Encouraging Employee Engagement: The Role of Equity Pay

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Abstract

This paper presents a principal-agent model of workplace engagement linking information and production. Engaged workers are more productive and better informed about the firm. Equity pay imposes income risk that correlates with firm performance. When workers value information that improves decision making under uncertainty, they are motivated to exert productive effort that generates that information. This novel incentive mechanism suggests that—contrary to the conventional view—equity pay can motivate non-executive workers to exert productive effort, even in large firms with severe free-rider problems. The model also offers novel testable implications linking workers' financial planning to firm outcomes.

Keywords: Incentives, broad-based equity pay, worker engagement, income-risk resolution, non-executive compensation.

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Large firms routinely give equity pay (e.g., restricted stock, stock options, profit-sharing) to their non-executive workers (Kruse et al. 2010, Blasi et al. 2016). They often do so for the explicit purpose of providing incentives.[1] Many empirical studies also document evidence consistent with an incentivization role of equity pay for non-executive workers (e.g., Jones and Kato 1995, Babenko 2009, Hochberg and Lindsey 2010, Kim and Ouimet 2014). However, traditional economic theory asserts that equity pay cannot provide meaningful incentives to non-executive workers in large firms due to the free-rider problem (see Bergman and Jenter 2007). This paper argues that equity pay can motivate non-executive workers to exert productive effort that also helps resolve income risk.

To clarify the concept, consider a software engineer at a large software-as-a-service (SaaS) company with equity pay that vests over several years. Unable to sell her equity before vesting, she faces income risk—due to random shocks to firm value—that makes her interim financial planning (e.g., buying a home) more difficult. Exerting effort to better understand the firm's strategy not only makes her better able to assess the firm's prospects, but also more productive. For instance, a deeper understanding of the firm's strategy—say, providing value through automated work flow—allows her to create features that more accurately reflect that strategy, such as the automatic reporting of expenses. Feedback on such features (e.g., usage data) provides her with more information about the fit between the firm's value proposition and client needs. It is this information, which improves her financial planning, that motivates her to exert costly effort. This *income-risk resolution* mechanism may help explain some puzzling findings such as a positive correlation between the prevalence of equity pay and idiosyncratic stock return volatility. It also generates novel testable implications linking the worker's interim financial planning decisions (e.g., home purchase) to future firm performance.

[1] For instance, Morgan Stanley's *The State of Equity Plan Management 2022* report notes that a majority of surveyed firms identify aligning employees' financial interest and goals with those of the company as the number one or two objective of equity compensation.

I formalize this intuition in a parsimonious principal-agent model of workplace engagement featuring a risk-neutral firm, a continuum of risk-averse workers, and three dates: $t = 0$, $t = 1$, and $t = 2$. The firm's output at $t = 2$ aggregates the workers' individual contributions, which are not verifiable. A worker's individual contribution depends on that worker's effort, idiosyncratic worker-level noise, and a firm-level productivity shock, such as the effectiveness of the firm's strategy. The firm maximizes its output net of compensation costs. The model's main results rely on two innovations to this otherwise standard principal-agent setup: effort as workplace engagement and the worker's interim planning problem.

To emphasize the connection between information and production (e.g., Garicano 2000), I conceptualize effort as workplace engagement. Highly engaged workers generate individual contributions that are more valuable and more accurately reflect the effectiveness of the firm's strategy, which is ex-ante unknown to all.[2] Workers privately observes feedback from their individual contribution, which provides a more precise signal of the firm-level productivity shock when they are highly engaged.[3] This conceptualization of effort as workplace engagement links productive effort with information that helps resolve income risk due to uncertain firm performance.

The key economic force underlying the incentive provision role of equity pay for non-executive workers in this setting is the value they place on information that improves decision-making under uncertainty (e.g., Hirshleifer 1971). In the model, workers face a consumption-smoothing problem at $t = 1$.[4] Uncertainty about their income at $t = 2$, introduced by equity pay, reduces the efficiency of their consumption decisions at $t = 1$. The workers may consume either too much or too little in the first period relative to the

[2] For example, studies conducted by the Gallup organization document that measures of worker engagement are positively correlated with future performance (see Gallup's *State of the American Workplace Reports*).

[3] Many empirical studies document evidence consistent with non-executive workers being able to obtain private information that allow them to better predict the firm's future performance than the market can (e.g., Huddart and Lang 2003, Babenko et al. 2011, Farhadi and Nanda 2021).

[4] Alternatively, workers may derive psychic benefits from the early resolution of uncertainty resolution (e.g., Epstein and Zin 1989).

realized income in the second period. Workplace engagement effort generates information that helps resolve the income risk associated with equity pay and improve their interim consumption decisions. It is the value of this information that allows equity pay to incentivize workplace engagement effort in my framework.

The income-risk resolution mechanism in my framework has several implications. First, the theoretical moral hazard literature emphasizes that risky performance pay provides incentives because the underlying performance measure increases with effort (i.e., high likelihood ratio for high performance). However, as firm size grows, each worker's individual impact on firm performance diminishes, thereby weakening their incentives to exert effort in response to equity pay. As Holmström (1982, p. 325) points out, the cost effectiveness of equity pay becomes "limited if there are many agents and if the agents are risk averse." From this perspective, evidence consistent with equity pay for non-executive workers providing significant incentives even at large firms (e.g., Klein 1987, Jones and Kato 1995, Kroumova and Lazarova 2009, Hochberg and Lindsey 2010, Kim and Ouimet 2014, Chang et al. 2015, Blasi et al. 2016) is puzzling because the effort of an individual non-executive worker is unlikely to meaningfully change firm value. In my framework, the individual worker exerts effort not to increase firm value—though their combined efforts do—but to generate information that resolves income risk. In stark contrast to the informativeness principle of Holmström (1979), this income-risk resolution mechanism implies that performance pay can incentivize some types of effort even if the underlying performance measure is uninformative.

Second, my framework suggests a more nuanced relationship between the prevalence of risky incentive pay and risk-related characteristics. In conventional principal-agent models, imposing income risk on risk-averse workers is an unintended consequence of providing incentives. These models consistently predict that an increase in the cost of imposing risk—whether due to a more volatile performance measure or greater worker risk aversion—discourages the use of incentive pay. In contrast, the imposition of income risk that productive effort helps resolve plays a central role in motivating workers in my framework. As a result, incentive pay is more likely when the performance measure is more exposed to factors that workers can learn about through their engagement efforts, such as the effectiveness of the firm's strategy, and less likely when driven by other factors like macroeconomic conditions. Consistent with this prediction, Spalt (2013) documents that firms with more idiosyncratic stock return volatility are more likely to give stock option pay to non-executive workers. Greater worker risk aversion raises the per-unit cost of equity pay, but also makes workers more responsive to the income-risk resolution mechanism. These opposing forces create an ambiguous relationship between worker risk aversion and the prevalence of risky incentive pay, consistent with the findings of Kruse et al. (2010), who report that worker risk aversion correlates positively with equity pay in some instances but negatively in others.

Third, my framework predicts a distinct relationship between the cost effectiveness of the equity pay as an incentive and many aspects of the firm's information environment. When workplace engagement generates more information that resolve income risk, workers derive larger benefits from engagement effort, making them easier to incentivize via the income-risk resolution mechanism. However, due to diminishing marginal returns to information, additional freely available information that substitutes the information generated by engagement effort lowers the worker's incremental benefit from exerting engagement effort. This complexity potentially helps explain the mixed evidence about the complementarity between equity pay and other employee participatory practices.[5]

Fourth, the income-risk resolution mechanism suggests an alternative explanation for the practice of time-based vesting for equity pay beyond retention (e.g., Oyer 2004, Kedia and Rajgopal 2009, Aldatmaz et al. 2018, Lie and Que 2020, Hoffmann and Vladimirov

[5] For instance, Kato and Morishima (2002) analyze a large panel data of Japanese firms and document a complementarity between the firm's information sharing practices and its use of equity pay in improving productivity. In contrast, using data from the British Workplace Employment Relations Survey, Pendleton and Robinson (2010) challenge the complementarity theory, noting that excessive employee involvement practices lower the productivity gains due to equity pay.

2023) and employee-supplied financing (e.g., Babenko et al. 2011, Sun and Xiaolan 2019). Because the income risk associated with equity pay increases with its horizon, one can view the vesting period as a tool for the firm to adjust the amount of income risk due to equity pay.

Finally, my framework also generates many novel testable implications linking the workers' financial planning decisions to the firm's compensation practices. The model predicts that equity pay increases the positive correlation between the worker's interim consumption choices (e.g., the price of a newly purchased home) and subsequent firm performance (e.g., earnings, stock returns). This effect should be more pronounced for younger, less-established workers who are more likely to face major financial decisions, such as family planning and home purchase.[6] This effect should also be more pronounced when workers have better access to financial markets that enhance their ability to smooth consumption, which makes information that resolves income risk more valuable. Researchers could further test this relationship by exploiting changes to the effectiveness of engagement effort, such as those brought about remote work during the COVID-19 pandemic, or by evaluating changes in the information environment due to the introduction of virtual "water-cooler" platforms (e.g., Blind, Company Bowl), which facilitate the exchange of information.[7]

Related Literature. This paper primarily relates to the literature that considers equity pay for non-executive workers. Studies in this literature typically assume that the free-riding problem makes equity pay an ineffective tool for incentivizing effort. Instead, they emphasize other benefits to justify the cost of imposing risk on workers, who are likely less diversified and more risk averse than the firm is. Some benefits include reduced financial constraints (Core and Guay 2001, Kim and Ouimet 2014, Sun and Xiaolan 2019), favorable accounting and tax treatment (Hall and Murphy 2003, Babenko and Tserlukevich 2009), employee retention (Oyer 2004, Kedia and Rajgopal 2009, Aldatmaz

[6] I thank Rui C. Silva for this suggestion.

[7] I thank Thierry Foucault for these suggestions.

et al. 2018, Lie and Que 2020, Hoffmann and Vladimirov 2023), employee sorting (Lazear 2004, Bergman and Jenter 2007), improved product market competitiveness (Bova and Yang 2017), mutual monitoring and cooperation (Hochberg and Lindsey 2010), insurance against promotion risk (Chen 2024), discouragement of whistle-blowing (Call et al. 2016), and lower recapitalization costs (Efing et al. 2023). I contribute to this literature by highlighting how equity pay *can* meaningfully incentivize costly effort from non-executive workers despite their marginal impact on overall firm value.

A separate contribution of this paper is to study optimal contracting with an emphasis on a novel incentive mechanism based on income-risk resolution. In my framework, equity pay incentivizes effort even when the performance measure is uninformative—a stark contrast to the informativeness principle of Holmström (1979)—because it makes information that productive effort generates valuable to the worker. This results implies that the free-rider problem may be less detrimental than previously posited in the moral hazard literature (e.g., Alchian and Demsetz 1972, Holmström 1982, Kandel and Lazear 1992), as performance pay may sometimes provide incentives in ways other than those traditionally assumed.

