Complementarity of Bank and Market Debt in Dynamic Debt Structure[†]

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Abstract

Firms often issue both bank debt and market debt. I develop a dynamic model of firm debt structure, in which market debt investors are arm's-length and competitive, whereas the bank lender forms a strategic bilateral relationship with the firm. Hence, bank debt is periodically renegotiated by the bank lender and the firm. The firm strategically adjusts its market debt issuance to position its equity holders for renegotiation, while anticipating the effect of renegotiation priced in by the market debt investors. The prevailing bank debt level shapes this strategic incentive by influencing the sensitivity of the equity holders' expected payoff from renegotiation to market debt issuance, thus affecting the firm's market debt issuance intensity. A key novel prediction of the model is the dynamic complementarity of bank and market debt: a higher (lower) amount of bank debt in the debt structure leads to more (less) market debt issuance. The dynamic complementarity is robust to the allocation of bargaining power between the bank lender and the firm, though the distribution of bargaining power affects both the degree of complementarity and the total firm value. Unlike the leverage ratchet effect, the firm in the model may repurchase its market debt. In equilibrium, bank debt is senior to market debt.

Keywords: Debt Structure, Debt Complementarity, Dynamics, Bargaining

1   Introduction

Firms obtain debt financing through two main sources: bank debt and market debt. A firm's debt choice is not a binary decision. Instead, the vast majority of firms issue both bank debt and market debt. For example, Rauh and Sufi (2010) document that 70% of firm-year observations in their sample have debt structures comprising at least two different types of debt instruments, with bank loans and corporate bonds being the two most commonly used debt types. Despite the prevalence of debt structures that include both bank debt and market debt, a satisfactory theory of debt composition has yet to be fully developed. Existing theories typically focus on the choice between bank debt and market debt rather than how firms combine them in their debt structures. A few models that address the latter either are static in nature or rule out incremental adjustment in debt amounts over time (e.g., Park (2000), Hackbarth et al. (2007), Crouzet (2018)).

However, firms in practice dynamically adjust their levels of bank debt and market debt over time, leading to fluctuations in their overall debt composition. In this paper, I develop a dynamic model of firm debt structure. In reality, bank debt and market debt could be different on many dimensions such as monitoring incentives, market segmentation, and contract flexibility. Nonetheless, my model is deliberately parsimonious and abstracts from such dimensions. Instead, I focus on one key distinction between bank debt and market debt: the renegotiability of bank debt. Since bank debt is obtained through a strategic bilateral relationship between the firm and a bank lender, it is subject to frequent renegotiations between them. These renegotiations result in changes to the face amount of bank debt and payments between the bargaining parties. By contrast, market debt is issued or repurchased from arm's-length market debt investors at competitive prices.

The paper focuses on the dynamic interaction between bank debt and market debt over time. A key prediction of the model is the *dynamic complementarity* of bank and market debt, in the sense that a higher current level of bank debt is followed by a greater intensity of market debt issuance by the firm. This prediction is novel to the literature. It is in contrast to the conventional wisdom that bank debt and market debt are substitutes. Intuitively, market debt issuance changes

firm leverage and thereby agents' current values, which in turn affect the equity holders' expected payoff in renegotiation with the bank lender. The firm therefore issues or repurchases market debt strategically to position itself for upcoming renegotiation. The prevailing bank debt level shapes this strategic incentive by influencing how sensitive the equity holders' renegotiation payoff is to market debt issuance, and thus impacts the firm's market debt issuance intensity.

For example, consider the simpler case with a strong bank lender who holds all the bargaining power. When the bank lender and the firm renegotiate, the firm captures no bargaining surplus and receive their current value in the status quo, which is their outside option and credible threat point during renegotiation. Market debt issuance raises leverage and lowers equity value, weakening the equity holders' bargaining position in renegotiation and imposing a strategic cost on the firm. When the firm already carries a high level of bank debt and is close to default, additional leverage from market debt issuance has minimal impact on the equity holders' already diminished value, since the default option allows the equity holders to walk away if repayment is no longer optimal. In other words, additional market debt does little to further weaken the equity holders' already constrained bargaining position, and thus there is limited cost associated with market debt issuance. Consequently, the firm is motivated to increase market debt issuance to capture the proceeds. In effect, a high level of bank debt leaves the equity holders with little to lose, encouraging issuance of market debt to dynamically shift risk onto creditors.

The dynamic complementarity between bank and market debt is robust to how bargaining power is allocated between the bank lender and the firm. That being said, the degree of this complementarity decreases with the firm's bargaining power. When the firm has little bargaining power, the equity holders' outside option is more important as a credible threat during renegotiation. As the firm's bargaining power increases, however, shifting surplus by reducing the bank lender's outside option becomes more pertinent. Since equity holders are the residual claimants, the impact of additional market debt on the equity holders' value is more sensitive to the firm's distance to default compared to the bank debt. When the firm has little bargaining power, it has more incentive to use market debt issuance strategically to influence renegotiation with the bank lender, leading to stronger dynamic complementarity between bank debt and market debt.

I provide suggestive empirical evidence supporting the above predictions. Using granular firm-level data, I show that controlling for firm characteristics, a greater amount of bank debt in the debt structure predicts higher rate of market debt issuance in the subsequent year. This pattern remains robust to alternative empirical specifications as well as using time-series aggregate-level balance sheet data. Moreover, I show that the degree of complementarity between bank and market debt varies across firm characteristics and time periods. The pattern is more pronounced for smaller, less profitable and riskier firms as well as during periods of market stress. I consider potential explanations for this heterogeneity across firms and time, and find that the pattern appears to be driven, at least in part, by differences in firm bargaining power.

Moreover in this model, market debt repurchases are possible. This is in contrast to the leverage ratchet effect of Admati et al. (2018) and DeMarzo and He (2021), which states that the firm never voluntarily repurchases its debt. If the firm is under-levered, bank-firm renegotiation is expected to lead to an increase in the amount of bank debt. Because the equity holders are guaranteed their continuation value (which is their outside option) in the event of renegotiation, the market debt investors bear the brunt of the dilution caused by the increase in bank debt due to renegotiation. The market debt is priced to reflect the expected dilution from bank debt renegotiation and trades at a discount. This creates an opportunity for the firm to strengthen its equity holders' outside option by repurchasing market debt at the depressed price, thereby improving their bargaining position ahead of bank debt renegotiation.

The model also implies that bank debt is senior to market debt in equilibrium.[1] Since the firm cannot commit to future debt choices, it issues market debt until the tax benefit is fully dissipated.[2] If the bank debt is not senior, additional bank debt raises the firm's leverage and shortens its distance to default, thereby hurting the joint value of the equity holders and the bank lender. Thus, it is not optimal for them to issue bank debt that is pari passu with or junior to the market debt. By contrast, when the bank debt is senior, the market debt acts as a buffer upon

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[1]While other mechanisms may also account for the seniority of bank debt, the model offers another plausible explanation.

[2]For example, see DeMarzo and He (2021).

default, absorbing losses before the bank debt is impaired. In this case, issuing bank debt enables the bank lender and the equity holders to extract value by diluting the market debt investors.

At a conceptual level, the model speaks to the efficiency of bank debt. One might expect the renegotiability of bank debt to make it a potentially expensive source of financing due to hold-up problems. Why, then, do firms still rely on banks for borrowing? In this paper, I show that bank debt renegotiations are firm value enhancing not only because periodic renegotiations allow part of the tax benefit to be captured through joint surplus-maximizing bargaining between the bank lender and the firm, but also because the firm's strategic trading of market debt to position for renegotiation mitigates the externalities of bank-firm bargaining on the silent non-bargaining market debt investors.

For instance, when the firm is over-levered, it is in the joint interest of the bank lender and the equity holders to lower leverage through partial forgiveness of the bank debt upon renegotiation. However, part of the benefit from bank debt forgiveness accrues to the non-bargaining market debt investors, who effectively free-ride on the efforts of the two bargaining parties, thereby reducing their incentive to cut leverage during renegotiation. However, the firm's ability to trade market debt to position for renegotiation allows it to extract value from the market debt investors ahead of bank debt renegotiation. In this example, the firm issues additional market debt at a favorable price and captures value by diluting the existing market debt investors. The strategic issuance of market debt in anticipation of bank debt renegotiation thus mitigates the free-riding problem by the market debt investors, thus increasing the total firm value. Since the strategic incentive to trade market debt to position for renegotiation is stronger when the firm has lower bargaining power, the model suggests that assigning bargaining power to the firm reduces the total firm value.

2   The Model

I develop a model of dynamic debt structure. In the model, a firm borrows from both a relationship bank lender and arm's-length market debt investors, while maximizing its equity

holders' value. The firm cannot commit to future debt choices ex-ante. Instead, the firm adjusts its market debt and bank debt over time, leading to dynamic changes in the firm's total leverage and debt composition.

2.1 General Setup

The model is set in continuous time, with $t \in [0, \infty)$. All agents are risk-neutral and discount the future at a constant rate $r > 0$. A firm maximizes the value of its deep-pocketed equity holders. Let Y_t denote the firm's instantaneous earnings before interest and taxes (EBIT) at time t, which evolves according to the geometric Brownian motion

$$\frac{dY_t}{Y_t} = \mu dt + \sigma dZ_t, \tag{2.1}$$

where the drift μ and the volatility σ are positive constants, and dZ_t is the increment of a standard Brownian motion. I assume that $r > \mu$.

