Ownership and Competition*

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Abstract

We model the trade-offs of an investor who builds positions and exerts governance in competing firms. The implicit cost of doing traditional governance is under-diversification, since her incentives to cut slack and improve efficiency are low when she has similar exposure to all firms in an industry. The benefit is to escape the incentive to push firms to compete less aggressively, and avoid the potential litigation and reputational costs. We study how these trade-offs shape the equilibrium interactions of ownership, governance, and competition, and the role of competition policy in a world where investors influence the objectives of competing firms.

Keywords: corporate governance, competition, horizontal shareholding.

JEL Classification: D82, D83, G34.

1 Introduction

Large shareholders play an important role in corporate governance since, in virtue of their stakes, they have significant incentives to monitor and (where possible) improve firm performance. In recent years, however, their governance role has come under scrutiny: due to a broader rise in diversified investment strategies, many institutional investors have become increasingly invested in groups of competing firms, in both public and private equity markets (Backus, Conlon, and Sinkinson 2021; Scheffler, Alexander, Fulton, Arnold, and Abdelhadi 2023; Eldar and Grennan 2024). This has raised questions among academics, policy-makers, and the general public about whether large investors influence competition and other important corporate decisions (e.g., He and Huang 2017; Azar, Schmalz, and Tecu 2018; Dennis, Gerardi, and Schenone 2022; Lewellen and Lewellen 2022a; Antón, Ederer, Giné, and Schmalz 2023) and may have contributed to the sharp rise in market power and concentration in the last four decades (Covarrubias, Gutiérrez, and Philippon 2020; De Loecker, Eeckhout, and Unger 2020).[1]

To make progress on this question, we model the trade-offs faced by an investor who builds positions and exerts governance in competing firms. The existing literature has studied how holding large stakes in competing firms may impact governance and competition (e.g., Rubinstein and Yaari 1983; Rotemberg 1984; López and Vives 2019; Azar and Vives 2021; Ederer and Pellegrino 2024), but how this feeds back into an investor's incentives to acquire such stakes in the first place is less understood. Our objective is to understand how these trade-offs shape the equilibrium interactions of ownership, governance, and competition, and how to think about competition policy in a world where investors influence the objectives of competing firms.

We extend conventional models of ownership and governance decisions (e.g., Admati, Pfleiderer, and Zechner 1994; DeMarzo and Urošević 2006) to a setting with competing firms. A large investor (ℓ) acquires positions in competing firms and may exert effort to influence how they are managed.[2] We refer to any effort ℓ exerts to influence how firms operate as *governance*,

[1]As of today, the US Federal Trade Commission (FTC), the US Department of Justice (DOJ), the OECD, and the European Commission have conducted hearings about the potential anti-competitive effects of horizontal shareholding (Azar and Schmalz 2017). For examples of the public discourse around these issues, see 'What BlackRock, Vanguard and State Street Are Doing to the Economy,' *New York Times*, May 2022, and 'Who Employs Your Doctor? Increasingly, a Private Equity Firm,' *New York Times*, July 2023.

[2]The closest interpretation for ℓ is a large, actively managed fund picking stocks in an industry. An alternative interpretation is a large asset manager who offers both passive (e.g., industry ETFs) and active funds, and controls its overall exposure to each individual firm through the actively managed fund. Specific examples include Berkshire Hathaway and SoftBank, who hold major stakes in some of the largest firms in the airline, banking, and food delivery sectors (Azar et al. 2018; Shekita 2022), BlackRock, Vanguard, and State Street, who are typically among the largest shareholders in an industry (see, e.g., Table I of Azar et al. 2018), and the private equity firm KKR, who owns one of the largest networks of physician groups in the U.S. (Envision Healthcare; see Scheffler et al. 2023).

and consider two different types of such governance efforts. First, a more traditional one, where ℓ reduces managerial slack and improves firm efficiency (e.g., by cutting production costs or improving product quality). Second, one where ℓ tries to soften competition among firms (e.g., by pushing for consolidation among commonly-owned firms, or by facilitating information sharing and coordination in their pricing strategies).[3] Engaging in either type of governance entails private costs (e.g., the time spent gathering information on how to reduce production costs and monitoring management, or the legal and reputational risk of influencing competition), and small shareholders *free-ride* on these efforts. Finally, the large investor cannot credibly commit to a particular level or type of governance, so her governance and portfolio decisions must be incentive-compatible with each other in equilibrium.

We formulate ℓ's choice problem in a way similar to Grossman and Hart's approach to the principal-agent problem (Grossman and Hart 1983): we first characterize the optimal portfolios compatible with any given governance efforts, and then let the investor pick the best governance-portfolio pair. The first step in understanding our results is to describe the types of portfolio and governance actions that are incentive-compatible with each other. Abstracting from the governance cost and consistent with traditional investment theory, a balanced, diversified portfolio gives ℓ the highest (risk-adjusted) returns.[4] However, such a portfolio also aligns the investor's payoff with industry profits, which creates an incentive for her to try to soften competition among firms.

An alternative is to build a portfolio that induces the traditional governance type. Suppose ℓ knows how to implement changes to cut production costs. This type of governance generates *negative* spillovers across firms: a firm with lower costs would try to gain more market share or push its rivals to set lower prices. If ℓ's portfolio is sufficiently *tilted* toward a given firm, say j, she cares little about the spillovers on other firms, so she is willing to cut j's costs. By contrast, she wants j to maintain its competitive edge, so she prefers to keep its competitors' costs high.

In equilibrium, ℓ picks the strategy that yields the highest returns net of any governance costs: she either builds an undiversified portfolio, tilted toward the firms she pushes to be more efficient, or a balanced one, which may eventually lead her to push firms to compete less

[3] Shekita (2022) provides a collection of examples of the second type of governance. Instances include large institutional investors, like SoftBank and BlackRock, pushing for consolidation among commonly-owned firms in the food delivery, pharmaceutical, and banking industries. In a survey of institutional investors, McCahery, Sautner, and Starks (2016) found that 63% of respondents had engaged in discussions with top management over the previous five years, which is a possible channel for either type of governance.

[4] Investors are risk-averse and firms' profits are risky in our model, so the utility investors get from holding portfolios of stocks can be represented as a certainty equivalent. Hereafter, "returns" refers to the value of this certainty equivalent gross of the governance costs.

aggressively. In a world where large investors pick stocks of competing firms, the implicit cost of traditional governance is then under-diversification, since investors have fewer incentives to exert this type of governance when they have a similar exposure to all firms in an industry. The benefit is to avoid the potential litigation and reputational costs of influencing competition, so investors may forgo some diversification to avoid being exposed to such costs.[5]

We use the model to explore how this trade-off and, thus, the investor's preferred strategy, change with the primitives of the model. The objective is to explore how recent trends in equity and product markets may relate to each other and revisit some of the typical intuitions about the links between governance and competition and the regulation of market power.

We begin by studying how the market powers of firms and large investors interact with each other: how firms' ability to raise prices above costs reflects the investors' capacity to generate returns, and vice-versa. In the last two decades, the sharp rise in markups in product markets was accompanied by an increased concentration of assets under management in a small group of institutional investors (Bebchuk, Cohen, and Hirst 2017; Coates 2019; Kacperczyk, Nosal, and Sundaresan 2024).[6] Intriguingly, we show that these two trends can be connected in our model, as shocks to competition in one market tend to spill over to the other, and potentially amplify their effects in the process.

First, we show that ℓ is more likely to try to soften competition among firms when she has more market power in the equity market. Stock prices are close to their fair value when the equity market is very competitive (e.g., when there are many small shareholders, so more investors compete for the same stocks). ℓ can then make little returns from buying stocks, so she holds a small, balanced portfolio and does not exert governance. As the market becomes less competitive, the potential for returns increases, so ℓ buys larger stakes and begins to exert governance in some firms. Deviations from a balanced portfolio, which are compatible with traditional governance, also become less attractive. As she gains market power, the investor becomes increasingly more concerned with maximizing returns and relatively less worried about governance costs. So, when she has sufficient market power in equity markets, ℓ switches to holding a large balanced portfolio, which creates an incentive to influence competition and

[5]A large literature in finance investigates why observed portfolios often display surprisingly low levels of diversification, seemingly inconsistent with traditional investment theory (Merton 1987; Mitton and Vorkink 2007; Barberis and Huang 2008; Van Nieuwerburgh and Veldkamp 2009, 2010). We are not aware of any theories that link under-diversification to the governance of competing firms.

[6]In 2019, the institutional ownership of an average stock in the US equaled around 60%. This ownership structure is heavily skewed, with the ten largest investors holding, on average, 35% of total shares outstanding, and it varies greatly across individual assets.

push firms to set higher markups.

Second, we show that ℓ may generate higher returns and hold larger portfolios in less competitive industries. Suppose that, for some exogenous reason, the industry becomes *ex-ante* less competitive. This has two contrasting effects on ℓ's incentives to influence competition. On the one hand, softening competition becomes less tempting, since firms compete less to begin with. On the other hand, it may also become cheaper if there are fewer firms to coordinate. Either way, building a large, diversified portfolio becomes more attractive for ℓ, because either influencing competition is no longer incentive compatible, or it is cheaper to do. In the latter case, ℓ's endogenous response *amplifies* the exogenous decline in competition, as she is now more willing to pressure firms to cooperate, which further reduces competition.

The mechanisms described above require that ℓ is able or willing to influence competition. Perhaps not surprisingly, institutional investors typically deny this.[7] In the next set of results, we take their word at face value and focus on the more traditional type of governance, which is improving firm efficiency. In the literature, competition is typically seen as a *substitute* for this type of governance: in competitive industries, firms already have strong incentives to maximize profits, or else they will go out of business, so there is little room for improvement (Hart 1983; Shleifer and Vishny 1997; Allen and Gale 2000). Consistent with this idea, there is evidence that governance matters less in more competitive industries (Giroud and Mueller 2010, 2011).

In our model, the link between governance and competition is more nuanced. Competition may also *deter* more traditional governance: if the negative spillovers of governance are larger in more competitive industries, a diversified investor (who internalizes the spillovers as much as the direct effects) has little incentive to do governance there. So she can build a large balanced portfolio without having to worry about governance costs: since ℓ cannot soften competition in this version of the model, she simply stays *passive*, even if firms are inefficient. This *crowding out* of traditional governance may explain why, even though competition in itself disciplines managers, bad management practices are still surprisingly common even in more competitive sectors (Bloom and Van Reenen 2007). Our results contribute to a large literature on the effects of competition on firm efficiency (e.g., Nickell 1996; Allen and Gale 2000; Syverson 2004).

The last set of results speaks to how to safeguard competition in a world like the one

[7] For an example, see the following excerpt from Warren Buffett's interview with Becky Quick (CNBC, February 2017):
Quick: *You know, Warren, it does occur to me, though, if you're building up such a significant stake in all the major players, is that anything that's, like, monopolistic behavior?* ***Buffett:*** *I've never met the CEOs of any of the four airlines . . . But– no– have no communication with them.*

described in our model. Because of the free-riding of small investors, the large investor never takes governance actions that *reduce* overall industry profits in equilibrium. Although ℓ may choose not to intervene to soften competition, she would then also never actively promote it, even when doing so has no direct costs. Since we cannot expect investors to *promote* competition, competition policy takes center stage in the last step of the analysis. An interesting aspect of our model is that, by impacting the governance actions that are incentive-compatible with certain portfolios, competition policy *indirectly* affects the investor's portfolio decisions, creating spillovers to equity markets. We evaluate different policy measures in light of these spillovers.

The alleged anticompetitive effects of horizontal shareholdings have caught the attention of policymakers and antitrust scholars. The common proposals for a policy response span from outright prohibition of such investments to limiting their ability to influence management (Elhauge 2015, 2017; Posner, Scott Morgan, and Weyl 2016; Posner 2021). These have received staunch opposition from institutional investors, who claim such measures would cause significant disruptions to equity markets.[8] In our model, these policies safeguard competition, but may also distort investors' portfolios and crowd out some socially desirable governance.

We argue that a more traditional antitrust approach may be as effective at promoting competition, but also create positive spillovers to investors. Larger penalties on firms for anti-competitive conduct and stricter enforcement of antitrust rules *indirectly* increase ℓ's cost of influencing competition. This reduces the need to use under-diversification as a commitment *not* to influence competition, which allows ℓ to reach better portfolio returns in equilibrium. Sometimes, this type of intervention may also push the investor toward the socially desirable governance type. Overall, our results suggest that if policymakers are worried about the effects of horizontal shareholding on competition, strengthening the traditional competition authorities (e.g., the DOJ's Antitrust Division and the FTC's Bureau of Competition in the US, which have seen a significant decrease in enforcement actions and funding in the last few decades (Morton 2020; Gilbert 2023; Babina, Barkai, Jeffers, Karger, and Volkova 2023)) may be a sensible and less controversial first step.

[8] See, e.g., 'Investment Giants Raise Voices in Debate Over Their Impact on Competition,' *Wall Street Journal*, 2019, and BlackRock's letter to the FTC in response to its hearing on Competition and Consumer Protection in the 21st Century (https://www.blackrock.com/corporate/literature/publication/ftc-hearing-8-competition-consumer-protection-21st-century-011419.pdf).

Contributions. We make three main contributions to the literature. First, we extend the models of ownership and governance decisions (Admati, Pfleiderer, and Zechner 1994; Burkart, Gromb, and Panunzi 1997; Maug 1998; DeMarzo and Urošević 2006; Levit, Malenko, and Maug 2024a) to a setting with competing firms. Our framework provides an account of how the ownership and governance of competing firms are jointly determined in equilibrium, contributing to a large literature on the interaction between governance and competition (Hart 1983; Shleifer and Vishny 1997; Allen and Gale 2000; Raith 2003; Giroud and Mueller 2010, 2011). From a more normative angle, the model speaks to the role of competition policy in a world where financial markets both shape, and are affected by, the intensity of competition in product markets. Through this angle, our results contribute to the literature on the regulation of market power (e.g., Motta 2004; Tirole 2015; Gutiérrez and Philippon 2019, 2023; Chassang and Ortner 2023).

Second, we contribute to a large and growing theoretical literature studying how large institutional investors shape the objectives of competing firms. Much of economic theory is based on the assumption that firms maximize their own profits: by and large, investors would discipline firms that do not at least mimic profit-maximizing behavior (Friedman 1953; Grossman and Hart 1980; Shleifer and Vishny 1997). Several more recent papers have challenged this assumption, questioning its validity in a world where investors own shares of competing firms. Our contribution is to formalize the investors' portfolio and governance choices. Existing work either takes their portfolios as given (López and Vives 2019; Azar and Vives 2021; Ederer and Pellegrino 2024; Antón, Ederer, Giné, and Schmalz 2023) or does not explicitly model how investors influence firms and/or how stock prices reflect governance (Rubinstein and Yaari 1983; Rotemberg 1984; O'Brien and Salop 1999; Moreno and Petrakis 2022; Denicolò and Panunzi 2024).[9] By contrast, we add the governance of competing firms to a canonical rational expectations model of financial markets.

Finally, we provide a framework for studying the ownership choices of large investors in the presence of governance spillovers. While we focus on competition, the model is general enough to encompass other types of spillovers, making it adaptable for a variety of applications. There is growing evidence that institutional investors influence and redirect innovation among

[9] Rotemberg (1984) considers a model of trading similar to ours but abstracts away from governance, assuming that firms maximize shareholders' payoff. Other papers consider governance but simplify the trading model, assuming Nash-bargaining among investors (Rubinstein and Yaari 1983; Moreno and Petrakis 2022; Denicolò and Panunzi 2024).

the firms in their portfolios (Aghion, Van Reenen, and Zingales 2013; Li, Liu, and Taylor 2023; Antón, Ederer, Giné, and Schmalz 2024; Kini, Lee, and Shen 2024). Similar effects have been documented for other corporate policies, like mergers and acquisitions, and supply-chain relationships (Matvos and Ostrovsky 2008; Lindsey 2008; He and Huang 2017). Our model can be used to study how the anticipation of these spillovers affects investors' portfolio choices.

2 The model

The model consists of two dates, $t \in \{1, 2\}$, and a collection of firms $j \in \mathcal{J} \equiv \{1, \ldots, N\}$. At time $t = 1$, a large strategic investor ($i = \ell$) and a mass $m > 0$ of small, competitive investors ($i = S$) trade claims to the firms' profits in a financial market.[10] At time $t = 2$, the large investor can engage in governance to influence the distribution of firm profits, which are then paid out to shareholders at the end of the period. All agents in the model are rational and risk-averse; for simplicity, we assume there is no discounting.

2.1 Governance and firm values

We consider $N \geq 2$ ex-ante identical firms. Firm j generates profits v_j, which are distributed to its shareholders at $t = 2$. The large investor ℓ can influence the expected level of firm profits through her governance choices $g_j \in \{0, 1\}$. If ℓ exerts governance in firm j (i.e., if $g_j = 1$), she incurs a private cost $\kappa \geq 0$. To simplify the exposition, we assume that the per-firm governance cost κ does not depend on the number of firms where ℓ exerts governance, or the type of governance she engages in (more on this shortly). In Online Appendix C.2, we show that our qualitative results continue to hold when we consider a general cost function that also accounts for these two aspects.[11]

We let $v_j = \pi_j + \varepsilon_j$, where π_j denotes expected firm profits. The random variable $\varepsilon_j \sim \mathcal{N}(0, \sigma)$ represents a shock to the firm's liquidation value. This shock is independent and identically distributed across firms, and realized after all the actions in the model are taken. We denote the collection of realized and expected profits by $\mathbf{v} \equiv (v_1, \ldots, v_N)$ and $\boldsymbol{\pi} \equiv (\pi_1, \ldots, \pi_N)$, respectively.

Expected firm profits π_j depend on the large investor's governance efforts $\mathbf{g} \equiv (g_1, \ldots, g_N)$.[12]

[10] Online Appendix C.1 explores the robustness to a setting with multiple large investors.

[11] We also assume that the cost of doing governance does not depend on the investor's stake (similar to the *allocation-neutral* specification in Admati, Pfleiderer, and Zechner (1994)). Our qualitative results also go through if κ is a decreasing function of ℓ's stake.

[12] Consistent with the existing literature, we focus on the large investor's governance efforts, since each small investor's stake in the firm is too small for the investor to be willing to engage in governance.

More specifically, given $\boldsymbol{\iota} \equiv (1, \ldots, 1)$, we assume that

$$\pi_j = \pi(g_j, \boldsymbol{\iota}'\mathbf{g}). \tag{1}$$

Firm j's expected profits depend both on ℓ's governance in that firm, through g_j, as well as her governance in the other firms, through the aggregate governance $n \equiv \boldsymbol{\iota}'\mathbf{g} = \sum_j g_j$.

Governance has a direct positive effect on firm value, that is $\pi(1, n) \geq \pi(0, n)$ and $\pi(1, n) \geq \pi(0, n-1)$, for any $n \geq 1$.[13] The indirect effect operates through the aggregate governance n and is the critical novel element of our model, as it captures the governance spillovers. We consider positive and negative spillovers, that is, $\pi(g_j, n)$ increasing and decreasing with n, respectively. The interpretation we have in mind for the positive spillovers is ℓ's efforts to limit competition (e.g., by facilitating consolidation or price agreements), which benefit all firms. The negative spillover case captures the more traditional type of governance: efforts to improve firm j's productivity (e.g., by reducing production costs and/or improving product quality). Improving firm j strengthens its product market position, negatively affecting its competitors.

The sum of the direct and indirect effects of governance determines its aggregate effects on expected industry profits, denoted by $\Pi(n) \equiv \boldsymbol{\iota}'\boldsymbol{\pi}$. Let $\mathbf{y}_\ell \equiv (y_{1\ell}, \ldots, y_{N\ell})$ describe the large investor's portfolio; at the beginning of $t = 2$, ℓ chooses $\mathbf{g}$ to maximize her portfolio payoff:[14]

$$\max_{\mathbf{g}} \mathbf{y}_\ell'\boldsymbol{\pi} - \kappa\boldsymbol{\iota}'\mathbf{g}. \tag{2}$$

We establish the main properties of the equilibrium for a generic function π_j, without having to specify the details of how firms compete and how governance works. To illustrate the equilibrium characterization and to derive some of the comparative statics results, we put more structure on π_j by considering two conventional models of competition (see Section 2.4).

2.2 Ownership market

At time $t = 1$, investors trade claims to the firms' terminal values. For simplicity, we consider a fixed unit supply of shares for each firm. All investors have CARA preferences; the risk-aversion

[13] When ℓ goes from doing governance in $n-1$ to n firms, the profit of the n-th firm increases from $\pi(0, n-1)$ to $\pi(1, n)$, so the second inequality is also necessary for the direct effect of governance to be positive.

[14] At the beginning of $t = 2$, ℓ holds a position $y_{j\ell}$ in firm j and expects a payoff $y_{j\ell}\pi_j$ for each j. Strictly speaking, ℓ's utility also includes an additional term capturing the risk of her position. However, this term does not depend on $\mathbf{g}$, and, as a result, it does not affect the governance choice.

coefficient is $\gamma_S > 0$ for the small investors (S), and $\gamma_\ell \geq 0$ for the large investor (ℓ).[15]

When choosing her optimal portfolio $\mathbf{y}_\ell$, the large investor anticipates the effect of her trades on the stock prices. The small investors trade competitively as price takers, but they understand how ℓ's trading decisions will affect her governance choices at $t = 2$, and the implications for firms' expected profits $\boldsymbol{\pi}$. Like in Admati, Pfleiderer, and Zechner (1994), DeMarzo and Urošević (2006), and Levit, Malenko, and Maug (2024b), we assume that ℓ moves first (chooses $\mathbf{y}_\ell$), and her trading choices are observable by S before they submit their demands. Investors form rational expectations about expected profits $\boldsymbol{\pi}$ and stock prices $\mathbf{p}$ when choosing their demands.

The optimal demand vector for each small investor is given by $\mathbf{y}_S \equiv (y_{1S}, \ldots, y_{NS})$ with

$$\mathbf{y}_S = \frac{1}{\gamma_S \sigma^2} (\boldsymbol{\pi} - \mathbf{p}) . \tag{3}$$

Market clearing requires $\mathbf{y}_\ell + m\mathbf{y}_S = \boldsymbol{\iota}$. We can, therefore, write the equilibrium stock price vector as a function of expected firm profits and the large investor's demand:

$$\mathbf{p} = \boldsymbol{\pi} - \frac{\gamma_S \sigma^2}{m}(\boldsymbol{\iota} - \mathbf{y}_\ell). \tag{4}$$

The equilibrium stock price for firm j reflects the expected firm profits net of a risk premium, which is proportional to a small investor's stake $\mathbf{y}_S = \frac{1}{m}(\boldsymbol{\iota} - \mathbf{y}_\ell)$ in the firm. It increases with their risk aversion γ_S and the volatility of the firm's profits σ.

The large investor chooses $\mathbf{y}_\ell$ to maximize the certainty equivalent measure representing her valuation of the risky portfolio $\mathbf{y}_\ell$, net of the implied governance cost. In the text, we refer to the certainty equivalent measure as the investor's "returns."

$$\max_{\mathbf{y}_\ell} \mathbf{y}_\ell' (\boldsymbol{\pi} - \mathbf{p}) - \frac{\gamma_\ell \sigma^2}{2} \mathbf{y}_\ell' \mathbf{y}_\ell - \kappa n \tag{5}$$

where $n = \boldsymbol{\iota}'\mathbf{g}$ is evaluated at the optimal governance choice given $\mathbf{y}_\ell$, that is, at the value of $\mathbf{g}$ that solves Program (2). Notice that the large investor has price impact, so she anticipates how the equilibrium prices $\mathbf{p}$ will depend on her portfolio choices through Eqn. (4).

At time $t = 1$, ℓ chooses her portfolio, anticipating the governance choice that each portfolio

[15] An alternative formulation of the model, which yields identical results and expressions, is to assume that investors are risk-neutral but incur a trading cost $\frac{1}{2}\gamma_i \sigma^2 y^2$ from acquiring an amount y of shares (as in, e.g., Banerjee, Davis, and Gondhi 2018; Dávila and Parlatore 2021; Levit, Malenko, and Maug 2024b). In addition, this alternative formulation also accommodates applications where there is strategic uncertainty or non-normal distributions at the governance stage (e.g., mixed-strategy equilibria and voting), which may be useful for alternative applications of our framework.

will induce at time $t = 2$. Since the equilibrium stock prices reflect expected firm profits, $\boldsymbol{\pi}$ cancels out from her objective in Program (5).[16] So, the governance choice only affects ℓ's time $t = 1$ payoffs through the cost κn. The shocks to the liquidation values (ε_j) are independent across firms. So, holding *n fixed*, Program (5) collapses to a firm-by-firm problem, since ℓ's exposure to firm j does not affect her payoffs from acquiring positions in other firms. However, the optimal governance choice depends on ℓ's entire portfolio, since her exposure to j does affect her willingness to exert governance in the other firms. As shown below, this means that the investor may choose different positions and governance actions across firms, even though firms are ex-ante identical.

2.3 Sequence of events and equilibrium definition

The timing of the model is summarized in what follows.

t = 1: Investors trade and form their portfolios $\{y_{ij}\}$ for $i \in \{\ell, S\}$ and $j \in \mathcal{J}$; equilibrium stock prices p_j clear the market.

t = 2: ℓ chooses governance effort g_j; profits realize and are distributed to shareholders.

We use *subgame perfect equilibrium* as the solution concept and restrict our attention to pure-strategy equilibria. An equilibrium is a collection $\{\{y_{ij}\}, \mathbf{g}\}$ for $i \in \{\ell, S\}$ and $j \in \mathcal{J}$, that jointly solves Programs (2) and (5) and satisfies sequential rationality.

2.4 Applied examples

Before analyzing the model, we describe how our general framework maps into two simple models of competition and governance. The examples we develop here will help illustrate the equilibrium characterization, as well as some of our comparative statics and welfare results.

In both examples, firms compete to sell a homogeneous good to consumers, and ℓ chooses between two governance actions: she can (a) reduce production costs in an arbitrary number of firms, who will then compete with each other (*cut costs*), or (b) pressure all firms to cooperate and

[16] Like Admati, Pfleiderer, and Zechner (1994) and DeMarzo and Urošević (2006), we consider a model with complete information, where governance efforts are thus fully reflected in prices. If ℓ had private information about governance (e.g., whether she can cut costs or soften competition), the price in Program (5) would reflect the small investors' expectation about $\boldsymbol{\pi}$ given $\mathbf{y}_\ell$. The main tradeoffs we highlight in our model carry through in this type of model, with the added feature that now ℓ considers how much information each portfolio reveals about her governance action (details about this model extension are available upon request).

charge higher markups (*facilitate collusion*).[17] The objective is to obtain equilibrium expressions for $\pi(1, n)$ and $\pi(0, n)$. For simplicity, we assume that the firms' production costs are linear. $c > 0$ denotes the firms' marginal cost of production when ℓ does not exert governance to cut costs; if she does, the cost becomes $c' \in [0, c)$. Example 1 describes the first model, where firms simultaneously set prices and consumers choose how much to buy from each firm.

Example 1 (Bertrand competition) *Firms simultaneously set product prices ρ_j, and consumers buy from those charging the lowest price. Let $\rho^m(x) = \arg\max_\rho D(\rho)(\rho - x)$, where $D(\rho)$ is the product-market demand, which is decreasing and concave in ρ and satisfies $D(c) > 0$. In equilibrium:*

1. *Firm j's expected profits π_j are strictly positive only if the large investor (ℓ) either **(i)** facilitates collusion ($n = N$), in which case all firms set the monopoly price $\rho^m(c)$, where $\rho^m(c) > c$, and make expected profits $\frac{1}{N}D(\rho^m)(\rho^m - c)$, or **(ii)** cuts costs only in firm j ($n = 1$), in which case $\pi_j = D(\rho')(\rho' - c')$, where $\rho' = \min\{\rho^m(c'), c\}$ and $\pi_{-j} = 0$ for all other firms.*
2. *Depending on the parameters and her portfolio, ℓ chooses one of the following governance actions: staying passive ($n = 0$), cutting costs in one firm only ($n = 1$), and facilitating collusion ($n = N$).*

In the Bertrand model, when firms compete, they all price at the marginal cost and make zero profits if at least two firms have the same cost (if one firm were to charge a positive markup, it would be profitable for the other to charge a slightly lower price and capture the entire demand). If only *one* firm has a lower cost, however, it can charge slightly below the others' cost and still make a positive profit. Finally, if the investor convinces firms to cooperate, they maximize industry profits: They all set the monopoly price and split the resulting profits.[18]

Of course, ℓ only exerts governance if it increases the value of some firms in equilibrium. If she exerts governance, she thus chooses between cutting costs in one firm only ($n = 1$) and pressuring all firms to collude ($n = N$); otherwise, she stays passive and chooses $n = 0$. This simple structure for the governance choice yields closed-form solutions for any given N. For this reason, we use Example 1 for some of our formal results.

[17]Since our focus is on competition, our applied examples consider cases where firms' cooperation leads to higher markups. However, it is worth emphasizing that (a) our model is general enough to capture other types of positive spillovers, such as technological improvements that enhance product quality and benefit both firms and consumers, and (b) except for the regulatory analysis in Section 5.3, our results do not depend on the specific mechanisms of firm competition or governance.

[18]In Example 1, $\pi(1, n)$ goes from strictly positive to zero when n goes from 1 to $n' \in [2, N-1]$, since cutting costs in more than one firm dissipates its competitive advantage. So, governance generates a negative spillover in this range for n. When n goes from $n' \in [2, N-1]$ to N, however, the spillovers are positive, since $\pi(1, n)$ goes from zero to strictly positive for all firms.

In Example 2, firms choose their supply, and the product price adjusts to clear the market. Here, firms make strictly positive profits even when they do not cooperate, but their profits decrease with the number of firms N in the industry. When firms collude, each one produces a fraction $\frac{1}{N}$ of the quantity that maximizes industry profits. Equilibrium governance is richer in this example, with ℓ potentially cutting costs in multiple firms.