1 Model Setup

The model has three dates ($t = 0$, $t = 1$, $t = 2$) and features a risk-neutral firm that employs a unit-mass continuum of risk-averse workers, indexed by $i \in [0, 1]$. The workers have time-separable, constant absolute risk aversion (CARA) utility over consumption at $t = 1$ and $t = 2$.[8] Given the consumption of c_1 and c_2 at $t = 1$ and $t = 2$, respectively, a worker's lifetime utility at $t = 0$ can be expressed as

$$U_0 = \frac{1 - e^{-\gamma c_1}}{\gamma} + \delta \left(\frac{1 - e^{-\gamma c_2}}{\gamma} \right), \tag{1}$$

[8] For simplicity, the workers do not consume at $t = 0$.

where $\gamma > 0$ represents the magnitude of the worker's absolute risk aversion. For simplicity, the worker's subjective discount factor (δ) is set to 1. As long as the discount factor is positive, its exact magnitude does not alter the main results of the paper. The worker's outside option at $t = 0$ corresponds to consuming u_0 at $t = 1$ and $t = 2$.

The firm's production technology is modeled as a linear function of its workers' efforts $e_i \in \{e_L, e_H\}$ and normally distributed productivity shocks $\tilde{\varepsilon}_f, \tilde{\varepsilon}_m \overset{i.i.d.}{\sim} N(0, 1)$. Specifically, the firm's output at $t = 2$ is given by

$$V = \left(\int_0^1 e_i d_i\right)\theta + \sigma_f \tilde{\varepsilon}_f + \sigma_m \tilde{\varepsilon}_m. \tag{2}$$

The parameter $\theta > 0$ represents the importance of aggregate worker effort ($\int_0^1 e_i di$) to the firm's output. The firm-level productivity shock ($\tilde{\varepsilon}_f$) reflects the effectiveness of the firm's overall strategy, such as the fit between its product offering and customer preferences, while the market-level productivity shock ($\tilde{\varepsilon}_m$) captures broader economic conditions. The parameters $\sigma_f > 0$ and $\sigma_m > 0$ indicate the relative importance of firm- and market-level productivity shocks, respectively, in determining firm output. Exerting effort is costly, reducing the worker's consumption at $t = 1$ and $t = 2$ by $\bar{C}$ units:

$$\bar{C}(e_i) = \begin{cases} 0 & e_i = e_L \\ \frac{c}{2} & e_i = e_H \end{cases}$$

The model aims to describe a setting in which the firm-level outcome aggregates the contribution of many workers, and as a result, provides little information about any individual worker's effort. To simplify the exposition, the model captures this setting in the extreme by assuming a continuum of workers, which implies that the firm's output (V) provides no information about the effort of any individual worker. This assumption helps distinguish the income-risk resolution mechanism underlying the incentive-provision role of equity pay in this framework from those in conventional principal-agent models (e.g., Holmström 1979).

As in Oyer (2004), the firm offers linear compensation contracts to its workers (for other examples, see Holmström and Tirole (1993), Bénabou and Tirole (2016)). Specifically, a feasible compensation contract for worker i takes the form

$$W_i = \alpha_i + \beta_i V, \tag{3}$$

where α_i represents a fixed wage and $\beta_i \geq 0$ captures the degree to which that worker's pay depends on the firm-level outcome (V). The focus on linear contracts simplifies the analysis and allows me to illustrate how one form of contingent pay, though perhaps not the best form, can be used to incentivize worker effort even when the underlying performance measure contains no information about the worker's effort. For simplicity, the main specification assumes that V is the only verifiable measure of performance. The income-risk resolution mechanism does not rely on this assumption. Section 4.2 relaxes this assumption by allowing for a verifiable but still noisy measure of individual performance. The firm maximizes its expected output net of compensation costs. The firm and the workers share a common discount factor. This assumption helps isolate the incentive-provision role of equity pay from ones based on financing from employees (e.g., Babenko et al. 2011, Sun and Xiaolan 2019).

The model's main results rely on two innovations to an otherwise standard principal-agent setup: effort as workplace engagement and the worker's interim planning problem. First, the framework emphasizes the relationship between information and production. Workers who are more informed about their firm's strategy are more productive and can better assess its future performance. Here, I conceptualize effort as workplace engagement, where highly engaged workers gain a deeper understanding of how their tasks fit into the firm's overall strategy, making their individual contributions a more accurate reflection of the strategy's effectiveness. Specifically, each worker i generates an individual output

$$\tilde{v}_i = e_i\theta + \sigma_f\left(\tilde{\varepsilon}_f + \sqrt{\tfrac{1}{e_i\phi}}\tilde{\epsilon}_i\right), \tag{4}$$

where $e_i \in \{e_L, e_H\}$ represents the worker's level of engagement and $\tilde{\epsilon}_i \overset{i.i.d.}{\sim} N(0,1)$ reflects individual variability in output. The contribution of a highly engaged worker ($e_i = e_H > e_L$) is more valuable in expectation and less subject to idiosyncratic noise. I assume that workers privately observe the value of their individual output $\tilde{v}_i$, which is informationally equivalent to a signal $\tilde{s}_i$ about the firm-level productivity shock:

$$\tilde{s}_i = \tfrac{\tilde{v}_i - e_i\theta}{\sigma_f} = \tilde{\varepsilon}_f + \sqrt{\tfrac{1}{e_i\phi}}\tilde{\epsilon}_i, \tag{5}$$

which has a precision of $e_i\phi$ that increases with the worker's engagement effort. The parameter $\phi > 0$ captures the extent to which workplace engagement allows the worker to learn about the firm-level productivity shock. This conceptualization of effort as workplace engagement links individual productivity with information about firm-level output.

Second, workers face an interim consumption-smoothing problem, deciding how much to consume at $t = 1$ (a_i) out of their expected income at $t = 2$. For simplicity, I assume that they can freely borrow and save at a gross rate $R = 1$.[9] Uncertainty about future income complicates this consumption-smoothing problem. As a result, workers derive a benefit from information that resolves income risk and improves their financial planning (e.g., Hirshleifer 1971). Together, these two innovations in the model form the basis of the income-risk resolution mechanism that underlies the incentive-provision role of equity pay in this framework. Figure 1 summarizes the timing of the model.

[9] The exact magnitude of the borrowing/saving rate does not qualitatively alter the paper's main results.

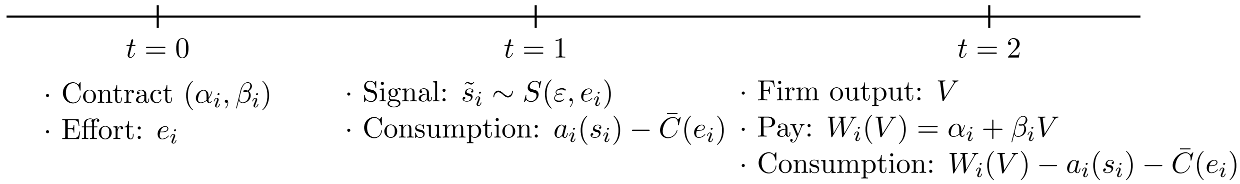

Figure 1. Model timing.

2 Incentive Compatibility

To begin, I solve the worker's consumption-smoothing problem at $t = 1$ taking the compensation contract (α_i, β_i), the worker's $t = 0$ engagement effort (e_i), and the aggregate level of worker engagement $(\bar{e} = \int_0^1 e_i di)$ as given. For the remainder of the main analysis, I suppress the subscript identifying the worker and focus on symmetric contracts.

The exponential form of CARA utility allows the worker's consumption-smoothing problem at $t = 1$ to be expressed in terms of $\bar{W}_s$, the worker's certainty-equivalent pay at $t = 2$ conditional on the realized signal s:

$$e^{-\gamma \bar{W}_s} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\varepsilon_f, s) m(\varepsilon_m) e^{-\gamma \overbrace{(\alpha + \beta(\bar{e}\theta + \sigma_f \varepsilon_f + \sigma_m \varepsilon_m))}^{=W(V)}} d\varepsilon_m d\varepsilon_f, \tag{6}$$

where $m(\varepsilon_m)$ is the distribution of $\tilde{\varepsilon}_m$ and $f(\varepsilon_f, s)$ is the conditional distribution of $\tilde{\varepsilon}_f$ given s. At $t = 1$, the worker solves

$$\underset{a_s}{\text{maximize}} \quad \underbrace{\frac{1 - e^{-\gamma(a_s - \bar{C})}}{\gamma}}_{\text{utility of } t=1 \text{ consumption}} + \underbrace{\frac{1 - e^{-\gamma(\bar{W}_s - \bar{C} - a_s)}}{\gamma}}_{\text{expected utility of } t=2 \text{ consumption}}, \tag{7}$$

where $\bar{C}(e)$ is the reduction in per-period consumption due to the worker's engagement effort at $t = 0$. Because the worker does not face borrowing and saving constraints, the solution to the worker's consumption-smoothing problem at $t = 1$ satisfies the first-order condition:

$$e^{-\gamma(a_s^* - \bar{C})} - e^{-\gamma(\bar{W}_s - \bar{C} - a_s^*)} = 0. \tag{8}$$

Lemma 1 *Conditional on a realized signal s, the optimal consumption amount at $t = 1$ is characterized by*

$$a_s^* = \tfrac{1}{2}\bar{W}_s = \tfrac{1}{2}\Big[\underbrace{\alpha + \beta(\bar{e}\theta)}_{E[\tilde{W}]} + \beta\sigma_f \underbrace{\big(\tfrac{\Phi}{1+\Phi}s\big)}_{E[\tilde{\varepsilon}_f|s]} - \frac{\gamma}{2}\beta^2\sigma_f^2 \underbrace{\big(\tfrac{1}{1+\Phi}\big)}_{Var(\tilde{\varepsilon}_f|s)} - \frac{\gamma}{2}\beta^2\sigma_m^2\Big], \tag{9}$$

where $\Phi = e\phi$ is the precision of the worker's signal ($\tilde{s}$) about the firm-level productivity shock ($\tilde{\varepsilon}$).

The first-order condition (8) implies that the worker's optimal consumption at $t = 1$ equalizes the marginal utility of consumption across the first and second periods. Because of the worker's CARA utility, the reduction in consumption at $t = 1$ and $t = 2$ due to the worker's $t = 0$ engagement effort ($\bar{C}$) simply scales the first-order condition. As a result, the optimal consumption at $t = 1$ only depends on the certainty-equivalent pay at $t = 2$. Because the borrowing and saving rate is $R = 1$, the worker consumes an equal fraction of the certainty-equivalent pay in each period.[10] The normality assumption, combined with linear compensation contracts, allows for a closed-form expression of the optimal consumption amount based on standard properties of normal-normal Bayesian updating.

