The Quiet Hand of Regulation: Harnessing Uncertainty and Disagreement

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Abstract

Regulating externalities is particularly challenging in the presence of uncertainty and disagreement among economic agents. Traditional Pigouvian and Coasean approaches often fail because they require either precise knowledge of externality costs or frictionless bargaining. We propose an "uncertainty-based regulation" (UBR) mechanism that harnesses heterogeneous information and disagreement among firms to achieve socially efficient outcomes without requiring explicit information revelation. UBR modifies firms' payoffs based on their deviation from the aggregate action, weighted by observable outcomes, effectively creating a synthetic market that internalizes externalities. This mechanism implicitly defines property rights, aligns incentives, and elicits private information without direct negotiation. We show that UBR achieves team efficiency, dominates conventional regulation, incentivizes information acquisition, and remains robust even when firms distrust each others' signals. Moreover, if brought to a vote, it would receive unanimous support, making it politically viable.

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1 Introduction

A central challenge in economics is the design of effective regulation for activities that generate externalities. This task is made all the more difficult in the presence of uncertainty and disagreement among economic agents. Nowhere is this more evident than in the current climate-change debate, where uncertainty about future climate outcomes and disagreement among stakeholders have led to political gridlock and stalled policy adoption. Traditional approaches, rooted in the work of Pigou (1920) and Coase (1960), often falter in these complex environments. Pigouvian taxes or subsidies, while theoretically appealing, require the regulator to possess precise knowledge of the externality's magnitude (such as the social cost of carbon)—a requirement rendered unattainable by the combination of pervasive uncertainty and firms' private information (Mirrlees, 1971; Weitzman, 1974; Stiglitz, 1982). Similarly, Coasean bargaining relies on well-defined property rights and negligible transaction costs, conditions rarely met in practice, especially when dealing with diffuse externalities and large numbers of actors holding private information (Kwerel, 1977; Farrell, 1987).

The literature has responded to this challenge through two main approaches. The first approach, mechanism design, consists of developing sophisticated schemes to elicit private information from firms (e.g., Dasgupta, Hammond, and Maskin, 1979). These mechanisms are often complex, difficult to implement, and vulnerable to collusion or strategic misreporting, particularly when trust among agents is limited (Laffont and Tirole, 1986; Laffont, 1994). The second approach explores second-best policies that rely solely on publicly available information, such as uniform standards or taxes based on aggregate outcomes (e.g., Roberts and Spence, 1976; Weitzman, 1974). While simpler, these approaches sacrifice efficiency by failing to utilize the valuable private information held by firms. The regulation of externalities is then confronted with a challenging trade off: either complex mechanisms with potential fragility or simpler mechanisms with inherent inefficiency.

In this paper we address this challenge by proposing a novel regulatory approach that *harnesses* uncertainty and disagreement to achieve efficiency *without* explicit information revelation. We label this approach "Uncertainty-Based Regulation" (UBR). We consider a setting where firms undertake socially beneficial actions (e.g., investing in green technologies, contributing to public goods, enhancing cybersecurity) whose effectiveness is uncertain and about which firms possess heterogeneous private information.

Instead of directly controlling prices or quantities, UBR adjusts firms' payoffs based on (i) the gap between their individual actions and the average, and (ii) observable aggregate outcomes such as temperature changes or total emissions. We show that this seemingly minor adjustment—a "quiet hand" of regulation—is equivalent to creating a *synthetic Coasean market* for the externality. An endogenous "shadow price" emerges for deviating from the average, reflecting both the overall level of socially beneficial activity and the true, underlying state of the environment. By maximizing their profits under this regulation, firms implicitly act as Cournot competitors responding to this endogenous price. This implicit price mimics the outcome of Coasean bargaining, but without requiring explicit negotiation or ex-ante defined property rights. It induces firms to internalize the externality and coordinate their actions in a way that implicitly incorporates their private information.

Our key result is to show that the proposed UBR achieves the *team-efficient* allocation in the economy, that is, the best outcome a benevolent social planner could attain, given the constraint that private information remains decentralized and cannot be directly communicated among agents (Radner, 1962; Angeletos and Pavan, 2007, 2009). This shows that efficiency can be achieved without explicit information revelation or centralized control, unlike standard regulatory instruments such as Pigouvian taxes, which often fall short of even the second-best outcome under uncertainty and heterogeneous information. UBR strictly dominates both the status quo (no regulation) and a social planner with only public information. This stems from two sources: a beneficial "tilting" of aggregate activity and the freedom of firms to choose actions tailored to their private information, which is absent under uniform policies. UBR *elicits* firms' dispersed knowledge, thereby boosting efficiency. This mechanism remains effective even when agents distrust the quality of others' information; in fact, distrust further strengthens their incentive to acquire more precise private signals. Finally, we examine the political economy implications of the proposed regulation, showing that in a voting setting, all firms would ex-ante prefer this approach, making it politically viable.

In sum, UBR bridges the Pigouvian goal of internalizing externalities and the Coasean principle of decentralized efficiency, achieving both without the precise ex-ante knowledge demanded by Pigou (1920) or the restrictive conditions required by Coase (1960). We show that the proposed mechanism is formally equivalent to a Cournot game in a hypothetical market for the externality, where the equilibrium price is determined by observable outcomes, connecting our results to the broader literature on market-based mechanisms and information aggregation in Cournot settings (e.g., Vives, 1988).

Literature. The work most closely related to ours is Angeletos and Pavan (2009), who analyze economies with dispersed information about aggregate shocks and characterize the resulting inefficiencies, suggesting that taxes contingent on aggregate outcomes could improve welfare. We build directly on their framework and propose an implementable mechanism. Unlike standard regulatory tools, UBR achieves team efficiency by creating a synthetic market that aligns private incentives with social goals without the need to observe firms' private information. We prove this is impossible with standard Pigouvian taxation or a cap-and-trade scheme, and offer a new economic interpretation, showing how the mechanism creates implicit property rights that mimics the outcome of Coasean bargaining.

Our work is also related to the literature on information economics and heterogeneous beliefs. Our model builds on Romer (1986), where agents take an aggregate variable—knowledge in Romer's model, environmental quality in ours—as given. While individual agents cannot directly influence environmental quality, their collective actions shape it, generating positive externalities. We extend this framework by introducing belief heterogeneity and a regulatory design that elicits private information, demonstrating how a regulator can leverage uncertainty and disagreement to align individual incentives with societal goals. This places our work within the literature on regulation under information asymmetry, e.g., Roberts and Spence (1976); Kwerel (1977); Montero (2008), though we focus specifically on leveraging agents' disagreements and private signals to coordinate actions toward a social optimum.

Additionally, by incorporating agents with diverse private signals, our work is related to the literature on heterogeneous beliefs and market dynamics (Keynes, 1964; Bikhchandani, Hirshleifer, and Welch, 1992; Morris and Shin, 2002; Veldkamp, 2011; Myatt and Wallace, 2008). As in Morris and Shin (2002), agents anticipate others' actions, with a firm's profit depending on its deviation from aggregate actions, a form of "beauty contest". Furthermore, similar to Banerjee (2011), our model includes agents who may distrust others' signals. Unlike most of this literature, however, our focus is on how regulation can exploit uncertainty and disagreement as policy tools to align individual actions with societal goals, even amid agent distrust.

Finally, our work contributes to the broader literature on uncertainty, economic decision-making, and regulatory design. Since the seminal work of Knight (1921), uncertainty has often been viewed as a barrier to effective decision-making (e.g., Bernanke, 1983; Rodrik, 1991; Dixit and Pindyck, 1994; Caballero and Pindyck, 1996). We challenge this perspective by demonstrating that uncertainty can be harnessed to achieve desirable societal goals. This insight aligns with Wang (2022), who shows that environmental regulatory uncertainty can incentivize firms to adopt greener practices preemptively, and with Pindyck (2007, 2022), who explore uncertainty's role in prompting proactive environmental policies. In the context of climate policy and regulatory economics (Nordhaus, 2019; Stern, 2007; Heal, 2009), our approach offers an alternative to traditional regulatory tools. While Pigouvian taxes (Pigou, 1920) require precise estimates of externalities and Coasean bargaining (Coase, 1960) relies on well-defined property rights and frictionless negotiations—conditions rarely met in practice (Farrell, 1987)—our framework bridges these perspectives by designing a regulatory mechanism that internalizes externalities without explicit information revelation or centralized control. In so doing, our work connects to the literature on market-based mechanisms and information aggregation in strategic settings (Vives, 1988).

In summary, our findings contribute to the long-standing debate on optimal regulation under uncertainty and asymmetric information, and have broad potential applications, ranging from environmental policy and the promotion of innovation to public health and cybersecurity. By demonstrating that it *is* possible to achieve a high degree of efficiency without resorting to complex, potentially fragile revelation mechanisms or sacrificing the benefits of decentralized decision-making, we offer a practical and theoretically sound alternative to traditional regulatory tools.

The paper proceeds as follows. Section 2 introduces the model, presents benchmark allocations, and characterizes the equilibrium allocation under UBR. Section 3 analyzes welfare and political viability. Section 4 studies the impact of distrust. Section 5 concludes. Appendix A contains all proofs.

2 The Model

Consider an economy with two dates, $t \in \{0, 1\}$, populated by a continuum of households indexed by $i \in [0, 1]$. Each household acts as both a consumer and a producer (owning and operating a firm). There is a single consumption good, which serves as the numéraire. Each household i is endowed with e units of the consumption good at time 0.

Actions and payoffs. At time $t = 1$, consumers derive utility from consumption and from the overall quality of the environment. We begin with a general utility function, $\mathcal{U}(c_i, q)$, where c_i represents consumer i's consumption and q represents *environmental quality*. We assume that consumers prefer higher consumption and higher environmental quality ($\mathcal{U}_{c_i}(c_i, q) > 0$ and $\mathcal{U}_q(c_i, q) > 0$), and that there are diminishing marginal returns (or increasing marginal damages) to environmental quality ($\mathcal{U}_{qq}(c_i, q) < 0$).[1] The utility function incorporates environmental quality as a non-pecuniary factor. This is consistent with integrated climate assessment models (Nordhaus, 1991, 2015), which incorporate the effects of climate change and emissions into the utility function.[2]

Each household i operates a firm. At $t = 0$, household i chooses γ_i, which represents its firm's contribution to a socially desirable activity (e.g., investment in green technology). We denote by γ_i the firm's level of "greenness." While individual households choose γ_i, *aggregate greenness*, $\tilde{\Gamma} = \int_0^1 \gamma_i di$, is determined by the combined actions of all households and

[1] Throughout this paper, we use subscripts to denote partial derivatives. For example, given a function $f(x, y)$, we have $f_x \equiv \partial f/\partial x$, $f_y \equiv \partial f/\partial y$, $f_{xx} \equiv \partial^2 f/\partial x^2$, $f_{xy} \equiv \partial^2 f/\partial x \partial y$, and so on.

[2] See, e.g., Michel and Rotillon (1995) and Baker, Hollifield, and Osambela (2022) for further discussion, and Acemoglu, Aghion, Bursztyn, and Hemous (2012) for an additional application.

is uncertain at $t = 0$.[3]

The firm's profit, $\pi(\gamma_i)$, depends on the household's choice of greenness, γ_i. We assume that all firms initially operate at a *status quo* level of greenness, Γ_0. The profit function is quadratic, concave (i.e., $\pi_{\gamma\gamma} < 0$ is a negative constant), and maximized at $\gamma_i = \Gamma_0$, reflecting diminishing returns and adjustment costs incurred for changing the level of greenness. Consequently, time-1 consumption for household i is:

$$c_i = e + \pi(\gamma_i). \tag{1}$$

Environmental quality is measured through changes in an Environmental Performance Index (EPI), where a higher EPI reflects a healthier environment.[4] A higher EPI reflects reductions in greenhouse gas emissions, carbon dioxide levels, deforestation, and other climate risks. We define $\tilde{q}$ as the change in the EPI from time 0 to time 1. To link environmental quality to aggregate greenness, we assume the following relationship:

$$\tilde{q} = \tilde{\theta} + a(\tilde{\Gamma} - \Gamma_0), \tag{2}$$

where $\tilde{\theta}$ is a random variable representing the underlying uncertain environmental trend absent human interventions and $a > 0$ is a "climate sensitivity" parameter. Equation (2) implies that increased aggregate greenness relative to the status quo improves environmental quality. We assume that the expected environmental trend $\mu_\theta \equiv \mathbb{E}[\tilde{\theta}]$ is public information. Although $\tilde{\theta}$ and $\tilde{\Gamma}$ are uncertain at $t = 0$, the realized value of q is publicly observed at $t = 1$.

At $t = 1$, the values of $\tilde{\theta}$, $\tilde{\Gamma}$, and c_i are realized. At $t = 0$, household i chooses γ_i to maximize its expected utility. Ideally, the household would choose γ_i to maximize $\mathbb{E}_i[\mathcal{U}(c_i, \tilde{q})]$, where c_i is given in equation (1), $\tilde{q}$ is defined in equation (2), and the expectation $\mathbb{E}_i[\cdot]$ is based on the information available to household i at $t = 0$. However, because of the general form of $\mathcal{U}$ and the uncertainty surrounding $\tilde{q}$, this optimization problem is analytically intractable.

To proceed, we approximate the general utility function $\mathcal{U}(c_i, q)$ using a Taylor expansion around (c_0, q_0), where c_0 is a baseline level of consumption and $q_0 = \mu_\theta$ is the expected value

[3] We use a tilde (e.g., $\tilde{\Gamma}$) to denote variables that are uncertain at $t = 0$, when households make their decisions. Variables without tildes (e.g., q, c_i, γ_i, Γ_0, a) represent either quantities known at $t = 0$ or values realized at $t = 1$.

[4] See https://epi.yale.edu/ for an example.

of environmental quality in the status quo (both corresponding to the status quo greenness level Γ_0 and the expected environmental trend μ_θ). Specifically,

$$U(c_i, \theta, \Gamma) := \mathcal{U}(c_0, \mu_\theta) + \mathcal{U}_c(c_0, \mu_\theta)(c_i - c_0) + \mathcal{U}_q(c_0, \mu_\theta)(q - \mu_\theta) + \frac{1}{2}\mathcal{U}_{qq}(c_0, \mu_\theta)(q - \mu_\theta)^2. \quad (3)$$

Substituting the expression for c_i from equation (1) and $\tilde{q}$ from equation (2) into the Taylor expansion (3), we obtain a new utility function, $U(c_i, \tilde{\theta}, \tilde{\Gamma})$, that is quadratic in $\tilde{\theta}$ and $\tilde{\Gamma}$, and linear in c_i.[5] This linear-quadratic form is commonly used in macroeconomic models with dispersed information (Angeletos and Pavan, 2007, 2009) and renders the model analytically tractable.

Therefore, at $t = 0$, household i chooses γ_i to maximize:

$$\max_{\gamma_i} \mathbb{E}_i[U(c_i, \tilde{\theta}, \tilde{\Gamma})], \quad (4)$$

subject to equation (1).

The Taylor approximation (3) and our assumptions on $\mathcal{U}$ yield the following properties for U:

Property 1 $U_c = \mathcal{U}_c(c_0, \mu_\theta) > 0$ is a positive constant. The first-order approximation of $\mathcal{U}$ in c_i implies U is linear in c_i.

Property 2 $U_\Gamma(\tilde{\theta}, \tilde{\Gamma})$ is linear in $\tilde{\theta}$ and $\tilde{\Gamma}$. This follows from the second-order approximation of $\mathcal{U}$ in q and the linearity of (2).

Property 3 $U_{\Gamma\Gamma} = a^2\mathcal{U}_{qq}(c_0, \mu_\theta) < 0$ is a negative constant. Since $\mathcal{U}_{qq}(c_0, \mu_\theta) < 0$, we have $U_{\Gamma\Gamma} < 0$, reflecting diminishing marginal returns to aggregate greenness.

Property 4 $U_{c\Gamma} = 0$. Marginal utility of consumption is independent of aggregate greenness, following from U's linearity in c_i and the lack of $c_i\tilde{\Gamma}$ interaction after substituting (2).

Property 5 $U_{\theta\Gamma} = a\mathcal{U}_{qq}(c_0, \mu_\theta) < 0$ is a negative constant. Given $a > 0$ and $\mathcal{U}_{qq}(c_0, \mu_\theta) < 0$, we have $U_{\theta\Gamma} < 0$.

[5]The function $U(c_i, \tilde{\theta}, \tilde{\Gamma})$ represents the approximated utility, expressed as a function of consumption and the uncertain variables $\tilde{\theta}$ and $\tilde{\Gamma}$. The household's choice of γ_i directly affects its utility through its impact on c_i, where $c_i = e + \pi(\gamma_i)$.

Information. At time $t = 0$, all households share a common prior belief about the unobservable environmental trend $\tilde{\theta}$, which we assume to be represented by a normal distribution, $\tilde{\theta} \sim \mathcal{N}(\mu_\theta, \sigma_\theta^2)$. Each household i observes a private signal y_i and a public signal z about $\tilde{\theta}$:

$$y_i = \tilde{\theta} + \tilde{\varepsilon}_{y,i}, \tag{5}$$

$$z = \tilde{\theta} + \tilde{\varepsilon}_z, \tag{6}$$

where $\tilde{\varepsilon}_{y,i} \sim \mathcal{N}(0, \tau_y^{-1})$ and $\tilde{\varepsilon}_z \sim \mathcal{N}(0, \tau_z^{-1})$ are independent noise terms, also independent of $\tilde{\theta}$. The parameters τ_y and τ_z represent the precisions of the private and public signals, respectively. We assume throughout that households trust the informational content of others' signals. Section 4 relaxes this assumption and explores the implications of agents' distrust in the quality of each others' private signals.

Let $\mathbb{E}_i[\tilde{\theta}] := \mathbb{E}[\tilde{\theta}|y_i, z]$ denote household i's posterior expectation of $\tilde{\theta}$, conditional on its information set $\{y_i, z\}$. The posterior precision, τ, is defined as:

$$\tau := (\mathrm{Var}[\tilde{\theta}|y_i, z])^{-1} = \sigma_\theta^{-2} + \tau_y + \tau_z. \tag{7}$$

By Bayes' theorem, household i's posterior expectation is the prior mean plus precision-weighted deviations of the private and public signals from that mean:

$$\mathbb{E}_i[\tilde{\theta}] = \mu_\theta + \frac{\tau_y}{\tau}(y_i - \mu_\theta) + \frac{\tau_z}{\tau}(z - \mu_\theta). \tag{8}$$

2.1 Benchmark Allocations

Before introducing our regulatory mechanism, we define several benchmark allocations that will serve as points of comparison. These benchmarks help illustrate the inefficiencies that arise in the absence of regulation and the potential gains from our proposed approach.

