# The Quiet Hand of Regulation: Harnessing Uncertainty and Disagreement

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**Abstract**

Regulating externalities is a major challenge when economic agents face uncertainty and disagreement. Traditional Pigouvian and Coasean approaches often struggle because they require either precise knowledge of externality costs or frictionless bargaining. We propose an "uncertainty-based regulation" (UBR) mechanism that leverages both uncertainty and firms' heterogeneous information to achieve socially efficient outcomes without requiring disclosure of private information. UBR works by creating a fictitious market where firms' incentives are aligned with social objectives through a simple, state-contingent transfer rule. This framework provides a Coasean solution that internalizes the externality. We show that the equilibrium allocation induced by UBR is team efficient, dominates traditional regulatory tools, incentivizes information acquisition, and remains robust even when firms distrust each other's private information. Moreover, if brought to a vote, it would receive unanimous support, making it politically viable.

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# 1 Introduction

A central challenge in economics is the design of effective regulation for activities that generate externalities. This task is made all the more difficult in the presence of uncertainty and disagreement among economic agents. Traditional approaches, rooted in the work of Pigou (1920) and Coase (1960), often falter in such complex environments. Pigouvian taxes or subsidies, while theoretically appealing, require the regulator to possess precise knowledge of the externality's magnitude (such as the social cost of carbon)—knowledge unattainable under pervasive uncertainty and firm-specific private information (Mirrlees, 1971; Weitzman, 1974; Stiglitz, 1982). Similarly, Coasean bargaining relies on well-defined property rights and negligible transaction costs—conditions rarely met in practice, especially when dealing with a large number of agents holding private information (Kwerel, 1977; Farrell, 1987).

The literature has responded to this challenge through two main approaches. The first approach, mechanism design, consists of developing sophisticated schemes to elicit private information from firms (e.g., Dasgupta, Hammond, and Maskin, 1979). The need for truthful revelation makes these mechanisms often complex, difficult to implement, and vulnerable to collusion or strategic misreporting, particularly when trust among agents is limited (Laffont and Tirole, 1986; Laffont, 1994). The second approach explores second-best policies that rely solely on publicly available information, such as uniform standards or taxes based on aggregate outcomes (e.g., Roberts and Spence, 1976; Weitzman, 1974). For instance, a Pigouvian framework under uncertainty typically prescribes a tax equal to the *expected* marginal social cost, based on public information alone (e.g., Lemoine, 2021). While conceptually sound, this method still requires the regulator to act as a *central calculator*, solving a full model of the economy without access to firms' private knowledge. The regulation of externalities is then confronted with a challenging trade-off: either complex mechanisms with potential fragility or simpler mechanisms with inherent inefficiency.

In this paper, we address this challenge by proposing a novel regulatory approach that *harnesses* uncertainty and disagreement to achieve efficiency *without* requiring the disclosure of private information. Unlike second-best approaches that rely only on public information, our mechanism actively harnesses dispersed private information as the very driver of efficiency. We label this approach "Uncertainty-Based Regulation" (UBR). We consider a setting where firms undertake socially beneficial actions (e.g., investing in green technologies, contributing to public goods, enhancing cybersecurity) whose effectiveness is uncertain and about which firms possess heterogeneous private information.

Instead of controlling prices (e.g., through taxes) or quantities (e.g., through permits), UBR operates through a simple transfer rule with two components: (i) a *quantity component*, which depends on the gap between a firm's action and the observable average across all firms, and (ii) a *price component*, which is tied to a publicly observable aggregate outcome. This outcome might include, for example, temperature change or total emissions (in environmental regulation), system stability (in financial or cybersecurity regulation), or aggregate productivity (in regulation that promotes research and knowledge creation, e.g., Romer (1986)).

Formally, we consider a setting similar to the canonical quadratic-payoff "beauty contest" coordination game of Morris and Shin (2002). In this environment, the Coasean resolution of the externality can be understood as the creation of a synthetic competitive market, the limit of a large Cournot game (Vives, 1988). In such a market firms would "supply" their socially beneficial actions, while society's needs would determine the "demand" and, ultimately, the price. Firms behave as price takers, guided by an endogenous "shadow price" that reflects the true underlying outcome. The shadow price internalizes the externality and coordinates firms' actions in a way that aggregates private information, capturing the Coasean solution.

Our contribution is to show that UBR implements precisely this synthetic market. From a policy-design perspective, the transfer rule belongs to the class of outcome- and aggregate-activity-contingent policies studied by Angeletos and Pavan (2009). By implementing the synthetic market, UBR achieves the *team-efficient* allocation—the best outcome attainable by a benevolent social planner under decentralized private information (Radner, 1962; Angeletos and Pavan, 2007, 2009). Unlike Pigouvian taxes or cap-and-trade systems, which require precise ex-ante knowledge and are often suboptimal under uncertainty and disagreement, UBR reaches efficiency without information revelation or centralized control, and without negotiation or pre-defined property rights. The allocation induced by UBR strictly dominates both the status quo and the allocation based on public information alone. The improvement arises from two sources: *individual flexibility*, as firms tailor actions to their signals, and *collective adaptability*, as aggregate behavior adjusts to the true state. Paradoxically, greater uncertainty strengthens incentives under UBR, amplifying the marginal value of information and thus inducing a stronger collective response. UBR also strengthens incentives under distrust of others' signals, as distrust raises perceived noise in aggregates and increases the marginal value of private information. Finally, UBR is politically viable, since in a simple vote all agents would ex-ante prefer it.

In sum, UBR bridges the Pigouvian goal of internalizing externalities and the Coasean principle of decentralized efficiency, achieving both without the precise ex-ante knowledge demanded by Pigou (1920) or the restrictive conditions required by Coase (1960). Crucially, the price component in UBR is not a fixed ex-ante Pigouvian rate but an *outcome-contingent schedule*, determined ex post from observable outcomes after firms have acted on both private and public information. This design links our analysis to formal work on information aggregation in strategic environments (e.g., Vives, 1988) and to the classic idea that decentralized actions can harness dispersed knowledge (Hayek, 1945).

**Literature.** Our work is closely related to Angeletos and Pavan (2009), who establish that taxes contingent on realized fundamentals and aggregate activity can restore efficiency in economies with dispersed information, and to Colombo, Femminis, and Pavan (2025), who show that Pigouvian-style schedules indexed to fundamentals and aggregate actions can also correct under-investment in information. While these papers identify the scope for outcome- and activity-contingent policies, our contribution is threefold. First, we prove a *market-equivalence* result: a simple transfer rule—Uncertainty-Based Regulation (UBR)—recreates the missing competitive market for the externality and thereby achieves the team-efficient allocation without information disclosure. Second, UBR is directly implementable: transfers depend only on ex post public observables (the realized outcome and the cross-sectional aggregate action), require no private reports, and do not rely on the regulator's knowledge of firms' cost structures or the dispersed information environment, but only on preference parameters. Third, UBR harnesses uncertainty as a driver of efficiency; it strengthens incentives under distrust of others' signals, which raises the marginal value of private information; and it is politically viable, since in a simple vote all agents would ex-ante prefer it. UBR thus connects the Pigouvian logic of corrective taxation with the Coasean principle of decentralized markets, showing how dispersed private information can be harnessed without central disclosure, in the spirit of Hayek (1945).

Recent work on state-contingent policies to address externalities has advanced along two distinct lines: creating new market-based assets and designing corrective payment schedules. The first approach follows the vision of Shiller (1994) by creating new financial securities to manage societal risks. A recent example is Lemoine (2024), who suggests "carbon shares" whose market price internalizes the externality. The second, exemplified by Colombo et al. (2025), shows that a subsidy indexed to aggregate investment and fundamentals can correct for both real spillovers and under-investment in information. Our framework shows that these two paradigms are equivalent: a state-contingent Pigouvian schedule can be interpreted as the creation of a synthetic market for the externality. UBR provides a simple regulatory mechanism that makes this equivalence operational, implementing the allocation that would prevail if the missing market actually existed. This trend towards outcome-contingent regulation is already reflected in major policy innovations. For instance, the European Union's Market Stability Reserve adjusts the future supply of emission permits based on the current market surplus, creating a dynamic cap that automatically responds to realized outcomes.

Our analysis also connects to the broader "global games" literature on coordination under heterogeneous private information, as described by Veldkamp (2011, Ch. 4, p. 45). Unlike models where inefficiency arises solely from information externalities (e.g., Amador and Weill 2010), our framework features a real payoff externality, which regulation directly addresses. We adopt the canonical public-private signal structure of Morris and Shin (2002) for clarity, while noting that the literature has since developed along several dimensions. One important line relaxes the signal environment beyond the public-private dichotomy by allowing multiple, partially correlated sources (e.g., Myatt and Wallace, 2008; Hellwig and Veldkamp, 2009), and, in parallel, by treating the information structure itself as a design object (Bergemann and Morris, 2016). Another line endogenizes information choice and the publicity of information, studying how agents allocate attention and how this shapes coordination (Myatt and Wallace, 2012; Morris and Yang, 2022). We retain the canonical representation with a public-private signal structure for simplicity. In linear-quadratic environments, additional (possibly correlated) signals enter only through posterior expectations and leave best-response slopes unchanged; hence implementation of the team-efficient allocation via UBR is unaffected. Moreover, because UBR induces strategic substitutability, it increases the marginal value of private information—consistent with comparative statics that predict less correlated information use when actions are substitutes (Hellwig and Veldkamp, 2009; Bergemann and Morris, 2016; Myatt and Wallace, 2012).

The strategic interdependence in our model connects it to the literature on heterogeneous beliefs and market dynamics (Keynes, 1964; Bikhchandani, Hirshleifer, and Welch, 1992; Morris and Shin, 2002; Veldkamp, 2011; Myatt and Wallace, 2008). As in the "beauty contest" framework of Morris and Shin (2002), agents must anticipate the actions of others. We extend this idea, in the spirit of Banerjee (2011), by allowing agents to distrust one another's signals—a mechanism that introduces a risk component analogous to the "sentiment risk" studied by Dumas, Kurshev, and Uppal (2009). While this literature provides deep insights into market dynamics, our contribution is to use these insights as a foundation for policy design. We ask how a regulator can exploit these very strategic interactions as policy tools to align individual actions with societal goals, even amid agents' distrust.

Our model's structure, in which agents treat an aggregate variable as a public input, shares features with the endogenous growth framework of Romer (1986). We depart from that perfect-foresight setting by introducing asymmetric information about the state of the economy and designing a regulatory mechanism to contend with it. This focus on designing incentive-compatible rules under uncertainty places our work at the intersection of endogenous growth and a classic literature on regulation. That tradition, exemplified by Roberts and Spence (1976), Kwerel (1977), and Montero (2008), has long sought to design mechanisms that function effectively when the regulator has imperfect information about firms. Our specific contribution is to show how regulation can create a missing market that leverages informational frictions as a source of efficiency.

Finally, our work contributes to the broader literature on uncertainty, economic decision-making, and regulatory design. Since the seminal work of Knight (1921), uncertainty has often been viewed as a barrier to effective decision-making (e.g., Bernanke, 1983; Rodrik, 1991; Dixit and Pindyck, 1994; Caballero and Pindyck, 1996). We challenge this perspective by demonstrating that uncertainty can be harnessed to achieve desirable societal goals. This insight aligns with Wang (2022), who shows that environmental regulatory uncertainty can incentivize firms to adopt greener practices preemptively, and with Pindyck (2007, 2022), who explore uncertainty's role in prompting proactive environmental policies. In the context of climate policy and regulatory economics (Nordhaus, 2019; Stern, 2007; Heal, 2009), our approach offers an alternative to traditional regulatory tools. UBR bridges the gap between Pigouvian taxation and Coasean bargaining by providing a regulatory mechanism that internalizes externalities without requiring explicit information disclosure or centralized control. In so doing, our work connects to the literature on market-based mechanisms and information aggregation in strategic settings (Vives, 1988).

Taken together, our findings contribute to the long-standing debate on optimal regulation under uncertainty and asymmetric information, with broad potential applications, ranging from environmental policy and innovation to financial stability, public health, and cybersecurity. By demonstrating that it *is* possible to achieve efficient allocations without resorting to complex, potentially fragile revelation mechanisms or sacrificing the benefits of decentralized decision-making, we offer a practical alternative to existing regulatory tools.

To ground the theory, Section 5 presents a stylized example that applies our framework to cybersecurity investment. Each firm chooses an investment to protect itself from cyber attacks, which benefits the rest of the system by reducing breach spillovers. This setting shows how UBR endogenously induces the optimal "diversity of defense" strategy, eliminating the coordination failure that leaves an unregulated market exposed to correlated cyber risks.

The paper proceeds as follows. Section 2 introduces the model, derives benchmark allocations, and characterizes the UBR equilibrium. Section 3 studies welfare and political feasibility. Section 4 examines the role of distrust. Section 5 applies the framework to cybersecurity and systemic risk, drawing out policy lessons. Section 6 concludes.

# 2 The Model

Consider an economy with two dates, $t \in \{0,1\}$, populated by a continuum of households indexed by $i \in [0,1]$. Each household, or "agent", acts as both consumer and producer, owning and operating a firm. There is a single consumption good, which serves as the numéraire. At time $t = 1$, agents derive utility from consumption and from a non-pecuniary component $q$. We refer to $q$ as the *aggregate outcome*. Our analysis applies broadly to settings where a non-pecuniary $q$ captures either a positive or negative externality.[1]

**Preferences.** Each agent's preferences are represented by a utility function $\mathcal{U}(c_i, q)$, where $c_i$ is agent $i$'s consumption. We assume that agents prefer both higher consumption and a higher outcome ($\mathcal{U}_{c_i}(c_i, q) > 0$, $\mathcal{U}_q(c_i, q) > 0$), and that the marginal utility of $q$ is decreasing ($\mathcal{U}_{qq}(c_i, q) < 0$).[2]

Each household $i$ operates a firm. At $t = 0$, household $i$ chooses an "action" $a_i$, representing the firm's contribution to a socially desirable activity that improves the aggregate outcome $q$. Since each household owns and operates a firm, we can equivalently refer to the household's decision $a_i$ as the firm's action. The *aggregate action*, $\tilde{A} = \int_0^1 a_i \, di$, is the sum of all actions. Because choices are made simultaneously and are unobservable to others, the aggregate $\tilde{A}$ is uncertain at $t = 0$.[3]

The firm's profit, $\pi(a_i)$, depends on the household's action $a_i$. We assume all firms begin at a *status quo* level of action, denoted $A_0$. The profit function is quadratic and concave (i.e., $\pi_{aa} < 0$ is a negative constant), and is maximized at $a_i = A_0$. This reflects diminishing

[1] For instance, $q$ may represent "environmental quality," as in integrated assessment models that incorporate climate change and emissions into utility (Nordhaus, 1991, 2015). See also Michel and Rotillon (1995), Baker, Hollifield, and Osambela (2022), and Acemoglu, Aghion, Bursztyn, and Hemous (2012) for related applications.

[2] Throughout, subscripts denote partial derivatives. For a function $f(x, y)$, we write $f_x \equiv \partial f/\partial x$, $f_y \equiv \partial f/\partial y$, $f_{xx} \equiv \partial^2 f/\partial x^2$, $f_{xy} \equiv \partial^2 f/\partial x \partial y$, etc.

[3] We use a tilde (e.g., $\tilde{A}$) to denote variables that are uncertain at $t = 0$, when households make decisions. Variables without tildes (e.g., $q$, $c_i$, $a_i$, $A_0$) are either known at $t = 0$ or realized at $t = 1$.

returns and adjustment costs from deviating from the status quo. As a result, household $i$'s consumption at time 1 is:

$$c_i = e + \pi(a_i), \tag{1}$$

where $e$ is an exogenous endowment of the consumption good, common to all agents.[4]

We assume that the outcome $q$ is given by:

$$\tilde{q} = \tilde{\theta} + (\tilde{A} - A_0), \tag{2}$$

where $\tilde{\theta}$ is a random variable capturing the underlying uncertain state of $q$. Equation (2) implies that higher aggregate action $\tilde{A}$, relative to the *status quo* $A_0$, raises the outcome $q$.[5] We assume the expected value $\mu_\theta := \mathbb{E}[\tilde{\theta}]$ is public information. Although both $\tilde{\theta}$ and $\tilde{A}$ are uncertain at $t = 0$, the realized value of $\tilde{q}$ is publicly observed at $t = 1$.[6]

At $t = 1$, the values of $\tilde{\theta}$, $\tilde{A}$, and $c_i$ are realized. At $t = 0$, household $i$ chooses $a_i$ to maximize its expected utility. Ideally, the household would solve $\max_{a_i} \mathbb{E}_i[\mathcal{U}(c_i, \tilde{q})]$, where $c_i$ is given in equation (1), $\tilde{q}$ is defined in equation (2), and the expectation $\mathbb{E}_i[\cdot]$ reflects the information available to household $i$ at $t = 0$. Since this problem cannot be solved in closed form under general assumptions, we adopt a tractable approximation.

Specifically, we approximate the utility function $\mathcal{U}(c_i, q)$ using a Taylor expansion around $(c_0, q_0)$, where $c_0$ is a baseline level of consumption and $q_0 = \mu_\theta$ is the expected value of the aggregate outcome in the status quo (both corresponding to the status quo action level $A_0$ and the expected state $\mu_\theta$). We define $U(c_i, \theta, A)$ as the utility obtained by taking a first-order expansion of $\mathcal{U}(c_i, q)$ in $c_i$ and a second-order expansion in $q$:

$$U(c_i, \theta, A) := \mathcal{U}(c_0, \mu_\theta) + \mathcal{U}_c(c_0, \mu_\theta)(c_i - c_0) + \mathcal{U}_q(c_0, \mu_\theta)(q - \mu_\theta) + \frac{1}{2}\mathcal{U}_{qq}(c_0, \mu_\theta)(q - \mu_\theta)^2. \tag{3}$$

[4] We assume $e$ is large enough that the probability of negative realized consumption is negligible in equilibrium.

[5] For expositional clarity, we present the simplest specification where the aggregate outcome depends linearly on the fundamental state and aggregate action. Our results extend to more general specifications, such as $\tilde{q} = g(\tilde{\theta}, \tilde{A}) + \tilde{\nu}$, where $g(\cdot)$ is an affine function and $\tilde{\nu}$ represents additional measurement noise. In this case, although $q$ is observable ex-post, $\theta$ remains unobservable and the regulator cannot simply infer and use the true state $\theta$ for direct Pigouvian taxation.

[6] The assumption that $\tilde{q}$ is observable is plausible. In an environmental context, for instance, $\tilde{q}$ could correspond to the measured change in a public index like the Environmental Performance Index (EPI), which tracks environmental health and ecosystem vitality across countries. See, e.g., https://epi.yale.edu/.

Substituting $c_i$ from equation (1) and $\tilde{q}$ from equation (2) into the Taylor expansion (3), we obtain an approximated utility function $U(c_i, \tilde{\theta}, \tilde{A})$ that is linear in $c_i$ and quadratic in $\tilde{\theta}$ and $\tilde{A}$. The function $U(c_i, \tilde{\theta}, \tilde{A})$ thus approximates utility in terms of consumption and the uncertain variables $\tilde{\theta}$ and $\tilde{A}$. The household's action $a_i$ directly affects utility through its effect on $c_i$, where $c_i = e + \pi(a_i)$. This linear-quadratic form, standard in macroeconomic models with dispersed information (Angeletos and Pavan, 2007, 2009), allows us to derive closed-form results.[7]

Therefore, at $t = 0$, household $i$ chooses $a_i$ to maximize:

$$\max_{a_i} \mathbb{E}_i[U(c_i, \tilde{\theta}, \tilde{A})], \tag{4}$$

subject to equation (1). The Taylor approximation (3) and our assumptions on $\mathcal{U}$ yield an approximated utility function $U$ with properties that we summarize in the following Lemma.

**Lemma 1.** *The approximated utility function $U(c_i, \theta, A)$ has the following properties:*

*(a)* $U_c = \mathcal{U}_c(c_0, \mu_\theta) > 0$ *is a positive constant;*

*(b)* $U_A(\tilde{\theta}, \tilde{A})$ *is linear in its arguments;*

*(c)* $U_{AA} = \mathcal{U}_{qq}(c_0, \mu_\theta) < 0$ *is a negative constant;*

*(d)* $U_{cA} = 0$*;*

*(e)* $U_{\theta A} = \mathcal{U}_{qq}(c_0, \mu_\theta) < 0$ *is a negative constant.*

Properties (b) and (d) are particularly informative. Property (b) shows where the externality lies: utility depends directly on the aggregate action $A$. Property (d) indicates that, in the baseline environment, there is no cross-effect between consumption and the aggregate action; this absence creates room for policy, as it defines the margin along which regulation can operate.

[7] Appendix C revisits the model using an exponential-utility specification. We solve for both individual and aggregate actions—analytically when possible, numerically otherwise—and show that the core equilibrium relationships in the main text remain valid. The linear-quadratic form simplifies exposition without affecting the results.

**Information.** At time $t = 0$, all households share a common prior belief about the unobservable state $\tilde{\theta}$, which is assumed to follow a normal distribution: $\tilde{\theta} \sim \mathcal{N}(\mu_\theta, \sigma_\theta^2)$. Each household $i$ observes a private signal $y_i$ and a public signal $z$ about $\tilde{\theta}$, both acquired at no cost. The signals are given by:

$$y_i = \tilde{\theta} + \tilde{\varepsilon}_{y,i}, \tag{5}$$

$$z = \tilde{\theta} + \tilde{\varepsilon}_z, \tag{6}$$

where $\tilde{\varepsilon}_{y,i} \sim \mathcal{N}(0, \tau_y^{-1})$ and $\tilde{\varepsilon}_z \sim \mathcal{N}(0, \tau_z^{-1})$ are independent noise terms, also independent of $\tilde{\theta}$. The parameters $\tau_y$ and $\tau_z$ denote the precisions of the private and public signals, respectively.[8] We assume households trust the informational content of others' signals. Section 4 relaxes this assumption and explores the implications of agents' distrust in the quality of others' private signals.[9]

Let $\mathbb{E}_i[\tilde{\theta}] := \mathbb{E}[\tilde{\theta}|y_i, z]$ denote household $i$'s posterior expectation of $\tilde{\theta}$, conditional on its information set $\{y_i, z\}$. The posterior precision, $\tau$, is defined as:

$$\tau := (\mathrm{Var}[\tilde{\theta}|y_i, z])^{-1} = \sigma_\theta^{-2} + \tau_y + \tau_z. \tag{7}$$

By Bayes' theorem, household $i$'s posterior expectation is the prior mean plus precision-weighted deviations of the private and public signals from that mean:

$$\mathbb{E}_i[\tilde{\theta}] = \mu_\theta + \frac{\tau_y}{\tau}(y_i - \mu_\theta) + \frac{\tau_z}{\tau}(z - \mu_\theta). \tag{8}$$

## 2.1 Benchmark Allocations

An allocation in this economy is a set of actions $\{a_i\}_{i\in[0,1]}$ undertaken by all households. Before introducing our regulatory mechanism, we define three benchmark allocations that

[8] Throughout this paper, the Greek letter $\sigma$ denotes all forms of prior uncertainty (e.g., $\sigma_\theta$), referring to variables or processes that are inherently uncertain before any information is observed. By contrast, the Greek letter $\tau$ represents the precision (the inverse of variance) of information signals, such as $\tau_y$ and $\tau_z$.