Lemma 1 implies that the worker's expected lifetime utility conditional on the realized signal s at $t = 1$ is given by

$$U_{1s} = \frac{1 - e^{-\gamma(a_s^* - \bar{C})}}{\gamma} + \frac{1 - e^{-\gamma(a_s^* - \bar{C})}}{\gamma}. \tag{10}$$

Taking the integral of (10) with respect to s yields the worker's expected lifetime utility $t = 0$:

$$U_0 = 2\left[\frac{1 - e^{-\gamma\left(\frac{1}{2}\left[\alpha+\beta(\bar{e}\theta) - \frac{\gamma}{2}\beta^2\sigma_f^2\left(\frac{2+\Phi}{2+2\Phi}\right) - \frac{\gamma}{2}\beta^2\sigma_m^2\right] - \bar{C}\right)}}{\gamma}\right]. \tag{11}$$

It is convenient to express the worker's expected lifetime utility at $t = 0$ in terms of $\bar{u}$,

[10] A larger rate R lowers the optimal consumption at $t = 1$ but does not affect the linear structure of a_s^* as a function of $\bar{W}_s$.

the worker's certainty-equivalent consumption at $t = 1$ and $t = 2$:

$$\bar{u} = \frac{1}{2}\left(\alpha + \beta(\bar{e}\theta) - \frac{\gamma}{2}\beta^2\sigma_f^2\left(\tfrac{2+\Phi}{2+2\Phi}\right) - \frac{\gamma}{2}\beta^2\sigma_m^2\right) - \bar{C}, \tag{12}$$

where both the signal precision (Φ) and effort cost ($\bar{C}$) depend on the worker's engagement effort at $t = 0$.

In this setting, a worker receiving equity pay ($\beta > 0$) faces the following trade-off at $t = 0$. A high level of engagement ($e = e_H$) incurs an effort cost that reduces per-period consumption by $\frac{c}{2}$ but generates more information about the firm-level productivity shock. A low level of engagement ($e = e_L$) avoids the effort cost but generates less information about the firm-level productivity shock. The worker's incentive-compatibility (IC) constraint can be expressed as

$$\underbrace{\frac{1}{2}\left(\alpha + \beta\bar{e}\theta - \tfrac{\gamma}{2}\beta^2\sigma_f^2\left(\tfrac{2+e_H\phi}{2+2e_H\phi}\right) - \tfrac{\gamma}{2}\beta^2\sigma_m^2\right) - \frac{c}{2}}_{\bar{u}(e_H)} \geq \underbrace{\frac{1}{2}\left(\alpha + \beta\bar{e}\theta - \tfrac{\gamma}{2}\beta^2\sigma_f^2\left(\tfrac{2+e_L\phi}{2+2e_L\phi}\right) - \tfrac{\gamma}{2}\beta^2\sigma_m^2\right)}_{\bar{u}(e_L)},$$

which simplifies to

$$\frac{\gamma}{2}\beta^2\sigma_f^2\left(\frac{e_H\phi - e_L\phi}{2(1+e_H\phi)(1+e_L\phi)}\right) \geq c. \tag{13}$$

The worker's IC constraint (13) implies that the firm can induce the worker to exert costly engagement effort with sufficient equity pay ($\beta > 0$) despite the underlying performance measure (V) containing no information about the worker's engagement effort, in sharp contrast to the informativeness principle of Holmström (1979). Moreover, when equity pay is provided ($\beta > 0$), the worker's IC constraint is easier to satisfy when the worker is more risk-averse (larger γ) and the firm is more exposed to firm-level productivity shocks (larger σ_f).

Proposition 1 *A necessary condition for incentive compatibility is that workers face income risk ($\beta > 0$) and value information that helps resolve that risk ($\frac{\partial\bar{u}}{\partial\Phi} > 0$).*

The economic force behind the incentive-provision role of equity pay in this framework is the benefit that workers derive from information about their future income. Equity pay ($\beta > 0$) exposes workers to income risk that correlates with firm output (V). Highly engaged workers gain a deeper understanding of the firm's strategy and the factors driving its performance. This knowledge not only enhances their productivity but also provides valuable insights into their future earnings, allowing for more informed interim consumption choices. It is this latter benefit—the resolution of income risk for improved financial planning—that incentivizes workers to exert costly engagement effort in this framework.

Proposition 2 *The optimal linear incentive-compatible (IC) compensation contract is given by*

$$\beta^*_{IC} = \sqrt{\frac{4c(1+e_L\phi)(1+e_H\phi)}{\gamma\sigma_f^2(e_H\phi - e_L\phi)}}$$

and

$$\alpha^*_{IC} = 2u_0 + c + \frac{\gamma}{2}\beta^{*2}_{IC}\sigma_f^2\left(\tfrac{2+e_H\phi}{2+2e_H\phi}\right) + \frac{\gamma}{2}\beta^{*2}_{IC}\sigma_m^2 - \beta^*_{IC}(\bar{e}\theta).$$

Given the worker's IC constraint (13), the results of Proposition 2 follow from standard optimal contracting arguments in a CARA-normal principal-agent model. First, exposure to risky firm performance is costly because workers are risk-averse. Therefore, the optimal IC contract sets the equity-pay component (β^*) just high enough to bind the IC constraint. Second, the optimal IC contract sets the fixed-pay component (α^*) sufficiently high to bind the worker's participation constraint. The firm's optimal compensation policy balances the cost of incentivizing high workplace engagement against the productivity gains that result from it.

3 Optimal Compensation

In this framework, the firm's objective of maximizing its expected output net of compensation costs simplifies to choosing between inducing high or low workplace engagement.

If the firm opts for low workplace engagement ($e = e_L$), it optimally offers workers a fixed wage that satisfies their outside option ($\alpha_0^* = 2u_0$), and no equity pay ($\beta_0^* = 0$) given their risk aversion. In this case, the firm's expected payoff is

$$\Pi_L = \underbrace{e_L\theta}_{\text{baseline productivity}} - \underbrace{2u_0}_{\text{workers' outside option}}.$$

If the firm opts for high workplace engagement ($e = e_H$), it offers the optimal IC compensation contract described in Proposition 2. In this case, its expected payoff can be expressed as

$$\Pi_H = \underbrace{e_H\theta}_{\text{enhanced productivity}} - \underbrace{2u_0}_{\text{workers' outside option}} - \underbrace{Cost_{IC}}_{\text{cost of providing incentives}},$$

where

$$Cost_{IC} = \underbrace{c}_{\text{effort cost}} + \underbrace{\frac{\gamma}{2}\beta_{IC}^{*2}\sigma_f^2\left(\frac{2+e_H\phi}{2+2e_H\phi}\right)}_{\text{compensation for firm-level risk}} + \underbrace{\frac{\gamma}{2}\beta_{IC}^{*2}\sigma_m^2}_{\text{compensation for market-level risk}}. \tag{14}$$

The condition under which the firm prefers to incentivize high workplace engagement is characterized by the following proposition.

Proposition 3 *The optimal contract features equity pay ($\beta^* = \beta_{IC}^* > 0$) if and only if*

$$(e_H - e_L)\theta \geq \left(1 + \frac{(2+e_H\phi)(1+e_L\phi)}{e_H\phi - e_L\phi} + \frac{2(1+e_L\phi)(1+e_H\phi)}{e_H\phi - e_L\phi}\frac{\sigma_m^2}{\sigma_f^2}\right)c.$$

Proposition 3 highlights the main trade-off faced by the firm in this setting. Incentivizing high workplace engagement with equity pay increases productivity by $(e_H - e_L)\theta$. However, doing so involves compensating workers not only for the effort required to maintain high engagement but also for the income risk tied to satisfying incentive compatibility. As a result, incentivizing high workplace engagement increases expected compensation costs by $\left(1+\frac{(2+e_H\phi)(1+e_L\phi)}{e_H\phi-e_L\phi}+\frac{2(1+e_L\phi)(1+e_H\phi)}{e_H\phi-e_L\phi}\frac{\sigma_m^2}{\sigma_f^2}\right)c$.

3.1 Comparative Statics

This section provides additional insights into how the prevalence of equity pay varies with firm and worker characteristics in this framework, focusing on the trade-off highlighted in Proposition 3. Let $\Delta\Pi = \Pi_H - \Pi_L$ be the increase in the firm's expected payoff from inducing high workplace engagement with the optimal incentive-compatible (IC) contract, which captures the cost effectiveness of equity pay:

$$\Delta\Pi = (e_H - e_L)\theta - \left(1+\frac{(2+e_H\phi)(1+e_L\phi)}{e_H\phi-e_L\phi}+\frac{2(1+e_H\phi)(1+e_L\phi)}{e_H\phi-e_L\phi}\frac{\sigma_m^2}{\sigma_f^2}\right)c. \tag{15}$$

Proposition 4 *Equity pay is more likely at firms where workplace engagement is more productive (higher θ) and less costly (lower c): $\frac{d\Delta\Pi}{d\theta} > 0$ and $\frac{d\Delta\Pi}{dc} < 0$.*

Intuitively, the firm is more likely to incentivize high workplace engagement when the benefits of doing so are higher and the costs are lower. Consistent with this prediction, many empirical studies find that equity pay for non-executive workers is more common in knowledge-intensive firms, where workers can more easily learn about their firm and where such information is more important for their productivity (e.g., Core and Guay 2001, Pendleton 2006, Kruse et al. 2010).

While most principal-agent models feature a cost-benefit analysis that generates the same implications highlighted by Proposition 4, this framework suggests a substantively different relationship between the prevalence of equity pay, risk-related characteristics, and the firm's information environment.

Proposition 5 *In a CARA-normal framework, as long as workers are risk-averse ($\gamma > 0$), the exact magnitude of their absolute risk aversion does not affect the prevalence of equity pay: $\frac{d\Delta\Pi}{d\gamma} = 0$.*

In conventional principal-agent models, high worker risk aversion increases the cost of imposing income risk, making risky incentive pay less attractive. This effect is also present in my framework. The compensation-for-risk components of (14) imply that an increase in worker risk aversion (γ) raises the per-unit cost of equity pay. However, my framework introduces a novel offsetting effect. Workers who are more risk-averse benefit more from the resolution of income risk. As a result, an increase in the workers' risk aversion also makes them easier to incentivize via the income-risk resolution mechanism, reducing the amount of equity pay required for incentive compatibility. In a CARA-normal framework, these two opposing effects exactly offset.