Example 2 (Cournot-quantity game) *Firms simultaneously choose quantities q_j, and the market clearing price ρ equates aggregate supply $\sum_j q_j$ and aggregate demand $A - b\rho$, with $A > bc$. In equilibrium:*

1. *Firms' expected profits satisfy $\pi(1, n) > \pi(0, n) \geq 0$ for any $n \in \{1, \ldots, N-1\}$, and $\pi(1, N) > 0$;*
2. *Depending on the parameters and her portfolio, ℓ chooses $n \in \{0, \ldots, N\}$, where $n = N$ represents cutting cost in all firms or facilitating collusion, depending on which leads to higher industry payoffs.*

3 Equilibrium analysis

We work our way backward and start with deriving ℓ's governance choices for a given portfolio. We then determine her portfolio choice given the correctly anticipated governance efforts.

3.1 Preliminaries

Lemma 1 describes the investor's governance choice for a given portfolio $\mathbf{y}_\ell$.

Lemma 1 *For a given portfolio $\mathbf{y}_\ell$, a governance choice $\mathbf{g}^*$ always exists. Let $\overline{n} \equiv \arg\max_n \{\frac{\gamma_S}{m\gamma_\ell + 2\gamma_S}\boldsymbol{\iota}'\boldsymbol{\pi} - n\kappa\}$; in equilibrium, the large investor ℓ:*

1. *never exerts governance in more than $\overline{n}$ firms (i.e., $n^* \leq \overline{n}$);*
2. *chooses $n^* = \overline{n}$ iff she holds the "unconstrained optimal" portfolio $\mathbf{y}_\ell = \frac{\gamma_S}{m\gamma_\ell + 2\gamma_S}\boldsymbol{\iota} \equiv y^u\boldsymbol{\iota}$.*

To build intuition, consider the case where ℓ's governance choice is exogenous, meaning that it does not depend on her portfolio choice. In that case, the investor holds a balanced portfolio, with positions $y^u = \frac{\gamma_S}{m\gamma_\ell + 2\gamma_S}$ in each firm, to maximize her risk-return trade-off. We refer to this portfolio as the "unconstrained optimal" portfolio, since it represents the solution to ℓ's portfolio choice problem in Program (5) for a *fixed* n. That is, the optimal $\mathbf{y}_\ell$ without the incentive-compatibility constraint implied by the optimal governance choice in Program (2).

Now consider the case where governance is endogenous. If the investor sticks to the unconstrained optimal portfolio $\mathbf{y}_\ell = y^u \boldsymbol{\iota}$, she ends up choosing $\overline{n} \equiv \arg\max_n \{y^u \boldsymbol{\iota}' \boldsymbol{\pi} - n\kappa\}$, as this is the incentive-compatible governance choice. Since governance is costly, ℓ may want to hold a different portfolio to induce a different governance choice. However, she cannot get a higher payoff by choosing a $\mathbf{y}_\ell$ that induces governance in more than $\overline{n}$ firms: such a portfolio would have lower risk-adjusted returns and higher governance costs. So ℓ always chooses $n \leq \overline{n}$ in equilibrium, and she chooses $n = \overline{n}$ only when she holds the unconstrained optimal portfolio.

In general, when choosing her positions, the large investor weighs in the returns and governance costs associated with each potential portfolio. In some cases, she may be indifferent between a portfolio that induces more governance but yields higher (gross) returns and one that leads to less governance but also lower returns. To simplify the exposition, we assume ℓ breaks ties in favor of the portfolios that induce *less* governance. Without a tie-breaking assumption, however, the equilibrium characterization changes only slightly: the equilibrium is then unique except for the specific parameter values for which the investor is indifferent between portfolios.

3.2 Benchmark without competition

To understand how competition among firms affects large investors' portfolio and governance decisions, it is useful to first briefly study a benchmark *without* competition. In this scenario, there are no governance spillovers across firms. The expected firm profits π_j are thus only a function of g_j and do not depend on ℓ's governance efforts in other firms, that is, $\pi_j(g_j, \boldsymbol{\iota}'\mathbf{g}) = \pi_j^{nc}(g_j)$. In this case, the investor's portfolio choice collapses into a firm-by-firm problem.

Proposition 1 (No competition benchmark) *A financial market equilibrium always exists and is unique. In equilibrium, the large investor (ℓ) chooses the same positions and governance actions in all firms:*

1. *If the governance cost κ is sufficiently large ($\kappa \geq y^u [\pi^{nc}(1) - \pi^{nc}(0)]$), ℓ does not exert governance ($n^* = 0$) and holds positions $\mathbf{y}_\ell = y^u \boldsymbol{\iota}$;*
2. *If $\underline{\kappa}^{nc} \leq \kappa < y^u [\pi^{nc}(1) - \pi^{nc}(0)]$, ℓ chooses $n^* = 0$ and $\mathbf{y}_\ell = \frac{\kappa}{\pi^{nc}(1) - \pi^{nc}(0)} \boldsymbol{\iota}$;*
3. *If $\kappa < \underline{\kappa}^{nc}$, ℓ exerts governance in all firms ($n^* = N$) and holds positions $\mathbf{y}_\ell = y^u \boldsymbol{\iota}$.*

Without governance, ℓ would hold y^u in each firm. She then chooses this portfolio whenever it is incentive-compatible with *not* exerting governance, which is the case when the governance cost κ is very high. ℓ still does not exert governance if κ is in an intermediate range. However, she must hold back on her positions to satisfy the incentive-compatibility constraint: If $y_{j\ell} > \frac{\kappa}{\pi^{nc}(1)-\pi^{nc}(0)}$, ℓ's stake in the firm is too large to not exercise governance at $t = 2$. So, the investor chooses to invest less at $t = 1$, sacrificing some trading returns to save on the governance costs at $t = 2$. Finally, if the governance cost is lower than $\underline{\kappa}^{nc}$, ℓ becomes active and holds again y^u in all firms, since the governance cost is too small for the investor to be willing to distort her portfolio.[19]

ℓ's governance in firm j only depends on her stake in that firm, as g_j has no indirect effects on the other firms. This means that the investor's choice of how much to invest and engage in firm j does not interact with how much ℓ holds or engages in the other firms, and vice versa. Since the firms are ex-ante identical, ℓ chooses the same positions and takes the same governance action in all firms. As shown below, this is not the case in the presence of governance spillovers.

3.3 Equilibrium characterization

Having described how the large investor's governance choice depends on firm ownership, we can now solve for the investors' optimal portfolio choices in the model with competition.

Proposition 2 (Equilibrium) *A financial market equilibrium always exists and is unique. In equilibrium, the large investor (ℓ) may choose different positions and governance actions across firms:*

1. *If it is incentive-compatible to not exert governance at the unconstrained optimal portfolio (i.e., $\overline{n} = 0$), ℓ chooses $n^* = 0$ and holds that portfolio ($\mathbf{y}_\ell = y^u \boldsymbol{\iota}$);*
2. *Otherwise, ℓ exerts governance in a subset of firms $n^* \in \{0, \ldots, N\}$ and holds larger positions ($\overline{y}$) in those firms, and a smaller position ($\underline{y}$, with $\underline{y} \leq \overline{y}$) in the others.*

Allowing for competition introduces two novel features to the equilibrium characterization. First, ℓ may now take different governance actions across firms, even though firms are ex-ante identical. So any $n \in \{0, \ldots, N\}$ can arise in equilibrium, while we only had $n \in \{0, N\}$ in the benchmark model. Second, the investor may now deviate from holding a balanced

[19] At $\kappa = \underline{\kappa}^{nc}$, ℓ is indifferent between $\mathbf{y}_\ell = \frac{\kappa}{\pi^{nc}(1)-\pi^{nc}(0)} \boldsymbol{\iota}$ and the unconstrained optimal portfolio $\mathbf{y}_\ell = y^u \boldsymbol{\iota}$. So, without a tie-breaking assumption, the equilibrium is unique except at $\kappa = \underline{\kappa}^{nc}$, where both strategies represent an equilibrium.

portfolio, even though this was never the case in the benchmarks without governance or without competition.

To understand our results, it is helpful to think of how ℓ chooses her portfolio as a two-step process. First, for any value of n, she finds the best portfolio for which it is incentive-compatible to exert governance in exactly n firms. The governance choice problem at time $t = 2$ (Program 2) defines a set of portfolios for which $\boldsymbol{\iota}'\mathbf{g}^* = n$, and ℓ chooses the best portfolio within that set. The investor's expected utility at $t = 1$ is concave in her positions,[20] so she would like to hold similar stakes in all firms. However, this may not be consistent with incentive-compatibility. So ℓ settles for the second-best: holding only two different types of positions, $\overline{y}$ and $\underline{y}$, the larger in the firms where she exerts governance. In the second step, ℓ compares all portfolios resulting from the first step and chooses the one with the highest return net of the governance cost.

The two-step procedure described above is similar to Grossman and Hart's approach to the principal-agent problem (Grossman and Hart 1983), where the principal first finds the optimal way to induce any possible action by the agent, and then chooses the one that maximizes her payoff net of the agent's pay.[21] Once the optimal incentive-compatible portfolios are determined, comparing them is a more straightforward step. So here we focus on illustrating the first step, and return to the second when we describe our comparative statics results (Section 4).

Proposition 3 (Optimal incentive-compatible portfolios) *The optimal portfolio that is incentive-compatible with exerting governance in firms $j \leq n$ solves the following problem:*

$$\max_{\underline{y},\overline{y}} \sigma^2 \left[n \left(\frac{\gamma_s}{m} \overline{y}(1-\overline{y}) - \frac{\gamma_\ell}{2} \overline{y}^2 \right) + (N-n) \left(\frac{\gamma_s}{m} \underline{y}(1-\underline{y}) - \frac{\gamma_\ell}{2} \underline{y}^2 \right) \right] \tag{6}$$

$$s.t.\ \ n\overline{y}\pi(1,n) + (N-n)\underline{y}\pi(0,n) - n\kappa \geq \begin{cases} (n\overline{y} + (\tilde{n}-n)\underline{y})\pi(1,\tilde{n}) + (N-\tilde{n})\underline{y}\pi(0,\tilde{n}) - \tilde{n}\kappa & \text{if } \tilde{n} \geq n \\ \tilde{n}\overline{y}\pi(1,\tilde{n}) + ((n-\tilde{n})\overline{y} + (N-n)\underline{y})\pi(0,\tilde{n}) - \tilde{n}\kappa & \text{if } \tilde{n} < n \end{cases}$$

$$\forall \tilde{n} \in \{0,1,\ldots,N\}, \quad \text{and} \quad \overline{y} \geq \underline{y}.$$

Provided that the set of $(\overline{y}, \underline{y})$ that satisfies the incentive-compatibility constraint above is non-empty, a

[20] For the large investor, this concavity comes from two different channels. First, her risk aversion makes it increasingly more costly to hold larger stakes in each firm j, as doing so increases ℓ's exposure to the firm's idiosyncratic risk. Second, since ℓ has market power, $i = S$ is the marginal investor, which implies that $\pi_j - p_j$ increases with the small investors' aggregate holding in firm j, as compensation for their risk exposure. So, on the one hand, ℓ wants to increase $y_{j\ell}$ to get a larger fraction of the return. On the other hand, increasing $y_{j\ell}$ reduces the small investors' holding and, thus, $\pi_j - p_j$.

[21] In our setting, ℓ has different objective functions at the portfolio ($t = 1$) and governance ($t = 2$) stages. This is because small investors have rational expectations, so stock prices at $t = 1$ reflect firm profits at $t = 2$. This means that π_j only affects ℓ's choice at $t = 2$, so there is a misalignment between ℓ's preferred choices across the two periods. In the analogy with the principal-agent problem, the investor at the portfolio stage is thus the principal, and the investor at the governance stage is the agent.

solution to Program (6) always exists and is unique.

Given that ℓ wants to do governance in n firms for at least some portfolio, a solution to Program (6) always exists and is unique. Since the constrained set is compact and the objective is continuous, this result follows directly from the Weierstrass Theorem. To understand why the optimal portfolio may be tilted (that is, feature $\overline{y} > \underline{y}$), it is helpful to zoom in on some of the incentive-compatibility (IC) constraints. For example, consider the IC that ensures ℓ prefers exerting governance in n firms over $\tilde{n} > n$. Rearranging this constraint, we obtain:

$$\overline{y}n \underbrace{[\pi(1,\tilde{n}) - \pi(1,n)]}_{\text{Indirect effect on } j \leq n} + \underline{y} \left\{ (\tilde{n} - n) \underbrace{[\pi(1,\tilde{n}) - \pi(0,n)]}_{\text{Direct effect on } j \in (n,\tilde{n}]} + (N - \tilde{n}) \underbrace{[\pi(0,\tilde{n}) - \pi(0,n)]}_{\text{Indirect effect on } j > \tilde{n}} \right\} - \kappa(\tilde{n} - n) \leq 0. \tag{7}$$

Increasing aggregate governance from n to $\tilde{n}$ has four different effects. First, an indirect effect on the firms where ℓ was already doing governance ($j \leq n$), and those where she continues to stay passive ($j > \tilde{n}$). These correspond to the first and third terms in Inequality (7), respectively. There is also a direct effect on the firms $j \in (n, \tilde{n}]$, since ℓ exerts governance in these firms at $\tilde{n}$, but not at n. Finally, the governance cost increases from κn to $\kappa \tilde{n}$. The direct effect and the cost increase correspond to the second and last terms in Inequality (7), respectively.

If the indirect effects are zero, the IC above simplifies to $\underline{y}\,[\pi(1) - \pi(0)] - \kappa \leq 0$, which implies $\underline{y} \leq \frac{\kappa}{\pi(1)-\pi(0)}$, since π_j no longer depends on aggregate governance. In this case, we are back to the benchmark without competition, where whether ℓ chooses to do governance also in firms $j > n$ only depends on her stake in those firms. If the indirect effects are different from zero, both $\underline{y}$ and $\overline{y}$ affect the IC constraint, in potentially different directions. For example, consider an industry with only two firms, and set $\tilde{n} = 2$ and $n = 1$; the IC constraint becomes:

$$\overline{y} \underbrace{[\pi(1,2) - \pi(1,1)]}_{\text{Indirect effect on } j = 1} + \underline{y} \underbrace{[\pi(1,2) - \pi(0,1)]}_{\text{Direct effect on } j = 2} - \kappa \leq 0. \tag{8}$$

The direct effect on firm 2 is always positive, that is $\pi(1,2) > \pi(0,1)$. If the governance spillovers are negative, the indirect effect on firm 1 is instead negative, that is, $\pi(1,1) > \pi(1,2)$.[22]

In this case, $\underline{y}$ and $\overline{y}$ have opposite effects on the IC constraint. Similar to the benchmark

[22]This is the case in the Bertrand model (Example 1) if being the only firm with lower costs is better than splitting the monopoly profits, and in the Cournot model (Example 2) when ℓ prefers cutting both firms' production costs to pushing them to cooperate.

without competition, increasing $\underline{y}$ makes the constraint harder to satisfy, since ℓ becomes more exposed to firm 2 and has thus more incentives to do governance in it. Increasing $\overline{y}$, however, reduces these incentives and, thus, relaxes the IC constraint: the larger the investor's stake in firm 1, the more she internalizes the negative spillovers of doing governance also in firm 2. This second effect creates an incentive for ℓ to tilt her portfolio towards firm 1, so that she can hold overall larger positions in both firms (that is, $\overline{y} > \underline{y} > \frac{\kappa}{\pi(1,2)-\pi(0,1)}$) while still doing governance in only one.

More generally, $\underline{y}$ and $\overline{y}$ having different effects on the IC constraint means that tilting her portfolio may allow ℓ to reach higher returns overall, by being able to hold larger stakes without having to engage in more governance. The cost of this strategy is less effective diversification, since the investor is then relatively more exposed to the firms where she exerts governance.

So far, our discussion of incentive compatibility has focused on the case where ℓ is tempted to do governance in more than n firms, that is, when the relevant IC constraint in Program (6) is one with $\tilde{n} > n$. The next proposition shows that this is always the case in equilibrium, and generalizes some of the insights discussed above.

Proposition 4 (Equilibrium portfolios) *If it is incentive-compatible to exert some governance at the unconstrained optimal portfolio ($\overline{n} > 0$), the large investor (ℓ) picks the best between this portfolio and those that solve Program (6) for $n < \overline{n}$. In equilibrium, if ℓ exerts governance in $n^* \in [0, \overline{n})$ firms, she must be indifferent between doing governance in n or $\tilde{n}$ firms, for some $\tilde{n} \in (n^*, \overline{n}]$.*

1. *If $n^* = 0$, ℓ holds a balanced portfolio: $y_{j\ell} = \min_{\tilde{n} \in \mathcal{N}_0^+} \frac{\kappa \tilde{n}}{\Pi(\tilde{n})-\Pi(0)}$ in all $j \in \mathcal{J}$ where $\mathcal{N}_0^+ = \{n \leq \overline{n} \mid \Pi(n) > \Pi(0)\}$.*

2. *If $n^* > 0$ and doing governance in more than n firms has a negative indirect effect on firms $j \leq n^*$, that is, $\pi(1, \tilde{n}) < \pi(1, n^*)$, ℓ holds strictly larger positions in those firms: $\overline{y} > \underline{y}$.*

For the sake of contradiction, suppose that, in equilibrium, ℓ is indifferent between doing governance in n or $\tilde{n} < n$ firms (that is, the IC constraint in Program (6) is binding for some $\tilde{n} < n$). Since the effect of governance on firm profits is correctly priced-in, even though ℓ may be indifferent between n and $\tilde{n}$ at the governance stage ($t = 2$), she strictly prefers $\tilde{n}$ at the portfolio-choice stage ($t = 1$), as the governance cost is lower for $\tilde{n}$. So, choosing the same portfolio and doing less governance (i.e., $\tilde{n} < n$) would be incentive-compatible and strictly preferred by ℓ. That, however, violates the initial equilibrium conjecture, since ℓ must choose an optimal portfolio in equilibrium.

The discussion above implies that if an IC constraint is binding in equilibrium, it must be for some $\tilde{n}$ greater than n. Moreover, the equilibrium portfolio cannot feature $n < \overline{n}$ and have none of the IC constraints binding: in that case, ℓ's portfolio-choice problem would be equivalent to an unconstrained one; however, absent any IC constraints, ℓ would choose $\mathbf{y}_\ell = y^u \boldsymbol{\iota}$, which is then incentive-compatible with $n^* = \overline{n}$. Knowing which constraints bind in equilibrium helps put more structure on the equilibrium portfolios, which yields two interesting economic implications.

First, the optimal *passive* portfolio (the one that induces $n = 0$) has a simple structure: when it is different from the unconstrained one, i.e., when $\overline{n} > 0$, this portfolio is the largest balanced portfolio that is incentive-compatible with not exerting governance in any firms.[23] The size of this portfolio depends on how governance affects *industry* profits $\Pi(n)$, rather than how it affects the individual firms. So changes in $\Pi(n)$ will indirectly affect also *passive* diversified investors, that is, those that hold a balanced portfolio but do not exert governance in any firm.

Second, ℓ tilts her portfolio in equilibrium only when she is willing to do some governance, but not too much ($n^* \in (0, \overline{n})$). So, the implicit benefit of under-diversification is always lower governance costs. Also, ℓ is relatively more likely to tilt her portfolio when governance generates negative spillovers on other firms. So, the typical undiversified investor in our model is one that pushes a subset of firms to be more efficient and, by doing so, imposes a cost on other firms.

3.4 Equilibrium illustration

We conclude this section with an illustration of the equilibrium through numerical simulations of the applied examples in Section 2.4. Figure 1 illustrates the optimal portfolio that is incentive-compatible with ℓ cutting costs in a subset of firms in the Cournot model. In the left panel, the 45-degree line reflects the constraint $\overline{y} \geq \underline{y}$, and the others represent each an incentive compatibility constraint in Program (6) for a specific value of $\tilde{n}$. The set of incentive-compatible portfolios corresponds to the convex set in $(\overline{y}, \underline{y})$ where all constraints are simultaneously satisfied. The right panel shows that the optimal incentive-compatible portfolio is tilted, that

[23] We describe the IC constraint for $n = 0$ and the intuition behind the structure of the optimal passive portfolio in Section 4.3.

is, features $\overline{y} > \underline{y}$.

Figure 2 describes ℓ's equilibrium choices in the Bertrand model with $N = 2$ firms. The left panel plots ℓ's portfolio and the corresponding governance choice as a function of the governance cost κ. For small values of κ, ℓ holds the unconstrained-optimal portfolio and exerts governance in both firms (facilitating collusion). As κ increases, ℓ tilts her portfolio toward one firm, to make it incentive-compatible to exert governance only in this firm (cutting costs). By doing so, she sacrifices portfolio returns but saves on the governance costs. As κ increases further, ℓ is better off holding a lower, balanced position in both firms, which makes it incentive-compatible to stay passive. Since exerting governance becomes less attractive as κ increases, the distortion in ℓ's passive portfolio decreases, and $y_{j\ell}$ converges to the unconstrained optimal portfolio as $\kappa \to \infty$.

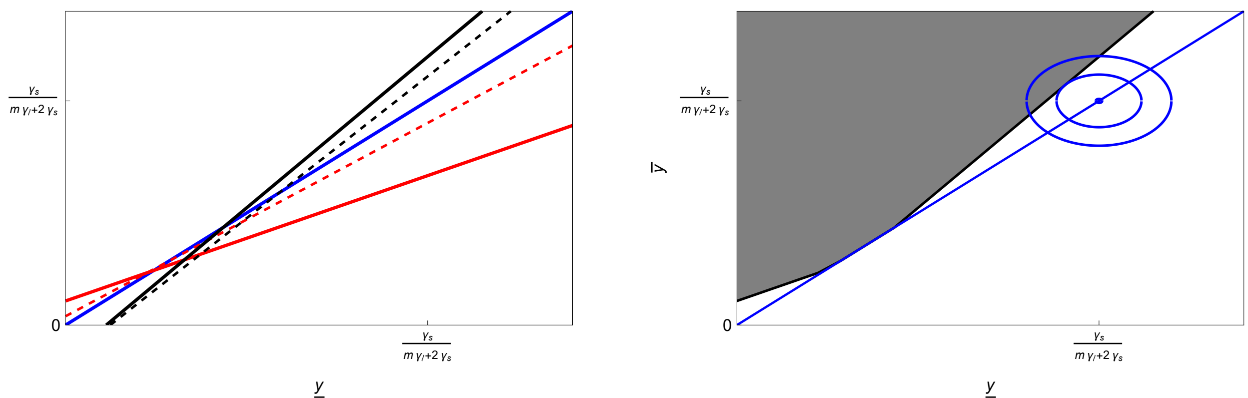

Figure 1: The left panel describes the set of portfolios that are incentive-compatible with cutting costs in $n = 2$ out of $N = 4$ firms in a numerical example of the Cournot model. The black and red lines describe the values of $(\overline{y}, \underline{y})$ for which ℓ is indifferent between doing governance in n or $\tilde{n}$ firms, for $\tilde{n} \in \{0, 1, 3, 4\}$, and the 45-degree line reflects the constraint $\overline{y} \geq \underline{y}$. The set of incentive-compatible portfolios is the convex set where all the constraints are simultaneously satisfied, that is, the shaded gray area in the right panel. The right panel shows that the optimal incentive-compatible portfolio (the one reaching the highest indifference curve within the constrained set) is tilted, that is, features $\overline{y} > \underline{y}$. Parameters: $A = 15$, $m = 2$, $c' = 0$, $\kappa = 0.1$; all other parameters are set to 1.

The right panel displays ℓ's expected utility at the optimal portfolios that are incentive-compatible with exerting governance in $n \in \{0, 1, 2\}$ firms. For a given κ, ℓ chooses the portfolio that yields the highest utility, so her expected utility in equilibrium is the upper envelope of U_ℓ. An increase in the governance cost reduces U_ℓ if ℓ exerts governance in at least one firm. Since the reduction scales with n, ℓ optimally transitions from $n = 2$ to $n = 1$ for $\kappa \approx 0.04$. An increase in κ decreases the distortion in $y_{j\ell}$ that is needed for ℓ to stay passive ($n = 0$). Therefore, U_ℓ always *increases* with κ at this portfolio. For $\kappa \approx 0.14$, then it becomes optimal for ℓ to remain passive.

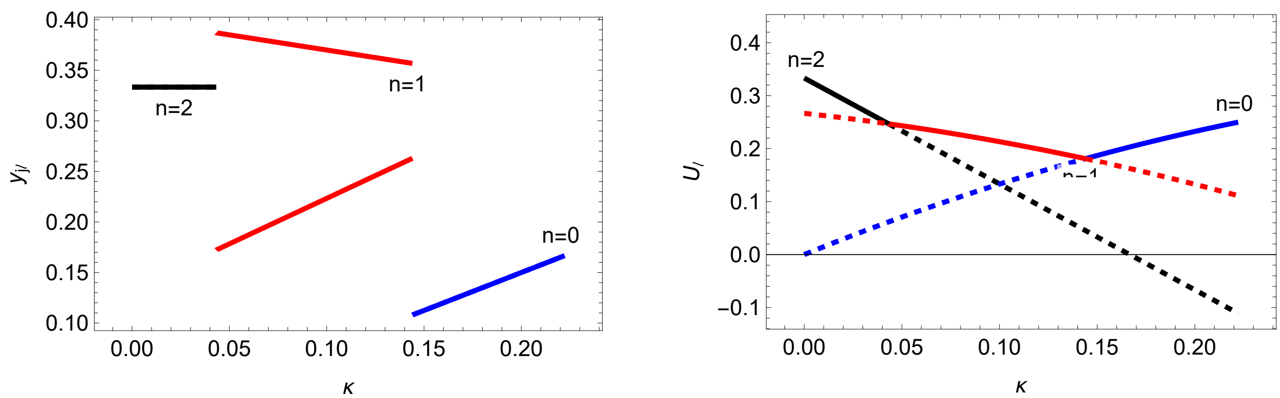

Figure 2: The left panel plots ℓ's equilibrium portfolio and governance choices as a function of the governance cost κ in a numerical example of the Bertrand model with $N = 2$ firms. For $n = 2$ and $n = 0$, ℓ holds the same position in both firms, so there is only one line. The right panel describes ℓ's expected utility at the optimal incentive-compatible portfolios for each value of n. A description of how ℓ transitions from one portfolio to another in equilibrium is given in the text. Parameters: $\pi(1,1) = \frac{4}{3}$; all other parameters are set to 1.

4 Implications

We now analyze the effect of varying some of the key parameters of the model. We first look at their impact on the equilibrium portfolio and governance decisions, and then explore their implications for welfare and product market surplus in Section 5. We discuss some testable predictions of these implications in our concluding remarks in Section 6.

4.1 Market power

We begin by studying how the market power of firms and investors relate to each other: how firms' ability to raise prices above costs reflects ℓ's ability to generate returns, and vice versa.

First, we analyze how ℓ's market power in the equity market affects the equilibrium outcomes. We follow the literature (e.g., Kyle 1985; Kacperczyk, Nosal, and Sundaresan 2024) and measure ℓ's market power by the impact of her trades on equilibrium returns $\pi_j - p_j$.[24] We have

$$\pi_j - p_j = \frac{\gamma_s}{m}\sigma^2(1 - y_{j\ell}) \tag{9}$$

in equilibrium, so the absolute value of the return impact is $\frac{\gamma_s}{m}\sigma^2$. Since the volatility of firm value (σ^2) also affects ℓ's payoffs through her risk exposure, we use the risk-adjusted measure $\lambda \equiv \frac{\gamma_s}{m}$.[25]

[24]Firm value π_j is typically exogenous in the literature, so the impact on returns is the same as the price impact in these papers. In our model, π_j is endogenous (and depends on $\mathbf{y}_\ell$ through the incentive-compatible governance choice), so here we use the "return" impact to capture market power, similar to Back, Collin-Dufresne, Fos, Li, and Ljungqvist (2018)

[25]γ_s and m only enter ℓ's equilibrium payoffs (the objective in Program 6) through their ratio $\frac{\gamma_s}{m}$, so the risk-adjusted measure is easier to work with. In terms of λ, the unconstrained optimal portfolio can be written as $y^u = \frac{\lambda}{\gamma_\ell + 2\lambda}$.

λ increases when the mass of small shareholders m decreases. Since small and large investors compete to buy a fixed supply of firms' shares, a decrease in m can be used to capture an increase in ownership concentration, similarly to Kacperczyk, Nosal, and Sundaresan (2024) and the dominant firm models in industrial organization (see, e.g., Bain 1956; Worcester Jr 1957; Stigler 1965; Tirole 1988): When m is smaller, the aggregate demand from small investors is lower and, thus, all else equal, ℓ owns a larger fraction of total shares. Also, since $y_{j\ell}$ is a larger fraction of total demand, the equilibrium returns become more sensitive to it (λ increases). In Online Appendix C.1, we consider a model with multiple large investors, and show that a decrease in the number of large investors leads to similar results as a decrease in m.

Proposition 5 describes how ℓ's portfolio and governance choices change with λ.

Proposition 5 (Equity market concentration) *The large investor (ℓ) exerts the most governance when her trades have the largest impact on returns, that is, when λ is large, and the least when λ is small. That is, if $\lim_{\lambda\to\infty} \overline{n} \equiv \overline{n}_\lambda > 0$, then there exist two thresholds $\overline{\lambda} \geq \underline{\lambda}$ for λ such that, in equilibrium, ℓ:*

1. *does not exert governance and holds the same position in all firms iff $\lambda \leq \underline{\lambda}$;*
2. *holds the unconstrained optimal portfolio and exerts governance in $\overline{n}_\lambda$ firms if $\lambda > \overline{\lambda}$;*
3. *may hold an unbalanced portfolio and always exerts governance in $n \leq \overline{n}_\lambda$ firms if $\lambda \in (\underline{\lambda}, \overline{\lambda}]$.*

If the volatility of firm value is small ($\sigma^2 < \frac{2\kappa}{\gamma_\ell N}$), the statements above read as "iff $\lambda > \overline{\lambda}$" and "iff $\lambda \in (\underline{\lambda}, \overline{\lambda}]$." If $\overline{n}_\lambda = 0$, ℓ always holds the unconstrained optimal portfolio and never exerts governance.