[9] We adopt the canonical private-public signal structure of Morris and Shin (2002) for expositional clarity. Our results are robust to richer information environments with multiple signals, such as those considered in Myatt and Wallace (2012) and Hellwig and Veldkamp (2009). The linearity of our framework ensures that additional signals—whether public, private, or with correlated noise—would simply be aggregated into the agents' posterior beliefs according to Bayes' rule. This would not alter the fundamental strategic structure or the design of the UBR mechanism, which operates on those posterior beliefs irrespective of the number or correlation of underlying signals.

serve as points of comparison. These benchmarks highlight the potential efficiency gains from our proposed approach.

We begin with the *status quo* allocation, which describes the outcome in the absence of regulation, when households do not internalize the externality.

**Lemma 2** (**Status quo allocation**). *In the absence of regulation or policy intervention, each firm chooses its action to maximize the household's utility, ignoring the impact on the aggregate outcome. This results in all firms choosing the status quo action:*

$$a_i^{sq} = A_0, \quad \forall i \in [0,1]. \tag{9}$$

*Hence, the aggregate action is also the status quo action:*

$$\tilde{A}^{sq} = A_0. \tag{10}$$

Given the assumption that $\pi(a_i)$ is a quadratic concave function maximized at $A_0$, the status quo ($a_i^{sq} = A_0$) is, by construction, privately optimal. It is, however, socially inefficient because $a_i^{sq}$ does not account for the positive externality of individual actions, which contribute to the aggregate outcome.

We next consider the *first-best* allocation. In this benchmark a social planner observes, at time $t = 0$, the realized value $\theta$ of the random variable $\tilde{\theta}$ and chooses the entire vector of firms' actions. With full knowledge of the state, the planner maximizes social welfare and fully internalizes the externality that each firm's action imposes on the aggregate outcome.

Since the planner observes $\theta$ directly, and utility is symmetric across agents, the problem reduces to choosing a common action $a$ for all firms. Formally, the planner solves:

$$\max_a \quad U(e + \pi(a), \theta, a). \tag{11}$$

Because $\pi(\cdot)$ is quadratic and $U$ is linear-quadratic in its arguments by Lemma 1 (a), (b), and (c), the objective function in (11) is quadratic in $a$. Hence, the resulting first-best action rule, $a^*(\theta)$, is linear in the state $\theta$. This leads to the following lemma.

**Lemma 3** (**First-best allocation**). *The first-best action $a^*(\theta)$ is a linear function of the* *realized underlying state:*

$$a^*(\theta) = \kappa_0^* + \kappa_1^*(\theta - \mu_\theta), \tag{12}$$

*where*

$$\kappa_0^* := A_0 - \frac{U_A(\mu_\theta, A_0)}{U_c\pi_{aa} + U_{AA}} \qquad \textit{and} \qquad \kappa_1^* := -\frac{U_{\theta A}}{U_c\pi_{aa} + U_{AA}}. \tag{13}$$

This first-best allocation is unattainable in our main setting because $\theta$ is unobservable at $t = 0$. In the first-best, all firms choose $a^*(\theta)$ contingent on the realized value of $\theta$. Since $U_{\theta A} < 0$ by Lemma 1 (e), $U_c > 0$ by Lemma 1 (a), $\pi_{aa} < 0$ by assumption, and $U_{AA} < 0$ by Lemma 1 (c), we have $\kappa_1^* < 0$. Therefore, the first-best action $a^*(\theta)$ is decreasing in $\theta$: a more adverse realization of the underlying state (a lower $\theta$) requires a higher action.

We finally consider the *team-efficient* allocation. Following the seminal team-theory results of Radner (1962) and Marschak and Radner (1972), and their macro applications in Angeletos and Pavan (2007, 2009), this benchmark represents the best outcome achievable when a social planner specifies a decision rule that maps each firm's private and public information into a recommended action, but cannot observe individual firms' private signals. Firms are assumed to commit to this decision rule. The team-efficient allocation differs from the first-best because actions are now decentralized.

To formalize this, ex-ante expected social welfare is given by:

$$\mathbb{E}[U] := \int_{(\tilde{\theta}, z)} \int_{y_i} U(e + \pi(a_i), \tilde{\theta}, \tilde{A}) dP(y_i|\tilde{\theta}, z) dP(\tilde{\theta}, z), \tag{14}$$

where $dP(y_i|\tilde{\theta}, z)$ is the conditional probability distribution of the private signal $y_i$ given $\tilde{\theta}$ and $z$, and $dP(\tilde{\theta}, z)$ is the joint probability distribution of $\tilde{\theta}$ and the public signal $z$.

The team-efficient allocation is then defined as the strategy $a_i^{te}(y_i, z)$ that solves the following maximization problem:

$$\max_{a_i^{te}(\cdot)} \int_{(\tilde{\theta}, z)} \int_{y_i} U\Big(e + \pi\Big(a_i^{te}(y_i, z)\Big), \tilde{\theta}, \tilde{A}^{te}(\tilde{\theta}, z)\Big) dP(y_i|\tilde{\theta}, z) dP(\tilde{\theta}, z), \tag{15}$$

subject to $\tilde{A}^{te}(\tilde{\theta}, z) = \int_{y_i} a_i^{te}(y_i, z) dP(y_i|\tilde{\theta}, z)$.

To provide intuition for the concept of team-efficient allocation, it is critical to clarify the nature of the social planner's problem. Unlike in the first-best case, the planner does not choose a single action level but instead designs a *decision rule*, $a_i(y_i, z)$, for all firms to follow. Crucially, the planner must choose this rule *without* observing any firm's private signal, $y_i$. With a continuum of firms, the Law of Large Numbers delivers a key simplification: the empirical distribution of private signals across the economy converges almost surely to the true conditional distribution $P(y_i \mid \tilde{\theta}, z)$. This convergence allows the planner to map any candidate rule to its implied aggregate action and thus choose the one that maximizes ex-ante welfare.

**Lemma 4** (**Team-efficient allocation**). *The team-efficient allocation is characterized by each firm choosing an action, $a_i^{te}$, that is a linear function of its conditional expectations of the first-best action, $a^*(\theta)$, and the aggregate action, $\tilde{A}^{te}$:*

$$a_i^{te} = (1-\alpha)\mathbb{E}_i[a^*(\theta)] + \alpha\mathbb{E}_i[\tilde{A}^{te}], \tag{16}$$

*where $\mathbb{E}_i[\cdot]$ denotes the expectation conditional on household $i$'s information set $\{y_i, z\}$, $a^*(\theta)$ is the first-best action defined in Lemma 3, $\tilde{A}^{te} = \int_0^1 a_i^{te} di$ is the aggregate action, and $\alpha$ is defined as*

$$\alpha := -\frac{U_{AA}}{U_c \pi_{aa}}. \tag{17}$$

*Since $U_{AA} < 0$, $U_c > 0$ and $\pi_{aa} < 0$, we have $\alpha < 0$, implying that in the team-efficient allocation firms' actions are strategic substitutes.*

In the team-efficient allocation, each firm's action is a weighted average of its expected first-best action and its expected aggregate action—an affine combination implied by the model's linear-quadratic structure. This functional form reflects the planner's need to balance two competing objectives: (i) steering the aggregate action as close as possible to the first-best and (ii) coordinating individual actions. Over-emphasizing the first objective makes firms' choices overly sensitive to their idiosyncratic private signals, creating costly dispersion. Over-emphasizing the second induces firms to ignore private signals in favor of public information—the standard "beauty contest" distortion—producing inefficient conformity. The decision rule (16) resolves this trade-off by directing each firm to target a weighted average of its estimate of the first-best, $\mathbb{E}_i[a^*(\theta)]$, and its estimate of the aggregate action, $\mathbb{E}_i[\tilde{A}^{te}]$.

The weight $\alpha$ captures the *socially optimal degree of coordination* (Angeletos and Pavan, 2007, 2009). Equivalently, $\alpha$ is the slope of a firm's best-response function with respect to its expected aggregate action. Because $\alpha < 0$ in our setting, actions are strategic substitutes: in the team-efficient allocation, firms take actions that diverge from the aggregate and place greater weight on their private information. Strategic substitutability emerges from society's aversion to outcome volatility ($U_{AA} < 0$): excessive coordination creates correlated risk, making diversification socially valuable. The cybersecurity example in Section 5 further illustrates this mechanism.

The team-efficient benchmark represents the best society could achieve if agents could internalize their payoff interdependencies and commit ex ante to a joint strategy for processing information before any signals are realized. This is a demanding standard: it presumes the kind of coordination that is infeasible in practice, making it a useful but stringent benchmark for evaluating equilibria. Importantly, this benchmark differs from standard constrained-efficiency notions that emphasize incentive compatibility under costless communication (e.g. Mirrlees, 1971; Holmström and Myerson, 1983). Instead, it is rooted in the team-theoretic tradition of (Radner, 1962) and Marschak and Radner (1972), and shares with Hayek (1945) the insight that information is dispersed and cannot be centralized.

We now establish a simple welfare ranking of the three benchmark allocations.

**Lemma 5** (**Welfare ranking**). *Let $\mathbb{E}[U^{sq}]$, $\mathbb{E}[U^{te}]$, and $\mathbb{E}[U^*]$ denote the ex-ante expected social welfare defined in* (14) *under the status quo, team-efficient, and first-best allocations, respectively. Then,*

$$\mathbb{E}[U^{sq}] \leq \mathbb{E}[U^{te}] \leq \mathbb{E}[U^*]. \tag{18}$$

The *team-efficient* allocation improves on the status quo by leveraging firms' dispersed private information even though no signal sharing or centralized control takes place. Because the underlying state remains imperfectly observed, however, this allocation still falls short of the unattainable first-best benchmark, where the planner has perfect information about the realized state.

## 2.2 Uncertainty-Based Regulation (UBR)

Because the team-efficient allocation sets such an exacting standard, the challenge is to design a rule that can bring decentralized behavior as close as possible to this benchmark without requiring the planner's information or coordination powers. To achieve this goal, in this section, we propose a regulatory mechanism, which we label "Uncertainty-Based Regulation (UBR)." The key idea underlying UBR is to implement a transfer scheme that creates private incentives for the team-efficient outcome to arise as a competitive equilibrium—without the regulator requiring the disclosure of any private information. Crucially, this is *not* a command-and-control approach: firms retain full discretion over their actions. Unlike in the theoretical team-efficient allocation, the regulator does not impose a decision rule. Instead, UBR works through incentives, steering firms toward socially optimal actions while leaving their choices entirely unconstrained. We show that under UBR, the team efficient allocation is incentive compatible.

Formally, let $f(q)$ be a *state-contingent* function of the realized aggregate outcome $q$. This function, known to all households at $t = 0$, is part of the regulatory framework, even though its value depends on the ex-post realization of $q$. The regulator implements a transfer scheme at time $t = 1$ through the following regulatory term:

$$\mathcal{R}(q, A, a_i) = (a_i - A)f(q). \tag{19}$$

This regulatory term creates a strategic interdependence among firms. The component $(a_i - A)$ captures a firm's deviation from the average action, while $f(q)$ serves as a *shadow price* for that deviation, determined ex post by the realized aggregate outcome $q$. As we will show below, a firm must forecast this shadow price when choosing its optimal action. Because the price depends on the realized outcome $\tilde{q}$, which in turn depends on the aggregate action $\tilde{A}$ via equation (2), each firm must form beliefs about the collective behavior of others. This need to anticipate the aggregate action—without being able to influence it—introduces a *beauty contest* element into firms' decisions, echoing the strategic structure studied by Morris and Shin (2002).

Accounting for the regulatory term, the household's time-1 consumption becomes:

$$\tilde{c}_i = e + \pi(a_i) + \mathcal{R}(\tilde{q}, \tilde{A}, a_i). \tag{20}$$

At $t = 0$, firm $i$ chooses the action $a_i$ to solve the following maximization problem:

$$\max_{a_i} \mathbb{E}_i[U(\tilde{c}_i, \tilde{\theta}, \tilde{A})], \tag{21}$$

where by equation (20), the consumption at time $t = 1$ is $\tilde{c}_i = e + \pi(a_i) + \mathcal{R}(\tilde{q}, \tilde{A}, a_i)$. Note that, from the household's perspective at $t = 0$, consumption at $t = 1$ is now uncertain due to the regulatory term $\mathcal{R}(\tilde{q}, \tilde{A}, a_i)$.

The central design question is how to specify the regulatory function $f(\tilde{q})$ to induce firms to *voluntarily* choose the team-efficient actions defined in Lemma 4. This requires aligning private incentives with the social optimum, even in the presence of an unobservable aggregate state and dispersed private information. The next proposition identifies the exact form of $f(\tilde{q})$ that achieves team efficiency.

The regulator's task is constrained by the fact that firms' private signals are non-verifiable and their cost structures are private knowledge. A regulatory mechanism must therefore operate without this private information, conditioning only on ex-post observable outcomes: the aggregate outcome $q$, the aggregate action $A$, and individual firms' actions $a_i$. For the policy to be self-financing, the mechanism must also be *neutral*, with all transfers summing to zero across firms. The design problem is thus to identify whether *any* transfer scheme, restricted to observables and neutrality, can uniquely align decentralized incentives with the team-efficient allocation—and if so, to characterize its form.

**Proposition 1** (**Implementation of team efficiency under UBR**). *Suppose each firm is subject to a regulatory transfer* $\mathcal{R}(\tilde{q}, \tilde{A}, a_i) = (a_i - \tilde{A})f(\tilde{q})$*, where the function* $f(\tilde{q})$ *takes the form:*

$$f(\tilde{q}) = \text{MRS} - \text{ERA}(\tilde{q} - \mu_\theta), \tag{22}$$

*with*

$$\text{MRS} := \frac{\mathcal{U}_q(c_0, \mu_\theta)}{\mathcal{U}_c(c_0, \mu_\theta)} > 0 \quad \textit{and} \quad \text{ERA} := -\frac{\mathcal{U}_{qq}(c_0, \mu_\theta)}{\mathcal{U}_c(c_0, \mu_\theta)} > 0 \tag{23}$$

*denoting, respectively, the marginal rate of substitution between the aggregate outcome q and consumption, and society's aversion to risk in the aggregate outcome (both measured at the status quo $A_0$). Then, the unique linear equilibrium of the game induced by this transfer scheme implements the team-efficient allocation characterized in Lemma 4.*

In the remainder of the paper we refer to UBR as a regulatory framework that operates through the transfer term $\mathcal{R}(\tilde{q}, \tilde{A}, a_i) = (a_i - \tilde{A})f(\tilde{q})$ with $f(\tilde{q})$ defined in equation (22). This transfer scheme is neutral: because individual deviations from the average sum to zero (since $f(\tilde{q})$ is common to all firms and $\int_0^1 (a_i - \tilde{A})di = 0$ by definition), the mechanism is self-financing. The key insight in Proposition 1 is that the form of the regulatory function $f(\tilde{q})$ aligns private and social incentives such that each firm's optimal action coincides with the team-efficient benchmark. To choose its action at time $t = 0$, a firm must forecast the realization of the shadow price $f(\tilde{q})$, which depends on the realized aggregate outcome at time $t = 1$. As we show in the proof of Proposition 1, team efficiency is achieved when the following condition holds:

$$\underbrace{A_0 - \frac{\mathbb{E}_i[f(\tilde{q})]}{\pi_{aa}}}_{\text{Privately Optimal Action}} \quad = \quad \underbrace{(1-\alpha)\mathbb{E}_i[a^*(\theta)] + \alpha \mathbb{E}_i[\tilde{A}^{te}]}_{\text{Team-Efficient Action}} \tag{24}$$

On the left-hand side is the action that maximizes firm $i$'s expected payoff under UBR. On the right-hand side is the team-efficient action, as defined in Lemma 4. The regulatory function in Proposition 1 is constructed precisely to equate these two expressions. This regulatory function is not arbitrary; its form is uniquely determined by the requirement of team efficiency. Thus, by aligning the firm's private first-order condition with the team-efficient rule, UBR induces the exact degree of coordination—captured by $\alpha$—required for team efficiency. The resulting action is a weighted average of the firm's forecast of the first-best allocation and its forecast of the aggregate action, with weights $(1 - \alpha)$ and $\alpha$, respectively.

The regulatory function $f(\tilde{q})$ embeds two key components that reflect society's preferences. The first is the Marginal Rate of Substitution (MRS), which sets the baseline price for deviations from the average action. It captures the value society places on marginal improvements in the aggregate outcome relative to consumption. The second is a state-contingent adjustment, $-\text{ERA}(\tilde{q} - \mu_\theta)$, which increases the price when the realized outcome is worse than expected and lowers it when the outcome is better. Together, these components generate a pricing structure that varies with the state of the world and thereby creates the state-contingent incentives detailed in Table 1.

These incentives are the heart of the mechanism. As the table shows, when the aggregate outcome is low ($q < \mu_\theta$), the shadow price $f(q)$ is high. This rewards above-average actions ($a_i > A$) and penalizes below-average ones ($a_i < A$), prompting leaders to *Advance* and laggards to *Catch up*. When the outcome is high ($q > \mu_\theta$), the price is low, relaxing this pressure: leaders are nudged to *Slow down* and laggards to *Linger*. Firms, guided by private signals about the underlying state, are thus steered to act precisely when their contributions are most valuable.

**Table 1: State-Contingent Incentives under UBR**

The table shows a firm's incentive contingent on the realized state of the aggregate outcome ($q$ vs. $\mu_\theta$) and the firm's action relative to the average ($a_i$ vs. $A$). The quantity $f(q)$ represents the shadow price firms face when deviating from the average action, as discussed in equation (19).

| | **Realized Aggregate Outcome** | |
|---|---|---|
| **Firm $i$'s Action** | $q < \mu_\theta$, high $f(q)$ | $q > \mu_\theta$, low $f(q)$ |
| $a_i > A$ (leader) | *Advance* | *Slow down* |
| $a_i < A$ (laggard) | *Catch up* | *Linger* |

The structure of the shadow price $f(\tilde{q})$ depends entirely on the parameters MRS and ERA, which encode society's preferences over risk and aggregate outcomes. The practical implementation of UBR therefore hinges on how these parameters are specified. This can be approached in two main ways. They can be estimated from data, for example, from the vast literature in environmental and public economics that uses methods such as contingent valuation (e.g., Arrow, Solow, Portney, Leamer, Radner, and Schuman, 1993; Carson, Flores, and Meade, 2001) or revealed preferences, which infer values from observed behavior like the housing choices in hedonic models (e.g., Rosen, 1974; see Freeman, Herriges, and Kling, 2014 for a comprehensive review). Alternatively, they can be set as explicit policy choices reflecting societal values, much like how policymakers choose social discount rates for cost-benefit analysis. While calibrating these preference parameters is necessary, a key advantage of UBR—a direct consequence of the model's linear-quadratic structure—is that it achieves the team-efficient outcome without requiring the regulator to know firms' private signals, their heterogeneous cost structures (i.e., $\pi_{aa}$), or the variance of the underlying state ($\sigma_\theta^2$). This is because the coefficients MRS and ERA are determined by the properties of the societal utility function, $\mathcal{U}(c, q)$, which are separate from the parameters governing firm costs or the information structure. This property can be viewed as a *separation result*: implementation depends only on preference parameters (MRS, ERA), while structural details drop out.

The UBR mechanism has several attractive features. Because it requires only the aggregate outcome $q$ to be observable ex post, it is both transparent and straightforward to implement. UBR thus adapts to shocks and promotes efficient behavior without requiring the regulator to collect firm-specific cost data or rely on complex revelation mechanisms.

### 2.2.1 UBR as a Synthetic Market Implementation

We have so far approached UBR from a "quantity" perspective: designing a rule that aligns each firm's private action with a socially optimal target. An alternative way to approach UBR is from a "price" perspective, relying on the Coasean insight that externalities can be internalized through well-designed markets (Coase, 1960). For this purpose, we construct a thought experiment: consider a synthetic *competitive market* for the social contribution—an environment that can be understood as the limit of a large Cournot market (Vives, 1988)—in which the "action" is a tradable good. We show that the equilibrium in this fictitious market exactly replicates the outcome induced by UBR.

We begin by deriving the market demand curve, which captures a consumer's marginal willingness to pay for the aggregate outcome, $q$. Consider a hypothetical problem where, at time $t = 1$ (when the state $\theta$ is revealed), a consumer $i$ chooses a desired level of the aggregate action, denoted $a_i^d$, at a market price $p$. Formally, each consumer solves:

$$\max_{\{c_i, a_i^d\}} U(c_i, \theta, a_i^d) \tag{25}$$

subject to the budget constraint

$$c_i + pa_i^d = e + \pi_i, \tag{26}$$

where $\pi_i$ represents the profit the consumer receives from its production activities. In this part of the thought experiment, we focus only on the household's role as a consumer; the profit $\pi_i$ is therefore treated as exogenous income, analogous to the endowment $e$.

Suppose consumers can buy their preferred level of the aggregate outcome, $a_i^d$, at a market price $p$. This requires a specific interpretation of the utility function $U(c_i, \theta, a_i^d)$ in equation (25): the function $U$ is the same as in the main model, but its third argument—the aggregate action—is now treated as a choice variable. This setup allows us to use the first-order condition to trace out a consumer's marginal willingness to pay for the aggregate outcome $q$.

Substituting the budget constraint (26) in the utility function and taking the derivative with respect to $a_i^d$ gives the familiar condition that price must equal the marginal rate of substitution:

$$p = \frac{U_A(\theta, a_i^d)}{U_c}, \tag{27}$$

where, as in Lemma 1, $U_A$ refers to the derivative with respect to the third argument of the utility function (the aggregate action), but it is evaluated at the level $a_i^d$ demanded by the consumer in this hypothetical problem.