More generally, the income-risk resolution mechanism introduces a countervailing force that results in an ambiguous relationship between the prevalence of equity pay and worker risk aversion. Consistent with this ambiguity, Kruse et al. (2010) find that worker risk aversion correlates positively with the prevalence of some forms of equity pay and negatively with others.

Proposition 6 *Firms that are more exposed to market-level productivity shocks (higher σ_m) are less likely to use equity pay, while those that are more exposed to firm-level productivity shocks (higher σ_f) are more likely to use equity pay: $\frac{d\Delta\Pi}{d\sigma_m} < 0$ and $\frac{d\Delta\Pi}{d\sigma_f} > 0$.*

In conventional principal-agent models, exposing risk-averse workers to income risks due to the random productivity shocks that affect performance is an unintended consequence of using incentive pay. A more volatile performance measure raises the cost of imposing risk and makes incentive pay less attractive. Consequently, conventional models consistently predict a negative relationship between the volatility of firm performance and the prevalence of equity pay for incentive purposes.[11] The first part of Proposition 6 reflects this logic: an increase in the firm's exposure to market-level productivity

[11] One notable exception is Prendergast (2000), who argues out that alternative governance mechanisms to incentive pay, such as input monitoring and sorting, may also be less effective in uncertain environments.

shocks means that each unit of equity pay carries more income risk, which the firm must compensate, making equity pay more costly to use.

In my framework, income risk is a key feature of the incentive mechanism, rather than an unfortunate byproduct. Specifically, the imposition of income risk, which workers can resolve through productive effort, plays a central role in motivating workers. While an increase in the firm's exposure to firm-level productivity shocks raises the per-unit cost of equity pay due to additional income risk, it also enhances the effectiveness of equity pay in incentivizing workplace engagement effort, thereby reducing the amount of equity pay required for incentive compatibility. These two opposing forces offset in a CARA-normal framework such that the compensation for income risk due to $\tilde{\varepsilon}_f$ remains constant in the firm's exposure to that risk (σ_f). However, when the firm is also exposed to other productivity shocks (i.e., $\sigma_m > 0$), the reduction in incentive-compatible equity pay (β^*_{IC}) also reduces the compensation for income risk due to $\tilde{\varepsilon}_m$. As a result, additional exposure to firm-level productivity shocks makes equity pay more attractive for the firm. Consistent with this prediction, Spalt (2013) documents that firms with stock returns that have higher idiosyncratic volatility are more likely to offer stock option pay to non-executive workers (see also Oyer and Schaefer 2005 for related evidence).

Proposition 7 *The cost effectiveness of equity pay has a non-monotonic relationship with ϕ, increasing in ϕ when ϕ is less than a threshold $\bar{\phi} > 0$, and decreasing in ϕ thereafter: $\frac{d\Delta\Pi}{d\phi} > 0$ for $\phi < \bar{\phi}$ and $\frac{d\Delta\Pi}{d\phi} < 0$ for $\phi \geq \bar{\phi}$. The threshold ($\bar{\phi}$) increases in σ_f and e_H and decreases in σ_m and e_L: $\frac{d\bar{\phi}}{d\sigma_f} > 0$, $\frac{d\bar{\phi}}{de_H} > 0$, $\frac{d\bar{\phi}}{d\sigma_m} < 0$, $\frac{d\bar{\phi}}{de_L} < 0$.*

Recall that ϕ captures the effectiveness of workplace engagement in generating information about the firm-level productivity shock. An increase in ϕ represents a change in the firm's information environment that makes it easier for workers to learn about the firm through workplace engagement. This change affects the cost effectiveness of the income-risk resolution mechanism in two opposing ways.

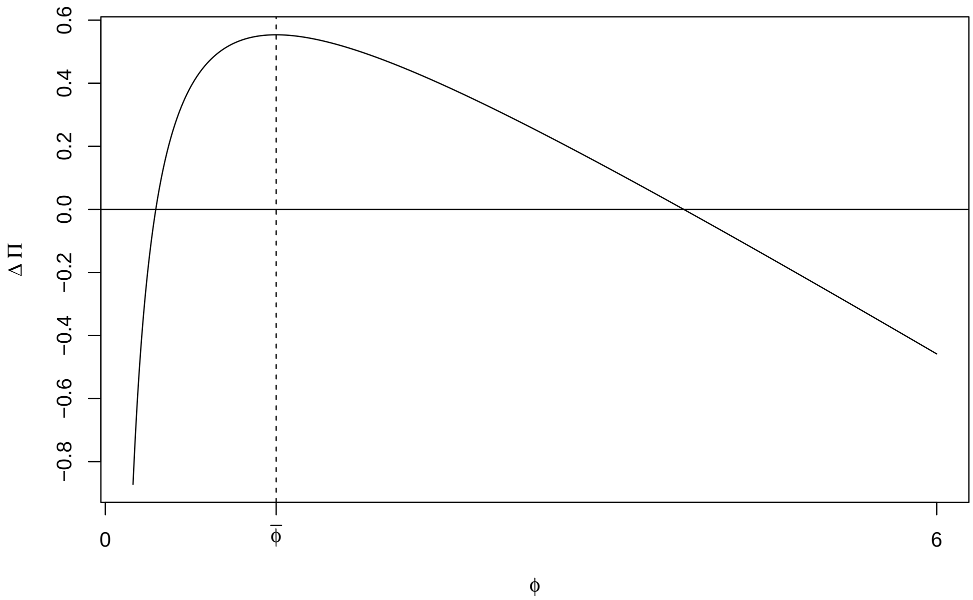

Figure 2. Cost Effectiveness of Equity Pay. This figure plots an example of how the cost effectiveness ($\Delta\Pi$) varies with the information environment within the firm (ϕ). A larger value of ϕ corresponds to engagement effort producing more information about the firm-level productivity shock. The figure is generated using parameters $c = 0.05$, $e_H = 1.5$, $e_L = 0.5$, $\theta = 2$, $\gamma = 2$, $\sigma_f = 1$, $\sigma_m = 1.75$, and $\phi \in (0.2, 6)$. The vertical dashed black line indicates $\bar{\phi}$, where the relationship between $\Delta\Pi$ and ϕ changes sign.

On the one hand, a larger ϕ means that high engagement effort generates more information resolving income risk, increasing the benefit that workers derive from high workplace engagement. This first effect makes it easier to be incentivize workers with equity pay. On the other, a larger ϕ also implies that even low engagement effort generates more information, allowing workers to resolve significant income risk without exerting high engagement effort. This second effect makes workers harder to incentivize with equity pay. At low levels of ϕ, the first effect dominates, making equity pay more cost-effective. However, as $\phi \to \infty$, the incentive provision from equity pay vanishes because workers can resolve all income risk stemming from the firm-level productivity shock regardless of their engagement effort level. Consequently, the second effect dominates at higher levels of ϕ, rendering equity pay less cost-effective. Figure 2 illustrates this non-monotonic relationship.

The comparative statics of $\bar{\phi}$ further highlight the interaction between these two opposing forces. A larger σ_f, corresponding to greater exposure to firm-level productivity shocks, magnifies resolvable income risk and thereby reinforcing the first effect. A larger e_H implies that high engagement effort generates relatively more information, weakening the second effect. Thus, increases in σ_f and e_H push the threshold to the right.

In contrast, a larger σ_m, representing greater exposure to market-level productivity shocks that workers cannot resolve through their engagement efforts, dampens the first effect. A larger e_L means that low engagement effort generates relatively more information, amplifying the second effect. Consequently, increases in σ_m and e_L push the threshold to the left.

It is worth noting that even if workers can fully resolve the income risk due to the firm-level productivity shock at $t = 1$ through high engagement effort, they would still perceive equity pay to be risky at $t = 0$. Hence, the compensation for risk stemming from equity pay is bounded below even as $\phi \to \infty$. This positive lower bound has implications for how other information about firm performance affects the prevalence of equity pay.[12]

4 Extensions and Robustness

This section studies several extensions to the baseline model and discusses how the income-risk resolution mechanism interacts with other economic forces in this setting.

4.1 Consumption-Smoothing Frictions

The main specification of the model assumes that workers are unconstrained when smoothing their consumption over time. However, workers may face frictions that prevent them from fully adjusting their consumption in response to new information at $t = 1$. This section examines the effects of consumption-smoothing frictions on the income-risk

[12] See Section 4.3.

resolution mechanism.

Recall from Section 2 that in the absence of consumption-smoothing frictions, the worker optimally consumes an equal proportion of her certainty-equivalent $t = 2$ pay in each period. Hence, without additional interim information, the worker's optimal consumption amount at $t = 1$ is half of $\bar{W}$, the unconditional certainty-equivalent pay at $t = 2$:

$$\bar{W} = \alpha + \beta\bar{e}\theta - \tfrac{\gamma}{2}\beta^2\sigma_f^2 - \tfrac{\gamma}{2}\beta^2\sigma_m^2.$$

Lemma 1 indicates that the worker updates the optimal consumption amount at $t = 1$ based on the information contained in the realized signal s:

$$a_s^* = \tfrac{1}{2}\bar{W}_s = \tfrac{1}{2}\bar{W} + \tfrac{1}{2}\left(\bar{W}_s - \bar{W}\right).$$

To model the frictions that hinder the worker's consumption smoothing, I assume that the worker consumes a constrained amount at $t = 1$ given by

$$a_s(\kappa) = \tfrac{1}{2}\bar{W} + \underbrace{\tfrac{\kappa}{1+\kappa}}_{\leq \frac{1}{2}}\left(\bar{W}_s - \bar{W}\right), \tag{16}$$

where the parameter $\kappa \in [0, 1]$ captures how much the worker can alter her consumption at $t = 1$ in response to interim information. When $\kappa = 0$, the worker cannot update her consumption decision. For instance, there may be consumption inertia due to unmodeled consumption choices made prior to $t = 1$ or there may be financial frictions that limit additional borrowing. An increase in κ allows the worker more flexibility in adjusting her consumption in response to interim information. The baseline model corresponds to $\kappa = 1$. Let $\Delta\Pi_\kappa$ be the cost effectiveness of equity pay under this extension.

Proposition 8 *When the worker cannot adjust her consumption at $t = 1$ in response to interim information (i.e., $\kappa = 0$), equity pay provides no incentives ($\Delta\Pi_\kappa < 0$). An* *increase in κ makes equity pay more cost effective:* $\frac{d\Delta\Pi_\kappa}{d\kappa} > 0$.

Intuitively, equity pay incentivizes workplace engagement in this framework because the worker's engagement effort generates valuable information. In the model, this value arises from a more efficient consumption-smoothing decision. When the worker cannot adjust her $t = 1$ consumption in response to new information, she derives no benefit from the additional information generated by high engagement, effectively shutting down the income-risk resolution mechanism. The greater her flexibility in adjusting consumption in response to interim information, the more valuable information about future income becomes.

