels of Maturity (a and b) and Volatility (c and d) and for a Call OTM and a Call ITM option respectively.

$$f^* = N(d_1) - (1 - N(d_1)) \frac{C(S, M)}{S_M - K}$$

Panel A: Optimal investment f^* considering different levels of Maturity (M)



Panel B: Optimal investment f^* considering different levels of Volatility (σ)

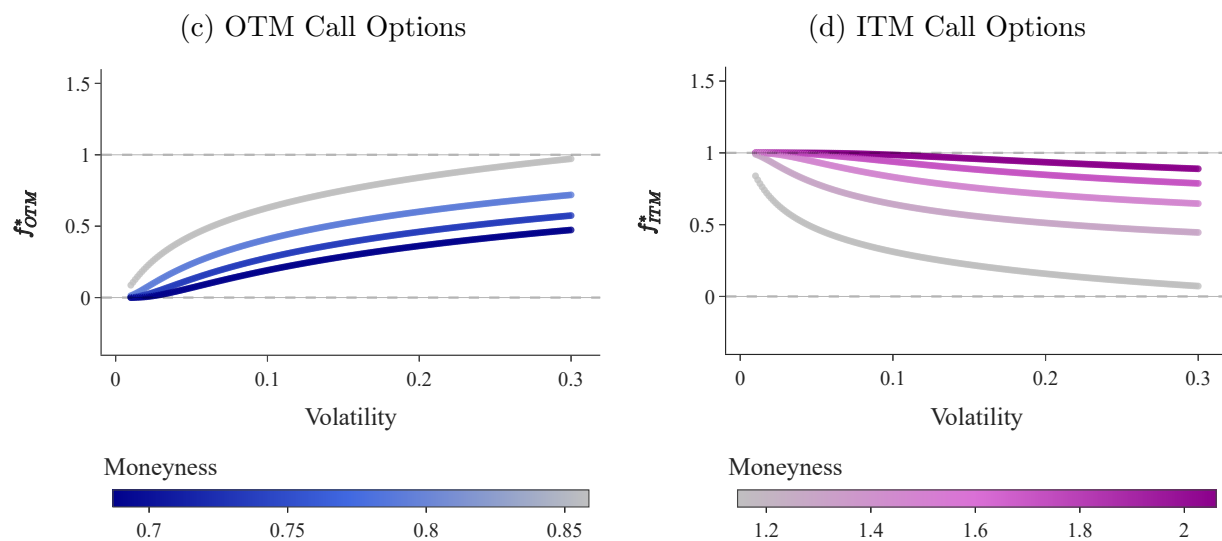


Table 1
Database

This table reports by year the number of unique option contracts, observations and unique stocks considered in the database constructed from 8 different exchanges. The sample period is from January 1, 2014, to December 31, 2022.

Year	# of unique option contracts	# of option observations	# of unique stocks
2014	2,327,362	110,472,482	2,861
2015	2,738,454	126,585,514	3,070
2016	2,740,471	126,354,540	3,003
2017	2,732,423	125,651,794	2,920
2018	3,034,317	134,938,416	2,884
2019	3,032,442	139,029,998	2,840
2020	3,660,093	169,408,389	2,931
2021	4,018,838	200,778,913	3,501
2022	3,907,593	193,689,127	3,481

Table 2
Summary Statistics of Equity Options Dollar Volume

This table reports the summary statistics the daily-stock average of the dollar volume traded in equity options traded by Small-size Customers, Professionals and Firms. The sample period is from January 1, 2014, to December 31, 2022.

<i>A. Call Options</i>									
	Small Customers			Professionals			Firms		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
Mean	201,629	102,435	172,036	28,755	23,490	26,556	51,416	42,830	47,811
5th	305	52	180	320	52	174	254	60	120
25th	1,890	550	1,335	1,438	526	1,233	1,275	560	900
Median	8,970	3,255	6,910	5,355	2,685	5,018	5,541	3,540	4,945
75th	49,285	20,590	39,912	20,000	13,660	19,825	29,242	22,250	27,318
95th	645,062	292,972	516,900	143,414	121,125	125,842	312,400	260,402	290,176
Total Percentage	(42%)	(29%)	(29%)	(21%)	(42%)	(38%)	(23%)	(44%)	(33%)

<i>B. Put Options</i>									
	Small Customers			Professionals			Firms		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
Mean	147,026	87,552	125,147	33,813	24,987	28,541	67,924	50,753	51,799
5th	242	50	145	368	73	212	277	62	127
25th	1,410	472	983	1,865	742	1,470	1,725	652	1,095
Median	6,450	2,700	4,890	7,121	3,572	5,762	9,350	4,465	6,225
75th	35,185	17,000	28,872	26,790	16,125	22,530	53,808	29,235	34,258
95th	459,870	237,745	347,150	154,402	128,696	136,442	314,512	312,400	312,400
Total Percentage	(38%)	(32%)	(29%)	(24%)	(39%)	(37%)	(24%)	(45%)	(31%)

Table 3
Dollar Volume by Maturity and type of Investor

This table reports the daily-stock average dollar volume traded in equity options traded by Small-size Customers, Professionals, and Firms, for different maturities. Panel A reports data for call options, while Panel B focuses on put options. Panels C and D display the percentage distribution of the average dollar volume reported in Panels A and B, respectively. The sample period spans from January 1, 2014, to December 31, 2022.