We begin by characterizing the *status quo* allocation, which represents the outcome when there is no regulation and households do not internalize the externality. Since each household owns and operates a single firm, we can equivalently refer to the household's decision as the firm's decision.

Lemma 1 (**Status quo allocation**). *In the absence of regulation or policy intervention, each* *firm chooses its greenness level to maximize the household's utility, ignoring the environmental impact. This results in all firms choosing the status quo level of greenness:*

$$\gamma_i^{sq} = \Gamma_0, \quad \forall i \in [0,1]. \tag{9}$$

Hence, aggregate greenness is also at the status quo level:

$$\tilde{\Gamma}^{sq} = \Gamma_0. \tag{10}$$

Given the assumption that $\pi(\gamma_i)$ is a quadratic concave function maximized at Γ_0, the status quo ($\gamma_i^{sq} = \Gamma_0$) is, by construction, privately optimal. It is, however, socially inefficient because γ_i^{sq} does not accounts for the positive externality of green investments, which enhance environmental quality.

Next, consider the *first-best* allocation. This allocation is defined as the solution to a social planner's problem, where the planner possesses perfect information regarding the environmental trend $\tilde{\theta}$ at time $t = 0$, that is, it observes θ directly and has direct control over all firms' greenness choices. The planner's objective is to maximize social welfare, subject to the constraint that the planner internalizes the externality arising from the impact of individual greenness choices on aggregate greenness.

Since the social planner observes θ directly, all households are effectively homogeneous. Consequently, the planner's problem reduces to choosing a uniform greenness level, γ, for all firms. The first-best allocation is thus the solution to the following:

$$\max_{\gamma} \; U(c(\gamma), \theta, \Gamma(\gamma)), \tag{11}$$

subject to $c(\gamma) = e + \pi(\gamma)$ and $\Gamma(\gamma) = \gamma$.

Lemma 2 (**First-best allocation**). *The first-best allocation is characterized by a uniform greenness level γ^* across all firms that is a linear function of the realized environmental trend:*

$$\gamma^*(\theta) = \kappa_0^* + \kappa_1^*(\theta - \mu_\theta), \tag{12}$$

where

$$\kappa_0^* := \Gamma_0 - \frac{U_\Gamma(\mu_\theta, \Gamma_0)}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} \qquad \textit{and} \qquad \kappa_1^* := -\frac{U_{\theta\Gamma}}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}}. \tag{13}$$

This first-best allocation is unattainable in our main setting because θ is unobservable at $t = 0$. In the first-best, all firms choose $\gamma^*(\theta)$ contingent on the realized value of θ. Since $U_{\theta\Gamma} < 0$ (by Property 5), $U_c > 0$ (by Property 1), $\pi_{\gamma\gamma} < 0$ (by assumption), and $U_{\Gamma\Gamma} < 0$ (by Property 3), we have $\kappa_1^* < 0$. Therefore, the first-best level of greenness, γ^*, is decreasing in θ: a worse environmental trend (lower θ) requires a higher level of green investment.

Finally, consider the *team-efficient* allocation. Following Radner (1962), and Angeletos and Pavan (2007, 2009), this benchmark represents the best achievable outcome when a social planner can specify a decision rule that maps each firm's private and public information to a recommended greenness level, but cannot directly observe individual firms' information sets. Firms are assumed to commit to this rule, maximizing ex-ante expected social welfare. This differs from the first-best because actions are decentralized.

To formalize this, the ex-ante expected social welfare in this economy is given by:

$$\mathbb{E}[U] := \int_{(\tilde{\theta},z)} \int_{y_i} U(e + \pi(\gamma_i), \tilde{\theta}, \tilde{\Gamma}) dP(y_i|\tilde{\theta}, z) dP(\tilde{\theta}, z), \tag{14}$$

where $dP(y_i|\tilde{\theta}, z)$ is the conditional probability distribution of the private signal y_i given $\tilde{\theta}$ and z, and $dP(\tilde{\theta}, z)$ is the joint probability distribution of $\tilde{\theta}$ and the public signal z.

The team-efficient allocation is then defined as the strategy $\gamma_i^{te}(y_i, z)$ that solves the following maximization problem:

$$\max_{\gamma_i^{te}(\cdot)} \int_{(\tilde{\theta},z)} \int_{y_i} U\Big(e + \pi\Big(\gamma_i^{te}(y_i, z)\Big), \tilde{\theta}, \tilde{\Gamma}^{te}(\tilde{\theta}, z)\Big) dP(y_i|\tilde{\theta}, z) dP(\tilde{\theta}, z), \tag{15}$$

subject to $\tilde{\Gamma}^{te}(\tilde{\theta}, z) = \int_{y_i} \gamma_i^{te}(y_i, z) dP(y_i|\tilde{\theta}, z)$.

Lemma 3 (**Team-efficient allocation**). *The team-efficient allocation is characterized by each firm choosing a greenness level, γ_i^{te}, that is a linear function of its conditional expectations of the first-best level of individual greenness, γ^*, and the aggregate greenness, $\tilde{\Gamma}^{te}$:*

$$\gamma_i^{te} = (1 - \alpha)\mathbb{E}_i[\gamma^*] + \alpha \mathbb{E}_i[\tilde{\Gamma}^{te}], \tag{16}$$

where $\mathbb{E}_i[\cdot]$ denotes the expectation conditional on household i's information set $\{y_i, z\}$, γ^* is the first-best level of greenness defined in Lemma 2, $\tilde{\Gamma}^{te} = \int_0^1 \gamma_i^{te} di$ is the aggregate greenness, *and α is defined as*

$$\alpha := -\frac{U_{\Gamma\Gamma}}{U_c \pi_{\gamma\gamma}}. \tag{17}$$

Since $U_{\Gamma\Gamma} < 0$, $U_c > 0$ and $\pi_{\gamma\gamma} < 0$, we have $\alpha < 0$, implying that firms' actions are strategic substitutes in equilibrium.

In the team-efficient allocation, each firm's action, γ_i^{te}, is a weighted average of its expectation of the first-best action, $\mathbb{E}_i[\gamma^*]$, and its expectation of the aggregate action, $\mathbb{E}_i[\tilde{\Gamma}^{te}]$. This reflects the dual goals of responding to private information and coordinating with others. The weight α governs the balance between these considerations and represents the *equilibrium degree of coordination.* When $\alpha < 0$, as in this case, actions are strategic substitutes, pushing firms to differentiate from others and rely more on their private information. As shown by Angeletos and Pavan (2007, 2009), this optimal action, shaped by expectations of both individual and aggregate targets, is a hallmark of economies with dispersed information and strategic interactions.

We now compare the welfare implications of the three benchmark allocations.

Lemma 4 (**Welfare ranking**). *The ex-ante expected social welfare, defined in* (14)*, is ranked across the three benchmark allocations as follows:*

$$\mathbb{E}[U^{sq}] \leq \mathbb{E}[U^{te}] \leq \mathbb{E}[U^*], \tag{18}$$

where $\mathbb{E}[U^{sq}]$, $\mathbb{E}[U^{te}]$, and $\mathbb{E}[U^*]$ denote the ex-ante expected social welfare under the status quo, team-efficient, and first-best allocations, respectively.

The team-efficient allocation improves upon the status quo by leveraging dispersed private information, even without direct information sharing or centralized control. However, because information remains dispersed, the team-efficient allocation remains inferior to the unattainable first-best, where θ is perfectly observed.

2.2 Uncertainty-Based Regulation (UBR)

The goal of the regulatory mechanism we propose, UBR, is to harness uncertainty and disagreement without requiring the regulator to have precise knowledge of the externality's magnitude or to observe firms' private information. The central idea of UBR is to modify firms' payoffs by adding a term that depends on both the firm's individual action and observable aggregate outcomes. Crucially, this modification is *not* a command-and-control approach; firms retain their freedom to choose their greenness levels. Instead, as we will show below, the mechanism creates an implicit "market" for the externality, where the "shadow price" of deviating from the average action is determined *ex post* by realized environmental outcomes.

Formally, let $f(q)$ be a function of the *realized* environmental quality, q. This function is known to all agents at $t = 0$ and is part of the regulatory framework, even though its value depends on the ex-post realizations of q. The regulator modifies each firm's profit at time $t = 1$ by adding the following term:

$$\mathcal{T}(q, \Gamma, \gamma_i) = (\gamma_i - \Gamma) f(q). \tag{19}$$

This regulatory term has a specific interpretation. The factor $(\gamma_i - \Gamma)$ represents the difference between firm i's greenness choice and aggregate greenness, measuring the firm's deviation from the average action. The function $f(q)$ acts as a "shadow price" for this deviation, determined ex post by the realized environmental quality, q. Although the *value* of this price depends on a realized outcome, the *function* $f(q)$ itself is fully specified ex ante as part of the regulatory framework. This term introduces a *beauty contest* element into the firms' decisions, creating incentives similar to those studied by Morris and Shin (2002).

Accounting for the regulatory term, the household's time-1 consumption becomes:

$$c_i = e + \pi(\gamma_i) + \mathcal{T}(q, \Gamma, \gamma_i). \tag{20}$$

At $t = 0$, firm i chooses the level of greenness γ_i to solve the following maximization problem:

$$\max_{\gamma_i} \mathbb{E}_i[U(\tilde{c}_i, \tilde{\theta}, \tilde{\Gamma})], \tag{21}$$

where by equation (20), the consumption at time $t = 1$ is $\tilde{c}_i = e + \pi(\gamma_i) + \mathcal{T}(\tilde{q}, \tilde{\Gamma}, \gamma_i)$. Note that, from the household's perspective at $t = 0$, consumption at $t = 1$ is now uncertain due to the regulatory term $\mathcal{T}(\tilde{q}, \tilde{\Gamma}, \gamma_i)$.

What functional form for $f(\tilde{q})$ will induce firms to choose the team-efficient allocation? In other words, how can we design the regulatory mechanism to align individual incentives with the socially desirable outcome, despite the presence of asymmetric information and uncertainty? As Angeletos and Pavan (2009) suggest, appropriately designed interventions that are contingent on aggregate outcomes can, in principle, achieve team efficiency in economies with dispersed information. In the following proposition we provide a concrete mechanism that achieves team efficiency.

Proposition 1 (**Team efficiency of UBR**). *Let the regulatory function $f(\tilde{q})$ be given by:*

$$f(\tilde{q}) = a \left[\mathrm{MRS} - \mathrm{ERA}(\tilde{q} - \mu_\theta) \right], \tag{22}$$

where MRS is the marginal rate of substitution between environmental quality and consumption (evaluated at the status quo) and ERA represents society's aversion to environmental risk (also evaluated at the status quo):

$$\mathrm{MRS} := \frac{\mathcal{U}_q(c_0, \mu_\theta)}{\mathcal{U}_c(c_0, \mu_\theta)} > 0, \tag{23}$$

$$\mathrm{ERA} := -\frac{\mathcal{U}_{qq}(c_0, \mu_\theta)}{\mathcal{U}_c(c_0, \mu_\theta)} > 0. \tag{24}$$

Then, the unique linear equilibrium in an economy with UBR is for each firm to choose the team-efficient level of greenness, γ_i^{te}, as defined in Lemma 3.

The marginal rate of substitution (MRS) between environmental quality and consumption represents the baseline social cost of reducing environmental quality—or equivalently, the social benefit of improving it—expressed in units of consumption. Society's aversion to environmental risk (ERA) reflects the concavity of the utility function with respect to environmental quality; a more negative $\mathcal{U}_{qq}$ (i.e., stronger diminishing marginal utility of environmental quality) implies a higher ERA. The term $(\tilde{q} - \mu_\theta)$, unknown at $t = 0$, captures the deviation of realized environmental quality from its expected status quo level.

Proposition 1 shows that the regulatory function acts as an endogenous price for deviations from average greenness. This price consists of two components: a baseline term, $a \times \text{MRS}$, which captures the fundamental trade-off between consumption and environmental quality, and a risk adjustment term, $-a \times \text{ERA} \times (q - \mu_\theta)$, which scales with both society's aversion to environmental risk (ERA) and the environmental surprise $(q - \mu_\theta)$. For instance, if the realized value of $q - \mu_\theta$ is strongly negative, meaning environmental quality is much worse than expected, the price of greenness rises significantly. In such bad times, greenness becomes especially valuable, as mitigating environmental damage is more urgent.

Because q is publicly observable at $t = 1$ and all other components are predetermined, the UBR mechanism is implementable. The regulator specifies the function $f(\cdot)$ ex ante, announcing calibrated values for a, MRS, and ERA. The climate sensitivity a is informed by scientific evidence,[6] while MRS and ERA can be either estimated from economic data or set as policy choices. Firms then choose γ_i, incorporating the regulatory rule and their private information. Ex post, q is observed and the regulatory adjustment $\mathcal{T}(q, \Gamma, \gamma_i)$ is applied to each firm's realized profit. As such, UBR requires no firm-specific cost data, adapts to shocks, incentivizes efficient action, and remains transparent. Moreover, the regulation is *neutral*, solely redistributing profits among firms—that is, from equation (19), $\int_0^1 \mathcal{T}(q, \Gamma, \gamma_i) di = 0$, hence the regulatory term cancels out when integrated across firms. Finally, its flexibility allows policymakers to embed different degrees of environmental risk aversion through ERA.

Proposition 1 establishes that the appropriately defined regulatory function, $f(\tilde{q})$, achieves the team-efficient allocation even without the regulator accessing firms' private signals. This function creates an endogenous "price" on deviations from aggregate greenness. This "price" induces firms to internalize the externality and encourages them to act based on their private information. The UBR needs no complex revelation mechanisms, or even advanced knowledge of externality levels. In the next section we show that this regulatory mechanism is equivalent to a *synthetic* Cournot market for greenness.

[6] The parameter a is informed by estimates of the relationship between greenhouse gas emissions and environmental quality, in the broader context of limiting global warming. See Meinshausen, Meinshausen, Hare, Raper, Frieler, Knutti, Frame, and Allen (2009) and the references therein.

2.2.1 Equivalence Between UBR and a Synthetic Cournot Market

Consider a hypothetical Cournot market where firms produce and trade "units of greenness" at an endogenous price, p. At $t = 1$, after uncertainty is resolved, each consumer i solves the following optimization problem:

$$\max_{\{c_i, g_i\}} U(c_i, \theta, g_i) \tag{25}$$

subject to

$$c_i + pg_i = e + \pi_i, \tag{26}$$

where the utility function $U(\cdot,\cdot,\cdot)$ is defined as in equation (3), c_i is consumption of the numéraire good, g_i is the quantity of greenness purchased, e is an endowment of the numéraire, and π_i represents consumer i's share of firm profits, which the consumer treats as exogenous. This framework is analogous to a consumer maximizing utility over greenness and the numéraire, with the utility function remaining consistent with the main model. Equation (26) is an *imaginary* budget constraint where the consumer would pay a price p for greenness.

At time $t = 0$, before the environmental state $\tilde{\theta}$ and aggregate greenness $\tilde{\Gamma}$ are revealed, each firm i chooses its level of greenness, γ_i^c (where the superscript 'c' denotes the Cournot analogy), to maximize household i's expected utility:

$$\max_{\gamma_i} \mathbb{E}_i \left[U(e + \pi(\gamma_i^c) + p\gamma_i^c, \tilde{\theta}, \tilde{\Gamma}^c) \right], \tag{27}$$

where $\tilde{\Gamma}^c = \int_i \gamma_i^c di$. This objective function includes the firm's revenue from selling greenness, $p\gamma_i^c$, in addition to its standard profit function, $\pi(\gamma_i^c)$ (which remains unchanged from the original model). Because each firm is infinitesimally small, it acts as a price-taker, treating the market price of greenness, p, as given.

In essence, the above analysis describes a market where firms supply greenness, and consumers demand it, even though this market doesn't *literally* exist. The following proposition shows that this synthetic Cournot market achieves the same outcome as UBR in equilibrium.

Proposition 2 (Cournot equivalence). *The UBR mechanism with the regulatory function $f(\tilde{q})$ defined in Proposition 1 is equivalent to a Cournot game in which firms supply greenness* *to a market with an inverse demand curve given by $p = f(q)$. In this Cournot game:*

1. *The realized equilibrium price of greenness at time $t = 1$ is given by $p = f(q)$.*
2. *Each firm's optimal greenness level is given by $\gamma_i^c = \gamma_i^{te}$, as defined in Lemma 3.*
3. *The realized aggregate greenness level at time $t = 1$ is given by $\Gamma^{te} = \int_0^1 \gamma_i^{te} di$.*

Proposition 2 reveals the underlying mechanism of UBR: the regulatory function, $f(q)$, serves as the inverse demand curve in a synthetic Cournot market for greenness. By maximizing their augmented profits, firms implicitly act as Cournot competitors responding to this endogenous "price," which reflects both aggregate greenness, Γ, and the realized environmental state, θ. This construction shares conceptual similarities with the information aggregation properties explored in Cournot models (see, e.g., Vives, 1988), but within a *synthetically* created market. Consequently, UBR achieves the Coasean ideal of efficient resource allocation in the presence of externalities (Coase, 1960), yet circumvents the need for explicit bargaining or perfectly defined property rights.

2.3 Strategic Behavior and Information Acquisition

Having established that UBR can achieve the team-efficient allocation, we now characterize the equilibrium behavior of firms and their incentives regarding information acquisition.

Proposition 3 (**Equilibrium greenness**). *Under UBR, with the regulatory function defined in Proposition 1, a linear equilibrium is for each firm i to choose a greenness level, γ_i, that is a linear function of its private signal, y_i, and the public signal, z:*

$$\gamma_i = \beta_0 + \beta_y(y_i - \mu_\theta) + \beta_z(z - \mu_\theta), \tag{28}$$

where

$$\beta_0 := \Gamma_0 - \frac{U_\Gamma(\mu_\theta, \Gamma_0)}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}}, \tag{29}$$

$$\beta_y := -\frac{U_{\theta\Gamma}\tau_y}{U_c \pi_{\gamma\gamma}\tau + U_{\Gamma\Gamma}\tau_y} < 0, \tag{30}$$

$$\beta_z := -\frac{1}{1-\alpha} \frac{U_{\theta\Gamma}\tau_z}{U_c \pi_{\gamma\gamma}\tau + U_{\Gamma\Gamma}\tau_y} < 0, \tag{31}$$

and where α is defined in Lemma 3.[7]

The coefficients β_y and β_z are negative. Consequently, firms increase their greenness investments in response to either private or public signals that indicate a deterioration in the expected environmental trend. The following corollary examines the relative weight firms place on private versus public information, as reflected in the ratio β_y/β_z.