4.2 Individual Performance Measure

To highlight how the income-risk resolution mechanism can provide incentives even when the underlying performance measure is uninformative, the baseline model assumes that the firm can only offer incentive pay based on the firm output (V). In this section, I relax this assumption by introducing a verifiable but noisy measure of individual engagement effort:

$$\tilde{y} = e + \sqrt{\frac{1}{\phi_y}}\tilde{\epsilon}_y,$$

where $\tilde{\epsilon}_y \sim N(0,1)$ and is uncorrelated with other shocks in the model. The parameter ϕ_y captures the precision of the signal $\tilde{y}$ about the worker's engagement effort. The firm can offer compensation of the form

$$W = \alpha + \beta V + \eta\tilde{y},$$

which consists of a fixed wage (α), equity pay ($\beta \geq 0$), and individual performance bonuses ($\eta \geq 0$).

The arguments in Sections 2 imply that the worker's IC constraint can be expressed as

$$\frac{\gamma}{2}\beta^2\sigma_f^2\left(\frac{e_H\phi - e_L\phi}{2(1+e_H\phi)(1+e_L\phi)}\right) + (e_H - e_L)\eta \geq c, \tag{17}$$

which suggests that equity pay and individual performance pay act as substitutes. Each squared unit of equity pay relaxes the worker's IC constraint by $\frac{\gamma}{2}\sigma_f^2\left(\frac{e_H\phi - e_L\phi}{2(1+e_H\phi)(1+e_L\phi)}\right)$ and increases the firm's compensation costs by $\frac{\gamma}{2}(\sigma_f^2(\frac{2+e_H\phi}{2+2e_H\phi}) + \sigma_m^2)$. In contrast, each unit of individual performance bonus relaxes the worker's IC constraint by $(e_H - e_L)$, but increases compensation costs at an increasing rate, with each additional unit of individual performance bonus raising costs by $\gamma\eta\phi_y^{-1}$. This cost structure implies that providing incentives through individual performance bonuses becomes more expensive when high engagement effort is more costly and when the signal of individual worker engagement is noisier. Consequently, the optimal IC contract continues to feature some equity pay, despite the firm performance measure (V) being uninformative about individual worker engagement effort, as long as high engagement effort is sufficiently costly or when the verifiable individual performance measure ($\tilde{y}$) is sufficiently noisy.[13] These results are summarized by the following proposition.

Proposition 9 *As long as the signal of individual worker engagement is sufficiently imprecise or high engagement effort is sufficiently costly, the optimal IC contract features some equity pay ($\beta^* > 0$).*

4.3 Other Information about Firm Performance

The baseline model assumes that workers can only learn about the firm's productivity shocks by observing the feedback from their individual tasks. However, workers may also learn about the firm in other ways. If this learning activity also makes them more productive, then it simply represents an alternative formulation of the link between indi-

[13] For discussions of the challenges of measuring employee engagement in practice, see Kahn (1990) and Saks (2006).

vidual productivity and information about firm performance.[14] As a result, the economic intuition underlying the model's main results continue to hold.

This analysis focuses on learning activities that do not make workers more productive for the firm. I model this unproductive learning activity in reduced form. Specifically, I modify the information structure such that the worker receives interim signals $\tilde{s}_f$ and $\tilde{s}_m$ about the firm- and market-level productivity shocks, $\tilde{\varepsilon}_f$ and $\tilde{\varepsilon}_m$, respectively:

$$\tilde{s}_f = \tilde{\varepsilon}_f + \sqrt{\tfrac{1}{\phi_f + e_i \phi}}\tilde{\epsilon} \tag{18}$$

and

$$\tilde{s}_m = \tilde{\varepsilon}_m + \sqrt{\tfrac{1}{\phi_m}}\tilde{\zeta}, \tag{19}$$

where $\tilde{\epsilon}, \tilde{\zeta} \overset{i.i.d.}{\sim} N(0,1)$ represent independent noise terms. The precision of the worker's interim signal about the firm-level productivity shock depends on the worker's productive engagement effort (e) and unproductive learning activities (ϕ_f). The precision of the worker's interim signal about the market-level productivity shock only depends on the worker's unproductive learning activities (ϕ_m).

If effort costs were convex in total information acquisition effort, then the possibility of unproductive learning activities increases the worker's marginal cost of engagement effort, making high workplace engagement harder to induce as in conventional multi-tasking models (e.g., Holmström and Milgrom 1991). If instead, the worker's outside learning activities made it easier to exert engagement effort, then the possibility of unproductive learning activities makes productive workplace engagement easier to incentivize. To emphasize how other information about firm performance affects the income-risk resolution mechanism, I abstract from these other effects due to changes in the marginal cost of effort and assume that unproductive information acquisition is not costly.

Proposition 10 *The cost effectiveness of equity pay decreases in the availability of other*

[14] I thank Jessica Jeffers for noting this connection.

information about the firm-level productivity shock, and it increases in the availability of information about the market-level productivity shock: $\frac{d\Delta\Pi}{d\phi_f} < 0$ *and* $\frac{d\Delta\Pi}{d\phi_m} > 0$.

How additional information affects the income-risk resolution mechanism depends on whether it acts as a substitute to information generated by productive engagement effort. Recall that workplace engagement effort generates information about firm-level productivity shocks but not about market-level productivity shocks.

Intuitively, additional information that resolves income risk due to the firm-level productivity shock reduces the value of information generated by high workplace engagement because of the diminishing marginal benefit of information. To illustrate, consider the case where $\phi_f \to \infty$, meaning that no income risk due to $\tilde{\varepsilon}_f$ remains unresolved at $t = 1$. In this scenario, workers derive no benefit from the information generated by workplace engagement effort. However, because workers still perceive equity pay to be risky at $t = 0$, the firm must compensate them for the not-yet-resolved income risk stemming from firm-level productivity shocks. Consequently, additional information that substitutes the information generated by workplace engagement lowers the cost effectiveness of equity pay. This result is reminiscent of Goldstein and Yang (2019), who show that the firm's disclosure policy could crowd out investor information acquisition. Similarly, in this setting, improved access to information about firm-level productivity shocks from unproductive learning activities crowds out workers' information acquisition through workplace engagement.

In contrast, additional information about market-level productivity shocks does not act as a substitute because workplace engagement does not generate information about those shocks. Consequently, this information does not affect the worker's IC constraint. However, it does reduce the residual income risk that workers bear due to market-level productivity shocks. Consequently, an increase in this type of information makes equity pay more cost-effective.

4.4 Consumption Periods Prior to Vesting

This section studies how the number of consumption periods prior to the worker's equity pay vesting affects the income-risk resolution mechanism. Specifically, suppose the firm outcome realizes and the equity pay vests at $t = n$. Worker engagement effort at $t = 0$ lowers per-period consumption by $\frac{c}{n}$; this formulation keeps the total cost of high engagement effort fixed at c. Workers privately observe feedback from their individual tasks at $t = 1$. The baseline model corresponds to $n = 2$.

Proposition 11 *All else equal, an increase in the number of consumption periods before the worker's pay is realized increases the cost effectiveness of equity pay.*

For any given positive amount of equity pay, the worker's incentive compatibility condition is easier to satisfy when there are more consumption periods before the realization of equity pay. Intuitively, all else equal, an increase in the number of consumption periods makes information that resolves income risk more valuable for planning purposes. Hence, the income-risk resolution mechanism is more effective when the worker has more interim financial planning decisions.

It is important to note that Proposition 11 holds the other aspects of the model fixed as n increases. An increase in the number of periods also implies that firm output at $t = n$ is more uncertain, potentially increasing the cost of imposing risk. In such instances, my framework implies an optimal length of the vesting period that balances this trade-off.[15]

[15] I thank my discussant Basil Williams for this insight. For instance, suppose the exposure of firm output to market-level productivity shocks increases with the number of periods: $\hat{\sigma}_m = \sqrt{n-1}\sigma_m$. In this case, the optimal vesting period length that maximizes the cost effectiveness of equity pay is given by $n^* = 1 + \frac{\sigma_f}{\sigma_m}$.

5 Discussions and Additional Empirical Implications

5.1 Value of Income Risk Resolution

The income-risk resolution mechanism is driven by the value workers place on the information that their productive effort generates. Equity pay imposes income risk on workers, and it is the value of the information that resolves this risk that incentivizes workers to exert costly effort in my framework.

In the model, workers derive instrumental value from information that helps resolve this risk due to their interim consumption-smoothing problem. Specifically, additional information about future income allows them to better smooth consumption over time, increasing their expected lifetime utility. This economic intuition also extends to other planning problems that depend on future income. For example, workers may receive alternative employment offers in the interim period. The efficiency of the stay-or-go decision depends on the potential value of unvested equity pay, which they would forfeit by leaving the firm.

Alternatively, workers may derive psychological benefits from information that helps resolve income risk due to their innate preferences. For instance, workers may have recursive utility over consumption at $t = 1$ and $t = 2$ (e.g., Epstein and Zin 1989) and prefer the early resolution of uncertainty. Regardless of the underlying rationale, as long as workers value information that helps resolve income risk, equity pay can incentivize the productive effort needed to generate that information.

5.2 Human Capital

The income-risk resolution mechanism in my framework can also operate through the worker's human capital in some cases. For instance, suppose the worker's future income depends on the labor market's inference of the quality of the worker's human capital, with higher-quality workers earning more. Because the labor market often infers that workers at high-performing generally have better human capital (e.g., Oyer 2004), the worker's future income becomes correlated with firm performance. In this scenario, productive engagement effort continues to generate information about the worker's future income. As a result, having human capital tied to firm performance would also motivate the worker to exert costly engagement effort in my framework. The main implication of this result is that firms can incentivize workplace engagement by investing in the development of its workers' human capital.[16]

5.3 Cost of Incentivization in Large Firms

Equity pay in my framework incentivizes effort in a fundamentally different way than performance pay in typical principal-agent models. Consequently, my framework suggests a substantively different relative magnitude between the cost of effort and the cost of imposing risk.

In a typical principal-agent model, equity pay motivates the worker to exert effort because doing so increases the expected value of the firm, which in turn raises the expected value of the worker's compensation. However, as the size of the firm increases, the sensitivity of the worker's equity pay to their individual effort diminishes, making shirking more tempting. As a result, the worker's incentive compatibility constraint implies that either the cost of effort is small or the equity pay is large, which requires high compensation for risk. Consequently, typical principal-agent models predict a large difference between the required compensation for risk and the cost of effort. For instance, Oyer and Schaefer (2005) calibrate a standard principal-agent model with compensation data for mid-level managers at large firms and report that \$22,600 in compensation for risk is required to incentivize effort that costs \$0.023.