Equation (27), derived from a consumer's ex-post decision at $t = 1$, characterizes individual demand for $a_i^d$. Because consumers are symmetric, individual demand $a_i^d$ coincides with aggregate demand, $\tilde{A}^d \equiv \int_0^1 \tilde{a}_i^d \, di$. The resulting market price is therefore a state-contingent function of this aggregate demand, which firms must forecast.

At time $t = 0$, each firm selects a quantity $a_i^s$ to produce and sell before the realization of the price $p$ at time $t = 1$. Its total payoff in this synthetic market consists of the profit,

$\pi(a_i^s)$, plus the revenue from selling its action, $\tilde{p}a_i^s$. The firm treats the future price $\tilde{p}$ and aggregate demand $\tilde{A}^d$ as exogenous, both being functions of the unobserved state $\tilde{\theta}$. Because household utility depends on the aggregate action—which in this synthetic market is $\tilde{A}^d$—the firm solves:

$$\max_{a_i^s} \mathbb{E}_i \left[ U\left(e + \pi(a_i^s) + \tilde{p}a_i^s, \tilde{\theta}, \tilde{A}^d\right)\right]. \tag{28}$$

Being infinitesimally small, each firm acts as a price-taker, treating both the market price $\tilde{p}$ and the aggregate demand $\tilde{A}^d$ as exogenous. The synthetic market clears at time $t = 1$, when the aggregate action supplied by firms, $\int a_i^s \, di$, equals the aggregate quantity demanded by households, $A^d$. This pins down the equilibrium price $\tilde{p}$. The following proposition establishes that the equilibrium allocation in this hypothetical market corresponds to the team efficient allocation achieved through UBR in Proposition 1.

**Proposition 2** (**Synthetic Market Equivalence**). *The UBR mechanism implements an allocation equivalent to that of a competitive market where firms supply "units of action" to a market whose inverse demand satisfies the function $\tilde{p} = f(\tilde{q})$ with $f(\tilde{q})$ given by equation (22) of Proposition 1. In this equilibrium:*

1. *Each firm's optimal action is the team-efficient action, $a_i^s = a_i^{te}$.*

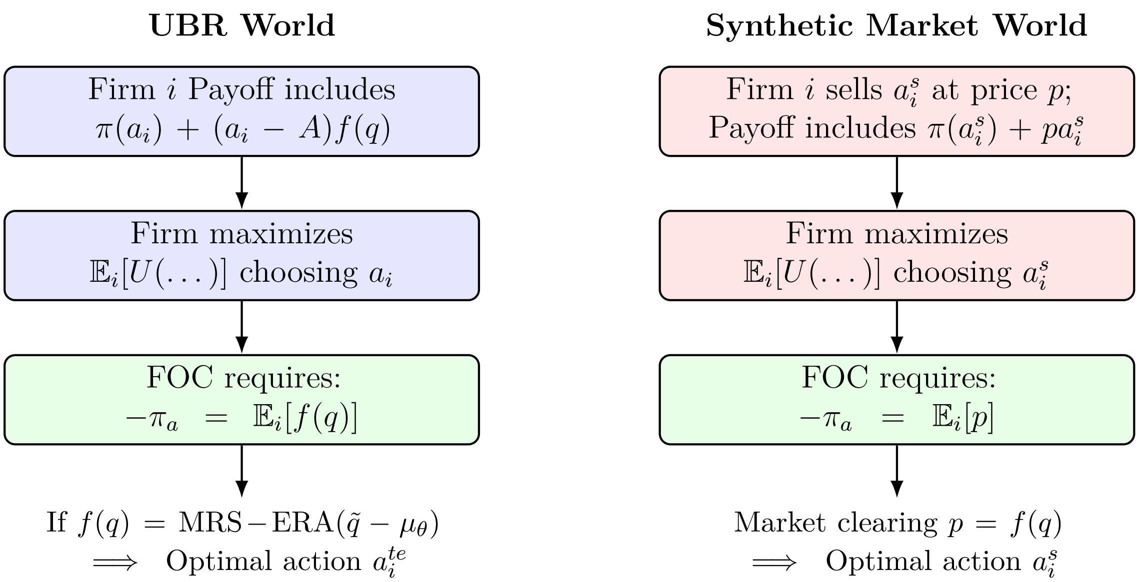
2. *The realized aggregate action is the team-efficient aggregate action, $A^{te}$.*

Proposition 2 reveals the underlying price mechanism of UBR. The regulatory function, $f(q)$, serves as the inverse demand curve in a synthetic market for action. By setting this price, the UBR mechanism induces firms to behave exactly as if they were *price-taking competitors* in this synthetic market, leading them to collectively choose the team-efficient quantity. This achieves a Coasean, market-based solution without requiring literal trade or pre-defined property rights. In so doing, UBR enacts Hayek's insight by creating the "missing market" whose quiet hand channels dispersed private information into a coherent collective outcome (Hayek, 1945).

Figure 1 illustrates the equivalence between UBR and a synthetic market. In the left panel, UBR modifies a firm's payoff with the term $(a_i - A)f(q)$, while in the right panel, a firm in the synthetic market earns revenue $pa_i^s$. In both settings, the firm's optimality condition is given by the same first-order condition (FOC): "marginal cost = expected price." Since the equilibrium price satisfies $p = f(q)$, the identical FOCs determine the optimal action in both the UBR and the Cournot worlds. The two diagrams thus present alternative representations of the same underlying incentive problem.

**Figure 1: Equivalence between UBR and a Synthetic Market**

The figure compares firm $i$'s optimization under UBR (left panel), where its payoff includes the term $(a_i - A)f(q)$ involving the shadow price $f(q)$, with its optimization in a hypothetical competitive market (right panel), where the payoff includes revenue $pa_i^s$ from market price $p$. If the market price equals the shadow price in equilibrium ($p = f(q)$), the FOCs (green boxes) align, leading to the same optimal action ($a_i^{te} = a_i^s$). (Note: in the figure we suppress tildes; $q$ denotes the realized value of $\tilde{q}$ and $A$ the realized value of $\tilde{A}$.)




## 2.3 Strategic Behavior and Information Acquisition

Having shown that UBR achieves the team-efficient allocation, we turn next to the equilibrium behavior it induces. In particular, we characterize how firms respond to their information and whether they have incentives to acquire more of it. The next proposition describes how each firm's action depends on its private and public signals.

**Proposition 3 (Firms' Equilibrium Strategy).** *Under UBR, with the regulatory function defined in Proposition 1, a linear equilibrium is for each firm $i$ to choose an action, $a_i$, that* *is a linear function of its private signal, $y_i$, and the public signal, $z$:*

$$a_i = \beta_0 + \beta_y(y_i - \mu_\theta) + \beta_z(z - \mu_\theta), \tag{29}$$

*where*

$$\beta_0 := A_0 - \frac{U_A(\mu_\theta, A_0)}{U_c \pi_{aa} + U_{AA}}, \tag{30}$$

$$\beta_y := -\frac{U_{\theta A}\tau_y}{U_c \pi_{aa}\tau + U_{AA}\tau_y} < 0, \tag{31}$$

$$\beta_z := -\frac{1}{1-\alpha}\frac{U_{\theta A}\tau_z}{U_c \pi_{aa}\tau + U_{AA}\tau_y} < 0, \tag{32}$$

*and where $\alpha < 0$ is the equilibrium degree of coordination defined in Lemma 4.*[10]

The coefficients $\beta_y$ and $\beta_z$ are negative. That is, firms raise their actions in response to either private or public signals that point to a worse underlying state. The next corollary examines how much weight firms place on each type of signal, as captured by the ratio $\beta_y/\beta_z$.

**Corollary 3.1 (Strategic substitutability and information weighting).** *In the linear equilibrium of Proposition 3, firms' actions are strategic substitutes. Moreover, the ratio of the weights placed on private and public signals is:*

$$\frac{\beta_y}{\beta_z} = (1-\alpha)\frac{\tau_y}{\tau_z} > \frac{\tau_y}{\tau_z}, \tag{33}$$

*implying that firms overweight their private information relative to the public signal, compared to the Bayesian posterior weights in equation (8).*

As established in Lemma 4, the parameter $\alpha$—which captures the nature of strategic interaction—is negative. Following Angeletos and Pavan (2007), $\alpha$ reflects the slope of a firm's best response to its expected aggregate action and represents the optimal degree of coordination. A negative $\alpha$ implies *strategic substitutability*: firms raise (lower) their action when they expect others to lower (raise) theirs. In other words, each firm has an incentive to move in the opposite direction of the crowd. This incentive to differentiate is what leads

[10]We restrict attention to linear strategies, consistent with the linear-quadratic framework. See, e.g., Angeletos and Pavan (2007, 2009) for similar approaches.

firms to place more weight on their private signal, as shown in the corollary. The idea that strategic motives shape information use is central to the literature on coordination games, beginning with the seminal work of Morris and Shin (2002), extended by Angeletos and Pavan (2007), and explored at length in Veldkamp (2011).

While strategic substitutability might suggest a free-rider problem—firms underinvesting in socially valuable actions because they rely on others to act—it has a key offsetting benefit: it strengthens the role of private information in firms' decisions. This happens because each firm puts more weight on its own signal than it would under standard Bayesian updating, due to the $(1 - \alpha) > 1$ term in the ratio $\beta_y/\beta_z$. As we show next, this makes the aggregate response more closely tied to the true state, in the spirit of Hayek's knowledge-aggregation via prices (Hayek, 1945).

### 2.3.1 Aggregate Action and its Determinants

We now examine the aggregate implications of firms' equilibrium actions, focusing on a key feature of UBR: its ability to transform dispersed information into a collective response that tracks the underlying state of the economy. Substituting the equilibrium expression for $a_i$ from Proposition 3 into the definition of the aggregate action, $\tilde{A} = \int_0^1 a_i \, di$, and noting that idiosyncratic private noise averages out across firms ($\int_0^1 (y_i - \mu_\theta) \, di = \tilde{\theta} - \mu_\theta$ by the Law of Large Numbers), we obtain the aggregate action:

$$\tilde{A} = \beta_0 + \beta_y(\tilde{\theta} - \mu_\theta) + \beta_z(z - \mu_\theta). \tag{34}$$

To better understand the components of the aggregate action, we use the definition of the public signal ($z = \tilde{\theta} + \tilde{\varepsilon}_z$) to rewrite this as:

$$\tilde{A} = \beta_0 + (\beta_y + \beta_z)(\tilde{\theta} - \mu_\theta) + \beta_z \tilde{\varepsilon}_z. \tag{35}$$

This decomposition reveals two distinct components: a fundamental part that reflects the true state, $\tilde{\theta}$, and a noise-driven part that reflects errors in the public signal, $\tilde{\varepsilon}_z$. UBR succeeds in channeling private information so that the aggregate action adjusts in response to shocks in the underlying state (since $\beta_y + \beta_z < 0$). This desirable feature, however, entails a tradeoff: the aggregate action also responds to public noise.

We define the *information sensitivity* of the aggregate action, denoted by $\mathcal{B}$, as the magnitude of its response to the true underlying state:

$$\mathcal{B} := |\beta_y + \beta_z|. \tag{36}$$

This measure captures how strongly the aggregate action responds to available information about the state. A higher $\mathcal{B}$ means that UBR translates dispersed private and public signals into a collective action more closely aligned with the (unobserved) state of the world. This improved alignment contrasts sharply with the unresponsive status quo. The next corollary identifies the key determinants of $\mathcal{B}$.

**Corollary 3.2** (**Information sensitivity**). *The information sensitivity, $\mathcal{B}$, of the aggregate action increases with:*

*(a) the level of uncertainty about $\tilde{\theta}$ ($\sigma_\theta$).*

*(b) the precision of private information ($\tau_y$).*

*(c) the precision of public information ($\tau_z$).*

Point (a) of Corollary 3.2 highlights a key feature of UBR: it turns uncertainty about the state—typically a challenge for policy—into a force that sharpens the system's response to information. A higher $\sigma_\theta$ makes the aggregate action more responsive to informative signals, improving the overall adjustment to the underlying state.

Points (b) and (c) of Corollary 3.2 further clarify how UBR channels information into aggregate behavior. Increased precision of either private information ($\tau_y$) or public information ($\tau_z$) also raises the information sensitivity, $\mathcal{B}$. While seemingly distinct from the effect of uncertainty in (a), these results are complementary. Higher $\tau_y$ and $\tau_z$ increase the weights, $\beta_y$ and $\beta_z$ respectively, placed on the corresponding signals in the equation for the aggregate action (34). Thus, both greater uncertainty (higher $\sigma_\theta$) and greater precision (higher $\tau_y$ or $\tau_z$) enhance the information sensitivity of the system, albeit through different channels.

### 2.3.2 Incentives for Information Acquisition

We now ask whether individual firms that are subject to UBR have an incentive to acquire more precise private information about the underlying state, $\tilde{\theta}$. To isolate this incentive, suppose one firm can, unilaterally and at zero cost, improve the precision of its private signal. This allows us to focus solely on the benefit of information acquisition.

Suppose firm $i$ improves the precision of its private signal $y_i$ from $\tau_y$ to $\tau_y' = \xi\tau_y$, with $\xi > 1$. We treat this as a unilateral deviation: the firm holds fixed the actions of others and the parameters of the UBR mechanism. The next corollary characterizes how this affects the firm's action and its ex-ante expected utility, evaluated *after* the public signal $z$ is observed, but *before* the private signal is realized. Conditioning on $z$ lets us isolate the incremental value of improved private information, over and above what is already publicly known.

**Corollary 3.3** (**Incentives for Information Acquisition**). *Under the UBR mechanism of Proposition 1, consider a unilateral increase in the precision of firm i's private signal about $\tilde{\theta}$ from $\tau_y$ to $\tau_y' = \xi\tau_y$, where $\xi > 1$. The firm's action, $a_i'$, is then given by:*

$$a_i' = \beta_0 + \beta_y \left[1 + \frac{(\xi - 1)(1 + \sigma_\theta^2 \tau_z)}{1 + \sigma_\theta^2(\xi\tau_y + \tau_z)}\right](y_i - \mu_\theta) + \beta_z \left[1 - \frac{(\xi - 1)(1 - \alpha)\sigma_\theta^2 \tau_y}{1 + \sigma_\theta^2(\xi\tau_y + \tau_z)}\right](z - \mu_\theta), \quad (37)$$

*where the coefficients $\beta_0$, $\beta_y$, and $\beta_z$ are defined in Proposition 3. The resulting change in ex-ante expected utility, conditional on the public signal $z$, is:*

$$\Delta\mathbb{E}[U|z] = -\frac{U_c}{2\pi_{aa}} \left( Var[\mathbb{E}_i'[f(\tilde{q})]|z] - Var[\mathbb{E}_i[f(\tilde{q})]|z] \right) > 0, \quad (38)$$

*where $\mathbb{E}_i'[f(\tilde{q})]$ denotes the expectation of $f(\tilde{q})$ under the more precise private signal. Consequently, the firm's ex-ante expected utility rises with signal precision.*

This corollary highlights a central feature of UBR: it gives firms a clear incentive to improve their private information. Because the payoff depends on the realized aggregate outcome, $\tilde{q}$, through the shadow price $f(\tilde{q})$, more accurate forecasting leads to actions that yield higher expected utility. Since the gain is strictly positive for any realization of $z$, the incentive to acquire better information holds unconditionally.

This marks a departure from the status quo. Under the status quo, firms lack any reason to improve their information, because their payoffs do not depend on the realized state. UBR changes this by linking payoffs to outcomes. It not only supports the team-efficient allocation but also motivates firms to acquire better information. It aligns private incentives with public goals and gives firms a reason to know more.

This finding connects our work to a key insight from the literature on endogenous information acquisition: the nature of strategic interaction determines whether agents want to "know what others know" (Hellwig and Veldkamp, 2009; Myatt and Wallace, 2012). When actions are strategic complements, agents prefer common information to better coordinate their actions. When actions are strategic substitutes, they instead prefer private information to better differentiate themselves. Because UBR endogenously creates an environment of strategic substitutability (Corollary 3.1), the incentive it provides for acquiring *private* signals (Corollary 3.3) is a direct consequence of this principle. Our mechanism is thus not only robust to endogenous information choice but also naturally promotes the decentralized knowledge acquisition essential for an efficient aggregate response.

# 3 Welfare Analysis and Political Viability of UBR

We now evaluate UBR's welfare effects. We first compare it to the outcome chosen by a social planner with limited information, then examine its political feasibility.

## 3.1 The Social Planner's Allocation

Consider a benevolent social planner whose information set consists solely of publicly available information—the prior distribution of the state $\tilde{\theta}$ and the public signal $z$. Because the planner cannot observe firms' private signals, the only feasible policy is to set a uniform action, $a_{sp}$, for all firms. Setting policy based on expectations formed from public information is the standard approach for a regulator facing uncertainty (Lemoine, 2021). This allocation serves as a natural benchmark for evaluating UBR.

The planner's problem is to choose the action $a_{sp}$ that maximizes ex-ante expected social welfare:

$$\max_{a_{sp}} \mathbb{E}_{sp}\left[U(c(a_{sp}), \tilde{\theta}, a_{sp})\right], \tag{39}$$

where $c(a_{sp}) = e + \pi(a_{sp})$ and the expectation is over the planner's information set.

**Proposition 4** (**Social Planner's solution**). *Let $a^*(\theta)$ be the first-best action from Lemma 3. The solution to the social planner's problem is the expected first-best action, conditional on public information:*

$$a_{sp} = \mathbb{E}_{sp}[a^*(\theta)] = A_0 - \frac{\mathbb{E}_{sp}[U_A(\tilde{\theta}, A_0)]}{U_c\pi_{aa} + U_{AA}}. \tag{40}$$

The social planner's optimal action, $a_{sp}$, reflects the best achievable outcome given the absence of firms' private information. This allocation improves upon the status quo ($a_i = A_0$), since at $A_0$, the marginal social benefit of action, $U_A(\mu_\theta, A_0)$, is generally non-zero. By adjusting away from $A_0$, the planner moves the economy closer to a point where this marginal benefit is internalized. For the remainder of this section, without loss of generality, we focus on the case of a positive externality. We therefore assume that the marginal social benefit at the status quo is positive ($U_A(\mu_\theta, A_0) > 0$), which by equation (40) implies that $a_{sp} > A_0$.

### 3.1.1 Application: Implementation via Standard Instruments

To illustrate how the social planner's allocation maps into real-world instruments, we show that $a_{sp}$ can be implemented through either a Pigouvian tax or a cap-and-trade system, as in the classic "prices-versus-quantities" framework of Weitzman (1974). We consider an environmental application in which a firm's action, $a_i$, represents its level of "greenness" or abatement effort. Let $\mathcal{E}(a_i)$ denote the emissions produced by a firm choosing $a_i$, with $\mathcal{E}(a_i) > 0$, $\mathcal{E}'(a_i) < 0$, and $\mathcal{E}''(a_i) > 0$. That is, greater abatement reduces emissions, but at a diminishing rate. The next corollary shows how the planner's allocation in Proposition 4 can be implemented using a standard Pigouvian tax on emissions.

**Corollary 4.1** (**Pigouvian tax on emissions**). *The social planner's optimal uniform* *action, $a_{sp}$, can be implemented by imposing a Pigouvian tax, $T$, per unit of emissions:*

$$T = \frac{(a_{sp} - A_0)\pi_{aa}}{\mathcal{E}'(a_{sp})} > 0. \tag{41}$$

*When the action generates a positive externality, $a_{sp} > A_0$, the emissions tax is strictly positive.*

Intuitively, the tax internalizes the expected externality as perceived by the social planner. When a firm considers deviating its action from $a_{sp}$, it must now account for the tax on the resulting emissions. Because $T$ depends solely on public information, all firms respond uniformly by choosing $a_{sp}$, and the aggregate emissions coincide with those in the planner's constrained optimum, as derived in Proposition 4. This is consistent with the classic insight that the regulator should set the tax equal to the expected marginal social cost of emissions under uncertainty (see Lemoine, 2021).

An alternative approach to implementing the social planner's solution is through a system of tradable permits, or "cap-and-trade." As before, let each firm's emissions be given by $\mathcal{E}(a_i)$, and suppose the regulator issues a total quantity of permits, $Q$, where each permit allows one unit of emissions. Firms must acquire permits equal to the emissions they produce, and the equilibrium permit price is denoted by $p_{permit}$.[11]

**Corollary 4.2 (Cap-and-Trade implementation).** *The social planner's optimal uniform action, $a_{sp}$, can be implemented by issuing a quantity of tradable permits, $Q$, equal to the aggregate emissions associated with $a_{sp}$, i.e., $Q = \mathcal{E}(a_{sp})$. The resulting equilibrium permit price, $p_{permit}$, will be identical to the Pigouvian tax, $T$, derived in Corollary 4.1, and will induce all firms to choose $a_i = a_{sp}$.*

By setting $Q = \mathcal{E}(a_{sp})$, the regulator ensures that the equilibrium permit price—identical to the Pigouvian tax $T$ from Corollary 4.1—induces all firms to choose the social planner's action $a_{sp}$. This equivalence arises because the profit function $\pi(\cdot)$ and the emissions technology

[11] We note that the term $\mathcal{R}$ in equation (19) admits a cap-and-trade interpretation: each firm's net position $a_i - A$ plays the role of tradable permits, and $f(q)$ the state-contingent permit price. Crucially, however, both the aggregate "cap" $A$ and the price schedule $f(q)$ are determined *ex post* by the realized aggregate outcome $q$, rather than fixed *ex ante* by the regulator.

$\mathcal{E}(\cdot)$ are common knowledge and deterministic, so there is no firm-specific cost uncertainty. Consequently, the classic *"price vs. quantity"* wedge identified by Weitzman (1974) does not apply. Since uncertainty arises solely on the benefit side—through the state $\tilde{\theta}$—it does not affect the choice or level of the instrument needed to implement $a_{sp}$.

Critically, however, neither $Q$ nor $p_{permit}$ can depend on firms' private information. Both instruments are based solely on public information, which precludes the firm-specific, information-contingent responses required for team efficiency. Unlike these standard instruments, UBR harnesses uncertainty and dispersed private signals, thereby generating social value from private information and improving welfare outcomes (see, e.g., Angeletos and Pavan, 2007; Vives, 1988; Hayek, 1945). Given these theoretical advantages, a natural question arises concerning the political viability of UBR, which we analyze next.

## 3.2 Is UBR Politically Viable?

Suppose households, after observing their private signals $y_i$ and the public signal $z$, must vote on whether to adopt UBR or implement the social planner's allocation. The following proposition establishes the political viability of UBR.