Corollary 3.1 (**Strategic substitutability and information weighting**). *In the linear equilibrium of Proposition 3, firms' actions are strategic substitutes. Furthermore, the ratio of the weights placed on private and public signals is:*

$$\frac{\beta_y}{\beta_z} = (1-\alpha)\frac{\tau_y}{\tau_z} > \frac{\tau_y}{\tau_z}, \tag{32}$$

implying that firms overweight their private information relative to the public signal, compared to the Bayesian posterior weights in equation (8).

As established in Lemma 3, the parameter α, which determines the nature of strategic interaction, is negative. Following Angeletos and Pavan (2007), α represents the equilibrium degree of coordination and reflects the slope of a firm's best response function with respect to the average action of other firms. A negative α implies strategic substitutability: firms increase (decrease) their greenness when they expect others to decrease (increase) theirs. In essence, firms have an incentive to differentiate their actions from their competitors. This drive to differentiate is what motivates firms overweight on their private information, as shown in the corollary.

While strategic substitutability might suggest a free-rider problem, it has a key offsetting benefit: it amplifies the informational content of firms' actions. Because each firm reacts more strongly to its private signal than it would in a purely Bayesian update, aggregate greenness, $\tilde{\Gamma}$, becomes more sensitive to the dispersed private information in the economy. This heightened responsiveness, driven by the $(1-\alpha)$ term in the ratio β_y/β_z, improves the informational efficiency of the aggregate outcome. We show in Section 3 that this effect contributes to the welfare gains under UBR.

[7]We restrict attention to linear strategies, consistent with the linear-quadratic framework. See, e.g., Angeletos and Pavan (2007, 2009) for similar approaches.

2.3.1 Aggregate Greenness and its Determinants

We now examine the aggregate implications of firms' equilibrium behavior. Substituting the equilibrium expression for γ_i from Proposition 3 into the definition of aggregate greenness, $\tilde{\Gamma} = \int_0^1 \gamma_i di$, and noting that $\int_0^1 (y_i - \mu_\theta) di = \tilde{\theta} - \mu_\theta$ by the law of large numbers, we obtain:

$$\tilde{\Gamma} = \beta_0 + \beta_y(\tilde{\theta} - \mu_\theta) + \beta_z(z - \mu_\theta). \tag{33}$$

The coefficients β_y and β_z (given in Proposition 3) govern the sensitivity of aggregate greenness to the unobservable environmental trend, $\tilde{\theta}$, and the public signal, z, respectively. Thus, UBR induces firms to collectively respond to the underlying environmental state, $\tilde{\theta}$, even though this state is not directly observable by the regulator. As shown in Proposition 3, the sum $\beta_y + \beta_z$ is negative. Equation (33) thus reveals that aggregate green investment *compensates* for adverse environmental shocks: a lower $\tilde{\theta}$ (worse underlying conditions) triggers a rise in $\tilde{\Gamma}$.

We define the *information sensitivity* of aggregate greenness, denoted by $\mathcal{B}$, as the absolute magnitude of the combined response to the environmental trend and the public signal:

$$\mathcal{B} := |\beta_y + \beta_z|. \tag{34}$$

This measure captures the overall responsiveness of aggregate green investment to changes in available information about the environmental state. A higher $\mathcal{B}$ implies that UBR more effectively translates dispersed private and public signals into collective action, better aligning green investment with the (unobserved) environmental needs. This improved alignment contrasts with the unresponsive status quo. The regulator's goal is to achieve the team-efficient level of responsiveness, meaning that $\mathcal{B}$ should be high enough to reflect the available information, but not so high as to overreact to noise.

The following corollary characterizes the key determinants of this information sensitivity.

Corollary 3.2 (**Information sensitivity**). *The information sensitivity, $\mathcal{B}$, of aggregate green investment has the following properties:*

(a) It increases with greater uncertainty about $\tilde{\theta}$ (higher σ_θ).

(b) It increases with the precision of private information (τ_y).

(c) It increases with the precision of public information (τ_z).

Point (a) of Corollary 3.2 reveals a key feature of UBR: it transforms environmental uncertainty, typically an impediment to effective policy, into a mechanism for enhancing information sensitivity. Specifically, a higher σ_θ *increases* the responsiveness of aggregate greenness to informative signals, promoting a more efficient aggregate response.

Points (b) and (c) of Corollary 3.2 further demonstrate UBR's effectiveness in aggregating information. Increased precision of either private information (τ_y) or public information (τ_z) also increases the information sensitivity, $\mathcal{B}$. While seemingly distinct from the effect of uncertainty in (a), these results are complementary. Higher τ_y and τ_z increase the weights, β_y and β_z respectively, placed on the corresponding signals in the aggregate greenness equation (33). Thus, both greater uncertainty (higher σ_θ) and greater precision (higher τ_y or τ_z) enhance the information sensitivity of the system, albeit through different channels.

2.3.2 Incentives for Information Acquisition

We now examine whether individual firms have an incentive to acquire more precise private information about the underlying environmental trend, $\tilde{\theta}$, when the UBR mechanism is in place. To analyze this, we consider a unilateral deviation by a single firm. We assume this firm can exogenously increase the precision of its private signal and we examine the impact on its ex-ante expected utility.

Suppose firm i increases the precision of its private information from τ_y to $\tau_y' = \xi\tau_y$, where $\xi > 1$. This represents an exogenous increase in the quality of the firm's private signal, y_i. We assume this is a unilateral deviation: firm i takes the actions of all other firms, and the parameters of the UBR mechanism, as given. It does not anticipate that its action will change the equilibrium behavior of other firms or the aggregate outcome, except insofar as its own action affects its payoff. To isolate the effect of increased signal precision, the following corollary characterizes its impact on the firm's action and its ex-ante expected utility—evaluated before the firm observes its private signal y_i but after the public signal z

has been revealed.

Corollary 3.3 (**Incentives for Information Acquisition**). *Under the UBR mechanism, consider a unilateral increase in the precision of firm i's private signal about $\tilde{\theta}$ from τ_y to $\tau'_y = \xi\tau_y$, where $\xi > 1$. The firm's action, γ'_i, is then given by:*

$$\gamma'_i = \beta_0 + \beta_y \left[1 + \frac{(\xi - 1)(1 + \sigma_\theta^2 \tau_z)}{1 + \sigma_\theta^2(\xi\tau_y + \tau_z)}\right](y_i - \mu_\theta) + \beta_z \left[1 - \frac{(\xi - 1)(1 - \alpha)\sigma_\theta^2 \tau_y}{1 + \sigma_\theta^2(\xi\tau_y + \tau_z)}\right](z - \mu_\theta), \quad (35)$$

where the coefficients β_0, β_y, and β_z are determined in Proposition 3. The resulting change in ex-ante expected utility, conditional on the public signal z, is:

$$\Delta\mathbb{E}[U|z] = -\frac{U_c}{2\pi_{\gamma\gamma}} \left(Var[\mathbb{E}'_i[f(\tilde{q})]|z] - Var[\mathbb{E}_i[f(\tilde{q})]|z] \right) > 0, \quad (36)$$

where $\mathbb{E}'_i[f(\tilde{q})]$ denotes the expectation of $f(\tilde{q})$ conditional on the more precise private signal. Consequently, the firm's ex-ante expected utility is strictly increasing in the precision of its private signal.

Equation (35) shows how the firm's optimal action adjusts in response to the increased precision. If $\xi = 1$, then $\gamma'_i = \gamma_i$, and the firm's action reverts to the original equilibrium. However, if $\xi > 1$, the firm places relatively more weight on its (now more precise) private signal, $(y_i - \mu_\theta)$, and relatively less weight on the public signal, $(z - \mu_\theta)$, reflecting the enhanced informational content of its private signal.

Equation (36) shows a strict increase in ex-ante expected utility due to the enhanced signal precision. A more precise private signal leads to a more precise posterior belief about $f(\tilde{q})$, which is reflected in a higher variance of the *expectation* of $f(\tilde{q})$ conditional on z. This, in turn, allows a more efficient response to the underlying environmental state, providing a clear incentive for information acquisition.

In sum, UBR achieves the team-efficient allocation *and* incentivizes firms to acquire more precise private information about the environmental trend, $\tilde{\theta}$. Therefore, UBR has the potential to not only align current actions with social objectives but also to encourage firms to actively seek more accurate information about environmental trends.

3 Welfare Analysis and Political Viability of UBR

We now turn to a welfare analysis, comparing UBR to alternative policy approaches and subsequently examining its political feasibility. We begin with a comparison to a social planner constrained by limited information.

3.1 The Social Planner's Allocation

Consider a benevolent social planner who seeks to maximize ex-ante expected social welfare. The planner's information set consists solely of publicly available information—the prior distribution of $\tilde{\theta}$ and, potentially, the public signal z—excluding individual private signals, y_i, and the true state, θ. Therefore, the only feasible policy is to set a uniform greenness level, γ_{sp}, for all firms.

The social planner's problem can be formally stated as:

$$\max_{\gamma_{sp}} \mathbb{E}_{sp}\left[U(c(\gamma_{sp}), \tilde{\theta}, \Gamma(\gamma_{sp}))\right], \tag{37}$$

where $c(\gamma_{sp}) = e + \pi(\gamma_{sp})$, $\Gamma(\gamma_{sp}) = \gamma_{sp}$, and $\mathbb{E}_{sp}[\cdot]$ denotes the expectation taken with respect to the social planner's information set. We now characterize the solution to this constrained optimization problem and its welfare implications.

Proposition 4 (**Social Planner's solution**). *The solution to the social planner's problem in equation (37) is given by:*

$$\gamma_{sp} = \Gamma_0 + \frac{-\mathbb{E}_{sp}[U_\Gamma(\tilde{\theta}, \Gamma_0)]}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} = \mathbb{E}_{sp}[\gamma^*], \tag{38}$$

where γ^* is the first-best allocation derived in Lemma 2.

The social planner allocation γ_{sp} yields a higher ex-ante expected utility than the status quo ($\gamma_i = \Gamma_0$ for all i). Specifically, define the welfare gain relative to the status quo by $\Delta W_{sp} := \mathbb{E}\left[\Delta U_i\right]$, where $\Delta U_i := \mathbb{E}_i[U(e + \pi(\gamma_{sp}), \tilde{\theta}, \gamma_{sp})] - \mathbb{E}_i[U(e + \pi(\Gamma_0), \tilde{\theta}, \Gamma_0)]$. Then the welfare gain can be written as

$$\Delta W_{sp} = \frac{-1}{2(U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma})}\left[U_\Gamma(\mu_\theta, \Gamma_0)^2 + U_{\theta\Gamma}^2 \frac{\sigma_\theta^2 \tau_z}{\sigma_\theta^{-2} + \tau_z}\right] > 0. \tag{39}$$

The welfare gain is increasing in the precision of the public signal , τ_z. However, the first-best allocation γ^* (Lemma 2) and the team-efficient allocation γ_i^{te} (Lemma 3) both yield strictly higher ex-ante welfare than the social planner allocation γ_{sp}.

The constrained social planner's optimal uniform greenness level, γ_{sp}, given by (38), is equivalent to the expected first-best level of greenness, conditional on public information. This reflects the planner's inability to condition policy on private signals; the best achievable outcome is thus the average first-best, given the public signal z and the prior on $\tilde{\theta}$.

The welfare gain ΔW_{sp} arises from two distinct channels. The first term, $U_\Gamma(\mu_\theta, \Gamma_0)^2$, captures the welfare improvement from adjusting the allocation away from the status quo (Γ_0) toward the social planner's chosen allocation (γ_{sp}). This improvement occurs because at the status quo, the marginal social benefit $U_\Gamma(\mu_\theta, \Gamma_0)$ differs from zero. The second term, $U_{\theta\Gamma}^2 \frac{\sigma_\theta^2 \tau_z}{\sigma_\theta^{-2} + \tau_z}$, captures the value of adjusting greenness based on the public signal z. This term increases with both the precision of the public signal (τ_z) and the responsiveness of the marginal social benefit to the underlying state ($U_{\theta\Gamma}$). Hence, the social planner's intervention unambiguously improves welfare relative to the status quo. Nonetheless, welfare remains strictly below the team-efficient and first-best allocations because the planner cannot use private information.

Going forward, we assume that the social planner's chosen allocation γ_{sp} exceeds the status quo Γ_0, reflecting the presence of a positive externality from green investment.

3.1.1 Implementing the Social Planner's Solution

We next demonstrate that the social planner's allocation, γ_{sp}, can be implemented through either a Pigouvian tax or a cap-and-trade system, as in the classic "prices-versus-quantities" framework of Weitzman (1974). For this purpose, let $\mathcal{E}(\gamma_i)$ denote the emissions generated by a firm investing γ_i in greenness. We assume $\mathcal{E}(\gamma_i) > 0$, $\mathcal{E}'(\gamma_i) < 0$, and $\mathcal{E}''(\gamma_i) > 0$. Thus, all firms generate some negative environmental impact, but emissions decrease (at a diminishing rate) as firm greenness increases.

A suitably designed Pigouvian tax on emissions can replicate the social planner's outcome.

Corollary 4.1 (**Pigouvian tax on emissions**). *The social planner's optimal uniform greenness level, γ_{sp}, can be implemented by imposing a Pigouvian tax, T, per unit of emissions:*

$$T = \frac{(\gamma_{sp} - \Gamma_0)\pi_{\gamma\gamma}}{\mathcal{E}'(\gamma_{sp})}. \tag{40}$$

Since we have assumed that green investment generates a positive externality, i.e., $\gamma_{sp} > \Gamma_0$, it follows that the emissions tax is strictly positive: $T > 0$.

Intuitively, the tax internalizes the expected environmental externality perceived by the social planner, who observes only public signals and the prior distribution of $\tilde{\theta}$. When a firm considers deviating its greenness from γ_{sp}, it must now account for the tax on the emissions thereby generated. Because T depends solely on public information, all firms respond uniformly by choosing γ_{sp}, and the resulting aggregate emissions coincide with those in the social planner's constrained optimum.

An alternative approach to implementing the social planner's solution is through a system of tradable permits, or "cap-and-trade." As before, let each firm's emissions be given by $\mathcal{E}(\gamma_i)$, and suppose the regulator issues a total quantity of permits, Q, where each permit allows one unit of emissions. Firms must acquire permits equal to the emission they produce, and the equilibrium permit price is denoted by p_{permit}.

Corollary 4.2 (**Cap-and-Trade implementation**). *The social planner's optimal uniform greenness level, γ_{sp}, can be implemented by issuing a quantity of tradable permits, Q, equal to the aggregate emissions associated with γ_{sp}, i.e., $Q = \mathcal{E}(\gamma_{sp})$. The resulting equilibrium permit price, p_{permit}, will be identical to the Pigouvian tax, T, derived in Corollary 4.1, and will induce all firms to choose $\gamma_i = \gamma_{sp}$.*

By setting $Q = \mathcal{E}(\gamma_{sp})$, the regulator ensures that the equilibrium permit price—identical to the Pigouvian tax T from Corollary 4.1—induces all firms to choose the social planner's greenness level γ_{sp}. Critically, however, neither Q nor p_{permit} can be contingent on firms' private information. Thus, the permit price reflects only public information available to the social planner, precluding firm-specific, information-contingent responses required for team efficiency.

Unlike standard Pigouvian taxes or cap-and-trade, which rely exclusively on ex-ante public information, UBR harnesses uncertainty and dispersed private signals, thereby generating social value from private information and improving welfare outcomes (see, e.g., Angeletos and Pavan, 2007; Vives, 1988; Hayek, 1945). Given these theoretical advantages, a natural question arises concerning the political viability of UBR, which we analyze next.

3.2 Is UBR Politically Viable?

Suppose households, after observing their private signals, y_i, and the public signal, z, must vote on whether to adopt UBR or implement the social planner's allocation (as shown in Lemma 4 and Proposition 4, both these alternative dominate the status quo). The next proposition establishes the political viability of UBR.

Proposition 5 (**Political viability of UBR**). *After observing their private signals, y_i, and the public signal, z, all households compare UBR to the social planner's uniform policy. The expected utility gain from UBR, compared to the social planner's allocation, is given by:*

$$\mathbb{E}_i[\Delta U_i] = \underbrace{-\frac{1}{2}U_c\pi_{\gamma\gamma}(\gamma_i^{te}-\gamma_{sp})^2}_{\textit{Gain from individualized greenness}} \underbrace{-\frac{1}{2}U_{\Gamma\Gamma}\mathbb{E}_i[(\tilde{\Gamma}^{te}-\gamma_{sp})^2]}_{\textit{Gain from aggregate greenness}}. \tag{41}$$

Since both terms are positive, every household experiences a strictly positive utility gain under UBR. Therefore, in a direct vote between UBR and the social planner's uniform policy, UBR would receive unanimous support.

The difference in equation (41) is strictly positive due to two distinct mechanisms. The first term, $-\frac{1}{2}U_c\pi_{\gamma\gamma}(\gamma_i^{te}-\gamma_{sp})^2 > 0$, captures the value of *individualized* greenness choices permitted under UBR, allowing firms to tailor their actions to their private signals. The second term, $-\frac{1}{2}U_{\Gamma\Gamma}\mathbb{E}_i[(\tilde{\Gamma}^{te}-\gamma_{sp})^2] > 0$, reflects the utility gain from aggregate greenness adjusting to the unobservable environmental state, $\tilde{\theta}$. This latter mechanism can be viewed as a form of social insurance: adopting UBR allows aggregate greenness to adjust to the realized environmental state, thereby insuring society against environmental risk.

Since both terms in equation (41) are positive, regardless of the specific values of y_i and z, all households strictly prefer UBR over the social planner's allocation. This unanimous preference, driven by both firm-level benefits and aggregate risk mitigation, holds despite dispersed private information and heterogeneous beliefs. It suggests that UBR is not only theoretically superior but also politically viable.

4 Distrust and UBR

Thus far, our analysis has assumed that households have rational expectations and fully trust the informational content of others' signals. We now relax this assumption, introducing the possibility of *distrust* in the signals received by other agents. This allows us to explore the robustness of UBR in a setting where agents may have differing, and potentially biased, views about the informativeness of others' observations. It is particularly relevant in the context of climate change, where public opinion is often polarized and trust in scientific data can vary significantly across groups.[8]

To capture disagreement that may emerge from difference of opinion, we build on insights from Morris (1995) and introduce the concept of trust in others' private signals. While each agent i observes its own private signal y_i, it believes that the private signals of other agents (which it does not observe) follow a different structure:

$$y_j^i = \varphi\tilde{\theta} + \sqrt{1-\varphi^2}\tilde{\phi}_i + \varepsilon_{y,j}, \tag{42}$$

where, $\tilde{\phi}_i \sim \mathcal{N}(0, \sigma_\mu^2)$ captures added noise or bias in others' signals (see, e.g. Banerjee, 2011). The parameter $\varphi \in [0, 1]$ measures the extent to which agent i trusts that others' signals are correlated with the true environmental trend $\tilde{\mu}$. A higher φ indicates greater trust in others' private information.