In my framework, however, equity pay incentivizes the worker to exert productive effort because doing so generates information that resolves income risk. When holding

[16] See also Sun and Xiaolan (2019) for more discussion on how firm investment into employee human capital affects compensation and incentives.

the information structure fixed, the severity of the free-rider problem has a relatively minor impact on the cost effectiveness of equity pay. To provide a sense of the relative magnitude, I conduct a back-of-the-envelope calculation for the cost effectiveness of equity pay in my framework. The worker's IC constraint (13) implies that a given amount of equity pay (β) can incentivize workplace engagement with total effort cost up to

$$C_{max} = \beta^2 \left(\frac{\gamma}{2}\sigma_f^2 \frac{e_H\phi - e_L\phi}{2(1+e_H\phi)(1+e_L\phi)} \right).$$

Moreover, expression (14) implies that the required compensation for risk is given by

$$Cost_{risk} = \beta^2 \left(\frac{\gamma}{2}\sigma_f^2 \left(\frac{2+e_H\phi}{2+2e_H\phi} \right) + \frac{\gamma}{2}\sigma_m^2 \right).$$

For simplicity, I assume that firm output has equal exposure to the firm- and market-level productivity shocks (i.e., $\sigma_f = \sigma_m = \sigma$) and no workplace engagement occurs without effort (i.e., $e_L = 0$). Suppose the worker's engagement effort generates information that resolves 10% of the income risk stemming from firm-level productivity shocks.[17] In this case, the model implies that equity pay that requires \$6,855 in compensation for risk can incentivize engagement effort that costs \$175.[18]

5.4 Direct Monitoring

The main specification of the model assumes that the firm can only induce high workplace engagement effort through optimal compensation contracting. However, in general, the availability of substitute governance mechanisms tends to reduce the firm's reliance on incentive pay. In the model, one can interpret e_L as the amount of effort the firm can

[17] This assumption corresponds to $\frac{e_H\phi}{1+e_H\phi} = 0.1$.

[18] This calculation assumes that the worker earns \$80,000 in salary and \$30,000 in equity pay. Following Oyer and Schaefer (2005), I set the worker's coefficient of absolute risk aversion at 2.5 divided by the worker's salary ($\gamma = \frac{2.5}{80000}$) and the volatility of equity pay at 50% ($\beta\sigma = 15000$). Although different assumptions result in different costs, in general, the ratio of risk compensation to total effort cost is much smaller in my framework than in other models with severe free-rider problems.

induce through direct monitoring. Improvements in the direct monitoring mechanism increase the baseline level of engagement (e_L), which reduces the additional productivity gains from inducing high workplace engagement, lowering the benefit of equity pay. Furthermore, an increase in e_L implies that workers receive more information about the firm-level productivity shock without needing to exert high engagement effort, which makes incentive compatibility more difficult. These factors suggest that improvements in monitoring mechanisms reduce the cost effectiveness of equity pay for the purpose of incentivizing engagement effort. Hence, my framework predicts that firms with better monitoring are less likely to use equity pay.

5.5 Additional Testable Implications

The income-risk resolution mechanism in this framework generates several novel testable implications linking the firm's compensation policy and its non-executive workers' interim planning problem. First, the model predicts that equity pay for non-executive workers induces a stronger positive correlation between the worker's interim consumption decision and the firm's future performance (Lemma 1 and Proposition 1). Researchers could test the income-risk resolution mechanism by examining whether equity pay increases the correlation between workers' interim consumption decisions (e.g., acquiring a car, renovating an existing home, purchasing a new home, enrolling children in private schools) and subsequent firm performance measures (e.g., earnings, sales, stock returns).[19]

Second, the income-risk resolution mechanism is most applicable in settings where workplace engagement is important for the firm and information that resolves income risk is highly valuable for its workers. Hence. the model predicts that the positive correlation between workers' interim consumption decisions and future firm performance following the institution of equity pay should be more pronounced for knowledge-intensive firms (Propositions 1 and 3). Younger or newly relocated workers are more likely to face

[19] For example, many Nordic countries maintain detailed administrative databases at the worker level that are available for research (e.g., Calvet et al. 2009, Bos et al. 2018).

significant financial decisions, such as family size planning or buying a home, whereas older or more established workers typically rely less on future income and have likely made most of the major financial decisions in their lifetime. Hence, the model's predictions should be more pronounced for younger, less-established workers (Proposition 11). Additionally, improved access to financial markets enhances workers' ability to smooth consumption. Consequently, the model's predictions should be more relevant in regions and time periods with better financial market access (Proposition 8).

Finally, the income-risk resolution mechanism implies a specific relationship between the firm's information environment and equity pay's cost effectiveness. The model predicts that workers in roles that offer moderate access to information are most likely to respond to incentives based on the income-risk resolution mechanism (Proposition 7). Researchers could explore this relationship by characterizing how job roles tie productivity and information together, such as through the job diagnostic surveys (Hackman and Oldham 1975) frequently used in the management research. Organizations like Gallup and Burning Glass also routinely collect detailed employee engagement data. Researchers could further test the relationship by exploiting changes to the effectiveness of engagement effort, such as those induced by remote work during the COVID-19 pandemic, or by evaluating shifts in information availability through the introduction of virtual "water cooler" platforms that allow workers to exchange information, vent about workplace issues, and seek advice (e.g., Blind, Company Bowl). To my knowledge, no empirical research has directly tested these implications.

6 Conclusion

This paper presents a principal-agent model of workplace engagement that highlights a novel incentive mechanism based on income-risk resolution. Equity pay induces income risk that correlates with firm performance. When workers value information that helps resolve income risk (e.g., for better financial planning), they are motivated to exert productive effort that generates that information. This income-risk resolution mechanism implies that performance pay can incentive some types of effort even when the underlying performance measure is uninformative, challenging the conventional view of the moral hazard literature.

The income-risk resolution mechanism offers a cohesive framework to understand many observations about non-executive compensation. First, it provides a bridge between economic theory and the popular perception of firms that equity pay for non-executive workers provides incentives. Second, its predictions are consistent with the prevalence of equity pay for non-executive workers in knowledge-intensive firms. Finally, it provides a potential explanation for some puzzling stylized facts such as the positive correlation between a firm's idiosyncratic risk and its likelihood of offering equity pay broadly to all employees.

This paper also underscores a link across a broad of literature. Specifically, the income-risk resolution mechanism in my framework depends crucially on the information environment within firm and how the worker's tasks combine information processing with production. The organizational economics literature often studies optimal hierarchies for managing information flow throughout the organization for production (e.g., Bolton and Dewatripont 1994, Garicano 2000, Skrastins and Vig 2019). The management literature has a long history of exploring how job design influences incentives (e.g., Hackman and Oldham 1975). In addition, the non-executive pay literature highlights how equity pay for non-executive workers impacts other firm policies such as disclosure (Bova et al. 2015b), employee retention (Aldatmaz et al. 2018) innovation and investment (Babenko et al. 2011, Chang et al. 2015, Sun and Xiaolan 2019), payout (Babenko 2009), and risk management (Bova et al. 2015a). Consequently, this paper emphasizes an under-explored connection between research in management and organizational economics, and finance.

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A Proofs

Proof of Lemma 1. Recall that the worker's compensation contract is $W = \alpha + \beta V = \alpha + \beta(\bar{e}\theta + \sigma_f \tilde{\varepsilon}_f + \sigma_m \tilde{\varepsilon}_m)$, the worker's interim signal is $\tilde{s} = \tilde{\varepsilon}_f + \sqrt{\frac{1}{e\phi}}\tilde{\epsilon}$, and the standard normal random variables $\tilde{\varepsilon}_f$, $\tilde{\varepsilon}_m$, and $\tilde{\epsilon}$ are independent. Hence, the worker's certainty-equivalent pay at $t = 2$ conditional on a realized signal s is given by

$$\begin{aligned}
\frac{1 - e^{-\gamma \bar{W}_s}}{\gamma} &= \int_{-\infty}^{\infty} f(\varepsilon_f, s) \Bigg(\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \left(e^{-\frac{\varepsilon_m^2}{2}} \right) \left[\frac{1 - e^{-\gamma(\alpha + \beta(\bar{e}\theta) + \sigma_f \varepsilon_f + \sigma_m \varepsilon_m)}}{\gamma} \right] d\varepsilon_m \Bigg) d\varepsilon_f \\
e^{-\gamma \bar{W}_s} &= e^{-\gamma(\alpha + \beta \bar{e}\theta)} \left(\int_{-\infty}^{\infty} f(\varepsilon_f, s) e^{-\gamma \sigma_f \varepsilon_f} d\varepsilon_f \right) \left(\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \left(e^{-\frac{\varepsilon_m^2}{2}} \right) e^{-\gamma \sigma_m \varepsilon_m} d\varepsilon_m \right),
\end{aligned}$$

where $f(\varepsilon_f, s)$ is the conditional distribution of $\tilde{\varepsilon}_f$ given a realized signal s. Integrating with respect to ε_m and completing the squares in the exponential simplifies the expression for $\bar{W}_s$ into

$$e^{-\gamma \bar{W}_s} = e^{-\gamma(\alpha + \beta \bar{e}\theta - \frac{\gamma}{2}\beta^2 \sigma_m^2)} \int_{-\infty}^{\infty} f(\varepsilon_f, s) e^{-\gamma \sigma_f \varepsilon_f} d\varepsilon_f.$$

Normal-normal Bayesian updating implies that the distribution of $\tilde{\varepsilon}_f$ conditional on a realized signal s is also normal with posterior mean, $\hat{\mu} = \frac{e\phi}{1+e\phi}s$, and posterior variance, $\hat{\sigma}^2 = \frac{1}{1+e\phi}$. Hence, integrating with respect to ε_f and completing the squares yields

$$e^{-\gamma \bar{W}_s} = e^{-\gamma\left(\alpha + \beta \bar{e}\theta - \frac{\gamma}{2}\beta^2 \sigma_m^2 + \beta \sigma_f \left(\frac{e\phi}{1+e\phi}s\right) - \frac{\gamma}{2}\beta^2 \sigma_f^2 \left(\frac{1}{1+e\phi}\right)\right)}. \tag{20}$$

Substituting the simplified expression of $e^{-\gamma \bar{W}_s}$ (20) into the worker's first-order condition (8), and then solving for a_s^* completes the proof of the lemma.