When λ is small, the equity market is very competitive, and stock prices are close to their fair value.[26] ℓ can then make little returns from buying stocks, so she holds a small, balanced portfolio and stays passive. As the market becomes less competitive, ℓ buys more stocks, but may still hold large positions only in some firms, as a commitment to do less governance and save on the governance costs. As λ continues to increase, however, the potential for returns increases, and deviations from a balanced portfolio (which maximizes trading profits) become

[26]It is worth emphasizing that the link between price impact and trading profits is different in our model compared to the Kyle (1985) type of models. In these models, the strategic investor (ℓ) has private information, and a competitive market maker sets the share price p_j. A higher λ means the market maker can better identify ℓ's trades and, thus, set p_j closer to the fair value. So, ℓ's trading profits are lower when λ is larger. In our model, there is no private information, but p_j adjusts to equate demand and (fixed) supply of shares. ℓ has more price impact when $y_{j\ell}$ is a larger fraction of total demand, which is when small investors' aggregate demand my_S is smaller. When my_S is small, there is also less price pressure, so p_j is smaller and, thus, ℓ's trading profits are *larger* when λ is larger.

increasingly more costly. When λ is sufficiently large, ℓ is too concerned with returns to worry about governance costs. So she always holds a large, *balanced* portfolio in this case, which aligns her incentives with the industry profits and induces the most governance. Corollary 1 explores the product market implications.[27]

Corollary 1 (Equity market concentration in the Bertrand model) *Consider the setting in Example 1 and suppose* $\lim_{\lambda \to \infty} \overline{n} = N$*; there always exist two thresholds* $\overline{\lambda}^b \geq \underline{\lambda}^b$ *such that, in equilibrium,* ℓ*:*

1. *does not exert governance and holds the same position in all firms iff* $\lambda \leq \underline{\lambda}^b$*;*
2. *cuts costs only in firm* 1 *and holds a larger position in this firm iff* $\underline{\lambda}^b < \lambda \leq \overline{\lambda}^b$*;*
3. *facilitates collusion among firms and holds the unconstrained optimal portfolio iff* $\lambda > \overline{\lambda}^b$*.*

In the Bertrand model, ℓ does not exert governance if λ is small, pushes to reduce a firm's production costs for intermediate values of λ, and facilitates collusion when λ is large. So, the investor becomes increasingly more likely to build a large, balanced portfolio and facilitate collusion in the product market when she has more market power in the equity market.

A natural counterpart to the results in Proposition 5 is what happens to the equity market when product markets become less competitive. A number of recent papers have shown that average markups and market concentration have significantly increased over the last four decades (De Loecker, Eeckhout, and Unger 2020; Autor, Dorn, Katz, Patterson, and Van Reenen 2020). In our model, we can capture an exogenous increase in markups and concentration in product markets as a decrease in the number of active firms N in the Cournot model.[28]

Proposition 6 (Product market concentration) *A decrease in the number of firms* N *can increase the large investor's ownership of the industry (*$\frac{\sum_j y_{j\ell}}{N}$*) and her willingness to facilitate collusion.*

To build intuition, suppose the only governance type available to ℓ is to facilitate collusion, so she chooses between doing that or staying passive. A lower N has two contrasting effects on ℓ's choice. On the one hand, softening competition becomes less tempting, since firms compete less to begin with. This first effect makes it more likely that ℓ prefers to stay passive in equilibrium.

[27]The proofs for Corollaries 1 and 2 can be found in the Online Appendices B.2 and B.5, respectively.

[28]In the Bertrand model, firms price at the marginal cost whenever (a) they are not colluding and (b) at least two firms have the same costs, so N is not a clear measure of competition (Bertrand 1883). For this reason, we focus on the Cournot model for this result.

On the other hand, softening competition also becomes cheaper, as there are now fewer firms to coordinate and pressure to collude. This second effect makes the strategy of building a diversified portfolio and facilitating collusion more attractive. If the latter effect dominates, ℓ is *more* likely to influence competition in industries that are *less* competitive to begin with.

Figures 3 and 4 illustrate, respectively, the results in Propositions 5 and 6 in two numerical examples of the model. Putting the results in this section together yields an important implication. Through their effects on the portfolio and governance decisions of large investors, exogenous shifts in the competitiveness of equity markets spill over to product markets, and vice versa, potentially amplifying their effects in the process. This formalizes a link between the trends in ownership consolidation and rising markups we discussed above. It also suggests that the impact of each trend extends beyond the market in which it originates, leading to broader effects on the overall economy. We return to these points in our analysis of welfare and regulation in Section 5.

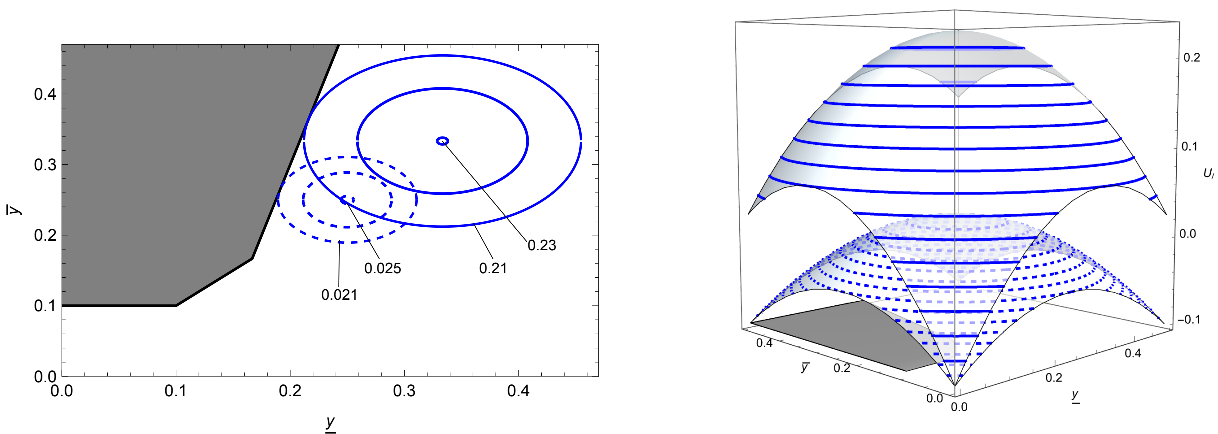

Figure 3: The shaded-gray areas in both panels represent the set of portfolios that are incentive-compatible with cutting costs in one out of $N = 2$ firms in a numerical example of the Bertrand model. The solid (dashed) blue ellipses represent iso-utility curves for $\lambda = 1$ ($\lambda = 0.5$). The difference in ℓ's utility at the unconstrained optimal portfolio and the optimal incentive-compatible one is larger when $\lambda = 1$, since U_ℓ is more sensitive to changes in $(\overline{y}, \underline{y})$ when λ is larger (it drops faster as one moves away from the unconstrained optimal portfolio). Parameters: $\pi(1,2) = 0.8$, $\kappa = 0.1$; all other parameters are set to 1.

4.2 Shareholder engagement

Shareholder engagement has become increasingly important and widespread in recent years, especially among large investors.[29] In our model, shareholder engagement occurs when ℓ

[29] In a survey of institutional investors, McCahery, Sautner, and Starks (2016) found that 63% of respondents had engaged in discussions with top management over the previous five years. BlackRock's Investment Stewardship Annual Report reports that in 2020, the company "had over 3,000 in-depth conversations with corporate leadership." Similar figures apply to Vanguard and State Street.

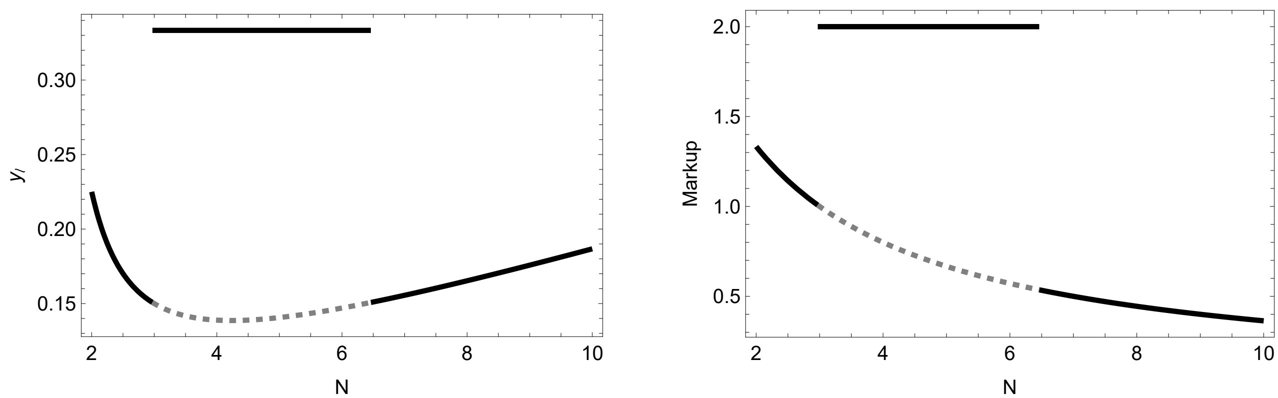

Figure 4: The left (right) panel plots ℓ's position in each firm (equilibrium markups) as a function of the number of firms N in a numerical example of the Cournot model. To ease the exposition, we assume ℓ chooses between facilitating collusion ($n = N$) and not exerting any governance ($n = 0$), and neglect the integer constraint on N. ℓ holds the unconstrained optimal portfolio and facilitates collusion only for intermediate values of N, since facilitating collusion has little value when N is small (as markups are already high), and is too costly when N is large. Parameters: $A = 5$, $\kappa = 0.05$; all other parameters are set to 1.

exerts either type of governance. This is more likely to occur when ℓ has more market power (Proposition 5) or the governance cost decreases. Proposition 7 studies the latter case.

Proposition 7 (Shareholder engagement) *The large investor (ℓ) exerts the most governance when the governance cost κ is small, and the least when κ is large. That is, if $\lim_{\kappa \to 0} \overline{n} \equiv \overline{n}_\kappa > 0$, then there exist two thresholds $\overline{\kappa} \geq \underline{\kappa}$ for κ such that, in equilibrium, ℓ:*

1. *holds the unconstrained optimal portfolio and exerts governance in $\overline{n}_\kappa$ firms iff $\kappa < \underline{\kappa}$;*
2. *does not exert governance and holds the same position in all firms if $\kappa \geq \overline{\kappa}$;*
3. *may hold an unbalanced portfolio and exerts governance in $n \leq \overline{n}_\kappa$ firms if $\kappa \in [\underline{\kappa}, \overline{\kappa})$.*

If $\overline{n}_\kappa = 0$, ℓ always holds the unconstrained optimal portfolio and never exerts governance.

When κ is large, doing governance is too costly and, thus, never incentive-compatible. So ℓ can hold the unconstrained optimal portfolio without having to worry about governance. As κ becomes smaller, some governance actions become incentive-compatible. Like before, here ℓ may choose to hold larger positions in some firms and do governance only in those, to save on the governance costs. As κ continues to decrease and governance becomes cheaper, the investor becomes increasingly less concerned with minimizing governance costs and more concerned with maximizing returns. So when κ is small enough, she switches to holding a large balanced portfolio, which aligns her incentives with the industry profits and induces the most governance. Corollary 2 explores the product market implications of these dynamics.

Corollary 2 (Shareholder engagement in Bertrand model) *Consider the setting in Example 1 and suppose* $\lim_{\kappa \to 0} \overline{n} = N$*; there always exist two thresholds* $\overline{\kappa}^b \geq \underline{\kappa}^b$ *such that, in equilibrium,* ℓ*:*

1. *does not exert governance and holds the same position in all firms if* $\kappa \geq \overline{\kappa}^b$*;*

2. *cuts costs only in firm* 1*, and holds a larger position in this firm if* $\underline{\kappa}^b \leq \kappa < \overline{\kappa}^b$*;*

3. *holds the unconstrained optimal portfolio and facilitates collusion among firms if* $\kappa < \underline{\kappa}^b$*.*

ℓ does not do governance if κ is large, pushes to reduce a firm's production costs for intermediate values of κ, and pressures firms to compete less aggressively when κ is small. Overall, our results suggest that the recent trends of increased engagement and concentration in the asset management industry may have ambiguous effects on product markets and welfare, depending on which type of governance they stimulate. They are also increasingly more likely to stimulate the socially undesirable type (softening of product market competition) if they continue in the long run.

4.3 Competition and traditional governance

There is a lively debate about the extent to which large investors can influence competition. The idea that they monitor and try to reduce managerial slack is less controversial. Proposition 8 explores how investing in competing firms affects this more traditional type of governance.

Proposition 8 (Complements vs. substitutes) *Consider only governance actions with negative spillovers (e.g., cost-cutting):*

1. *Compared to the benchmark without competition, the large investor may exert more or less governance in the equilibrium of the model with competing firms;*

2. *In the limit where competition is a zero-sum game* ($\Pi(\tilde{n}) - \Pi(0) \to 0$)*,* ℓ *is always passive only in the model with competing firms.*

If the negative spillovers are larger in more competitive industries, a diversified investor (who internalizes the spillovers as much as the direct effects) has less incentive to do governance in such industries. So, it is easier for ℓ to build a relatively large, balanced portfolio without having to exert any governance. On the other hand, large negative spillovers also allow ℓ to build large positions in all firms while doing governance *only* in a subset, provided that the investor is willing to tilt her portfolio toward the firms where she exerts governance. Depending on which of these two options – the balanced or tilted portfolios – becomes more attractive when competition increases, governance and competition can be either *substitutes* or *complements* in our model, respectively.

To build intuition for the results described above, it is useful to look at the IC constraint for a passive portfolio, that is, one that induces $n = 0$. Since the optimal passive portfolio is balanced (that is, $\overline{y} = \underline{y}$), we can write the IC constraint as:

$$N\underline{y}\pi(0,0) \geq \underline{y}\left[\tilde{n}\pi(1,\tilde{n}) + (N-\tilde{n})\pi(0,\tilde{n})\right] - \tilde{n}\kappa, \tag{10}$$

for any $\tilde{n} \in (0, \overline{n}]$. Using the definition of industry profits $\Pi(n)$, we can rewrite Inequality (10) as:

$$\underbrace{\tilde{n}\kappa}_{\text{Governance cost}} \geq \underline{y} \underbrace{\left[\Pi(\tilde{n}) - \Pi(0)\right]}_{\text{Effect of governance on }\Pi}. \tag{11}$$

Since the optimal passive portfolio is balanced, ℓ's payoff is aligned with the industry profits also in this case. For a given stake $\underline{y}$, the IC constraint then boils down to whether cutting costs in $\tilde{n}$ firms increases industry profits enough compared to the governance cost $\tilde{n}\kappa$. So ℓ internalizes the aggregate effects of governance, which includes the negative spillovers to the other $N - \tilde{n}$ firms.[30] In the extreme case where product market competition is a zero-sum game (that is, the direct positive effect of doing governance in firms $j \leq \tilde{n}$ is fully offset by the negative spillover on the other firms), $\Pi(\tilde{n}) - \Pi(0)$ is zero. In this case, the IC constraint in Inequality (10) is always satisfied for any $\underline{y}$, so ℓ always chooses to be passive, even when κ is arbitrarily close to zero.

In less extreme environments (where competition is *not* a zero-sum game), ℓ may do more or less governance in more competitive industries. Figure 5 illustrates the sets of portfolios that are incentive-compatible with different governance choices ($n = 0$ and $n = 1$) as the intensity of competition changes in a numerical example of the Cournot model. Competition and governance are *substitutes* if ℓ prefers the optimal portfolio with $n = 0$ when competition is high, and $n = 1$ when it is low. They are instead *complements* in the opposite case.

[30]The relevant (that is, most stringent) IC constraint here is the one for $\tilde{n} = \arg\min_{n' \in \mathcal{N}_0^+} \frac{\kappa n'}{\Pi(n') - \Pi(0)}$, which yields the expression for the optimal passive portfolio in Proposition 4

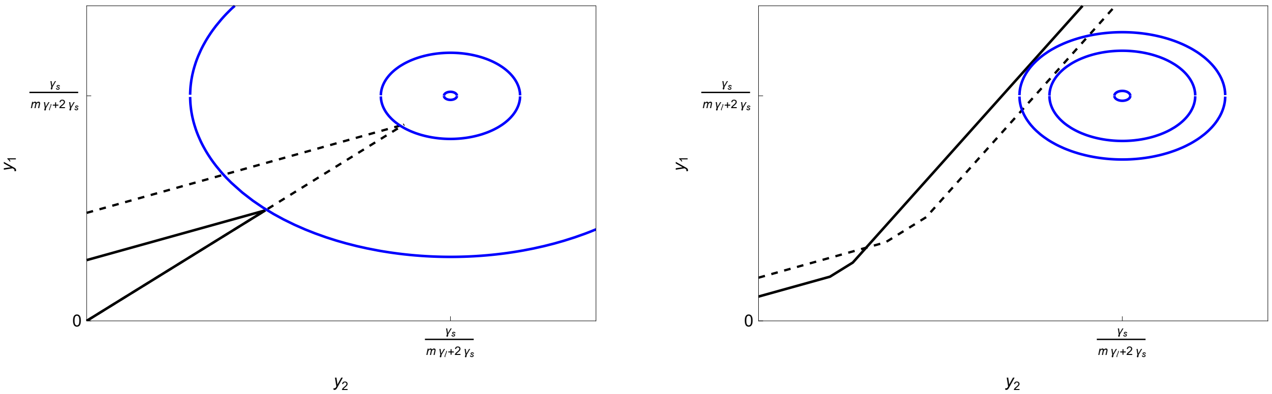

Figure 5: The left (right) panel plots the set of portfolios that are incentive-compatible with $n = 0$ ($n = 1$) in a numerical example of the Cournot model with $N = 2$ firms. The solid (dashed) black lines are obtained setting $b = 1.5$ ($b = 2$), where b is the slope of the demand function in the product market. Competition is more intense when b is larger. The governance spillovers are larger in more competitive industries, so both IC sets increase with b, which allows ℓ to build larger positions and reach higher iso-utility curves (the blue ellipses) in both panels. Parameters: $A = 15, m = 2, c' = 0$; in the left (right) panel we set $\kappa = 0.20$ ($\kappa = 0.08$); all other parameters are set to 1.

5 Regulating horizontal shareholding

This section explores some normative implications of our results. First, we study how the equilibrium choices differ from those of a social planner to shed light on the equilibrium inefficiencies. Second, we use the model to evaluate policies to alleviate some of these inefficiencies.

5.1 Planner's solution

We consider a setting in which a social planner chooses $\{\mathbf{y}_\ell, \mathbf{g}\}$ to maximize total surplus. The planner's and ℓ's choice problems differ in three ways. First, the planner internalizes the benefits of sharing risk between large and small investors, while each individual investor only cares about their own risk exposure. Second, in addition to the impact of governance on firm profits, the planner also internalizes its impact on consumers. Third, we assume that the planner can commit to any governance policy $\mathbf{g}$, while ℓ's governance must be incentive-compatible with her portfolio.

We define product market surplus PMS as the sum of industry profits Π and consumer surplus CS. Like Π, PMS only depends on n, i.e., the number of firms where ℓ exerts governance. Next, we distinguish between governance actions based on their impact on PMS.

Definition 1 *A governance vector* $\mathbf{g}$ *with* $\boldsymbol{\iota}'\mathbf{g} = n$ *is socially desirable if it increases product market surplus net of the governance cost, that is, if* $PMS(n) - \kappa n \geq PMS(0)$*, and socially undesirable otherwise.*

The planner chooses ℓ's portfolio $\mathbf{y}_\ell$ and governance efforts $\mathbf{g}$ to maximize total surplus:

$$\max_{\mathbf{y}_\ell,\mathbf{g}} \mathcal{S} = PMS(n) - \kappa n - \frac{\sigma^2}{2}\left(\gamma_\ell \mathbf{y}_\ell' \mathbf{y}_\ell + m\gamma_S \mathbf{y}_S' \mathbf{y}_S\right), \tag{12}$$

where the small investors' portfolio holdings follow from market clearing: $\mathbf{y}_S = \frac{1}{m}(\boldsymbol{\iota} - \mathbf{y}_\ell)$.

Total surplus consists of three components: product market surplus, ℓ's governance cost, and the degree of risk sharing between ℓ and the mass m of small investors S.[31] Proposition 9 formally characterizes the solution to the social planner's problem.

Proposition 9 (Planner's solution) *The social planner sets* $\mathbf{y}_\ell^{SP} = \frac{\gamma_S}{m\gamma_\ell+\gamma_S}\boldsymbol{\iota}$. *The socially optimal governance vector is given by* $\mathbf{g}^{SP} = \arg\max_{\mathbf{g}} PMS(n) - \kappa n$ *where* $n = \boldsymbol{\iota}'\mathbf{g}$.

The planner can separate the choices of $\mathbf{y}_\ell$ and $\mathbf{g}$. So, she sets $\mathbf{y}_\ell^{SP} = \frac{\gamma_S}{m\gamma_\ell+\gamma_S}\boldsymbol{\iota}$ to efficiently share risk between ℓ and S, and $\mathbf{g}^{SP}$ to maximize the product market surplus net of the governance cost.

5.2 Market failures

In a next step, we compare the planner's solution to the main equilibrium in Section 3.3.

Proposition 10 (Sources of inefficiency) *In equilibrium, the large investor* ℓ *never chooses the pair* $(\mathbf{y}_\ell, \mathbf{g})$ *that solves the social planner's problem. Furthermore, (a)* ℓ *may exert socially undesirable governance (Definition 1), and (b) for a fix* $\mathbf{g}$*, she always trades too little.*

Proposition 10 describes two types of equilibrium inefficiencies. The first inefficiency relates to ℓ's governance choice, and arises because ℓ cares about the effects of governance on the expected profits of the firms in her portfolio (through her objective function at time $t = 2$), while the planner cares about the effects of governance on PMS. This means that ℓ may engage in socially-undesirable governance, while the planner never wants that type of governance. The second inefficiency relates to ℓ's portfolio choice, and arises because ℓ considers her price impact when choosing $\mathbf{y}_\ell$. For a fixed governance choice $\mathbf{g}$, ℓ always trades too little (to mitigate her price impact) and exposes small investors to too much risk, compared to the planner's solution.[32]

[31] The expression for $\mathcal{S}$ does not include the share prices $\mathbf{p}$, since they represent transfers from the investors to the original firms' owners (who supply the shares that are traded in the model), and so cancel out when we consider total surplus.

[32] When we allow $\mathbf{g}$ to change with the investor's portfolio $\mathbf{y}_\ell$, whether ℓ trades too little or too much depends on the effect of $\mathbf{y}_\ell$ on $\mathbf{g}$, through the IC constraint. For example, when ℓ holds a large, diversified portfolio and facilitates collusion, total surplus may be larger if ℓ were to hold smaller positions and stay passive, if the gains in PMS exceed the losses in risk-sharing.

In the analysis above, we have focused on the inefficiencies that directly stem from ℓ's behavior. If firms have, *ex-ante*, some degree of market power, that also translates into lower PMS and overall welfare. A natural question is, then, when large investors do engage in socially desirable governance, do they help promote competition among firms? If they did, they would *indirectly* contribute to increase welfare by reducing the deadweight losses associated with the exercise of firms' market power. Lemma 2 shows that, under some mild conditions on the industry profits $\Pi(n)$, ℓ never engages in governance actions that reduce $\Pi(n)$ in equilibrium. So, although ℓ may choose not to soften competition across firms, she would also never actively promote it.

Lemma 2 (Not promoting competition) *Suppose industry profits $\Pi(n)$ are concave in n. In equilibrium, the large investor never exerts governance that reduces industry profits, that is, such that $\Pi(n^*) < \Pi(0)$.*

5.3 Policy intervention

The growth of horizontal shareholding in the last four decades (Backus, Conlon, and Sinkinson 2021) and its potential anticompetitive effects have caught the attention of policymakers and antitrust scholars. Next, we use our model to evaluate some of the most commonly discussed policies related to this issue. Definition 2 introduces the policy interventions we will consider.

Definition 2 (Types of regulations) *We consider the following types of policy interventions:*

1. ***Limiting engagement:*** *this regulation increases the governance cost κ.*
2. ***Limiting horizontal shareholding:*** *this regulation limits the investors' ability to build large stakes in all firms, that is, given positions $y_{1\ell} \geq y_{2\ell} \geq ... \geq y_{N\ell}$, if $y_{1\ell} > \overline{y}^r$, then $y_{j\ell} \leq \underline{y}^r$ for all $j > 1$.*
3. ***Traditional competition policy:*** *this regulation decreases a firm's profit (e.g., by imposing fines) if its product price is above the competitive equilibrium level.*

The first policy intervention makes it more costly for ℓ to engage in governance and, therefore, discourages her from doing so. The second intervention takes a more *indirect* approach, limiting ℓ's positions in competing firms. The third intervention represents a more traditional anti-trust policy targeted at firms, instead of investors. The following Proposition evaluates these policy interventions and discusses their impact on the equilibrium outcomes.

Proposition 11 (Analysis of regulation) *Consider the setting with Bertrand competition (Example 1), where facilitating collusion is always socially undesirable, and cutting costs is always socially desirable when it is selected in equilibrium. The following results hold in equilibrium:*

1. *Limiting engagement always decreases socially-undesirable governance, but may also decrease socially-desirable governance and ℓ's utility.*
2. *Limiting horizontal shareholding may increase or decrease both socially-undesirable and socially-desirable governance, and always reduces ℓ's utility.*
3. *Traditional competition policy always increases socially-desirable governance, decreases socially-undesirable governance, and increases ℓ's utility.*

Limiting engagement always discourages ℓ from exerting socially-undesirable governance, but it might also crowd out socially-desirable governance. The net effect on ℓ's expected utility is ambiguous. On the one hand, there is a negative direct effect if ℓ continues to choose the same n in equilibrium after the increase in κ. On the other hand, exerting *less* governance becomes incentive-compatible for a larger set of portfolios. So, after the increase in κ, ℓ may switch to a lower value of n and, thus, incur a *lower* governance cost, or reach a better risk-adjusted return.

Limiting horizontal shareholdings always makes ℓ worse off by shrinking the set of feasible portfolios, and it may also *increase* socially-undesirable or decrease socially-desirable governance. This is the case when the policy is such that ℓ can take a large position in a firm *only if* she owns minimal positions in all other firms. In this case, the cost-cutting portfolio becomes less attractive, and ℓ might switch to a smaller but balanced portfolio, which may still be incentive-compatible with socially-undesirable governance. Finally, traditional competition policy indirectly increases ℓ's cost of influencing competition, discouraging socially-undesirable governance. As ℓ moves away from influencing competition, she is relatively more likely to engage in socially-desirable governance under this policy. Perhaps surprisingly, ℓ always benefits from this intervention: since facilitating collusion is now less attractive at time $t = 2$, ℓ can build larger and more balanced portfolios without worrying about the governance costs.

6 Conclusions

This paper explored the choices of an investor who builds positions and exerts governance in competing firms. We highlighted a novel trade-off related to the investor's governance efforts. On the one hand, doing traditional governance (such as cutting managerial slack and improving firm efficiency) requires the investor to sacrifice diversification benefits, since her incentives to exert this type of governance are low when she has similar exposure to all firms in an industry. On the other hand, holding a large diversified portfolio increases the investor's incentives to pressure firms to compete less aggressively, increasing her exposure to regulation and reputational costs.

We study the implications of these trade-offs for the equilibrium interactions of ownership, governance, and competition. We show that they create a channel for market power and concentration to spread *across* markets, that is, from equity to product markets, and vice-versa. When equity markets become less competitive, in particular, large investors are more likely to hold large, diversified portfolios, and, as a consequence, to try soften competition among firms. Some papers have studied how concentration affects funds' performance (e.g., Pástor, Stambaugh, and Taylor 2015), but we are not aware of any paper studying how concentration shapes investors' portfolio choices. One way to potentially test this prediction would be to use shocks to consolidations in the asset management industry, which have been used for different questions in the literature (e.g., He and Huang 2017; Azar, Schmalz, and Tecu 2018; Massa, Schumacher, and Wang 2021).

We also show that the incentives to improve firm efficiency may be lower in more competitive industries, since the negative spillovers of governance across firms are stronger in such industries, and so this type of governance requires larger deviations from a diversified portfolio. This *crowding out* of traditional governance may explain why bad management practices and poor governance are surprisingly persistent even in more competitive sectors (Bloom and Van Reenen 2007), and provides new insights into the role of competition in promoting efficiency.

Another related insight is that investors may be more willing to build large, diversified portfolios in less competitive industries, even when they *cannot* influence competition. This generates a *spurious* correlation between competition and horizontal shareholding, which should be considered when interpreting correlations between these two variables in the data. Finally, we use the model to evaluate different policy responses to the large growth in horizontal shareholding and, more generally, to explore the role of competition policy in a world where investors influence the objectives of competing firms.

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A Proofs

A.1 Proof of Lemma 1.

For a given ownership vector $\mathbf{y}_\ell = (y_{1\ell}, \ldots, y_{N\ell}\}$ with $y_{1\ell} \geq \ldots \geq y_{N\ell}$, ℓ's objective function in Program (2) is a real-valued function for all $\mathbf{g} \in \{0,1\}^N$. As a result, there always exists a maximum. Note that $y^u\boldsymbol{\iota}$ maximizes ℓ's expected utility for a fixed governance choice n. Suppose there exists a portfolio vector $\mathbf{y}_\ell$ that is incentive-compatible with $n > \overline{n}$. ℓ would be strictly better off choosing $\mathbf{y}_\ell = y^u\boldsymbol{\iota}$ and $n^* = \overline{n}$, since that gives her a higher certainty equivalent and lower governance cost. If ℓ holds the unconstrained optimal portfolio $\mathbf{y}_\ell = y^u\boldsymbol{\iota}$, she will choose $n^* = \overline{n}$ because $\overline{n} \equiv \arg\max_n\{y^u\boldsymbol{\iota}'\boldsymbol{\pi} - n\kappa\}$. Similarly, if she chooses $n^* = \overline{n}$, she will hold the unconstrained optimal portfolio because it maximizes ℓ's expected utility for a given n.