**Proposition 5** (**Political viability of UBR**). *Consider two regulatory platforms: UBR, from Proposition 1 and the social planner solution, from Proposition 4. Let $\mathbb{E}_i[\Delta U_i]$ denote agent $i$'s expected utility gain from implementing UBR instead of the social planner's solution, conditional on the information set $\{y_i, z\}$. Then the expected utility of every agent $i$ is higher under UBR than under the social planner's solution, that is,*

$$\mathbb{E}_i[\Delta U_i] = \underbrace{-\frac{1}{2}U_c\pi_{aa}(a_i^{te} - a_{sp})^2}_{\text{Gain from Individual Flexibility}} + \underbrace{\left(-\frac{1}{2}U_{AA}\right)\mathbb{E}_i[(\tilde{A}^{te} - a_{sp})^2]}_{\text{Gain from Aggregate Adaptiveness}} > 0. \tag{42}$$

*Hence, if asked to vote between the two platforms, agents would unanimously support UBR.*

Crucially, the utility gain in Proposition 5 is evaluated from each household's perspective, that is, after observing their own private signal. This is a stronger criterion than the ex-ante welfare gain considered in the social planner's problem (Proposition 4), as it requires each individual to prefer UBR based on their own information. This stronger condition underpins UBR's ability to garner unanimous political support.

The expected utility gain in equation (42) arises from two distinct sources of value that UBR creates, both of which are inaccessible under a standard policy. The first, *Gain from Individual Flexibility*, reflects the benefit of allowing each firm to use its private signal to choose an action tailored to its own information, rather than conforming to the planner's one-size-fits-all decision. The second, *Gain from Aggregate Adaptiveness*, captures the value of letting the collective action respond to the true underlying state of the world, thereby providing a form of social insurance against aggregate risk that a fixed policy cannot offer.

Since both sources of utility gain in equation (42) are non-negative, and their sum is strictly positive, regardless of the specific values of $y_i$ and $z$, all households strictly prefer UBR over the social planner's allocation. This unanimous preference holds despite dispersed private information and heterogeneous beliefs. It suggests that UBR is politically viable.

# 4 Distrust and UBR

Thus far, our analysis has assumed that households have rational expectations and fully trust the informational content of others' signals. We now relax this assumption by introducing the possibility of *distrust*—that is, agents may question the reliability of information held by others. This builds on the literature examining how agents may differ in their beliefs about others' information (Banerjee, 2011), and is conceptually related to the "sentiment risk" channel in Dumas et al. (2009). Our goal is to explore the robustness of UBR when agents hold divergent, and potentially biased, views about the informational environment. This issue is especially relevant in domains like climate policy, where public opinion is polarized and trust in scientific data varies significantly.[12]

To model heterogeneous beliefs about others' information, we build on Morris (1995) and Banerjee (2011) by introducing the notion of *trust* in others' private signals. Each agent

[12]Research documents significant political polarization in public views on climate change, highlighting differing beliefs about environmental data across groups (McCright and Dunlap, 2011). For further evidence on polarized beliefs and skepticism about environmental data, see Dunlap and McCright (2008) and Douglas, Uscinski, Sutton, Cichocka, Nefes, Ang, and Deravi (2019).

$i$ observes its own private signal $y_i$, but forms beliefs about the structure of other agents' signals $y_j$ ($j \neq i$), which it does not observe. Specifically, agent $i$ believes that the signal of any other agent $j \neq i$ is given by

$$y_j^i = \mu_\theta + \varphi(\tilde{\theta} - \mu_\theta) + \sqrt{1-\varphi^2}\tilde{\phi}_i + \varepsilon_{y,j}, \tag{43}$$

where $\tilde{\phi}_i \sim \mathcal{N}(0, \sigma_\theta^2)$ represents noise or distortion in others' signals. For $j = i$ we write $y_i^i \equiv y_i$ and set $\varphi = 1$, so each agent fully trusts its own signal. The trust parameter $\varphi \in [0, 1]$ for others' signals ($j \neq i$) captures the degree of trust, with $\varphi = 1$ meaning full trust and $\varphi = 0$ meaning complete distrust (pure noise). Agent $i$ does not observe $\tilde{\phi}_i$, but believes it exists and shapes others' information. Importantly, the distribution of $\tilde{\phi}_i$ is constructed so that the unconditional moments of $y_j^i$, from agent $i$'s perspective, are independent of $\varphi$: that is, $\mathbb{E}_i[y_j^i] = \mu_\theta$ and $\mathrm{Var}_i[y_j^i] = \sigma_\theta^2 + \tau_y^{-1}$. This ensures that changes in $\varphi$ reflect beliefs about correlation with the state, not signal precision.[13]

By varying the parameter $\varphi$, the model nests both the standard rational expectations benchmark ($\varphi = 1$) and settings with limited trust ($\varphi < 1$). This allows us to examine the robustness of UBR when agents hold different views about the reliability of others' information. We now characterize equilibrium under this modified belief structure.

**Impact on strategy and information.** Distrust alters the information environment, with implications for equilibrium actions, the information sensitivity $\mathcal{B}$, and firms' incentives to acquire private information. While several results from Section 2 continue to hold, distrust introduces novel effects. Crucially, the definitions of ex-ante expected social welfare (equation (14)) and the team-efficient allocation (equation (15) and its constraint) remain unchanged, as they are grounded in the true distribution of signals rather than agents' subjective beliefs. The main results are summarized in Proposition 6. Appendix B provides a detailed discussion of how each result from Section 2 is affected by distrust.

[13]Our main analysis focuses on the case where agents believe others' signals are noisy but unbiased (i.e., $\mathbb{E}_i[\tilde{\phi}_i] = 0$). Two valuable extensions are left for future work. First, one could allow distrust in the public signal $z$. We conjecture that UBR would remain effective in such a setting, as it continues to aggregate dispersed private beliefs. Second, one could model systematic bias in private signals, where an agent believes others are, on average, overly optimistic or pessimistic ($\mathbb{E}_i[\tilde{\phi}_i] \neq 0$).

**Proposition 6.** *Suppose that agents distrust the information of others, i.e., $i$ believes that the signal $y_j^i$ of any other agent $j \neq i$ is given by equation (43). Then:*

*(a) UBR implemented through the function $f(\tilde{q})$ defined in Proposition 1 achieves a team-efficient allocation.*

*(b) Each firm $i$ chooses an action $a_i$ that is a linear function of its private signal $y_i$ and the public signal $z$:*

$$a_i = \widehat{\beta}_0 + \widehat{\beta}_y(y_i - \mu_\theta) + \widehat{\beta}_z(z - \mu_\theta), \tag{44}$$

*where*

$$\widehat{\beta}_0 := A_0 - \frac{U_A(\mu_\theta, A_0)}{U_c\pi_{aa} + U_{AA}}, \tag{45}$$

$$\widehat{\beta}_y := -\frac{U_{\theta A}\tau_y}{U_c\pi_{aa}\tau + U_{AA}\tau_y\varphi} < 0, \tag{46}$$

$$\widehat{\beta}_z := -\frac{1}{1-\alpha}\frac{U_{\theta A}\tau_z}{U_c\pi_{aa}\tau + U_{AA}\tau_y\varphi} < 0, \tag{47}$$

*(c) The information sensitivity $\mathcal{B}$ increases with the precision of public information $\tau_z$ if $\varphi > 1 - \frac{1}{(-\alpha)\sigma_\theta^2\tau_y}$.*

*(d) The information sensitivity $\mathcal{B}$ increases with distrust, meaning it decreases with $\varphi$:*

$$\frac{\partial \mathcal{B}}{\partial \varphi} = -\frac{\alpha\kappa_1^*\sigma_\theta^4\tau_y(\tau_y - \alpha\tau_y + \tau_z)}{(1 + \sigma_\theta^2(\tau_y + \tau_z - \alpha\tau_y\varphi))^2} < 0. \tag{48}$$

*(e) Incentives for information acquisition increase with the degree of distrust.*

Point (a) highlights the robustness of UBR's design. Although distrust alters equilibrium behavior, the functional form of the payment function $f(\tilde{q})$ remains optimal for achieving team efficiency. Equation (44) shows that, in equilibrium, each firm's action $a_i$ is a linear function of its private signal $y_i$ and the public signal $z$. Setting $\varphi = 1$ (full trust) recovers the coefficients from Proposition 3.

Point (b) shows that lower trust (smaller $\varphi$) has several effects. It increases the magnitudes of $\widehat{\beta}_y$ and $\widehat{\beta}_z$, making firms more responsive to their private and public signals, respectively.

This seemingly counterintuitive result arises because lower trust leads each firm to believe that competitors' private signals—and thus their actions—are less correlated with the true state $\tilde{\theta}$ and more influenced by noise. Given strategic substitutability ($\alpha < 0$), a firm that expects others to act less in line with fundamentals relies more heavily on the signals it trusts: its own private signal and the public signal. As a result, the absolute values of $\widehat{\beta}_y$ and $\widehat{\beta}_z$ rise.

Points (c) through (e) show how distrust reinforces the role of private information in three distinct ways. First, the impact of public information precision ($\tau_z$) on information sensitivity ($\mathcal{B}$) becomes conditional on trust: $\tau_z$ increases $\mathcal{B}$ only when $\varphi$ exceeds a critical threshold. Second, distrust directly raises information sensitivity, as firms place more weight on their own signals. Third, it strengthens incentives for information acquisition, since the strategic value of private signals increases. Ultimately, distrust amplifies the importance of private signals across all dimensions of firm behavior.

**Impact on Welfare and Political Viability.** We now examine how distrust affects welfare and the political viability of UBR. Since UBR achieves team efficiency for any level of trust (Proposition 6), it continues to dominate the social planner's allocation in terms of ex-ante welfare. The next proposition confirms that these welfare gains translate into unanimous support.

**Proposition 7.** *Under UBR, and for any trust parameter $\varphi \in [0, 1]$, all households strictly prefer UBR to the social planner's allocation.*

In summary, while distrust introduces complexities into the equilibrium, it does not undermine the fundamental advantages of UBR. The mechanism's design—in particular, the functional form of $f(\tilde{q})$—remains optimal for achieving team efficiency regardless of the level of trust. Moreover, UBR continues to dominate the social planner's allocation, preserving its political viability. Distrust does, however, affect firm behavior: it increases information sensitivity by inducing greater reliance on private signals, and it strengthens incentives for information acquisition. Thus, UBR remains a robust and effective regulatory tool even in the presence of significant disagreement among agents.

# 5 Illustrative Example: Cybersecurity

We apply our framework to a stylized model of cybersecurity investment. Firms' defensive efforts generate a positive externality by improving network security and lowering systemic risk. This setting illustrates how UBR can create incentives for coordination in a context where firms would otherwise act in isolation.

**An Economy with Cybersecurity Externalities.** Consider a continuum of firms, each choosing a cybersecurity investment $a_i$. A firm's profit is $\pi(a_i) = \tilde{\theta} a_i - \frac{1}{2}a_i^2$, where the common shock $\tilde{\theta}$ measures the baseline intensity of cyber threats. For simplicity, we set the endowment $e = 0$, so the household's consumption $c_i$ is equal to the firm's profit.

The key externality is that total investment, $\tilde{A} = \int_0^1 a_i \, di$, creates the public good of "network security," modeled as $\tilde{q} = \tilde{A}$. For pedagogical clarity, this specification isolates the coordination effects that emerge from dispersed private information about the threat environment $\tilde{\theta}$, in contrast to our general model where the aggregate outcome also depends directly on the fundamental shock. Society values this good through the utility function:

$$\mathcal{U}(c_i, q) = c_i + \beta q - \frac{\delta}{2}(q - \mu_q)^2, \tag{49}$$

where $\beta > 0$ is the baseline social value of security, $\delta > 0$ captures aversion to risk in outcomes, and $\mu_q \equiv \mathbb{E}[\tilde{q}] = \mathbb{E}[\tilde{A}]$. This setup maps to our general model with: $U_c = 1$, $\pi_{aa} = -1$, $\text{MRS} = \beta$, and $\text{ERA} = \delta$.

**Diagnosing the Inefficiency.** In the unregulated market, each firm chooses its security investment $a_i$ to maximize expected profit. The first-order condition yields:

$$a_i = \mathbb{E}_i[\tilde{\theta}]. \tag{50}$$

This choice is independent of total investment $\tilde{A}$, so there is no strategic interaction—the *status quo* equilibrium coordination parameter is $\alpha_{sq} = 0$.

We compare the unregulated market to the efficient benchmark by computing the *team-*

*efficient* degree of coordination, $\alpha_{te}$, from Lemma 4. A direct calculation yields:[14]

$$\alpha_{te} = -\delta. \tag{51}$$

Strategic substitutability ($\alpha < 0$) is optimal because society is averse to volatility in outcomes ($\delta > 0$). While the externality term $\beta q$ makes investment valuable, risk aversion penalizes variability in the aggregate outcome $\tilde{q}$. This variability stems from correlated individual actions.[15] Strategic substitutability gives firms a reason to lean on their private signals rather than mimic others, thereby reducing correlation. The unregulated market provides no such incentive ($\alpha_{sq} = 0$), while the optimum requires it ($\alpha_{te} = -\delta$).

**The UBR Solution and Policy Implications.** UBR restores the missing strategic link by adding a transfer $(a_i - \tilde{A})f(\tilde{q})$ to each firm's payoff, with pricing rule $f(\tilde{q}) = \beta - \delta(\tilde{q} - \mu_q)$. Here, $\beta$ and $\delta$ correspond to the marginal rate of substitution (MRS) and exposure risk aversion (ERA), respectively, as in Proposition 1. The first-order condition for a firm that maximizes expected payoff is $\mathbb{E}_i[\tilde{\theta} - a_i + f(\tilde{q})] = 0$, which yields:

$$a_i = \mathbb{E}_i[\tilde{\theta}] + \beta - \delta(\mathbb{E}_i[\tilde{A}] - \mu_q). \tag{52}$$

The coordination parameter is therefore $\alpha_{\text{UBR}} = -\delta$, which matches the team-efficient value $\alpha_{te} = -\delta$ and restores efficiency.

This result is a direct application of the team-efficient condition in Lemma 4. In this example, the first-best action is $a^*(\theta) = (\theta + \beta + \delta\mu_q)/(1+\delta)$. Substituting this expression and the optimal coordination parameter $\alpha = -\delta$ into the general formula $a_i^{te} = (1-\alpha)\mathbb{E}_i[a^*(\theta)] + \alpha\mathbb{E}_i[\tilde{A}^{te}]$ recovers precisely the firm's equilibrium action under UBR. This confirms the mechanism's efficiency and clarifies its strategic channel—each firm's action adjusts based on how its forecast of the aggregate action, $\mathbb{E}_i[\tilde{A}]$, deviates from the public mean, $\mu_q = \mathbb{E}[\tilde{A}]$.

This application offers direct policy insights. If society is risk-neutral ($\delta = 0$), optimal coordination is zero ($\alpha_{te} = 0$), and UBR acts as a Pigouvian subsidy ($\beta$) that raises investment

[14] As defined in Lemma 4, $\alpha := -U_{AA}/(U_c\pi_{aa})$. In this example, where $q = A$, we have $U_{AA} = -\delta$, $U_c = 1$, and $\pi_{aa} = -1$, which gives $\alpha = -(-\delta)/(1 \cdot -1) = -\delta$.

[15] For a finite number of firms $N$, the variance of the average action $A_N = \frac{1}{N}\sum a_i$ is $\text{Var}(A_N) = \frac{1}{N}\text{Var}(a_i) + \left(1 - \frac{1}{N}\right)\text{Cov}(a_i, a_j)$ under symmetry. As $N \to \infty$, the first term vanishes, so aggregate volatility equals $\text{Cov}(a_i, a_j)$.

without altering strategic behavior. If society is risk-averse ($\delta > 0$), UBR then induces strategic substitutability to reduce aggregate volatility. This distinction echoes real-world debates—for instance, between imposing uniform cybersecurity standards, which can heighten correlated risks (Kunreuther and Heal, 2003), and fostering a *diversity of defense* to build resilience (Jajodia, Ghosh, Swarup, Wang, and Wang, 2011). Finally, as in our general model (Corollary 3.2), the information-sensitivity index $\mathcal{B}$ rises with the prior standard deviation $\sigma_\theta$.[16]

# 6 Conclusion

How can societies effectively regulate complex externalities amidst pervasive uncertainty and disagreement? We argue the solution lies not in waiting for consensus, but in designing policies that leverage these forces directly. We develop a regulatory mechanism that aligns decentralized decisions with the team-efficient outcome by creating a synthetic competitive market for the externality. This approach harnesses the power of dispersed private information to turn uncertainty and disagreement from barriers into catalysts for efficient coordination, all without requiring direct disclosure.

Although our framework is broadly applicable, it is particularly relevant in domains where externalities interact with dispersed and privately held information. Examples include climate policy, cybersecurity, public health, financial stability and artificial intelligence—settings where uncertainty and disagreement are pervasive and where centralized control is often impractical. In each case, linking individual payoffs to deviations from an aggregate benchmark channels disagreement and uncertainty into productive coordination, allowing firms to respond to implicit prices that emerge from others' actions.

For simplicity, we assume that each household owns a single firm, isolating the core incentives behind our regulatory mechanism. A natural next step is to incorporate financial markets, which aggregate dispersed information across investors and generate price signals that can complement or reinforce the mechanism's effects. These signals may amplify incentives for

[16] This follows from Corollary 3.2. In this cybersecurity example, $\alpha = -\delta$ and $\kappa_1^* = -\delta/(1+\delta)$. Substituting into the general formula $\mathcal{B} = \kappa_1^* \left( \frac{1}{1+\sigma_\theta^2(\tau_y(1-\alpha)+\tau_z)} - 1 \right)$ yields $\mathcal{B} = -\frac{\delta}{1+\delta} \left( \frac{1}{1+\sigma_\theta^2(\tau_y(1+\delta)+\tau_z)} - 1 \right)$. Since $\frac{\partial \mathcal{B}}{\partial \sigma_\theta} = \frac{2\sigma_\theta \delta(\tau_y(1+\delta)+\tau_z)}{(1+\delta)(1+\sigma_\theta^2(\tau_y(1+\delta)+\tau_z))^2} > 0$, information sensitivity increases with prior uncertainty.

firms to pursue socially beneficial investments, such as green innovation or robustness against systemic risk. Portfolio mandates—an increasingly discussed policy tool—could further shape how these financial signals influence firms' behavior. Studying the interaction between our mechanism and asset prices offers a promising direction for future research.

By showing how uncertainty and disagreement can be managed productively, our work contributes to the broader discussion on governance in complex environments. Rather than treating imperfect information as a barrier, we show it can support market-based regulatory solutions. Outcome-responsive mechanisms that coordinate agents through endogenous incentives can achieve decentralized efficiency, even when central authorities lack detailed knowledge and agents disagree. The future of regulation, therefore, may lie not in overcoming uncertainty and disagreement, but in harnessing them to elicit and aggregate dispersed information.

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# A Proofs

## A.1 Proof of Lemma 1

The properties listed in the lemma follow directly from the Taylor expansion of the utility function $\mathcal{U}(c_i, q)$ around the point $(c_0, \mu_\theta)$, as given in equation (3), and the definition of the aggregate outcome $q$ in equation (2).

The approximated utility function is provided in (3). We prove each property in turn.

(a) $U_c = \mathcal{U}_c(c_0, \mu_\theta) > 0$ *is a positive constant.* The approximation of $\mathcal{U}$ is first-order (linear) in consumption $c_i$. Taking the partial derivative of $U$ with respect to $c_i$ yields $U_c = \mathcal{U}_c(c_0, \mu_\theta)$, which is a constant evaluated at the expansion point. By assumption, utility is increasing in consumption, so this constant is positive.

(b) $U_A(\tilde{\theta}, \tilde{A})$ *is linear in its arguments.* The aggregate action $\tilde{A}$ affects utility through the aggregate outcome $\tilde{q}$. By the chain rule and equation (2),

$$U_A = \frac{\partial U}{\partial q} = \mathcal{U}_q(c_0, \mu_\theta) + \mathcal{U}_{qq}(c_0, \mu_\theta)(\tilde{\theta} + \tilde{A} - A_0 - \mu_\theta).$$

Since $\mathcal{U}_q(c_0, \mu_\theta)$ and $\mathcal{U}_{qq}(c_0, \mu_\theta)$ are constants, this expression is linear in $\tilde{\theta}$ and $\tilde{A}$.

(c) $U_{AA} = \mathcal{U}_{qq}(c_0, \mu_\theta) < 0$ *is a negative constant.* Taking the partial derivative of $U_A$ from the previous point with respect to $\tilde{A}$ yields $U_{AA} = \mathcal{U}_{qq}(c_0, \mu_\theta)$. By assumption, the marginal utility of the aggregate outcome is declining, so $U_{AA} < 0$.

(d) $U_{cA} = 0$. The Taylor expansion in equation (3) contains no cross-terms between consumption and the aggregate outcome, so the cross-partial derivative $U_{cA}$ is zero.

(e) $U_{\theta A} = \mathcal{U}_{qq}(c_0, \mu_\theta) < 0$ *is a negative constant.* Taking the partial derivative of $U_A$ from point (b) with respect to $\tilde{\theta}$ yields $U_{\theta A} = \mathcal{U}_{qq}(c_0, \mu_\theta)$, which is negative as established in point (c). □

## A.2 Proof of Lemma 2

In the status quo, household $i$ maximizes its expected utility:

$$\max_{a_i} \mathbb{E}_i[U(c_i, \tilde{\theta}, \tilde{A})], \tag{A1}$$

subject to $c_i = e + \pi(a_i)$. Because household $i$ is infinitesimal, its choice of $a_i$ does not affect the aggregate action $\tilde{A} = \int_0^1 a_i di$. Therefore, $\tilde{A}$ can be treated as a constant with respect to the maximization over $a_i$.