The firm borrows from both arm's-length market debt investors and a "relationship" bank lender. The key distinction between the bank lender and the market debt investors is that the former maintains a strategic relationship with the firm. As the firm's circumstances change, the existing bank debt contract can be improved upon to generate gains that benefit both the bank lender and the firm's equity holders, creating an incentive for them to renegotiate. By contrast, the market debt is held by dispersed institutional investors such as mutual funds, insurance companies, pension funds and so on. The dispersed nature of market debt investors gives rise to free-riding and coordination problems, preventing them from engaging in negotiations with the firm. Hence, the market debt investors are arm's-length and competitive.

The firm chooses when to default. Upon default, the firm is assumed to have liquidation value $\alpha Y^\star$, where $Y^\star$ is the firm's EBIT at default and $0 < \alpha < \frac{1-\pi}{r-\mu}$. That is, the liquidation value is a fraction of the unlevered firm value at default. If the firm defaults, the equity holders forfeit their equity ownership and receive nothing in recovery. The liquidation value is divided by the bank lender and the market debt investors subject to the absolute priority rule. The firm pays

corporate taxes on its earnings after interest at a constant rate $\pi \in (0, 1)$. The benefit of debt arises from the tax deductibility of interest expenses.

A. Market Debt

The firm issues a homogeneous class of market debt. The market debt is modeled as a consol with fixed coupon rate $c > 0$ and no stated maturity. Absent frictions, the firm may continuously adjust its market debt outstanding by issuing additional market debt or repurchasing existing market debt. Let M_t denote the firm's face amount of market debt outstanding at time t. The firm issues market debt at an endogenous rate G_t^m such that the firm's market debt outstanding increases by $G_t^m dt$ over the time interval dt. The firm optimally chooses the market debt issuance rate such that the equity holders' value is maximized. Note that G_t^m may be positive or negative, with $G_t^m > 0$ denoting market debt issuance and $G_t^m < 0$ denoting market debt repurchase. The market debt is priced rationally by the market debt investors, and any issuance or repurchase of the market debt occurs at the prevailing competitive price p_t^m.

B. Bank Debt

The bank debt is borrowed from the bank lender through a bilateral financing relationship that is inherently strategic. The bank debt can be renegotiated at Poisson arrival times with intensity $\lambda > 0$.[3] Let B_t denote the firm's face amount of bank debt at time t. Renegotiation of the bank debt leads to an amended face amount $\bar{B}_t$ and a payment $\mathcal{T}_t$ from the bank lender to the equity holders. The payment reflects the net proceeds from issuing (or repurchasing) the bank debt, after accounting for pricing adjustments and amendment fees. Both $\bar{B}_t$ and $\mathcal{T}_t$ are determined through Nash bargaining between the bank lender and the firm, with the firm's bargaining power given by $\theta \in [0, 1]$. The bank debt is also modeled as a consol with fixed coupon rate c. However, the bank debt's face amount can be periodically adjusted through renegotiations between the bank lender and the firm.

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[3]As the firm's circumstances evolve, the bank lender and the firm have an incentive to renegotiate the bank debt contract whenever there are mutual gains from trade. However, due to bargaining frictions, renegotiations can only occur at Poisson times. More generally, the intensity parameter λ is state-dependent (e.g., as a function of the firm's leverage). In the main analysis, I treat λ as a constant, which imparts all the key economics of the model. In Appendix ??, I extend the analysis to state-dependent λ and consider endogenization of λ.

2.2 Value Functions

The model's equilibrium concept is that of Markov perfect equilibrium. There are three payoff-relevant states, namely the firm's EBIT Y_t, the face amount of market debt M_t, and the face amount of bank debt B_t. Agents' values are functions of these three state variables. Let $V^e(Y_t, M_t, B_t)$ denote the equity holders' value function, then

$$
V^e(Y_t, M_t, B_t) = \sup_{\tau^\star, \{G_s^m\}} \mathbb{E}_t \bigg\{ \int_t^{\min\{\tau^\star, \bar{\tau}\}} e^{-r(s-t)} \Big[\underbrace{(1-\pi)\big(Y_s - c(B_s + M_s)\big)}_{\substack{\text{net income} \\ \text{after interest and taxes}}} + \underbrace{p_s^m G_s^m}_{\substack{\text{market debt} \\ \text{issuance}}} \Big] ds
$$
$$
+ e^{-r(\bar{\tau}-t)} \mathbf{1}_{\{\bar{\tau} < \tau^\star\}} \Big[\underbrace{V^e(Y_{\bar{\tau}}, M_{\bar{\tau}}, \bar{B}_{\bar{\tau}}) + \mathcal{T}_{\bar{\tau}}}_{\substack{\text{total payoff from} \\ \text{bank debt renegotiation}}} \Big] \bigg\},
\tag{2.2}
$$

where $\tau^\star$ is the endogenous default time chosen by the firm, and $\bar{\tau}$ is the (stochastic) arrival time of the next bank debt renegotiation.

Given the current states Y_t, M_t, and B_t, the firm chooses the default time $\tau^\star$ and the path of market debt issuance rate $\{G_s^m\}$ to maximize the equity holder' value. Prior to default or bank debt renegotiation, equity holders receive the firm's net income after interest and taxes, and the proceeds from continuous market debt issuance.[4] If the bank debt renegotiation occurs before default, the equity holders receive a payment $\mathcal{T}_{\bar{\tau}}$ from the bank lender, and their continuation value becomes $V^e(Y_{\bar{\tau}}, M_{\bar{\tau}}, \bar{B}_{\bar{\tau}})$ reflecting the amended face amount of bank debt post-renegotiation. If the firm declares default, the equity holders lose their ownership stake and receive zero value in return.

Similarly, let $V^b(Y_t, M_t, B_t)$ denote the bank lender's value function, then

$$
V^b(Y_t, M_t, B_t) = \mathbb{E}_t \bigg\{ \int_t^{\min\{\tau^\star, \bar{\tau}\}} e^{-r(s-t)} c B_s ds + e^{-r(\tau^\star - t)} \mathbf{1}_{\{\tau^\star \le \bar{\tau}\}} V^{b,\star}(Y_{\tau^\star}, M_{\tau^\star}, B_{\tau^\star})
$$
$$
+ e^{-r(\bar{\tau}-t)} \mathbf{1}_{\{\bar{\tau} < \tau^\star\}} \Big[V^b(Y_{\bar{\tau}}, M_{\bar{\tau}}, \bar{B}_{\bar{\tau}}) - \mathcal{T}_{\bar{\tau}} \Big] \bigg\},
\tag{2.3}
$$

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[4]In the rest of the paper, I refer to debt issuance and repurchase collectively as *debt issuance*, since debt repurchase can be thought of as negative debt issuance.

where $V^{b,\star}$ denotes the bank lender's recovery value upon default, which is a function of the state variables at the time of default. Prior to default or bank debt renegotiation, the bank lender receives coupon payments on the outstanding bank debt. If the bank debt renegotiation occurs before default, the bank lender makes a payment $\mathcal{T}_{\bar{\tau}}$ to the equity holders, and their continuation value becomes $V^b(Y_{\bar{\tau}}, M_{\bar{\tau}}, \bar{B}_{\bar{\tau}})$. Upon default, the firm's liquidation value is distributed between the bank lender and the market debt investors and the bank lender receives $V^{b,\star}(Y_{\tau^\star}, M_{\tau^\star}, B_{\tau^\star})$ in recovery.

The firm continuously issues market debt at the prevailing market debt price. In equilibrium, the market debt price p_t^m is a function of the state variables. Let $p^m(Y_t, M_t, B_t)$ denote the equilibrium price of the market debt. Let $p^{m,\star}$ be the market debt's recovery upon default, then

$$
\begin{aligned}
p^m(Y_t, M_t, B_t) = \mathbb{E}_t\bigg\{ &\int_t^{\min\{\tau^\star, \bar{\tau}\}} e^{-r(s-t)} c\, ds + e^{-r(\tau^\star - t)} \mathbf{1}_{\{\tau^\star \leq \bar{\tau}\}} p^{m,\star}(Y_{\tau^\star}, M_{\tau^\star}, B_{\tau^\star}) \\
&+ e^{-r(\bar{\tau}-t)} \mathbf{1}_{\{\bar{\tau} < \tau^\star\}} p^m(Y_{\bar{\tau}}, M_{\bar{\tau}}, \bar{B}_{\bar{\tau}}) \bigg\}.
\end{aligned}
\tag{2.4}
$$

2.3 Bank Debt Renegotiation

When the bank lender and the firm renegotiate the bank debt, they Nash bargain over the face amount of the bank debt and the payment from the bank lender to the firm's equity holders. Suppose that $\{\bar{B}_t, \mathcal{T}_t\}$ are the proposed terms in the bargaining problem. If the firm and the bank lender agree on the terms, the equity holders' value and the bank lender's value become $V^e(Y_t, M_t, \bar{B}_t)$ and $V^b(Y_t, M_t, \bar{B}_t)$ respectively, and the equity holders receive the payment $\mathcal{T}_t$ from the bank lender. By contrast, if no agreement is reached, the amount of bank debt stays unchanged. In this case, the values to the equity holders and the bank lender remain at $V^e(Y_t, M_t, B_t)$ and $V^b(Y_t, M_t, B_t)$, which are their respective outside options upon renegotiation.