A. Call Options (dollars)

	Small Customers			Professionals			Firms		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
0 to 7 days	123,715	36,451	106,212	18,981	4,824	10,881	34,395	11,984	25,050
7 to 30 days	81,959	38,556	74,996	17,772	8,663	15,443	32,955	18,790	28,051
30 to 90 days	73,651	39,787	58,317	17,393	11,690	13,745	34,559	26,288	31,294
> 90 days	102,614	56,515	60,156	27,249	20,024	15,916	52,712	44,288	45,178

B. Put Options (dollars)

	Small Customers			Professionals			Firms		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
0 to 7 days	102,652	31,645	84,277	19,386	4,569	10,256	35,896	11,737	24,351
7 to 30 days	64,041	33,508	56,127	18,130	8,800	15,727	39,547	19,260	29,637
30 to 90 days	53,007	35,121	41,855	18,461	11,996	14,889	48,043	28,062	35,010
> 90 days	77,500	51,093	45,333	32,517	21,847	18,189	79,725	61,681	59,445

C. Call Options (percentage)

	Small Customers			Professionals			Firms		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
0 to 7 days	21	10	28	15	5	17	21	5	21
7 to 30 days	23	22	30	23	18	36	24	18	30
30 to 90 days	22	27	23	24	30	29	24	31	28
> 90 days	33	41	19	37	47	18	31	47	21

D. Put Options (percentage)

	Small Customers			Professionals			Firms		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
0 to 7 days	26	11	33	10	4	14	14	4	18
7 to 30 days	25	24	31	18	17	33	19	15	28
30 to 90 days	21	28	21	25	28	30	24	26	27
> 90 days	28	37	15	47	51	23	43	54	27

Table 4
Dollar Volume by Market Capitalization and type of Investor

This table reports the daily-stock average dollar volume traded in equity options traded by Small-size Customers, Professionals, and Firms, for stocks with different market capitalizations. Panel A reports data for call options, while Panel B focuses on put options. Panels C and D display the percentage distribution of the average dollar volume reported in Panels A and B, respectively. The sample period spans from January 1, 2014, to December 31, 2022.

A. Call Options (dollars)

	Small Customers			Professionals			Firms		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
1	41,879	23,812	26,259	13,186	6,993	7,788	21,042	14,227	14,101
2	57,110	35,429	28,831	16,236	9,809	8,634	26,382	18,460	15,526
3	75,224	44,877	34,757	16,096	11,567	9,095	30,591	22,756	19,513
4	104,334	57,294	52,293	17,947	14,820	11,717	36,344	28,627	24,267
5	579,608	332,962	427,803	40,852	36,698	38,009	85,345	74,455	71,928

B. Put Options (dollars)

	Small Customers			Professionals			Firms		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
1	36,687	17,177	17,513	15,363	8,054	8,108	43,099	18,630	18,770
2	55,159	21,485	18,907	17,609	8,673	8,897	50,363	19,654	18,525
3	69,298	28,426	23,944	19,216	10,443	9,928	50,840	23,632	23,149
4	90,229	38,710	34,537	24,331	13,554	12,963	52,487	30,612	26,488
5	371,996	252,278	283,723	47,864	37,066	40,233	95,653	79,540	72,528

C. Call Options (percentage)

	Small Customers			Professionals			Firms		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
1	5	5	5	13	9	10	11	9	10
2	7	7	5	16	12	11	13	12	11
3	9	9	6	15	14	12	15	14	13
4	12	12	9	17	19	16	18	18	17
5	68	67	75	39	46	51	43	47	49

D. Put Options (percentage)

	Small Customers			Professionals			Firms		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
1	6	5	5	12	10	10	15	11	12
2	9	6	5	14	11	11	17	11	12
3	11	8	6	15	13	12	17	14	15
4	14	11	9	20	17	16	18	18	17
5	60	70	75	38	48	50	33	46	45

Table 5
Dollar Volume by Market Capitalization and type of Investor

This table reports the daily-stock average of the difference between dollar volume of ITM minus OTM options ($DollarVolume^{ITM-OTM}$) traded by Small-size Customers, categorized by market capitalization quintiles. Panel A reports data for call options, while Panel B focuses on put options. The sample period spans from January 1, 2018, to December 31, 2022.

A. Call options

Top 25			Bottom 25		
Underlying ticker	Market Capitalization	$DollarVolume^{ITM-OTM}$	Underlying ticker	Market Capitalization	$DollarVolume^{ITM-OTM}$
AAPL	5	4,728,702	OCGN	1	-133,689
FB	5	3,572,510	CRSR	2	-139,494
MSFT	5	3,256,499	HOOD	4	-143,732
GOOG	5	3,189,875	GHIV	2	-175,912
AMZN	5	2,862,505	UPST	3	-191,884
GOOGL	5	2,768,412	WISH	1	-199,673
NVDA	5	2,225,773	FMCI	1	-200,800
BRK	5	2,064,271	SPCE	2	-207,165
NFLX	5	1,703,764	BFT	3	-219,696
CMG	5	1,703,036	RKT	2	-231,727
TSLA	5	1,271,174	CCIV	3	-239,534
ADBE	5	1,193,587	DPHC	3	-251,829
TTD	5	1,155,177	CLOV	1	-261,981
AMD	5	1,150,152	RBLX	5	-264,911
CRM	5	1,140,469	FUBO	2	-315,851
GS	5	1,089,719	SHLL	2	-316,542
COST	5	1,087,942	ABNB	5	-337,648
MA	5	1,081,564	RIVN	5	-489,506
QCOM	5	1,024,994	SNOW	5	-528,648
PYPL	5	1,004,975	SPAQ	1	-533,808
BAC	5	991,456	AMC	2	-643,089
HD	5	960,998	GME	3	-787,364
LRCX	5	932,338	COIN	5	-797,551
UNH	5	905,450	META	5	-831,550
BKNG	5	851,684	PLTR	5	-857,568