A.2 Proof of Proposition 1.

Here, ℓ's problem collapses into a firm-by-firm problem, so she holds the same position (y_ℓ) and exerts the same governance (g) in all firms. If $g = 0$ is incentive-compatible at $y_\ell = y^u$, i.e., if $\kappa \geq y^u [\pi^{nc}(1) - \pi^{nc}(0)]$, ℓ chooses $y_\ell^* = y^u$ and $g^* = 0$ because y^u maximizes the portfolio return and $g = 0$ minimizes governance costs. Otherwise, ℓ chooses between the best portfolio that is incentive-compatible with $g = 1$ and that incentive-compatible with $g = 0$. The first portfolio is equal to $y_\ell = y^u$ and leads to an objective function (per-firm) equal to $\sigma^2 \left[\frac{\gamma_S}{m} y_\ell(1-y_\ell) - \frac{\gamma_\ell}{2} y_\ell^2\right] - \kappa$. The second portfolio maximizes $\sigma^2 \left[\frac{\gamma_S}{m} y_\ell(1-y_\ell) - \frac{\gamma_\ell}{2} y_\ell^2\right]$ subject to the incentive-compatibility constraint $y_\ell \pi^{nc}(0) \geq y_\ell \pi^{nc}(1) - \kappa$. Note that ℓ's objective function is concave in y_ℓ. So, $y_\ell \leq \frac{\kappa}{\pi^{nc}(1)-\pi^{nc}(0)} < y^u$ implies $\frac{dU_\ell}{dy_\ell} > 0$. The optimal passive portfolio is thus equal to $\frac{\kappa}{\pi^{nc}(1)-\pi^{nc}(0)}$. Therefore, ℓ chooses $y_\ell^* = y^u$ and $g^* = 1$ iff:

$$\frac{\gamma_S}{m} y^u \left(1 - y^u\right) - \frac{\gamma_\ell}{2} \left(y^u\right)^2 - \frac{\kappa}{\sigma^2} > \frac{\gamma_S}{m} \frac{\kappa}{\pi^{nc}(1) - \pi^{nc}(0)} \left(1 - \frac{\kappa}{\pi^{nc}(1) - \pi^{nc}(0)}\right) - \frac{\gamma_\ell}{2} \left(\frac{\kappa}{\pi^{nc}(1) - \pi^{nc}(0)}\right)^2 \tag{A.1}$$

The LHS of Inequality (A.1) decreases in κ. Let the RHS of the inequality be $U_\ell^{g=0}$, which satisfies $\frac{dU_\ell^{g=0}}{d\kappa} = \frac{dU_\ell^{g=0}}{dy_\ell} \frac{dy_\ell}{d\kappa} > 0$, since U_ℓ increases in y_ℓ for any $y_\ell < y^u$ (as shown above), and $y_\ell = \frac{\kappa}{\pi^{nc}(1)-\pi^{nc}(0)}$ increases with κ. Inequality (A.1) holds at $\kappa = 0$, but it does not hold at $\kappa = y^u [\pi^{nc}(1) - \pi^{nc}(0)]$. Hence, Inequality (A.1) holds, i.e., ℓ chooses $y_\ell^* = y^u$ and $g^* = 1$, iff $\kappa < \underline{\kappa}^{nc}$, with:

$$\frac{\gamma_S}{m} y^u \left(1 - y^u\right) - \frac{\gamma_\ell}{2} \left(y^u\right)^2 - \frac{\underline{\kappa}^{nc}}{\sigma^2} = \frac{\gamma_S}{m} \frac{\underline{\kappa}^{nc}}{\pi^{nc}(1) - \pi^{nc}(0)} \left(1 - \frac{\underline{\kappa}^{nc}}{\pi^{nc}(1) - \pi^{nc}(0)}\right) - \frac{\gamma_\ell}{2} \left(\frac{\underline{\kappa}^{nc}}{\pi^{nc}(1) - \pi^{nc}(0)}\right)^2 \tag{A.2}$$

A.3 Proof of Proposition 2 and 3.

We prove Proposition 2 and 3 together. We separate them in the text to make the exposition clearer.

Proposition 2. To prove equilibrium existence, we reformulate ℓ's choice problem as a two-step process (similar to Grossman and Hart's approach to the principal-agent problem (Grossman and Hart 1983)). First, for a given $n \in \mathcal{J}_0$ where $\mathcal{J}_0 \equiv \mathcal{J} \cup \{0\}$, ℓ selects the optimal $\mathbf{y}_\ell$ to maximize her objective function subject to N incentive-compatibility (IC) constraints. The IC constraints are linear in $y_{j\ell}$, while the objective function is concave in $y_{j\ell}$. Provided that the constrained set is non-empty, the solution to this first maximization problem is thus unique. In the second step, ℓ chooses among the $N+1$ portfolios that result from the first step to maximize her utility.

The equilibrium exists if the constrained set is non-empty for at least one n. To see that this is satisfied, consider $\overline{n} = \arg\max_n \{y^u \boldsymbol{\iota}' \boldsymbol{\pi} - n\kappa\}$. $n = \overline{n}$ is always incentive compatible for ℓ at $\mathbf{y}_\ell = y^u \boldsymbol{\iota}$. Therefore, an equilibrium must always exist. If the investor obtains the same expected utility at n' and $n'' \neq n'$ with corresponding portfolios $\mathbf{y}'_\ell$ and $\mathbf{y}''_\ell$, we assume she breaks ties in favor of the portfolio that induces less governance. So, the equilibrium is unique.

We continue with the proof of Parts 1 and 2 in Proposition 2. First, note that for a given n, ℓ's expected utility in Program (5) is maximized at $n = 0$ and $\mathbf{y}_\ell = y^u \boldsymbol{\iota}$. So, if staying passive is incentive-compatible at the unconstrained optimal portfolio, ℓ will choose this portfolio. Second, suppose that ℓ's expected utility in Program (5) is maximized at an optimal portfolio vector $\mathbf{y}_\ell$, with $y_{1\ell} \geq \ldots \geq y_{N\ell}$, which is incentive compatible with $n \in \mathcal{J}_0$. We will prove by contradiction that ℓ holds the same positions for all $j \leq n$ and $j > n$, respectively, in this case. Without loss of generality, we order firms based on ℓ's position from large to small. For all alternative governance choices $\tilde{n} \in \mathcal{J}_0$, ℓ's IC constraint is given by:

$$\pi(1,n)\sum_{j=1}^{n} y_{j\ell} + \pi(0,n)\sum_{j=n+1}^{N} y_{j\ell} - n\kappa \geq \pi(1,\tilde{n})\sum_{j=1}^{\tilde{n}} y_{j\ell} + \pi(0,\tilde{n})\sum_{j=\tilde{n}+1}^{N} y_{j\ell} - \tilde{n}\kappa. \tag{A.3}$$

Consider two firms $j' < j'' \leq n$ and suppose $y_{j'\ell}$ is *strictly* greater than $y_{j''\ell}$. Next, we consider a deviation portfolio vector with $y^d_{j'\ell} = y^d_{j''\ell} = \frac{y^d_{j'\ell}+y^d_{j''\ell}}{2}$ which keeps the left-hand side of the constraint above unchanged. For all $\tilde{n} < j'$ and $\tilde{n} > j''$, the left-hand side is unchanged as well. For $j' \leq \tilde{n} \leq j''$, the right-hand side decreases because $y^d_{j'\ell} < y_{j'\ell}$, $y^d_{j''\ell} > y_{j''\ell}$, and $\pi(1,\tilde{n}) > \pi(0,\tilde{n})$. The deviation portfolio then makes ℓ better off because (a) n continues to be incentive-compatible, and (b) ℓ achieves a greater expected utility because, for a given n, her objective function is concave, which implies that $u(y_{j'\ell}) + u(y_{j''\ell}) < 2u\left(\frac{y^d_{j'\ell}+y^d_{j''\ell}}{2}\right)$, where $u(\cdot)$ denotes the per-firm certainty equivalent. We conclude that $y_{j'\ell} = y_{j''\ell} \forall j' < j'' \leq n$. Following the same steps, it follows that for $n < j' < j''$ and $y_{j'\ell} > y_{j''\ell}$, ℓ is better off selecting the deviation portfolio. These two results imply that ℓ always holds the same position in firms where she exerts the same governance action.

Proposition 3. We use the results above in Proposition 3 to re-formulate ℓ's problem. For a fixed $n \in \mathcal{J}_0$, ℓ selects an optimal pair $\{\underline{y}, \overline{y}\}$ to maximize her objective function subject to the N incentive-compatibility constraints and $\overline{y} \geq \underline{y}$.

A.4 Proof of Proposition 4.

Preliminaries. We first show that ℓ's incentive-compatibility (IC) constraint must bind for at least one $\tilde{n} > n$ if $n < \overline{n}$. We define a function $\hat{G}$ (G) that captures ℓ's (net) payoff from exerting governance in n firms for a fixed portfolio vector $\mathbf{y}_\ell$ with elements $y_{j\ell}$:

$$G(n, \mathbf{y}_\ell) = \hat{G}(n, \mathbf{y}_\ell) - \kappa n = \pi(1, n) \sum_{j=1}^{n} y_{j\ell} + \pi(0, n) \sum_{j=n+1}^{N} y_{j\ell} - \kappa n. \tag{A.4}$$

If ℓ chooses n in equilibrium, she must prefer n to all other governance choices $\tilde{n} \in \{0, \ldots, N\}$, i.e., $G(n, \mathbf{y}_\ell) \geq G(\tilde{n}, \mathbf{y}_\ell)$. For the sake of contradiction, suppose the constraint is slack for *all* $\tilde{n} > n$: $G(n, \mathbf{y}_\ell) > G(\tilde{n}, \mathbf{y}_\ell)$. Then the constraint must hold with equality for at least one $\tilde{n} < n$: Otherwise, ℓ faces an unconstrained problem and chooses the unconstrained optimal portfolio, which leads to $n = \overline{n}$. So, we would need to have $G(n, \mathbf{y}_\ell) = G(\tilde{n}, \mathbf{y}_\ell)$ for at least one $\tilde{n} < n$.

ℓ can then choose an arbitrarily close $\mathbf{y}'_\ell$ such that she prefers exerting governance in $\tilde{n} < n$ firms: $G(n, \mathbf{y}'_\ell) < G(\tilde{n}, \mathbf{y}'_\ell)$. Since $G(\cdot, \cdot)$ is continuous in y_ℓ, the constraint continues to be slack for all $\tilde{n} > n$. By continuity of ℓ's certainty-equivalent return, ℓ is strictly better off at deviation portfolio $\mathbf{y}'_\ell$ because it achieves (i) the same certainty equivalent and (ii) a lower governance cost. We conclude that the IC constraint must be binding for at least one $\tilde{n} > n$.

Next, we show that, in equilibrium, the constraint only binds for an $n' > n$ but not for an $n'' < n$. Again, for the sake of contradiction, suppose that the IC constraint is binding for both, n' and n'', i.e., $G(n', \mathbf{y}_\ell) = G(n, \mathbf{y}_\ell) = G(n'', \mathbf{y}_\ell)$. It directly follows from $n' > n > n''$ that $\hat{G}(n', \mathbf{y}_\ell) > \hat{G}(n, \mathbf{y}_\ell) > \hat{G}(n'', \mathbf{y}_\ell)$. We consider a deviation portfolio $\mathbf{y}^d_\ell$, which is identical to $\mathbf{y}_\ell$ except for element $n'' + 1$ which is replaced by $y^d_{n''+1} = y_{n''+1} - \epsilon$ where $\epsilon > 0$ is arbitrarily small. Note that $\frac{dG(n, \mathbf{y}_\ell)}{dy_{n''+1}} = \pi(1, n)$, $\frac{dG(n', \mathbf{y}_\ell)}{dy_{n''+1}} = \pi(1, n')$, and $\frac{dG(n'', \mathbf{y}_\ell)}{dy_{n''+1}} = \pi(0, n'')$. Moreover, we have that

$\pi(0,n'') < \pi(1,n)$ and $\pi(0,n'') < \pi(1,n')$ because:

$$\begin{aligned} & \pi(0,n'')[n\overline{y} + (N-n)\underline{y}] = \pi(0,n'')n''\overline{y} + \pi(0,n'')\left[(n-n'')\overline{y} + (N-n)\underline{y}\right] \\ < \; & \pi(1,n'')n''\overline{y} + \pi(0,n'')\left[(n-n'')\overline{y} + (N-n)\underline{y}\right] = \hat{G}(n'', \mathbf{y}_\ell) < \hat{G}(n, \mathbf{y}_\ell) \\ = \; & \pi(1,n)n\overline{y} + \pi(0,n)(N-n)\underline{y} < \pi(1,n)[n\overline{y} + (N-n)\underline{y}] \end{aligned} \tag{A.5}$$

and

$$\begin{aligned} & \pi(0,n'')[n\overline{y} + (N-n)\underline{y}] = \pi(0,n'')n''\overline{y} + \pi(0,n'')\left[(n-n'')\overline{y} + (N-n)\underline{y}\right] \\ < \; & \pi(1,n'')n''\overline{y} + \pi(0,n'')\left[(n-n'')\overline{y} + (N-n)\underline{y}\right] = \hat{G}(n'', \mathbf{y}_\ell) < \hat{G}(n', \mathbf{y}_\ell) \\ = \; & \pi(1,n')\left[n\overline{y} + (n'-n)\underline{y}\right] + \pi(0,n')(N-n')\underline{y} < \pi(1,n')[n\overline{y} + (N-n)\underline{y}]. \end{aligned} \tag{A.6}$$

The deviation portfolio $\mathbf{y}_\ell^d$ makes n'' strictly preferred at the governance stage: $G(n'', \mathbf{y}_\ell^d) > G(n, \mathbf{y}_\ell^d)$ and $G(n'', \mathbf{y}_\ell^d) > G(n', \mathbf{y}_\ell^d)$. As above, by continuity of ℓ's certainty-equivalent return, ℓ is strictly better off at deviation portfolio $\mathbf{y}_\ell^d$ because it achieves (i) the same certainty equivalent and (ii) a lower governance cost. We conclude that the IC constraint must be binding for at least one $n' > n$ but not simultaneously for $n'' < n$.

Part 1 of Proposition 4. We have shown in the proof of Proposition 2 and 3 that $y_{j\ell} = \underline{y}\ \forall j \in \mathcal{J}$ if $n^* = 0$. The IC constraint at $n^* = 0$ becomes:

$$\underline{y}\Pi(0) \geq \underline{y}\Pi(\tilde{n}) - \kappa\tilde{n} \quad \text{for } \tilde{n} \in \{1, \ldots, \overline{n}\}. \tag{A.7}$$

Solving this inequality for $\underline{y}$ leads to:

$$\max_{\tilde{n} \in \mathcal{N}_0^-} \frac{-\kappa\tilde{n}}{\Pi(0) - \Pi(\tilde{n})} \leq \underline{y} \leq \min_{\tilde{n} \in \mathcal{N}_0^+} \frac{\kappa\tilde{n}}{\Pi(\tilde{n}) - \Pi(0)} \tag{A.8}$$

where the sets $\mathcal{N}_0^-, \mathcal{N}_0^+$ are defined as follows: $\mathcal{N}_0^+ = \{n \leq \overline{n} \mid \Pi(n) > \Pi(0)\}$ and $\mathcal{N}_0^- = \{n \leq \overline{n} \mid \Pi(n) < \Pi(0)\}$. Note that $\mathcal{N}_0^+ \neq \emptyset$ because $\overline{n} > 0$ implies that $\Pi(\overline{n}) > \Pi(0)$. The concavity of U_ℓ implies that $\frac{dU_\ell}{dy_{j\ell}} > 0$ for all $y_{j\ell} < y^u$. Moreover, $\overline{n} > 0$ implies that $\underline{y} < y^u$ so that ℓ optimally chooses $y^* = \min_{\tilde{n} \in \mathcal{N}_0^+} \frac{\kappa\tilde{n}}{\Pi(\tilde{n}) - \Pi(0)}$ in all firms.

Part 2 of Proposition 4. We consider $n^* \in (0, \overline{n})$ and negative spillovers, i.e., $\pi(1,n') - \pi(1,n'') < 0$ for $n' > n''$. The goal is to show that, under these assumptions, ℓ always holds a tilted portfolio with $\overline{y} > \underline{y}$. As above, we use $G(n, \mathbf{y}_\ell)$ to denote ℓ's payoff from holding portfolio $\mathbf{y}_\ell$ and exerting governance in n firms. We also use the result that ℓ's IC constraint always binds for some $\tilde{n} > n^*$.

For the sake of contradiction, suppose that ℓ holds a balanced portfolio, i.e., $\overline{y} = \underline{y}$. It follows from $n^* < \overline{n}$ that $\overline{y}$ and $\underline{y}$ would have to be below the unconstrained optimal portfolio y^u. Below, we will show that a small deviation from this balanced portfolio makes ℓ strictly better off. In particular, we consider the deviation portfolio $\mathbf{y}^d$ with $y_{j\ell} = \overline{y} + \varepsilon$ in firms $j \leq n^*$ and and $y_{j\ell} = \underline{y}$ for all other firms. Let $\epsilon > 0$ be arbitrarily close to zero. Note that the deviation portfolio leads to a strictly higher certainty-equivalent return because $\frac{dU_\ell}{dy_{j\ell}} > 0$ for all $y_{j\ell} < y^u$. Next, we show that this deviation portfolio makes ℓ better off because either n^* remains incentive-compatible or an $\tilde{n} < n^*$ becomes incentive-compatible (which leads to a strictly lower governance cost).

We first focus on $\tilde{n}$ values for which ℓ's constraint is slack. The continuity of the G function with respect to the portfolio vector implies that if $G(n^*, \mathbf{y}_\ell) > G(\tilde{n}, \mathbf{y}_\ell)$, then we also have $G(n^*, \mathbf{y}^d) \geq G(\tilde{n}, \mathbf{y}^d)$. Next, consider $\tilde{n}$ such that $G(n^*, \mathbf{y}_\ell) = G(\tilde{n}, \mathbf{y}_\ell)$, with $\tilde{n} > n^*$. In this case, $G(n^*, \mathbf{y}^d) > G(\tilde{n}, \mathbf{y}^d)$ because Inequality (7) implies that $G(n^*, \mathbf{y}^d) - G(\tilde{n}, \mathbf{y}^d)$ increases with $\overline{y}$.

We conclude that under negative spill-overs, $\overline{n} > 0$, and $n^* \in (0, \overline{n})$, there always exists a profitable deviation from a balanced portfolio so that ℓ holds a tilted portfolio with $\overline{y} > \underline{y}$.

A.5 Proof of Proposition 5.

Preliminaries. Note that $\overline{n} \equiv \arg\max_n\{y^u \boldsymbol{\iota}'\boldsymbol{\pi} - n\kappa\} = \arg\max_n\{\frac{\lambda}{\gamma_\ell + 2\lambda}\boldsymbol{\iota}'\boldsymbol{\pi} - n\kappa\}$ increases with λ, because y^u increases with λ. Since $\overline{n} \leq \overline{n}_\lambda$ and we must have $n^* \leq \overline{n}$ in equilibrium (see Lemma 1), ℓ remains passive for any λ and holds the unconstrained optimal portfolio $y^u\boldsymbol{\iota}$ when $\overline{n}_\lambda = 0$.

Since ℓ never exerts governance when $\overline{n}_\lambda = 0$, the rest of the proof focuses on the case $\overline{n}_\lambda \geq 1$. In this case, since $\overline{n}$ increases with λ, it is incentive-compatible for ℓ to exert governance in $\overline{n}_\lambda$ firms at $y^u\boldsymbol{\iota}$ if λ is sufficiently large. By a similar logic, if λ is sufficiently close to 0, ℓ does not exert any governance at the unconstrained optimal portfolio, that is, $\overline{n} = 0$. In what follows, we use the notation $A \succeq B$ ($A \succ B$) to signify that ℓ weakly (strictly) prefers portfolio A to B.

Part 1 of Proposition 5. The optimal portfolio that is incentive compatible with ℓ not exerting governance in any firm is $y_0\boldsymbol{\iota}$, where $y_0 \equiv \min\{y^u, \frac{\kappa\tilde{n}}{\Pi(\tilde{n}) - \Pi(0)}\}$, $y^u = \frac{\lambda}{\gamma_\ell + 2\lambda}$, $\tilde{n} = \arg\min_{n' \in \mathcal{N}_0^+} \frac{\kappa n'}{\Pi(n') - \Pi(0)}$, and $\mathcal{N}_0^+ = \{n \leq \overline{n} \mid \Pi(n) > \Pi(0)\}$ (see Proposition 4).

$\overline{n}_\lambda \geq 1$ implies that ℓ is willing to do some governance at $\lim_{\lambda\to\infty} y^u$, so for λ sufficiently large, we must have $y^u > \frac{\kappa\tilde{n}}{\Pi(\tilde{n})-\Pi(0)}$. Let $\lambda^* \equiv \frac{\gamma_\ell\kappa\tilde{n}}{\Pi(\tilde{n})-\Pi(0)-2\kappa\tilde{n}} > 0$ denote the value of λ at which y_0 switches from $\frac{\kappa\tilde{n}}{\Pi(\tilde{n})-\Pi(0)}$ to y^u (i.e., λ such that $\frac{\kappa\tilde{n}}{\Pi(\tilde{n})-\Pi(0)} = \frac{\lambda}{\gamma_\ell+2\lambda}$). So, we have $y_0 = y^u$ for $\lambda \leq \lambda^*$, and $y_0 = \frac{\kappa\tilde{n}}{\Pi(\tilde{n})-\Pi(0)}$ otherwise. If ℓ holds the optimal passive portfolio, her expected utility is:

$$U_\ell(y_0\boldsymbol{\iota}) = \gamma_\ell\sigma^2 N\left(\frac{y_0(1-y_0)}{\frac{\gamma_\ell}{\lambda}} - \frac{1}{2}y_0^2\right). \tag{A.9}$$

Step One. Next, consider an alternative portfolio $\mathbf{y}^{\text{alt}}$, with positions $\overline{y}$ in firms $j \leq n$ and $\underline{y}$ in firms $j > n$, with $\overline{y} \geq \underline{y}$. Let $\mathcal{IC}(n)$ denote the set of alternative portfolios $\mathbf{y}^{\text{alt}}$ that are incentive compatible with exerting governance in $n > 0$ firms. Since $y_0\boldsymbol{\iota}$ is the optimal portfolio in $\mathcal{IC}(0)$, in this first step we compare $U_\ell(y_0\boldsymbol{\iota})$ and $U_\ell(\mathbf{y}^{\text{alt}})$, for all $\mathbf{y}^{\text{alt}} \in \bigcup_{n=1}^{N}\mathcal{IC}(n)$.

The investor's expected utility from holding $\mathbf{y}^{\text{alt}}$ is

$$U_\ell(\mathbf{y}^{\text{alt}}) = \gamma_\ell\sigma^2\left[n\left(\frac{\overline{y}(1-\overline{y})}{\frac{\gamma_\ell}{\lambda}} - \frac{1}{2}\overline{y}^2\right) + (N-n)\left(\frac{\underline{y}(1-\underline{y})}{\frac{\gamma_\ell}{\lambda}} - \frac{1}{2}\underline{y}^2\right)\right] - \kappa n. \tag{A.10}$$

If we define the difference in expected utilities as $\Delta(0) \equiv U_\ell(y_0\boldsymbol{\iota}) - U_\ell(\mathbf{y}^{\text{alt}})$, ℓ prefers the passive portfolio to the alternative one iff $\Delta(0) \geq 0$. If $y_0 = y^u$ (that is, for $\lambda \leq \lambda^*$), we must have $\Delta(0) > 0$, because the optimal passive portfolio is also the unconstrained optimal portfolio. If $y_0 = \frac{\kappa\tilde{n}}{\Pi(\tilde{n})-\Pi(0)}$ (that is, for $\lambda > \lambda^*$), the equation $\Delta(0) = 0$ can be rewritten as a linear equation in λ, which implies $\Delta(0)$ has (at most) one root in $\lambda \in (0, \infty)$.

From the discussion above, it follows that $U_\ell(y_0\boldsymbol{\iota})$ is always above $U_\ell(\mathbf{y}^{\text{alt}})$ for $\lambda \leq \lambda^*$, and $U_\ell(y_0\boldsymbol{\iota})$ may cross $U_\ell(\mathbf{y}^{\text{alt}})$ at most once and from above for $\lambda > \lambda^*$. Therefore, ℓ either always prefers $y_0\boldsymbol{\iota}$ to $\mathbf{y}^{\text{alt}}$, or there exists a threshold $\underline{\lambda}(\mathbf{y}^{\text{alt}})$ such that ℓ prefers $y_0\boldsymbol{\iota}$ for $\lambda \leq \underline{\lambda}(\mathbf{y}^{\text{alt}})$, and $\mathbf{y}^{\text{alt}}$ otherwise. This is described in Figure A.1.

Step Two. By Proposition 2, the optimal portfolio that is incentive-compatible with governance in n firms (i.e., the solution to Program 6) has the same structure as $\mathbf{y}^{\text{alt}}$. So, in the second step of the two-step process that determines the equilibrium portfolio, for all $n \in \mathcal{J}$, ℓ compares the passive portfolio $y_0\boldsymbol{\iota}$ with a special case of $\mathbf{y}^{\text{alt}}$ (that is, where $\overline{y}$ and $\underline{y}$ maximize ℓ's utility subject to the incentive-compatibility constraint). Let $\mathbf{y}^{\text{alt}}(\lambda)^*$ denote, for a given value of λ, the best of these alternative portfolios. Next, we show that ℓ either always prefers $y_0\boldsymbol{\iota}$ to $\mathbf{y}^{\text{alt}}(\lambda)^*$, or there

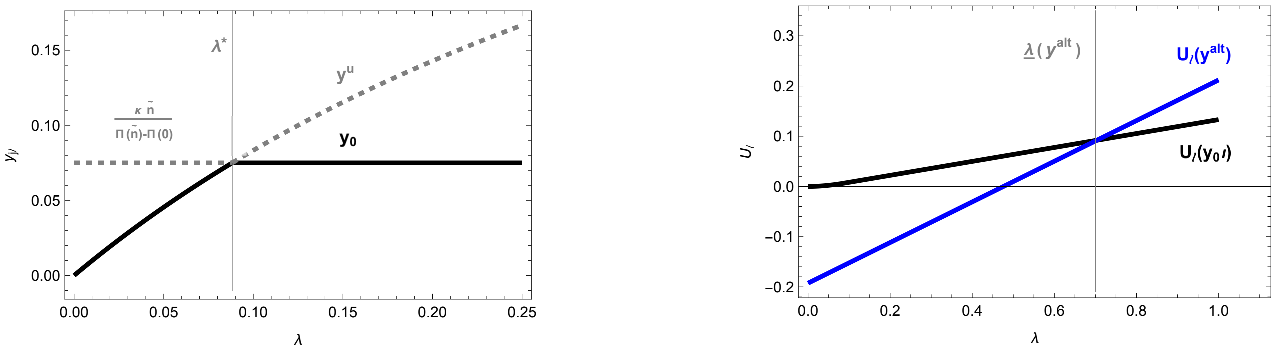

Figure A.1: The left panel plots the optimal passive portfolio $y_0\boldsymbol{\iota}$ (solid black line). The right panel compares ℓ's expected utility at this portfolio ($U_\ell(y_0\boldsymbol{\iota})$) with her expected utility at an alternative portfolio $\mathbf{y}^{\text{alt}}$ ($U_\ell(\mathbf{y}^{\text{alt}})$). In this example, $\mathbf{y}^{\text{alt}}$ has $\overline{y} = 0.37$ and $\underline{y} = 0.22$, and is incentive-compatible with $n = 1$, $\kappa = 0.1$, and the other parameters are the same as in Figure 2.

exists a threshold $\underline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$ such that ℓ prefers $y_0\boldsymbol{\iota}$ for $\lambda \leq \underline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$, and $\mathbf{y}^{\text{alt}}(\lambda)^*$ otherwise.

For the sake of contradiction, suppose there are two values $\lambda'' > \lambda' > 0$, such that ℓ strictly prefers $\mathbf{y}^{\text{alt}}(\lambda)^*$ at λ', and $y_0\boldsymbol{\iota}$ at λ'' (that is, $\mathbf{y}^{\text{alt}}(\lambda')^* \succ y_0\boldsymbol{\iota}$ and $y_0\boldsymbol{\iota} \succ \mathbf{y}^{\text{alt}}(\lambda'')^*$). Consider the best alternative portfolio at λ', that is, $\mathbf{y}^{\text{alt}}(\lambda')^*$. We know (as shown in Step One above) that ℓ prefers $y_0\boldsymbol{\iota}$ for $\lambda \leq \underline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda')^*)$, and $\mathbf{y}^{\text{alt}}(\lambda')^*$ otherwise, with $\underline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda')^*) \leq \lambda'$. However, at λ'', we must have $\mathbf{y}^{\text{alt}}(\lambda'')^* \succeq \mathbf{y}^{\text{alt}}(\lambda')^*$, which contradicts the initial conjecture $\mathbf{y}^{\text{alt}}(\lambda')^* \succ y_0\boldsymbol{\iota}$.