If $\{\bar{B}_t, \mathcal{T}_t\}$ are the outcomes of the Nash bargaining problem between the bank lender and the firm with the firm having bargaining power θ, then

$$\{\bar{B}_t, \mathcal{T}_t\} = \underset{B'_t, \mathcal{T}'_t}{\arg\max} \; \left[V^e(Y_t, M_t, B'_t) + \mathcal{T}'_t - V^e(Y_t, M_t, B_t)\right]^\theta \tag{2.5}$$

$$\left[V^b(Y_t, M_t, B'_t) - \mathcal{T}'_t - V^b(Y_t, M_t, B_t)\right]^{1-\theta},$$

subject to $V^e(Y_t, M_t, B'_t) + \mathcal{T}'_t - V^e(Y_t, M_t, B_t) \geq 0$ and $V^b(Y_t, M_t, B'_t) - \mathcal{T}'_t - V^b(Y_t, M_t, B_t) \geq 0$. The bracketed terms on the right-hand side are the trade surpluses captured by the equity holders and the bank lender from the bank debt renegotiation.

The solution to the Nash bargaining problem (2.5) must satisfy two conditions. First, the transaction terms $\{\bar{B}_t, \mathcal{T}_t\}$ are bilaterally Pareto optimal. Intuitively, if the transaction terms are inefficient, one party can always propose an offer that benefits themselves without making the other party worse off. Second, the transaction terms $\{\bar{B}_t, \mathcal{T}_t\}$ are such that the equity holders receive θ fraction of the joint surplus, while the bank lender receives $1 - \theta$ fraction of the joint surplus. The solution to the bargaining problem is summarized in the following proposition, with derivations contained in Appendix A.1.

Lemma 1. *Let $\{\bar{B}_t, \mathcal{T}_t\}$ denote the outcomes of the Nash bargaining problem where $\bar{B}_t$ is the bank debt amount post-renegotiation and $\mathcal{T}_t$ is the payment from the bank lender to the equity holders. Then*

$$\bar{B}_t = \underset{B'_t}{\arg\max} \; \left[V^e(Y_t, M_t, B'_t) + V^b(Y_t, M_t, B'_t)\right], \tag{2.6}$$

and

$$\mathcal{T}_t = \theta\left[V^b(Y_t, M_t, \bar{B}_t) - V^b(Y_t, M_t, B_t)\right] - (1-\theta)\left[V^e(Y_t, M_t, \bar{B}_t) - V^e(Y_t, M_t, B_t)\right]. \tag{2.7}$$

Let $V(Y_t, M_t, B_t)$ denote the sum of the equity holders' value and the bank lender's value, that is $V(Y_t, M_t, B_t) \equiv V^e(Y_t, M_t, B_t) + V^b(Y_t, M_t, B_t)$. The condition (2.6) says that given the firm's EBIT Y_t and market debt outstanding M_t, the renegotiated bank debt amount $\bar{B}_t$ must

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maximize the joint value of the equity holders and the bank lender. $\bar{B}_t$ is thus a function of Y_t and M_t. Using the condition (2.7), one can rewrite the equity holders' payoff from the bank debt renegotiation as

$$V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t = \underbrace{V^e(Y_t, M_t, B_t)}_{\text{outside option}} + \theta \underbrace{\left[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t) \right]}_{\text{joint bargaining surplus}}. \qquad (2.8)$$

In the bargaining game, the equity holders' outside option equals their continuation value under the current bank debt contract, $V^e(Y_t, M_t, B_t)$. This is because if the firm and the bank lender fail to reach an agreement, the status quo prevails and the amount of bank debt remains unchanged. Any renegotiated agreement must provide the equity holders with at least this value in order for them to accept the new terms. That is, the equity holders' outside option (or current continuation value) serves as a credible threat point during renegotiation of the bank debt. Analogously, the bank lender's outside option is also given by its continuation value under the current bank debt contract, $V^b(Y_t, M_t, B_t)$. The joint surplus from bargaining for the equity holders and the bank lender is then the total value generated under the renegotiated agreement in excess of the sum of the two parties' outside options.

The equity holders' total payoff from the renegotiation consists of their outside option $V^e(Y_t, M_t, B_t)$ plus a fraction θ of the joint bargaining surplus they capture, where θ is their bargaining power. It is worth noting that, in the dynamic model, the equity holders' outside option and the joint bargaining surplus are both functions of the prevailing state variables. When the firm has weaker bargaining power, its equity holders capture a smaller share of the joint surplus, making their outside option more important as a credible threat during bargaining. If the bank lender has all the bargaining power (i.e., $\theta = 0$), the equity holders' payoff from the bank debt renegotiation is simply their outside option.

2.4 Market Debt Issuance

The equilibrium market debt price $p^m(Y_t, M_t, B_t)$ is pinned down by the equity holders' first-order condition with respect to market debt issuance, given by

$$p^m(Y_t, M_t, B_t) = -V_M^e(Y_t, M_t, B_t). \tag{2.9}$$

That is, the firm issues market debt until the marginal devaluation of the equity holders' value due to market debt issuance fully offsets the price at which the market debt is issued. In equilibrium, the equity holders gain no value from market debt issuance as long as the firm cannot commit to a market debt leverage policy ex-ante.

Using the pricing condition (2.9), one can obtain the equity holders' optimal market debt issuance rate G_t^m. The result is given in the proposition below, with derivations in Appendix A.2.

Proposition 2. *The firm's optimal rate of market debt issuance is*

$$
\begin{aligned}
G_t^m = &\frac{\pi c}{-p_M^m(Y_t, M_t, B_t)} \\
&+ \lambda \frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t) + \theta\big[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t)\big]}{-p_M^m(Y_t, M_t, B_t)}.
\end{aligned}
\tag{2.10}
$$

The first term on the right-hand side of (2.10) corresponds to the market debt issuance rate if the bank lender and the firm commit never to renegotiate the bank debt once the bank debt is in place, that is $\lambda = 0$. This expression also appears in DeMarzo and He (2021). Absent bank debt renegotiations, the firm issues market debt at a rate such that the tax benefit of market debt issuance is fully dissipated by the negative valuation effect on the market debt investors due to an increase in leverage. However, as the firm's earnings and leverage evolve, the existing amount of bank debt may deviate from the surplus-maximizing level for the bank lender and the firm, creating an incentive for them to renegotiate. Accordingly, the bank lender and the firm cannot credibly commit to refrain from renegotiation ex-ante.

The second term on the right-hand side of (2.10) represents a dynamic correction, capturing how bank debt renegotiation affects the equilibrium rate of market debt issuance. With periodic renegotiations of the bank debt, the firm's market debt issuance policy must balance the marginal proceeds from issuing additional market debt against its impact on the equity holders' payoff from potential renegotiation, which is in turn priced in by the market debt investors. Thus, the firm not only relies on market debt as a source of financing, but also strategically adjusts its market debt to position the equity holders for more favorable outcomes in future bank debt renegotiations. Specifically, the firm takes into account how its market debt issuance influences the bargaining outcome for its equity holders, while anticipating the effect of that bargaining outcome on the market debt price.

2.5 Discrete-Time Example

To build intuition, I present a heuristic discrete-time example. For expositional simplicity, I suppose that the firm's EBIT Y remains constant over the time interval dt. I also set $\pi = 0$ thereby abstracting from any tax benefit of debt. The timing is as follows. At time t, the firm has market debt outstanding M and bank debt outstanding B. It issues an incremental amount of market debt Δ, which is determined endogenously. At time $t + dt$, the bank lender and the firm renegotiate the bank debt contract, resulting in an amended face amount $\bar{B}$ and a payment $\mathcal{T}$ from the bank lender to the equity holders.

The bank debt renegotiation at $t + dt$ determines $\bar{B}$ and $\mathcal{T}$ via Nash bargaining, where the firm has bargaining power θ. As shown in Appendix A.3, the renegotiated bank debt amount $\bar{B}$ maximizes the combined value of the equity holders and the bank lender, and depends on the amount of market debt $M + \Delta$ going into the renegotiation. The payment $\mathcal{T}$, in turn, ensures that the equity holders receive their outside option plus their share of the bargaining surplus.

Thus, the firm's problem is

$$\max_{\Delta} \underbrace{V^e(M+\Delta, B) + \theta\big[V(M+\Delta, \bar{B}(M+\Delta)) - V(M+\Delta, B)\big]}_{\text{equity holders' payoff from bank debt renegotiation}}$$

$$+ \underbrace{\Delta p^m(M+\Delta, \bar{B}(M+\Delta))}_{\text{market debt issuance proceeds}}. \qquad (2.11)$$

At time t, when choosing how much market debt to issue, the firm maximizes its equity holders' total expected value, including their anticipated payoff from the upcoming bank debt renegotiation as well as the proceeds from market debt issuance. The first-order condition implies that the firm's optimal market debt issuance Δ at time t satisfies

$$0 = \underbrace{V_M^e(M+\Delta, B)}_{\text{effect on outside option}} + \underbrace{\theta\big[V_M(M+\Delta, \bar{B}(M+\Delta)) - V_M(M+\Delta, B)\big]}_{\text{effect on bargaining surplus}}$$

$$+ \underbrace{p^m(M+\Delta, \bar{B}(M+\Delta)) + \Delta\big[p_M^m(M+\Delta, \bar{B}(M+\Delta)) + p_B^m(M+\Delta, \bar{B}(M+\Delta))\bar{B}'(M+\Delta)\big]}_{\text{marginal proceeds from market debt issuance}}.$$

$$(2.12)$$

In equilibrium, the proceeds from issuing an additional unit of market debt must be offset by its impact on the equity holders' payoff from the upcoming bank debt renegotiation, consisting of their outside option and their share of the bargaining surplus as captured by the first two terms of (2.12). If the bank lender has all the bargaining power (i.e., $\theta = 0$), the equity holders receive no bargaining surplus and their renegotiation payoff is simply their outside option $V^e(M+\Delta, B)$, which equals the equity holders' value if the amount of bank debt remains unchanged. Issuing additional market debt reduces the firm's distance to default, which lowers the equity holders' current value thereby reducing their outside option in the upcoming bank debt renegotiation. Since the outside option serves as the equity holders' credible threat point in bargaining, a lower outside option implies a weaker bargaining position, which represents a strategic cost of market debt issuance.