By varying the parameter φ, our model encompasses both the standard rational expectations framework and scenarios with different level of trust. When $\varphi = 1$, agents fully trust that others' private signals are as informative as their own, consistent with rational expectations. In contrast, when $\varphi = 0$, agents regard others' signals as pure noise, representing a complete

[8] Research documents significant political polarization in public views on climate change, highlighting differing beliefs about environmental data across groups (McCright and Dunlap, 2011). For further evidence on polarized beliefs and skepticism about environmental data, see Dunlap and McCright (2008) and Douglas, Uscinski, Sutton, Cichocka, Nefes, Ang, and Deravi (2019).

lack of trust. Examining our regulation under varying trust levels helps us understand its robustness in settings where agents are not fully rational.

Impact on strategy and information. Distrust among agents fundamentally alters the information environment, with consequences for equilibrium greenness, the information sensitivity $\mathcal{B}$, and firms' incentives for private information acquisition. While several results from Section 2 continue to hold, the presence of distrust introduces important changes and new results. These modifications are summarized in Proposition 6 below. A detailed discussion of how each result from Section 2 is affected by distrust is provided in the Appendix.

Proposition 6. *When agents distrust the information of others:*

(a) The implementation of UBR, specifically the functional form of $f(\tilde{q})$ defined in Proposition 1, remains valid for achieving team efficiency.

(b) The information sensitivity $\mathcal{B}$ increases with the precision of public information, τ_z, if and only if $\varphi > 1 + \frac{1}{\alpha\sigma_\theta^2\tau_y}$.

(c) The information sensitivity of aggregate greenness, $\mathcal{B}$, increases with the degree of distrust, that is, it decreases with φ:

$$\frac{\partial \mathcal{B}}{\partial \varphi} = -\frac{\alpha\kappa_1^*\sigma_\theta^4\tau_y(\tau_y - \alpha\tau_y + \tau_z)}{(1 + \sigma_\theta^2(\tau_y + \tau_z - \alpha\tau_y\varphi))^2} < 0. \tag{43}$$

(d) Incentives for information acquisition increase with the degree of distrust.

Point (a) highlights the robustness of the UBR mechanism's design. While the presence of distrust alters equilibrium outcomes, the functional form of the payment function, $f(\tilde{q})$, remains optimal for achieving team efficiency.

However, distrust significantly impacts firm behavior and information aggregation. Increased distrust (represented by a lower φ) induces firms to rely more heavily on their private signals, y_i, and to discount the influence of public information precision, denoted by τ_z. This shift in information weighting has three key consequences. First, the impact of public information precision (τ_z) on information sensitivity ($\mathcal{B}$) is contingent on the level of trust;

specifically, τ_z increases $\mathcal{B}$ only when φ exceeds a critical threshold (point (b)). Second, increased distrust (a decrease in φ) raises the information sensitivity, $\mathcal{B}$ (point (c)), as firms place greater weight on their private information. Third, distrust strengthens incentives for firms to acquire private information (point (d)), as the relative value of their private signals increases. Ultimately, distrust makes private signals matter more.

Impact on Welfare and Political Viability. We now examine the effect of distrust on welfare and the political viability of UBR. Because UBR achieves team efficiency for any level of distrust (Proposition 6), it continues to dominate the social planner's allocation in terms of ex-ante welfare. The following proposition formalizes the implications for political viability.

Proposition 7. *Under UBR, and with a trust parameter $\varphi \in [0, 1]$, all households strictly prefer UBR to the social planner's allocation, irrespective of the value of φ.*

In summary, while distrust among introduces complexities into the equilibrium, it does not undermine the fundamental advantages of UBR. The mechanism's design, specifically the functional form of $f(\tilde{q})$, remains optimal for achieving team efficiency regardless of the level of distrust. Furthermore, UBR continues to dominate the social planner's allocation, ensuring its political viability. Distrust does, however, affect firm behavior: it increases information sensitivity by inducing greater reliance on private signals, and it strengthens incentives for information acquisition. Thus, UBR remains a robust and effective regulatory tool even in the presence of significant disagreement among economic agents.

5 Conclusion

We challenge the common view that uncertainty and disagreement hinder good decision-making. Instead, we show how both can drive markets toward efficient and socially desirable outcomes. We propose a new regulatory mechanism that embeds a *synthetic market* for the externality, ensuring firms internalize social costs and benefits—without requiring direct information disclosure. By harnessing private signals and differences in opinion, this approach turns perceived obstacles into policy tools.

Although our main implementation focuses specifically on climate-change policies, we believe that the framework we propose applies more broadly whenever there are externalities and private information varies across agents. For example, it could guide regulation in areas like cybersecurity, public health, or artificial intelligence, where uncertainty and ethical or practical disagreements are widespread. In each case, the same principle—linking individual payoffs to deviations from an aggregate benchmark—channels uncertainty and disagreement into productive coordination.

For simplicity, we assume each household owns a single firm, isolating the core incentives behind our regulation. A valuable next step is to relax this assumption and incorporate financial markets. Share prices would provide an extra public signal about firms' prospects and the broader environment. This could strengthen firms' incentives to invest in green innovation (or make other beneficial investments) since financial markets aggregate information beyond what regulators observe. Portfolio mandates—an increasingly debated policy tool—could also shape how market signals influence firms' decisions. Studying how our mechanism interacts with financial markets, particularly in pricing climate or other external risks, is a promising avenue for future research.

By showing how uncertainty and disagreement can be managed, our work contributes to the broader discussion on governance in complex environments. Rather than seeing imperfect information as a barrier, we show it can fuel market-based regulatory solutions. We hope this offers a useful template not only for climate policy but for any regulation where private information is hard to gather yet crucial for efficient outcomes.

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A Proofs

A.1 Proof of Lemma 1

In the status quo, household i maximizes its expected utility:

$$\max_{\gamma_i} \mathbb{E}_i[U(c_i, \tilde{\theta}, \tilde{\Gamma})], \tag{A1}$$

subject to $c_i = e + \pi(\gamma_i)$. Because household i is infinitesimal, its choice of γ_i does not affect $\tilde{\Gamma} = \int_0^1 \gamma_i di$. Therefore, $\tilde{\Gamma}$ can be treated as a constant with respect to the maximization over γ_i. Substituting the constraint, then taking the first-order condition with respect to γ_i is leads to:

$$\mathbb{E}_i \left[U_c(e + \pi(\gamma_i), \tilde{\theta}, \tilde{\Gamma}) \cdot \pi_\gamma(\gamma_i) \right] = 0. \tag{A2}$$

By Property 1, U_c is a positive constant. Therefore, the first-order condition simplifies to:

$$\pi_\gamma(\gamma_i) = 0. \tag{A3}$$

By assumption, the profit function $\pi(\gamma_i)$ is maximized at $\gamma_i = \Gamma_0$. Therefore, $\gamma_i^{sq} = \Gamma_0$. Since all households are identical ex-ante and face the same optimization problem, they all choose the same level of greenness. Aggregate greenness is then:

$$\tilde{\Gamma}^{sq} = \int_0^1 \gamma_i^{sq} di = \int_0^1 \Gamma_0 di = \Gamma_0. \quad \square \tag{A4}$$

A.2 Proof of Lemma 2

The perfectly informed social planner maximizes social welfare, while internalizing the externality. Due to household homogeneity, $\gamma_i = \gamma$ for all i, and thus $\tilde{\Gamma} = \gamma$. The planner's problem is then:

$$\max_{\gamma} U(c(\gamma), \theta, \Gamma(\gamma)), \tag{A5}$$

subject to $c(\gamma) = e + \pi(\gamma)$ and $\Gamma(\gamma) = \gamma$. Substituting the constraints, we have:

$$\max_{\gamma} U(e + \pi(\gamma), \theta, \gamma). \tag{A6}$$

The first-order condition with respect to γ is:

$$\frac{dU}{d\gamma} = U_c \pi_\gamma(\gamma) + U_\Gamma(\theta, \gamma) = 0. \tag{A7}$$

By Property 1, U_c is a constant. The quadratic profit function, maximized at Γ_0, implies $\pi_\gamma(\gamma) = \pi_{\gamma\gamma}(\gamma - \Gamma_0)$. Applying a first-order Taylor expansion to $U_\Gamma(\theta, \gamma)$ around (μ_θ, Γ_0), which is justified by Property 2, yields:

$$U_\Gamma(\theta,\gamma) = U_\Gamma(\mu_\theta,\Gamma_0) + U_{\theta\Gamma}(\theta-\mu_\theta) + U_{\Gamma\Gamma}(\gamma-\Gamma_0). \tag{A8}$$

Substituting into the first-order condition:

$$U_c\pi_{\gamma\gamma}(\gamma-\Gamma_0) + U_\Gamma(\mu_\theta,\Gamma_0) + U_{\theta\Gamma}(\theta-\mu_\theta) + U_{\Gamma\Gamma}(\gamma-\Gamma_0) = 0. \tag{A9}$$

Solving for γ, which we denote as $\gamma^*(\theta)$ in the first-best:

$$\gamma^*(\theta) = \Gamma_0 - \frac{U_\Gamma(\mu_\theta,\Gamma_0)}{U_c\pi_{\gamma\gamma}+U_{\Gamma\Gamma}} - \frac{U_{\theta\Gamma}}{U_c\pi_{\gamma\gamma}+U_{\Gamma\Gamma}}(\theta-\mu_\theta). \quad \square \tag{A10}$$

A.3 Proof of Lemma 3

An efficient allocation is a strategy $\gamma_i^{te}(y_i,z)$ that maximizes

$$\mathbb{E}[U] = \int_{(\tilde\theta,z)}\int_{y_i} U\big(e+\pi(\gamma_i^{te}(y_i,z)),\tilde\theta,\tilde\Gamma^{te}(\tilde\theta,z)\big)dP(y_i|\tilde\theta,z)dP(\tilde\theta,z), \tag{A11}$$

subject to $\tilde\Gamma^{te}(\tilde\theta,z) = \int_{y_i}\gamma_i^{te}(y_i,z)dP(y_i|\tilde\theta,z)$. Note that given the continuum of firms, the empirical distribution of private signals converges to the conditional distribution $P(y_i|\tilde\theta,z)$ by the Law of Large Numbers. Consequently, the aggregate greenness, $\tilde\Gamma(\tilde\theta,z)$, can be represented both as an integral over the signal distribution and as an integral over the firm index: $\int_{y_i}\gamma_i(y_i,z)dP(y_i|\tilde\theta,z) = \int_i \gamma_i di$. Write the Lagrangian:

$$\begin{aligned}\Lambda = &\int_{(\tilde\theta,z)}\int_{y_i} U\big(e+\pi(\gamma_i^{te}(y_i,z)),\tilde\theta,\tilde\Gamma^{te}(\tilde\theta,z)\big)dP(y_i|\tilde\theta,z)dP(\tilde\theta,z)\\ &+\int_{(\tilde\theta,z)}\lambda(\tilde\theta,z)\Big[\tilde\Gamma^{te}(\tilde\theta,z) - \int_{y_i}\gamma_i^{te}(y_i,z)dP(y_i|\tilde\theta,z)\Big]dP(\tilde\theta,z).\end{aligned} \tag{A12}$$

The first order condition for $\tilde\Gamma^{te}(\tilde\theta,z)$,

$$\int_{y_i} U_\Gamma\Big(\tilde\theta,\tilde\Gamma^{te}(\tilde\theta,z)\Big)dP(y_i|\tilde\theta,z) + \lambda(\tilde\theta,z) = 0, \tag{A13}$$

must hold for almost all $(\tilde\theta,z)$. Thus:

$$U_\Gamma\Big(\tilde\theta,\tilde\Gamma^{te}(\tilde\theta,z)\Big) + \lambda(\tilde\theta,z) = 0. \tag{A14}$$

The chain rule of probability states that:

$$dP(y_i,\tilde\theta,z) = dP(y_i|\tilde\theta,z)dP(\tilde\theta,z) = dP(\tilde\theta|y_i,z)dP(y_i,z). \tag{A15}$$

By the chain rule and Fubini's theorem (to change the order of integration), the Lagrangian becomes:

$$\Lambda = \int_{(y_i,z)} \int_{\tilde{\theta}} U\left(e + \pi(\gamma_i^{te}(y_i,z)), \tilde{\theta}, \tilde{\Gamma}^{te}(\tilde{\theta},z)\right) dP(\tilde{\theta}|y_i,z) dP(y_i,z)$$
$$+ \int_{(y_i,z)} \int_{\tilde{\theta}} \lambda(\tilde{\theta},z) \Big[\tilde{\Gamma}^{te}(\tilde{\theta},z) - \int_{y_i} \gamma_i^{te}(y_i,z) dP(y_i|\tilde{\theta},z)\Big] dP(\tilde{\theta}|y_i,z) dP(y_i,z). \tag{A16}$$

The first order condition for $\gamma_i^{te}(y_i,z)$ is then:

$$\int_{\tilde{\theta}} \Big[U_c \pi_\gamma(\gamma_i^{te}(y_i,z)) - \lambda(\tilde{\theta},z)\Big] dP(\tilde{\theta}|y_i,z) = 0, \tag{A17}$$

which must hold for almost all (y_i,z). Replacing the first-order condition for $\tilde{\Gamma}^{te}$, we obtain:

$$\mathbb{E}_i\Big[U_c \pi_\gamma(\gamma_i^{te}(y_i,z))\Big] + \mathbb{E}_i\Big[U_\Gamma\Big(\tilde{\theta}, \tilde{\Gamma}^{te}(\tilde{\theta},z)\Big)\Big] = 0. \tag{A18}$$

Since π_γ and U_Γ are linear in their arguments, we can write:

$$\pi_\gamma(\gamma_i^{te}(y_i,z)) = \pi_\gamma(\gamma^*) + \pi_{\gamma\gamma}(\gamma_i^{te}(y_i,z) - \gamma^*), \tag{A19}$$

$$U_\Gamma\Big(\tilde{\theta}, \tilde{\Gamma}^{te}(\tilde{\theta},z)\Big) = U_\Gamma(\tilde{\theta},\gamma^*) + U_{\Gamma\Gamma}(\tilde{\Gamma}^{te}(\tilde{\theta},z) - \gamma^*). \tag{A20}$$

By definition of the first-best allocation (see equation (A7)):

$$U_c \pi_\gamma(\gamma^*) + U_\Gamma(\tilde{\theta},\gamma^*) = 0, \tag{A21}$$

and thus one can rewrite equation (A18) as

$$U_c \pi_{\gamma\gamma} \mathbb{E}_i[\gamma_i^{te}(y_i,z) - \gamma^*] + U_{\Gamma\Gamma} \mathbb{E}_i[\tilde{\Gamma}^{te}(\tilde{\theta},z) - \gamma^*] = 0. \tag{A22}$$

Solving for $\gamma_i^{te}(y_i,z)$ yields:

$$\gamma_i^{te}(y_i,z) = (1-\alpha)\mathbb{E}_i[\gamma^*] + \alpha \mathbb{E}_i[\tilde{\Gamma}^{te}(\tilde{\theta},z)], \tag{A23}$$

where α is defined as in Lemma 3, equation (17). $\square$

A.4 Proof of Lemma 4

The ranking $\mathbb{E}[U^{sq}] \leq \mathbb{E}[U^{te}]$ follows from the definition of the team-efficient allocation as the solution to the welfare maximization problem. The team-efficient allocation, $\gamma_i^{te}(y_i,z)$, is chosen to maximize $\mathbb{E}[U]$ subject to the constraint that actions depend only on the available information (private and public signals). The status quo allocation, $\gamma_i^{sq} = \Gamma_0$, is not the solution to this maximization problem, and thus cannot yield a higher level of ex-ante expected welfare.

We now prove the statement that welfare is highest in the first-best. The ex-ante expected social welfare in this economy is given by equation (14). Recall that $\tilde{\Gamma} = \int_0^1 \gamma_i di$ and that U is linear in $c_i = e + \pi(\gamma_i)$ and $\pi(\gamma_i)$ is quadratic in γ_i. Thus, U can be written as:

$$U(e + \pi(\gamma_i), \tilde{\theta}, \tilde{\Gamma}) = U(e + \pi(\tilde{\Gamma}), \tilde{\theta}, \tilde{\Gamma}) + U_c \pi_\gamma(\tilde{\Gamma}) \cdot (\gamma_i - \tilde{\Gamma}) + \frac{1}{2} U_c \pi_{\gamma\gamma} \cdot (\gamma_i - \tilde{\Gamma})^2. \tag{A24}$$

This is a Taylor series expansion around the point $\gamma_i = \tilde{\Gamma}$. The transformation is exact, since U is quadratic in γ_i. We omit the dependence of $\pi_{\gamma\gamma}$ on $\tilde{\Gamma}$ since $\pi(\cdot)$ is a quadratic function.