Therefore, for any λ' and λ'' in $(0, \infty)$ with $\lambda'' > \lambda'$, we must either have that ℓ prefers $y_0\boldsymbol{\iota}$ or $\mathbf{y}^{\text{alt}}(\lambda)^*$ at both λ' and λ'', or $y_0\boldsymbol{\iota} \succeq \mathbf{y}^{\text{alt}}(\lambda')^*$ and $\mathbf{y}^{\text{alt}}(\lambda'')^* \succeq y_0\boldsymbol{\iota}$. For $\lambda \leq \lambda^*$, ℓ strictly prefers $y_0\boldsymbol{\iota}$. In the proof of Part 2 of Proposition 5, we show that there exist some $\lambda > \lambda^*$ for which ℓ prefers the unconstrained portfolio $y^u\boldsymbol{\iota}$ (note that, for $\lambda > \lambda^*$, y_0 is different from y^u, so the unconstrained portfolio is now one of the alternative portfolios we described above). It follows that a threshold $\underline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$ such that ℓ prefers $y_0\boldsymbol{\iota}$ for $\lambda \leq \underline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$, and $\mathbf{y}^{\text{alt}}(\lambda)^*$ otherwise, always exists. For brevity, we write $\underline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$ as $\underline{\lambda}$ in the statement of Proposition 5.

Part 2 of Proposition 5. The proof for Part 2 follows similar steps as the one for Part 1. First, under the assumption that $\overline{n}_\lambda \geq 1$, when λ is sufficiently large, ℓ exerts governance in at least some firms when it holds the unconstrained optimal portfolio $y^u\boldsymbol{\iota}$. So, for λ sufficiently large, the investor's expected utility from holding the unconstrained portfolio $y^u = \frac{\lambda}{\gamma_\ell + 2\lambda}$ is:

$$U_\ell(\overline{n}_\lambda) = \sigma^2 N \left(\lambda y^u(1 - y^u) - \frac{\gamma_\ell}{2}(y^u)^2\right) - \kappa\overline{n}_\lambda = \frac{\sigma^2 N}{\frac{2}{\lambda}(2 + \frac{\gamma_\ell}{\lambda})} - \kappa\overline{n}_\lambda \tag{A.11}$$

Step One. Next, we compare the expected utility in Eqn. (A.11) to the one the investor gets from holding the alternative portfolio $\mathbf{y}^{\text{alt}}$, which is described in Eqn. (A.10). For the values of λ such that $y^u \boldsymbol{\iota} \in \mathcal{IC}(\overline{n}_\lambda)$ (that is, for λ large), $y^u \boldsymbol{\iota}$ is the optimal portfolio in that set. In this first step, here we then want to compare $U_\ell(\overline{n}_\lambda)$ and $U_\ell(\mathbf{y}^{\text{alt}})$, for all $\mathbf{y}^{\text{alt}} \in \bigcup_{n=0}^{\overline{n}_\lambda - 1} \mathcal{IC}(n)$.

The difference between these two expressions, $\Delta(\overline{n}_\lambda) \equiv U_\ell(\overline{n}_\lambda) - U_\ell(\mathbf{y}^{\text{alt}})$, writes as

$$\begin{aligned} \Delta(\overline{n}_\lambda) &= \sigma^2 n \left(\lambda y^u(1-y^u) - \frac{\gamma_\ell}{2}(y^u)^2 - \lambda \overline{y}(1-\overline{y}) + \frac{\gamma_\ell}{2}\overline{y}^2 \right) \\ &+ \sigma^2 (N-n) \left(\lambda y^u(1-y^u) - \frac{\gamma_\ell}{2}(y^u)^2 - \lambda \underline{y}(1-\underline{y}) + \frac{\gamma_\ell}{2}\underline{y}^2 \right) - \kappa(\overline{n}_\lambda - n). \end{aligned} \tag{A.12}$$

Note that $(\underline{y}, \overline{y})$ do not depend on λ and so $U_\ell(\mathbf{y}^{\text{alt}})$ is linear in λ. It follows that:

$$\frac{d^2 \Delta(\overline{n}_\lambda)}{d\lambda^2} = \frac{d^2 U_\ell(\overline{n}_\lambda)}{d\lambda^2} = \frac{\gamma_\ell^2 \sigma^2 N}{(\gamma_\ell + 2\lambda)^3} > 0. \tag{A.13}$$

Hence, $\Delta(\overline{n}_\lambda)$ strictly convex in $\lambda \in (0, \infty)$. Next, consider the limits $\lambda \to 0$ and $\lambda \to \infty$:

$$\lim_{\lambda \to 0} \Delta(\overline{n}_\lambda) = \sigma^2 \frac{\gamma_\ell}{2} \left(n\overline{y}^2 + (N-n)\underline{y}^2 \right) - \kappa(\overline{n}_\lambda - n) \tag{A.14}$$

$$\begin{aligned} \lim_{\lambda \to \infty} \Delta(\overline{n}_\lambda) &= \sigma^2 \left[\lim_{\lambda \to \infty} \{ n\lambda \left(y^u(1-y^u) - \overline{y}(1-\overline{y}) \right) + (N-n)\lambda \left(y^u(1-y^u) - \underline{y}(1-\underline{y}) \right) \} \right] \\ &+ \sigma^2 \left[\frac{n\gamma_\ell}{2} \left(\overline{y}^2 - \frac{1}{4} \right) + \frac{(N-n)\gamma_\ell}{2} \left(\underline{y}^2 - \frac{1}{4} \right) \right] - \kappa(\overline{n}_\lambda - n), \end{aligned} \tag{A.15}$$

where we have used $\lim_{\lambda \to \infty} y^u = \frac{1}{2}$. Moreover, note that:

$$\lim_{\lambda \to \infty} \{ n\lambda \left(y^u(1-y^u) - \overline{y}(1-\overline{y}) \right) + (N-n)\lambda \left(y^u(1-y^u) - \underline{y}(1-\underline{y}) \right) \} = \infty \tag{A.16}$$

as long as $\underline{y}$ and $\overline{y}$ are not both equal to $\lim_{\lambda \to \infty} y^u = \frac{1}{2}$.

At $\overline{y} = \underline{y} = \frac{1}{2}$, the incentive-compatible action is $n = \overline{n}_\lambda$ (since $\lim_{\lambda \to \infty} y^u = \frac{1}{2}$), so $\frac{1}{2}\boldsymbol{\iota} \notin \bigcup_{n=0}^{\overline{n}_\lambda - 1} \mathcal{IC}(n)$. In the limit $(\overline{y}, \underline{y}) \to (\frac{1}{2}, \frac{1}{2})$, the incentive compatibility constraint for exerting governance in $n < \overline{n}_\lambda$ firms would be violated. As a result, $(\overline{y}, \underline{y})$ must be bounded away from $(\frac{1}{2}, \frac{1}{2})$ and $\lim_{\lambda \to \infty} \{ n \left(y^u(1-y^u) - \overline{y}(1-\overline{y}) \right) + (N-n) \left(y^u(1-y^u) - \underline{y}(1-\underline{y}) \right) \} > 0$. So, we must have $\lim_{\lambda \to \infty} \Delta(\overline{n}_\lambda) = \infty$ for all $\mathbf{y}^{\text{alt}} \in \bigcup_{n=0}^{\overline{n}_\lambda - 1} \mathcal{IC}(n)$.

Since $\lim_{\lambda \to \infty} \Delta(\overline{n}_\lambda) = \infty$, and $\Delta(\overline{n}_\lambda)$ is a continuous, convex function of λ, $\Delta(\overline{n}_\lambda)$ has at most two roots in $\lambda \in (0, \infty)$. From the properties of $\Delta(\overline{n}_\lambda)$ discussed above, it follows that $U_\ell(\overline{n}_\lambda)$ is always above $U_\ell(\mathbf{y}^{\text{alt}})$ if λ is sufficiently large. So, there exists a threshold $\overline{\lambda}(\mathbf{y}^{\text{alt}})$ such that ℓ prefers $y^u \boldsymbol{\iota}$ to $\mathbf{y}^{\text{alt}}$ for all $\lambda > \overline{\lambda}(\mathbf{y}^{\text{alt}})$. When λ is large, we also know that $n = \overline{n}_\lambda$ is incentive-compatible at the unconstrained optimal portfolio $y^u\boldsymbol{\iota}$. We can then define a threshold, $\overline{\lambda}(\overline{n}_\lambda)$, such that ℓ exerts governance in $\overline{n}_\lambda$ firms at $y^u\boldsymbol{\iota}$ for all $\lambda > \overline{\lambda}(\overline{n}_\lambda)$. It follows that, for $\lambda > \max\{\overline{\lambda}(\mathbf{y}^{\text{alt}}), \overline{\lambda}(\overline{n}_\lambda)\}$, we know that $y^u\boldsymbol{\iota} \succ \mathbf{y}^{\text{alt}}$ and ℓ exerts governance in $\overline{n}_\lambda$ firms at $y^u\boldsymbol{\iota}$.

Step Two. Next, we focus on the range for λ where $n = \overline{n}_\lambda$ is the incentive-compatible governance action at $y^u\boldsymbol{\iota}$, that is, $\lambda > \overline{\lambda}(\overline{n}_\lambda)$. Within this range, we compare the unconstrained optimal portfolio $y^u\boldsymbol{\iota}$ with the best alternative portfolio $\mathbf{y}^{\text{alt}}(\lambda)^*$. Here $\mathbf{y}^{\text{alt}}(\lambda)^*$ describes, for any given λ, the best portfolio that is incentive-compatible with doing governance in less than $\overline{n}_\lambda$ firms, that is, the best portfolio in the set $\mathbf{y}^{\text{alt}} \in \bigcup_{n=0}^{\overline{n}_\lambda - 1} \mathcal{IC}(n)$. In Step One above, we have shown that, for any alternative portfolio $\mathbf{y}^{\text{alt}}$, we must have $y^u\boldsymbol{\iota} \succ \mathbf{y}^{\text{alt}}$ if λ is sufficiently large. It follows that there always exists a threshold $\overline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$, with $\overline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*) > \overline{\lambda}(\overline{n}_\lambda)$,[33] such that ℓ chooses $y^u\boldsymbol{\iota}$ and exerts governance in $\overline{n}_\lambda$ firms for $\lambda > \overline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$. For brevity, we write $\overline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$ as $\overline{\lambda}$ in the statement of Proposition 5. Figure A.2 illustrates the properties of the $\Delta(\overline{n}_\lambda)$ function.

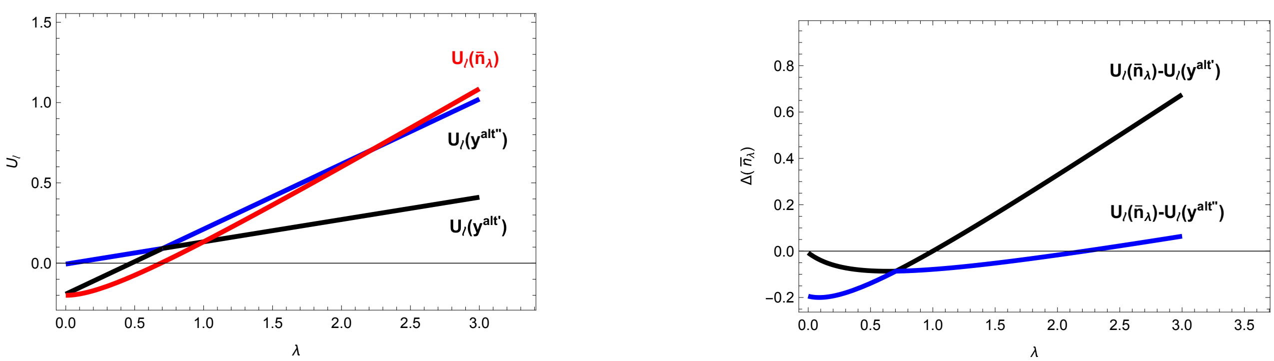

Figure A.2: The left panel plots ℓ's expected utilities when she holds the optimal unconstrained portfolio $y^u\boldsymbol{\iota}$ and exerts governance in $\overline{n}_\lambda$ firms ($U_\ell(\overline{n}_\lambda)$) and at two alternative portfolios $\mathbf{y}^{\text{alt}\prime}$ and $\mathbf{y}^{\text{alt}\prime\prime}$. The solid blue line represents the utility at the best of these two alternative portfolios (the upper envelope of $U_\ell(\mathbf{y}^{\text{alt}\prime})$ and $U_\ell(\mathbf{y}^{\text{alt}\prime\prime})$), to illustrate the logic by which we obtain $U_\ell(\mathbf{y}^{\text{alt}}(\lambda)^*)$. The right panel plots the utility differences $U_\ell(\overline{n}_\lambda) - U_\ell(\mathbf{y}^{\text{alt}})$. The solid blue line here represents the difference between $U_\ell(\overline{n}_\lambda)$ and ℓ's expected utility at the best alternative portfolios (the lower envelope of $U_\ell(\overline{n}_\lambda) - U_\ell(\mathbf{y}^{\text{alt}\prime})$ and $U_\ell(\overline{n}_\lambda) - U_\ell(\mathbf{y}^{\text{alt}\prime\prime})$), to illustrate the logic by which we obtain $\min_{\mathbf{y}^{\text{alt}} \in \mathcal{Y}^{\text{alt}}} \Delta(\overline{n}_\lambda)$. In this example, $\mathbf{y}^{\text{alt}\prime}$ has $\overline{y} = \underline{y} = 0.075$, and is incentive-compatible with $n = 0$, $\mathbf{y}^{\text{alt}\prime\prime}$ has $\overline{y} = 0.37$ and $\underline{y} = 0.22$, and is incentive-compatible with $n = 1$, $\kappa = 0.1$, and the other parameters are the same as in Figures 2 and A.1.

Step Three. Finally, we prove the "iff" condition in the statement of Proposition 5. First, we derive a condition such that, for a given portfolio, $\Delta(\overline{n}_\lambda)$ has a unique root in $(0, \infty)$. Due to the convexity of $\Delta(\overline{n}_\lambda)$, this requires $\lim_{\lambda \to 0} \Delta(\overline{n}_\lambda) < 0$. By Eqn. (A.14), this is equivalent to:

$$\kappa > \frac{\gamma_\ell \sigma^2}{2(\overline{n}_\lambda - n)} \left(n\overline{y}^2 + (N - n)\underline{y}^2 \right). \tag{A.17}$$

[33]For $\lambda \leq \overline{\lambda}(\overline{n}_\lambda)$, exerting governance in $\overline{n}_\lambda$ firms is not incentive-compatible at the unconstrained optimum. So, $y^u\boldsymbol{\iota}$ also belongs to the set of alternative portfolios here, which means that we cannot have $y^u\boldsymbol{\iota} \succ \mathbf{y}^{\text{alt}}(\lambda^*)$. It follows that we must have $\overline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*) > \overline{\lambda}(\overline{n}_\lambda)$.

The right-hand side of this condition is maximized at $\overline{y} = \underline{y} = 1$ and $\overline{n}_\lambda - n = 1$. Therefore a sufficient condition for $\lim_{\lambda \to 0} \Delta(\overline{n}_\lambda) < 0$ is $\kappa > \frac{\gamma_\ell \sigma^2}{2} N$ or equivalently $\sigma^2 < \frac{2\kappa}{\gamma_\ell N}$.

If Inequality (A.17) holds, the difference between $U_\ell(\overline{n}_\lambda)$ and $U_\ell(\mathbf{y}^{\text{alt}})$ is convex in λ, is negative at $\lambda = 0$, and is positive in the limit where λ approaches ∞. It follows that there exists a threshold $\overline{\lambda}(\mathbf{y}^{\text{alt}})$ such that $\mathbf{y}^{\text{alt}} \succeq y^u \boldsymbol{\iota}$ for $\lambda \leq \overline{\lambda}(\mathbf{y}^{\text{alt}})$, and $y^u \boldsymbol{\iota} \succ \mathbf{y}^{\text{alt}}$ otherwise.

Like Step Two of Part 1, we know that the optimal portfolio that is incentive-compatible with doing governance in n firms has the same structure as $\mathbf{y}^{\text{alt}}$. Let $\mathbf{y}^{\text{alt}}(\lambda)^*$ denote, for a given value of λ, the best of these alternative portfolios. Since $\mathbf{y}^{\text{alt}}(\lambda)^*$ is the best alternative portfolio in the set $\mathbf{y}^{\text{alt}} \in \bigcup_{n=0}^{\overline{n}_\lambda - 1} \mathcal{IC}(n) \equiv \mathcal{Y}^{\text{alt}}$, it minimizes the difference $\Delta(\overline{n}_\lambda) \equiv U_\ell(\overline{n}_\lambda) - U_\ell(\mathbf{y}^{\text{alt}})$. So, for a given λ, we can write the difference between $U_\ell(\overline{n}_\lambda)$ and $U_\ell(\mathbf{y}^{\text{alt}}(\lambda)^*)$ as $\min_{\mathbf{y}^{\text{alt}} \in \mathcal{Y}^{\text{alt}}} \Delta(\overline{n}_\lambda)$, where $y^u \boldsymbol{\iota} \succ \mathbf{y}^{\text{alt}}(\lambda)^*$ iff $\min_{\mathbf{y}^{\text{alt}} \in \mathcal{Y}^{\text{alt}}} \Delta(\overline{n}_\lambda) > 0$. Next, we show that the function $\min_{\mathbf{y}^{\text{alt}} \in \mathcal{Y}^{\text{alt}}} \Delta(\overline{n}_\lambda)$ crosses zero only once, and from below, in $\lambda \in (0, \infty)$.

For the sake of contradiction, suppose there are two values λ' and λ'', with $\lambda'' > \lambda' > 0$, such that $\min_{\mathbf{y}^{\text{alt}} \in \mathcal{Y}^{\text{alt}}} \Delta(\overline{n}_\lambda) \geq 0$ at λ' and $\min_{\mathbf{y}^{\text{alt}} \in \mathcal{Y}^{\text{alt}}} \Delta(\overline{n}_\lambda) \leq 0$ at λ''. At λ', we have that $\Delta(\overline{n}_\lambda) \geq 0$ and $\frac{d\Delta(\overline{n}_\lambda)}{d\lambda} > 0$ for each alternative portfolio because each $\Delta(\overline{n}_\lambda)$ intersects the zero axis only once from below, as shown above. As a result, we have that at $\lambda'' > \lambda'$, $\min_{\mathbf{y}^{\text{alt}} \in \mathcal{Y}^{\text{alt}}} \Delta(\overline{n}_\lambda) > 0$ because each individual $\Delta(\overline{n}_\lambda)$ increases in this range. However, this contradicts the initial conjecture that $\min_{\mathbf{y}^{\text{alt}} \in \mathcal{Y}^{\text{alt}}} \Delta(\overline{n}_\lambda) \leq 0$ at λ''. Therefore, we must have that $\min_{\mathbf{y}^{\text{alt}} \in \mathcal{Y}^{\text{alt}}} \Delta(\overline{n}_\lambda)$ has a unique root (from below) in $(0, \infty)$, which we denote by $\overline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*) \in (\overline{\lambda}(\overline{n}_\lambda), \infty)$.[34] So, we have $\min_{\mathbf{y}^{\text{alt}} \in \mathcal{Y}^{\text{alt}}} \Delta(\overline{n}_\lambda) > 0$ for $\lambda > \overline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$, and $\min_{\mathbf{y}^{\text{alt}} \in \mathcal{Y}^{\text{alt}}} \Delta(\overline{n}_\lambda) < 0$ if $\lambda < \overline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$.

It follows that ℓ prefers $y^u \boldsymbol{\iota}$ and exerts governance in $\overline{n}_\lambda$ firms iff $\lambda > \overline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$, since, only for these values of λ, we have that both $\overline{n}_\lambda$ is the incentive-compatible governance action at the portfolio $y^u \boldsymbol{\iota}$ and ℓ prefers this portfolio to any possible alternative portfolio. Like before, we write $\overline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$ as $\overline{\lambda}$ in the statement of Proposition 5.

Part 3 of Proposition 5. If λ lies between $\underline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$ and $\overline{\lambda}(\mathbf{y}^{\text{alt}}(\lambda)^*)$, then ℓ exerts governance in at least one firm, i.e., $n^* \geq 1$. As shown above, she will not exert governance in more than $\overline{n}_\lambda$ firms. Moreover, her portfolio may be unbalanced (see the example in Corollary 1).

[34] Like before, for $\lambda \leq \overline{\lambda}(\overline{n}_\lambda)$, exerting governance in $\overline{n}_\lambda$ firms is not incentive-compatible at the unconstrained optimum. So, $y^u \boldsymbol{\iota}$ also belongs to the set of alternative portfolios here, which means that we cannot have $y^u \boldsymbol{\iota} \succ \mathbf{y}^{\text{alt}}(\lambda^*)$.

Online Appendix

Table of Contents

B Additional proofs

B.1 Equilibrium in Example 1

In the Bertrand example, ℓ chooses between $n \in \{0, 1, N\}$, and the equilibrium characterization depends on the ordering of the profit parameters $\pi(1,1)$ and $\pi(1,N)$. We must consider three orderings of these parameters (Cases #1 to #3). Below, we describe the equilibrium characterization in each case as a function of κ.[1] We let $\hat{U}_\ell(\mathbf{y})$ denote ℓ's certainty equivalent (its expected utility gross of the governance cost κn) for a given portfolio $\mathbf{y}$, and set $\tilde{m} \equiv m\frac{\gamma_\ell}{\gamma_s}$.

Case #1. If $\pi(1,N) \leq \frac{1}{N}\pi(1,1)$, we have to distinguish two different scenarios:

1.a If $\kappa \geq y^u \pi(1,1)$, staying passive is incentive compatible at $y^u \boldsymbol{\iota}$, so ℓ chooses $n = 0$ and holds $y^u \boldsymbol{\iota}$.

[1] The proof of Corollary 1 in Online Appendix B.2 describes the equilibrium characterization in Case #2 and Case #3 as a function of the price impact parameter λ.

1.b If $\kappa < y^u\pi(1,1)$, $n = 1$ is incentive compatible at the unconstrained optimal portfolio. ℓ thus compares her expected utility from (i) holding $y^u\boldsymbol{\iota}$ and cutting cost ($n = 1$), and (ii) holding $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$ and staying passive ($n = 0$). It follows that ℓ stays passive iff $\hat{U}_\ell(\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}) \geq \hat{U}_\ell(y^u\boldsymbol{\iota}) - \kappa$, which can be rewritten as

$$\kappa \geq \frac{m\pi(1,1)^2 + N\pi(1,1)\gamma_s - \pi(1,1)\sqrt{m^2\pi(1,1)^2 + 2mN\pi(1,1)\gamma_s}}{N\left(m\gamma_\ell + 2\gamma_s\right)}. \tag{B1}$$

Case #2. If $\frac{1}{N}\pi(1,1) < \pi(1,N) \leq \pi(1,1)$, we have to distinguish three different scenarios.

2.a If $\kappa \geq y^u\pi(1,1)$, $n = 0$ is incentive compatible at $y^u\boldsymbol{\iota}$, so ℓ chooses $n = 0$ and holds $y^u\boldsymbol{\iota}$.

2.b If $y^u\pi(1,1) > \kappa \geq \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u$, $n = 1$ is incentive compatible at $y^u\boldsymbol{\iota}$. ℓ chooses between two options: (i) $y^u\boldsymbol{\iota}$ and $n = 1$, and (ii) $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$ and $n = 0$. It follows that ℓ stays passive iff $\hat{U}_\ell(\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}) \geq \hat{U}_\ell(y^u\boldsymbol{\iota}) - \kappa$.

2.c If $\kappa < \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u$, facilitating collusion is incentive compatible at y^u. ℓ compares the following three options.

(i) If ℓ stays passive ($n = 0$), she holds a balanced portfolio $y_{j\ell}$, which satisfies $y_{j\ell}\pi(1,1) \leq \kappa$. The parameter constraints imply that $\kappa < y^u\pi(1,1)$. So, ℓ holds $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$ if she chooses $n = 0$. The associated value for the objective function $U_\ell(n)$ is:

$$U_\ell(0) = N\gamma_\ell\sigma^2\left[\frac{1}{\tilde{m}}\frac{\kappa}{\pi(1,1)}\left(1 - \frac{\kappa}{\pi(1,1)}\right) - \frac{1}{2}\frac{\kappa^2}{\pi(1,1)^2}\right] \tag{B2}$$

(ii) If ℓ facilitates collusion ($n = N$), she holds $y^u\boldsymbol{\iota}$. The associated value for the objective function equals:

$$U_\ell(N) = N\gamma_\ell\sigma^2\left[\frac{1}{\tilde{m}}y^u\left(1 - y^u\right) - \frac{1}{2}(y^u)^2\right] - N\kappa \tag{B3}$$

(iii) If ℓ becomes active in one firm, we have shown in Proposition 2 that she holds $\overline{y}$ in firm $j = 1$ and $\underline{y} \leq \overline{y}$ in all other firms. We obtain the following optimization problem:

$$\begin{aligned} \max_{\overline{y},\underline{y}} \quad & \frac{1}{\tilde{m}}\overline{y}(1-\overline{y}) - \frac{1}{2}\overline{y}^2 + (N-1)\left[\frac{1}{\tilde{m}}\underline{y}(1-\underline{y}) - \frac{1}{2}\underline{y}^2\right] \\ \text{s.t.} \quad & \overline{y}\pi(1,1) - \kappa > 0, \\ & \overline{y}\pi(1,1) - \kappa \geq (\overline{y} + (N-1)\underline{y})\pi(1,N) - N\kappa, \\ & \overline{y} \geq \underline{y}. \end{aligned} \tag{B4}$$

We have shown in Proposition 4 that the second constraint will hold with equality at the optimum. We can then solve for $\overline{y}$ and $\underline{y}$:

$$\begin{aligned} \overline{y} &= \frac{(N-1)(\kappa(\tilde{m}+2)(\pi(1,N) - \pi(1,1)) + \pi(1,N)\pi(1,1))}{(\tilde{m}+2)\left(N\pi(1,N)^2 + \pi(1,1)(\pi(1,1) - 2\pi(1,N))\right)} \\ \underline{y} &= \frac{\kappa(\tilde{m}+2)(N-1)\pi(1,N) - \pi(1,N)\pi(1,1) + \pi(1,1)^2}{(\tilde{m}+2)\left(N\pi(1,N)^2 + \pi(1,1)(\pi(1,1) - 2\pi(1,N))\right)} \end{aligned} \tag{B5}$$

which leads to the following objective function:

$$U_\ell(1) = \gamma_\ell \sigma^2 \left(\frac{1}{\tilde{m}}\overline{y}(1-\overline{y}) - \frac{1}{2}\overline{y}^2 + (N-1)\left[\frac{1}{\tilde{m}}\underline{y}(1-\underline{y}) - \frac{1}{2}\underline{y}^2\right]\right) - \kappa. \tag{B6}$$

In the Proof of Corollary 2, we show that also in this interval, the best portfolio depends on the value of κ, with ℓ choosing $n = N$ if κ is small, $n = 0$ if κ is large, and $n = 1$ otherwise.

Case #3. If $\pi(1,N) > \pi(1,1)$, we have to distinguish two cases.

3.a If $\kappa \geq y^u\pi(1,N)$, $n = 0$ is incentive compatible at $y^u\boldsymbol{\iota}$, so ℓ chooses $n = 0$ and holds $y^u\boldsymbol{\iota}$.

3.b If $\kappa < y^u\pi(1,N)$, facilitating collusion is incentive compatible at y^u. Since we can show that cost-cutting is never optimal in this case, we have to compare ℓ's expected utility for (i) $n = N$ and $y^u\boldsymbol{\iota}$, and (ii) $n = 0$ and $\frac{\kappa}{\pi(1,N)}\boldsymbol{\iota}$. It follows that ℓ facilitates collusion iff $\hat{U}_\ell(y^u\boldsymbol{\iota}) - N\kappa > \hat{U}_\ell(\frac{\kappa}{\pi(1,N)}\boldsymbol{\iota})$, which can be rewritten as

$$\kappa < \frac{m\pi(1,\mathrm{N})^2 + N\pi(1,\mathrm{N})\gamma_s - \pi(1,\mathrm{N})\sqrt{m^2\pi(1,\mathrm{N})^2 + 2mN\pi(1,\mathrm{N})\gamma_s}}{N\left(m\gamma_\ell + 2\gamma_s\right)}. \tag{B7}$$

B.2 Proof of Corollary 1

The condition $\overline{n}_\lambda = N$ implies that we need to consider only Case #2 and Case #3 of the characterization in Online Appendix B.1 (in Case #1, the optimal governance action at the unconstrained optimal portfolio is always $n = 1$). Since we have $\lim_{\lambda\to\infty} y^u = \frac{1}{2}$, this condition also implies $N\frac{1}{2}\pi(1,N) - N\kappa > \max\{0, \frac{1}{2}\pi(1,1) - \kappa\}$. Next, we consider these two cases separately.

Case #2. If $\frac{1}{N}\pi(1,1) < \pi(1,N) \leq \pi(1,1)$, we have to distinguish three different scenarios.

2.a If $\lambda \leq \frac{\gamma_\ell\kappa}{\pi(1,N)-2\kappa}$, $n = 0$ is incentive compatible at $y^u\boldsymbol{\iota}$, so ℓ chooses $n = 0$ and holds $y^u\boldsymbol{\iota}$.

2.b If $\frac{\gamma_\ell\kappa(N-1)}{2\kappa(N-1)-N\pi(1,N)+\pi(1,1)} \geq \lambda > \frac{\gamma_\ell\kappa}{\pi(1,N)-2\kappa}$, $n = 1$ is incentive compatible at $y^u\boldsymbol{\iota}$. ℓ chooses between two options: (i) $y^u\boldsymbol{\iota}$ and $n = 1$, and (ii) $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$ and $n = 0$. It follows that ℓ stays passive iff $\hat{U}_\ell(\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}) \geq \hat{U}_\ell(y^u\boldsymbol{\iota}) - \kappa$. Let $\Delta(\lambda)$ denote the difference $\hat{U}_\ell(y^u\boldsymbol{\iota}) - \kappa - \hat{U}_\ell(\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota})$. We can write

$$\Delta(\lambda) = \frac{1}{2}\left(\frac{\lambda(\lambda N - 4\kappa) - 2\gamma_\ell\kappa}{\gamma_\ell + 2\lambda} + \frac{N\kappa(\gamma\kappa + 2\lambda(\kappa - \pi(1,1)))}{\pi(1,1)^2}\right), \tag{B8}$$

which implies $\Delta(\frac{\gamma_\ell\kappa}{\pi(1,N)-2\kappa}) = -\kappa < 0$ and $\Delta(\infty) > 0$. Also, the equation $\Delta(\lambda) = 0$ can be rearranged as a quadratic equation in λ, which implies that $\Delta(\lambda) = 0$ has only two roots. $\Delta(\frac{\gamma_\ell\kappa}{\pi(1,N)-2\kappa}) < 0$ and $\Delta(\infty) > 0$ implies that at most one of these roots is in $\lambda \in (\frac{\gamma_\ell\kappa}{\pi(1,N)-2\kappa}, \frac{\gamma_\ell\kappa(N-1)}{2\kappa(N-1)-N\pi(1,N)+\pi(1,1)})$.