When the equity holders possess some bargaining power ($\theta > 0$), market debt issuance affects both their outside option and their share of the bargaining surplus. Issuing market debt brings

the firm closer to default, which tends to lower the amount of bargaining surplus by reducing the combined post-renegotiation value of the equity holders and the bank lender. At the same time, it also decreases the outside options of both parties, which increases the size of the surplus to be split. In the limiting case with $\theta = 1$, the bank lender gets its outside option and the equity holders capture the remaining value. In this case, the equity holders' outside option does not matter and they benefit from actions that raise the combined value or reduce the bank lender's outside option. Compared to the market debt issuance policy that is optimal for the combined value, the equity holders have a strategic incentive to issue more market debt to squeeze the bank lender's bargaining position.

Proposition 3. *The firm's optimal market debt issuance Δ at time t is given by*

$$\Delta = \frac{p^m(M+\Delta, \bar{B}) + V_M^e(M+\Delta, B) + \theta\left[V_M(M+\Delta, \bar{B}) - V_M(M+\Delta, B)\right]}{-p_M^m(M+\Delta, \bar{B})\left[1 + \frac{p_B^m(M+\Delta, \bar{B})}{p_M^m(M+\Delta, \bar{B})}\bar{B}'(M+\Delta)\right]}. \tag{2.13}$$

The equation (2.13) mirrors the market debt issuance policy (2.10) in the continuous-time model.[5] The numerator reflects the net marginal value from issuing market debt, and equals the sum of the forward-looking market debt price (which incorporates the impact due to the upcoming bank debt renegotiation) and the marginal effect of market debt issuance on the equity holders' renegotiation payoff (including both their outside option and their share of the bargaining surplus). The denominator is the sensitivity of market debt price to market debt issuance, incorporating the direct price impact from dilution and the indirect price impact through its effect on the renegotiated bank debt amount.[6]

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[5]The denominator of (2.10) captures the total effect of market debt issuance, as the equilibrium market debt price function $p^m(Y_t, M_t, B_t)$ incorporates the anticipated outcomes of future bank debt renegotiations. Since Δ is of dt order, the second term in the brackets in the denominator is of higher dt order.

[6]Note that (2.13) resembles the markup rule in a standard monopoly problem, as the firm may be viewed as a monopolistic supplier of its risky market debt. However, an important difference is that renegotiation of the bank debt introduces a strategic effect both on the net benefit of market debt issuance and on the sensitivity of market debt price to its issuance. Hence, the problem in this paper is conceptually different.

2.6 Discussion

The model is developed to study debt structure dynamics, with a focus on the dynamic interaction between bank debt and market debt. However, the framework is sufficiently general and can be applied to a variety of other settings. In a nutshell, the model is about the interplay among three parties: two bargaining parties and a silent non-bargaining party. Of the two bargaining parties, one is a "control" bargaining party who can trade directly with the non-bargaining party, while the other is a non-control bargaining party who cannot. In the context of this paper, the equity holders act as the control bargaining party, the bank lender as the non-control bargaining party, and the market debt investors as the non-bargaining party.

The central idea of the model is that the non-bargaining party prices in the expected outcome of the bargaining game between the two bargaining parties, thus shaping its trade with the control bargaining party. Trade between the non-bargaining party and the control bargaining party, in turn, affects the bargaining outcome by altering the bargaining parties' outside options and joint value. This feedback mechanism is illustrated in Figure 1. The control bargaining party trades with the non-bargaining party to strategically position itself for bargaining with the non-control bargaining party, while anticipating how this trade will influence the bargaining outcome, which in turn feeds back into its current trade price with the non-bargaining party.

3 Solution with a Strong Bank

In this section, I specialize the model to the case with a strong bank lender who holds all the bargaining power, that is $\theta = 0$. The bank lender can be thought to make a take-it-or-leave-it offer to the firm when they renegotiate the bank debt. This corner case admits several analytical results while preserving the key strategic forces at play in the general setting, thereby clarifying the economic intuition.

When the bank lender has all the bargaining power, it captures the entire joint surplus from renegotiation, leaving the equity holders with only their outside option – namely their continuation
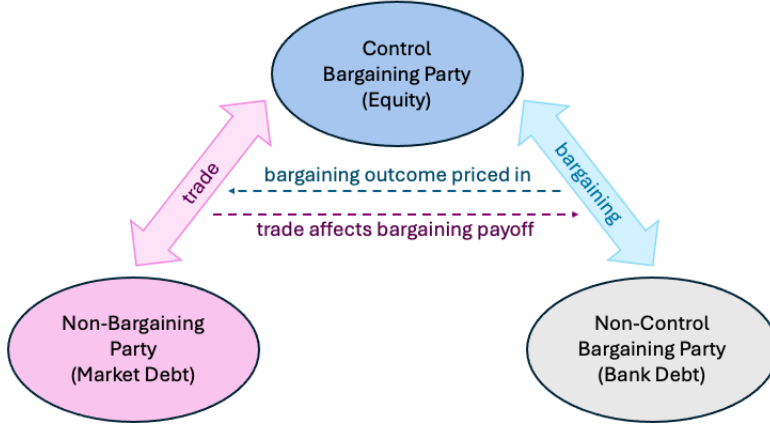
Figure 1: Feedback between trade and bargaining. The stylized diagram illustrates the feedback mechanism between trade and bargaining. The outcome of the bargaining game between the control bargaining party (e.g., equity holders) and the non-control bargaining party (e.g., bank lender) is priced in by the non-bargaining party (e.g., market debt investors), thereby affecting trade between the non-bargaining party and the control bargaining party, which in turn influences the bargaining outcome by altering the bargaining parties' outside options and joint value.

value under the status quo. That is, bank debt renegotiations do not affect the equity holders' value. Moreover, the equity holders do not gain from market debt issuance in equilibrium, as any potential tax benefit is fully dissipated by the adverse valuation impact of increased leverage. Consequently, the equity holders' value can be solved as if the firm neither issues market debt nor renegotiates its bank debt.

With levels of bank debt and market debt fixed over time, the only state variable is the firm's EBIT Y_t. The equity holders' value is then characterized by an ordinary differential equation (ODE), subject to the no-bubble condition as $Y_t \to \infty$ as well as the value matching and smooth-pasting conditions at the endogenous default boundary $Y^\star(M_t, B_t)$ – the EBIT threshold at which the equity holders optimally default. The following proposition provides the closed-form expression for the equity holders' value when $\theta = 0$. Detailed derivations are provided in Appendix A.4.

Proposition 4. *In the case with $\theta = 0$, the equity holders' value is given by*

$$
\begin{aligned}
V^e(Y_t, M_t, B_t) =& \frac{1-\pi}{r-\mu} Y_t - \frac{(1-\pi)c}{r}(B_t + M_t) \\
& + \frac{1}{1+\gamma} \frac{(1-\pi)c}{r} \left[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r} \right]^\gamma (B_t + M_t)^{1+\gamma} Y_t^{-\gamma},
\end{aligned}
\tag{3.1}
$$

16

where γ is a positive constant given in the Appendix, and the default boundary is

$$Y^{\star}(M_t, B_t) = \frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r}(B_t + M_t). \tag{3.2}$$

In the case with a strong bank lender, the equity holders do not capture any tax benefit from either market debt issuance or bank debt renegotiation. Hence, as is obvious from (3.1), the equity holders' value depends only on the total debt outstanding $B_t + M_t$ and not on the composition of debt. This result is special to the case of $\theta = 0$. However, despite the equity holders' indifference toward debt composition, the firm's market debt issuance decision does depend on it. This is because the firm's current level of market debt influences the renegotiated face amount of bank debt upon renegotiation, which in turn is priced in and affects the firm's current incentives to issue market debt. Thus, small differences in the initial choices of debt composition between the market debt and the bank debt can lead to very different debt structure and leverage dynamics.

Corollary 4.1. *In the case with $\theta = 0$, the equity holders' value satisfies $V_M^e(Y_t, M_t, B_t) < 0$, $V_B^e(Y_t, M_t, B_t) < 0$, and $V_{MB}^e(Y_t, M_t, B_t) > 0$.*

The equity holders' value comprises the discounted present value of future cash flows plus the value of their option to default. Higher debt levels imposes a greater debt servicing burden on the firm, reducing the residual cash flows available to the firm's equity holders and lowering their continuation value. On the other hand, the equity holders' default option gives them the right to walk away from the firm's debt obligations when repayment is no longer in their interest. This default option is valuable to the equity holders because it limits their downside.