The linear term, $U_c \pi_\gamma(\tilde{\Gamma}) \cdot (\gamma_i - \tilde{\Gamma})$, vanishes when integrated over the conditional distribution of private signals in equation (14), $\int_{y_i} U_c \pi_\gamma(\tilde{\Gamma}) \cdot (\gamma_i - \tilde{\Gamma}) dP(y_i|\tilde{\theta}, z)$, because $\int_{y_i} \gamma_i dP(y_i|\tilde{\theta}, z) = \tilde{\Gamma}$. The quadratic term, when integrated over the conditional distribution of private signals, is the variance of γ_i conditional on $\tilde{\theta}$ and z: $\int_{y_i} (\gamma_i - \tilde{\Gamma})^2 dP(y_i|\tilde{\theta}, z) = \sigma_\gamma^2$, where $\sigma_\gamma^2 := \int_{y_i} (\gamma_i - \int_{y_i} \gamma_i' dP(y_i'|\tilde{\theta}, z))^2 dP(y_i|\tilde{\theta}, z)$. Thus, defining $W(\tilde{\theta}, \tilde{\Gamma}, \sigma_\gamma) := U(e + \pi(\tilde{\Gamma}), \tilde{\theta}, \tilde{\Gamma}) + \frac{1}{2} U_c \pi_{\gamma\gamma} \sigma_\gamma^2$, ex-ante welfare can be rewritten as:

$$\mathbb{E}[U] = \int_{(\tilde{\theta}, z)} W(\tilde{\theta}, \tilde{\Gamma}, \sigma_\gamma) dP(\tilde{\theta}, z). \tag{A25}$$

We notice that the first-best γ^* is the unique solution to $W_\Gamma(\tilde{\theta}, \gamma^*, 0) = 0$. A second-order Taylor expansion of $W(\tilde{\theta}, \tilde{\Gamma}, \sigma_\gamma)$ around $\tilde{\Gamma} = \gamma^*$ and $\sigma_\gamma = 0$ gives:

$$\begin{aligned} W(\tilde{\theta}, \tilde{\Gamma}, \sigma_\gamma) = {} & W(\tilde{\theta}, \gamma^*, 0) + W_\Gamma(\tilde{\theta}, \gamma^*, 0) \cdot (\tilde{\Gamma} - \gamma^*) + W_{\sigma_\gamma}(\tilde{\theta}, \gamma^*, 0) \cdot \sigma_\gamma \\ & + \frac{1}{2} W_{\Gamma\Gamma}(\tilde{\theta}, \gamma^*, 0) \cdot (\tilde{\Gamma} - \gamma^*)^2 + \frac{1}{2} W_{\sigma_\gamma \sigma_\gamma}(\tilde{\theta}, \gamma^*, 0) \cdot \sigma_\gamma^2. \end{aligned} \tag{A26}$$

Replacing $W_\Gamma(\tilde{\theta}, \gamma^*, 0) = 0$ and $W_{\sigma_\gamma}(\tilde{\theta}, \gamma^*, 0) = 0$ and recognizing that $\int_{(\tilde{\theta}, z)} (\tilde{\Gamma} - \gamma^*)^2 dP(\tilde{\theta}, z) = \mathbb{E}[(\tilde{\Gamma} - \gamma^*)^2]$ and $\int_{(\tilde{\theta}, z)} \sigma_\gamma^2 dP(\tilde{\theta}, z) = \mathbb{E}[(\gamma_i - \tilde{\Gamma})^2]$, we obtain

$$\mathbb{E}[U] = \mathbb{E}[W(\tilde{\theta}, \gamma^*, 0)] + \frac{1}{2}(U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}) \cdot \mathbb{E}[(\tilde{\Gamma} - \gamma^*)^2] + \frac{1}{2} U_c \pi_{\gamma\gamma} \cdot \mathbb{E}[(\gamma_i - \tilde{\Gamma})^2]. \tag{A27}$$

Since $U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma} < 0$ and $U_c \pi_{\gamma\gamma} < 0$, it implies that welfare is highest in the first-best:

$$\mathbb{E}[U] \leq \mathbb{E}[W(\tilde{\theta}, \gamma^*, 0)]. \tag{A28}$$

The additional terms, $\frac{1}{2}(U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}) \cdot \mathbb{E}[(\tilde{\Gamma} - \gamma^*)^2]$ and $U_c \pi_{\gamma\gamma} \cdot \mathbb{E}[(\gamma_i - \tilde{\Gamma})^2]$, measure welfare losses due to volatility and dispersion, as $-(U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma})$ can be interpreted as "social aversion to volatility" and $-U_c \pi_{\gamma\gamma}$ can be interpreted as "social aversion to dispersion." See Angeletos and Pavan (2007) for similar interpretations. $\square$

A.5 Proof of Proposition 1

Under UBR, firm i's maximization problem leads to the first-order condition:

$$\mathbb{E}_i[U_c(\pi_\gamma(\gamma_i) + f(\tilde{q}))] = 0. \tag{A29}$$

Substituting $\pi_\gamma(\gamma_i) = \pi_{\gamma\gamma}(\gamma_i - \Gamma_0)$, then solving for γ_i:

$$\gamma_i = \Gamma_0 - \frac{\mathbb{E}_i[f(\tilde{q})]}{\pi_{\gamma\gamma}}. \tag{A30}$$

Consider the team-efficient allocation from Lemma 3:

$$\gamma_i^{te} = \left(1 + \frac{U_{\Gamma\Gamma}}{U_c \pi_{\gamma\gamma}}\right) \mathbb{E}_i[\gamma^*] - \frac{U_{\Gamma\Gamma}}{U_c \pi_{\gamma\gamma}} \mathbb{E}_i[\tilde{\Gamma}^{te}]. \tag{A31}$$

where γ^* is the first-best greenness level (obtained in Lemma 2):

$$\gamma^* = \Gamma_0 - \frac{U_\Gamma(\mu_\theta, \Gamma_0)}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} - \frac{U_{\theta\Gamma}}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} (\tilde{\theta} - \mu_\theta). \tag{A32}$$

We want to find a regulatory function $f(\tilde{q})$ such that the firm's optimal choice under UBR given by equation (A30) equals the team-efficient greenness, γ_i^{te} (given by the general form of equation (A31) without the *'te'* superscript on the last term). That is, we seek to satisfy:

$$\Gamma_0 - \frac{\mathbb{E}_i[f(\tilde{q})]}{\pi_{\gamma\gamma}} = \left(1 + \frac{U_{\Gamma\Gamma}}{U_c \pi_{\gamma\gamma}}\right) \mathbb{E}_i[\gamma^*] - \frac{U_{\Gamma\Gamma}}{U_c \pi_{\gamma\gamma}} \mathbb{E}_i[\tilde{\Gamma}]. \tag{A33}$$

Substituting in the expression for γ^* from equation (A32) and taking expectations:

$$\Gamma_0 - \frac{\mathbb{E}_i[f(\tilde{q})]}{\pi_{\gamma\gamma}} = \left(1 + \frac{U_{\Gamma\Gamma}}{U_c \pi_{\gamma\gamma}}\right) \left(\Gamma_0 - \frac{U_\Gamma(\mu_\theta, \Gamma_0)}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} - \frac{U_{\theta\Gamma}}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} \mathbb{E}_i[\tilde{\theta} - \mu_\theta]\right) - \frac{U_{\Gamma\Gamma}}{U_c \pi_{\gamma\gamma}} \mathbb{E}_i[\tilde{\Gamma}]. \tag{A34}$$

Multiplying through and simplifying, we want (A33) to hold, meaning we need:

$$\mathbb{E}_i[f(\tilde{q})] = \frac{U_\Gamma(\mu_\theta, \Gamma_0)}{U_c} + \frac{U_{\theta\Gamma}}{U_c} \mathbb{E}_i[\tilde{\theta} - \mu_\theta] + \frac{U_{\Gamma\Gamma}}{U_c} \mathbb{E}_i[\tilde{\Gamma} - \Gamma_0]. \tag{A35}$$

From the Taylor expansion of the utility function $\mathcal{U}$ provided in equation (3), we can derive:

$$\frac{U_{\theta\Gamma}}{U_c} = a \frac{\mathcal{U}_{qq}(c_0, \mu_\theta)}{\mathcal{U}_c(c_0, \mu_\theta)} = -a \cdot \text{ERA}, \tag{A36}$$

$$\frac{U_{\Gamma\Gamma}}{U_c} = a^2 \frac{\mathcal{U}_{qq}(c_0, \mu_\theta)}{\mathcal{U}_c(c_0, \mu_\theta)} = -a^2 \cdot \text{ERA}, \tag{A37}$$

and

$$\frac{U_\Gamma(\mu_\theta, \Gamma_0)}{U_c} = a\frac{\mathcal{U}_q(c_0, \mu_\theta)}{\mathcal{U}_c(c_0, \mu_\theta)} = a \cdot \text{MRS}. \tag{A38}$$

Consider now the following candidate function:

$$f(\tilde{q}) = \frac{U_\Gamma(\mu_\theta, \Gamma_0)}{U_c} + \frac{U_{\theta\Gamma}}{U_c}(\tilde{\theta} - \mu_\theta) + \frac{U_{\Gamma\Gamma}}{U_c}(\tilde{\Gamma} - \Gamma_0). \tag{A39}$$

Using equations (A36), (A37), and (A38), and recalling the definition of $\tilde{q}$ from (2), leads to:

$$f(\tilde{q}) = a\left[\text{MRS} - \text{ERA}(\tilde{q} - \mu_\theta)\right]. \tag{A40}$$

Take the expectation of this candidate function, conditional on household i's information:

$$\mathbb{E}_i[f(\tilde{q})] = \frac{U_\Gamma(\mu_\theta, \Gamma_0)}{U_c} + \frac{U_{\theta\Gamma}}{U_c}\mathbb{E}_i[\tilde{\theta} - \mu_\theta] + \frac{U_{\Gamma\Gamma}}{U_c}\mathbb{E}_i[\tilde{\Gamma} - \Gamma_0]. \tag{A41}$$

Equation (A41) is identical to equation (A35), which is the required condition for the firm's optimal choice under UBR to coincide with the general form of the team-efficient action. Because the candidate $f(\tilde{q})$ given by equation (A39) (or equivalently, (A40)) satisfies the necessary condition (A35), it follows that under UBR, each firm's optimal choice of greenness, γ_i, will be given by:

$$\gamma_i = \left(1 + \frac{U_{\Gamma\Gamma}}{U_c\pi_{\gamma\gamma}}\right)\mathbb{E}_i[\gamma^*] - \frac{U_{\Gamma\Gamma}}{U_c\pi_{\gamma\gamma}}\mathbb{E}_i[\tilde{\Gamma}]. \tag{A42}$$

This is the defining characteristic of the team-efficient equilibrium. Since each firm i is acting according to this rule, the aggregate outcome $\tilde{\Gamma}$ will be the team-efficient aggregate outcome, $\tilde{\Gamma}^{te}$. Therefore, the regulatory function defined by (A39) (or (A40)) ensures that each firm chooses the team-efficient level of greenness, γ_i^{te}, in the unique linear equilibrium. □

A.6 Proof of Proposition 2

Substituting the budget constraint (26) into the utility maximization problem (25) yields:

$$\max_{g_i} \; U(e + \pi_i - pg_i, \theta, g_i). \tag{A43}$$

Taking the first-order condition with respect to g_i, then rearranging, we find the condition for optimal greenness choice:

$$p = \frac{U_\Gamma(\theta, g_i)}{U_c}. \tag{A44}$$

Because U_Γ is linear in g_i and U_c is constant, condition (A44) implies that all consumers choose the same level of greenness, $g_i = G$ for all i. Aggregating across all consumers, the total demand for greenness is then $\int_i g_i di = G$. Substituting G into (A44), we obtain the inverse demand function for greenness:

$$p = \frac{U_\Gamma(\theta, G)}{U_c}. \tag{A45}$$

Expanding $U_\Gamma(\theta, G)$ around the status quo (μ_θ, Γ_0) using a first-order Taylor expansion (consistent with Property 2), we have:

$$p = \frac{U_\Gamma(\mu_\theta, \Gamma_0)}{U_c} + \frac{U_{\theta\Gamma}(\mu_\theta, \Gamma_0)}{U_c}(\theta - \mu_\theta) + \frac{U_{\Gamma\Gamma}(\mu_\theta, \Gamma_0)}{U_c}(G - \Gamma_0). \tag{A46}$$

We notice that the "price of greenness" in the hypothetical Cournot market has the same structure as the UBR regulatory function, $f(\tilde{q})$, from Proposition 1:

$$f(\tilde{q}) = a \cdot \mathrm{MRS} - a \cdot \mathrm{ERA}(\tilde{\theta} - \mu_\theta) - a^2 \cdot \mathrm{ERA}(\tilde{\Gamma}^{te} - \Gamma_0). \tag{A47}$$

Consider now firms' problem in this hypothetical Cournot market. Firm i chooses its "greenness output," denoted by γ_i^c, to maximize household i's expected utility:

$$\max_{\gamma_i^c} \mathbb{E}_i \left[U(e + \pi(\gamma_i^c) + p\gamma_i^c, \tilde{\theta}, \tilde{\Gamma}^c) \right], \tag{A48}$$

where $\pi(\gamma_i^c)$ is the firm's profit (exactly the same function as before) to which we add the revenues that the firm gets from selling "greenness," and hence the new term $p\gamma_i^c$. The aggregate supply of greenness is $\tilde{\Gamma}^c = \int_i \gamma_i^c di$. Note that the firm is taking as given p and $\tilde{\Gamma}^c$ (firms are infinitesimally small). The first-order condition leads to:

$$0 = \mathbb{E}_i[p + \pi_{\gamma\gamma}(\gamma_i^c - \Gamma_0)], \tag{A49}$$

and thus

$$\gamma_i^c = \Gamma_0 - \frac{\mathbb{E}_i[p]}{\pi_{\gamma\gamma}}. \tag{A50}$$

Substituting the expression for p from the inverse demand function (A46) into (A50), and imposing the market-clearing condition $\tilde{\Gamma}^c = G$ (since aggregate supply of "greenness" must equal aggregate demand in equilibrium), we have:

$$\gamma_i^c = \Gamma_0 - \frac{1}{\pi_{\gamma\gamma}} \mathbb{E}_i \left[\frac{U_\Gamma(\mu_\theta, \Gamma_0)}{U_c} + \frac{U_{\theta\Gamma}}{U_c}(\tilde{\theta} - \mu_\theta) + \frac{U_{\Gamma\Gamma}}{U_c}(\tilde{\Gamma}^c - \Gamma_0) \right]. \tag{A51}$$

Substituting, we get:

$$\gamma_i^c = \Gamma_0 - \frac{1}{\pi_{\gamma\gamma}} \Big[a \cdot \mathrm{MRS} - a \cdot \mathrm{ERA}\, \mathbb{E}_i[\tilde{\theta} - \mu_\theta] - a^2 \cdot \mathrm{ERA}\, \mathbb{E}_i[\tilde{\Gamma}^c - \Gamma_0] \Big]. \tag{A52}$$

Equation (A52) is identical to the first-order condition for γ_i under UBR (equation (A30) in Proposition 1), implying $\gamma_i^c = \gamma_i^{te}$. Thus, the UBR mechanism effectively creates a synthetic Cournot market, achieving the team-efficient outcome. □

A.7 Proof of Proposition 3

The firm's problem is to maximize (21), where consumption is given by:

$$\tilde{c}_i = e + \pi(\gamma_i) + (\gamma_i - \tilde{\Gamma}) f(\tilde{q}). \tag{A53}$$

Substituting (A53) into the objective function (21) and taking the first-order condition with respect to γ_i gives $\gamma_i = \Gamma_0 - \frac{\mathbb{E}_i[f(\tilde{q})]}{\pi_{\gamma\gamma}}$. Substituting $f(\tilde{q})$ from equation (A39) leads to:

$$\gamma_i = \Gamma_0 - \frac{1}{\pi_{\gamma\gamma} U_c} \Big[U_\Gamma(\mu_\theta, \Gamma_0) + U_{\Gamma\Gamma}(\mathbb{E}_i[\tilde{\Gamma}] - \Gamma_0) + U_{\theta\Gamma}(\mathbb{E}_i[\tilde{\theta}] - \mu_\theta) \Big]. \tag{A54}$$

From equations (8) and (33), we are given that:

$$\mathbb{E}_i[\tilde{\theta}] - \mu_\theta = \frac{\tau_y}{\tau}(y_i - \mu_\theta) + \frac{\tau_z}{\tau}(z - \mu_\theta), \tag{A55}$$

$$\mathbb{E}_i[\tilde{\Gamma}] = \beta_0 + \beta_y \mathbb{E}_i[\tilde{\theta} - \mu_\theta] + \beta_z (z - \mu_\theta), \tag{A56}$$

where the last equation follows from aggregating the conjectured form in equation (28).
Substituting (A55) and (A56) into (A54), and using our conjecture (28) yields:

$$\begin{aligned} \beta_0 + \beta_y(y_i - \mu_\theta) + \beta_z(z - \mu_\theta) = \Gamma_0 - \frac{1}{\pi_{\gamma\gamma} U_c} \Big[& U_\Gamma(\mu_\theta, \Gamma_0) \\ & + U_{\Gamma\Gamma} \left(\beta_0 + \beta_y \frac{\tau_y}{\tau}(y_i - \mu_\theta) + \left(\beta_y \frac{\tau_z}{\tau} + \beta_z \right)(z - \mu_\theta) - \Gamma_0 \right) \\ & + U_{\theta\Gamma} \left(\frac{\tau_y}{\tau}(y_i - \mu_\theta) + \frac{\tau_z}{\tau}(z - \mu_\theta) \right) \Big]. \end{aligned} \tag{A57}$$

We equate coefficients for the constant term, the coefficients for $(y_i - \mu_\theta)$, and the coefficients for $(z - \mu_\theta)$. First, solving for β_0:

$$\beta_0 = \Gamma_0 - \frac{1}{\pi_{\gamma\gamma} U_c} \left[U_\Gamma(\mu_\theta, \Gamma_0) + U_{\Gamma\Gamma}(\beta_0 - \Gamma_0) \right], \tag{A58}$$

and thus:

$$\beta_0 = \Gamma_0 - \frac{U_\Gamma(\mu_\theta, \Gamma_0)}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}}. \tag{A59}$$

Second, solving for β_y:

$$\beta_y = -\frac{1}{U_c \pi_{\gamma\gamma}} \left[U_{\Gamma\Gamma} \beta_y \frac{\tau_y}{\tau} + U_{\theta\Gamma} \frac{\tau_y}{\tau} \right], \tag{A60}$$

yields:

$$\beta_y = -\frac{U_{\theta\Gamma} \tau_y}{U_c \pi_{\gamma\gamma} \tau + U_{\Gamma\Gamma} \tau_y} < 0. \tag{A61}$$

Third and finally, solving for β_z:

$$\beta_z = -\frac{1}{U_c \pi_{\gamma\gamma}} \left[U_{\Gamma\Gamma} \left(\beta_y \frac{\tau_z}{\tau} + \beta_z \right) + U_{\theta\Gamma} \frac{\tau_z}{\tau} \right]. \tag{A62}$$

which, after substituting the solution for β_y from (A61), leads to

$$\beta_z = -\frac{U_c \pi_{\gamma\gamma}}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} \frac{U_{\theta\Gamma} \tau_z}{U_c \pi_{\gamma\gamma} \tau + U_{\Gamma\Gamma} \tau_y}. \tag{A63}$$

and thus, recognizing that $\frac{U_c \pi_{\gamma\gamma}}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} = \frac{1}{1-\alpha}$ (Lemma 3, equation (17)):

$$\beta_z = -\frac{1}{1-\alpha} \frac{U_{\theta\Gamma} \tau_z}{U_c \pi_{\gamma\gamma} \tau + U_{\Gamma\Gamma} \tau_y} < 0. \tag{A64}$$

Equations (A59), (A61), and (A64) characterize the coefficients β_0, β_y, and β_z in terms of the model parameters, and confirm equations (29) through (31) in Proposition 3. □

A.8 Proof of Corollary 3.1

From Lemma 3, we know that $\alpha = -\frac{U_{\Gamma\Gamma}}{U_c \pi_{\gamma\gamma}}$. We also know that $U_{\Gamma\Gamma} < 0$, $U_c > 0$, and $\pi_{\gamma\gamma} < 0$. Therefore, $\alpha < 0$. Consequently, as shown in equation (16) of Lemma 3, firm i's expectation of the average action of others, $\mathbb{E}_i[\tilde{\Gamma}^{te}]$, enters with a negative coefficient, implying that firms' actions are strategic substitutes.