Proof of Proposition 1. Recall that the worker's expected lifetime utility at $t = 0$ can be summarized by the certainty-equivalent per-period consumption $\bar{u}$, which can be expressed as a function of the precision of the worker's interim signal (Φ) and effort cost due to workplace engagement ($\bar{C}$):

$$\bar{u}(\Phi, \bar{C}) = \frac{1}{2}\Bigg(\alpha + \beta\bar{e}\theta - \frac{\gamma}{2}\beta^2\sigma_m^2 - \frac{\gamma}{2}\beta^2\sigma_f^2\left(\tfrac{2+\Phi}{2+2\Phi}\right)\Bigg) - \bar{C}.$$

The free-rider problem in the model implies that aggregate worker engagement ($\bar{e}$) does not depend on the engagement of any individual worker. As a result, individual engagement effort only affects the worker's expected lifetime utility ($\bar{u}$) by influencing the worker's signal precision and effort cost. High engagement effort corresponds to a more precise signal (i.e., $\Phi(e_H) > \Phi(e_L)$) and higher effort costs (i.e., $\bar{C}(e_H) > \bar{C}(e_L)$). Consequently, incentive compatibility requires that the worker's expected lifetime utility increase in Φ to offset its decrease due to the cost of effort (i.e., $\frac{\partial \bar{u}}{\partial \bar{C}} < 0$). The expression for $\bar{u}$ implies that $\frac{\partial \bar{u}}{\partial \Phi} > 0$ if and only if $\beta > 0$, corresponding to income risk from equity pay.

Proof of Proposition 2. The construction of the optimal IC contract follows from standard optimal contracting arguments in a CARA-normal principal-agent model. First, equity pay ($\beta > 0$) imposes income risk, which is costly due to the worker's risk aversion. As a result, the optimal IC contract minimizes β, binding the worker's IC constraint:

$$\begin{aligned} \frac{\gamma}{2}\beta_{IC}^{*2}\sigma_f^2\left(\frac{e_H\phi - e_L\phi}{2(1+e_H\phi)(1+e_L\phi)}\right) &= c \\ \beta_{IC}^* &= \sqrt{\frac{4c(1+e_H\phi)(1+e_L\phi)}{\gamma\sigma_f^2(e_H\phi - e_L\phi)}}. \end{aligned}$$

Second, the firm uses the fixed-pay component (α) to compensate the worker for bearing income risk and for their outside option, binding the worker's participation constraint (IR):

$$\begin{aligned} \bar{u}(e_H\phi, \tfrac{c}{2}) &= \frac{1}{2}\left(\alpha^* + \beta_{IC}^*\bar{e}\theta - \frac{\gamma}{2}\beta_{IC}^{*2}\sigma_f^2\left(\tfrac{2+e_H\phi}{2+2e_H\phi}\right) - \frac{\gamma}{2}\beta_{IC}^{*2}\sigma_m^2\right) - \tfrac{c}{2} = u_0 \\ \alpha_{IC}^* &= 2u_0 + c - \beta_{IC}^*\bar{e}\theta + \frac{\gamma}{2}\beta_{IC}^{*2}\sigma_f^2\left(\tfrac{2+e_H\phi}{2+2e_H\phi}\right) + \frac{\gamma}{2}\beta_{IC}^{*2}\sigma_m^2. \end{aligned}$$

Proof of Proposition 3. Because equity pay is risky and increases expected compensation costs, the firm offers equity pay only if inducing worker engagement is optimal. Hence, the firm offers equity pay if and only if $\Pi_H \geq \Pi_L$, which is equivalent to

$$
\begin{aligned}
(e_H - e_L)\theta \geq &\; Cost_{IC} \\
(e_H - e_L)\theta \geq &\; \left(1 + \frac{\gamma}{2}\beta_{IC}^{*2}\sigma_f^2 \left(\tfrac{2+e_H\phi}{2+2e_H\phi}\right) + \frac{\gamma}{2}\beta_{IC}^{*2}\sigma_m^2\right) c \\
(e_H - e_L)\theta \geq &\; \left(1 + \tfrac{(1+e_L\phi)(2+e_H\phi)}{e_H\phi - e_L\phi} + \tfrac{2(1+e_H\phi)(1+e_L\phi)}{e_H\phi - e_L\phi}\tfrac{\sigma_m^2}{\sigma_f^2}\right) c.
\end{aligned}
$$

Proof of Propositions 4, 5, 6. Proposition 3 implies that the firm induces worker engagement with equity pay if and only if $\Delta\Pi \geq 0$. Note that

$$\frac{d\Delta\Pi}{d\theta} = e_H - e_L > 0,$$

$$\frac{d\Delta\Pi}{dc} = -\left(1 + \frac{(1+e_L\phi)(2+e_H\phi)}{e_H\phi - e_L\phi} + \frac{2(1+e_H\phi)(1+e_L\phi)}{e_H\phi - e_L\phi}\frac{\sigma_m^2}{\sigma_f^2}\right) < 0,$$

$$\frac{d\Delta\Pi}{d\gamma} = 0,$$

$$\frac{d\Delta\Pi}{d\sigma_m} = -\frac{4(1+e_H\phi)(1+e_L\phi)}{e_H\phi - e_L\phi}\frac{\sigma_m}{\sigma_f^2} < 0,$$

and

$$\frac{d\Delta\Pi}{d\sigma_f} = \frac{4(1+e_H\phi)(1+e_L\phi)}{e_H\phi - e_L\phi}\frac{\sigma_m^2}{\sigma_f^3} > 0.$$

Proof of Propositions 7. Recall that the increase in the firm's expected payoff due to inducing high workplace engagement with optimal equity pay is given by

$$\Delta\Pi = (e_H - e_L)\theta - \left(1 + \tfrac{(1+e_L\phi)(2+e_H\phi)}{e_H\phi - e_L\phi} + \tfrac{2(1+e_H\phi)(1+e_L\phi)}{e_H\phi - e_L\phi}\tfrac{\sigma_m^2}{\sigma_f^2}\right) c.$$

Taking the derivative of $\Delta\Pi$ with respect to ϕ yields

$$\begin{aligned}\frac{d\Delta\Pi}{d\phi} =& \frac{c}{e_H - e_L}\left(\frac{2 + e_H\phi + 2e_L\phi + e_He_L\phi^2}{\phi^2} + \frac{2\sigma_m^2}{\sigma_f^2}\frac{1 + e_H\phi + e_L\phi + e_He_L\phi^2}{\phi^2}\right)\\ &- \frac{c}{e_H - e_L}\left(\frac{e_H + e_L + 2e_He_L\phi}{\phi} + \frac{2\sigma_m^2}{\sigma_f^2}\frac{e_H + e_L + 2e_He_L\phi}{\phi}\right)\\ =& \frac{c}{e_H - e_L}\left(\frac{1}{\phi^2}\left(2 + \frac{2\sigma_m^2}{\sigma_f^2}\right) - e_He_L - \frac{2\sigma_m^2 e_He_L}{\sigma_f^2}\right),\end{aligned}$$

which is positive if and only if

$$\begin{aligned}\frac{1}{\phi^2}\left(2 + \frac{2\sigma_m^2}{\sigma_f^2}\right) >& \; e_He_L\left(1 + \frac{2\sigma_m^2}{\sigma_f^2}\right)\\ \phi <& \sqrt{\frac{1}{e_He_L}\frac{2\sigma_f^2 + 2\sigma_m^2}{\sigma_f^2 + 2\sigma_m^2}} = \bar{\phi}.\end{aligned}$$

Note that $\bar{\phi} > 0$ because $e_H, e_L, \sigma_f, \sigma_m > 0$. Hence, $sign(\frac{d\bar{\phi}^2}{d\sigma_f}) = sign(\frac{d\bar{\phi}}{d\sigma_f})$ and $sign(\frac{d\bar{\phi}^2}{d\sigma_m}) = sign(\frac{d\bar{\phi}}{d\sigma_m})$ because $\bar{\phi} > 0$. The derivatives of $\bar{\phi}^2$ with respect to σ_f and σ_m are

$$\frac{d\phi^2}{d\sigma_f} = \frac{4\sigma_f\sigma_m^2}{e_He_L(\sigma_f^2 + 2\sigma_m^2)^2} > 0$$

and

$$\frac{d\phi^2}{d\sigma_m} = -\frac{4\sigma_f^2\sigma_m}{e_He_L(\sigma_f^2 + 2\sigma_m^2)^2} < 0,$$

respectively, implying that $\frac{d\bar{\phi}}{d\sigma_f} > 0$ and $\frac{d\bar{\phi}}{d\sigma_m} < 0$.

Proof of Propositions 8. Given a realized signal s, the worker consumes a constrained amount

$$a_s(\kappa) = \frac{1}{2}\underbrace{\left(\alpha + \beta\bar{e}\theta - \tfrac{\gamma}{2}\beta^2\sigma_f^2 - \tfrac{\gamma}{2}\beta^2\sigma_m^2\right)}_{=\bar{W}} + \frac{\kappa}{1+\kappa}\underbrace{\left(\beta\sigma_f\left(\tfrac{\Phi}{1+\Phi}\right)s + \tfrac{\gamma}{2}\beta^2\sigma_f^2\left(\tfrac{1}{1+\Phi}\right)\right)}_{=\bar{W}_s - \bar{W}}$$

at $t = 1$ out of her conditional certainty-equivalent pay at $t = 2$ ($\bar{W}_s$). Her expected lifetime utility at $t = 1$ given a realized signal s is

$$\begin{aligned} U_{1s}(\kappa) = & \frac{1 - e^{-\gamma\left(\overbrace{\frac{1}{2}\bar{W}_s + \frac{\kappa}{1+\kappa}(\bar{W}_s - \bar{W})}^{a_s(\kappa)} - \bar{C}\right)}}{\gamma} + \frac{1 - e^{-\gamma\left(\bar{W}_s - \overbrace{(\frac{1}{2}\bar{W} + \frac{\kappa}{1+\kappa}(\bar{W}_s - \bar{W}))}^{a_s(\kappa)} - \bar{C}\right)}}{\gamma} \\ = & \frac{2}{\gamma} - \frac{1}{\gamma}\left(e^{-\gamma\left(\frac{1}{2}\bar{W} - \bar{C} + \frac{\kappa}{1+\kappa}[\bar{W}_s - \bar{W}]\right)} + e^{-\gamma\left(\frac{1}{2}\bar{W} - \bar{C} + \frac{1}{1+\kappa}[\bar{W}_s - \bar{W}]\right)}\right) \\ = & \frac{2}{\gamma} - \frac{e^{-\gamma\left(\frac{1}{2}\bar{W} - \bar{C}\right)}}{\gamma}\left(e^{-\gamma\left(\frac{\gamma}{2}\frac{\kappa}{1+\kappa}\beta^2\sigma_f^2\left(\frac{\Phi}{1+\Phi}\right) + \frac{\kappa}{1+\kappa}\beta\sigma_f\left(\frac{\Phi}{1+\Phi}\right)s\right)} + e^{-\gamma\left(\frac{\gamma}{2}\frac{1}{1+\kappa}\beta^2\sigma_f^2\left(\frac{\Phi}{1+\Phi}\right) + \frac{1}{1+\kappa}\beta\sigma_f\left(\frac{\Phi}{1+\Phi}\right)s\right)}\right) \end{aligned}$$