The downside protection afforded by the default option makes the equity holders' value resemble that of a call option on the firm value. When the firm already carries a high balance of bank debt, the equity (as a call option on the firm value) has low moneyness, and further increasing the firm's leverage through market debt issuance has little impact on the equity holders' already diminished value. That is, when the equity is sufficiently out of the money, additional leverage adds little further harm. Consequently, market debt issuance has a less negative impact

on the equity holders' value when the firm already carries a high level of bank debt, that is $V_{MB}^e(Y_t, M_t, B_t) > 0$.

3.1 Complementarity of Bank and Market Debt

The equilibrium market debt price follows from the first-order condition (2.9), obtained by differentiating (3.1). The market debt issuance rate is then derived using (2.10) of Proposition 2. To allow for a more meaningful interpretation of issuance behavior across leverage levels, I scale the market debt issuance rate by the total debt outstanding $B_t + M_t$, and define $g^m(Y_t, M_t, B_t) \equiv G^m(Y_t, M_t, B_t)/(B_t + M_t)$, which is henceforth referred to as the *market debt issuance intensity*. The following proposition provides the closed-form expression for the firm's market debt issuance intensity, with derivations in Appendix A.5.

Proposition 5. *In the case with $\theta = 0$, the equilibrium market debt issuance intensity is*

$$g^m(Y_t, M_t, B_t) = \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \left[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t + M_t}\right]^\gamma + \underbrace{\frac{\lambda}{\gamma}\left[1 - \left(\frac{\bar{B}(Y_t, M_t) + M_t}{B_t + M_t}\right)^\gamma\right]}_{positioning\ for\ renegotiation}. \quad (3.3)$$

The second component of the market debt issuance intensity corresponds to the second term on the right-hand side of (2.10). It reflects the part of the market debt issuance policy driven by the firm's strategic incentive to position for future bank debt renegotiation. I thus refer to this component as the *strategic* market debt issuance intensity, denoted by $g^{m,S}(Y_t, M_t, B_t)$.

Corollary 5.1. *In the case with $\theta = 0$, the firm's strategic market debt issuance intensity is*

$$g^{m,S}(Y_t, M_t, B_t) = \frac{\lambda}{\gamma}\left[1 - \left(\frac{\bar{B}(Y_t, M_t) + M_t}{B_t + M_t}\right)^\gamma\right]. \quad (3.4)$$

It is increasing in the amount of bank debt, that is $\partial g^{m,S}(Y_t, M_t, B_t)/\partial B_t > 0$.

Intuitively, the firm's strategic market debt issuance balances the issuance price against its impact on the equity holders' outside option. Conditional on bank debt renegotiation, the market

debt price reflects the post-renegotiation bank debt level rather than the current amount of bank debt. However, the prevailing bank debt amount affects the sensitivity of the equity holders' outside option (or current continuation value) to market debt issuance, thereby influencing the firm's market debt issuance intensity.

As discussed earlier, the equity holders have the option to default when repayment is no longer optimal, making their value resemble that of a call option on the firm value. When the firm has a high level of bank debt and is close to default, further increasing its leverage through market debt issuance has minimal impact on the equity holders' already diminished value, which is their outside option and bargaining threat point. Put differently, when the firm has more bank debt, additional market debt does little to further weaken the equity holders' already constrained bargaining position, thus reducing the strategic cost of issuing market debt and incentivizing greater market debt issuance. In effect, a high level of bank debt leaves the equity holders with little to lose, encouraging issuance of market debt to shift risk onto creditors.

Hence, the strategic component of the market debt issuance intensity increases with the amount of bank debt B_t. If the strategic component is large enough in magnitude, the overall market debt issuance intensity as given by (3.3) also increases with B_t, suggesting that a higher current level of bank debt is followed by a greater intensity of market debt issuance by the firm. I describe this pattern as the *dynamic complementarity* between bank debt and market debt.

In Figure 2, I illustrate the dynamic complementarity of bank and market debt by plotting the market debt issuance intensity as a function of the bank debt level B_t using the following parameter values: $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = c(1 - \pi) = 0.05$, $\alpha = 2.33$, and $\lambda = 0.2$ (Panel A) or 1 (Panel B). I normalize the face amount of market debt to one, and plot the market debt issuance intensity for various levels of EBIT. The rationale for these parameter choices is discussed in Section 3.3. For comparison, the dashed lines represent $\lambda = 0$ which is the case without bank debt renegotiation such that the strategic market debt issuance intensity drops out.

The dashed lines in Figure 2 all slope downward, implying that more bank debt is associated with lower market debt issuance intensity absent strategic bank-firm renegotiations of the bank
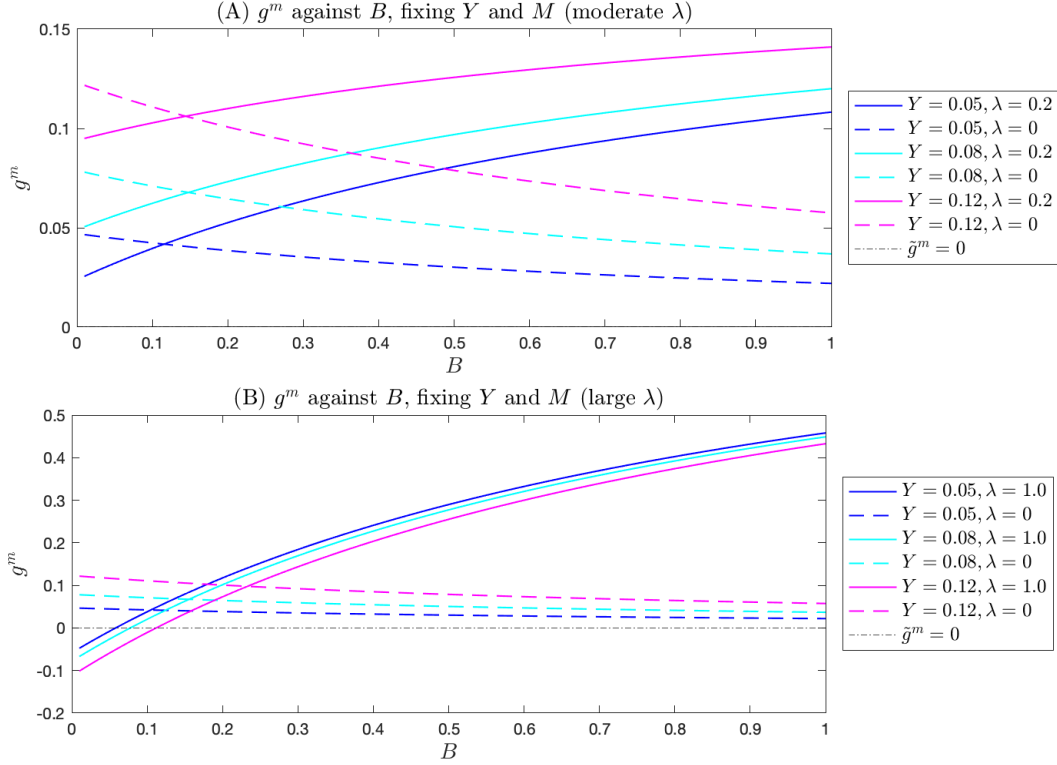
Figure 2: Market debt issuance intensity $g^m(Y_t, M_t, B_t)$**.** The figure plots the market debt issuance intensity as a function of the face amount of bank debt B_t, holding the firm's EBIT Y_t and market debt outstanding M_t constant. Specifically, I plot the relationship between the market debt issuance intensity and the bank debt outstanding for $Y_t = 0.05$, 0.08, and 0.12 while normalizing the amount of market debt outstanding to one. Panel A plots this relationship for $\lambda = 0.2$, while Panel B plots this relationship for $\lambda = 1$. The other parameters take the values: $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = (1 - \pi)c = 0.05$, and $\alpha = 2.33$.

debt. With periodic bank debt renegotiations, the market debt issuance intensity may increase or decrease with the amount of bank debt, depending on λ.[7] That being said, the figure shows that when the bank debt is renegotiated relatively frequently, the market debt issuance intensity increases with the bank debt level. That is, the bank debt and the market debt are dynamically complementary and the firm issues market debt more aggressively when it has more bank debt in its debt structure. Furthermore, the solid lines in Panel A are flatter than those in Panel B, indicating that the dynamic complementarity between bank debt and market debt is more pronounced as λ increases. Intuitively, more frequent bank debt renegotiations increase the firm's strategic incentive to use market debt issuance to position for renegotiations.

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[7]For example, I show in Figure B.1 that when λ is very small (e.g., $\lambda = 0.02$), the firm's market debt issuance intensity decreases with the amount of bank debt, but at a slower rate than in the $\lambda = 0$ case.

3.2 Repurchase of Market Debt

The dynamic complementarity between bank debt and market debt implies low market debt issuance when there is little bank debt in the debt structure. Could the firm repurchase its market debt? The answer is yes. While the market debt issuance intensity (3.3) is always positive when $\lambda = 0$, the strategic component of the market debt issuance intensity may be positive or negative.

Corollary 5.2. *In the case with $\theta = 0$ and given Y_t and M_t,*

$$
g^{m,S}(Y_t, M_t, B_t) \begin{cases} = 0 & \text{if } B_t = \bar{B}(Y_t, M_t) \\ > 0 & \text{if } B_t > \bar{B}(Y_t, M_t) \\ < 0 & \text{if } B_t < \bar{B}(Y_t, M_t) \end{cases}.
\tag{3.5}
$$

When the bank debt is already at the surplus-maximizing level, bank debt renegotiation would result in no change to the firm's debt structure. It is as if renegotiation does not occur and the strategic market debt issuance intensity is zero. By contrast, if $B_t > \bar{B}(Y_t, M_t)$, renegotiation would reduce the amount of bank debt through partial forgiveness by the bank lender. The equity holders do not capture any surplus from renegotiation, and the benefit of leverage reduction accrues to the market debt investors who in turn value their claims more highly. Recognizing this, the firm is willing to slightly weaken its equity holders' outside option (or current continuation value) by issuing additional market debt at a favorable price, so as to dilute the existing market debt investors and capture part of the value that would otherwise accrue to them.