From Proposition 3, using the expressions for β_y and β_z, we can directly calculate the ratio $\frac{\beta_y}{\beta_z}$:

$$\frac{\beta_y}{\beta_z} = \frac{\tau_y}{\frac{\tau_z}{1-\alpha}} = (1-\alpha) \frac{\tau_y}{\tau_z}. \tag{A65}$$

Since $\alpha < 0$, we have $(1-\alpha) > 1$. Therefore:

$$\frac{\beta_y}{\beta_z} = (1-\alpha)\frac{\tau_y}{\tau_z} > \frac{\tau_y}{\tau_z}. \tag{A66}$$

This confirms that firms overweight their private information relative to the public signal. □

A.9 Proof of Corollary 3.2

We start first by writing the coefficients β_0, β_y, and β_z as functions of κ_0^*, κ_1^*, and α, noting that these latter three coefficients *do not* depend on σ_θ, τ_y, and τ_z:

$$\beta_0 = \kappa_0^*, \tag{A67}$$

$$\beta_y = \frac{(1-\alpha)\kappa_1^*\sigma_\theta^2\tau_y}{1+\sigma_\theta^2(\tau_y - \alpha\tau_y + \tau_z)}, \tag{A68}$$

$$\beta_z = \frac{\kappa_1^*\sigma_\theta^2\tau_z}{1+\sigma_\theta^2(\tau_y - \alpha\tau_y + \tau_z)}. \tag{A69}$$

We aim to show that $\frac{\partial \mathcal{B}}{\partial \sigma_\theta} > 0$, where $\mathcal{B} = |\beta_y + \beta_z|$. Since $\beta_y < 0$ and $\beta_z < 0$, we have $\mathcal{B} = -(\beta_y + \beta_z)$:

$$\mathcal{B} = \kappa_1^* \left(\frac{1}{1+\sigma_\theta^2(\tau_y(1-\alpha)+\tau_z)} - 1 \right) \tag{A70}$$

Noting that $\kappa_1^* < 0$ and $\alpha < 0$, points (a), (b), and (c) of Corollary 3.2 result immediately. □

A.10 Proof of Corollary 3.3

Consider a firm i that unilaterally increases its private signal precision to $\tau_y' = \xi\tau_y$, where $\xi > 1$. Let $\mathbb{E}_i'[\cdot]$ denote expectations conditional on this more precise signal. The first-order condition implies the firm's optimal greenness level, denoted γ_i':

$$\gamma_i' = \Gamma_0 - \frac{\mathbb{E}_i'[f(\tilde{q})]}{\pi_{\gamma\gamma}}. \tag{A71}$$

We know from Proposition 1 that $f(\tilde{q}) = \frac{U_\Gamma(\mu_\theta,\Gamma_0)}{U_c} + \frac{U_{\theta\Gamma}}{U_c}(\tilde{\theta} - \mu_\theta) + \frac{U_{\Gamma\Gamma}}{U_c}(\tilde{\Gamma} - \Gamma_0)$, which remains the same as in the main model. Replacing this in (A71) yields:

$$\gamma_i' = \Gamma_0 - \frac{U_\Gamma(\mu_\theta,\Gamma_0)}{U_c\pi_{\gamma\gamma}} - \frac{U_{\theta\Gamma}}{U_c\pi_{\gamma\gamma}}(\mathbb{E}_i'[\tilde{\theta}] - \mu_\theta) - \frac{U_{\Gamma\Gamma}}{U_c\pi_{\gamma\gamma}}(\mathbb{E}_i'[\tilde{\Gamma}] - \Gamma_0). \tag{A72}$$

Make the following substitutions:

$$\mathbb{E}_i'[\tilde{\theta}] = \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2} + \xi\tau_y + \tau_z}\mu_\theta + \frac{\xi\tau_y}{\sigma_\theta^{-2} + \xi\tau_y + \tau_z}y_i + \frac{\tau_z}{\sigma_\theta^{-2} + \xi\tau_y + \tau_z}z, \tag{A73}$$

$$\mathbb{E}_i'[\tilde{\Gamma}] = \beta_0 + \beta_y(\mathbb{E}_i'[\tilde{\theta}] - \mu_\theta) + \beta_z(z - \mu_\theta), \tag{A74}$$

and replace the solutions for β_0, β_y, and β_z given in equations (29)–(31) of Proposition 3. Straightforward but tedious algebra (details omitted) then leads to equation (35) of Corollary 3.3:

$$\gamma_i' = \beta_0 + \beta_y\left[1 + \frac{(\xi-1)(1+\sigma_\theta^2\tau_z)}{1+\sigma_\theta^2(\xi\tau_y+\tau_z)}\right](y_i - \mu_\theta) + \beta_z\left[1 - \frac{(\xi-1)(1-\alpha)\sigma_\theta^2\tau_y}{1+\sigma_\theta^2(\xi\tau_y+\tau_z)}\right](z-\mu_\theta). \tag{A75}$$

We now demonstrate that firm i's ex-ante expected utility (conditional on z, but before observing y_i) is strictly increasing in the precision of its private signal. From Proposition 1, we note that the regulatory function can also be written as:

$$f(\tilde{q}) = \frac{U_\Gamma(\tilde{\theta},\tilde{\Gamma})}{U_c} = \frac{U_\Gamma(\tilde{\theta},\Gamma_0) + U_{\Gamma\Gamma}(\tilde{\Gamma}-\Gamma_0)}{U_c}. \tag{A76}$$

Consider a second-order Taylor expansion of the utility function $U(e + \pi(\gamma_i) + (\gamma_i - \tilde{\Gamma})f(\tilde{q}), \tilde{\theta}, \tilde{\Gamma})$ around the status quo point $(\gamma_i, \tilde{\Gamma}) = (\Gamma_0, \Gamma_0)$:

$$\begin{aligned} U\left(e + \pi(\gamma_i) + (\gamma_i - \tilde{\Gamma})\frac{U_\Gamma(\tilde{\theta},\Gamma_0) + U_{\Gamma\Gamma}(\tilde{\Gamma}-\Gamma_0)}{U_c}, \tilde{\theta}, \tilde{\Gamma}\right) &= U\left(e + \pi(\Gamma_0), \tilde{\theta}, \Gamma_0\right) \\ &+ 0\cdot(\tilde{\Gamma}-\Gamma_0) + U_\Gamma(\tilde{\theta},\Gamma_0)\cdot(\gamma_i - \Gamma_0) \\ &- \frac{1}{2}U_{\Gamma\Gamma}\cdot(\tilde{\Gamma}-\Gamma_0)^2 + \frac{1}{2}U_c\pi_{\gamma\gamma}\cdot(\gamma_i-\Gamma_0)^2 + U_{\Gamma\Gamma}\cdot(\tilde{\Gamma}-\Gamma_0)(\gamma_i-\Gamma_0). \end{aligned} \tag{A77}$$

This expansion is exact, given the linear-quadratic structure of the model. The expected utility (conditional on both y_i and z) is then

$$\begin{aligned} \mathbb{E}_i[U] &= U\left(e + \pi(\Gamma_0), \tilde{\theta}, \Gamma_0\right) + \mathbb{E}_i[U_\Gamma(\tilde{\theta},\Gamma_0)](\gamma_i - \Gamma_0) \\ &- \frac{1}{2}U_{\Gamma\Gamma}\mathbb{E}_i[(\tilde{\Gamma}-\Gamma_0)^2] + \frac{1}{2}U_c\pi_{\gamma\gamma}(\gamma_i-\Gamma_0)^2 + U_{\Gamma\Gamma}\mathbb{E}_i[\tilde{\Gamma}-\Gamma_0](\gamma_i-\Gamma_0). \end{aligned} \tag{A78}$$

Recall from Lemma 2 that the first-best greenness level is given by $\gamma^* = \kappa_0^* + \kappa_1^*(\tilde{\theta} - \mu_\theta)$, and that $\kappa_0^* = \Gamma_0 - \frac{U_\Gamma(\mu_\theta,\Gamma_0)}{U_c\pi_{\gamma\gamma}+U_{\Gamma\Gamma}}$ and $\kappa_1^* = -\frac{U_{\theta\Gamma}}{U_c\pi_{\gamma\gamma}+U_{\Gamma\Gamma}}$. We can therefore rewrite the expression for γ^* as

$$\gamma^* = \Gamma_0 + \frac{-U_\Gamma(\mu_\theta,\Gamma_0) - U_{\theta\Gamma}(\tilde{\theta}-\mu_\theta)}{U_c\pi_{\gamma\gamma}+U_{\Gamma\Gamma}} = \Gamma_0 - \frac{U_\Gamma(\tilde{\theta},\Gamma_0)}{U_c\pi_{\gamma\gamma}+U_{\Gamma\Gamma}}. \tag{A79}$$

Thus, we can write $\mathbb{E}_i[U_\Gamma(\tilde{\theta}, \Gamma_0)] = -(U_c\pi_{\gamma\gamma} + U_{\Gamma\Gamma})\mathbb{E}_i[\gamma^* - \Gamma_0]$, which leads to:

$$
\begin{aligned}
\mathbb{E}_i[U] = U\left(e + \pi(\Gamma_0), \tilde{\theta}, \Gamma_0\right) - \frac{1}{2}U_{\Gamma\Gamma}\mathbb{E}_i[(\tilde{\Gamma} - \Gamma_0)^2] + \frac{1}{2}U_c\pi_{\gamma\gamma}(\gamma_i - \Gamma_0)^2 \\
- \left((U_c\pi_{\gamma\gamma} + U_{\Gamma\Gamma})\mathbb{E}_i[\gamma^* - \Gamma_0] - U_{\Gamma\Gamma}\mathbb{E}_i[\tilde{\Gamma} - \Gamma_0]\right)(\gamma_i - \Gamma_0),
\end{aligned}
\tag{A80}
$$

or, using that $\alpha = -\frac{U_{\Gamma\Gamma}}{U_c\pi_{\gamma\gamma}}$,

$$
\begin{aligned}
\mathbb{E}_i[U] = U\left(e + \pi(\Gamma_0), \tilde{\theta}, \Gamma_0\right) - \frac{1}{2}U_{\Gamma\Gamma}\mathbb{E}_i[(\tilde{\Gamma} - \Gamma_0)^2] + \frac{1}{2}U_c\pi_{\gamma\gamma}(\gamma_i - \Gamma_0)^2 \\
- U_c\pi_{\gamma\gamma}\left((1-\alpha)\mathbb{E}_i[\gamma^* - \Gamma_0] + \alpha\mathbb{E}_i[\tilde{\Gamma} - \Gamma_0]\right)(\gamma_i - \Gamma_0).
\end{aligned}
\tag{A81}
$$

Finally, recognizing from Lemma 3 that $(1-\alpha)\mathbb{E}_i[\gamma^* - \Gamma_0] + \alpha\mathbb{E}_i[\tilde{\Gamma} - \Gamma_0] = \gamma_i - \Gamma_0$:

$$
\mathbb{E}_i[U] = U\left(e + \pi(\Gamma_0), \tilde{\theta}, \Gamma_0\right) - \frac{1}{2}U_{\Gamma\Gamma}\mathbb{E}_i[(\tilde{\Gamma} - \Gamma_0)^2] + \frac{1}{2}U_c\pi_{\gamma\gamma}(\gamma_i - \Gamma_0)^2 - U_c\pi_{\gamma\gamma}(\gamma_i - \Gamma_0)^2 \tag{A82}
$$

$$
= U\left(e + \pi(\Gamma_0), \tilde{\theta}, \Gamma_0\right) - \frac{1}{2}U_{\Gamma\Gamma}\mathbb{E}_i[(\tilde{\Gamma} - \Gamma_0)^2] - \frac{1}{2}U_c\pi_{\gamma\gamma}(\gamma_i - \Gamma_0)^2. \tag{A83}
$$

Taking the *ex-ante* expectation (conditional on z), for both γ_i and γ_i', yields:

$$
\mathbb{E}[\mathbb{E}_i[U]|z] = \mathbb{E}[U(e + \pi(\Gamma_0), \tilde{\theta}, \Gamma_0)|z] - \frac{1}{2}U_c\pi_{\gamma\gamma}\mathbb{E}[(\gamma_i - \Gamma_0)^2|z] - \frac{1}{2}U_{\Gamma\Gamma}\mathbb{E}[(\tilde{\Gamma} - \Gamma_0)^2|z], \tag{A84}
$$

$$
\mathbb{E}[\mathbb{E}_i'[U]|z] = \mathbb{E}[U(e + \pi(\Gamma_0), \tilde{\theta}, \Gamma_0)|z] - \frac{1}{2}U_c\pi_{\gamma\gamma}\mathbb{E}[(\gamma_i' - \Gamma_0)^2|z] - \frac{1}{2}U_{\Gamma\Gamma}\mathbb{E}[(\tilde{\Gamma} - \Gamma_0)^2|z]. \tag{A85}
$$

The difference in *ex-ante* expected utilities is therefore:

$$
\Delta\mathbb{E}[U|z] := \mathbb{E}[\mathbb{E}_i'[U]|z] - \mathbb{E}[\mathbb{E}_i[U]|z] = -\frac{1}{2}U_c\pi_{\gamma\gamma}\left(\mathbb{E}[(\gamma_i' - \Gamma_0)^2|z] - \mathbb{E}[(\gamma_i - \Gamma_0)^2|z]\right). \tag{A86}
$$

From Propositions 2 and 3, we have $\gamma_i - \Gamma_0 = -\frac{\mathbb{E}_i[f(\tilde{q})]}{\pi_{\gamma\gamma}}$ and $\gamma_i' - \Gamma_0 = -\frac{\mathbb{E}_i'[f(\tilde{q})]}{\pi_{\gamma\gamma}}$. Substituting these into the expression for $\Delta\mathbb{E}[U|z]$, we obtain:

$$
\Delta\mathbb{E}[U|z] = \frac{-U_c}{2\pi_{\gamma\gamma}}\left(\mathbb{E}[\mathbb{E}_i'[f(\tilde{q})]^2|z] - \mathbb{E}[\mathbb{E}_i[f(\tilde{q})]^2|z]\right). \tag{A87}
$$

Applying the definition of variance, $\text{Var}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$, and the Law of Iterated Expectations:

$$
\mathbb{E}[\mathbb{E}_i'[f(\tilde{q})]^2|z] = \text{Var}[\mathbb{E}_i'[f(\tilde{q})]|z] + \mathbb{E}[f(\tilde{q})|z]^2, \tag{A88}
$$

$$
\mathbb{E}[\mathbb{E}_i[f(\tilde{q})]^2|z] = \text{Var}[\mathbb{E}_i[f(\tilde{q})]|z] + \mathbb{E}[f(\tilde{q})|z]^2. \tag{A89}
$$

Substituting these into the expression for the difference in expected utilities, we get

$$\Delta\mathbb{E}[U|z] = \frac{-U_c}{2\pi_{\gamma\gamma}}\left(\mathrm{Var}[\mathbb{E}'_i[f(\tilde{q})]|z] - \mathrm{Var}[\mathbb{E}_i[f(\tilde{q})]|z]\right). \tag{A90}$$

The term $\frac{-U_c}{2\pi_{\gamma\gamma}}$ is positive ($\pi_{\gamma\gamma} < 0$ and $U_c > 0$). The increased precision ($\tau'_y = \xi\tau_y$, $\xi > 1$) implies that the posterior belief about $f(\tilde{q})$ based on y'_i varies more than the posterior based on y_i. Thus, a more precise signal leads to a larger variance of the conditional expectation. This can be understood from the Law of Total Variance (LTV). Denoting $\mathrm{Var}_i[f(\tilde{q})]$ as shorthand notation for $\mathrm{Var}[f(\tilde{q})|y_i, z]$, the LTV states that:

$$\mathrm{Var}[f(\tilde{q})|z] = \mathbb{E}[\mathrm{Var}_i[f(\tilde{q})]|z] + \mathrm{Var}[\mathbb{E}_i[f(\tilde{q})]|z]. \tag{A91}$$

In this setting, $\mathrm{Var}_i[f(\tilde{q})]$ is a constant with respect to both y_i and z. Thus, we can write:

$$\mathrm{Var}[f(\tilde{q})|z] = \mathrm{Var}[f(\tilde{q})|y'_i, z] + \mathrm{Var}[\mathbb{E}'_i[f(\tilde{q})]|z], \tag{A92}$$

$$\mathrm{Var}[f(\tilde{q})|z] = \mathrm{Var}[f(\tilde{q})|y_i, z] + \mathrm{Var}[\mathbb{E}_i[f(\tilde{q})]|z]. \tag{A93}$$

$\mathrm{Var}[f(\tilde{q})|z]$ is the same in both LTV equations as it depends on the distribution of $f(\tilde{q})$ given the public signal, z, which is unchanged by firm i's private signal precision. Since the precision of the private signal is increased, we have $\mathrm{Var}[f(\tilde{q})|y'_i, z] < \mathrm{Var}[f(\tilde{q})|y_i, z]$. Therefore, it must be that:

$$\mathrm{Var}[\mathbb{E}'_i[f(\tilde{q})]|z] > \mathrm{Var}[\mathbb{E}_i[f(\tilde{q})]|z]. \tag{A94}$$

Plugging this result into equation (A90), we conclude that $\Delta\mathbb{E}[U|z] > 0$. Thus, the firm has a strict incentive to increase its private information precision, τ_y. □

A.11 Proof of Proposition 4

The social planner's problem is:

$$\max_{\gamma_{sp}} \mathbb{E}_{sp}\left[U(e + \pi(\gamma_{sp}), \tilde{\theta}, \gamma_{sp})\right]. \tag{A95}$$

The first-order condition (FOC) with respect to γ_{sp} is:

$$0 = \mathbb{E}_{sp}\left[U_c\pi_\gamma(\gamma_{sp}) + U_\Gamma(\tilde{\theta}, \gamma_{sp})\right]. \tag{A96}$$