B. Put options

Top 25			Bottom 25		
Underlying ticker	Market Capitalization	$DollarVolume^{ITM-OTM}$	Underlying ticker	Market Capitalization	$DollarVolume^{ITM-OTM}$
META	5	1,783,892	AMD	5	-63,517
UPST	3	1,447,092	PANW	5	-64,620
BA	5	1,398,464	FSLR	5	-65,083
COIN	5	1,271,515	CAR	4	-68,477
RIVN	5	972,828	ENPH	5	-82,625
ROKU	4	956,867	LULU	5	-85,445
HOOD	4	864,491	UNH	5	-87,407
PLTR	5	852,339	XLNX	5	-88,332
PYPL	5	826,436	MDCO	4	-88,998
CCIV	3	817,786	SPRT	1	-99,045
DKNG	4	805,457	NOW	5	-99,222
QS	3	727,157	GREE	1	-103,824
SOFI	4	692,701	SEDG	3	-113,378
BYND	1	675,030	AVGO	5	-114,799
LCID	4	668,683	CRWD	5	-136,216
AFRM	4	668,510	COST	5	-143,172
RBLX	5	631,168	ZS	5	-160,843
DWAC	1	624,715	SHLL	2	-173,905
MELI	5	579,657	CELG	5	-195,438
TLRY	2	567,384	NFLX	5	-209,939
PARA	4	516,000	GOOGL	5	-495,647
SPCE	2	498,223	FB	5	-500,369
MARA	3	483,940	TSLA	5	-648,907
VIAC	4	456,016	AAPL	5	-1,493,776
SNAP	5	451,253	NVDA	5	-1,637,700

Table 10
Dollar Return of options traded by Small-size Customers

This table reports the coefficients of the following regression

$$\$CumPerf(j, M)_{t,h} = AbnPost(j, \tau)_{t,h} + \alpha_j + \alpha_t + \varepsilon_{j,t}$$

Where $\$CumPerf(j, M)_{t,h}$ corresponds to the cummulative performance in millions of dollars, over the horizon of h days $t = [t, t - h]$ of stock j , for different type of moneyeness $M = \{ITM, OTM, ATM\}$. $AbnPost(j, \tau)_{t,h}$ is the abnormal average of number of posts related to stock j over the horizon of h days $[t - h, t]$, minus the average on $[t - 60, t - h]$. α_s and α_t correspond to stock and day fixed effects, respectively. Newey-West corrected standard errors are clustered by stock and day, and are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The sample period is from January 1, 2014, to December 31, 2022.

A: Horizon $h = 5$ days						
	Call options			Put options		
	ITM	OTM	ATM	ITM	OTM	ATM
$AbnPosts(\tau)$	-0.019*** (0.001)	-0.010*** (0.001)	-0.006*** (0.000)	-0.007*** (0.001)	-0.004*** (0.000)	-0.002*** (0.000)
Intercept	-0.051*** (0.000)	-0.021*** (0.000)	-0.012*** (0.000)	-0.022*** (0.000)	-0.010*** (0.000)	-0.005*** (0.000)
N	5,794,351	5,794,351	5,794,351	5,794,179	5,794,179	5,794,179
R^2	0.04	0.05	0.08	0.01	0.01	0.03
B: Horizon $h = 10$ days						
	Call options			Put options		
	ITM	OTM	ATM	ITM	OTM	ATM
$AbnPosts(\tau)$	-0.035*** (0.002)	-0.017*** (0.001)	-0.010*** (0.000)	-0.014*** (0.002)	-0.007*** (0.001)	-0.004*** (0.000)
Intercept	-0.100*** (0.000)	-0.041*** (0.000)	-0.022*** (0.000)	-0.043*** (0.000)	-0.019*** (0.000)	-0.009*** (0.000)
N	5,794,351	5,794,351	5,794,351	5,794,179	5,794,179	5,794,179
R^2	0.03	0.05	0.09	0.02	0.01	0.03
C: Horizon $h = 30$ days						
	Call options			Put options		
	ITM	OTM	ATM	ITM	OTM	ATM
$AbnPosts(\tau)$	-0.060*** (0.006)	-0.027*** (0.002)	-0.017*** (0.001)	-0.028*** (0.004)	-0.011*** (0.002)	-0.007*** (0.001)
Intercept	-0.288*** (0.000)	-0.119*** (0.000)	-0.064*** (0.000)	-0.122*** (0.000)	-0.053*** (0.000)	-0.026*** (0.000)
N	5,796,286	5,796,286	5,796,286	5,796,114	5,796,114	5,796,114
R^2	0.01	0.02	0.04	0.01	0.01	0.02

Table 11
Dollar Return of options traded by Small-size Customers
Before and After 2018

This table reports the coefficients of the following regression

$$\$CumPerf(j, M)_{t,h} = AbnPost(j, \tau)_{t,h} + \alpha_j + \alpha_t + \varepsilon_{j,t}$$

Where $\$CumPerf(j, M)_{t,h}$ corresponds to the cumulative performance in millions of dollars, over the horizon of h days $t = [t - h, t]$ of stock j , for different type of moneyness $M = \{ITM, OTM, ATM\}$. $AbnPost(j, \tau)_{t,h}$ is the abnormal average of number of posts related to stock j over the horizon of h days $[t - h, t]$, minus the average on $[t - 60, t - h]$. α_s and α_t correspond to stock and day fixed effects, respectively. Newey-West corrected standard errors are clustered by stock and day, and are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The sample periods are from January 1, 2014, to December 31, 2017, and from January 1, 2018, to December 31, 2022.

A: Horizon $h = 5$ days

	From 2014 to 2017						From 2018 to 2022					
	Call options			Put options			Call options			Put options		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
$AbnPosts(\tau)$	-0.010*** (0.001)	-0.005*** (0.000)	-0.005*** (0.000)	-0.007*** (0.001)	-0.004*** (0.001)	-0.002*** (0.000)	-0.023*** (0.002)	-0.012*** (0.001)	-0.006*** (0.000)	-0.003* (0.001)	-0.001 (0.001)	-0.001** (0.000)
Intercept	-0.017*** (0.000)	-0.006*** (0.000)	-0.005*** (0.000)	-0.035*** (0.000)	-0.017*** (0.000)	-0.007*** (0.000)	-0.075*** (0.000)	-0.032*** (0.000)	-0.016*** (0.000)	-0.031*** (0.000)	-0.016*** (0.000)	-0.007*** (0.000)
N	2,391,060	2,391,060	2,391,060	2,757,101	2,757,101	2,757,101	3,403,291	3,403,291	3,403,291	3,404,507	3,404,507	3,404,507
R^2	0.09	0.11	0.13	0.01	0.01	0.02	0.05	0.06	0.08	0.00	0.00	0.00