If $B_t < \bar{B}(Y_t, M_t)$, renegotiation would lead to an increase in the amount of bank debt thereby diluting the market debt. The equity holders are guaranteed their current continuation value (which is their outside option) in the event of renegotiation, and the market debt investors bear the brunt of the dilution caused by bank debt renegotiation. The market debt investors price in the expected dilution and value the market debt less. This creates an opportunity for the firm to strengthen the equity holders' outside option by repurchasing market debt at a discount, improving their bargaining position ahead of renegotiation. Thus, in contrast to the leverage

ratchet effect of Admati et al. (2018) and DeMarzo and He (2021), the firm in this model may voluntarily repurchase its market debt if the strategic motive is sufficiently strong. This is seen from Panel B of Figure 2, which shows that the market debt issuance intensity can be negative.

3.3 Seniority of Bank Debt

So far, I have not specified the relative seniority of bank debt and market debt, as the results presented in the previous sections do not rely on any particular assumption about debt seniority. Instead, I have treated the firm's EBIT Y_t, market debt outstanding M_t, and bank debt outstanding B_t as the relevant state variables, and derived the firm's decisions as functions of these variables. The surplus-maximizing level of bank debt $\bar{B}(Y_t, M_t)$, which results from bank debt renegotiation, has been left as a general function of the state variables Y_t and M_t.

To characterize the debt structure dynamics, it is necessary to solve for the bank lender's value and the bank debt level that maximizes the bargaining surplus in renegotiation. The solution exploits the firm's scale invariance in the setting, which helps reduce the dimensionality of the state space. For instance, the bank lender's value is homogeneous of degree one, that is

$$V^b(Y_t, M_t, B_t) = V^b\Big(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\Big) M_t \equiv v^b(y_t, b_t) M_t, \tag{3.6}$$

where $y_t \equiv Y_t/M_t$ is the firm's EBIT-to-market-debt ratio and $b_t \equiv B_t/M_t$ is the *bank debt ratio* which reflects the firm's debt structure. $v^b(y_t, b_t)$ is the bank lender's value scaled by the market debt outstanding, and is referred to as the *bank debt value*. The partial differential equation (PDE) for the bank debt value is given in Appendix A.6. The surplus-maximizing bank debt ratio is

$$\bar{b}(y_t) \equiv \frac{\bar{B}(Y_t, M_t)}{M_t} = \arg\max_{b'_t} \ v(y_t, b'_t), \tag{3.7}$$

where $v(y_t, b_t)$ is the joint value of the equity holders and the bank lender scaled by the amount of market debt.

An implication of the model is that bank debt is senior to market debt in equilibrium. To demonstrate this, I first numerically solve for the bank debt value and the post-renegotiation bank debt ratio assuming that the bank debt is senior. I then show that $\bar{b}(y_t) = 0$ if the bank debt is pari passu with or junior to the market debt. Consequently, any bank debt that arises in equilibrium must be senior.

A. Numerical Solution with Senior Bank Debt

The firm chooses to default when its EBIT-to-market-debt ratio $y_t < y^\star(b_t)$, where the default boundary $y^\star(b_t)$ depends on the bank debt ratio b_t. To facilitate numerical solution, it is more convenient to express the value functions in terms of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ along with the bank debt ratio b_t. The firm defaults when its distance to default reaches zero from above, regardless of its debt structure.

The boundary value of the bank debt depends on its seniority. If the bank debt is senior to the market debt, the bank lender's recovery value upon default is $\min\{\alpha Y^\star, B_{\tau^\star}\}$. When the liquidation value is enough to cover the principal amount of the bank debt at the time of default, that is $\alpha Y^\star \geq B_{\tau^\star}$, the bank lender receives the face amount of the bank debt and the bank debt is defeased. Otherwise, the bank lender gets the entire liquidation value under absolute priority, leaving the market debt investors with nothing in recovery.

I present a numerical solution using the following baseline parameter values: $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = c(1 - \pi) = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$. The Poisson intensity $\lambda = 0.2$ implies that the bank debt is renegotiated, on average, every five years – roughly the maturity of a typical term bank loan. $\alpha = 2.33$ implies a liquidation value that is 10% of the unlevered firm value at default, indicating a relatively large cost of default. The remaining parameters are taken from the literature (e.g., DeMarzo and He (2021), Greenwood et al. (2023)).

Panel A of Figure 3 plots the bank debt value as a function of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ and its bank debt ratio b_t within the no-default region. Panels B and C fix one variable and plot the bank debt value against the other. Panel B shows that given the bank debt ratio b_t, the bank debt value increases with the distance to default $\tilde{y}_t$, as the firm is further from

financial distress and more likely to continue servicing its debt. Panel C shows that given the distance to default $\tilde{y}_t$, the bank debt value increases with the bank debt ratio b_t, since the bank lender has a larger claim on the firm's value.
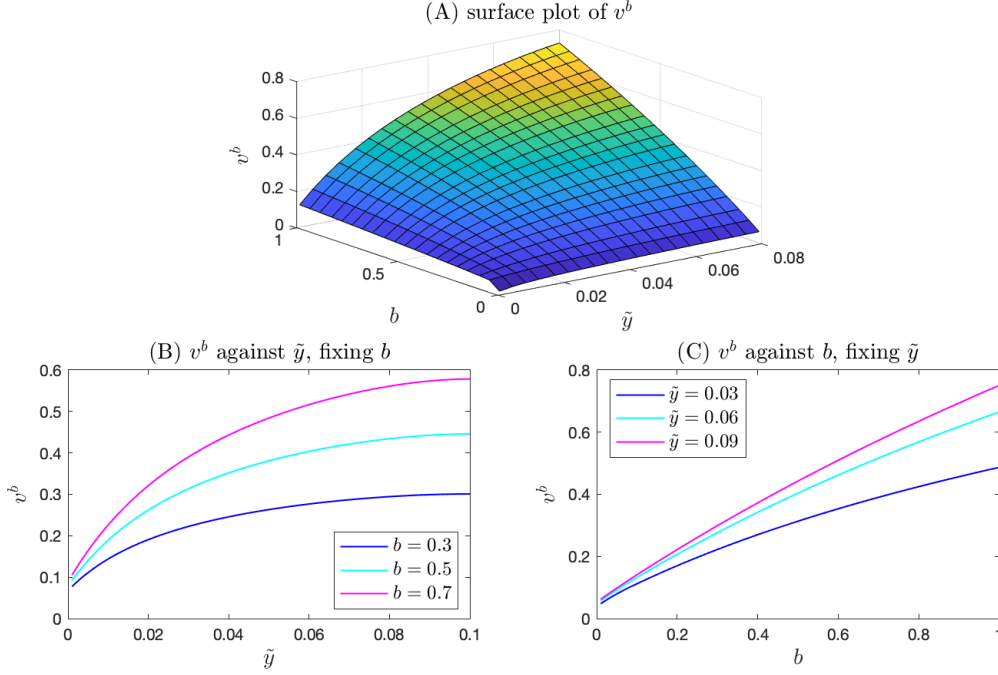

Figure 3: Bank debt value $v^b(\tilde{y}_t, b_t)$**.** The figure plots the bank debt value v^b as a function of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ and the bank debt ratio b_t within the no-default region. Panel A is a two-dimensional surface plot. Panel B plots v^b against $\tilde{y}_t$ for fixed levels of b_t, while Panel C plots v^b against b_t for fixed levels of $\tilde{y}_t$. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = c(1-\pi) = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

Given the firm's EBIT Y_t and market debt outstanding M_t, an increase in the bank debt ratio b_t (i.e., more bank debt) also raises the default boundary thereby shortening the firm's distance to default. Thus, the bank debt value is generally non-monotonic in b_t. Panel A of Figure 4 illustrates this by showing that the bank debt value is hump-shaped in the bank debt ratio. If there is no bank debt, the bank lender has no claim on the firm's value and the bank debt value is zero. Increasing b_t raises the bank lender's claim and hence the bank debt value. However, once bank debt becomes excessive, further adding bank debt makes default imminent, thus driving the bank debt value down toward its boundary value.