2.c If $\lambda > \frac{\gamma_\ell\kappa(N-1)}{2\kappa(N-1)-N\pi(1,N)+\pi(1,1)}$, $n = N$ is incentive compatible at $y^u\boldsymbol{\iota}$. ℓ compares the following three options.

(i) If ℓ stays passive ($n = 0$), she holds a balanced portfolio $y_{j\ell}$, which satisfies $y_{j\ell}\pi(1,1) \leq \kappa$. The parameter constraints imply that $\kappa < y^u\pi(1,1)$. So, ℓ holds $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$ if she chooses $n = 0$. The associated value for the objective function $U_\ell(n)$ is:

$$U_\ell(0) = N\sigma^2\left[\lambda\frac{\kappa}{\pi(1,1)}\left(1 - \frac{\kappa}{\pi(1,1)}\right) - \frac{\gamma_\ell}{2}\frac{\kappa^2}{\pi(1,1)^2}\right] \tag{B9}$$

(ii) If ℓ facilitates collusion ($n = N$), she holds $y^u\boldsymbol{\iota}$. The associated value for the objective function equals:

$$U_\ell(N) = N\sigma^2\left[\lambda y^u\left(1 - y^u\right) - \frac{\gamma_\ell}{2}(y^u)^2\right] - N\kappa \tag{B10}$$

(iii) If ℓ becomes active in one firm, we have shown in Proposition 2 that she holds $\overline{y}$ in firm $j = 1$ and $\underline{y} \leq \overline{y}$ in all other firms. We obtain the following optimization problem:

$$\begin{aligned} \max_{\overline{y},\underline{y}} \quad & \lambda\overline{y}(1-\overline{y}) - \frac{\gamma_\ell}{2}\overline{y}^2 + (N-1)\left[\lambda\underline{y}(1-\underline{y}) - \frac{\gamma_\ell}{2}\underline{y}^2\right] \\ \text{s.t.} \quad & \overline{y}\pi(1,1) - \kappa > 0, \\ & \overline{y}\pi(1,1) - \kappa \geq (\overline{y} + (N-1)\underline{y})\pi(1,N) - N\kappa, \\ & \overline{y} \geq \underline{y}. \end{aligned} \tag{B11}$$

We have shown in Proposition 4 that the second constraint will hold with equality at the optimum. We can then solve for $\overline{y}$ and $\underline{y}$:

$$\begin{aligned} \overline{y} &= \frac{(N-1)\left(\lambda\pi(1,1)(\pi(1,\mathrm{N}) - 2\kappa) + 2\kappa\lambda\pi(1,\mathrm{N}) + \kappa(\pi(1,\mathrm{N}) - \pi(1,1))\gamma_\ell\right)}{\left(N\pi(1,\mathrm{N})^2 + \pi(1,1)^2 - 2\pi(1,1)\pi(1,\mathrm{N})\right)(2\lambda + \gamma_\ell)} \\ \underline{y} &= \frac{\lambda\left(2\kappa(N-1)\pi(1,\mathrm{N}) + \pi(1,1)^2 - \pi(1,1)\pi(1,\mathrm{N})\right) + \kappa(N-1)\pi(1,\mathrm{N})\gamma_\ell}{\left(N\pi(1,\mathrm{N})^2 + \pi(1,1)^2 - 2\pi(1,1)\pi(1,\mathrm{N})\right)(2\lambda + \gamma_\ell)} \end{aligned} \tag{B12}$$

which leads to the following objective function:

$$U_\ell(1) = \sigma^2\left(\lambda\overline{y}(1-\overline{y}) - \frac{\gamma_\ell}{2}\overline{y}^2 + (N-1)\left[\lambda\underline{y}(1-\underline{y}) - \frac{\gamma_\ell}{2}\underline{y}^2\right]\right) - \kappa. \tag{B13}$$

Next, we derive the thresholds $\underline{\lambda}^b$ and $\overline{\lambda}^b$ in Corollary 1. First, consider scenario 2.c. The differences $U_\ell(N) - U_\ell(1)$ and $U_\ell(N) - U_\ell(0)$ obtained from the expressions above have the following properties: (i) they are strictly convex and (ii) positive in the limit $\lambda \to \infty$. Moreover, at $\lambda = \frac{\gamma_\ell\kappa(N-1)}{2\kappa(N-1)-N\pi(1,N)+\pi(1,1)}$, ℓ prefers cutting cost and holding $\{\overline{y}, \underline{y}\}$ to facilitating collusion and holding $y^u\boldsymbol{\iota}$, which implies that $U_\ell(N) - U_\ell(1) < 0$ at this point. Hence, there exists a threshold $\overline{\lambda}_1^b > \frac{\gamma_\ell\kappa(N-1)}{2\kappa(N-1)-N\pi(1,N)+\pi(1,1)}$ such that ℓ prefers facilitating collusion to cost-cutting if and only if $\lambda > \overline{\lambda}_1^b$. Similarly, $U_\ell(N) - U_\ell(0)$ decreases with λ and so there either exists a threshold $\overline{\lambda}_2^b$ such that ℓ prefers facilitating collusion to staying passive if and only if $\lambda > \overline{\lambda}^b$ or ℓ prefers facilitating collusion to staying passive for all $\lambda > \frac{\gamma_\ell\kappa(N-1)}{2\kappa(N-1)-N\pi(1,N)+\pi(1,1)}$, in which case we set $\overline{\lambda}_2^b = 0$. Since facilitating collusion is never incentive-compatible for $\lambda < \frac{\gamma_\ell\kappa(N-1)}{2\kappa(N-1)-N\pi(1,N)+\pi(1,1)}$, we conclude that ℓ facilitates collusion and holds $y^u\boldsymbol{\iota}$ if and only if $\lambda > \overline{\lambda}^b \equiv \max\{\overline{\lambda}_1^b, \overline{\lambda}_2^b\}$.

Next, we focus on $\lambda \leq \overline{\lambda}^b$ so we can focus on the two governance choices $n \in \{0, 1\}$. If $0 < \lambda \leq$

$\frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}$ (i.e., Scenario 2.a), ℓ stays passive and holds $y^u \boldsymbol{\iota}$. If $\frac{\gamma_\ell \kappa(N-1)}{2\kappa(N-1)-N\pi(1,N)+\pi(1,1)} \geq \lambda > \frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}$ (i.e., Scenario 2.b), ℓ chooses between staying passive and holding $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$ and cutting cost and holding $y^u \boldsymbol{\iota}$. As shown above, the difference between the two corresponding expected utilities has at most one root in this interval. If this root exists in $\frac{\gamma_\ell \kappa(N-1)}{2\kappa(N-1)-N\pi(1,N)+\pi(1,1)} \geq \lambda > \frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}$, we refer to the corresponding value as $\underline{\lambda}^b$. In this case, ℓ prefers staying passive for all $\lambda \leq \overline{\lambda}^b$. For $\lambda > \overline{\lambda}^b$, ℓ prefers cost-cutting. If the root does not exist in the interval associated with Scenario 2.b., then ℓ prefers staying passive for all $\lambda > \frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}$. For $\lambda \leq \frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}$, we can again show that the difference $U_\ell(1) - U_\ell(0)$ is convex. Since $U_\ell(1) - U_\ell(0) < 0$ at $\lambda = \frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}$, there exists a unique threshold $\underline{\lambda}_1^b > \frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}$, such that ℓ prefers cost-cutting to staying passive if and only if $\lambda > \underline{\lambda}_1^b$. If $\underline{\lambda}_1^b \geq \overline{\lambda}^b$, then cost-cutting is never chosen in equilibrium and we set $\underline{\lambda}^b = \overline{\lambda}^b$. If, instead, $\underline{\lambda}_1^b < \overline{\lambda}^b$, then ℓ chooses cost cutting if $\underline{\lambda}^b < \lambda \leq \overline{\lambda}^b$ where $\underline{\lambda}^b = \underline{\lambda}_1^b$.

Case #3. If $\pi(1,N) > \pi(1,1)$, we have to distinguish two cases.

3.a If $\lambda \leq \frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}$, $n = 0$ is incentive compatible at $y^u \boldsymbol{\iota}$, so ℓ chooses $n = 0$ and holds $y^u \boldsymbol{\iota}$.

3.b If $\lambda > \frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}$, $n = N$ is incentive compatible at y^u. Cost-cutting is never incentive-compatible in this case, so ℓ compares her expected utility from (i) choosing $n = N$ and holding $y^u \boldsymbol{\iota}$, and (ii) choosing $n = 0$ and holding $\frac{\kappa}{\pi(1,N)}\boldsymbol{\iota}$. It follows that ℓ facilitates collusion iff $\hat{U}_\ell(y^u \boldsymbol{\iota}) - N\kappa > \hat{U}_\ell(\frac{\kappa}{\pi(1,N)}\boldsymbol{\iota})$. Let $\Delta(\lambda)$ denote the difference $\hat{U}_\ell(y^u \boldsymbol{\iota}) - N\kappa - \hat{U}_\ell(\frac{\kappa}{\pi(1,N)}\boldsymbol{\iota})$. We can write

$$\Delta(\lambda) = \frac{N}{2}\left(\frac{\lambda(\lambda - 4\kappa) - 2\gamma_\ell \kappa}{\gamma_\ell + 2\lambda} + \frac{\kappa(\gamma_\ell \kappa + 2\lambda(\kappa - \pi(1,N)))}{\pi(1,N)^2}\right), \quad \text{(B14)}$$

which implies $\Delta(\frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}) = -N\kappa < 0$ and $\Delta(\infty) > 0$. Also, the equation $\Delta(\lambda) = 0$ can be rearranged as a quadratic equation in λ, which implies that $\Delta(\lambda) = 0$ has two roots. $\Delta(\frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}) < 0$ and $\Delta(\infty) > 0$ implies that only one of these roots is in $\lambda \in (\frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}, \infty)$.

It follows that, also in Case #3, there exists a unique threshold $\overline{\lambda}_b$, with $\overline{\lambda}_b > \frac{\gamma_\ell \kappa}{\pi(1,N)-2\kappa}$, such that $\Delta(\lambda) \leq 0$ and, thus, ℓ chooses $n = 0$ for all $\lambda \leq \overline{\lambda}_b$, and $\Delta(\lambda) > 0$ and ℓ chooses $n = N$ for $\lambda > \overline{\lambda}_b$. Since here ℓ never chooses $n = 1$, we have $\underline{\lambda}_b = \overline{\lambda}_b$.

B.3 Proof of Proposition 6.

We impose conditions on π_j such that ℓ either becomes active in all firms (and facilitates collusion) or stays passive. We also define $\tilde{m} = m\frac{\gamma_\ell}{\gamma_S}$, $\pi^H = \pi(1, N)$, $\pi^L = \pi(0, 0)$, and $\Delta = \pi^H - \pi^L$.

1. If ℓ stays passive, she will hold a balanced portfolio $y_{j\ell}$, which satisfies $\pi^L \sum_{j=1}^N y_{j\ell} \geq \pi^H \sum_{j=1}^N y_{j\ell} - N\kappa$. If $\kappa \geq y^u\Delta$, then ℓ holds y^u in each firm. Otherwise, she holds $\frac{\kappa}{\Delta}$. The associated value for the objective function equals:

$$U_\ell(0) = \begin{cases} N\gamma_\ell\sigma^2\left[\frac{1}{\tilde{m}}y^u\left(1-y^u\right)-\frac{1}{2}(y^u)^2\right] & \text{if } \kappa \geq y^u\Delta \\ N\gamma_\ell\sigma^2\left[\frac{1}{\tilde{m}}\frac{\kappa}{\Delta}\left(1-\frac{\kappa}{\Delta}\right)-\frac{1}{2}\frac{\kappa^2}{\Delta^2}\right] & \text{if } \kappa < y^u\Delta. \end{cases} \tag{B15}$$

2. If ℓ becomes active in all firms, she will hold y^u in all firms to maximize her risk-return trade-off. The associated value for the objective function equals:

$$U_\ell(N) = N\gamma_\ell\sigma^2\left[\frac{1}{\tilde{m}}y^u\left(1-y^u\right)-\frac{1}{2}(y^u)^2\right] - N\kappa. \tag{B16}$$

As a last step, we compare the values for U_ℓ. It follows that ℓ chooses $n = 0$ if $\kappa \geq \underline{\kappa}^{(2)} \equiv \frac{\gamma_\ell\Delta\sigma^2-\sqrt{\Delta^4\tilde{m}^2+2\gamma_\ell\Delta^3\tilde{m}\sigma^2+\Delta^2\tilde{m}}}{\gamma_\ell(\tilde{m}+2)\sigma^2}$ and $n = N$ otherwise. We have that $\frac{d\underline{\kappa}^{(2)}}{d\Delta} > 0$. To see that N can increase or decrease the range for collusion, see the left Panel in Figure 4.

B.4 Proof of Proposition 7.

Preliminaries. First, note that $\overline{n} \equiv \arg\max_n\{y^u\boldsymbol{\iota}'\boldsymbol{\pi} - n\kappa\}$ decreases with κ, since y^u and $\boldsymbol{\iota}'\boldsymbol{\pi}$ do not depend on κ, while the governance cost increases when κ goes up. Since $\overline{n} \leq \overline{n}_\kappa$ and we must have $n^* \leq \overline{n}$ in equilibrium (see Lemma 1), ℓ remains passive for any κ and holds the unconstrained optimal portfolio $y^u\boldsymbol{\iota}$ when $\overline{n}_\kappa = 0$. Since ℓ never exerts governance when $\overline{n}_\kappa = 0$, the proposition focuses on the case $\overline{n}_\kappa \geq 1$. In this case, since $\overline{n}$ decreases with κ, it is incentive-compatible for ℓ to exert governance in $\overline{n}_\kappa$ firms at $y^u\boldsymbol{\iota}$ if κ is sufficiently small. By a similar logic, if κ is sufficiently large, ℓ does not exert any governance at the unconstrained optimal portfolio, that is, $\overline{n} = 0$. We use the notation $A \succeq B$ ($A \succ B$) to signify that ℓ weakly (strictly) prefers portfolio A to portfolio B. We let $\mathcal{IC}(n)$ denote the set of alternative portfolios that are incentive compatible with exerting governance in $n > 0$ firms and define $\tilde{m} \equiv \frac{\gamma_\ell}{\gamma_S}m$

Part 1 of Proposition 7. Under the assumption that $\overline{n}_\kappa \geq 1$, when κ is sufficiently small, ℓ exerts governance in at least some firms at the unconstrained optimal portfolio $y^u\boldsymbol{\iota}$. So, for κ sufficiently small, the investor's expected utility from holding $y^u\boldsymbol{\iota}$ is:

$$U_\ell(\overline{n}_\kappa) = \frac{\gamma_\ell \sigma^2 N}{2\tilde{m}(2+\tilde{m})} - \kappa \overline{n}_\kappa. \tag{B17}$$

We focus on the range for κ where $n = \overline{n}_\kappa$ is the incentive-compatible governance action at $y^u\boldsymbol{\iota}$, that is, $\kappa < \underline{\kappa}(\overline{n}_\kappa)$. Within this range, we compare the unconstrained optimal portfolio $y^u\boldsymbol{\iota}$ with the best alternative portfolio $\mathbf{y}^{\text{alt}}(\kappa)^*$, that is, the best of all portfolios that are incentive compatible with exerting governance in $n < \overline{n}_\kappa$ firms. Next, we show that ℓ either always prefers $y^u\boldsymbol{\iota}$ to $\mathbf{y}^{\text{alt}}(\kappa)^*$, or there exists a threshold $\underline{\kappa}(\mathbf{y}^{\text{alt}}(\kappa)^*)$ such that ℓ prefers $y^u\boldsymbol{\iota}$ for $\kappa \leq \underline{\kappa}(\mathbf{y}^{\text{alt}}(\kappa)^*)$, and $\mathbf{y}^{\text{alt}}(\kappa)^*$ otherwise. It follows that, for $\kappa < \min\{\underline{\kappa}(\mathbf{y}^{\text{alt}}(\kappa)^*), \underline{\kappa}(\overline{n}_\kappa)\}$, we know that $y^u\boldsymbol{\iota}$ is preferred to any other portfolio and ℓ exerts governance in $\overline{n}_\lambda$ firms at $y^u\boldsymbol{\iota}$. For brevity, we write $\min\{\underline{\kappa}(\mathbf{y}^{\text{alt}}(\kappa)^*), \underline{\kappa}(\overline{n}_\kappa)\}$, as $\underline{\kappa}$ in the statement of Proposition 7.

By Proposition 2, we know that $\mathbf{y}^{\text{alt}}(\kappa)^*$ has positions $\overline{y}$ in firms $j \leq n$ and $\underline{y} \leq \overline{y}$ in firms $j > n$, where n is a function of κ. ℓ's expected utility from holding this alternative portfolio is

$$U_\ell(\mathbf{y}^{\text{alt}}(\kappa)^*) = \gamma_\ell \sigma^2 \left[n \left(\frac{\overline{y}(1-\overline{y})}{\tilde{m}} - \frac{1}{2}\overline{y}^2 \right) + (N-n) \left(\frac{\underline{y}(1-\underline{y})}{\tilde{m}} - \frac{1}{2}\underline{y}^2 \right) \right] - \kappa n. \tag{B18}$$

We define the difference between $U_\ell(\overline{n}_\kappa)$ and $U_\ell(\mathbf{y}^{\text{alt}}(\kappa)^*)$ as $\Delta(\overline{n}_\kappa)$. If κ is sufficiently small, we have $\Delta(\overline{n}_\kappa) > 0$, since ℓ's certainty equivalent is maximized at $y^u\boldsymbol{\iota}$, and we cannot have $\overline{y} = \underline{y} = y^u$, otherwise the incentive-compatible action would be $n = \overline{n}_\kappa$ also at $\mathbf{y}^{\text{alt}}(\kappa)^*$.

In what follows, we show that the $\Delta(\overline{n}_\kappa)$ function decreases with κ. Suppose κ increases from κ' to κ''. First, consider the case where ℓ does not reoptimize after the decrease in κ and so keeps the same portfolio, $\mathbf{y}^{\text{alt}}(\kappa')^*$, at both κ' and κ''. For this particular portfolio, let n' denote the incentive-compatible governance choice when $\kappa = \kappa'$, and n'' the incentive-compatible governance choice when $\kappa = \kappa''$. Let also $\hat{U}_\ell(\mathbf{y})$ denote ℓ's certainty equivalent (its expected utility gross of the governance cost κn) for a given portfolio $\mathbf{y}$. In this case, we can write

$$\begin{aligned} \Delta'' - \Delta' &= \left\{ \left[\hat{U}_\ell(y^u\boldsymbol{\iota}) - \overline{n}_\kappa \kappa'' \right] - \left[\hat{U}_\ell(\mathbf{y}^{\text{alt}}(\kappa')^*) - n''\kappa'' \right] \right\} - \left\{ \left[\hat{U}_\ell(y^u\boldsymbol{\iota}) - \overline{n}_\kappa \kappa' \right] - \left[\hat{U}_\ell(\mathbf{y}^{\text{alt}}(\kappa')^*) - n'\kappa' \right] \right\} \\ &= (n'' - \overline{n}_\kappa)\kappa'' - (n' - \overline{n}_\kappa)\kappa', \end{aligned} \tag{B19}$$

where Δ'' and Δ' indicate the values of $\Delta(\overline{n}_\kappa)$ at κ'' and κ', respectively.

When κ increases, ℓ exerts less governance at the portfolio $\mathbf{y}^{\text{alt}}(\kappa')^*$. So, we must have $n'' \le n' < \overline{n}_\kappa$, which implies the expression in Eqn. (B19) is negative.[2]

If ℓ can reoptimize after the increase in κ, it chooses the best alternative portfolio $\mathbf{y}^{\text{alt}}(\kappa'')^*$. Let $n(\kappa'')$ denote the incentive-compatible governance choice when $\kappa = \kappa''$ and ℓ's portfolio is $\mathbf{y}^{\text{alt}}(\kappa'')^*$. In this case, we can write

$$\begin{aligned} \Delta'' - \Delta' &= \left\{ \left[\hat{U}_\ell(y^u \boldsymbol{\iota}) - \overline{n}_\kappa \kappa'' \right] - \left[\hat{U}_\ell(\mathbf{y}^{\text{alt}}(\kappa'')^*) - n(\kappa'') \right] \right\} - \left\{ \left[\hat{U}_\ell(y^u \boldsymbol{\iota}) - \overline{n}_\kappa \kappa' \right] - \left[\hat{U}_\ell(\mathbf{y}^{\text{alt}}(\kappa')^*) - n'\kappa' \right] \right\} \\ &\le (n'' - \overline{n}_\kappa)\kappa'' - (n' - \overline{n}_\kappa)\kappa', \end{aligned} \tag{B20}$$

where the inequality follows from the fact that we must have $\hat{U}_\ell(\mathbf{y}^{\text{alt}}(\kappa'')^*) - n(\kappa'') \ge \hat{U}_\ell(\mathbf{y}^{\text{alt}}(\kappa')^*) - n''\kappa''$, since ℓ must be at least weakly better off when it can reoptimize and choose a different portfolio after the increase in κ.

It follows that $\Delta(\overline{n}_\kappa)$ always decreases with κ within the interval $\kappa < \underline{\kappa}(\overline{n}_\kappa)$, and it is positive for κ sufficiently close to 0. This implies that either ℓ either always prefers $y^u \boldsymbol{\iota}$ to $\mathbf{y}^{\text{alt}}(\kappa)^*$, or there exists a threshold $\underline{\kappa}(\mathbf{y}^{\text{alt}}(\kappa)^*)$ such that ℓ prefers $y^u \boldsymbol{\iota}$ for $\kappa \le \underline{\kappa}(\mathbf{y}^{\text{alt}}(\kappa)^*)$, and $\mathbf{y}^{\text{alt}}(\kappa)^*$ otherwise.

Part 2 of Proposition 7. If ℓ does not exert any governance, she holds a balanced portfolio with $y_0 \equiv \min\{y^u, \kappa \min_{\tilde{n} \in \mathcal{N}_0^+} \frac{\tilde{n}}{\Pi(\tilde{n}) - \Pi(0)}\}$ in all firms (see Proposition 4). Note that $\min_{\tilde{n} \in \mathcal{N}_0^+} \frac{\tilde{n}}{\Pi(\tilde{n}) - \Pi(0)}$ does not depend on κ. It follows that $y_0 = \min_{\tilde{n} \in \mathcal{N}_0^+} \frac{\kappa \tilde{n}}{\Pi(\tilde{n}) - \Pi(0)}$ for $\kappa < \frac{y^u}{\min_{\tilde{n} \in \mathcal{N}_0^+} \frac{\tilde{n}}{\Pi(\tilde{n}) - \Pi(0)}} \equiv \overline{\kappa}$ and $y_0 = y^u$ otherwise. Therefore, if κ is sufficiently large, i.e., $\kappa > \overline{\kappa}$, ℓ holds the unconstrained optimal portfolio at $n = 0$. This strategy maximizes her certainty equivalent return and minimizes the governance cost, which is thus preferred to all other feasible strategies.

Part 3 of Proposition 7. If κ lies between $\underline{\kappa}$ and $\overline{\kappa}$, then ℓ exerts governance in at most $\overline{n}_\kappa$ firms, as shown above. Moreover, her portfolio may be unbalanced (see the example in Corollary 2).

B.5 Proof of Corollary 2

The condition $\overline{n}_\kappa = N$ implies that we need to consider only Case #2 and Case #3 of the characterization in Online Appendix B.1 (in Case #1, the optimal governance action at the

[2] The proof of Proposition 4 shows that, at an optimal portfolio $\mathbf{y}_\ell$, the incentive-compatibility constraint holds with equality for some $\tilde{n} > n$ but is slack for all $\tilde{n} < n$. We can re-write the constraint as $\hat{G}(\tilde{n}, \mathbf{y}_\ell) - \hat{G}(n, \mathbf{y}_\ell) \le \kappa(\tilde{n} - n)$ and so we can see that an increase in κ makes it easier (harder) to satisfy the constraints for $\tilde{n} > n$ ($\tilde{n} < n$) given a fixed portfolio $\mathbf{y}_\ell$. As a result, an increase in κ leads to (weakly) less governance at a fixed portfolio.

unconstrained optimal portfolio is always $n = 1$). Next, we consider these two cases separately.[3]

Case #2. If $\frac{1}{N}\pi(1,1) \leq \pi(1,N) \leq \pi(1,1)$, ℓ never facilitates collusion if $\kappa \geq \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u$. If $\kappa < \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u$, ℓ facilitates collusion and holds $y^u\boldsymbol{\iota}$, if this strategy is preferred to the other two incentive-compatible strategies, (i) cost-cutting and holding $\{\overline{y}, \underline{y}\}$ and (ii) staying passive and holding $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$. Note that $\hat{U}_\ell(y^u\boldsymbol{\iota}) - N\kappa$ and $\hat{U}(\{\overline{y}, \underline{y}\}) - \kappa$ decrease with κ, while $\hat{U}_\ell(\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota})$ increases with κ. Moreover, we can show that the difference $\left(\hat{U}_\ell(y^u\boldsymbol{\iota}) - N\kappa\right) - \left(\hat{U}(\{\overline{y}, \underline{y}\}) - \kappa\right)$ is convex, positive at $\kappa \to 0$, and negative at $\kappa \to \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u$. Hence, there exists a unique threshold for κ, denoted by $\underline{\kappa}_1^b \in (0, \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u)$ such that ℓ prefers facilitating collusion and holding $y^u\boldsymbol{\iota}$ to cost cutting and holding $\{\overline{y}, \underline{y}\}$ iff $\kappa < \underline{\kappa}_1^b$. Next, note that the difference $\left(\hat{U}_\ell(y^u\boldsymbol{\iota}) - N\kappa\right) - \hat{U}(\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota})$ decreases with κ and is positive at $\kappa \to 0$. Hence, there either exists a $\underline{\kappa}_2^b \in (0, \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u)$ such that ℓ prefers facilitating collusion and holding $y^u\boldsymbol{\iota}$ to staying passive and holding $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$ iff $\kappa < \underline{\kappa}_2^b$ or staying passive is never optimal in this range (in this case, we set $\underline{\kappa}_2^b = \infty$). It follows that ℓ holds the unconstrained optimal portfolio and facilitates collusion if $\kappa < \underline{\kappa}^b \equiv \min\{\underline{\kappa}_1^b, \underline{\kappa}_2^b\}$.

Next, we focus on $\kappa \geq \underline{\kappa}^b$, so we only need to consider the governance choices $n \in \{0, 1\}$. As shown above, ℓ stays passive and holds $y^u\boldsymbol{\iota}$ if $\kappa \geq y^u\pi(1,1)$. If $y^u\pi(1,1) > \kappa \geq \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u$, cost-cutting becomes incentive-compatible at $y^u\boldsymbol{\iota}$ and staying passive requires a portfolio of $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$. Let $\Delta(\kappa)$ denote the difference $\hat{U}_\ell(\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}) - \left(\hat{U}_\ell(y^u\boldsymbol{\iota}) - \kappa\right)$. Note that $\Delta(\kappa)$ has the following properties: $\Delta''(\kappa) < 0$, $\Delta(0) < 0$, and $\lim_{\kappa \to y^u\pi(1,1)} \Delta(\kappa) > 0$ because, in this case, ℓ's portfolio when choosing $n = 0$ gets arbitrarily close to the unconstrained optimal portfolio. As a result $\Delta(\kappa)$ has a unique root $\overline{\kappa}_1^b$ in $\kappa \in (0, y^u\pi(1,1))$. If $\overline{\kappa}_1^b > \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u > \underline{\kappa}^b$, then ℓ chooses cost cutting if κ is between $\underline{\kappa}^b$ and $\overline{\kappa}^b \equiv \overline{\kappa}_1^b$. If, however, $\overline{\kappa}_1^b < \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u$ then ℓ prefers staying passive to cost-cutting in the entire range $y^u\pi(1,1) > \kappa \geq \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u$ and we must have $\overline{\kappa}^b < \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u$. As shown above, ℓ's utility from staying passive increases in κ for $\kappa \in (0, \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u)$, while the utility from cost-cutting decreases in this range. Moreover, at $\kappa \to 0$, ℓ strictly prefers cost cutting because $\frac{\kappa}{\pi(1,1)}$ goes to zero in this limit while $\lim_{\kappa \to 0} \hat{U}(\{\overline{y}, \underline{y}\}) > 0$. So, if $\overline{\kappa}_1^b < \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u$, there must exist a threshold $\overline{\kappa}_2^b \in (0, \frac{N\pi(1,N)-\pi(1,1)}{(N-1)}y^u)$ such that ℓ prefers cost cutting to staying passive if and only if κ is below the threshold. If $\overline{\kappa}_2^b \leq \underline{\kappa}^b$, cost cutting is never optimal and we set $\overline{\kappa}^b = \underline{\kappa}^b$. However, if $\overline{\kappa}_2^b > \underline{\kappa}^b$,

[3] In Case #1, we would have $\underline{\kappa}^{(b)} = 0$, since ℓ never chooses $n = N$, and $\overline{\kappa}^{(b)}$ equal to the right-hand side of Inequality (B1).

then ℓ chooses cost cutting if $\underline{\kappa}^b \leq \kappa < \overline{\kappa}_2^b \equiv \overline{\kappa}^b$.