B: Horizon $h = 10$ days

	From 2014 to 2017						From 2018 to 2022					
	Call options			Put options			Call options			Put options		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
$AbnPosts(\tau)$	-0.018*** (0.001)	-0.009*** (0.000)	-0.008*** (0.000)	-0.014*** (0.003)	-0.006*** (0.001)	-0.003*** (0.000)	-0.043*** (0.003)	-0.022*** (0.001)	-0.011*** (0.001)	-0.016*** (0.002)	-0.008*** (0.001)	-0.004*** (0.000)
Intercept	-0.033*** (0.000)	-0.011*** (0.000)	-0.010*** (0.000)	-0.069*** (0.000)	-0.034*** (0.000)	-0.014*** (0.000)	-0.146*** (0.000)	-0.063*** (0.000)	-0.031*** (0.000)	-0.065*** (0.000)	-0.032*** (0.000)	-0.014*** (0.000)
N	2,391,060	2,391,060	2,391,060	2,757,101	2,757,101	2,757,101	3,403,291	3,403,291	3,403,291	3,403,307	3,403,307	3,403,307
R^2	0.11	0.15	0.17	0.01	0.01	0.02	0.05	0.07	0.10	0.02	0.02	0.02

C: Horizon $h = 30$ days

	From 2014 to 2017						From 2018 to 2022					
	Call options			Put options			Call options			Put options		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
$AbnPosts(\tau)$	-0.033*** (0.002)	-0.016*** (0.001)	-0.014*** (0.001)	-0.027*** (0.007)	-0.009*** (0.003)	-0.005*** (0.001)	-0.080*** (0.008)	-0.036*** (0.003)	-0.021*** (0.001)	-0.017*** (0.006)	-0.009*** (0.003)	-0.005*** (0.001)
Intercept	-0.094*** (0.000)	-0.030*** (0.000)	-0.028*** (0.000)	-0.203*** (0.000)	-0.100*** (0.000)	-0.041*** (0.000)	-0.426*** (0.000)	-0.181*** (0.000)	-0.089*** (0.000)	-0.185*** (0.001)	-0.093*** (0.000)	-0.041*** (0.000)
N	2,391,795	2,391,795	2,391,795	2,758,116	2,758,116	2,758,116	3,404,491	3,404,491	3,404,491	3,403,307	3,403,307	3,403,307
R^2	0.07	0.12	0.11	0.01	0.00	0.01	0.02	0.03	0.07	0.00	0.00	0.01

Table 12
Percentage Return options traded by Small-size Customers

This table reports the coefficients of the following regression

$$\%CumPerf(j, M)_{[t, t-h]} = AbnPost(j, \tau)_{t,h} + \alpha_j + \alpha_t + \varepsilon_{j,t}$$

Where $\%CumPerf(j, t, M)$ is the cummulative sun of percentage returns over the horizon of h days $t = [t - h, t]$ of stock j , for different type of moneyeness $M = \{ITM, OTM, ATM\}$. $AbnPost(j, \tau)_{t,h}$ is the abnormal average of number of posts related to stock j over the horizon of h days $[t - h, t]$, minus the average on $[t - 60, t - h]$. α_s and α_t correspond to stock and day fixed effects, respectively. Newey-West corrected standard errors are clustered by stock and day, and are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The sample period is from January 1, 2014, to December 31, 2022.

A: Horizon $h = 5$ days						
	Call options			Put options		
	ITM	OTM	ATM	ITM	OTM	ATM
<i>AbnPosts</i> (τ)	-0.042*** (0.004)	-0.035*** (0.006)	0.079*** (0.008)	-0.014*** (0.002)	-0.036*** (0.005)	0.023*** (0.007)
Intercept	-0.039*** (0.001)	-0.222*** (0.001)	-0.022*** (0.001)	0.023*** (0.000)	0.017*** (0.001)	-0.065*** (0.001)
<i>N</i>	5,794,351	5,794,351	5,794,351	5,794,179	5,794,179	5,794,179
<i>R</i> ²	0.04	0.01	0.02	0.01	0.01	0.00

B: Horizon $h = 10$ days						
	Call options			Put options		
	ITM	OTM	ATM	ITM	OTM	ATM
<i>AbnPosts</i> (τ)	-0.402*** (0.032)	-0.367*** (0.053)	0.808*** (0.064)	-0.124*** (0.017)	-0.338*** (0.043)	0.244*** (0.065)
Intercept	-0.360*** (0.003)	-2.198*** (0.005)	-0.261*** (0.006)	0.241*** (0.002)	0.188*** (0.004)	-0.675*** (0.006)
<i>N</i>	5,794,351	5,794,351	5,794,351	5,794,179	5,794,179	5,794,179
<i>R</i> ²	0.07	0.02	0.03	0.02	0.02	0.00

C: Horizon $h = 30$ days						
	Call options			Put options		
	ITM	OTM	ATM	ITM	OTM	ATM
<i>AbnPosts</i> (τ)	-0.819*** (0.060)	-0.942*** (0.108)	1.431*** (0.107)	-0.260*** (0.033)	-0.764*** (0.095)	0.446*** (0.131)
Intercept	-0.957*** (0.001)	-6.446*** (0.001)	-0.996*** (0.001)	0.737*** (0.000)	0.632*** (0.001)	-2.161*** (0.001)
<i>N</i>	5,796,286	5,796,286	5,796,286	5,796,114	5,796,114	5,796,114
<i>R</i> ²	0.08	0.04	0.03	0.02	0.03	0.00

Table 13
Percentage Return options traded by Small-size Customers
Before and After 2018

This table reports the coefficients of the following regression

$$\%CumPerf(j, M)_{[t,t-h]} = AbnPost(j, \tau)_{t,h} + \alpha_j + \alpha_t + \varepsilon_{j,t}$$

Where $\%CumPerf(j, t, M)$ is the cummulative sun of percentage returns over the horizon of h days $t = [t - h, t]$ of stock j , for different type of moneyness $M = \{ITM, OTM, ATM\}$. $AbnPost(j, \tau)_{t,h}$ is the abnormal average of number of posts related to stock j over the horizon of h days $[t - h, t]$, minus the average on $[t - 60, t - h]$. α_s and α_t correspond to stock and day fixed effects, respectively. Newey-West corrected standard errors are clustered by stock and day, and are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The sample periods are from January 1, 2014, to December 31, 2017, and from January 1, 2018, to December 31, 2022.