Substituting the constraint into the objective and taking the first-order condition with respect to $a_i$ leads to:

$$\mathbb{E}_i \left[ U_c(e + \pi(a_i), \tilde{\theta}, \tilde{A}) \cdot \pi_a(a_i) \right] = 0. \tag{A2}$$

By Lemma 1 (a), $U_c$ is a positive constant. Therefore, the first-order condition simplifies to:

$$\pi_a(a_i) = 0. \tag{A3}$$

By assumption, the profit function $\pi(a_i)$ is maximized at $a_i = A_0$. Therefore, $a_i^{sq} = A_0$. Since all households are identical ex-ante and face the same optimization problem, they all choose the same action. Aggregate action is then:

$$\tilde{A}^{sq} = \int_0^1 a_i^{sq} di = \int_0^1 A_0 di = A_0. \quad \square \tag{A4}$$

## A.3 Proof of Lemma 3

The perfectly informed social planner maximizes social welfare, while internalizing the externality. Due to household homogeneity, $a_i = a$ for all $i$, and thus $\tilde{A} = a$. The planner's problem is then:

$$\max_a U(c(a), \theta, A(a)), \tag{A5}$$

subject to $c(a) = e + \pi(a)$ and $A(a) = a$. Substituting the constraints, we have:

$$\max_a U(e + \pi(a), \theta, a). \tag{A6}$$

The first-order condition with respect to $a$ is:

$$\frac{dU}{da} = U_c \pi_a(a) + U_A(\theta, a) = 0. \tag{A7}$$

By Lemma 1 (a), $U_c$ is a constant. The quadratic profit function, maximized at $A_0$, implies $\pi_a(a) = \pi_{aa}(a - A_0)$. Applying a first-order Taylor expansion to $U_A(\theta, a)$ around $(\mu_\theta, A_0)$, which is justified by Lemma 1 (b), yields:

$$U_A(\theta, a) = U_A(\mu_\theta, A_0) + U_{\theta A}(\theta - \mu_\theta) + U_{AA}(a - A_0). \tag{A8}$$

Substituting into the first-order condition:

$$U_c \pi_{aa}(a - A_0) + U_A(\mu_\theta, A_0) + U_{\theta A}(\theta - \mu_\theta) + U_{AA}(a - A_0) = 0. \tag{A9}$$

Solving for $a$, which we denote as $a^*(\theta)$ in the first-best:

$$a^*(\theta) = A_0 - \frac{U_A(\mu_\theta, A_0)}{U_c \pi_{aa} + U_{AA}} - \frac{U_{\theta A}}{U_c \pi_{aa} + U_{AA}}(\theta - \mu_\theta). \quad \square \tag{A10}$$

## A.4 Proof of Lemma 4

An efficient allocation is a strategy $a_i^{te}(y_i, z)$ that maximizes

$$\mathbb{E}[U] = \int_{(\tilde{\theta},z)} \int_{y_i} U\big(e + \pi(a_i^{te}(y_i, z)), \tilde{\theta}, \tilde{A}^{te}(\tilde{\theta}, z)\big) dP(y_i|\tilde{\theta}, z) dP(\tilde{\theta}, z), \tag{A11}$$

subject to $\tilde{A}^{te}(\tilde{\theta}, z) = \int_{y_i} a_i^{te}(y_i, z) dP(y_i|\tilde{\theta}, z)$. Note that given the continuum of firms, the empirical distribution of private signals converges to the conditional distribution $P(y_i|\tilde{\theta}, z)$ by the Law of Large Numbers. Consequently, the aggregate action, $\tilde{A}(\tilde{\theta}, z)$, can be represented both as an integral over the signal distribution and as an integral over the firm index: $\int_{y_i} a_i(y_i, z) dP(y_i|\tilde{\theta}, z) = \int_i a_i di$. Write the Lagrangian:

$$\begin{aligned} \Lambda = & \int_{(\tilde{\theta},z)} \int_{y_i} U\big(e + \pi(a_i^{te}(y_i, z)), \tilde{\theta}, \tilde{A}^{te}(\tilde{\theta}, z)\big) dP(y_i|\tilde{\theta}, z) dP(\tilde{\theta}, z) \\ & + \int_{(\tilde{\theta},z)} \lambda(\tilde{\theta}, z) \Big[\tilde{A}^{te}(\tilde{\theta}, z) - \int_{y_i} a_i^{te}(y_i, z) dP(y_i|\tilde{\theta}, z)\Big] dP(\tilde{\theta}, z). \end{aligned} \tag{A12}$$

The first order condition for $\tilde{A}^{te}(\tilde{\theta}, z)$,

$$\int_{y_i} U_A\Big(\tilde{\theta}, \tilde{A}^{te}(\tilde{\theta}, z)\Big) dP(y_i|\tilde{\theta}, z) + \lambda(\tilde{\theta}, z) = 0, \tag{A13}$$

must hold for almost all $(\tilde{\theta}, z)$. Thus:

$$U_A\Big(\tilde{\theta}, \tilde{A}^{te}(\tilde{\theta}, z)\Big) + \lambda(\tilde{\theta}, z) = 0. \tag{A14}$$

The chain rule of probability states that:

$$dP(y_i, \tilde{\theta}, z) = dP(y_i|\tilde{\theta}, z) dP(\tilde{\theta}, z) = dP(\tilde{\theta}|y_i, z) dP(y_i, z). \tag{A15}$$

By the chain rule and Fubini's theorem (to change the order of integration), the Lagrangian becomes:

$$\Lambda = \int_{(y_i,z)} \int_{\tilde{\theta}} U(e + \pi(a_i^{te}(y_i,z)), \tilde{\theta}, \tilde{A}^{te}(\tilde{\theta},z)) dP(\tilde{\theta}|y_i,z) dP(y_i,z)$$

$$+ \int_{(y_i,z)} \int_{\tilde{\theta}} \lambda(\tilde{\theta},z) \Big[\tilde{A}^{te}(\tilde{\theta},z) - \int_{y_i} a_i^{te}(y_i,z) dP(y_i|\tilde{\theta},z)\Big] dP(\tilde{\theta}|y_i,z) dP(y_i,z). \tag{A16}$$

The first order condition for $a_i^{te}(y_i,z)$ is then:

$$\int_{\tilde{\theta}} \Big[U_c \pi_a(a_i^{te}(y_i,z)) - \lambda(\tilde{\theta},z)\Big] dP(\tilde{\theta}|y_i,z) = 0, \tag{A17}$$

which must hold for almost all $(y_i,z)$. Replacing the first-order condition for $\tilde{A}^{te}$, we obtain:

$$\mathbb{E}_i\Big[U_c \pi_a(a_i^{te}(y_i,z))\Big] + \mathbb{E}_i\Big[U_A\Big(\tilde{\theta}, \tilde{A}^{te}(\tilde{\theta},z)\Big)\Big] = 0. \tag{A18}$$

Since $\pi_a$ and $U_A$ are linear in their arguments, we can write:

$$\pi_a(a_i^{te}(y_i,z)) = \pi_a(a^*(\theta)) + \pi_{aa}(a_i^{te}(y_i,z) - a^*(\theta)), \tag{A19}$$

$$U_A\Big(\tilde{\theta}, \tilde{A}^{te}(\tilde{\theta},z)\Big) = U_A(\tilde{\theta}, a^*(\theta)) + U_{AA}(\tilde{A}^{te}(\tilde{\theta},z) - a^*(\theta)). \tag{A20}$$

By definition of the first-best allocation (see equation (A7)):

$$U_c \pi_a(a^*(\theta)) + U_A(\tilde{\theta}, a^*(\theta)) = 0, \tag{A21}$$

and thus one can rewrite equation (A18) as

$$U_c \pi_{aa} \mathbb{E}_i[a_i^{te}(y_i,z) - a^*(\theta)] + U_{AA} \mathbb{E}_i[\tilde{A}^{te}(\tilde{\theta},z) - a^*(\theta)] = 0. \tag{A22}$$

Solving for $a_i^{te}(y_i,z)$ yields:

$$a_i^{te}(y_i,z) = (1-\alpha)\mathbb{E}_i[a^*(\theta)] + \alpha \mathbb{E}_i[\tilde{A}^{te}(\tilde{\theta},z)], \tag{A23}$$

where $\alpha$ is defined as in Lemma 4, equation (17). □

## A.5 Proof of Lemma 5

The ranking $\mathbb{E}[U^{sq}] \leq \mathbb{E}[U^{te}]$ follows from the definition of the team-efficient allocation as the solution to the welfare maximization problem. The team-efficient allocation, $a_i^{te}(y_i,z)$, is chosen to maximize $\mathbb{E}[U]$ subject to the constraint that actions depend only on the available information (private and public signals). The status quo allocation, $a_i^{sq} = A_0$, is not the solution to this maximization problem, and thus cannot yield a higher level of ex-ante expected welfare.

We now prove the statement that welfare is highest in the first-best. The ex-ante expected social welfare in this economy is given by equation (14). Recall that $\tilde{A} = \int_0^1 a_i di$ and that $U$ is linear in $c_i = e + \pi(a_i)$ and $\pi(a_i)$ is quadratic in $a_i$. Thus, $U$ can be written as:

$$U(e+\pi(a_i), \tilde{\theta}, \tilde{A}) = U(e+\pi(\tilde{A}), \tilde{\theta}, \tilde{A}) + U_c \pi_a(\tilde{A}) \cdot (a_i - \tilde{A}) + \frac{1}{2} U_c \pi_{aa} \cdot (a_i - \tilde{A})^2. \tag{A24}$$

This is a Taylor series expansion around the point $a_i = \tilde{A}$. The transformation is exact, since $U$ is quadratic in $a_i$. We omit the dependence of $\pi_{aa}$ on $\tilde{A}$ since $\pi(\cdot)$ is a quadratic function.

The linear term, $U_c \pi_a(\tilde{A}) \cdot (a_i - \tilde{A})$, vanishes when integrated over the conditional distribution of private signals in equation (14), $\int_{y_i} U_c \pi_a(\tilde{A}) \cdot (a_i - \tilde{A}) dP(y_i|\tilde{\theta}, z)$, because $\int_{y_i} a_i dP(y_i|\tilde{\theta}, z) = \tilde{A}$. The quadratic term, when integrated over the conditional distribution of private signals, is the variance of $a_i$ conditional on $\tilde{\theta}$ and $z$: $\int_{y_i} (a_i - \tilde{A})^2 dP(y_i|\tilde{\theta}, z) = \sigma_a^2$, with $\sigma_a^2 := \int_{y_i} (a_i - \int_{y_j} a_j dP(y_j|\tilde{\theta}, z))^2 dP(y_i|\tilde{\theta}, z)$. Thus, defining $W(\tilde{\theta}, \tilde{A}, \sigma_a) := U(e + \pi(\tilde{A}), \tilde{\theta}, \tilde{A}) + \frac{1}{2} U_c \pi_{aa} \sigma_a^2$, ex-ante welfare can be rewritten as:

$$\mathbb{E}[U] = \int_{(\tilde{\theta}, z)} W(\tilde{\theta}, \tilde{A}, \sigma_a) dP(\tilde{\theta}, z). \tag{A25}$$

We notice that the first-best $a^*(\theta)$ is the unique solution to $W_A(\tilde{\theta}, a^*(\theta), 0) = 0$. A second-order Taylor expansion of $W(\tilde{\theta}, \tilde{A}, \sigma_a)$ around $\tilde{A} = a^*(\theta)$ and $\sigma_a = 0$ gives:

$$\begin{aligned} W(\tilde{\theta}, \tilde{A}, \sigma_a) &= W(\tilde{\theta}, a^*(\theta), 0) + W_A(\tilde{\theta}, a^*(\theta), 0) \cdot (\tilde{A} - a^*(\theta)) + W_{\sigma_a}(\tilde{\theta}, a^*(\theta), 0) \cdot \sigma_a \\ &\quad + \frac{1}{2} W_{AA}(\tilde{\theta}, a^*(\theta), 0) \cdot (\tilde{A} - a^*(\theta))^2 + \frac{1}{2} W_{\sigma_a \sigma_a}(\tilde{\theta}, a^*(\theta), 0) \cdot \sigma_a^2. \end{aligned} \tag{A26}$$

Replacing $W_A(\tilde{\theta}, a^*(\theta), 0) = 0$ and $W_{\sigma_a}(\tilde{\theta}, a^*(\theta), 0) = 0$ and recognizing that $\int_{(\tilde{\theta}, z)} (\tilde{A} - a^*(\theta))^2 dP(\tilde{\theta}, z) = \mathbb{E}[(\tilde{A} - a^*(\theta))^2]$ and $\int_{(\tilde{\theta}, z)} \sigma_a^2 dP(\tilde{\theta}, z) = \mathbb{E}[(a_i - \tilde{A})^2]$, we obtain

$$\mathbb{E}[U] = \mathbb{E}[W(\tilde{\theta}, a^*(\theta), 0)] + \frac{1}{2}(U_c \pi_{aa} + U_{AA}) \cdot \mathbb{E}[(\tilde{A} - a^*(\theta))^2] + \frac{1}{2} U_c \pi_{aa} \cdot \mathbb{E}[(a_i - \tilde{A})^2]. \tag{A27}$$

Since $U_c \pi_{aa} + U_{AA} < 0$ and $U_c \pi_{aa} < 0$, it implies that welfare is highest in the first-best:

$$\mathbb{E}[U] \leq \mathbb{E}[W(\tilde{\theta}, a^*(\theta), 0)]. \tag{A28}$$

The additional terms, $\frac{1}{2}(U_c \pi_{aa} + U_{AA}) \cdot \mathbb{E}[(\tilde{A} - a^*(\theta))^2]$ and $\frac{1}{2} U_c \pi_{aa} \cdot \mathbb{E}[(a_i - \tilde{A})^2]$, measure welfare losses due to volatility and dispersion, as $-(U_c \pi_{aa} + U_{AA})$ can be interpreted as "social aversion to volatility" and $-U_c \pi_{aa}$ can be interpreted as "social aversion to dispersion". See Angeletos and

Pavan (2007) for similar interpretations. $\square$

## A.6 Proof of Proposition 1

Under UBR, firm $i$'s maximization problem leads to the first-order condition:

$$\mathbb{E}_i[U_c\left(\pi_a(a_i) + f(\tilde{q})\right)] = 0. \tag{A29}$$

Substituting $\pi_a(a_i) = \pi_{aa}(a_i - A_0)$, then solving for $a_i$:

$$a_i = A_0 - \frac{\mathbb{E}_i[f(\tilde{q})]}{\pi_{aa}}. \tag{A30}$$

Consider the team-efficient allocation from Lemma 4:

$$a_i^{te} = \left(1 + \frac{U_{AA}}{U_c \pi_{aa}}\right)\mathbb{E}_i[a^*(\theta)] - \frac{U_{AA}}{U_c \pi_{aa}}\mathbb{E}_i[\tilde{A}^{te}]. \tag{A31}$$

where $a^*(\theta)$ is the first-best action (obtained in Lemma 3):

$$a^*(\theta) = A_0 - \frac{U_A(\mu_\theta, A_0)}{U_c\pi_{aa} + U_{AA}} - \frac{U_{\theta A}}{U_c\pi_{aa} + U_{AA}}(\tilde{\theta} - \mu_\theta). \tag{A32}$$

We want to find a regulatory function $f(\tilde{q})$ such that the firm's optimal choice under UBR given by equation (A30) equals the team-efficient action, $a_i^{te}$ (given by the general form of equation (A31) without the *'te'* superscript on the last term). That is, we seek to satisfy:

$$A_0 - \frac{\mathbb{E}_i[f(\tilde{q})]}{\pi_{aa}} = \left(1 + \frac{U_{AA}}{U_c \pi_{aa}}\right)\mathbb{E}_i[a^*(\theta)] - \frac{U_{AA}}{U_c \pi_{aa}}\mathbb{E}_i[\tilde{A}]. \tag{A33}$$

Substituting in the expression for $a^*(\theta)$ from equation (A32) and taking expectations:

$$A_0 - \frac{\mathbb{E}_i[f(\tilde{q})]}{\pi_{aa}} = \left(1 + \frac{U_{AA}}{U_c \pi_{aa}}\right)\left(A_0 - \frac{U_A(\mu_\theta, A_0)}{U_c\pi_{aa} + U_{AA}} - \frac{U_{\theta A}}{U_c\pi_{aa} + U_{AA}}\mathbb{E}_i[\tilde{\theta} - \mu_\theta]\right) - \frac{U_{AA}}{U_c \pi_{aa}}\mathbb{E}_i[\tilde{A}]. \tag{A34}$$

Multiplying through and simplifying, we want (A33) to hold, meaning we need:

$$\mathbb{E}_i[f(\tilde{q})] = \frac{U_A(\mu_\theta, A_0)}{U_c} + \frac{U_{\theta A}}{U_c}\mathbb{E}_i[\tilde{\theta} - \mu_\theta] + \frac{U_{AA}}{U_c}\mathbb{E}_i[\tilde{A} - A_0]. \tag{A35}$$

From the Taylor expansion of the utility function $\mathcal{U}$ provided in equation (3) and the properties in

Lemma 1, we obtain

$$\frac{U_{\theta A}}{U_c} = \frac{\mathcal{U}_{qq}(c_0, \mu_\theta)}{\mathcal{U}_c(c_0, \mu_\theta)}, \tag{A36}$$

$$\frac{U_{AA}}{U_c} = \frac{\mathcal{U}_{qq}(c_0, \mu_\theta)}{\mathcal{U}_c(c_0, \mu_\theta)}, \tag{A37}$$

$$\frac{U_A(\mu_\theta, A_0)}{U_c} = \frac{\mathcal{U}_q(c_0, \mu_\theta)}{\mathcal{U}_c(c_0, \mu_\theta)}. \tag{A38}$$

We then recover society's two key parameters as

$$\text{ERA} := -\frac{U_{AA}}{U_c} = -\frac{U_{\theta A}}{U_c}, \tag{A39}$$

$$\text{MRS} := \frac{U_A(\mu_\theta, A_0)}{U_c}. \tag{A40}$$

Consider now the following candidate function:

$$f(\tilde{q}) = \frac{U_A(\tilde{\theta}, \tilde{A})}{U_c} = \frac{U_A(\mu_\theta, A_0)}{U_c} + \frac{U_{\theta A}}{U_c}(\tilde{\theta} - \mu_\theta) + \frac{U_{AA}}{U_c}(\tilde{A} - A_0). \tag{A41}$$

Using equations (A36), (A37), and (A38), and recalling the definition of $\tilde{q}$ from (2), leads to:

$$f(\tilde{q}) = \text{MRS} - \text{ERA}(\tilde{q} - \mu_\theta). \tag{A42}$$

Take the expectation of this candidate function, conditional on household $i$'s information:

$$\mathbb{E}_i[f(\tilde{q})] = \frac{U_A(\mu_\theta, A_0)}{U_c} + \frac{U_{\theta A}}{U_c}\mathbb{E}_i[\tilde{\theta} - \mu_\theta] + \frac{U_{AA}}{U_c}\mathbb{E}_i[\tilde{A} - A_0]. \tag{A43}$$

Equation (A43) is identical to equation (A35), which is the required condition for the firm's optimal choice under UBR to coincide with the general form of the team-efficient action. Because the candidate $f(\tilde{q})$ given by equation (A41) (or equivalently, (A42)) satisfies the necessary condition (A35), it follows that under UBR, each firm's optimal choice of action, $a_i$, will be given by:

$$a_i = \left(1 + \frac{U_{AA}}{U_c \pi_{aa}}\right)\mathbb{E}_i[a^*(\theta)] - \frac{U_{AA}}{U_c \pi_{aa}}\mathbb{E}_i[\tilde{A}]. \tag{A44}$$

This is the defining characteristic of the team-efficient equilibrium. Since each firm $i$ is acting according to this rule, the aggregate outcome $\tilde{A}$ will be the team-efficient aggregate outcome, $\tilde{A}^{te}$. Therefore, the regulatory function defined by (A41) (or (A42)) ensures that each firm chooses the team-efficient level of action, $a_i^{te}$, in the unique linear equilibrium. $\square$

## A.7 Proof of Proposition 2

The consumer's problem in the synthetic market yields an inverse demand for the aggregate action. Ex post, the consumer's FOC yields the inverse demand $p = U_A(\theta, A)/U_c$. Expanding $U_A(\theta, A)$ around the status quo $(\mu_\theta, A_0)$ using a first-order Taylor expansion (which is exact by Lemma 1 (b)), we have:

$$p = \frac{U_A(\mu_\theta, A_0)}{U_c} + \frac{U_{\theta A}}{U_c}(\theta - \mu_\theta) + \frac{U_{AA}}{U_c}(A - A_0). \tag{A45}$$

We note that the expression for the market price $p$ in equation (A45) is identical in form to the candidate function for $f(\tilde{q})$ derived in the proof of Proposition 1 (equation (A41)).
Consider now the firms' problem in this hypothetical Cournot market. Firm $i$ chooses its action, denoted by $a_i^s$, to maximize household $i$'s expected utility:

$$\max_{a_i^s} \mathbb{E}_i \left[ U\left( e + \pi(a_i^s) + \tilde{p}a_i^s, \tilde{\theta}, \tilde{A}^d \right) \right], \tag{A46}$$

where $\pi(a_i^s)$ is the firm's profit, to which we add the revenue from selling the action, $\tilde{p}a_i^s$. The aggregate action supplied, $\int_i a_i^s di$, must equal aggregate demand $\tilde{A}^d$ in equilibrium. Note that the firm takes $\tilde{p}$ and $\tilde{A}^d$ as given. The first-order condition leads to ($\pi(\cdot)$ is quadratic):

$$0 = \mathbb{E}_i[U_c(\tilde{p} + \pi_a(a_i^s))] = \mathbb{E}_i[U_c\left(\tilde{p} + \pi_{aa}(a_i^s - A_0)\right)], \tag{A47}$$

and thus ($U_c$ is constant and cancels out):

$$a_i^s = A_0 - \frac{\mathbb{E}_i[\tilde{p}]}{\pi_{aa}}. \tag{A48}$$

Substituting the expression for the price $p$ from the inverse demand function (A45) into (A48), and imposing the market clearing condition $\tilde{A}^d = \int_i a_i^s di$, we have:

$$a_i^s = A_0 - \frac{1}{\pi_{aa}} \mathbb{E}_i \left[ \frac{U_A(\mu_\theta, A_0)}{U_c} + \frac{U_{\theta A}}{U_c}(\tilde{\theta} - \mu_\theta) + \frac{U_{AA}}{U_c}\left( \int_i a_i^s di - A_0 \right) \right]. \tag{A49}$$

Using the definitions of MRS and ERA from equations (A38) and (A37), this becomes:

$$a_i^s = A_0 - \frac{1}{\pi_{aa}} \left( \text{MRS} - \text{ERA}\, \mathbb{E}_i[\tilde{\theta} - \mu_\theta] - \text{ERA}\, \mathbb{E}_i \left[ \int_i a_i^s di - A_0 \right] \right). \tag{A50}$$

The first-order condition for a firm in the synthetic market, given by equation (A50), is identical to the first-order condition for a firm under UBR (derived by substituting (A35) into (A30) in the proof of Proposition 1). Since the firms' optimization problems are identical, their chosen actions must also be identical, which implies $a_i^s = a_i^{te}$. Thus, the UBR mechanism effectively creates a synthetic Cournot market that achieves the team-efficient outcome. $\square$