On the other hand, more bank debt means greater leverage, which reduces the equity holders' value. In Panel B of Figure 4, I plot the (normalized) joint equity and bank debt value against

the bank debt ratio b_t for given levels of EBIT-to-market-debt ratio y_t.[8] When the bank debt is senior, it turns out that the joint equity and bank debt value is also hump-shaped in b_t. Since renegotiation determines a bank debt ratio that maximizes this joint value, the optimal post-renegotiation bank debt ratio $\bar{b}(y_t)$ is interior, implying a positive balance of bank debt post-renegotiation.
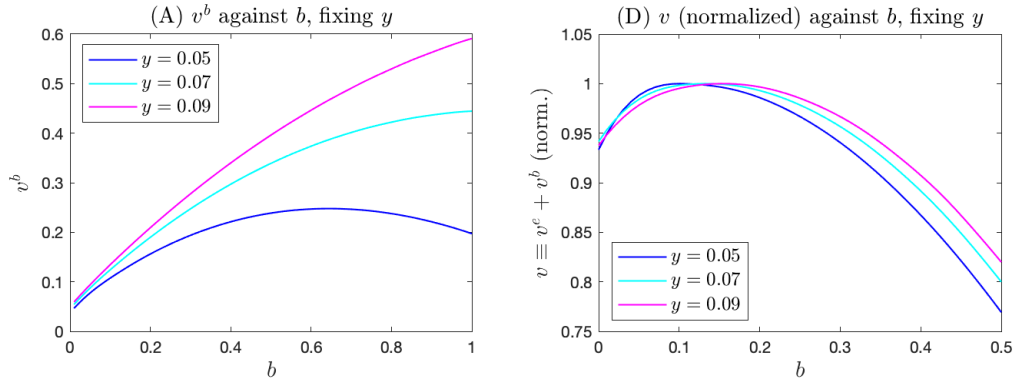

Figure 4: Bank debt value and joint equity and bank debt value as functions of bank debt ratio b_t. The figure plots the bank debt value v^b (Panel A) and the (normalized) joint equity and bank debt value v (Panel B) as functions of the firm's bank debt ratio b_t for given levels of EBIT-to-market-debt ratio y_t. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = c(1 - \pi) = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

However, as I will show shortly, the joint equity and bank debt value decreases with b_t if the bank debt is pari passu with or junior to the market debt. Since the firm cannot commit to future debt choices, it issues market debt until the tax benefit is fully dissipated. If the bank debt is not senior, additional bank debt raises the firm's leverage and shortens its distance to default, thereby reducing the joint value of the equity holders and the bank lender. By contrast, when the bank debt is senior, the market debt acts as a buffer upon default, absorbing losses before the bank debt is impaired. In this case, issuing bank debt enables the bank lender and the equity holders to extract value by diluting the market debt investors.

In Figure 5, I plot the surplus-maximizing bank debt ratio against the firm's EBIT-to-market-debt ratio y_t (Panel A) and the corresponding distance to default (Panel B). When the firm is far from default, it is in the joint interest of the bank lender and the equity holders to keep the

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[8]I normalize the joint equity and bank debt value by applying the formula $v^{norm} = \frac{v - v^{min}}{v^{max} - v^{min}}$ where v^{min} and v^{max} are the minimum and the maximum values of v in the given support.

bank debt ratio low and rely primarily on market debt financing. Since dilution works through default, the bank lender and the equity holders can extract little value from diluting the market debt investors, when both types of debt are relatively safe. As the firm gets closer to default, the incentive to dilute the market debt investors becomes stronger and the surplus-maximizing bank debt ratio increases. However, when default is imminent, it becomes optimal for the bank lender and the equity holders to reduce bank debt through renegotiation to avoid default.
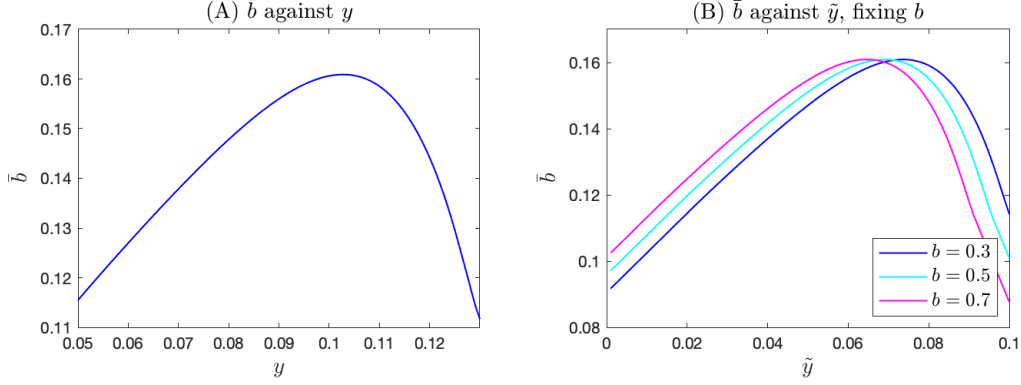

Figure 5: Surplus-maximizing senior bank debt ratio $\bar{b}(y_t)$ from renegotiation. Panel A of the figure plots the surplus-maximizing bank debt ratio $\bar{b}(y_t)$, which results from bank debt renegotiation, as a function of the firm's EBIT-to-market-debt ratio y_t when the bank debt is senior. Panel B plots the surplus-maximizing bank debt ratio post-renegotiation as a function of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ for given levels of bank debt ratio b_t. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = c(1 - \pi) = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

B. No Pari Passu or Junior Bank Debt

I now consider the case in which the bank debt is not senior to the market debt. Without commitment, the firm's equity holders are incentivized to dilute the existing lenders by issuing new market debt just before default. If the bank debt is pari passu with or junior to the market debt, the equity holders are able to fully dilute the existing lenders, and the bank lender receives no liquidation value upon default.

In Panel A of Figure 6, I plot the surplus-maximizing bank debt ratio $\bar{b}(y_t)$ when the bank debt is not senior. I also plot the (normalized) joint equity and bank debt value against b_t in Panel B. The figure shows that if the bank debt is not senior to the market debt, the joint equity and bank debt value is monotonically decreasing in the bank debt ratio b_t and the firm never

issues any bank debt. Hence, the optimal bank debt ratio $\bar{b}(y_t)$, which maximizes the joint equity and bank debt value, is zero.
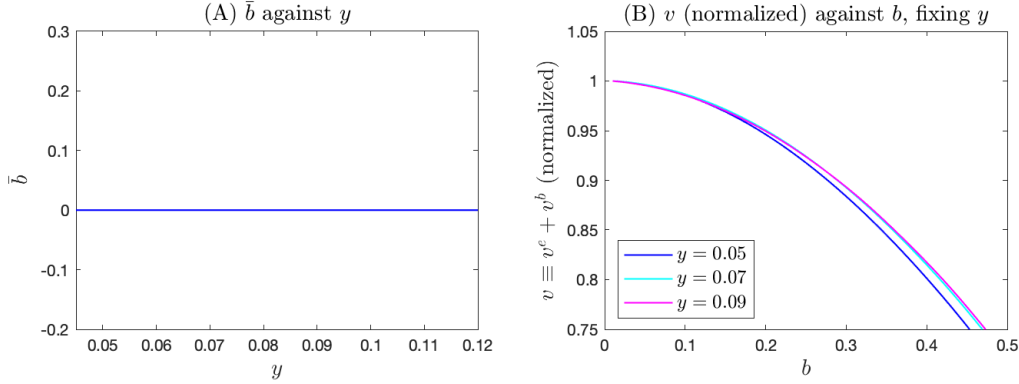

Figure 6: Surplus-maximizing non-senior bank debt ratio $\bar{b}(y_t)$ from renegotiation. Panel A of the figure plots the surplus-maximizing bank debt ratio $\bar{b}(y_t)$ as a function of the firm's EBIT-to-market-debt ratio y_t, when the bank debt is non-senior. Panel B plots the normalized joint equity and bank debt value against the bank debt ratio b_t for various y_t. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = (1 - \pi)c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

In practice, covenants and legal restrictions (e.g., fraudulent conveyance) help limit dilution due to new debt issuance just before default. In Appendix B, I consider an alternative regime where there is perfect legal protection and the liquidation value $\alpha Y^\star$ is split between the bank lender and the market debt investors according to their holdings pursuant to the absolute priority rule. The results are plotted in Figure B.2. Again, the joint equity and bank debt value is decreasing in b_t and the surplus-maximizing bank debt ratio is zero when the bank debt is not senior.

3.4 Efficiency of Bank Debt

When there is only market debt, the setting reduces to that in DeMarzo and He (2021), where the lack of commitment leads to full dissipation of the tax benefit. By contrast, when there is only bank debt, the bank lender and the firm periodically renegotiate to maximize the total value by trading off the tax benefit against the cost of default. The case with both competitive market debt and renegotiable bank debt lies between these extremes: market debt can be issued without commitment, but the presence of renegotiable bank debt partially restores commitment, as periodic renegotiations allow some of the tax benefit to be captured.

Bank debt renegotiations maximize the joint value of the senior bank lender and the junior equity holders. The mezzanine market debt investors remain the passive non-bargaining party, but their payoff is affected by the bargaining outcome between the bank lender and the firm. For instance, when the firm is over-levered, it is in the joint interest of the bank lender and the equity holders to lower leverage through partial forgiveness of the bank debt upon renegotiation. However, part of the benefit from bank debt forgiveness accrues to the non-bargaining market debt investors, who effectively free-ride on the efforts of the two bargaining parties, thereby reducing their incentive to cut leverage in renegotiation.

The firm's ability to strategically trade market debt to position for renegotiation allows it to extract value from the market debt investors ahead of bank debt renegotiation. In the above example where bank debt renegotiation is expected to result in a partial forgiveness of the bank debt, the firm issues additional market debt at a favorable price and captures value by diluting the existing market debt investors. The strategic issuance of market debt in anticipation of bank debt renegotiation thus mitigates the free-riding problem by the market debt investors, thereby increasing the total firm value.

Similarly, when the firm is under-levered, the bank lender and the equity holders wish to increase the amount of bank debt upon renegotiation. This increase in bank debt reduces the value of the market debt investors, but the bargaining parties do not internalize this effect, leading to excessive leverage in renegotiation. However, to position for renegotiation, the firm strategically issues less (or repurchases) market debt, partially internalizing the effect on the market debt investors and thereby constraining the increase in leverage from bank debt renegotiation.