A: Horizon $h = 5$ days

	From 2014 to 2017						From 2018 to 2022					
	Call options			Put options			Call options			Put options		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
<i>AbnPosts</i> (τ)	-0.121*** (0.019)	0.005 (0.032)	0.650*** (0.060)	-0.097*** (0.014)	-0.228*** (0.037)	0.038 (0.053)	-0.261*** (0.026)	-0.279*** (0.040)	0.246*** (0.048)	-0.082*** (0.013)	-0.199*** (0.033)	0.092* (0.048)
Intercept	-0.059*** (0.002)	-0.417*** (0.004)	0.019*** (0.007)	0.156*** (0.003)	0.079*** (0.007)	-0.196*** (0.009)	-0.289*** (0.005)	-1.601*** (0.007)	-0.208*** (0.008)	0.178*** (0.002)	0.159*** (0.006)	-0.128*** (0.008)
<i>N</i>	2,391,060	2,391,060	2,391,060	2,757,101	2,757,101	2,757,101	3,403,291	3,403,291	3,403,291	3,403,307	3,403,307	3,403,307
<i>R</i> ²	0.02	0.00	0.04	0.02	0.02	0.00	0.06	0.02	0.01	0.02	0.02	0.00

B: Horizon $h = 10$ days

	From 2014 to 2017						From 2018 to 2022					
	Call options			Put options			Call options			Put options		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
<i>AbnPosts</i> (τ)	-0.194*** (0.031)	0.045 (0.052)	1.280*** (0.101)	-0.188*** (0.026)	-0.457*** (0.072)	0.043 (0.097)	-0.531*** (0.047)	-0.610*** (0.075)	0.530*** (0.079)	-0.153*** (0.023)	-0.394*** (0.064)	0.158* (0.088)
Intercept	-0.104*** (0.002)	-0.822*** (0.003)	-0.017*** (0.006)	0.324*** (0.003)	0.186*** (0.008)	-0.400*** (0.011)	-0.545*** (0.005)	-3.173*** (0.008)	-0.442*** (0.009)	0.365*** (0.003)	0.339*** (0.007)	-0.276*** (0.010)
<i>N</i>	2,391,060	2,391,060	2,391,060	2,757,101	2,757,101	2,757,101	3,403,291	3,403,291	3,403,291	3,403,307	3,403,307	3,403,307
<i>R</i> ²	0.02	0.00	0.07	0.04	0.04	0.00	0.11	0.05	0.01	0.02	0.03	0.00

C: Horizon $h = 30$ days

	From 2014 to 2017						From 2018 to 2022					
	Call options			Put options			Call options			Put options		
	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM	ITM	OTM	ATM
<i>AbnPosts</i> (τ)	-0.257*** (0.056)	0.009 (0.112)	2.142*** (0.173)	-0.486*** (0.050)	-1.178*** (0.157)	-0.074 (0.204)	-1.163*** (0.084)	-1.462*** (0.148)	1.039*** (0.132)	-0.375*** (0.047)	-1.000*** (0.138)	0.177 (0.180)
Intercept	-0.252*** (0.002)	-2.446*** (0.003)	-0.440*** (0.005)	1.093*** (0.000)	0.808*** (0.000)	-1.046*** (0.000)	-1.464*** (0.000)	-9.276*** (0.001)	-1.402*** (0.000)	1.097*** (0.000)	1.076*** (0.001)	-1.025*** (0.001)
<i>N</i>	2,391,795	2,391,795	2,391,795	2,758,116	2,758,116	2,758,116	3,404,491	3,404,491	3,404,491	3,404,507	3,404,507	3,404,507
<i>R</i> ²	0.01	0.00	0.06	0.07	0.07	0.00	0.15	0.08	0.01	0.04	0.05	0.00

Table 14
Dollar Performance options traded by Small-size Customers according to Volatility

This table reports the coefficients of the following regression

$$\$CumPerf(j, M)_{[t, t-h]} = \mathbb{1}_{HighVol} + \mathbb{1}_{LowVol} + \alpha_j + \alpha_t + \varepsilon_{j,t}$$

Where $\$CumPerf(j, M)_{[t, t-h]}$ corresponds to the cumulative performance in millions of dollars, on the period $[t, t-h]$ days of stock j at time t , for different horizons $h = 5, 10, 30$ days, and different type of moneyness $M = \{ITM, OTM, ATM\}$. $\mathbb{1}_{HighVol}$ ($\mathbb{1}_{LowVol}$) is a dummy that takes the value of 1 if the stock's volatility during the period $[t-h, t]$ falls within the top (bottom) quintile of its time-series. Otherwise, it is set to 0. The volatility of stock j is defined as the standard deviation of its returns over the period $[t-h, t]$. α_s and α_t correspond to stock and day fixed effects, respectively. Newey-West corrected standard errors are clustered by stock and day, and are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The sample period is from January 1, 2014, to December 31, 2022.