## A.8 Proof of Proposition 3

The firm's problem is to maximize (21), where consumption is given by:

$$\tilde{c}_i = e + \pi(a_i) + (a_i - \tilde{A})f(\tilde{q}). \tag{A51}$$

Substituting (A51) into the objective function (21) and taking the first-order condition with respect to $a_i$ gives $a_i = A_0 - \frac{\mathbb{E}_i[f(\tilde{q})]}{\pi_{aa}}$. Substituting $f(\tilde{q})$ from equation (A41) leads to:

$$a_i = A_0 - \frac{1}{\pi_{aa}U_c}\left[U_A(\mu_\theta, A_0) + U_{AA}(\mathbb{E}_i[\tilde{A}] - A_0) + U_{\theta A}(\mathbb{E}_i[\tilde{\theta}] - \mu_\theta)\right]. \tag{A52}$$

To solve this equation, we use the conjectured linear form of the equilibrium strategy from equation (29). Aggregating these individual actions across all firms and applying the Law of Large Numbers (such that $\int_0^1 (y_i - \mu_\theta)di = \tilde{\theta} - \mu_\theta$) gives the aggregate action:

$$\tilde{A} = \beta_0 + \beta_y(\tilde{\theta} - \mu_\theta) + \beta_z(z - \mu_\theta). \tag{A53}$$

Firm $i$'s expectation of $\tilde{A}$ is therefore a function of its own expectation of $\tilde{\theta}$. From the model setup, we have:

$$\mathbb{E}_i[\tilde{\theta}] - \mu_\theta = \frac{\tau_y}{\tau}(y_i - \mu_\theta) + \frac{\tau_z}{\tau}(z - \mu_\theta), \tag{A54}$$

$$\mathbb{E}_i[\tilde{A}] = \beta_0 + \beta_y \mathbb{E}_i[\tilde{\theta} - \mu_\theta] + \beta_z(z - \mu_\theta). \tag{A55}$$

Substituting (A54) and (A55) into (A52), and using our conjecture (29) for the LHS, yields:

$$\begin{aligned}\beta_0 + \beta_y(y_i - \mu_\theta) + \beta_z(z - \mu_\theta) = A_0 - \frac{1}{\pi_{aa}U_c}\Big[&U_A(\mu_\theta, A_0) \\ &+ U_{AA}\left(\beta_0 + \beta_y\frac{\tau_y}{\tau}(y_i - \mu_\theta) + \left(\beta_y\frac{\tau_z}{\tau} + \beta_z\right)(z - \mu_\theta) - A_0\right) \\ &+ U_{\theta A}\left(\frac{\tau_y}{\tau}(y_i - \mu_\theta) + \frac{\tau_z}{\tau}(z - \mu_\theta)\right)\Big].\end{aligned} \tag{A56}$$

We equate coefficients for the constant term, the coefficients for $(y_i - \mu_\theta)$, and the coefficients for $(z - \mu_\theta)$. First, solving for $\beta_0$:

$$\beta_0 = A_0 - \frac{1}{\pi_{aa}U_c}\left[U_A(\mu_\theta, A_0) + U_{AA}(\beta_0 - A_0)\right], \tag{A57}$$

and thus:

$$\beta_0 = A_0 - \frac{U_A(\mu_\theta, A_0)}{U_c\pi_{aa} + U_{AA}}. \tag{A58}$$

Second, solving for $\beta_y$:

$$\beta_y = -\frac{1}{U_c\pi_{aa}}\left[U_{AA}\beta_y\frac{\tau_y}{\tau} + U_{\theta A}\frac{\tau_y}{\tau}\right], \tag{A59}$$

yields:

$$\beta_y = -\frac{U_{\theta A}\tau_y}{U_c\pi_{aa}\tau + U_{AA}\tau_y} < 0. \tag{A60}$$

Third and finally, solving for $\beta_z$:

$$\beta_z = -\frac{1}{U_c\pi_{aa}}\left[U_{AA}\left(\beta_y\frac{\tau_z}{\tau} + \beta_z\right) + U_{\theta A}\frac{\tau_z}{\tau}\right]. \tag{A61}$$

which, after substituting the solution for $\beta_y$ from (A60), leads to

$$\beta_z = -\frac{U_c\pi_{aa}}{U_c\pi_{aa} + U_{AA}}\frac{U_{\theta A}\tau_z}{U_c\pi_{aa}\tau + U_{AA}\tau_y}. \tag{A62}$$

and thus, recognizing that $\frac{U_c\pi_{aa}}{U_c\pi_{aa}+U_{AA}} = \frac{1}{1-\alpha}$ (Lemma 4, equation (17)):

$$\beta_z = -\frac{1}{1-\alpha}\frac{U_{\theta A}\tau_z}{U_c\pi_{aa}\tau + U_{AA}\tau_y} < 0. \tag{A63}$$

Equations (A58), (A60), and (A63) characterize the coefficients $\beta_0$, $\beta_y$, and $\beta_z$ in terms of the model parameters, and confirm equations (30) through (32) in Proposition 3. $\square$

## A.9 Proof of Corollary 3.1

From Lemma 4, we know that $\alpha = -\frac{U_{AA}}{U_c\pi_{aa}}$. We also know that $U_{AA} < 0$, $U_c > 0$, and $\pi_{aa} < 0$. Therefore, $\alpha < 0$. Consequently, as shown in equation (16) of Lemma 4, a firm's expectation of the aggregate action, $\mathbb{E}_i[\tilde{A}^{te}]$, enters its best-response function with a negative coefficient, implying that firms' actions are strategic substitutes.

From Proposition 3, using the expressions for $\beta_y$ and $\beta_z$, we can directly calculate the ratio $\frac{\beta_y}{\beta_z}$:

$$\frac{\beta_y}{\beta_z} = \frac{\tau_y}{\frac{\tau_z}{1-\alpha}} = (1-\alpha)\frac{\tau_y}{\tau_z}. \tag{A64}$$

Since $\alpha < 0$, we have $(1-\alpha) > 1$. Therefore:

$$\frac{\beta_y}{\beta_z} = (1-\alpha)\frac{\tau_y}{\tau_z} > \frac{\tau_y}{\tau_z}. \tag{A65}$$

This confirms that firms overweight their private information relative to the public signal. □

## A.10 Proof of Corollary 3.2

We start first by writing the coefficients $\beta_0$, $\beta_y$, and $\beta_z$ as functions of $\kappa_0^*$, $\kappa_1^*$, and $\alpha$, noting that these latter three coefficients *do not* depend on $\sigma_\theta$, $\tau_y$, and $\tau_z$:

$$\beta_0 = \kappa_0^*, \tag{A66}$$

$$\beta_y = \frac{(1-\alpha)\kappa_1^*\sigma_\theta^2\tau_y}{1+\sigma_\theta^2(\tau_y - \alpha\tau_y + \tau_z)}, \tag{A67}$$

$$\beta_z = \frac{\kappa_1^*\sigma_\theta^2\tau_z}{1+\sigma_\theta^2(\tau_y - \alpha\tau_y + \tau_z)}. \tag{A68}$$

We aim to show that $\frac{\partial \mathcal{B}}{\partial \sigma_\theta} > 0$, where $\mathcal{B} = |\beta_y + \beta_z|$. Since $\beta_y < 0$ and $\beta_z < 0$, we have $\mathcal{B} = -(\beta_y + \beta_z)$:

$$\mathcal{B} = \kappa_1^*\left(\frac{1}{1+\sigma_\theta^2(\tau_y(1-\alpha)+\tau_z)} - 1\right). \tag{A69}$$

Noting that $\kappa_1^* < 0$ and $\alpha < 0$, points (a), (b), and (c) of Corollary 3.2 result immediately. □

## A.11 Proof of Corollary 3.3

Consider a firm $i$ that unilaterally increases its private signal precision to $\tau_y' = \xi\tau_y$, where $\xi > 1$. Let $\mathbb{E}_i'[\cdot]$ denote expectations conditional on this more precise signal. The first-order condition implies the firm's optimal action, denoted $a_i'$:

$$a_i' = A_0 - \frac{\mathbb{E}_i'[f(\tilde{q})]}{\pi_{aa}}. \tag{A70}$$

We know from Proposition 1 that $f(\tilde{q}) = \frac{U_A(\mu_\theta, A_0)}{U_c} + \frac{U_{\theta A}}{U_c}(\tilde{\theta} - \mu_\theta) + \frac{U_{AA}}{U_c}(\tilde{A} - A_0)$, which remains the same as in the main model. Replacing this in (A70) yields:

$$a_i' = A_0 - \frac{U_A(\mu_\theta, A_0)}{U_c\pi_{aa}} - \frac{U_{\theta A}}{U_c\pi_{aa}}(\mathbb{E}_i'[\tilde{\theta}] - \mu_\theta) - \frac{U_{AA}}{U_c\pi_{aa}}(\mathbb{E}_i'[\tilde{A}] - A_0). \tag{A71}$$

Make the following substitutions:

$$\mathbb{E}_i'[\tilde{\theta}] = \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2} + \xi\tau_y + \tau_z}\mu_\theta + \frac{\xi\tau_y}{\sigma_\theta^{-2} + \xi\tau_y + \tau_z}y_i + \frac{\tau_z}{\sigma_\theta^{-2} + \xi\tau_y + \tau_z}z, \tag{A72}$$

$$\mathbb{E}_i'[\tilde{A}] = \beta_0 + \beta_y(\mathbb{E}_i'[\tilde{\theta}] - \mu_\theta) + \beta_z(z - \mu_\theta), \tag{A73}$$

and replace the solutions for $\beta_0$, $\beta_y$, and $\beta_z$ given in equations (30)–(32) of Proposition 3. Straightforward but tedious algebra (details omitted) then leads to equation (37) of Corollary 3.3:

$$a_i' = \beta_0 + \beta_y\left[1 + \frac{(\xi-1)(1+\sigma_\theta^2\tau_z)}{1+\sigma_\theta^2(\xi\tau_y+\tau_z)}\right](y_i - \mu_\theta) + \beta_z\left[1 - \frac{(\xi-1)(1-\alpha)\sigma_\theta^2\tau_y}{1+\sigma_\theta^2(\xi\tau_y+\tau_z)}\right](z - \mu_\theta). \tag{A74}$$

We now demonstrate that firm $i$'s ex-ante expected utility (conditional on $z$, but before observing $y_i$) is strictly increasing in the precision of its private signal. From Proposition 1 and equation (A41), we note that the regulatory function can also be written as:

$$f(\tilde{q}) = \frac{U_A(\tilde{\theta}, \tilde{A})}{U_c} = \frac{U_A(\tilde{\theta}, A_0) + U_{AA}(\tilde{A} - A_0)}{U_c}. \tag{A75}$$

Here we have used a first-order Taylor expansion of $U_A(\tilde{\theta}, \cdot)$ around $A_0$, holding $\tilde{\theta}$ fixed:

$$U_A(\tilde{\theta}, \tilde{A}) = U_A(\tilde{\theta}, A_0) + U_{AA}\,(\tilde{A} - A_0),$$

and in our linear-quadratic setup the higher-order terms drop out. Dividing by $U_c$ then yields the stated form.

Consider a second-order Taylor expansion of the utility function $U(e + \pi(a_i) + (a_i - \tilde{A})f(\tilde{q}), \tilde{\theta}, \tilde{A})$ around the status quo point $(a_i, \tilde{A}) = (A_0, A_0)$:

$$\begin{aligned}
U\left(e + \pi(a_i) + (a_i - \tilde{A})\frac{U_A(\tilde{\theta}, A_0) + U_{AA}(\tilde{A} - A_0)}{U_c}, \tilde{\theta}, \tilde{A}\right) &= U\left(e + \pi(A_0), \tilde{\theta}, A_0\right) \\
&+ 0 \cdot (\tilde{A} - A_0) + U_A(\tilde{\theta}, A_0) \cdot (a_i - A_0) \\
&- \frac{1}{2}U_{AA} \cdot (\tilde{A} - A_0)^2 + \frac{1}{2}U_c\pi_{aa} \cdot (a_i - A_0)^2 + U_{AA} \cdot (\tilde{A} - A_0)(a_i - A_0).
\end{aligned} \tag{A76}$$

This expansion is exact, given the linear-quadratic structure of the model. The expected utility (conditional on both $y_i$ and $z$) is then

$$\begin{aligned}
\mathbb{E}_i[U] = U\left(e + \pi(A_0), \tilde{\theta}, A_0\right) + \mathbb{E}_i[U_A(\tilde{\theta}, A_0)](a_i - A_0) \\
- \frac{1}{2}U_{AA}\mathbb{E}_i[(\tilde{A} - A_0)^2] + \frac{1}{2}U_c\pi_{aa}(a_i - A_0)^2 + U_{AA}\mathbb{E}_i[\tilde{A} - A_0](a_i - A_0).
\end{aligned} \tag{A77}$$

Recall from Lemma 3 that the first-best action is given by $a^*(\theta) = \kappa_0^* + \kappa_1^*(\tilde{\theta} - \mu_\theta)$, and that $\kappa_0^* = A_0 - \frac{U_A(\mu_\theta, A_0)}{U_c\pi_{aa}+U_{AA}}$ and $\kappa_1^* = -\frac{U_{\theta A}}{U_c\pi_{aa}+U_{AA}}$. We can therefore rewrite the expression for $a^*(\theta)$ as

$$a^*(\theta) = A_0 + \frac{-U_A(\mu_\theta, A_0) - U_{\theta A}(\tilde{\theta} - \mu_\theta)}{U_c\pi_{aa} + U_{AA}} = A_0 - \frac{U_A(\tilde{\theta}, A_0)}{U_c\pi_{aa} + U_{AA}}. \tag{A78}$$

Thus, we can write $\mathbb{E}_i[U_A(\tilde{\theta}, A_0)] = -(U_c\pi_{aa} + U_{AA})\mathbb{E}_i[a^*(\theta) - A_0]$, which leads to:

$$\begin{aligned}\mathbb{E}_i[U] = U\left(e + \pi(A_0), \tilde{\theta}, A_0\right) - \frac{1}{2}U_{AA}\mathbb{E}_i[(\tilde{A} - A_0)^2] + \frac{1}{2}U_c\pi_{aa}(a_i - A_0)^2 \\ - \left((U_c\pi_{aa} + U_{AA})\mathbb{E}_i[a^*(\theta) - A_0] - U_{AA}\mathbb{E}_i[\tilde{A} - A_0]\right)(a_i - A_0),\end{aligned} \tag{A79}$$

or, using the fact that, from equation (17), $\alpha = -\frac{U_{AA}}{U_c\pi_{aa}}$,

$$\begin{aligned}\mathbb{E}_i[U] = U\left(e + \pi(A_0), \tilde{\theta}, A_0\right) - \frac{1}{2}U_{AA}\mathbb{E}_i[(\tilde{A} - A_0)^2] + \frac{1}{2}U_c\pi_{aa}(a_i - A_0)^2 \\ - U_c\pi_{aa}\left((1-\alpha)\mathbb{E}_i[a^*(\theta) - A_0] + \alpha\mathbb{E}_i[\tilde{A} - A_0]\right)(a_i - A_0).\end{aligned} \tag{A80}$$

Finally, recognizing from Lemma 4 that $(1-\alpha)\mathbb{E}_i[a^*(\theta) - A_0] + \alpha\mathbb{E}_i[\tilde{A} - A_0] = a_i - A_0$:

$$\mathbb{E}_i[U] = U\left(e + \pi(A_0), \tilde{\theta}, A_0\right) - \frac{1}{2}U_{AA}\mathbb{E}_i[(\tilde{A} - A_0)^2] + \frac{1}{2}U_c\pi_{aa}(a_i - A_0)^2 - U_c\pi_{aa}(a_i - A_0)^2 \tag{A81}$$

$$= U\left(e + \pi(A_0), \tilde{\theta}, A_0\right) - \frac{1}{2}U_{AA}\mathbb{E}_i[(\tilde{A} - A_0)^2] - \frac{1}{2}U_c\pi_{aa}(a_i - A_0)^2. \tag{A82}$$

Taking the *ex-ante* expectation (conditional on $z$), for both $a_i$ and $a_i'$, yields:

$$\mathbb{E}[\mathbb{E}_i[U]|z] = \mathbb{E}[U(e + \pi(A_0), \tilde{\theta}, A_0)|z] - \frac{1}{2}U_c\pi_{aa}\mathbb{E}[(a_i - A_0)^2|z] - \frac{1}{2}U_{AA}\mathbb{E}[(\tilde{A} - A_0)^2|z], \tag{A83}$$

$$\mathbb{E}[\mathbb{E}_i'[U]|z] = \mathbb{E}[U(e + \pi(A_0), \tilde{\theta}, A_0)|z] - \frac{1}{2}U_c\pi_{aa}\mathbb{E}[(a_i' - A_0)^2|z] - \frac{1}{2}U_{AA}\mathbb{E}[(\tilde{A} - A_0)^2|z]. \tag{A84}$$

The difference in *ex-ante* expected utilities is therefore:

$$\Delta\mathbb{E}[U|z] := \mathbb{E}[\mathbb{E}_i'[U]|z] - \mathbb{E}[\mathbb{E}_i[U]|z] = -\frac{1}{2}U_c\pi_{aa}\left(\mathbb{E}[(a_i' - A_0)^2|z] - \mathbb{E}[(a_i - A_0)^2|z]\right). \tag{A85}$$

From Propositions 2 and 3, we have $a_i - A_0 = -\frac{\mathbb{E}_i[f(\tilde{q})]}{\pi_{aa}}$ and $a_i' - A_0 = -\frac{\mathbb{E}_i'[f(\tilde{q})]}{\pi_{aa}}$. Substituting these into the expression for $\Delta\mathbb{E}[U|z]$, we obtain:

$$\Delta\mathbb{E}[U|z] = \frac{-U_c}{2\pi_{aa}}\left(\mathbb{E}[\mathbb{E}_i'[f(\tilde{q})]^2|z] - \mathbb{E}[\mathbb{E}_i[f(\tilde{q})]^2|z]\right). \tag{A86}$$

Applying the definition of variance, $\text{Var}[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$, and the Law of Iterated Expectations:

$$\mathbb{E}[\mathbb{E}'_i[f(\tilde{q})]^2|z] = \text{Var}[\mathbb{E}'_i[f(\tilde{q})]|z] + \mathbb{E}[f(\tilde{q})|z]^2, \tag{A87}$$

$$\mathbb{E}[\mathbb{E}_i[f(\tilde{q})]^2|z] = \text{Var}[\mathbb{E}_i[f(\tilde{q})]|z] + \mathbb{E}[f(\tilde{q})|z]^2. \tag{A88}$$

Substituting these into the expression for the difference in expected utilities, we get

$$\Delta\mathbb{E}[U|z] = \frac{-U_c}{2\pi_{aa}} \left(\text{Var}[\mathbb{E}'_i[f(\tilde{q})]|z] - \text{Var}[\mathbb{E}_i[f(\tilde{q})]|z]\right). \tag{A89}$$

The term $\frac{-U_c}{2\pi_{aa}}$ is positive ($\pi_{aa} < 0$ and $U_c > 0$). The gain in utility therefore depends on the sign of the difference in the variances of the conditional expectations. Let this difference be denoted by $\Delta\mathcal{V}(\xi) \equiv \text{Var}[\mathbb{E}'_i[f(\tilde{q})]|z] - \text{Var}[\mathbb{E}_i[f(\tilde{q})]|z]$. We can show that $\Delta\mathcal{V}(\xi) > 0$ by the Law of Total Variance (LTV).

Denoting $\text{Var}_i[f(\tilde{q})]$ as shorthand for $\text{Var}[f(\tilde{q})|y_i, z]$, the LTV states that:

$$\text{Var}[f(\tilde{q})|z] = \mathbb{E}[\text{Var}_i[f(\tilde{q})]|z] + \text{Var}[\mathbb{E}_i[f(\tilde{q})]|z]. \tag{A90}$$

The conditional variance, $\text{Var}_i[f(\tilde{q})]$, is independent of the signal realizations $y_i$ and $z$. This is a standard feature of linear-Gaussian models, where the variance of a linear function of normal random variables, conditional on a set of signals, depends only on the known precisions of the model's shocks and signals. Thus, we can write:

$$\text{Var}[f(\tilde{q})|z] = \text{Var}[f(\tilde{q})|y'_i, z] + \text{Var}[\mathbb{E}'_i[f(\tilde{q})]|z], \tag{A91}$$

$$\text{Var}[f(\tilde{q})|z] = \text{Var}[f(\tilde{q})|y_i, z] + \text{Var}[\mathbb{E}_i[f(\tilde{q})]|z]. \tag{A92}$$

$\text{Var}[f(\tilde{q})|z]$ is the same in both LTV equations as it depends on the distribution of $f(\tilde{q})$ given the public signal, $z$, which is unchanged by firm $i$'s private signal precision. Since the precision of the private signal is increased, we have $\text{Var}[f(\tilde{q})|y'_i, z] < \text{Var}[f(\tilde{q})|y_i, z]$. Therefore, it must be that $\Delta\mathcal{V}(\xi) > 0$.