Case #3. If $\pi(1,N) > \pi(1,1)$, we have $\overline{\kappa}^b = \underline{\kappa}^b$, since ℓ never chooses $n = 1$ in equilibrium, and $\underline{\kappa}^b$ equal to the right-hand side of Inequality (B7), since that expression is lower than $y^u\pi(1,1)$.

B.6 Proof of Proposition 8.

Here we define $\tilde{m} = m\frac{\gamma_\ell}{\gamma_S}$. Consider the following setting with negative spillovers and $N = 2$ firms: $\pi(0,0) = \pi(0)^{nc}$, $\pi(1,1) = \pi(1)^{nc}$, and $\pi(1,2) = 0$. Without competition, ℓ cuts cost in both firms if and only if $\kappa < \underline{\kappa}^{nc}$ (see Proposition 1). With competition, ℓ holds the unconstrained optimal portfolio $y_{j\ell} = y^u$ and stays passive ($n = 0$) if $\kappa > y^u\pi(1,1)$. Otherwise, she chooses the best of the two following strategies: (i) $y_{j\ell} = \frac{\kappa}{\pi(1,1)}$ and $n = 0$, and (ii) $y_{j\ell} = y^u$ and $n = 1$. Comparing U_ℓ for these two strategies implies that ℓ prefers (ii) if and only if $\kappa < \frac{\pi(1,1)\left(2\gamma_\ell\sigma^2+\tilde{m}\pi(1,1)\right)-\sqrt{\tilde{m}\pi(1,1)^3(4\gamma_\ell\sigma^2+\tilde{m}\pi(1,1))}}{2\gamma_\ell(\tilde{m}+2)\sigma^2} \equiv \tilde{\kappa}$. We can then show that $\underline{\kappa}^{nc} < \tilde{\kappa}$ so that for $\kappa \in (\underline{\kappa}^{nc}, \tilde{\kappa})$, ℓ is active in one firm with competition but passive without competition. If $\kappa \in (0, \underline{\kappa}^{nc})$, however, ℓ is active in one firm with competition and in two firms without competition.

If competition becomes a zero-sum game, i.e., if $\Pi(\tilde{n}) - \Pi(0) \longrightarrow 0$, we have shown in the main text that it is always incentive-compatible for ℓ to stay passive at the unconstrained optimal portfolio. Therefore, ℓ always stays passive and hold y^u in each firm.

B.7 Proof of Proposition 9.

We start with the planner's choice for $\mathbf{y}_\ell$. The social planner chooses $\mathbf{y}_\ell$ to solve:

$$\min_{\mathbf{y}_\ell} \mathbf{y}_\ell'\mathbf{y}_\ell + \frac{1}{\tilde{m}}(\boldsymbol{\iota} - \mathbf{y}_\ell)'(\boldsymbol{\iota} - \mathbf{y}_\ell) \tag{B21}$$

where we have used the market clearing condition and $\tilde{m} = m\frac{\gamma_\ell}{\gamma_S}$. It follows that the social planner sets $\mathbf{y}_\ell = \frac{1}{\tilde{m}+1}\boldsymbol{\iota}$. Since the governance action does not affect $\mathbf{y}_\ell'\mathbf{y}_\ell + \frac{1}{\tilde{m}}(\boldsymbol{\iota} - \mathbf{y}_\ell)'(\boldsymbol{\iota} - \mathbf{y}_\ell)$, it follows that the social planner sets $n = \boldsymbol{\iota}'\mathbf{g}$ to solve:

$$\max_{\mathbf{g}} PMS(n) - \kappa n. \tag{B22}$$

B.8 Proof of Proposition 10.

We start with part (b). For a fixed $\mathbf{g}$, ℓ sets $\mathbf{y}_\ell$ to maximize $\frac{\gamma_S}{m}\mathbf{y}'_\ell(\boldsymbol{\iota}-\mathbf{y}_\ell)-\frac{\gamma_\ell}{2}\mathbf{y}'_\ell\mathbf{y}_\ell$, which leads to $\mathbf{y}_\ell = y^u\boldsymbol{\iota}$. We have shown in the Proof of Proposition 9 that the social planner sets $\mathbf{y}_\ell = \frac{\gamma_S}{m\gamma_\ell+\gamma_S}\boldsymbol{\iota}$.

Next, we prove the first part by contradiction. Suppose ℓ holds the planner's optimal portfolio $\mathbf{y}_\ell = \frac{\gamma_S}{m\gamma_\ell+\gamma_S}\boldsymbol{\iota}$. The incentive-compatible governance choice is $\hat{n} \equiv \arg\max_n\{\frac{\gamma_S}{m\gamma_\ell+\gamma_S}\boldsymbol{\iota}'\boldsymbol{\pi} - n\kappa\}$. Since $\frac{\gamma_S}{m\gamma_\ell+\gamma_S} > \frac{\gamma_S}{m\gamma_\ell+2\gamma_S} = y^u$, $\hat{n} \geq \overline{n}$, where $\overline{n} = \arg\max_n\{y^u\boldsymbol{\iota}'\boldsymbol{\pi} - n\kappa\}$. Hence, ℓ is strictly better off holding the unconstrained optimal portfolio because it leads to a strictly higher return and a weakly lower governance cost. So, ℓ never chooses $(\mathbf{y}_\ell, \mathbf{g})$ that solves the planner's problem.

Finally, consider part (a). For a fixed portfolio $\mathbf{y}_\ell$, ℓ sets $\mathbf{g}$ to maximize $\mathbf{y}'_\ell\boldsymbol{\pi} - \kappa n$, while the planner maximizes $PMS(n) - \kappa n = \boldsymbol{\iota}'\boldsymbol{\pi} + CS(n) - \kappa n$. The planner thus never chooses n such that $PMS(n) - \kappa n < PMS(0)$, while ℓ might do that if it increases $\mathbf{y}'_\ell\boldsymbol{\pi}$. For an example, see the equilibrium characterization in the Bertrand model (Online Appendix B.1), where ℓ facilitates collusion even if it is socially undesirable (since $PMS(N) < PMS(0)$, as in Appendix B.10).

B.9 Proof of Lemma 2.

Suppose industry profits $\Pi(n) = \sum_{j=1}^N \pi_j = n\pi(1,n) + (N-n)\pi(0,n)$ are concave and maximized at $\hat{n} \in \{1,\ldots,N\}$. Next, we consider the portfolio vector $\mathbf{y}' = y^u\boldsymbol{\iota}$. We will then proceed in two steps. First, we will show that with portfolio $\mathbf{y}'$, ℓ would never become active in more than $\hat{n}$ firms. Second, we will show that $\mathbf{y}'$ is always preferred to any $\mathbf{y}''$ that induces $n'' > \hat{n}$.

ℓ prefers becoming active in $\hat{n}$ firms over becoming active in $n'' > \hat{n}$ firms if:

$$y^u\left[\hat{n}\pi(1,\hat{n}) + (N-\hat{n})\pi(0,\hat{n})\right] - \hat{n}\kappa > y^u\left[\hat{n}\pi(1,n'') + (N-\hat{n})\pi(0,n'')\right] - n''\kappa. \tag{B23}$$

This condition always holds because industry profits are maximized at $\hat{n}$ and $n'' > \hat{n}$.

Next, we show that $\mathbf{y}'$ is preferred to any $\mathbf{y}''$, which is incentive-compatible with $n'' > \hat{n}$. Hence, our goal is to show that:

$$U_\ell(\mathbf{y}', n') > U_\ell(\mathbf{y}, n'') \tag{B24}$$

with $n^{'} \leq \hat{n}$ reflecting ℓ's incentive-compatible choice at $\mathbf{y}'$. We also know that $U_\ell(\mathbf{y}', n') \geq$

$U_\ell(\mathbf{y}', \hat{n})$. Note that:

$$U_\ell(\mathbf{y}', \hat{n}) = \gamma_\ell \sigma^2 N \left[\frac{1}{\tilde{m}} y^u \left(1 - y^u\right) - \frac{1}{2}(y^u)^2 \right] - \hat{n}\kappa. \tag{B25}$$

For any $\mathbf{y}''$ with elements $\overline{y}''$ and $\underline{y}''$, which is incentive-compatible with $n'' > \hat{n}$ we have that:

$$U_\ell(\mathbf{y}, n'') = \gamma_\ell \sigma^2 \left[n'' \frac{1}{\tilde{m}} \overline{y}''(1 - \overline{y}'') - \hat{n} \frac{1}{2} \overline{y}'' \overline{y}'' + (N - \hat{n}) \frac{1}{\tilde{m}} \underline{y}''(1 - \underline{y}'') - \frac{1}{2} \underline{y}'' \underline{y}'' \right] - n''\kappa. \tag{B26}$$

So $U_\ell(\mathbf{y}', \hat{n}) > U_\ell(\mathbf{y}, n'')$ because $n'' > \hat{n}$ and $y = y^u$ maximizes the risk-return trade-off for all j.

B.10 Proof of Proposition 11.

Preliminaries. We start by showing that in the Bertrand setting in Example 1, $n = N$ is always socially undesirable, and $n = 1$ is always socially desirable when it is selected in equilibrium. Governance choice n is socially desirable iff $PMS(n) - \kappa n \geq PMS(0)$ (Definition 1). The equilibrium price given governance choice $n \in \{0, 1, N\}$ is $\rho^*(n)$. For $n = N$, we have

$$PMS(N) - PMS(0) = \int_0^{D(\rho^*(N))} \left[D^{-1}(Q) - c\right] dQ - \int_0^{D(\rho^*(0))} \left[D^{-1}(Q) - c\right] dQ, \tag{B27}$$

where $D^{-1}(\cdot)$ represents the inverse of the product market demand $D(\rho)$.

$\rho^*(N) > \rho^*(0)$ implies $PMS(N) < PMS(0)$. It follows that $n = N$ is always socially undesirable, since $PMS(N) < PMS(0)$ implies $PMS(N) - \kappa N < PMS(0)$.

Next, we consider $n = 1$; we have

$$\begin{aligned} PMS(1) - \kappa - PMS(0) &= \pi(1,1) - \kappa + CS(1) - CS(0) \\ &= \pi(1,1) - \kappa + \int_0^{D(\rho^*(1))} D^{-1}(Q)dQ - \int_0^{D(\rho^*(0))} D^{-1}(Q)dQ. \end{aligned} \tag{B28}$$

$\rho^*(0) \leq \rho^*(1)$ implies $CS(1) > CS(0)$ (and, thus, $PMS(1) > PMS(0)$).

For $n = 1$ to be selected in equilibrium, we must have $y_{1\ell}\pi(1,1) - \kappa > 0$ (by the IC constraint $G(1, \mathbf{y}_\ell) > G(0, \mathbf{y}_\ell)$, where $G(n, \mathbf{y}_\ell)$ is ℓ's payoff from exerting governance in n firms, given portfolio $\mathbf{y}_\ell$, as described in Eqn. (A.4)), where $y_{1\ell} \leq 1$. Since $y_{1\ell}\pi(1,1) - \kappa > 0$ implies $\pi(1,1) - \kappa > 0$, it follows that the difference in Eqn. (B28) is always positive and, thus, $n = 1$ is always socially desirable when it is an equilibrium outcome.

For the remainder of this proof, we use the equilibrium characterization in Online Appendix B.1 for the details of how ℓ's equilibrium choices depend on the model parameters.

Limiting engagement. ℓ is only willing to facilitate collusion if κ is sufficiently small. Hence, an increase in the governance cost κ always (weakly) decreases socially undesirable governance.

Next, we show that the regulation may increase or decrease socially-desirable governance (*cost cutting*). If $\pi(1, N) \leq \frac{1}{N}\pi(1, 1)$, ℓ cuts costs if κ is sufficiently small. Increasing κ then (weakly) *decreases* socially-desirable governance. If $\frac{1}{N}\pi(1, 1) < \pi(1, N) \leq \pi(1, 1)$, ℓ facilitates collusion for κ small, cuts costs for intermediate values of κ, and stays passive otherwise. Increasing κ then (weakly) increases socially-desirable governance.

Last, we show that an increase in κ has an ambiguous effect on U_ℓ. Consider the case $\pi(1, N) > \pi(1, 1)$, where ℓ facilitates collusion if κ is sufficiently small, and stays passive otherwise. If ℓ continues to facilitate collusion after the increase in κ, ℓ is worse off. Otherwise (i.e., if the increase in κ leads to $\kappa \geq y^u \pi(1, N)$), ℓ switches to staying passive and holding $y^u \boldsymbol{\iota}$, making her better off.

Limiting horizontal shareholding. First, note that any additional constraint on ℓ's portfolio choice makes the investor worse off by revealed-preference.

Next, we show that the regulation may increase or decrease socially-desirable governance (*cost cutting*). If $\pi(1, N) \leq \frac{1}{N}\pi(1, 1)$, ℓ cuts costs in firm 1 and holds $y^u \boldsymbol{\iota}$ if κ is sufficiently small. Consider a regulation $\overline{y}^r = \underline{y}^r = \frac{\kappa}{\pi(1,1)}$ and suppose that ℓ cuts costs without the regulation. With the regulation, ℓ can then no longer hold $y^u \boldsymbol{\iota}$, so she might switch from $n = 1$ to $n = 0$, since the optimal passive portfolio is instead still feasible under the regulation. If $\frac{1}{N}\pi(1, 1) < \pi(1, N) \leq \pi(1, 1)$ and $\kappa < \frac{N\pi(1,N) - \pi(1,1)}{(N-1)} y^u$, ℓ facilitates collusion when she holds the portfolio $y^u \boldsymbol{\iota}$, and cuts costs when she holds $\overline{y}$ in firm 1 and $\underline{y}$ in the others, where $\overline{y}$ and $\underline{y}$ are described in Eqn. (B5), and satisfy $\overline{y} > y^u > \underline{y}$. Consider a regulation $\overline{y}^r = \underline{y}^r = \underline{y}$ and suppose that ℓ facilitates collusion without the regulation. With the regulation, ℓ can then no longer hold $y^u \boldsymbol{\iota}$, so she might switch from $n = N$ to $n = 1$, since the optimal cost-cutting portfolio is instead still feasible under the regulation.

Last, we show that the regulation may increase or decrease socially-undesirable governance (*facilitating collusion*). If $\pi(1, N) > \pi(1, 1)$, ℓ facilitates collusion and holds $y^u \boldsymbol{\iota}$ if κ is sufficiently small; otherwise, she remains passive and holds $\frac{\kappa}{\pi(1,N)}\boldsymbol{\iota}$. Consider a regulation $\overline{y}^r = \underline{y}^r = \frac{\kappa}{\pi(1,1)}$ and suppose that ℓ facilitates collusion without the regulation. With the regulation, ℓ can then no longer hold $y^u \boldsymbol{\iota}$, so she might switch from $n = N$ to $n = 0$, since the optimal passive portfolio is instead still feasible under the regulation. If $\frac{1}{N}\pi(1,1) < \pi(1,N) \leq \pi(1,1)$, when κ is in an intermediate range, ℓ cuts costs and holds $\overline{y}$ in firm 1 and $\underline{y}$ in the others, where $\overline{y}$ and $\underline{y}$ are described in Eqn. (B5). Consider a regulation $\overline{y}^r = y^u < \overline{y}$ and $\underline{y}^r = 0$ and suppose that ℓ cuts costs without the regulation. With the regulation, ℓ can then no longer hold the optimal cost-cutting portfolio, so she might switch from $n = 1$ to $n = N$, since the portfolio $y^u\boldsymbol{\iota}$ is instead still feasible under the regulation.

Traditional competition policy. In the Bertrand model, firms set prices above the competitive equilibrium level if ℓ facilitates collusion ($n = N$). So this regulation is equivalent to reducing $\pi(1,N)$. In what follows, $\hat{U}_\ell(\mathbf{y})$ denotes ℓ's certainty equivalent (its expected utility gross of the governance cost κn) for a given portfolio $\mathbf{y}$.

If $\pi(1,N) > \pi(1,1)$, ℓ stays passive ($n = 0$) and holds a portfolio $\frac{\kappa}{\pi(1,N)}\boldsymbol{\iota}$ if κ is sufficiently large, and facilitates collusion ($n = N$) and holds $y^u\boldsymbol{\iota}$ otherwise. Under the regulation, $\hat{U}_\ell(\frac{\kappa}{\pi(1,N)}\boldsymbol{\iota})$ goes up, since $\frac{\kappa}{\pi(1,N)}\boldsymbol{\iota}$ increases when $\pi(1,N)$ decreases, while $\hat{U}_\ell(y^u\boldsymbol{\iota})$ does not change. Within this interval, ℓ is then always (weakly) better off under the regulation, and may switch from $n = N$ to $n = 0$ in equilibrium, in which case socially-undesirable governance decreases.

If $\pi(1,N) \leq \frac{1}{N}\pi(1,1)$, ℓ chooses $n = 0$ and holds $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$ if κ is sufficiently large, and cuts costs in firm 1 ($n = 1$) and holds $y^u\boldsymbol{\iota}$ otherwise. Within this interval, a decrease in $\pi(1,N)$ has no impact on the equilibrium outcomes. If $\frac{1}{N}\pi(1,1) < \pi(1,N) \leq \pi(1,1)$, ℓ chooses $n = 0$ and holds $\frac{\kappa}{\pi(1,1)}$ if κ is sufficiently large, and chooses $n = N$ and holds $y^u\boldsymbol{\iota}$ if κ is sufficiently small; if κ is in an intermediate range, ℓ chooses $n = 1$ and holds $\overline{y}$ in firm 1 and $\underline{y}$ in the others, where $\overline{y}$ and $\underline{y}$ are described in Eqn. (B5). Under the regulation, the optimal cost-cutting portfolio becomes more attractive, since the IC set becomes larger.[4] So, ℓ may only switch from $n = N$ to $n = 1$, or from $n = 0$ to $n = 1$. Within this interval, ℓ is then always at least weakly better off under the regulation, and socially-desirable (socially-undesirable) governance increases (decreases).

Last, we need to consider the case where the regulation moves the parameters across intervals. Suppose $\pi(1,N)$ decreases from π' to π'' under the regulation. First, consider the case where we go from $\pi' > \pi(1,1)$ to $\pi'' < \frac{1}{N}\pi(1,1)$. In this case, socially-desirable governance increases and socially-undesirable governance decreases, since ℓ goes from choosing

[4] By Proposition 4, at the optimal cost-cutting portfolio, the IC constraint is binding for $\tilde{n} = N$, while is slack for all $\tilde{n} = 0$. We can rewrite the IC constraints as $G(n, \mathbf{y}_\ell) - G(\tilde{n}, \mathbf{y}_\ell)$, where the function $G(\cdot,\cdot)$ captures ℓ's payoff from exerting governance (see Eqn. A.4). $\pi(1,N)$ thus affects the optimal cost-cutting portfolio through the IC constraint $G(1,\mathbf{y}_\ell) \geq G(N,\mathbf{y}_\ell)$. Since this constraint becomes slacker when $\pi(1,N)$ decreases (since $G(N,\mathbf{y}_\ell)$ decreases), the set from which ℓ chooses the optimal cost-cutting becomes larger.

between $n = 0$ and $n = N$ for $\pi' > \pi(1,1)$, to choosing between $n = 0$ and $n = 1$ for $\pi'' < \frac{1}{N}\pi(1,1)$. ℓ is also better off, since her expected utility goes from $\max\{\hat{U}_\ell(\frac{\kappa}{\pi'}\boldsymbol{\iota}), \hat{U}_\ell(y^u\boldsymbol{\iota}) - N\kappa\}$ to $\max\{\hat{U}_\ell(\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}), \hat{U}_\ell(y^u\boldsymbol{\iota}) - \kappa\}$, where $\hat{U}_\ell(\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}) \geq \hat{U}_\ell(\frac{\kappa}{\pi'}\boldsymbol{\iota})$ since $\pi' > \pi(1,1)$, and $\frac{\kappa}{\pi'} < y^u$, so that $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$ is closer to the unconstrained optimum portfolio. Second, consider the case where we go from $\pi' > \pi(1,1)$ to $\frac{1}{N}\pi(1,1) < \pi'' < \pi(1,1)$. By the same logic as before, the optimal passive portfolio becomes more attractive under the regulation (increases from $\frac{\kappa}{\pi'}\boldsymbol{\iota}$ to $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$). Hence, if ℓ prefers choosing $n = 0$ and holding $\frac{\kappa}{\pi'}\boldsymbol{\iota}$ to choosing $n = N$ and holding $y^u\boldsymbol{\iota}$ before the regulation, it also prefers choosing $n = 0$ and holding $\frac{\kappa}{\pi(1,1)}\boldsymbol{\iota}$ to choosing $n = N$ and holding $y^u\boldsymbol{\iota}$ after the regulation. Also in this case, then socially-desirable governance increases ($n = 1$ is possible in equilibrium only under the regulation) and socially-undesirable governance decreases (since ℓ would not switch from $n = 0$ to $n = N$, but she may switch from $n = N$ to $n = 0$ or $n = 1$). ℓ is also weakly better off under the regulation. Finally, consider the case where we go from $\frac{1}{N}\pi(1,1) < \pi'' < \pi(1,1)$ to $\pi'' < \frac{1}{N}\pi(1,1)$. $n = N$ is no longer possible in equilibrium at π'', so socially-undesirable governance decreases. The optimal passive portfolio doesn't change across interval, but the optimal cost-cutting portfolio becomes more attractive, since it goes from the portfolio with $\overline{y}$ and $\underline{y}$ to the unconstrained optimum. So, ℓ is weakly better off and socially-desirable governance increases under the regulation.

C Model Extensions

In this appendix, we explore the robustness of our main results to alternative assumptions. Below, we briefly discuss the setup and main insights of each extension of our baseline model. We provide a complete analysis of each variation of the model in the remainder of the appendix.

Multiple large investors Our model studies the portfolio and governance choices in a setting with a single large investor, and a continuum of small investors who behave perfectly competitively. Even though ownership has become increasingly concentrated over the last two decades, most public firms have multiple large investors (see, e.g., Edmans and Holderness 2017; Lewellen and Lewellen 2022b), with potentially different exposures and incentives. Online Appendix C.1 considers a model variation featuring multiple large investors as well as small investors.

Holding the other large investors' choices fixed, ℓ's problem is the same as in the main model:[5] ℓ continues to have more incentives to influence competition when it holds a large diversified portfolio, and more incentives to do traditional governance when it is more exposed to a subset of firms. So, the key tradeoffs of the main model continue to hold in this extension.

When we let the other large investors choose their strategies, the equilibrium analysis becomes more complex, as we have to worry about unilateral deviations. For example, if all investors are expected to build a large diversified portfolio and soften competition, ℓ may want to deviate and build a portfolio that leads to a different governance action. Investors realize that ℓ deviated after submitting their demands, so they may have to reoptimize and change their governance choice. Two examples illustrate how these considerations affect the equilibrium characterization: one where the large investors choose between facilitating collusion among firms and staying passive; the other where they choose between promoting competition and staying passive. The analysis of these two polar cases offers a number of interesting insights.

In the first example, the number of large investors (L) plays a similar role to the mass of small investors (m) in the main model. When L increases, all investors hold smaller positions as the equity market becomes more competitive. So, the insight that large investors are more likely to influence firms' competition when the equity market is more concentrated (which now means when m and/or L are smaller) continues to hold here. Interestingly, this can be the case even when the investors split the governance cost of influencing competition (i.e., each investor pays a cost κ/L), as their returns may decrease faster than the governance cost when L increases.[6]

In the second example, we look at the reverse setting: Firms are colluding on their own, but large investors can push them to deviate and compete more aggressively. Our objective is to understand if equilibria where each investor buys a larger stake in a certain firm, and then pushes it to compete more aggressively later on, are easier to sustain here. We show that, under certain conditions, there is no equilibrium where investors choose to promote competition. The intuition is similar to the main model: investors make higher returns if they hold a diversified portfolio. So, any potential equilibrium where ℓ is expected to hold a larger stake in firm j and push it to compete more aggressively admits profitable deviation by the investor.

[5]The main difference is that some firms may now have higher profits than others even when ℓ does not exert governance, as the other large investors may exert governance only in a subset of firms. So, ℓ may now want to hold an unbalanced portfolio even when it does not exert any governance.

[6]The collusion example also highlights another reason why large investors may be more likely to soften competition when there is fewer of them: Coordination problems. Perhaps not surprisingly, we show that if all large investors need to be on board to soften competition (e.g., to push for an anti-competitive agreement between firms), there is always an equilibrium where they do not try to soften competition even if they hold a large diversified portfolio, as they expect the other investors to be against it.

Generalizing the governance cost In the main model, we impose few assumptions on how the firms' profits depend on ℓ's governance choice, but put more structure on the governance cost, which we assume is proportional to the number of firms n where ℓ exerts governance. In equilibrium, the share prices fully reflect the firms' profits, but do not incorporate the governance cost. So, the governance cost directly affects ℓ's portfolio choice at $t = 1$, while the profit function $\pi(g_j, n)$ only affects this choice *indirectly*, through the incentive-compatible governance actions at $t = 2$. In Online Appendix C.2, we study a variation of the model where the cost of governance is described by a generic cost function $\kappa(n)$.

The equilibrium machinery is similar to the main model. Let $\overline{n}_k$ denote the incentive-compatible governance choice at the unconstrained optimal portfolio. Since the unconstrained optimum maximizes her trading profits gross of the governance costs, ℓ chooses the best among this portfolio and all the others that induce a governance cost lower than $\kappa(\overline{n}_k)$. Since here $\kappa(n)$ may increase or decrease with n, however, ℓ may also choose $n > \overline{n}_k$ in equilibrium if $\kappa(n) < \kappa(\overline{n}_k)$.

Considering a generic cost function for governance allows us to capture a few interesting new effects. First, we can study the role that economies and diseconomies of scale in governance have on the equilibrium through the curvature of $\kappa(n)$. An intriguing insight is that economies of scale, which make it cheaper to do governance in multiple firms (e.g., proxy voting), may reduce overall governance. For example, in a setting with two firms, ℓ may switch from $n = 1$ to $n = 0$ when $\kappa(2)$ goes down: As $\kappa(2)$ decreases, ℓ needs larger deviations from a balanced portfolio to keep $n = 1$ incentive-compatible. So, the optimal portfolio that induces $n = 1$ becomes less attractive. If $\kappa(2)$ is still too high for ℓ to be willing to choose $n = 2$, she switches from $n = 1$ to $n = 0$ when $\kappa(2)$ decreases. Second, we can separate the costs of traditional governance and facilitating collusion (in the example with two firms, $\kappa(1)$ and $\kappa(2)$), which allows us to consider the case where facilitating collusion is the cheapest governance option.[7] Perhaps not surprisingly, ℓ chooses between facilitating collusion and staying passive if the former is the cheapest governance action. Otherwise, like in the main model, the investor trades off trading returns and governance costs across the two types of governance.

[7] This also allows to study the effect of directly imposing fines on investors that facilitate collusion among firms. In Section 5.3, we consider the case where the competition authority imposes fines on firms, but not on the investor, when the firms are caught colluding.

C.1 Multiple large investors

In this section, we analyze a variation of the main model featuring multiple large investors, indexed by $l \in \{1, \dots, L\} \equiv \mathcal{L}$. The large investors are ex-ante identical, so they have the same payoff functions, preferences, and strategy sets. The market clearing condition requires that:

$$\sum_{l=1}^{L} \mathbf{y}_l + m\mathbf{y}_S = \boldsymbol{\iota}. \tag{C1}$$

The small investors' optimal demand $\mathbf{y}_S$ is identical to that in the main model. Plugging in $\mathbf{y}_S$ into Eqn. (C1) and solving for the vector of stock prices $\mathbf{p}$ leads to the following expression:

$$\mathbf{p} = \boldsymbol{\pi} - \frac{\gamma_S \sigma^2}{m}(\boldsymbol{\iota} - \sum_{l=1}^{L} \mathbf{y}_l). \tag{C2}$$

Like in the main model, $\boldsymbol{\pi} = (\pi_1, \dots, \pi_N)$, where π_j represents firm j's expected profits.

Since here multiple investors may exert governance in firm j, we also have to specify how their governance efforts in the same firm aggregate. We denote the aggregate governance by all L investors in firm j by $G_j = G\left(g_{j1}, \dots, g_{jL}\right)$ where $G : \{0,1\}^L \to \{0,1\}$ is the aggregator function (more on this shortly). Firm j's expected profit depends on G_j and the aggregate governance efforts in the other firms: $\pi_j = \pi(G_j, \mathbf{G}'\boldsymbol{\iota})$ where $\mathbf{G} \equiv (G_1, \dots, G_N)$ and $\boldsymbol{\iota} = (1, \dots, 1)$.

We can write the expected utility for large investor l as:

$$U_l = \frac{\gamma_S \sigma^2}{m}\mathbf{y}_l'(\boldsymbol{\iota} - \sum_{l=1}^{L} \mathbf{y}_l) - \frac{\gamma_\ell \sigma^2}{2}\mathbf{y}_l{}'\mathbf{y}_l - \kappa_L n_l \tag{C3}$$

where $n_l \equiv \sum_{j=1}^{N} g_{jl}$ denotes investor l's aggregate governance effort across firms, and $\gamma_\ell \geq 0$ is the large investors' common risk-aversion parameter.

Lastly, since there are multiple large investors, an equilibrium here is a collection $\{\{y_{ij}\}, \mathbf{g_l}\}$ for $i \in \{l, S\}$, $l \in \mathcal{L}$, $j \in \mathcal{J}$ that is individually optimal and sequentially rational.

C.1.1 Preliminaries

The first step of the analysis is to characterize the equilibrium of a benchmark where the investors' governance choice is fixed, so there are no incentive-compatibility constraints to consider. This first step is similar to how we characterize the unconstrained optimal portfolio in the main model, except that here each large investor also needs to form conjectures about the other large investors' demand, and these conjectures need to be satisfied in equilibrium.

Proposition C.1 (Equilibrium without governance) *Holding the governance choices fixed, an equilibrium always exists and is unique. In equilibrium, all investors hold the same portfolio* $\mathbf{y}^* = \frac{\gamma_S}{\gamma_l m + (L+1)\gamma_S}\boldsymbol{\iota}$.