Rewrite $U_\Gamma(\tilde{\theta}, \gamma_{sp}) = U_\Gamma(\tilde{\theta}, \Gamma_0) + U_{\Gamma\Gamma}(\gamma_{sp} - \Gamma_0)$ and $U_c\pi_\gamma(\gamma_{sp}) = U_c\pi_{\gamma\gamma}(\gamma_{sp} - \Gamma_0)$, and substitute:

$$0 = \mathbb{E}_{sp}\left[U_c\pi_{\gamma\gamma}(\gamma_{sp} - \Gamma_0) + U_\Gamma(\tilde{\theta}, \Gamma_0) + U_{\Gamma\Gamma}(\gamma_{sp} - \Gamma_0)\right]. \tag{A97}$$

Solving for γ_{sp}, we obtain:

$$\gamma_{sp} = \Gamma_0 - \frac{\mathbb{E}_{sp}[U_\Gamma(\tilde{\theta}, \Gamma_0)]}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}}. \tag{A98}$$

Taking expectation of equation (A79) conditional only on public information gives:

$$\mathbb{E}_{sp}[\gamma^*] = \Gamma_0 - \frac{\mathbb{E}_{sp}[U_\Gamma(\tilde{\theta}, \Gamma_0)]}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}}. \tag{A99}$$

Comparing this with equation (A98), we see that:

$$\gamma_{sp} = \mathbb{E}_{sp}[\gamma^*]. \tag{A100}$$

We now compare the ex-ante expected social welfare under the social planner's allocation to the status quo. Define the difference in expected utility for household i, conditional on y_i and z as:

$$\Delta U_i := \mathbb{E}_i[U(e + \pi(\gamma_{sp}), \tilde{\theta}, \gamma_{sp})] - \mathbb{E}_i[U(e + \pi(\Gamma_0), \tilde{\theta}, \Gamma_0)]. \tag{A101}$$

Write a second-order Taylor expansion of U around the status quo point $(\tilde{\Gamma}, \gamma_i) = (\Gamma_0, \Gamma_0)$:

$$\begin{aligned} U(e + \pi(\gamma_i), \tilde{\theta}, \tilde{\Gamma}) &= U(\tilde{\theta}, \Gamma_0, e + \pi(\Gamma_0)) + U_\Gamma(\tilde{\theta}, \Gamma_0)(\tilde{\Gamma} - \Gamma_0) + U_c \pi_\gamma(\Gamma_0)(\gamma_i - \Gamma_0) \\ &\quad + \frac{1}{2} U_{\Gamma\Gamma}(\tilde{\Gamma} - \Gamma_0)^2 + \frac{1}{2} U_c \pi_{\gamma\gamma}(\gamma_i - \Gamma_0)^2. \end{aligned} \tag{A102}$$

Since $\pi_\gamma(\Gamma_0) = 0$, and evaluating the expression at $\gamma_i = \tilde{\Gamma} = \gamma_{sp}$, the expansion becomes:

$$U(e + \pi(\gamma_{sp}), \tilde{\theta}, \gamma_{sp}) = U(\tilde{\theta}, \Gamma_0, e + \pi(\Gamma_0)) + U_\Gamma(\tilde{\theta}, \Gamma_0)(\gamma_{sp} - \Gamma_0) + \frac{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}}{2}(\gamma_{sp} - \Gamma_0)^2. \tag{A103}$$

Thus, the difference ΔU_i simplifies to:

$$\Delta U_i = \mathbb{E}_i[U_\Gamma(\tilde{\theta}, \Gamma_0)](\gamma_{sp} - \Gamma_0) + \frac{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}}{2}(\gamma_{sp} - \Gamma_0)^2. \tag{A104}$$

From equation (A98), we know that $\gamma_{sp} - \Gamma_0 = \frac{-\mathbb{E}_{sp}[U_\Gamma(\tilde{\theta}, \Gamma_0)]}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} = \frac{-\mathbb{E}[U_\Gamma(\tilde{\theta}, \Gamma_0)|z]}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}}$, where the second equality holds because the social planner's information set consists only of the prior and z. Thus:

$$\Delta U_i = \mathbb{E}_i[U_\Gamma(\tilde{\theta}, \Gamma_0)] \frac{-\mathbb{E}[U_\Gamma(\tilde{\theta}, \Gamma_0)|z]}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} + \frac{U_{\Gamma\Gamma} + U_c \pi_{\gamma\gamma}}{2} \left(\frac{-\mathbb{E}[U_\Gamma(\tilde{\theta}, \Gamma_0)|z]}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} \right)^2 \tag{A105}$$

$$= \frac{-1}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} \left(\mathbb{E}_i[U_\Gamma(\tilde{\theta}, \Gamma_0)] \mathbb{E}[U_\Gamma(\tilde{\theta}, \Gamma_0)|z] - \frac{1}{2} \mathbb{E}[U_\Gamma(\tilde{\theta}, \Gamma_0)|z]^2 \right). \tag{A106}$$

The welfare difference is the the ex-ante expectation $\mathbb{E}[\Delta U_i] = \Delta W_{sp}$, which averages over y_i and z:

$$\Delta W_{sp} = \frac{-1}{U_c\pi_{\gamma\gamma} + U_{\Gamma\Gamma}}\mathbb{E}\left[\mathbb{E}_i[U_\Gamma(\tilde{\theta},\Gamma_0)]\mathbb{E}[U_\Gamma(\tilde{\theta},\Gamma_0)|z] - \frac{1}{2}\mathbb{E}[U_\Gamma(\tilde{\theta},\Gamma_0)|z]^2\right]. \tag{A107}$$

Recall that $U_\Gamma(\tilde{\theta},\Gamma_0) = U_\Gamma(\mu_\theta,\Gamma_0) + U_{\theta\Gamma}(\tilde{\theta}-\mu_\theta)$. Also, $\mathbb{E}_i[\tilde{\theta}-\mu_\theta] = \frac{\tau_y}{\tau}(y_i-\mu_\theta) + \frac{\tau_z}{\tau}(z-\mu_\theta)$ and $\mathbb{E}[\tilde{\theta}-\mu_\theta|z] = \frac{\tau_z}{\tau_{sp}}(z-\mu_\theta)$, where $\tau = \sigma_\theta^{-2} + \tau_y + \tau_z$ and $\tau_{sp} = \sigma_\theta^{-2} + \tau_z$. Therefore:

$$\mathbb{E}_i[U_\Gamma(\tilde{\theta},\Gamma_0)] = U_\Gamma(\mu_\theta,\Gamma_0) + U_{\theta\Gamma}\left(\frac{\tau_y}{\tau}(y_i-\mu_\theta) + \frac{\tau_z}{\tau}(z-\mu_\theta)\right), \tag{A108}$$

$$\mathbb{E}[U_\Gamma(\tilde{\theta},\Gamma_0)|z] = U_\Gamma(\mu_\theta,\Gamma_0) + U_{\theta\Gamma}\frac{\tau_z}{\tau_{sp}}(z-\mu_\theta). \tag{A109}$$

Substituting into the expression for ΔW_{sp}:

$$\begin{aligned}\Delta W_{sp} = \frac{-1}{U_c\pi_{\gamma\gamma} + U_{\Gamma\Gamma}}\mathbb{E}\Bigg\{&\left[U_\Gamma(\mu_\theta,\Gamma_0) + U_{\theta\Gamma}\left(\frac{\tau_y}{\tau}(y_i-\mu_\theta) + \frac{\tau_z}{\tau}(z-\mu_\theta)\right)\right] \\ &\times\left[U_\Gamma(\mu_\theta,\Gamma_0) + U_{\theta\Gamma}\frac{\tau_z}{\tau_{sp}}(z-\mu_\theta)\right] - \frac{1}{2}\left[U_\Gamma(\mu_\theta,\Gamma_0) + U_{\theta\Gamma}\frac{\tau_z}{\tau_{sp}}(z-\mu_\theta)\right]^2\Bigg\}.\end{aligned} \tag{A110}$$

Now, we expand the terms inside the expectation and apply the following rules:

$$\mathbb{E}[y_i-\mu_\theta] = \mathbb{E}[z-\mu_\theta] = 0, \tag{A111}$$

$$\mathbb{E}[(y_i-\mu_\theta)(z-\mu_\theta)] = \mathbb{E}[(\tilde{\theta}-\mu_\theta+\tilde{\varepsilon}_{y,i})(\tilde{\theta}-\mu_\theta+\tilde{\varepsilon}_z)] = \sigma_\theta^2, \tag{A112}$$

$$\mathbb{E}[(z-\mu_\theta)^2] = \mathrm{Var}(z) = \sigma_\theta^2 + 1/\tau_z. \tag{A113}$$

Expanding the product and taking expectations, then replacing $\tau_{sp} = \sigma_\theta^{-2} + \tau_z$ and $\tau = \sigma_\theta^{-2} + \tau_z + \tau_y$:

$$\Delta W_{sp} = \frac{-1}{2(U_c\pi_{\gamma\gamma} + U_{\Gamma\Gamma})}\left[U_\Gamma(\mu_\theta,\Gamma_0)^2 + U_{\theta\Gamma}^2\frac{\sigma_\theta^2\tau_z}{\sigma_\theta^{-2}+\tau_z}\right] > 0. \quad \square \tag{A114}$$

A.12 Proof of Corollary 4.1

Consider a firm facing a tax T per unit of emissions, $\mathcal{E}(\gamma_i)$. The firm's objective function is:

$$\max_{\gamma_i}\mathbb{E}_i\left[U(e + \pi(\gamma_i) - T\mathcal{E}(\gamma_i), \tilde{\theta}, \tilde{\Gamma})\right]. \tag{A115}$$

Taking the first-order condition with respect to γ_i (and noting that firm i takes $\tilde{\Gamma}$ as given) implies:

$$\mathbb{E}_i[\pi_\gamma(\gamma_i)] = T\mathbb{E}_i[\mathcal{E}'(\gamma_i)]. \tag{A116}$$

To implement the social planner's solution, γ_{sp}, we need all firms to choose $\gamma_i = \gamma_{sp}$. Since the tax is uniform and all firms are identical ex ante, we need to impose:

$$\pi_\gamma(\gamma_{sp}) = T\mathcal{E}'(\gamma_{sp}), \tag{A117}$$

and noting that $\pi_\gamma(\gamma_{sp}) = \pi_{\gamma\gamma}(\gamma_{sp} - \Gamma_0)$, we must have

$$T = \frac{(\gamma_{sp} - \Gamma_0)\pi_{\gamma\gamma}}{\mathcal{E}'(\gamma_{sp})}. \tag{A118}$$

Because $\pi_{\gamma\gamma} < 0$ and $\mathcal{E}'(\gamma_{sp}) < 0$, the sign of t is dictated by the difference $\gamma_{sp} - \Gamma_0$. Since we have assumed that green investment generates a positive externality, we have $\gamma_{sp} > \Gamma_0$ and thus $T > 0$. Consequently, the per-unit-emissions tax is strictly positive and ensures that all firms choose the social planner's greenness level γ_{sp}. $\square$

A.13 Proof of Corollary 4.2

When firm i purchases Q_i permits, its profit function is:

$$\pi_i = \pi(\gamma_i) - p_{permit}Q_i. \tag{A119}$$

Firms must hold enough permits to cover their emissions. Therefore, the quantity of permits purchased, Q_i, must equal the firm's emissions:

$$Q_i = \mathcal{E}(\gamma_i). \tag{A120}$$

The firm maximizes its expected utility. Substituting (A120) into the profit function:

$$\max_{\gamma_i} \mathbb{E}_i[U(e + \pi(\gamma_i) - p_{permit}\mathcal{E}(\gamma_i), \tilde{\theta}, \tilde{\Gamma})]. \tag{A121}$$

Taking the first-order condition with respect to γ_i (the firm takes p_{permit} and $\tilde{\Gamma}$ as given):

$$\mathbb{E}_i[U_c(\pi_\gamma(\gamma_i) - p_{permit}\mathcal{E}'(\gamma_i))] = 0, \tag{A122}$$

or

$$\pi_{\gamma\gamma}(\gamma_i - \Gamma_0) - p_{permit}\mathcal{E}'(\gamma_i) = 0. \tag{A123}$$

Since $\mathcal{E}'(\gamma_i) < 0$ and $\mathcal{E}''(\gamma_i) > 0$, the left-hand side of (A123) is strictly decreasing in γ_i. It is also strictly positive at $\gamma_i = \Gamma_0$. Thus, for a given p_{permit}, there's a unique $\gamma_i > \Gamma_0$ that solves the FOC. Because p_{permit} is uniform across firms, and $\pi(\cdot)$ and $\mathcal{E}(\cdot)$ are identical for all firms, all firms will choose the same level of greenness.

The social planner aims to implement γ_{sp}. In equilibrium, aggregate emissions must equal the total permits issued, Q. Since all firms choose γ_{sp}, aggregate emissions are:

$$\int_0^1 \mathcal{E}(\gamma_{sp})di = \mathcal{E}(\gamma_{sp}). \tag{A124}$$

Therefore, to implement the social planner's solution Q must solve:

$$Q = \mathcal{E}(\gamma_{sp}). \tag{A125}$$

Substituting γ_{sp} into the firm's FOC:

$$\pi_{\gamma\gamma}(\gamma_{sp} - \Gamma_0) - p_{permit}\mathcal{E}'(\gamma_{sp}) = 0. \tag{A126}$$

Solving for the equilibrium permit price, p_{permit} and using the result of Proposition 4:

$$p_{permit} = \frac{\pi_{\gamma\gamma}(\gamma_{sp} - \Gamma_0)}{\mathcal{E}'(\gamma_{sp})}. \tag{A127}$$

Thus, the equilibrium permit price, p_{permit}, is identical to the Pigouvian tax, T. □

A.14 Proof of Proposition 5

A household with signal y_i compares its expected utility under the two regimes:

1. **UBR:** $\mathbb{E}_i[U(c_i^{UBR}, \tilde{\theta}, \tilde{\Gamma}^{UBR})]$, where $c_i^{UBR} = e + \pi(\gamma_i) + (\gamma_i - \tilde{\Gamma})f(\tilde{q})$. As shown in Proposition 1, γ_i is the team-efficient greenness level under UBR. Furthermore, from Proposition 1, we notice that the regulatory function can also be written as:

$$f(\tilde{q}) = \frac{U_\Gamma(\tilde{\theta}, \tilde{\Gamma})}{U_c} = \frac{U_\Gamma(\tilde{\theta}, \gamma_{sp}) + U_{\Gamma\Gamma}(\tilde{\Gamma} - \gamma_{sp})}{U_c}. \tag{A128}$$

2. **Social Planner:** $\mathbb{E}_i[U(c_i^{sp}, \tilde{\theta}, \gamma_{sp})]$, where $c_i^{sp} = e + \pi(\gamma_{sp})$.

The household votes for the social planner if the second expression is greater than the first; otherwise, it votes for UBR. Define the difference in utility for household i as:

$$\Delta U_i := U(c_i^{UBR}, \tilde{\theta}, \tilde{\Gamma}^{UBR}) - U(c_i^{sp}, \tilde{\theta}, \gamma_{sp}). \tag{A129}$$

Take a second-order Taylor expansion of $U(c_i^{UBR}, \tilde{\theta}, \tilde{\Gamma}^{UBR})$ around the social planner's solution:

$$\begin{aligned} U(c_i^{UBR}, \tilde{\theta}, \tilde{\Gamma}^{UBR}) &= U(c_i^{sp}, \tilde{\theta}, \gamma_{sp}) + [U_\Gamma(\tilde{\theta}, \gamma_{sp}) + U_c\pi_\gamma(\gamma_{sp})](\gamma_i - \gamma_{sp}) \\ &\quad - \frac{1}{2}U_{\Gamma\Gamma}(\tilde{\Gamma} - \gamma_{sp})^2 + \frac{1}{2}U_c\pi_{\gamma\gamma}(\gamma_i - \gamma_{sp})^2 + U_{\Gamma\Gamma}(\tilde{\Gamma} - \gamma_{sp})(\gamma_i - \gamma_{sp}). \end{aligned} \tag{A130}$$

Substituting this expansion into ΔU_i, and using the fact that $\pi_\gamma(\gamma_{sp}) = \pi_{\gamma\gamma} \cdot (\gamma_{sp} - \Gamma_0)$, we get:

$$\begin{aligned} \Delta U_i = {} & [U_\Gamma(\tilde{\theta}, \gamma_{sp}) + U_c \pi_{\gamma\gamma}(\gamma_{sp} - \Gamma_0)](\gamma_i - \gamma_{sp}) \\ & - \frac{1}{2} U_{\Gamma\Gamma}(\tilde{\Gamma} - \gamma_{sp})^2 + \frac{1}{2} U_c \pi_{\gamma\gamma}(\gamma_i - \gamma_{sp})^2 + U_{\Gamma\Gamma}(\tilde{\Gamma} - \gamma_{sp})(\gamma_i - \gamma_{sp}). \end{aligned} \tag{A131}$$

The household's voting decision is based on $\mathbb{E}_i[\Delta U_i]$. Given household i's information set $\{y_i, z\}$, γ_{sp} is known, and so is γ_i. The only random variables are $\tilde{\theta}$ and $\tilde{\Gamma}$. Taking the expectation:

$$\begin{aligned} \mathbb{E}_i[\Delta U_i] = {} & \mathbb{E}_i[U_\Gamma(\tilde{\theta}, \gamma_{sp}) + U_c \pi_{\gamma\gamma}(\gamma_{sp} - \Gamma_0)](\gamma_i - \gamma_{sp}) \\ & - \frac{1}{2} U_{\Gamma\Gamma} \mathbb{E}_i[(\tilde{\Gamma} - \gamma_{sp})^2] + \frac{1}{2} U_c \pi_{\gamma\gamma}(\gamma_i - \gamma_{sp})^2 + U_{\Gamma\Gamma} \mathbb{E}_i[(\tilde{\Gamma} - \gamma_{sp})](\gamma_i - \gamma_{sp}). \end{aligned} \tag{A132}$$