Plugging this result into equation (A89), we conclude that $\Delta\mathbb{E}[U|z] > 0$. Thus, the firm has a strict incentive to increase its private information precision, $\tau_y$. □

## A.12 Proof of Proposition 4

The social planner's problem is:

$$\max_{a_{sp}} \mathbb{E}_{sp}\left[U(e + \pi(a_{sp}), \tilde{\theta}, a_{sp})\right]. \tag{A93}$$

The first-order condition (FOC) with respect to $a_{sp}$ is:

$$0 = \mathbb{E}_{sp}\left[U_c \pi_a(a_{sp}) + U_A(\tilde{\theta}, a_{sp})\right]. \tag{A94}$$

Rewrite $U_A(\tilde{\theta}, a_{sp}) = U_A(\tilde{\theta}, A_0) + U_{AA}(a_{sp} - A_0)$ and $U_c\pi_a(a_{sp}) = U_c\pi_{aa}(a_{sp} - A_0)$, and substitute:

$$0 = \mathbb{E}_{sp}\left[U_c\pi_{aa}(a_{sp} - A_0) + U_A(\tilde{\theta}, A_0) + U_{AA}(a_{sp} - A_0)\right]. \tag{A95}$$

Solving for $a_{sp}$, we obtain:

$$a_{sp} = A_0 - \frac{\mathbb{E}_{sp}[U_A(\tilde{\theta}, A_0)]}{U_c\pi_{aa} + U_{AA}}. \tag{A96}$$

Taking expectation of equation (A78) conditional only on public information gives:

$$\mathbb{E}_{sp}[a^*(\theta)] = A_0 - \frac{\mathbb{E}_{sp}[U_A(\tilde{\theta}, A_0)]}{U_c\pi_{aa} + U_{AA}}. \tag{A97}$$

Comparing this with equation (A96), we see that:

$$a_{sp} = \mathbb{E}_{sp}[a^*(\theta)]. \tag{A98}$$

We now compare the ex-ante expected social welfare under the social planner's allocation to the status quo. Define the difference in expected utility for household $i$, conditional on $y_i$ and $z$ as:

$$\Delta U_i := \mathbb{E}_i[U(e + \pi(a_{sp}), \tilde{\theta}, a_{sp})] - \mathbb{E}_i[U(e + \pi(A_0), \tilde{\theta}, A_0)]. \tag{A99}$$

Write a second-order Taylor expansion of $U$ around the status quo point $(\tilde{A}, a_i) = (A_0, A_0)$:

$$\begin{aligned} U(e + \pi(a_i), \tilde{\theta}, \tilde{A}) &= U(\tilde{\theta}, A_0, e + \pi(A_0)) + U_A(\tilde{\theta}, A_0)(\tilde{A} - A_0) + U_c\pi_a(A_0)(a_i - A_0) \\ &\quad + \frac{1}{2}U_{AA}(\tilde{A} - A_0)^2 + \frac{1}{2}U_c\pi_{aa}(a_i - A_0)^2. \end{aligned} \tag{A100}$$

Since $\pi_a(A_0) = 0$, and evaluating the expression at $a_i = \tilde{A} = a_{sp}$, the expansion becomes:

$$U(e+\pi(a_{sp}), \tilde{\theta}, a_{sp}) = U(\tilde{\theta}, A_0, e+\pi(A_0)) + U_A(\tilde{\theta}, A_0)(a_{sp} - A_0) + \frac{U_c\pi_{aa} + U_{AA}}{2}(a_{sp} - A_0)^2. \tag{A101}$$

Thus, the difference $\Delta U_i$ simplifies to:

$$\Delta U_i = \mathbb{E}_i[U_A(\tilde{\theta}, A_0)](a_{sp} - A_0) + \frac{U_c\pi_{aa} + U_{AA}}{2}(a_{sp} - A_0)^2. \tag{A102}$$

From equation (A96), we know that $a_{sp} - A_0 = \frac{-\mathbb{E}_{sp}[U_A(\tilde{\theta}, A_0)]}{U_c\pi_{aa} + U_{AA}} = \frac{-\mathbb{E}[U_A(\tilde{\theta}, A_0)|z]}{U_c\pi_{aa} + U_{AA}}$, where the second equality holds because the social planner's information set consists only of the prior and $z$. Thus:

$$\Delta U_i = \mathbb{E}_i[U_A(\tilde{\theta}, A_0)] \frac{-\mathbb{E}[U_A(\tilde{\theta}, A_0)|z]}{U_c\pi_{aa} + U_{AA}} + \frac{U_{AA} + U_c\pi_{aa}}{2} \left( \frac{-\mathbb{E}[U_A(\tilde{\theta}, A_0)|z]}{U_c\pi_{aa} + U_{AA}} \right)^2 \tag{A103}$$

$$= \frac{-1}{U_c\pi_{aa} + U_{AA}} \left( \mathbb{E}_i[U_A(\tilde{\theta}, A_0)]\mathbb{E}[U_A(\tilde{\theta}, A_0)|z] - \frac{1}{2}\mathbb{E}[U_A(\tilde{\theta}, A_0)|z]^2 \right). \tag{A104}$$

The welfare difference is the the ex-ante expectation $\mathbb{E}[\Delta U_i] = \Delta W_{sp}$, which averages over $y_i$ and $z$:

$$\Delta W_{sp} = \frac{-1}{U_c\pi_{aa} + U_{AA}} \mathbb{E}\left[ \mathbb{E}_i[U_A(\tilde{\theta}, A_0)]\mathbb{E}[U_A(\tilde{\theta}, A_0)|z] - \frac{1}{2}\mathbb{E}[U_A(\tilde{\theta}, A_0)|z]^2 \right]. \tag{A105}$$

Recall that $U_A(\tilde{\theta}, A_0) = U_A(\mu_\theta, A_0) + U_{\theta A}(\tilde{\theta} - \mu_\theta)$. Also, $\mathbb{E}_i[\tilde{\theta} - \mu_\theta] = \frac{\tau_y}{\tau}(y_i - \mu_\theta) + \frac{\tau_z}{\tau}(z - \mu_\theta)$ and $\mathbb{E}[\tilde{\theta} - \mu_\theta|z] = \frac{\tau_z}{\tau_{sp}}(z - \mu_\theta)$, where $\tau = \sigma_\theta^{-2} + \tau_y + \tau_z$ and $\tau_{sp} = \sigma_\theta^{-2} + \tau_z$. Therefore:

$$\mathbb{E}_i[U_A(\tilde{\theta}, A_0)] = U_A(\mu_\theta, A_0) + U_{\theta A} \left( \frac{\tau_y}{\tau}(y_i - \mu_\theta) + \frac{\tau_z}{\tau}(z - \mu_\theta) \right), \tag{A106}$$

$$\mathbb{E}[U_A(\tilde{\theta}, A_0)|z] = U_A(\mu_\theta, A_0) + U_{\theta A} \frac{\tau_z}{\tau_{sp}}(z - \mu_\theta). \tag{A107}$$

Substituting into the expression for $\Delta W_{sp}$:

$$\begin{aligned} \Delta W_{sp} = \frac{-1}{U_c\pi_{aa} + U_{AA}} \mathbb{E}\Bigg\{ & \left[ U_A(\mu_\theta, A_0) + U_{\theta A} \left( \frac{\tau_y}{\tau}(y_i - \mu_\theta) + \frac{\tau_z}{\tau}(z - \mu_\theta) \right) \right] \\ & \times \left[ U_A(\mu_\theta, A_0) + U_{\theta A} \frac{\tau_z}{\tau_{sp}}(z - \mu_\theta) \right] - \frac{1}{2} \left[ U_A(\mu_\theta, A_0) + U_{\theta A} \frac{\tau_z}{\tau_{sp}}(z - \mu_\theta) \right]^2 \Bigg\}. \end{aligned} \tag{A108}$$

Now, we expand the terms inside the expectation and apply the following rules:

$$\mathbb{E}[y_i - \mu_\theta] = \mathbb{E}[z - \mu_\theta] = 0, \tag{A109}$$

$$\mathbb{E}[(y_i - \mu_\theta)(z - \mu_\theta)] = \mathbb{E}[(\tilde{\theta} - \mu_\theta + \tilde{\varepsilon}_{y,i})(\tilde{\theta} - \mu_\theta + \tilde{\varepsilon}_z)] = \sigma_\theta^2, \tag{A110}$$

$$\mathbb{E}[(z - \mu_\theta)^2] = \mathrm{Var}(z) = \sigma_\theta^2 + 1/\tau_z. \tag{A111}$$

Expanding the product and taking expectations, then replacing $\tau_{sp} = \sigma_\theta^{-2} + \tau_z$ and $\tau = \sigma_\theta^{-2} + \tau_z + \tau_y$:

$$\Delta W_{sp} = \frac{-1}{2(U_c\pi_{aa} + U_{AA})} \left[ U_A(\mu_\theta, A_0)^2 + U_{\theta A}^2 \frac{\sigma_\theta^2 \tau_z}{\sigma_\theta^{-2} + \tau_z} \right] > 0. \quad \square \tag{A112}$$

## A.13 Proof of Corollary 4.1

Consider a firm facing a tax $T$ per unit of emissions, $\mathcal{E}(a_i)$. The firm's objective function is:

$$\max_{a_i} \mathbb{E}_i \left[ U(e + \pi(a_i) - T\mathcal{E}(a_i), \tilde{\theta}, \tilde{A}) \right]. \tag{A113}$$

Taking the first-order condition with respect to $a_i$ (and noting that firm $i$ takes $\tilde{A}$ as given) implies:

$$\mathbb{E}_i[\pi_a(a_i)] = T\mathbb{E}_i[\mathcal{E}'(a_i)]. \tag{A114}$$

Since both $\pi_a(a_i)$ and $\mathcal{E}'(a_i)$ are deterministic and all firms are ex-ante identical, we may drop the expectations by evaluating at $a_i = a_{sp}$, giving

$$\pi_a(a_{sp}) = T\,\mathcal{E}'(a_{sp}). \tag{A115}$$

To implement the social planner's solution, $a_{sp}$, we need all firms to choose $a_i = a_{sp}$. Since the tax is uniform and all firms are identical ex ante, we need to impose:

$$\pi_a(a_{sp}) = T\mathcal{E}'(a_{sp}), \tag{A116}$$

and noting that $\pi_a(a_{sp}) = \pi_{aa}(a_{sp} - A_0)$, we must have

$$T = \frac{(a_{sp} - A_0)\pi_{aa}}{\mathcal{E}'(a_{sp})}. \tag{A117}$$

Because $\pi_{aa} < 0$ and $\mathcal{E}'(a_{sp}) < 0$, the sign of $T$ is dictated by the difference $a_{sp} - A_0$. Since we have assumed that the action generates a positive externality, we have $a_{sp} > A_0$ and thus $T > 0$. Consequently, the per-unit-emissions tax is strictly positive and ensures that all firms choose the social planner's action level $a_{sp}$. □

## A.14 Proof of Corollary 4.2

When firm $i$ purchases $Q_i$ permits, its profit function is:

$$\pi_i = \pi(a_i) - p_{permit}Q_i. \tag{A118}$$

Firms must hold enough permits to cover their emissions. Therefore, the quantity of permits purchased, $Q_i$, must equal the firm's emissions:

$$Q_i = \mathcal{E}(a_i). \tag{A119}$$

The firm maximizes its expected utility. Substituting (A119) into the profit function:

$$\max_{a_i} \mathbb{E}_i[U(e + \pi(a_i) - p_{permit}\mathcal{E}(a_i), \tilde{\theta}, \tilde{A})]. \tag{A120}$$

Taking the first-order condition with respect to $a_i$ (the firm takes $p_{permit}$ and $\tilde{A}$ as given) implies:

$$\mathbb{E}_i[U_c(\pi_a(a_i) - p_{permit}\mathcal{E}'(a_i))] = 0, \tag{A121}$$

and since both $\pi_a(a_i)$ and $\mathcal{E}'(a_i)$ are deterministic functions of $a_i$ (and common across firms), and $U_c$ is a constant, we may drop the expectation to obtain

$$\pi_a(a_i) - p_{permit}\,\mathcal{E}'(a_i) = 0. \tag{A122}$$

or

$$\pi_{aa}(a_i - A_0) - p_{permit}\mathcal{E}'(a_i) = 0. \tag{A123}$$

Since $\mathcal{E}'(a_i) < 0$ and $\mathcal{E}''(a_i) > 0$, the left-hand side of (A123) is strictly decreasing in $a_i$. It is also strictly positive at $a_i = A_0$. Thus, for a given $p_{permit}$, there's a unique $a_i > A_0$ that solves the FOC. Because $p_{permit}$ is uniform across firms, and $\pi(\cdot)$ and $\mathcal{E}(\cdot)$ are identical for all firms, all firms will choose the same action level.

The social planner aims to implement $a_{sp}$. In equilibrium, aggregate emissions must equal the total permits issued, $Q$. Since all firms choose $a_{sp}$, aggregate emissions are:

$$\int_0^1 \mathcal{E}(a_{sp})di = \mathcal{E}(a_{sp}). \tag{A124}$$

Therefore, to implement the social planner's solution $Q$ must solve:

$$Q = \mathcal{E}(a_{sp}). \tag{A125}$$

Substituting $a_{sp}$ into the firm's FOC:

$$\pi_{aa}(a_{sp} - A_0) - p_{permit}\mathcal{E}'(a_{sp}) = 0. \tag{A126}$$

Solving for the equilibrium permit price, $p_{permit}$, and using the result of Proposition 4:

$$p_{permit} = \frac{\pi_{aa}(a_{sp} - A_0)}{\mathcal{E}'(a_{sp})}. \tag{A127}$$

Thus, the equilibrium permit price, $p_{permit}$, is identical to the Pigouvian tax, $T$. $\square$

## A.15 Proof of Proposition 5

A household with signal $y_i$ compares its expected utility under the two regimes:

1. **UBR:** $\mathbb{E}_i[U(c_i^{te}, \tilde{\theta}, \tilde{A}^{te})]$, where $c_i^{te} = e + \pi(a_i) + (a_i - \tilde{A})f(\tilde{q})$. As shown in Proposition 1, $a_i$ is the team-efficient action under UBR. Furthermore, from Proposition 1, we notice that the regulatory function can also be written as:

$$f(\tilde{q}) = \frac{U_A(\tilde{\theta}, \tilde{A})}{U_c} = \frac{U_A(\tilde{\theta}, a_{sp}) + U_{AA}(\tilde{A} - a_{sp})}{U_c}. \tag{A128}$$

2. **Social Planner:** $\mathbb{E}_i[U(c_i^{sp}, \tilde{\theta}, a_{sp})]$, where $c_i^{sp} = e + \pi(a_{sp})$.

The household votes for the social planner if the second expression is greater than the first; otherwise, it votes for UBR. Define the difference in utility for household $i$ as:

$$\Delta U_i := U(c_i^{te}, \tilde{\theta}, \tilde{A}^{te}) - U(c_i^{sp}, \tilde{\theta}, a_{sp}). \tag{A129}$$

Take a second-order Taylor expansion of $U(c_i^{te}, \tilde{\theta}, \tilde{A}^{te})$ around the social planner's solution:

$$\begin{aligned} U(c_i^{te}, \tilde{\theta}, \tilde{A}^{te}) &= U(c_i^{sp}, \tilde{\theta}, a_{sp}) + [U_A(\tilde{\theta}, a_{sp}) + U_c \pi_a(a_{sp})](a_i - a_{sp}) \\ &\quad - \frac{1}{2}U_{AA}(\tilde{A} - a_{sp})^2 + \frac{1}{2}U_c \pi_{aa}(a_i - a_{sp})^2 + U_{AA}(\tilde{A} - a_{sp})(a_i - a_{sp}). \end{aligned} \tag{A130}$$

Substituting this expansion into $\Delta U_i$, and using the fact that $\pi_a(a_{sp}) = \pi_{aa} \cdot (a_{sp} - A_0)$, we get:

$$\begin{aligned} \Delta U_i &= [U_A(\tilde{\theta}, a_{sp}) + U_c \pi_{aa}(a_{sp} - A_0)](a_i - a_{sp}) \\ &\quad - \frac{1}{2}U_{AA}(\tilde{A} - a_{sp})^2 + \frac{1}{2}U_c \pi_{aa}(a_i - a_{sp})^2 + U_{AA}(\tilde{A} - a_{sp})(a_i - a_{sp}). \end{aligned} \tag{A131}$$

The household's voting decision is based on $\mathbb{E}_i[\Delta U_i]$. Given household $i$'s information set $\{y_i, z\}$, $a_{sp}$ is known, and so is $a_i$. The only random variables are $\tilde{\theta}$ and $\tilde{A}$. Taking the expectation:

$$\begin{aligned} \mathbb{E}_i[\Delta U_i] &= \mathbb{E}_i[U_A(\tilde{\theta}, a_{sp}) + U_c \pi_{aa}(a_{sp} - A_0)](a_i - a_{sp}) \\ &\quad - \frac{1}{2}U_{AA}\mathbb{E}_i[(\tilde{A} - a_{sp})^2] + \frac{1}{2}U_c \pi_{aa}(a_i - a_{sp})^2 + U_{AA}\mathbb{E}_i[(\tilde{A} - a_{sp})](a_i - a_{sp}). \end{aligned} \tag{A132}$$

Make the following substitutions:

$$U_A(\tilde{\theta}, a_{sp}) = U_A(\mu_\theta, a_{sp}) + U_{\theta A}(\tilde{\theta} - \mu_\theta), \tag{A133}$$

$$\mathbb{E}_i[\tilde{A} - a_{sp}] = \frac{1}{\alpha}(a_i - a_{sp}) - \frac{1-\alpha}{\alpha}\mathbb{E}_i[a^*(\theta) - a_{sp}], \tag{A134}$$

where the the latter comes from Lemma 4: $a_i = (1-\alpha)\mathbb{E}_i[a^*(\theta)] + \alpha\mathbb{E}_i[\tilde{A}]$. Thus:

$$\mathbb{E}_i[\Delta U_i] = [U_A(\mu_\theta, a_{sp}) + U_{\theta A}\mathbb{E}_i[\tilde{\theta} - \mu_\theta] + U_c\pi_{aa}(a_{sp} - A_0)](a_i - a_{sp}) - \frac{1}{2}U_{AA}\mathbb{E}_i[(\tilde{A} - a_{sp})^2]$$
$$+ \frac{1}{2}U_c\pi_{aa}(a_i - a_{sp})^2 + U_{AA}\left[\frac{1}{\alpha}(a_i - a_{sp}) - \frac{1-\alpha}{\alpha}\mathbb{E}_i[a^*(\theta) - a_{sp}]\right](a_i - a_{sp}). \tag{A135}$$

Using the definition of $\alpha = -\frac{U_{AA}}{U_c\pi_{aa}}$ and simplifying:

$$\mathbb{E}_i[\Delta U_i] = \left\{U_A(\mu_\theta, a_{sp}) + U_{\theta A}\mathbb{E}_i[\tilde{\theta} - \mu_\theta] + U_c\pi_{aa}(a_{sp} - A_0) + (U_{AA} + U_c\pi_{aa})\mathbb{E}_i[a^*(\theta) - a_{sp}]\right\}(a_i - a_{sp})$$
$$- \frac{1}{2}U_c\pi_{aa}(a_i - a_{sp})^2 - \frac{1}{2}U_{AA}\mathbb{E}_i[(\tilde{A} - a_{sp})^2]. \tag{A136}$$

From the first-best solution (Lemma 3) and equation (A78), we can write:

$$\mathbb{E}_i[a^*(\theta) - a_{sp}] = \mathbb{E}_i[a^*(\theta) - A_0] - (a_{sp} - A_0) = \frac{-U_A(\mu_\theta, A_0) - U_{\theta A}\mathbb{E}_i[\tilde{\theta} - \mu_\theta]}{U_c\pi_{aa} + U_{AA}} - (a_{sp} - A_0). \tag{A137}$$

Substituting and simplifying leads to:

$$\mathbb{E}_i[\Delta U_i] = \{U_A(\mu_\theta, a_{sp}) - U_A(\mu_\theta, A_0) - U_{AA}(a_{sp} - A_0)\}(a_i - a_{sp})$$
$$- \frac{1}{2}U_c\pi_{aa}(a_i - a_{sp})^2 - \frac{1}{2}U_{AA}\mathbb{E}_i[(\tilde{A} - a_{sp})^2]. \tag{A138}$$

Because $U_A(\mu_\theta, a_{sp}) = U_A(\mu_\theta, A_0) + U_{AA}(a_{sp} - A_0)$ the first term in the above equation cancels out and we obtain

$$\mathbb{E}_i[\Delta U_i] = -\frac{1}{2}U_c\pi_{aa}(a_i - a_{sp})^2 - \frac{1}{2}U_{AA}\mathbb{E}_i[(\tilde{A} - a_{sp})^2]. \tag{A139}$$

Since $U_c > 0$, $\pi_{aa} < 0$, and $U_{AA} < 0$, we have $\mathbb{E}_i[\Delta U_i] > 0$. □

# B Derivations for the Model with Distrust (Section 4)

We examine how distrust affects the equilibrium. Individual learning about $\tilde{\theta}$ remains unchanged, but learning about the aggregate, $\tilde{A}$, is altered. Under the conjecture (29), where hats denote potentially modified coefficients, household $i$ perceives the aggregate action as:

$$\tilde{A}_i = \widehat{\beta}_0 + \widehat{\beta}_y \left( \int_j y_j^i dj - \mu_\theta \right) + \widehat{\beta}_z (z - \mu_\theta) \tag{B1}$$

$$= \widehat{\beta}_0 + \widehat{\beta}_y (\varphi(\tilde{\theta} - \mu_\theta) + \sqrt{1 - \varphi^2} \tilde{\phi}_i) + \widehat{\beta}_z (z - \mu_\theta). \tag{B2}$$

Consequently, household $i$'s expectation of the aggregate action is:

$$\mathbb{E}_i[\tilde{A}_i] = \widehat{\beta}_0 + \widehat{\beta}_y \varphi (\mathbb{E}_i[\tilde{\theta}] - \mu_\theta) + \widehat{\beta}_z (z - \mu_\theta). \tag{B3}$$

We proceed by revisiting the relevant results.

**Lemma 2 (Status quo allocation):** Unchanged.

**Lemma 3 (First-Best allocation):** Unchanged.

**Lemma 4 (Team-efficient allocation):** Equation (A18), reproduced here, remains valid:

$$\mathbb{E}_i \left[ U_c \pi_a (a_i^{te}(y_i, z)) \right] + \mathbb{E}_i \left[ U_A \Big( e + \pi(\tilde{A}^{te}(\tilde{\theta}, z)), \tilde{\theta}, \tilde{A}^{te}(\tilde{\theta}, z) \Big) \right] = 0. \tag{B4}$$

Taking the derivative and rearranging, we obtain

$$U_c \pi_{aa} \mathbb{E}_i[a_i^{te} - a^*(\theta)] + U_{AA} \mathbb{E}_i[\tilde{A}^{te} - a^*(\theta)] = 0, \tag{B5}$$

which, as in Lemma 4, implies

$$a_i^{te} = \left( 1 + \frac{U_{AA}}{U_c \pi_{aa}} \right) \mathbb{E}_i[a^*(\theta)] - \frac{U_{AA}}{U_c \pi_{aa}} \mathbb{E}_i[\tilde{A}^{te}]. \tag{B6}$$

Recall from Lemma 3 that $a^*(\theta) = \kappa_0^* + \kappa_1^*(\theta - \mu_\theta)$. Thus, $\mathbb{E}_i[a^*(\theta)]$ is unaffected by distrust. However, $\mathbb{E}_i[\tilde{A}^{te}]$ depends on the trust parameter, $\varphi$. In the limiting case of complete distrust ($\varphi = 0$), $\mathbb{E}_i[\tilde{A}^{te}]$ becomes independent of $\tilde{\theta}$, as seen from equation (B3). Still, the key result from Lemma 4, given in equation (16), holds, and firms' actions remain strategic substitutes.