Integrating U_{1s} with respect to s yields

$$U_0 = \frac{2}{\gamma} - \frac{e^{-\gamma\left(\frac{1}{2}\bar{W} - \bar{C}\right)}}{\gamma}\left[e^{-\gamma\left(\frac{\gamma}{2}\beta^2\sigma_f^2\left(\frac{\Phi}{1+\Phi}\right)\frac{\kappa}{(1+\kappa)^2}\right)} + e^{-\gamma\left(\frac{\gamma}{2}\beta^2\sigma_f^2\left(\frac{\Phi}{1+\Phi}\right)\frac{\kappa}{(1+\kappa)^2}\right)}\right]$$

Because $\frac{\kappa}{1+\kappa}(\bar{W}_s - \bar{W})$ and $\frac{1}{1+\kappa}(\bar{W}_s - \bar{W})$ result in the same certainty-equivalent amount, the worker's expected lifetime utility can again be expressed in terms of the per-period certainty equivalent consumption

$$\bar{u} = \frac{1}{2}\underbrace{\left(\alpha + \beta\bar{e}\theta - \frac{\gamma}{2}\beta^2\sigma_m^2 - \frac{\gamma}{2}\beta_f^2\right)}_{=\bar{W}} - \bar{C} + \frac{\gamma}{2}\beta^2\sigma_f^2\frac{\kappa}{(1+\kappa)^2}\left(\frac{\Phi}{1+\Phi}\right),$$

which implies that the worker's IC constraint can be expressed as

$$\frac{\gamma}{2}\beta^2\sigma_f^2\frac{\kappa}{(1+\kappa)^2}\frac{e_H\phi - e_L\phi}{(1+e_H\phi)(1+e_L\phi)} \geq \frac{c}{2}.$$

Note that the worker's IC cannot be satisfied when $\kappa = 0$ because $c > 0$. Additionally, following the logic underlying Propositions 2 and 4, we have

$$\beta^*_{IC}(\kappa) = \sqrt{\frac{(1+\kappa)^2}{\kappa}\frac{c(1+e_H\phi)(1+e_L\phi)}{\gamma\sigma_f^2(e_H\phi - e_L\phi)}}$$

and

$$Cost_{IC}(\kappa) = c + \frac{\gamma}{2}\beta^*_{IC}(\kappa)^2\sigma_m^2 + \frac{\gamma}{2}\beta^*_{IC}(\kappa)^2\sigma_f^2 \underbrace{\left(1 - \frac{\kappa}{(1+\kappa)^2}\frac{e_H\phi}{1+e_H\phi}\right)}_{>0}.$$

Note that $\beta^*_{IC}(\kappa)$ and $\left(1 - \frac{\kappa}{(1+\kappa)^2}\frac{e_H\phi}{1+e_H\phi}\right)$ decreases in κ. Because neither the direct effort cost ($\frac{c}{2}$) nor the productivity gain (θ) of engagement effort is affected by κ, a decrease in the cost of providing incentives implies an increase in the cost effectiveness of equity pay.

Proof of Propositions 9. Suppose for contradiction that the optimal IC contract does not feature equity pay (i.e., $\beta^* = 0$). The binding IC constraint implies that $\eta^* = \frac{c}{e_H - e_L}$. Note that the marginal cost of relaxing the worker's IC using equity pay is given by $\frac{1+e_L\phi}{(e_H-e_L)\phi}\left(2 + e_H\phi + 2(1+e_H\phi)\frac{\sigma_m^2}{\sigma_f^2}\right)$. At $\eta^* = \frac{c}{e_H-e_L}$, the marginal cost of relaxing the worker's IC using individual performance bonuses is given by $\frac{\gamma\phi_y^{-1}c}{(e_H-e_L)^2}$. When $\frac{\gamma\phi_y^{-1}c}{e_H-e_L} > \frac{1+e_L\phi}{\phi}\left(2 + e_H\phi + 2(1+e_H\phi)\frac{\sigma_m^2}{\sigma_f^2}\right)$, one can construct a cheaper IC contract by slightly increasing β and slightly decreasing η, which is a contradiction. Note that the condition is satisfied when c is sufficiently large or ϕ_y is sufficiently small.

Proof of Propositions 10. In the baseline specification of the model, the cost effectiveness of equity pay is

$$\Delta\Pi = (e_H - e_L)\theta - \left(1 + \frac{(2 + \overbrace{e_H\phi}^{=\Phi_H})(1 + \overbrace{e_L\phi}^{=\Phi_L})}{\underbrace{e_H\phi - e_L\phi}_{=\Phi_H - \Phi_L}} + \frac{2(1 + \overbrace{e_H\phi}^{=\Phi_H})(1 + \overbrace{e_L\phi}^{=\Phi_L})}{\underbrace{e_H\phi - e_L\phi}_{=\Phi_H - \Phi_L}}\frac{\sigma_m^2}{\sigma_f^2}\right)c.$$

In the extension with other information about firm performance, the precision of the worker's interim signal about the firm-level productivity shock is given by $\Phi = \phi_f + e\phi$, where ϕ_f captures the information that the worker can receive without exerting productive effort, implying that $\Phi_H = \phi_f + e_H\phi$ and $\Phi_L = \phi_f + e_L\phi$. The precision of the worker's interim signal about the market-level productivity shock is ϕ_m. Following the logic of Sections 2 and 3, one can derive the cost effectiveness of equity pay as a function of ϕ_f and ϕ_m:

$$\Delta\Pi = (e_H - e_L)\theta - \left(1 + \frac{(2 + \overbrace{\phi_f + e_H\phi}^{=\Phi_H})(1 + \overbrace{\phi_f + e_L\phi}^{=\Phi_L})}{\underbrace{e_H\phi - e_L\phi}_{=\Phi_H - \Phi_L}} + \frac{2(1 + \overbrace{\phi_f + e_H\phi}^{=\Phi_H})(1 + \overbrace{\phi_f + e_L\phi}^{=\Phi_L})}{\underbrace{e_H\phi - e_L\phi}_{=\Phi_H - \Phi_L}} \frac{\sigma_m^2 \left(\frac{2+\phi_m}{2+2\phi_m}\right)}{\sigma_f^2} \right) c.$$

Note that the term $\frac{2+\phi_m}{2+2\phi_m}$ after σ_m^2 reflects the fact that the new interim signal $\tilde{s}_m$ helps resolve some income risk stemming from the market-level productivity shock ($\tilde{\varepsilon}_m$). Taking the derivative of $\Delta\Pi$ with respect to ϕ_f and ϕ_m yields

$$\frac{d\Delta\Pi}{d\phi_f} = -\left(\frac{3 + 2\phi_f + e_H\phi + e_L\phi}{e_H\phi - e_L\phi} + \frac{2(2 + 2\phi_f + e_H\phi + e_L\phi)}{e_H\phi - e_L\phi} \frac{\sigma_m^2 \left(\frac{2+\phi_m}{2+2\phi_m}\right)}{\sigma_f^2} \right) c < 0$$

and

$$\frac{d\Delta\Pi}{d\phi_m} = \left(\frac{2(1 + \phi_f + e_H\phi)(1 + \phi_f + e_L\phi)}{e_H\phi - e_L\phi} \frac{\sigma_m^2}{\sigma_f^2} \right) \frac{2}{(2 + \phi_m)^2} > 0,$$

respectively.

Proof of Propositions 11. Following the arguments outlined in Section 2, a highly engaged ($e = e_H$) worker has a certainty-equivalent per-period consumption of

$$\bar{u}_n(e_H) = \tfrac{1}{n}\big(\alpha - \tfrac{\gamma}{2}\beta^2\sigma_f^2 \left(\tfrac{n+e_H\phi}{n+ne_H\phi}\right) - \tfrac{\gamma}{2}\beta^2\sigma_m^2 - c\big).$$

A less engaged ($e = e_L$) worker has a certainty-equivalent per-period consumption of

$$\bar{u}_n(e_L) = \tfrac{1}{n}\big(\alpha - \tfrac{\gamma}{2}\beta^2\sigma_f^2 \left(\tfrac{n+e_L\phi}{n+ne_L\phi}\right) - \tfrac{\gamma}{2}\beta^2\sigma_m^2\big).$$

Hence, the worker's IC constraint can be expressed as

$$\tfrac{1}{n}\big(\alpha - \tfrac{\gamma}{2}\beta^2\sigma_f^2 \left(\tfrac{n+e_H\phi}{n+ne_H\phi}\right) - \tfrac{\gamma}{2}\beta^2\sigma_m^2 - c\big) \geq \tfrac{1}{n}\big(\alpha - \tfrac{\gamma}{2}\beta^2\sigma_f^2 \left(\tfrac{n+e_L\phi}{n+ne_L\phi}\right) - \tfrac{\gamma}{2}\beta^2\sigma_m^2\big)$$

which simplifies to

$$\tfrac{\gamma}{2}\beta^2\sigma_f^2\left(\tfrac{n-1}{n}\tfrac{e_H\phi-e_L\phi}{(1+e_H\phi)(1+e_L\phi)}\right)\geq c.$$

When there are n consumption periods prior to vesting, the amount of equity pay in the optimal IC contract is

$$\beta_n^* = \sqrt{\frac{2c}{\gamma\sigma_f^2}\frac{n(1+e_H\phi)(1+e_L\phi)}{(n-1)(e_H\phi-e_L\phi)}}$$

and the cost effectiveness of equity pay is

$$\Delta\Pi_n = (e_H-e_L)\theta - c\left(1+\frac{(n+e_H\phi)(1+e_L\phi)}{(n-1)(e_H\phi-e_L\phi)}+\frac{\sigma_m^2}{\sigma_f^2}\frac{n(1+e_H\phi)(1+e_L\phi)}{(n-1)(e_H\phi-e_L\phi)}\right)$$

$$\Delta\Pi_n = (e_H-e_L)\theta - c - \frac{(1+e_L\phi)c}{e_H\phi-e_L\phi}\left(\frac{(n+e_H\phi)}{n-1}+\frac{\sigma_m^2}{\sigma_f^2}\frac{n(1+e_H\phi)}{n-1}\right).$$

Taking the derivative of $\Delta\Pi_n$ with respect to n yields

$$\frac{d\Delta\Pi_n}{dn} = \frac{(1+e_L\phi)c}{e_H\phi-e_L\phi}\left(\frac{1+e_H\phi}{(n-1)^2}+\frac{\sigma_m^2}{\sigma_f^2}\frac{1+e_H\phi}{(n-1)^2}\right) > 0.$$