A: Horizon $h = 5$ days

	Call options			Put options		
	Full sample	Small caps	Large caps	Full sample	Small caps	Large caps
$\mathbb{1}_{HighVol}$	-0.018*** (0.001)	-0.002*** (0.000)	-0.041*** (0.003)	-0.013*** (0.001)	-0.000** (0.000)	-0.024*** (0.002)
$\mathbb{1}_{LowVol}$	0.009*** (0.001)	0.000*** (0.000)	0.020*** (0.002)	0.003*** (0.001)	0.000 (0.000)	0.005*** (0.001)
Intercept	-0.045*** (0.001)	-0.002*** (0.000)	-0.100*** (0.001)	-0.017*** (0.001)	-0.004*** (0.000)	-0.036*** (0.001)
N	5,907,191	3,336,665	2,570,526	5,907,004	3,336,460	2,570,544
R^2	0.04	0.02	0.08	0.03	0.00	0.05

B: Horizon $h = 10$ days

	Call options			Put options		
	Full sample	Small caps	Large caps	Full sample	Small caps	Large caps
$\mathbb{1}_{HighVol}$	-0.035*** (0.003)	-0.003*** (0.000)	-0.078*** (0.006)	-0.031*** (0.002)	-0.002*** (0.000)	-0.057*** (0.005)
$\mathbb{1}_{LowVol}$	0.015*** (0.002)	0.000** (0.000)	0.034*** (0.004)	0.005*** (0.001)	0.000 (0.000)	0.009*** (0.002)
Intercept	-0.089*** (0.001)	-0.005*** (0.000)	-0.198*** (0.003)	-0.031*** (0.001)	-0.007*** (0.000)	-0.068*** (0.002)
N	5,907,191	3,336,665	2,570,526	5,907,004	3,336,460	2,570,544
R^2	0.04	0.04	0.07	0.06	0.01	0.08

C: Horizon $h = 30$ days

	Call options			Put options		
	Full sample	Small caps	Large caps	Full sample	Small caps	Large caps
$\mathbb{1}_{HighVol}$	-0.081*** (0.008)	-0.010*** (0.001)	-0.182*** (0.019)	-0.094*** (0.007)	-0.008*** (0.001)	-0.158*** (0.012)
$\mathbb{1}_{LowVol}$	0.044*** (0.005)	0.001*** (0.000)	0.085*** (0.012)	0.015*** (0.003)	0.000 (0.001)	0.032*** (0.005)
Intercept	-0.276*** (0.004)	-0.015*** (0.000)	-0.608*** (0.010)	-0.091*** (0.003)	-0.019*** (0.000)	-0.206*** (0.006)
N	5,907,191	3,336,665	2,570,526	5,907,004	3,336,460	2,570,544
R^2	0.03	0.08	0.05	0.10	0.04	0.12

Table 15
Dollar Performance options traded by Small-size Customers according to Volatility

This table reports the coefficients of the following regression

$$\%CumPerf(j, M)_{[t, t-h]} = \mathbb{1}_{HighVol} + \mathbb{1}_{LowVol} + \alpha_j + \alpha_t + \varepsilon_{j,t}$$

Where $\%CumPerf(j, M)_{[t, t-h]}$ corresponds to the cumulative performance in percentage, on the period $[t, t-h]$ days of stock j at time t , for different horizons $h = 5, 10, 30$ days, and different type of moneyness $M = \{ITM, OTM, ATM\}$. $\mathbb{1}_{HighVol}$ ($\mathbb{1}_{LowVol}$) is a dummy that takes the value of 1 if the stock's volatility during the period $[t-h, t]$ falls within the top (bottom) quintile of its time-series. Otherwise, it is set to 0. The volatility of stock j is defined as the standard deviation of its returns over the period $[t-h, t]$. α_s and α_t correspond to stock and day fixed effects, respectively. Newey-West corrected standard errors are clustered by stock and day, and are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. The sample period is from January 1, 2014, to December 31, 2022.

A: Horizon $h = 5$ days

	Call options			Put options		
	Full sample	Small caps	Large caps	Full sample	Small caps	Large caps
$\mathbb{1}_{HighVol}$	-0.316*** (0.017)	-0.398*** (0.024)	-0.187*** (0.013)	-0.071*** (0.014)	-0.095*** (0.013)	-0.050*** (0.019)
$\mathbb{1}_{LowVol}$	0.119*** (0.010)	0.141*** (0.013)	0.083*** (0.009)	0.045*** (0.009)	0.043*** (0.009)	0.052*** (0.013)
Intercept	-0.082*** (0.008)	-0.166*** (0.011)	0.024*** (0.007)	0.137*** (0.006)	0.318*** (0.006)	-0.095*** (0.008)
N	5,907,191	3,336,665	2,570,526	5,907,004	3,336,460	2,570,544
R^2	0.09	0.09	0.10	0.01	0.02	0.01

B: Horizon $h = 10$ days

	Call options			Put options		
	Full sample	Small caps	Large caps	Full sample	Small caps	Large caps
$\mathbb{1}_{HighVol}$	-0.519*** (0.029)	-0.658*** (0.037)	-0.306*** (0.023)	-0.076*** (0.023)	-0.127*** (0.022)	-0.043 (0.031)
$\mathbb{1}_{LowVol}$	0.263*** (0.018)	0.306*** (0.025)	0.191*** (0.015)	0.087*** (0.016)	0.083*** (0.015)	0.105*** (0.023)
Intercept	-0.219*** (0.013)	-0.391*** (0.018)	0.003 (0.011)	0.246*** (0.010)	0.618*** (0.011)	-0.227*** (0.014)
N	5,907,191	3,336,665	2,570,526	5,907,004	3,336,460	2,570,544
R^2	0.14	0.14	0.15	0.01	0.02	0.01

C: Horizon $h = 30$ days

	Call options			Put options		
	Full sample	Small caps	Large caps	Full sample	Small caps	Large caps
$\mathbb{1}_{HighVol}$	-0.943*** (0.054)	-1.161*** (0.071)	-0.601*** (0.037)	-0.085** (0.043)	-0.196*** (0.039)	-0.030 (0.058)
$\mathbb{1}_{LowVol}$	0.537*** (0.038)	0.588*** (0.055)	0.452*** (0.031)	0.096*** (0.033)	0.106*** (0.035)	0.148*** (0.043)
Intercept	-0.796*** (0.025)	-1.360*** (0.036)	-0.071*** (0.019)	0.725*** (0.020)	1.824*** (0.021)	-0.690*** (0.028)
N	5,907,191	3,336,665	2,570,526	5,907,004	3,336,460	2,570,544
R^2	0.15	0.15	0.21	0.01	0.01	0.01