If there is no governance choice and, thus, no IC constraints to satisfy, the large investors' objective functions are symmetric. The investors' demands are strategic substitutes, and equilibria where some investors have zero demand are not feasible. This implies that the equilibrium is unique and symmetric, similar to the traditional Cournot games (e.g., Novshek, 1985).

The next step is to show that, holding the other investors' governance and portfolio decisions fixed, l's problem is similar to the main model. The only difference is that her objective functions at both the portfolio and governance stages are no longer symmetric across firms since the other large investors may exert governance or acquire large positions in a subset of firms. This means that the returns from trading shares or doing governance for l may be different across firms.

Consider large investor l and fix the governance efforts of the other $L-1$ large investors. We collect their governance efforts in the vector $\mathbf{g}_{-l}$. We write firm j's expected profits as a function of investor l's governance effort in firm j and all other firms, as well as the governance efforts of all other investors in all firms: $\pi_j(g_{jl}, \mathbf{g}_l) = \pi(g_{jl}, \mathbf{g}_l; \mathbf{g}_{-l})$. The optimal portfolio that is incentive-compatible with a given governance strategy $\mathbf{g}_l$ solves the following problem.

Proposition C.2 (Optimal incentive-compatible portfolios; fixed $\mathbf{g}_{-l}$) *Investor l's optimal portfolio that is incentive-compatible with exerting governance in firms $j \leq n = \boldsymbol{\iota}'\tilde{\mathbf{g}}_l$ solves:*

$$\max_{\{y_{jl}\}} \sigma^2 \sum_{j=1}^{N} \left[\frac{\gamma_S}{m} y_{jl}(1-y_{jl}) - \frac{\gamma_\ell}{2} y_{jl}^2 \right] \tag{C4}$$

$$\text{s.t.} \sum_{j=1}^{n} y_{jl}\pi_j(1,\mathbf{g}_l) + \sum_{j=n+1}^{N} y_{jl}\pi_j(0,\mathbf{g}_l) - n\kappa \geq \sum_{j=1}^{\tilde{n}} y_{jl}\pi_j(1,\tilde{\mathbf{g}}_l) + \sum_{j=\tilde{n}+1}^{N} y_{jl}\pi_j(0,\tilde{\mathbf{g}}_l) - \tilde{n}\kappa$$

$$\forall \tilde{\mathbf{g}}_l \neq \mathbf{g}_l \quad \text{and } \tilde{n} = \boldsymbol{\iota}'\tilde{\mathbf{g}}_l.$$

Provided that the set of $\{y_{jl}\}$ that satisfies the incentive-compatibility constraint above is non-empty, a solution to Program (C4) always exists and is unique.

The properties of Program (C4) are similar to those in the main model (Proposition 3). The constrained set is compact, and the objective is continuous, so the existence and uniqueness of its solution follow from the Weierstrass Theorem. Also, as l's portfolio becomes more balanced, her payoff at the governance stage (i.e., the left-hand side of the incentive-compatibility constraint) becomes more aligned with industry profits ($\sum_{j=1}^{N} \pi_j$). Holding the other investors' strategies fixed, the key tradeoffs of the main model continue to hold here, since the investor will have more incentives to maximize industry profits when she holds a large, diversified portfolio.

When we let the other large investors choose their strategies, the equilibrium analysis becomes more complex, as we now have to worry about unilateral deviations. Suppose all investors are expected to build a large diversified portfolio and soften competition. l may now want to deviate and build a portfolio that leads to a different governance action. Investors realize that l deviated after submitting their demands, so they may have to reoptimize and change their governance choice. Next, we illustrate how these considerations affect the equilibrium characterization in two examples: facilitating collusion and promoting competition.

C.1.2 Facilitating collusion

First, we consider the case where large investors choose between staying passive and facilitating collusion. We assume that the N firms collude if and only if *all* L large investors choose to facilitate collusion. This is equivalent to setting the aggregator function to $G_j = \min_l(\{g_{jl}\})$.

In practice, this means that all the large shareholders of a firm must be on board for the firm to enter an anti-competitive agreement (e.g., price fixing) with other firms. The rationale for this assumption is that anti-competitive agreements are typically illegal under the law, and unpopular with the public, and a shareholder opposing the agreement may choose to expose it to the regulator or the public. This would then impose large legal and reputational costs on the firm and other large investors. So, if shareholders know that one of them opposes the anti-competitive agreement, they will prevent the agreement from going through.

The corresponding expected firm value is $\pi(1, N) \equiv \pi_{coll}$, and the per-firm cost for each investor is $\kappa_L = \chi\kappa$, where κ is the cost from the baseline model. The coefficient $\chi \in (0, 1]$ captures the idea that the cost of facilitating collusion might be shared among the L investors. If at least one large investor stays passive, the expected firm value is $\pi(0, 0) \equiv \pi_0 < \pi_{coll}$.

The following proposition characterizes the symmetric equilibria in this example.

Proposition C.3 (Facilitating collusion) *Let y^* denote the equilibrium portfolio when the governance choices $\{\{\mathbf{g}_l\}\}$ are fixed, $\Delta \equiv \pi_{coll} - \pi_0$, $\widetilde{m} \equiv \frac{\gamma_\ell}{\lambda}$, and $\lambda \equiv \frac{\gamma_s}{m}$. The symmetric equilibria of the game are:*

1. ***Unconstrained-passive***: *where ℓ holds y^* and stays passive. This equilibrium always exists, and is unique when $\kappa_L \geq \frac{\Delta}{1+L+\widetilde{m}}$.*

2. ***Collusion***: *where ℓ holds y^* and facilitates collusion among firms. This equilibrium exists iff the governance cost is small and l's price impact is large, that is, $\kappa_L < \frac{\Delta}{1+L+\widetilde{m}}$ and $\lambda \geq \underline{\lambda}_L$.*

The threshold $\underline{\lambda}_L$ is defined in Eqn. (C11).

Proposition C.3 calls for several comments. First, since all large investors need to be on board to facilitate collusion, there always exists an equilibrium where they stay passive even if they hold large diversified portfolios: if l expects the other investors to be against anti-competitive agreements, she has no incentive to try and push for one. Second, similar to the main model, the existence of an equilibrium where the large investors facilitate collusion depends on their price impact λ: when λ is small, the equity market is competitive, and the investors' returns are small. In this case, l is more concerned with governance costs than returns. So, even when she expects the others to facilitate collusion, she prefers to deviate and build a small portfolio that makes it incentive-compatible *not* to do so for her. If λ is large, the investors make significant returns, so they are relatively less concerned with the governance cost. So, the collusion equilibrium is only possible when λ is large and the equity market is not too competitive.

Lastly, an equilibrium where large investors escape any incentive to influence competition by building smaller portfolios (i.e., one where the IC constraint binds and the investors hold the largest portfolio compatible with not exerting governance, $\mathbf{y}_l = \frac{\Delta}{\kappa_L}\boldsymbol{\iota}$) no longer exists. The combination of strategic complementarity in the investors' governance choices and substitutability in their asset demands creates a free-riding problem: If l expects other investors to hold small portfolios and vote against collusion, she is better off building a large portfolio at $t = 1$, as she anticipates that collusion will *not* materialize, and she won't pay the governance cost at $t = 2$.

Lemma C.1 (Comparative statics) *Holding the governance cost κ_L fixed, the set of parameters for which the collusion equilibrium exists decreases with the number of large investors L. The opposite may hold when the investors share the governance costs, that is, when $\kappa_L = \frac{\kappa}{L}$, with $\kappa > 0$.*

The number of large investors (L) has a similar effect on the existence condition for the collusion equilibrium to that of the mass of small investors (m) in the main model. When L increases, all investors hold smaller positions as the equity market becomes more competitive. So, the insight that large investors are more likely to influence firms' competition when the equity market is more concentrated (which now means when m and/or L are smaller) continues to hold here. Interestingly, we show that this can be the case even when the investors split the governance cost of influencing competition (that is, each individual investor pays a cost κ/L), as their returns may decrease faster than the governance cost when L increases. It is also worth emphasizing that the coordination problem we described in our discussion of Proposition C.3 (the one where large investors stay passive when they expect the other to do so) is not present when there is a single large investor. So, also through this channel, multiple large investors make an equilibrium where investors facilitate collusion relatively less likely to occur.

C.1.3 (Not) Promoting competition

Next, we consider the opposite case where the investors choose between staying passive and *promoting* competition. This is equivalent to setting the aggregator function to $G_j = \max_l\{g_{jl}\}$.

The interpretation is that firms collude on their own (e.g., to keep prices higher than the competitive equilibrium), but a large investor l can push each firm j to deviate from the collusive agreement and maximize its profits. By the opposite logic as before, since anti-competitive agreements are typically illegal under the law, one large investor is enough to convince j to deviate from the agreement. Consistent with this interpretation, we assume: (a) industry profits Π decrease with $\sum_j G_j$, so that aggregate profits are larger when fewer firms deviate from the agreement; (b) unilaterally deviating from the collusive agreement is strictly profitable for the deviating firm; and (c) deviating is more profitable when fewer other firms deviate.

To simplify the exposition, we consider the case with as many large investors as firms, i.e., $L = N$. The mechanisms we describe here are also present with $L < N$ or $L > N$, but the analysis is more cumbersome. Our main objective is to understand if equilibria where l holds a larger stake in a specific firm and push it to compete more aggressively are easier to sustain.

Proposition C.4 (Not promoting competition) *Consider the set of equilibria where the large investors either play the exact same strategies ($\mathbf{y}_l = \mathbf{y}_{-l}$ and $\mathbf{g}_l = \mathbf{g}_{-l}$ for all $l \in \mathcal{L}$), or they play the same strategies but in different firms ($\mathbf{y}_l = \tilde{\mathbf{y}}_{-l}$ and $\mathbf{g}_l = \tilde{\mathbf{g}}_{-l}$ for all l where $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{g}}$ are permutations of $\mathbf{y}$ and $\mathbf{g}$). Within this set, the large investors never promote competition among firms.*

Proposition C.4 shows that, within the set of equilibria where the large investors either play the same strategies or they play the same strategies but in different firms, there is no equilibrium where investors choose to promote competition. The intuition is similar to the main model: investors make higher returns if they hold a diversified portfolio. So, any potential equilibrium where l is expected to hold a larger stake in firm j and push it to compete more aggressively admits profitable deviation by the investor.

Proof of Proposition C.1. First, note that in the absence of governance, the investors' problem collapses into a firm-by-firm problem. Differentiating U_l with respect to y_l leads to:

$$\frac{dU_l}{dy_l} = 1 - \sum_{l=1}^{L} y_l - y_l \left(1 + \frac{\gamma_\ell m}{\gamma_S}\right). \tag{C5}$$

At $y_l = 1$, we have $\frac{dU_l}{dy_l} < 0$. Similarly, $y_l = 0$ implies $\frac{dU_l}{dy_l} = 1 - \sum_{l=1}^{L} y_l < 0$. However, in this case, $\frac{dU_{-l}}{dy_{-l}} < 0$ so $\sum_{l=1}^{L} y_l = 0$, which leads to a contradiction. We conclude that $\frac{dU_l}{dy_l} = 0$ for all l.

Consider two investors, l and l'. Subtracting $\frac{dU_l}{dy_l}$ from $\frac{dU_{l'}}{dy_{l'}}$ leads to:

$$0 = (y_l - y_{l'}) \left(1 + \frac{\gamma_\ell m}{\gamma_S}\right) \tag{C6}$$

which implies that $y_l = y_{l'}$ for all $l, l' \in \mathcal{L}$ is the unique equilibrium. It follows that investor l's first-order condition equals:

$$1 - Ly^* - y^* \left(1 + \frac{\gamma_\ell m}{\gamma_S}\right) = 0 \Rightarrow y^* = \frac{1}{L + 1 + \frac{\gamma_\ell m}{\gamma_S}}. \tag{C7}$$

So, each investor holds $\frac{1}{L+1+\frac{\gamma_\ell m}{\gamma_S}}$ in each firm.

Proof of Proposition C.2. We reformulate the large investor's problem in the spirit of Grossman and Hart (1983). The objective function is identical to that in the main model and a concave function of l's position in firm j, y_{jl}. The incentive-compatibility constraints remain linear in y_{jl}, so we obtain a unique solution if the constrained set is non-empty. Unlike the main model, expected firm profits also depend on the governance efforts of other large investors.

Proof of Proposition C.3. First, consider an equilibrium where all L large investors hold the unconstrained optimal portfolio and stay passive. Given a position of y^* in each firm, investor l's incentive-compatibility constraint becomes:

$$Ny^*\pi_0 \geq Ny^*\pi_0 - N\kappa_L \Leftrightarrow N\kappa_L \geq 0. \tag{C8}$$

It follows that this equilibrium always exists.

Second, consider an equilibrium where large investors hold y^* and facilitate collusion. Investor l's incentive-compatibility constraint becomes:

$$Ny^*\pi_{coll} - N\kappa_L > Ny^*\pi_0 \Leftrightarrow \kappa_L < \frac{\Delta}{1+L+\tilde{m}}. \tag{C9}$$

To show that this is an equilibrium, we must rule out unilateral deviations at the initial stage when the investors choose their portfolios. Assuming that the other $L-1$ investors are willing to facilitate collusion, investor l is willing to do so as well as long as $y_l > \frac{\kappa_L}{\Delta}$. So, if the investor holds $y_l \leq \frac{\kappa_L}{\Delta}$, she is no longer willing to facilitate collusion and, thus, does not pay the governance cost κ_L. To rule out this deviation, we require that:

$$U_l(y_l = \frac{\kappa_l}{\Delta}, n_l = 0) < U_l(y^*, n_l = N) \Leftrightarrow \lambda > \underline{\lambda}_L \tag{C10}$$

where the threshold $\underline{\lambda}_L$ is defined as follows:

$$\underline{\lambda}_L \equiv \frac{2\Delta^2\kappa_L\left(\tilde{m}+L+1\right)^2}{\left(\tilde{m}+2\right)\left(\Delta-\kappa_L\left(\tilde{m}+L+1\right)\right)^2}. \tag{C11}$$

Proof of Lemma C.1. We first keep κ_L fixed and differentiate the threshold with respect to L:

$$\frac{d\underline{\lambda}_L}{dL} = -\frac{4\Delta^3\kappa\left(\tilde{m}+L+1\right)}{\left(\tilde{m}+2\right)\left(\kappa\tilde{m}-\Delta+\kappa+\kappa L\right)^3} > 0. \tag{C12}$$

Second, we assume that the cost is shared evenly, i.e., $\kappa_L = \frac{\kappa}{L}$. In this case, we find that:

$$\frac{d\underline{\lambda}_L}{dL} = \frac{2\Delta^2\kappa\left(\tilde{m}+L+1\right)\left(\tilde{m}\left(\kappa\tilde{m}+\Delta L+2\kappa(L+1)\right)+\kappa+L(\Delta(1-L)+\kappa(L+2))\right)}{\left(\tilde{m}+2\right)\left(\kappa\tilde{m}+\kappa+L(\kappa-\Delta)\right)^3} \tag{C13}$$

which can be positive or negative.

Proof of Proposition C.4. Note that there are only two candidates for symmetric equilibria if $L = N$. One in which all large investors remain passive and one in which each large investor becomes active in a different firm. Investor l's incentive-compatibility constraint implies that she would never become active in firm j if another investor l' is active in the same firm because it would only create costs. So, equilibria, in which investors become active in more than one firm, cannot exist because symmetry implies that multiple investors become active in the same firm. Next, we show that a symmetric equilibrium where l becomes active does not exist.

We proceed in three steps. We prove that (i) incentive-compatible portfolios must be unbalanced; (ii) the incentive-compatibility constraint must be binding for each investor; (iii) a profitable deviation from this candidate equilibrium always exists.

Consider a candidate equilibrium in which each investor becomes active in a separate firm. So, industry profits $\sum_j \pi_j = \Pi(N)$. Suppose investors hold a balanced portfolio with position y_l in each firm. Investor l's incentive compatibility constraint implies:

$$y_l \Pi(N) - \kappa_L \geq y_l \Pi(N-1). \tag{C14}$$

This constraint cannot hold because we assume that $\Pi(n') < \Pi(n'')$ for any $n' > n''$. So, by contradiction, we conclude that investors' incentive-compatible portfolios must be unbalanced.

Next, we rule out that the investors' incentive-compatibility constraint is slack. Again, we prove this by contradiction. Suppose the constraint is slack. In this case, the L large investors would face an unconstrained problem, which only admits an equilibrium in which all investors hold the same position in each firm, as shown above. By symmetry, the investors' portfolios must be balanced, which is a contradiction. Hence, each investor's constraint must be binding.

Finally, we show that a profitable deviation always exists. Consider investor l and suppose the investor deviates by choosing deviation portfolio $\mathbf{y}_l^{dev} \neq \mathbf{y}_l$. If the other investors continue to be active in their firms, this portfolio can never make it incentive-compatible for l to be more active than before. However, since l's incentive-compatibility constraint holds with equality, there must exist a portfolio $\mathbf{y}_l^{dev}$ that is (i) incentive-compatible with staying passive and (ii) arbitrarily close to $\mathbf{y}_l$. If this is the case, then it is always profitable for l to deviate and hold $\mathbf{y}_l^{dev}$. By doing so, she can save the governance cost $\kappa_L > 0$ while sacrificing an arbitrarily small amount of trading profits, i.e., U_l net of the governance cost. We conclude that a symmetric equilibrium does not exist in which large investors become active and promote competition.

C.2 Generalizing the governance cost

In this section, we explore a variation of our main model where the large investors' total governance cost is a general function $\kappa(n)$, where n is the number of firms where she exerts governance. In the main model, we have considered the special case where this function is linear, i.e., $\kappa(n) = \kappa n$. In this section, we only assume $\kappa(0) = 0$ and $\kappa(n) \geq 0 \forall n \in \mathcal{J}$.

C.2.1 Equilibrium analysis

Similar to the main model, we define $\overline{n}_k \equiv \arg\max_n \{y^u \boldsymbol{\iota}'\boldsymbol{\pi} - \kappa(n)\}$ as the incentive-compatible governance choice at the unconstrained optimal portfolio. ℓ thus only considers governance actions n such that $\kappa(n) \leq \kappa(\overline{n}_k)$, where the condition holds with inequality if and only if $n = \overline{n}_k$. Finding the optimal portfolios does not rely on the specific form for the cost. So, the equilibrium machinery is the same as in the main model. Proposition C.5 describes the investor's optimization problem when we use a general governance cost function $\kappa(n)$.

Proposition C.5 *The optimal portfolio that is incentive-compatible with exerting governance in firms $j \leq n$ solves the following problem:*

$$\max_{\underline{y},\overline{y}} \sigma^2 \left[n \left(\frac{\gamma_s}{m} \overline{y}(1-\overline{y}) - \frac{\gamma_\ell}{2} \overline{y}^2 \right) + (N-n) \left(\frac{\gamma_s}{m} \underline{y}(1-\underline{y}) - \frac{\gamma_\ell}{2} \underline{y}^2 \right) \right] \tag{C15}$$

$$s.t.\ \ n\overline{y}\pi(1,n) + (N-n)\underline{y}\pi(0,n) - \kappa(n) \geq \begin{cases} (n\overline{y} + (\tilde{n}-n)\underline{y})\pi(1,\tilde{n}) + (N-\tilde{n})\underline{y}\pi(0,\tilde{n}) - \kappa(\tilde{n}) & \text{if } \tilde{n} \geq n \\ \tilde{n}\overline{y}\pi(1,\tilde{n}) + ((n-\tilde{n})\overline{y} + (N-n)\underline{y})\pi(0,\tilde{n}) - \kappa(\tilde{n}) & \text{if } \tilde{n} < n \end{cases}$$

$$\forall \tilde{n} \in \{0, 1, \ldots, N\}, \quad \text{and} \quad \overline{y} \geq \underline{y}.$$

Provided that the set of $(\overline{y}, \underline{y})$ that satisfies the incentive-compatibility constraint above is non-empty, a solution to Program (C15) always exists and is unique.

Since the unconstrained optimum maximizes her trading profits gross of the governance costs, ℓ chooses the best among this portfolio and all those that induce a governance cost lower than $\kappa(\overline{n}_k)$. Since here $\kappa(n)$ may decrease with n, ℓ may choose $n > \overline{n}_k$ in equilibrium if $\kappa(n) < \kappa(\overline{n}_k)$. This also means that the relevant IC constraint in Program (6) may be one with $\tilde{n} < n$ (while it was always one with $\tilde{n} > n$ in the main model; see Proposition 4).

Proposition C.6 summarizes the key properties of the equilibrium choices.

Proposition C.6 *Let $\mathcal{N}_k$ be such that for any $n \in \mathcal{N}_k \subseteq \{0, \ldots, N\}$, then $\kappa(n) < \kappa(\overline{n}_k)$.*

1. *If it is incentive-compatible to not exert governance at the unconstrained optimal portfolio (i.e., $\overline{n}_\kappa = 0$), ℓ chooses $n^* = 0$ and holds that portfolio ($\mathbf{y}_\ell = y^u \boldsymbol{\iota}$);*

2. *Otherwise, ℓ picks the best between the unconstrained optimal portfolio and the portfolios that solve Program (C15) for $n \in \mathcal{N}_k$. If ℓ exerts governance in $n^* \neq \overline{n}_\kappa$ firms in equilibrium, she must be indifferent between doing governance in n^* or $\tilde{n}$ firms, for some $\tilde{n} \in \mathcal{N}_k \cup \overline{n}_\kappa$, with $\kappa(\tilde{n}) > \kappa(n^*)$.*

To illustrate the properties of the equilibrium, we describe an example with two firms. We set $\kappa(1) = \kappa$ and $\kappa(2) = \phi\kappa$ with $\kappa, \phi > 0$, and assume that industry profits Π are increasing in n.

We focus on the most interesting case where $\overline{n}_k = 2$, i.e., ℓ chooses $n = 2$ at the unconstrained optimal portfolio. If $\kappa(2) < \kappa(1) \Leftrightarrow \phi < 1$, ℓ chooses between the following two strategies: (i) $n^* = 2$ and $y^* = y^u$ in both firms; (ii) $n^* = 0$ and $y^* = \frac{\phi\kappa}{\Pi(2)-\Pi(0)}$ in both firms. The investor chooses $n^* = 2$ if $\kappa\phi$ is sufficiently small. $\phi < 1$ captures settings where facilitating collusion is the cheapest governance action available to the investor.

Next, suppose $\kappa(2) > \kappa(1) \Leftrightarrow \phi > 1$. This case can be used to capture two economic features. First, if we interpret $n = 2$ as ℓ's efforts to facilitate collusion, $\kappa(2) > \kappa(1)$ describes settings where facilitating collusion is more costly than the more traditional governance (e.g., cutting production costs). Second, if we interpret $n = 2$ as ℓ's efforts to do traditional governance in both firms, $\kappa(2) > \kappa(1)$ allows us to think about economies of scale ($\phi \in (1, 2)$) and diseconomies of scale ($\phi \in (2, \infty)$) in this type of governance. Economies of scale may be due to knowledge spillovers (e.g., ℓ learns about the industry from doing governance in firm 1, and can then apply the same knowledge when doing governance in firm 2). Diseconomies of scale may be due to resource constraints (e.g., the time the investor has to spend to engage in both firms).

With $\kappa(2) > \kappa(1)$, ℓ chooses between the following three strategies: (i) $n^* = 2$ and $y^* = y^u$ in both firms; (ii) $n^* = 0$ and $y^* = \min\{\frac{\phi\kappa}{\Pi(2)-\Pi(0)}, \frac{\kappa}{\Pi(1)-\Pi(0)}\}$ in both firms; (iii) $n^* = 1$ and $\overline{y}$ in firm 1 and $\underline{y} \le \overline{y}$ in firm 2, where $(\overline{y}, \underline{y})$ solve the following problem:

$$\max_{\underline{y},\overline{y}} \sigma^2 \left[\left(\frac{\gamma_s}{m} \overline{y}(1-\overline{y}) - \frac{\gamma_\ell}{2} \overline{y}^2 \right) + \left(\frac{\gamma_s}{m} \underline{y}(1-\underline{y}) - \frac{\gamma_\ell}{2} \underline{y}^2 \right) \right] \tag{C16}$$

$$\text{s.t.} \quad \overline{y}\pi(1,1) + \underline{y}\pi(0,1) - \kappa \ge (\overline{y} + \underline{y})\pi(1,2) - \phi\kappa$$

$$\overline{y}\pi(1,1) + \underline{y}\pi(0,1) - \kappa \ge (\overline{y} + \underline{y})\pi(0,0)$$

$$\overline{y} \ge \underline{y}.$$

Figure C1 illustrates the equilibrium in a numerical example. If ϕ is sufficiently small, ℓ is active in both firms and holds the unconstrained optimal portfolio. As ϕ increases, choosing $n = 2$ becomes increasingly expensive, and for a value of $\phi \approx 1.9$, ℓ becomes passive ($n = 0$) and holds a smaller, diversified portfolio. As ϕ increases further, the optimal position increases because $n = 2$ is a less attractive outside option for ℓ. For $\phi \approx 2.4$, the binding incentive constraint switches from $\tilde{n} = 2$ to $\tilde{n} = 1$ and (y_ℓ, U_ℓ) becomes invariant with respect to ϕ. Finally, for

$\phi > 3.2$, ℓ chooses $n = 1$ and tilts her portfolio toward the firm in which she is active. In this case, an increase in ϕ leads to an increase in $y_{1\ell}$ and $y_{2\ell}$, and to a decrease in the tilt, i.e., $y_{1\ell} - y_{2\ell}$.

Proposition C.7 below summarizes the insights described above.

Proposition C.7 *The large investor may exert governance in more or less firms when $\kappa(\overline{n}_\kappa)$ increases.*

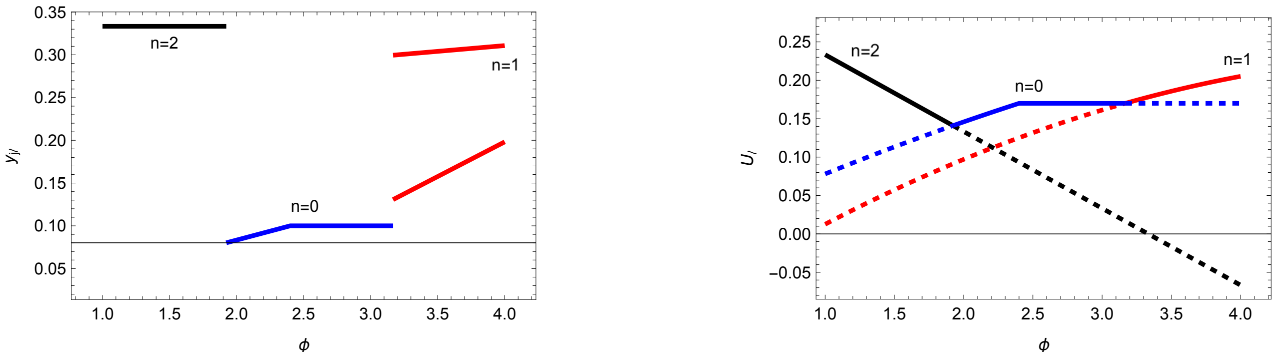

Figure C1: The left (right) panel plots ℓ's equilibrium portfolio (utility) and governance choices as a function of ϕ, where $\kappa(1) = \kappa$ and $\kappa(2) = \phi\kappa$, in an example with $N = 2$ firms. Parameters: $\pi(1, 2) = 1.2, \pi(0, 1) = \pi(0, 0) = 0, \kappa = 0.1$; all other parameters are set to 1.

Proof of Proposition C.5. We can consider the Proof of Proposition 2 and 3 and replace the linear cost κn by the generalized cost function $\kappa(n)$. It follows that the large investor holds $y_{j\ell} = \overline{y}$ in firms $j \leq n$ and $y_{j\ell} = \underline{y}$ in all other firms. As in the main model, we can show that if this is not the case, there is a profitable deviation for ℓ, which leaves the constraint unchanged but implies a higher portfolio return. For a fixed $n \in \mathcal{J} \cup \{0\}$, the large investor's objective function is thus the same as before. The incentive-compatibility constraints continue to be linear in $(\underline{y}, \overline{y})$. The only difference is the functional form for the governance costs.

Proof of Proposition C.6. It follows from the definition of $\overline{n}_k$ that the large investor holds $y^u \boldsymbol{\iota}$ at $n = \overline{n}_k$. Any governance choice n with $\kappa(n) > \kappa(\overline{n}_k)$ is strictly dominated because it yields (i) a lower portfolio return and (ii) higher governance costs. As a result, the large investor only considers governance options that are cheaper than $\overline{n}_k$, i.e., $n \in \mathcal{N}_k$. Suppose the large investor exerts governance in $n^* \in \mathcal{N}_k$ firms. The definition of $\mathcal{N}_k$ implies that at least one incentive-compatibility constraint must hold with equality. Otherwise, the investor would face an unconstrained problem and choose $n^* = \overline{n}_k$. As in the main model, the binding constraint must be $\tilde{n}$ such that $\kappa(\tilde{n}) > \kappa(n)$. The proof follows that for Proposition 4. If the incentive compatibility constraint was binding for an $\tilde{n}$ with $\kappa(\tilde{n}) < \kappa(n)$, the investor can choose an arbitrarily close portfolio vector such that the constraint is violated. By continuity, this deviation leads to the same risk-adjusted portfolio return for ℓ but a strictly lower governance cost.

Proof of Proposition C.7. This possibility result is shown in Figure C1. In this example, $\overline{n}_k = 2$ and so an increase in ϕ leads to an increase in $\kappa(\overline{n}_k)$. This example shows that an increase in ϕ leads to governance in fewer firms (i.e., it may lead to a transition from $n^* = 2$ to $n^* = 0$) for small values of ϕ. Vice versa, for larger values, an increase in ϕ can also lead to more governance.

References for Online Appendix

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