**Lemma 5 (Welfare Ranking):** Unchanged.

**Proposition 1 (Team efficiency of UBR):** The function $f(\tilde{q})$ must still satisfy (A43):

$$\mathbb{E}_i[f(\tilde{q})] = \frac{U_A(\mu_\theta, A_0)}{U_c} + \frac{U_{\theta A}}{U_c} (\mathbb{E}_i[\tilde{\theta}] - \mu_\theta) + \frac{U_{AA}}{U_c} (\mathbb{E}_i[\tilde{A}^{te}] - A_0). \tag{B7}$$

This yields the same functional form as in Proposition 1:

$$f(\tilde{q}) = \mathrm{MRS} - \mathrm{ERA}(\tilde{q} - \mu_\theta). \tag{B8}$$

Therefore, UBR is robust to distrust among economic agents.

**Proposition 2 (Cournot equivalence):** Unchanged.

**Proposition 3 (Equilibrium action):** Under distrust, the coefficients of the conjectured equilibrium action, given by equation (29), are modified. To further highlight the impact of distrust (see also equations (44)–(47)), we express these coefficients in terms of $\kappa_0^*$, $\kappa_1^*$, and $\alpha$, which remain unchanged from the no-distrust case. The modified coefficients $\widehat{\beta}_0$, $\widehat{\beta}_y$, and $\widehat{\beta}_z$ are:

$$\widehat{\beta}_0 = \kappa_0^*, \tag{B9}$$

$$\widehat{\beta}_y = \frac{(1-\alpha)\kappa_1^* \sigma_\theta^2 \tau_y}{1 + \sigma_\theta^2(\tau_y + \tau_z - \alpha\tau_y\varphi)}, \tag{B10}$$

$$\widehat{\beta}_z = \frac{\kappa_1^* \sigma_\theta^2 \tau_z}{1 + \sigma_\theta^2(\tau_y + \tau_z - \alpha\tau_y\varphi)}. \tag{B11}$$

When $\varphi = 1$, we fall back on the original coefficients of Proposition 3 (see equations (A66)-(A68)).

**Corollary 3.1 (Strategic substitutability and information weighting):** Unchanged.

**Corollary 3.2 (Information sensitivity):** Points (a) and (b) of the corollary are unchanged. For point (c), the sign of dependence of the information sensitivity on the precision of public information depends on the trust parameter $\varphi$. To see this, write

$$\frac{\partial \mathcal{B}}{\partial \tau_z} = \frac{\kappa_1^* \sigma_\theta^2(-1 + \alpha\sigma_\theta^2\tau_y(-1+\varphi))}{(1 + \sigma_\theta^2(\tau_y + \tau_z - \alpha\tau_y\varphi))^2}. \tag{B12}$$

The sign of this derivative depends on the sign of $(-1 + \alpha\sigma_\theta^2\tau_y(-1+\varphi))$. Since $\kappa_1^* < 0$, it follows that $\partial\mathcal{B}/\partial\tau_z > 0$ if and only if

$$(-1 + \alpha\sigma_\theta^2\tau_y(-1+\varphi)) < 0, \tag{B13}$$

or, equivalently, if and only if

$$\varphi > 1 - \frac{1}{(-\alpha)\sigma_\theta^2\tau_y}. \tag{B14}$$

Finally, we check the dependence of $\mathcal{B}$ on the trust parameter $\varphi$:

$$\frac{\partial \mathcal{B}}{\partial \varphi} = -\frac{\alpha \kappa_1^* \sigma_\theta^4 \tau_y (\tau_y - \alpha \tau_y + \tau_z)}{(1 + \sigma_\theta^2(\tau_y + \tau_z - \alpha \tau_y \varphi))^2} < 0. \tag{B15}$$

**Corollary 3.3 (Incentives for Information Acquisition):** Equation (A89) remains valid under distrust:

$$\Delta \mathbb{E}[U|z] = \frac{-U_c}{2\pi_{aa}} \left( \mathrm{Var}[\mathbb{E}_i'[f(\tilde{q})]|z] - \mathrm{Var}[\mathbb{E}_i[f(\tilde{q})]|z] \right) > 0. \tag{B16}$$

Thus, firms have a strict incentive to increase the precision of their private information.

The incentive for information acquisition is driven by the difference in the variance of conditional expectations, which we denote by $\Delta\mathcal{V}$. In the presence of distrust, this term becomes a function of both the improved precision, $\xi$, and the degree of trust, $\varphi$. We now analyze how this incentive, $\Delta\mathcal{V}(\xi, \varphi)$, changes with the trust parameter $\varphi$. Recall that $\mathbb{E}_i'$ denotes the expectation after the firm increases the precision of its private signal $y_i$ by a factor of $\xi > 1$, and $\mathbb{E}_i$ is the expectation with the original precision.

First, we substitute $\tilde{A}_i = \widehat{\beta}_0 + \widehat{\beta}_y(\varphi(\tilde{\theta} - \mu_\theta) + \sqrt{1-\varphi^2}\tilde{\phi}_i) + \widehat{\beta}_z(z - \mu_\theta)$ into the expression for $f(\tilde{q})$:

$$f(\tilde{q}) = \frac{U_A(\mu_\theta, A_0)}{U_c} + \frac{U_{\theta A}}{U_c}(\tilde{\theta} - \mu_\theta) + \frac{U_{AA}}{U_c}(\tilde{A} - A_0) \tag{B17}$$

$$\begin{aligned} &= \frac{U_A(\mu_\theta, A_0)}{U_c} + \frac{U_{\theta A}}{U_c}(\tilde{\theta} - \mu_\theta) \\ &+ \frac{U_{AA}}{U_c}\left( \widehat{\beta}_0 + \widehat{\beta}_z(z - \mu_\theta) + \widehat{\beta}_y(\varphi(\tilde{\theta} - \mu_\theta) + \sqrt{1-\varphi^2}\tilde{\phi}_i) - A_0 \right). \end{aligned} \tag{B18}$$

Next, we compute the conditional expectations $\mathbb{E}_i'[f(\tilde{q})|z]$ and $\mathbb{E}_i[f(\tilde{q})|z]$. Given that $z$ is observed, the only remaining random variables in (B18) are $\tilde{\theta}$ and $\tilde{\phi}_i$. We have $\mathbb{E}[\tilde{\phi}_i|z] = 0$. The conditional expectation of $\tilde{\theta}$ given $z$ and $y_i$ (with precision $\tau_y$) is given in equation (8):

$$\mathbb{E}[\tilde{\theta}|z, y_i] = \mu_\theta + \frac{\tau_y}{\sigma_\theta^{-2} + \tau_y + \tau_z}(y_i - \mu_\theta) + \frac{\tau_z}{\sigma_\theta^{-2} + \tau_y + \tau_z}(z - \mu_\theta). \tag{B19}$$

When the precision of $y_i$ increases by a factor of $\xi$, we replace $y_i$ with $y_i'$ and $\tau_y$ with $\xi\tau_y$. Thus:

$$\mathbb{E}'[\tilde{\theta}|z, y_i'] = \mu_\theta + \frac{\xi\tau_y}{\sigma_\theta^{-2} + \xi\tau_y + \tau_z}(y_i - \mu_\theta) + \frac{\tau_z}{\sigma_\theta^{-2} + \xi\tau_y + \tau_z}(z - \mu_\theta). \tag{B20}$$

Substituting these into the expression (B18) for $f(\tilde{q})$, we are interested in the coefficients of $y_i$ in the resulting expression, as the variance with respect to $y_i$ is what determines the difference

$\Delta\mathcal{V}(\xi,\varphi) = \mathrm{Var}[\mathbb{E}_i'[f(\tilde{q})]|z] - \mathrm{Var}[\mathbb{E}_i[f(\tilde{q})]|z]$. Let $c_i'$ and $c_i$ be the coefficients of $y_i$ in $\mathbb{E}_i'[f(\tilde{q})|z]$ and $\mathbb{E}_i[f(\tilde{q})|z]$, respectively. These coefficients are

$$c_i' = \frac{\xi\sigma_\theta^2\tau_y(U_{\theta A} + U_{AA}\widehat{\beta}_y\varphi)}{U_c(1+\sigma_\theta^2(\xi\tau_y+\tau_z))} \qquad \text{and} \qquad c_i = \frac{\sigma_\theta^2\tau_y(U_{\theta A} + U_{AA}\widehat{\beta}_y\varphi)}{U_c(1+\sigma_\theta^2(\tau_y+\tau_z))}. \tag{B21}$$

We also need the conditional variances of $y_i$ and $y_i'$ given $z$. Using standard results for conditional distributions of jointly normal variables:

$$\mathrm{Var}[y_i'|z] = \frac{1}{\xi\tau_y} + \frac{\sigma_\theta^2}{1+\sigma_\theta^2\tau_z} \qquad \text{and} \qquad \mathrm{Var}[y_i|z] = \frac{1}{\tau_y} + \frac{\sigma_\theta^2}{1+\sigma_\theta^2\tau_z}. \tag{B22}$$

The difference in variances of the expectations is then:

$$\Delta\mathcal{V}(\xi,\varphi) = (c_i')^2\mathrm{Var}[y_i'|z] - c_i^2\mathrm{Var}[y_i|z] \tag{B23}$$

$$= \frac{(\xi-1)\tau_y(U_{\theta A} + U_{AA}\widehat{\beta}_y\varphi)^2}{U_c^2(\sigma_\theta^{-2}+\tau_y+\tau_z)(\sigma_\theta^{-2}+\xi\tau_y+\tau_z)}. \tag{B24}$$

This expression is positive since $\xi > 1$. Substitute the solution for $\widehat{\beta}_y$ from equation (46):

$$\beta_y = -\frac{U_{\theta A}\tau_y}{U_c\pi_{aa}(\sigma_\theta^{-2}+\tau_y+\tau_z) + U_{AA}\tau_y\varphi}, \tag{B25}$$

simplifying, then taking the derivative of this expression with respect to $\varphi$ leads to:

$$\frac{\partial}{\partial\varphi}\Delta\mathcal{V}(\xi,\varphi) = -\frac{2U_{AA}U_{\theta A}^2(\xi-1)\pi_{aa}^2\tau_y^2(\sigma_\theta^{-2}+\tau_y+\tau_z)}{(\sigma_\theta^{-2}+\xi\tau_y+\tau_z)(U_c\pi_{aa}(\sigma_\theta^{-2}+\tau_y+\tau_z)+U_{AA}\tau_y\varphi)^3}. \tag{B26}$$

Under the assumptions $U_{\theta A} < 0$, $\pi_{aa} < 0$, $\sigma_\theta > 0$, $\tau_y > 0$, $\tau_z > 0$, $U_c > 0$, $0 < \varphi < 1$, $U_{AA} < 0$, and $\xi > 1$, this derivative is negative. Therefore, the incentive for information acquisition increases as trust ($\varphi$) decreases. $\square$

## B.1 Proof of Proposition 7

We examine how distrust affects the results from Section 3.

**Proposition 4 (Social Planner's solution):** Unchanged.

**Corollary 4.1 (Pigouvian tax on emissions):** Unchanged.

**Corollary 4.2 (Cap-and-Trade implementation):** Unchanged.

**Proposition 5 (Political viability of UBR):** Unchanged. □

# C Alternative Model with Exponential Utility

This appendix outlines an alternative model specification based on exponential utility to illustrate the robustness of the regulatory mechanism harnessing uncertainty and disagreement, similar in spirit to the main analysis. The detailed derivations and analysis for this model specification can be found in a previous version of this paper, available from the authors upon request.

**Model Setup** Consider a two-date economy ($t = 0, 1$) with a continuum of household-firm pairs indexed by $i \in [0, 1]$. Households have CARA utility over consumption ($c_{0i}, \tilde{c}_{1i}$) and the aggregate outcome ($\tilde{q}$):

$$\max_{k_i} -e^{-\rho_c c_{0i}} - \beta \mathbb{E}_i \left[ e^{-\rho_c \tilde{c}_{1i} - \rho_q \tilde{q}} \right] \tag{C1}$$

subject to budget constraints $c_{0i} = w_0 - k_i$ and $\tilde{c}_{1i} = (1 + r_i)k_i + \tilde{\pi}_i$, where $r_i$ is the firm-specific equilibrium cost of capital and $\tilde{\pi}_i$ is the firm's profit. The aggregate outcome evolves as (derived from an assumed GBM for the underlying state):

$$\tilde{q} = \mu_0 + \tilde{\mu}_a - \frac{1}{2}\sigma^2 + \tilde{\varepsilon}, \quad \text{with } \tilde{\varepsilon} \sim \mathcal{N}(0, \sigma^2). \tag{C2}$$

The underlying trend $\tilde{\mu}_a$ depends on the aggregate firm action $\tilde{\Gamma} \equiv \int_0^1 \gamma_i \, di$:

$$\tilde{\mu}_a \equiv \tilde{\mu} + (\tilde{\Gamma} - \Gamma_0). \tag{C3}$$

The intrinsic trend $\tilde{\mu} \sim \mathcal{N}(0, \sigma_\mu^2)$ is unknown. Similar to the main model, agents observe a private signal $y_i = \tilde{\mu} + \tilde{\varepsilon}_{y,i}$ and a public signal $z = \tilde{\mu} + \tilde{\varepsilon}_z$, where $\tilde{\varepsilon}_{y,i} \sim \mathcal{N}(0, \tau_y^{-1})$ and $\tilde{\varepsilon}_z \sim \mathcal{N}(0, \tau_z^{-1})$. The posterior precision and posterior mean are:

$$\tau = \sigma_\mu^{-2} + \tau_y + \tau_z \qquad \text{and} \qquad \mathbb{E}_i[\tilde{\mu} \mid y_i, z] = \frac{\tau_y}{\tau} y_i + \frac{\tau_z}{\tau} z. \tag{C4}$$

Agents may distrust others' signals ($\varphi \in [0, 1]$), believing $y_j^i = \varphi\tilde{\mu} + \sqrt{1 - \varphi^2}\tilde{\phi}_i + \tilde{\varepsilon}_{y,j}$. This specification mirrors the distrust mechanism introduced in Section 4 (cf. eq. (43)).
Firms choose capital $k_i$ and an action $\gamma_i$. Their realized profits include a regulatory term:

$$\tilde{\pi}_i = Ak_i^\alpha - (1 + r_i)k_i - \frac{g}{2}(\gamma_i - \Gamma_0)^2 + \zeta(\tilde{\Gamma} - \gamma_i)\tilde{q}, \quad \text{with } \; A, g, \zeta > 0. \tag{C5}$$

The regulatory term $\zeta(\tilde{\Gamma} - \gamma_i)\tilde{q}$ is analogous to the UBR mechanism (eq. (19)), with $\zeta$ modulating the regulatory intensity.

Firms maximize expected profits:

$$\mathbb{E}_i[\tilde{\pi}_i] = Ak_i^\alpha - (1 + r_i)k_i - \frac{g}{2}(\gamma_i - \Gamma_0)^2 + \zeta\mathbb{E}_i[\tilde{\Gamma}\tilde{q}] - \zeta\gamma_i\mathbb{E}_i[\tilde{q}]. \tag{C6}$$

**Equilibrium Firm Action** The firm's choice of action $\gamma_i^*$ is determined independently of capital investment. Taking the first-order condition of (C6) with respect to $\gamma_i$ yields:

$$\gamma_i^* - \Gamma_0 = -\frac{\zeta}{g}\mathbb{E}_i[\tilde{q}], \tag{C7}$$

where the expected aggregate outcome is

$$\mathbb{E}_i[\tilde{q}] = \mu_0 + \mathbb{E}_i[\tilde{\mu}] + (\mathbb{E}_i[\tilde{\Gamma}] - \Gamma_0) - \frac{\sigma^2}{2}. \tag{C8}$$

We conjecture a linear solution for $\gamma_i^*$:

$$\gamma_i^* = \theta_0 + \theta_\mu\mu_0 + \theta_y y_i + \theta_z z. \tag{C9}$$

**Proposition C.1.** *Firm i's optimal action $\gamma_i^*$ and aggregate action $\tilde{\Gamma} = \int_0^1 \gamma_i^* di$ are:*

$$\gamma_i^* = \theta_0 + \theta_\mu\mu_0 + \theta_y y_i + \theta_z z \tag{C10}$$

$$\tilde{\Gamma} = \theta_0 + \theta_\mu\mu_0 + \theta_y\tilde{\mu} + \theta_z z, \tag{C11}$$

*where*

$$\theta_0 = \Gamma_0 + \frac{\zeta\sigma^2}{2(\zeta + g)} > 0, \quad \theta_\mu = -\frac{\zeta}{\zeta + g} < 0, \tag{C12}$$

$$\theta_y = -\frac{\zeta\tau_y}{\varphi\zeta\tau_y + g\tau} < 0, \quad \textit{and} \quad \theta_z = -\frac{\zeta g\tau_z}{(\varphi\zeta\tau_y + g\tau)(\zeta + g)} < 0, \tag{C13}$$

*and where, by the definition of private signals $y_i$ in equation (5), $\tilde{\mu} = \int_0^1 y_i di$.*

Proposition C.1 shows that, as in the main model (Proposition 3), the optimal individual action strategy ($\gamma_i^*$) is linear in private ($y_i$) and public ($z$) signals. Aggregating these choices leads to an aggregate action level ($\tilde{\Gamma}$) also depending linearly on the underlying trend ($\tilde{\mu}$) and public signal ($z$), mirroring the structure for $\tilde{A}$ in the main text (eq. (34)). The coefficients governing these relationships ($\theta$'s vs $\beta$'s) reflect sensitivities to information and the impact of the regulatory design in each framework.

**Information Sensitivity and Strategic Interactions** The equilibrium aggregate action $\tilde{\Gamma}$ responds to the underlying trend $\tilde{\mu}$. We define the *information sensitivity* $\delta$ as the magnitude of this response (analogous to $\mathcal{B} := |\beta_y + \beta_z|$ used in the main model, eq. (36)):

$$\delta := |\theta_y + \theta_z| = \frac{\zeta[\zeta\tau_y + g(\tau_y + \tau_z)]}{(\zeta + g)[\varphi\zeta\tau_y + g(\sigma_\mu^{-2} + \tau_y + \tau_z)]}. \tag{C14}$$

The sensitivity $\delta$ increases with greater prior uncertainty about the trend ($\sigma_\mu$), higher precision of private information ($\tau_y$), stronger regulation ($\zeta$), and greater distrust among agents (lower $\varphi$). The effect of public information precision ($\tau_z$) is positive, provided private information precision or distrust are not excessively high (specifically, if $\tau_y < g/[\zeta\sigma_\mu^2(1-\varphi)]$). These findings parallel the results in the main model (Corollary 3.2 and Proposition 6), where sensitivity $\mathcal{B}$ also increases with prior uncertainty ($\sigma_\theta$), information precision ($\tau_y, \tau_z$ under conditions), and distrust.

Similar to the main model, firms' actions in this specification are strategic substitutes. This leads firms to overweight their private information relative to the public signal when making decisions about their action, compared to a simple Bayesian benchmark:

$$\frac{\theta_y}{\theta_z} = \frac{g + \zeta}{g}\frac{\tau_y}{\tau_z} > \frac{\tau_y}{\tau_z}. \tag{C15}$$

This mirrors the result in Corollary 3.1 of the main model, where the ratio $\beta_y/\beta_z = (1-\alpha)(\tau_y/\tau_z)$ also showed overweighting due to strategic substitutability (since $\alpha < 0$). The factor $(g + \zeta)/g$ here is analogous to $(1 - \alpha)$ in the main model.

In summary, this alternative CARA model yields equilibrium dynamics for the firm's action, information sensitivity, and strategic interactions structurally consistent with the findings from the main paper's linear-quadratic (LQ) approximation. This supports the paper's central theme: regulatory mechanisms can effectively harness uncertainty and disagreement.

**Capital Investment, Welfare, and Information Acquisition** Further analysis of this alternative CARA model, particularly for equilibrium capital investment ($k_i^*$), welfare comparisons, and information acquisition incentives, generally requires numerical methods due to complex expectations under exponential utility. This contrasts with the main paper's LQ framework, which often permits analytical solutions. Despite this difference in tractability, numerical investigation of the CARA model confirms the robustness of the main paper's core findings:

(a) **Equilibrium Capital Investment:** The equilibrium capital $k_i^*$ is determined implicitly and varies with agents' private information $y_i$. Numerical results suggest regulation ($\zeta > 0$) tends to crowd out capital investment in favour of investment in the action, an effect not explicitly captured in the main model's framework which focuses solely on the cost of the action $\pi(\gamma_i)$.

(b) **Welfare Analysis:** Numerical welfare comparisons with an information-constrained social planner show the decentralized regulated outcome can yield higher welfare, especially with adverse underlying trends ($\tilde{\mu} < 0$) or high distrust ($\varphi = 0$). There typically exists an optimal level of regulatory stringency ($\zeta^*$) maximizing welfare. This echoes the main model's result where UBR achieves team efficiency, dominating a planner restricted to public information (Lemma 5, Proposition 4).

(c) **Political Economy:** Numerical simulations suggest a majority of households may prefer this uncertainty-driven regulation over the planner's uniform policy, particularly under high distrust. This aligns qualitatively with the analytical result of unanimous support for UBR in the main model (Proposition 5), indicating the potential political viability of such mechanisms.

(d) **Information Acquisition:** The CARA model confirms that regulation provides incentives for firms to acquire more precise private information. The gain in expected profit from increasing precision from $\tau_y$ to $\tau_y' = \xi\tau_y$ is:

$$\mathbb{E}_{t<0}[\tilde{\pi}_i'^*] - \mathbb{E}_{t<0}[\tilde{\pi}_i^*] = (\xi - 1)\frac{g\zeta^2\tau_y(\sigma_\mu^{-2} + \tau_y + \tau_z)}{2[g(\sigma_\mu^{-2} + \tau_y + \tau_z) + \zeta\tau_y\varphi]^2(\sigma_\mu^{-2} + \tau_z + \xi\tau_y)}. \tag{C16}$$

This gain is positive if $\xi > 1$ and $\zeta > 0$, and it increases as trust decreases (lower $\varphi$). This directly parallels the findings in the main model (Corollary 3.3 and Proposition 6), where UBR incentivizes information acquisition, and this incentive is strengthened by distrust.

In conclusion, while the CARA model introduces complexities requiring numerical analysis for some aspects, its core results regarding the strategic effects of uncertainty-driven regulation on firms' actions, information aggregation, welfare, and information acquisition incentives align well with the analytical findings of the main paper's LQ framework. This supports the general validity of using regulatory mechanisms to harness uncertainty and disagreement. □