A. Appendix

A.1. Proofs of Propositions

A.1.1. Maturity

Proposition 4 *For ITM options, when $F_T - K > 0$, then $\frac{\partial f^*}{\partial T} < 0$ for all strike prices $K < S_0 \exp^{-(r+\sigma^2/2)T}$. That is, there is a negative relationship between the optimal fraction of investment and the maturity of ITM options.*

Proof: I start by examining the partial derivative of f^* with respect to time to maturity T :

$$\frac{\partial f^*}{\partial T} = \frac{-(\mathcal{N}(d_1) - 1) \frac{\partial C(S_0, T)}{\partial T} + (K - F_T - C(S_0, T)) \Phi(d_1) \frac{\partial d_1}{\partial T}}{K - F_T}$$

Where $\frac{\partial C(S_0, T)}{\partial T}$ represents the sensitivity of the call option value to the passage of time. Since $\frac{\partial C(S_0, T)}{\partial T} = -\Theta = rK \exp^{rT} \mathcal{N}(d_2) + \frac{\sigma}{2\sqrt{T}} > 0$, I have $-(\mathcal{N}(d_1) - 1) \frac{\partial C(S_0, T)}{\partial T} > 0$, given that $\mathcal{N}(d_1) \leq 1$.

For ITM options, $F_T - K > 0$ and the call price $C(S_0, T) \geq 0$, leading to $K - F_T - C(S_0, T) \leq 0$. Additionally, $(K - F_T - C(S_0, T)) \Phi(d_1) \leq 0$ since $\Phi(d_1) > 0$. Now, considering $\frac{\partial d_1}{\partial T}$, there are two possibilities:

- $\frac{\partial d_1}{\partial T} = \frac{T(2r+\sigma^2)-2\log\left(\frac{S_0}{K}\right)}{4T^{3/2}\sigma} > 0 \iff T(2r + \sigma^2) > 2\log\left(\frac{S_0}{K}\right) \iff K > S_0 \exp^{-T(r+\frac{\sigma^2}{2})}$
- $\frac{\partial d_1}{\partial T} = \frac{T(2r+\sigma^2)-2\log\left(\frac{S_0}{K}\right)}{4T^{3/2}\sigma} < 0 \iff T(2r + \sigma^2) < 2\log\left(\frac{S_0}{K}\right) \iff K < S_0 \exp^{-T(r+\frac{\sigma^2}{2})}$

Given that the condition $K < S_0 \exp^{-T(r+\frac{\sigma^2}{2})}$ encompasses a wider range of strike prices of ITM options, especially for deep in-the-money options, $\frac{\partial d_1}{\partial T} < 0$. Therefore, $(K - F_T - C(S_0, T)) \Phi(d_1) \frac{\partial d_1}{\partial T} > 0$, leading to $\frac{\partial f^*}{\partial T} < 0$. Thus, the optimal fraction of investment decreases with increasing maturity of ITM options.

Proposition 5 *For OTM options, when $F_T - K < 0$, then $\frac{\partial f^*}{\partial T} > 0$ for all strick prices $K > C(S_0, T) + F_T$. That is, there is a positive relationship between the optimal fraction of investment and the maturity of OTM options.*

Proof: I begin by examining the partial derivative of f^* with respect to time to maturity T :

$$\frac{\partial f^*}{\partial T} = \frac{-(\mathcal{N}(d_1) - 1) \frac{\partial C(S_0, T)}{\partial T} + (K - F_T - C(S_0, T)) \Phi(d_1) \frac{\partial d_1}{\partial T}}{K - F_T}$$

Where $\frac{\partial C(S_0, T)}{\partial T}$ represents the sensitivity of the call option value to the passage of time. Since

$\frac{\partial C(S_0, T)}{\partial T} = -\Theta = rK \exp^{rT} \mathcal{N}(d_2) + \frac{\sigma}{2\sqrt{T}} > 0$, I have $-(\mathcal{N}(d_1) - 1) \frac{\partial C(S_0, T)}{\partial T} > 0$, given that $\mathcal{N}(d_1) \leq 1$.

Considering $\frac{\partial d_1}{\partial T}$, there is two possibilities:

- $\frac{\partial d_1}{\partial T} = \frac{T(2r+\sigma^2)-2\log(\frac{S_0}{K})}{4T^{3/2}\sigma} > 0 \iff T(2r + \sigma^2) > 2\log\left(\frac{S_0}{K}\right) \iff K > S_0 \exp^{-T(r+\frac{\sigma^2}{2})}$
- $\frac{\partial d_1}{\partial T} = \frac{T(2r+\sigma^2)-2\log(\frac{S_0}{K})}{4T^{3/2}\sigma} < 0 \iff T(2r + \sigma^2) < 2\log\left(\frac{S_0}{K}\right) \iff K < S_0 \exp^{-T(r+\frac{\sigma^2}{2})}$

For OTM options, $K - F_T > 0$, which means $K > F_T > S_0 > S_0 \exp^{-T(r+\frac{\sigma^2}{2})}$. Therefore $\frac{\partial d_1}{\partial T} > 0$ and $\Phi(d_1) \frac{\partial d_1}{\partial T} > 0$, since $\Phi(d_1) > 0$.

Regarding $K - F_T - C(S_0, T)$, there are two possibilities:

- $K - F_T - C(S_0, T) > 0 \iff K > F_T + C(S_0, T)$
- $K - F_T - C(S_0, T) < 0 \iff F_T < K < F_T + C(S_0, T)$

Given that the condition $K > F_T + C(S_0, T)$ encompasses a wider range of strike prices of OTM options, especially for deep out-the-money options, then $K - F_T - C(S_0, T) > 0$, leading to $\frac{\partial f^*}{\partial T} > 0$. Thus, the optimal fraction of investment increases with increasing maturity of ITM options.