Make the following substitutions:

$$U_\Gamma(\tilde{\theta}, \gamma_{sp}) = U_\Gamma(\mu_\theta, \gamma_{sp}) + U_{\theta\Gamma}(\tilde{\theta} - \mu_\theta), \tag{A133}$$

$$\mathbb{E}_i[\tilde{\Gamma} - \gamma_{sp}] = \frac{1}{\alpha}(\gamma_i - \gamma_{sp}) - \frac{1-\alpha}{\alpha} \mathbb{E}_i[\gamma^* - \gamma_{sp}], \tag{A134}$$

where the the latter comes from Lemma 3: $\gamma_i = (1-\alpha)\mathbb{E}_i[\gamma^*] + \alpha \mathbb{E}_i[\tilde{\Gamma}]$. Thus:

$$\begin{aligned} \mathbb{E}_i[\Delta U_i] = {} & [U_\Gamma(\mu_\theta, \gamma_{sp}) + U_{\theta\Gamma} \mathbb{E}_i[\tilde{\theta} - \mu_\theta] + U_c \pi_{\gamma\gamma}(\gamma_{sp} - \Gamma_0)](\gamma_i - \gamma_{sp}) - \frac{1}{2} U_{\Gamma\Gamma} \mathbb{E}_i[(\tilde{\Gamma} - \gamma_{sp})^2] \\ & + \frac{1}{2} U_c \pi_{\gamma\gamma}(\gamma_i - \gamma_{sp})^2 + U_{\Gamma\Gamma} \left[\frac{1}{\alpha}(\gamma_i - \gamma_{sp}) - \frac{1-\alpha}{\alpha} \mathbb{E}_i[\gamma^* - \gamma_{sp}]\right](\gamma_i - \gamma_{sp}). \end{aligned} \tag{A135}$$

Using the definition of $\alpha = -\frac{U_{\Gamma\Gamma}}{U_c \pi_{\gamma\gamma}}$ and simplifying:

$$\begin{aligned} \mathbb{E}_i[\Delta U_i] = {} & \Big\{U_\Gamma(\mu_\theta, \gamma_{sp}) + U_{\theta\Gamma} \mathbb{E}_i[\tilde{\theta} - \mu_\theta] + U_c \pi_{\gamma\gamma}(\gamma_{sp} - \Gamma_0) + (U_{\Gamma\Gamma} + U_c \pi_{\gamma\gamma}) \mathbb{E}_i[\gamma^* - \gamma_{sp}]\Big\}(\gamma_i - \gamma_{sp}) \\ & - \frac{1}{2} U_c \pi_{\gamma\gamma}(\gamma_i - \gamma_{sp})^2 - \frac{1}{2} U_{\Gamma\Gamma} \mathbb{E}_i[(\tilde{\Gamma} - \gamma_{sp})^2]. \end{aligned} \tag{A136}$$

From the first-best solution (Lemma 2) and equation (A79), we can write:

$$\mathbb{E}_i[\gamma^* - \gamma_{sp}] = \mathbb{E}_i[\gamma^* - \Gamma_0] - (\gamma_{sp} - \Gamma_0) = \frac{-U_\Gamma(\mu_\theta, \Gamma_0) - U_{\theta\Gamma} \mathbb{E}_i[\tilde{\theta} - \mu_\theta]}{U_c \pi_{\gamma\gamma} + U_{\Gamma\Gamma}} - (\gamma_{sp} - \Gamma_0). \tag{A137}$$

Substituting and simplifying leads to:

$$\begin{aligned} \mathbb{E}_i[\Delta U_i] = {} & \{U_\Gamma(\mu_\theta, \gamma_{sp}) - U_\Gamma(\mu_\theta, \Gamma_0) - U_{\Gamma\Gamma}(\gamma_{sp} - \Gamma_0)\}(\gamma_i - \gamma_{sp}) \\ & - \frac{1}{2} U_c \pi_{\gamma\gamma}(\gamma_i - \gamma_{sp})^2 - \frac{1}{2} U_{\Gamma\Gamma} \mathbb{E}_i[(\tilde{\Gamma} - \gamma_{sp})^2]. \end{aligned} \tag{A138}$$

Recognizing that $U_\Gamma(\mu_\theta, \gamma_{sp}) = U_\Gamma(\mu_\theta, \Gamma_0) + U_{\Gamma\Gamma}(\gamma_{sp} - \Gamma_0)$ cancels the first term:

$$\mathbb{E}_i[\Delta U_i] = -\frac{1}{2} U_c \pi_{\gamma\gamma}(\gamma_i - \gamma_{sp})^2 - \frac{1}{2} U_{\Gamma\Gamma} \mathbb{E}_i[(\tilde{\Gamma} - \gamma_{sp})^2]. \tag{A139}$$

Since $U_c > 0$, $\pi_{\gamma\gamma} < 0$, and $U_{\Gamma\Gamma} < 0$, we have $\mathbb{E}_i[\Delta U_i] > 0$. □

A.15 Proof of Proposition 6

We examine how distrust affects the equilibrium. Individual learning about $\tilde{\theta}$ remains unchanged, but learning about the aggregate, $\tilde{\Gamma}$, is altered. Under the conjecture (28), household i perceives the aggregate greenness as:

$$\tilde{\Gamma}_i = \beta_0 + \beta_y \left(\int_j y_j^i dj - \mu_\theta \right) + \beta_z (z - \mu_\theta) \tag{A140}$$

$$= \beta_0 + \beta_y (\varphi \tilde{\theta} + \sqrt{1 - \varphi^2} \tilde{\phi}_i - \mu_\theta) + \beta_z (z - \mu_\theta). \tag{A141}$$

Consequently, household i's expectation of aggregate greenness is:

$$\mathbb{E}_i[\tilde{\Gamma}_i] = \beta_0 + \beta_y (\varphi \mathbb{E}_i[\tilde{\theta}] - \mu_\theta) + \beta_z (z - \mu_\theta). \tag{A142}$$

We proceed by revisiting the relevant results.

Lemma 1 (Status quo allocation): Unchanged.

Lemma 2 (First-Best allocation): Unchanged.

Lemma 3 (Team-efficient allocation): Equation (A18), reproduced here, remains valid:

$$\mathbb{E}_i \Big\{ U_c \pi_\gamma (\gamma_i^{te}(y_i, z)) \Big\} + \mathbb{E}_i \Big\{ U_\Gamma \Big(e + \pi(\tilde{\Gamma}^{te}(\tilde{\theta}, z)), \tilde{\theta}, \tilde{\Gamma}^{te}(\tilde{\theta}, z) \Big) \Big\} = 0. \tag{A143}$$

Taking the derivative and rearranging, we obtain

$$U_c \pi_{\gamma\gamma} \mathbb{E}_i[\gamma_i^{te} - \gamma^*] + U_{\Gamma\Gamma} \mathbb{E}_i[\tilde{\Gamma}^{te} - \gamma^*] = 0, \tag{A144}$$

which, as in Lemma 3, implies

$$\gamma_i^{te} = \left(1 + \frac{U_{\Gamma\Gamma}}{U_c \pi_{\gamma\gamma}} \right) \mathbb{E}_i[\gamma^*] - \frac{U_{\Gamma\Gamma}}{U_c \pi_{\gamma\gamma}} \mathbb{E}_i[\tilde{\Gamma}^{te}]. \tag{A145}$$

Recall from Lemma 2 that $\gamma^* = \kappa_0^* + \kappa_1^* (\theta - \mu_\theta)$. Thus, $\mathbb{E}_i[\gamma^*]$ is unaffected by distrust. However,

$\mathbb{E}_i[\tilde{\Gamma}^{te}]$ depends on the trust parameter, φ. In the limiting case of complete distrust ($\varphi = 0$), $\mathbb{E}_i[\tilde{\Gamma}^{te}]$ becomes independent of $\tilde{\theta}$, as seen from equation (A142). Still, the key result from Lemma 3, given in equation (16), holds, and firms' actions remain strategic substitutes.

Lemma 4 (Welfare Ranking): Unchanged.

Proposition 1 (Team efficiency of UBR): The function $f(\tilde{q})$ must still satisfy (A41):

$$\mathbb{E}_i[f(\tilde{q})] = \frac{U_\Gamma(\mu_\theta, \Gamma_0)}{U_c} + \frac{U_{\theta\Gamma}}{U_c}(\mathbb{E}_i[\tilde{\theta}] - \mu_\theta) + \frac{U_{\Gamma\Gamma}}{U_c}(\mathbb{E}_i[\tilde{\Gamma}^{te}] - \Gamma_0). \tag{A146}$$

This yields the same functional form as in Proposition 1:

$$f(\tilde{q}) = a\left[\mathrm{MRS} - \mathrm{ERA}(\tilde{q} - \mu_\theta)\right]. \tag{A147}$$

Therefore, UBR is robust to distrust among economic agents.

Proposition 2 (Cournot equivalence): Unchanged.

Proposition 3 (Equilibrium greenness): Under distrust, the coefficients of the conjectured equilibrium greenness, given by equation (28), are modified. To highlight the impact of distrust, we express these coefficients in terms of κ_0^*, κ_1^*, and α, which remain unchanged from the no-distrust case. The modified coefficients β_0, β_y, and β_z are:

$$\beta_0 = \kappa_0^* + \frac{\alpha\kappa_1^*\mu_\theta\sigma_\theta^2\tau_y(\varphi - 1)}{1 + \sigma_\theta^2(\tau_y + \tau_z - \alpha\tau_y\varphi)}, \tag{A148}$$

$$\beta_y = \frac{(1-\alpha)\kappa_1^*\sigma_\theta^2\tau_y}{1 + \sigma_\theta^2(\tau_y + \tau_z - \alpha\tau_y\varphi)}, \tag{A149}$$

$$\beta_z = \frac{\kappa_1^*\sigma_\theta^2\tau_z}{1 + \sigma_\theta^2(\tau_y + \tau_z - \alpha\tau_y\varphi)}. \tag{A150}$$

When $\varphi = 1$, we fall back on the original coefficients of Proposition 3 (see equations (A67)-(A69)).

Corollary 3.1 (Strategic substitutability and information weighting): Unchanged.

Corollary 3.2 (Information sensitivity): Points (a) and (b) of the corollary are unchanged. For point (c), the sign of dependence of the information sensitivity on the precision of public information depends on the trust parameter φ. To see this, write

$$\frac{\partial \mathcal{B}}{\partial \tau_z} = \frac{\kappa_1^* \sigma_\theta^2(-1+\alpha\sigma_\theta^2\tau_y(-1+\varphi))}{(1+\sigma_\theta^2(\tau_y+\tau_z-\alpha\tau_y\varphi))^2}. \tag{A151}$$

The sign of this derivative depends on the sign of $(-1+\alpha\sigma_\theta^2\tau_y(-1+\varphi))$. Since $\kappa_1^* < 0$, it follows that $\partial\mathcal{B}/\partial\tau_z > 0$ if and only if

$$(-1+\alpha\sigma_\theta^2\tau_y(-1+\varphi)) < 0, \tag{A152}$$

or, equivalently, if and only if

$$\varphi > 1 + \frac{1}{\alpha\sigma_\theta^2\tau_y}. \tag{A153}$$

Finally, we check the dependence of $\mathcal{B}$ on the trust parameter φ:

$$\frac{\partial \mathcal{B}}{\partial \varphi} = -\frac{\alpha\kappa_1^*\sigma_\theta^4\tau_y(\tau_y-\alpha\tau_y+\tau_z)}{(1+\sigma_\theta^2(\tau_y+\tau_z-\alpha\tau_y\varphi))^2} < 0. \tag{A154}$$

Corollary 3.3 (Incentives for Information Acquisition): Equation (A90) remains valid under distrust:

$$\Delta\mathbb{E}[U|z] = \frac{-U_c}{2\pi_{\gamma\gamma}}\left(\mathrm{Var}[\mathbb{E}_i'[f(\tilde{q})]|z] - \mathrm{Var}[\mathbb{E}_i[f(\tilde{q})]|z]\right) > 0. \tag{A155}$$

Thus, firms have a strict incentive to increase the precision of their private information.

Denote the difference in variances $\mathrm{Var}[\mathbb{E}_i'[f(\tilde{q})]|z] - \mathrm{Var}[\mathbb{E}_i[f(\tilde{q})]|z]$ by $\Delta\mathcal{V}(\xi,\varphi)$. We analyze how the incentive for information acquisition, driven by $\Delta\mathcal{V}(\xi,\varphi)$, changes with the trust parameter φ. Recall that $\mathbb{E}_i'$ denotes the expectation after the firm increases the precision of its private signal y_i by a factor of $\xi > 1$, and $\mathbb{E}_i$ is the expectation with the original precision.

First, we substitute $\tilde{\Gamma} = \beta_0 + \beta_y(\varphi\tilde{\theta} + \sqrt{1-\varphi^2}\tilde{\phi}_i - \mu_\theta) + \beta_z(z-\mu_\theta)$ into the expression for $f(\tilde{q})$:

$$f(\tilde{q}) = \frac{U_\Gamma(\mu_\theta,\Gamma_0)}{U_c} + \frac{U_{\theta\Gamma}}{U_c}(\tilde{\theta}-\mu_\theta) + \frac{U_{\Gamma\Gamma}}{U_c}(\tilde{\Gamma}-\Gamma_0) \tag{A156}$$

$$\begin{aligned} &= \frac{U_\Gamma(\mu_\theta,\Gamma_0)}{U_c} + \frac{U_{\theta\Gamma}}{U_c}(\tilde{\theta}-\mu_\theta) \\ &+ \frac{U_{\Gamma\Gamma}}{U_c}\left(\beta_0 + \beta_z(z-\mu_\theta) + \beta_y(\varphi\tilde{\theta} + \sqrt{1-\varphi^2}\tilde{\phi}_i - \mu_\theta) - \Gamma_0\right). \end{aligned} \tag{A157}$$

Next, we compute the conditional expectations $\mathbb{E}_i'[f(\tilde{q})|z]$ and $\mathbb{E}_i[f(\tilde{q})|z]$. Given that z is observed, the only remaining random variables in (A157) are $\tilde{\theta}$ and $\tilde{\phi}_i$. We have $\mathbb{E}[\tilde{\phi}_i|z] = 0$. The conditional expectation of $\tilde{\theta}$ given z and y_i (with precision τ_y) is given in equation (8):

$$\mathbb{E}[\tilde{\theta}|z, y_i] = \mu_\theta + \frac{\tau_y}{\sigma_\theta^{-2} + \tau_y + \tau_z}(y_i - \mu_\theta) + \frac{\tau_z}{\sigma_\theta^{-2} + \tau_y + \tau_z}(z - \mu_\theta). \tag{A158}$$

When the precision of y_i increases by a factor of ξ, we replace y_i with y_i' and τ_y with $\xi\tau_y$. Thus:

$$\mathbb{E}'[\tilde{\theta}|z, y_i'] = \mu_\theta + \frac{\xi\tau_y}{\sigma_\theta^{-2} + \xi\tau_y + \tau_z}(y_i - \mu_\theta) + \frac{\tau_z}{\sigma_\theta^{-2} + \xi\tau_y + \tau_z}(z - \mu_\theta). \tag{A159}$$

Substituting these into the expression (A157) for $f(\tilde{q})$, we are interested in the coefficients of y_i in the resulting expression, as the variance with respect to y_i is what determines the difference $\Delta\mathcal{V}(\xi, \varphi) = \mathrm{Var}[\mathbb{E}_i'[f(\tilde{q})]|z] - \mathrm{Var}[\mathbb{E}_i[f(\tilde{q})]|z]$. Let c_i' and c_i be the coefficients of y_i in $\mathbb{E}_i'[f(\tilde{q})|z]$ and $\mathbb{E}_i[f(\tilde{q})|z]$, respectively. These coefficients are

$$c_i' = \frac{\xi\sigma_\theta^2\tau_y(U_{\theta\Gamma} + U_{\Gamma\Gamma}\beta_y\varphi)}{U_c(1 + \sigma_\theta^2(\xi\tau_y + \tau_z))} \qquad \text{and} \qquad c_i = \frac{\sigma_\theta^2\tau_y(U_{\theta\Gamma} + U_{\Gamma\Gamma}\beta_y\varphi)}{U_c(1 + \sigma_\theta^2(\tau_y + \tau_z))}. \tag{A160}$$

We also need the conditional variances of y_i and y_i' given z. Using standard results for conditional distributions of jointly normal variables:

$$\mathrm{Var}[y_i'|z] = \frac{1}{\xi\tau_y} + \frac{\sigma_\theta^2}{1 + \sigma_\theta^2\tau_z} \qquad \text{and} \qquad \mathrm{Var}[y_i|z] = \frac{1}{\tau_y} + \frac{\sigma_\theta^2}{1 + \sigma_\theta^2\tau_z}. \tag{A161}$$

The difference in variances of the expectations is then:

$$\Delta\mathcal{V}(\xi, \varphi) = (c_i')^2\mathrm{Var}[y_i'|z] - c_i^2\mathrm{Var}[y_i|z] \tag{A162}$$

$$= \frac{(\xi - 1)\tau_y(U_{\theta\Gamma} + U_{\Gamma\Gamma}\beta_y\varphi)^2}{U_c^2(\sigma_\theta^{-2} + \tau_y + \tau_z)(\sigma_\theta^{-2} + \xi\tau_y + \tau_z)}. \tag{A163}$$

This expression is positive since $\xi > 1$. Substitute the solution for β_y:

$$\beta_y = -\frac{U_{\theta\Gamma}\tau_y}{U_c\pi_{\gamma\gamma}(\sigma_\theta^{-2} + \tau_y + \tau_z) + U_{\Gamma\Gamma}\tau_y\varphi}, \tag{A164}$$

simplifying, then taking the derivative of this expression with respect to φ leads to:

$$\frac{\partial}{\partial\varphi}\Delta\mathcal{V}(\xi, \varphi) = -\frac{2U_{\Gamma\Gamma}U_{\theta\Gamma}^2(\xi - 1)\pi_{\gamma\gamma}^2\tau_y^2(\sigma_\theta^{-2} + \tau_y + \tau_z)}{(\sigma_\theta^{-2} + \xi\tau_y + \tau_z)(U_c\pi_{\gamma\gamma}(\sigma_\theta^{-2} + \tau_y + \tau_z) + U_{\Gamma\Gamma}\tau_y\varphi)^3}. \tag{A165}$$

Under the assumptions $U_{\theta\Gamma} < 0$, $\pi_{\gamma\gamma} < 0$, $\sigma_\theta > 0$, $\tau_y > 0$, $\tau_z > 0$, $U_c > 0$, $0 < \varphi < 1$, $U_{\Gamma\Gamma} < 0$, and $\xi > 1$, this derivative is negative. Therefore, the incentive for information acquisition increases as trust (φ) decreases. □

A.16 Proof of Proposition 7

We examine how distrust affects the results from Section 3.

Proposition 4 (Social Planner's solution): Unchanged.

Corollary 4.1 (Pigouvian tax on emissions): Unchanged.

Corollary 4.2 (Cap-and-Trade implementation): Unchanged.

Proposition 5 (Political viability of UBR): Unchanged. □