A.1.2. Volatility

Proposition 6 *For ITM options, when $F_T - K > 0$, then $\frac{\partial f^*}{\partial \sigma} > 0$, for all strike prices $K < S \exp^{(r-\sigma^2/2)T}$ and $\sigma^2/2 > r$. That is, there is a negative relationship between the optimal fraction of investment and the volatility of the stock for ITM options.*

Proof: I start by examining the partial derivative of f^* with respect to time to volatility σ :

$$\frac{\partial f^*}{\partial \sigma} = \frac{-(\mathcal{N}(d_1) - 1) \frac{\partial C(S_0, T)}{\partial \sigma} + (K - F_T - C(S_0, T)) \Phi(d_1) \frac{\partial d_1}{\partial \sigma}}{K - F_T}$$

Where $\frac{\partial C(S_0, T)}{\partial \sigma}$ represents the sensitivity of the call option price with respect to the volatility of the underlying stock. Since $\frac{\partial C(S_0, T)}{\partial \sigma} = \nu = \sqrt{T} S_0 \Phi(d_1) > 0$, then $-(\mathcal{N}(d_1) - 1) \frac{\partial C(S_0, T)}{\partial \sigma} > 0$, given that $\mathcal{N}(d_1) \leq 1$ and $\Phi(d_1) > 0$.

For ITM options, $F_T - K > 0$ and the call price $C(S_0, T) \geq 0$, leading to $K - F_T - C(S_0, T) \leq 0$. Additionally, $(K - F_T - C(S_0, T)) \Phi(d_1) \leq 0$. Now, considering $\frac{\partial d_1}{\partial \sigma}$, there are two possibilities:

- $\frac{\partial d_1}{\partial \sigma} = \frac{T(-2r+\sigma^2)-2\log\left(\frac{S_0}{K}\right)}{2\sqrt{T}\sigma^2} > 0 \iff T(-2r+\sigma^2) > 2\log\left(\frac{S_0}{K}\right) \iff K > S_0 \exp^{T(r-\frac{\sigma^2}{2})}$
- $\frac{\partial d_1}{\partial \sigma} = \frac{T(-2r+\sigma^2)-2\log\left(\frac{S_0}{K}\right)}{2\sqrt{T}\sigma^2} < 0 \iff T(-2r+\sigma^2) < 2\log\left(\frac{S_0}{K}\right) \iff K < S_0 \exp^{T(r-\frac{\sigma^2}{2})}$

Given that the condition $K < S_0 \exp^{T(r-\frac{\sigma^2}{2})}$ encompasses a wider range of strike prices of ITM options, especially for deep in-the-money options, $\frac{\partial d_1}{\partial \sigma} < 0$. Therefore, $(K - F_T - C(S_0, T))\Phi(d_1)\frac{\partial d_1}{\partial \sigma} > 0$, leading to $\frac{\partial f^*}{\partial \sigma} < 0$. Thus, the optimal fraction of investment decreases with increasing volatility of ITM options.

Proposition 7 *For OTM options, when $F_T - K > 0$, then $\frac{\partial f^*}{\partial \sigma} > 0$, for all strike prices $K > C(S_0, T) + F_T$ and $\sigma^2/2 > r$. That is, there is a positive relationship between the optimal fraction of investment and the volatility of OTM options.*

Proof: I begin by examining the partial derivative of f^* with respect to time to volatility σ :

$$\frac{\partial f^*}{\partial \sigma} = \frac{-(\mathcal{N}(d_1) - 1)\frac{\partial C(S_0, T)}{\partial \sigma} + (K - F_T - C(S_0, T))\Phi(d_1)\frac{\partial d_1}{\partial \sigma}}{K - F_T}$$

Where $\frac{\partial C(S_0, T)}{\partial \sigma}$ represents the sensitivity of the call option price with respect to the volatility of the underlying stock. Since $\frac{\partial C(S_0, T)}{\partial \sigma} = \nu = \sqrt{T}S_0\Phi(d_1) > 0$, I have $-(\mathcal{N}(d_1) - 1)\frac{\partial C(S_0, T)}{\partial \sigma} > 0$, given that $\mathcal{N}(d_1) \leq 1$.

Considering $\frac{\partial d_1}{\partial \sigma}$, there is two possibilities:

- $\frac{\partial d_1}{\partial \sigma} = \frac{T(-2r+\sigma^2)-2\log\left(\frac{S_0}{K}\right)}{2\sqrt{T}\sigma^2} > 0 \iff T(-2r+\sigma^2) > 2\log\left(\frac{S_0}{K}\right) \iff K > S_0 \exp^{T(r-\frac{\sigma^2}{2})}$
- $\frac{\partial d_1}{\partial \sigma} = \frac{T(-2r+\sigma^2)-2\log\left(\frac{S_0}{K}\right)}{2\sqrt{T}\sigma^2} < 0 \iff T(-2r+\sigma^2) < 2\log\left(\frac{S_0}{K}\right) \iff K < S_0 \exp^{T(r-\frac{\sigma^2}{2})}$

For OTM options, $K - F_T > 0$, which means $K > F_T > S_0 > S_0 \exp^{T(r-\frac{\sigma^2}{2})}$. Therefore $\frac{\partial d_1}{\partial \sigma} > 0$ and $\Phi(d_1)\frac{\partial d_1}{\partial \sigma} > 0$, since $\Phi(d_1) > 0$.

Regarding $K - F_T - C(S_0, T)$, there are two possibilities:

- $K - F_T - C(S_0, T) > 0 \iff K > F_T + C(S_0, T)$
- $K - F_T - C(S_0, T) < 0 \iff F_T < K < F_T + C(S_0, T)$

Given that the condition $K > F_T + C(S_0, T)$ encompasses a wider range of strike prices of OTM options, especially for deep out-the-money options, then $K - F_T - C(S_0, T) > 0$, leading to $\frac{\partial f^*}{\partial \sigma} > 0$. Thus, the optimal fraction of investment increases with increasing volatility of ITM options.