Consequently, having renegotiable bank debt is efficient from a firm value perspective. Periodic renegotiations between the bank lender and the firm help restore commitment and preserve part of the tax benefit through joint value maximization. Indirectly, by strategically trading market debt to position for bank debt renegotiation, the firm captures value and mitigates spillovers to the non-bargaining market debt investors that are not internalized by the bargaining parties. This makes the bargaining coalition of the bank lender and the equity holders more representative of the total firm, further increasing the overall firm value. Hence, bank debt and market debt

are also "complementary" in a conceptual sense: bank debt restores commitment lacked by the market debt, while strategic trading of market debt mitigates spillovers to the non-bargaining market debt investors from renegotiations between the bank lender and the firm.

4   Effects of Firm Bargaining Power

I now turn to the general case where the firm has positive bargaining power, that is $\theta > 0$. In particular, I am interested in knowing how the firm's bargaining power affects the dynamic relationship between the firm's bank debt amount and its market debt issuance. As discussed in Sections 2.4 and 2.5, the firm's market debt issuance policy balances the issuance proceeds against its impact on the equity holders' payoff from bank debt renegotiation, which consists of the equity holders' outside option and the share of the bargaining surplus they capture. The equity holders' renegotiation payoff is given by (2.8), which can be rearranged as

$$V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t = \theta \underbrace{\left[V(Y_t, M_t, \bar{B}_t) - V^b(Y_t, M_t, B_t) \right]}_{\substack{\text{negotiable value above} \\ \text{bank lender's outside option}}} + (1 - \theta) \underbrace{V^e(Y_t, M_t, B_t)}_{\substack{\text{equity holders'} \\ \text{outside option}}}. \qquad (4.1)$$

The equity holders' payoff from renegotiation can be decomposed into (1) a fraction θ of the negotiable value after satisfying the bank lender's outside option, where θ is the firm's bargaining power; and (2) a fraction $1 - \theta$ of the equity holders' own outside option. When the firm has weaker bargaining power, its equity holders capture a smaller share of the negotiable value, making their outside option more important as a credible threat during bargaining. The prevailing bank debt level influences how sensitive the equity holders' and the bank lender's outside options are to market debt issuance, thereby affecting the firm's strategic incentive to issue market debt.

The equity holders' default option allows them to walk away from debt obligations, at the expense of the creditors. When the firm has a higher level of bank debt and is closer to default, the negative valuation impact of a further increase in leverage through market debt issuance becomes attenuated for the equity holders ($V_{MB}^e > 0$) but more pronounced for the bank lender

$(V_{MB}^b < 0)$.[9] Hence, additional market debt does little to further erode the equity holders' already weakened bargaining position but undermines the bank lender's bargaining position which helps shift more surplus toward the equity holders. This incentivizes the firm to issue more market debt, giving rise to dynamic complementarity between bank debt and market debt.

While the dynamic complementarity between bank and market debt remains robust, the degree of complementarity depends on the allocation of bargaining power between the bank lender and the firm. Specifically, increasing the firm's bargaining power weakens the dynamic complementarity of bank and market debt. The intuition is that because the equity holders are the residual claimants, the impact of additional leverage on the equity holders' value is more sensitive to the firm's distance to default compared to the bank debt. In other words, as the prevailing level of bank debt rises and pushes the firm closer to default, additional market debt issuance has a relatively stable impact on the bank lenders' value, while the impact on the equity holders' value diminishes significantly in magnitude.

When the firm has little bargaining power (i.e., θ is small), the equity holders' renegotiation payoff comes almost solely from their outside option, which equals their current continuation value. As the firm's bargaining power increases, shifting surplus by reducing the bank lender's outside option (or continuation value) becomes more pertinent. In this case, the sensitivity of the equity holders' value to market debt issuance becomes less dependent on the prevailing level of bank debt. As a result, the firm has less incentive to use market debt issuance strategically to influence renegotiation with the bank lender, thus weakening the dynamic complementarity of bank debt and market debt.

For illustration, I plot $g^m(Y_t, M_t, B_t) \equiv G^m(Y_t, M_t, B_t)/(B_t + M_t)$, the firm's market debt issuance intensity as defined earlier. The market debt issuance intensity is homogeneous of degree zero, and can thus be written as a function of the firm's distance to default $\tilde{y}_t$ and its bank debt ratio b_t. In Figure 7, I plot $g^m(\tilde{y}_t, b_t)$ against the bank debt ratio b_t at various levels of distance to default $\tilde{y}_t$, for both $\theta = 0.1$ (solid lines) and $\theta = 0$ (dashed lines). The general case can only

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[9]The bank lender can be viewed as having a negative exposure to the default option, and its value resembles that of a short put position on the firm value. When the firm already has a high level of bank debt and is close to default, the bank lender's value is more sensitive to further increases in the firm's leverage.

be solved numerically, with the corresponding PDEs provided in Appendix A.7. The parameter values are as follows: $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = c(1 - \pi) = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.
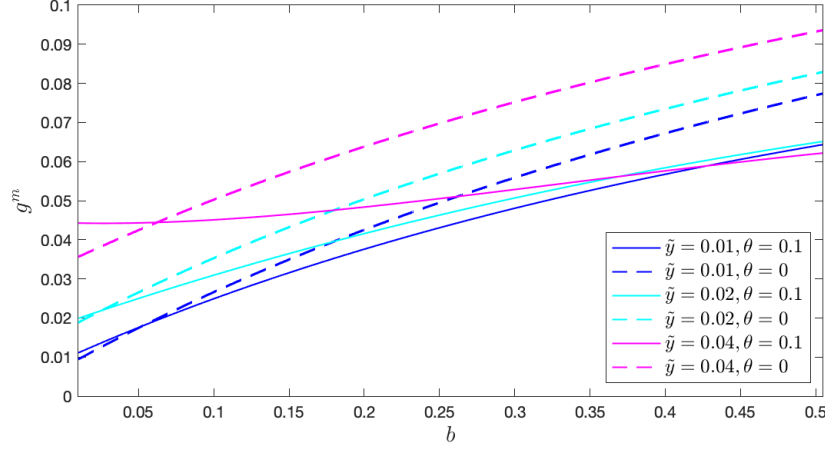

Figure 7: Effect of firm bargaining power θ on market debt issuance intensity $g^m(\tilde{y}_t, b_t)$. The figure plots the market debt issuance intensity $g^m(\tilde{y}_t, b_t)$ as a function of the bank debt ratio b_t at various levels of distance to default $\tilde{y}_t$ (including $\tilde{y}_t = 0.01, 0.02, 0.04$), for $\theta = 0.1$ and $\theta = 0$ respectively. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = (1 - \pi)c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

For each given distance to default, the slope of the solid line is also positive. The positive slope indicates that holding the amount of market debt constant, the firm's market debt issuance intensity increases with the level of bank debt, thus confirming the dynamic complementarity between bank debt and market debt for $\theta > 0$. However, the slope of the solid line is flatter than that of the dashed line. That is, fixing the amount of market debt, an increase in the firm's bank debt amount leads to a smaller increase in its market debt issuance intensity when the firm has positive bargaining power than when it has none. In other words, assigning bargaining power to the firm weakens the dynamic complementarity between bank debt and market debt.

How does the allocation of bargaining power between the bank lender and the firm affect the total firm value? As explained in Section 3.4, the firm's strategic trading of market debt in anticipation of bank debt renegotiation enhances efficiency and raises the overall firm value. Following the discussion above, this strategic incentive is stronger when the firm holds less bargaining power. Hence, the total firm value is decreasing in the firm's bargaining power. In Figure 8, I plot the total enterprise value against the firm's bank debt ratio for various levels of distance to default. The total enterprise value is defined as the firm value scaled by the amount of

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market debt outstanding. The figure shows that assigning bargaining power to the firm reduces the total enterprise value.
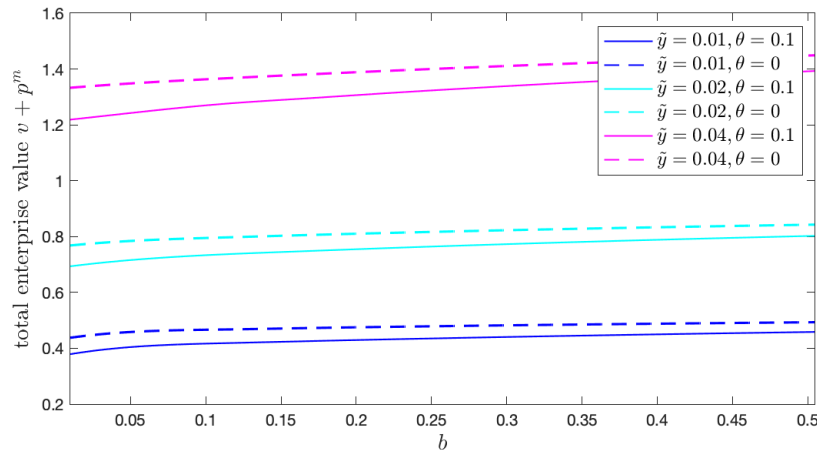

Figure 8: Total enterprise value $v + p^m$. This figure plots the total enterprise value $v + p^m$ as a function of the bank debt ratio b_t at various levels of distance to default $\tilde{y}_t$ (including $\tilde{y}_t = 0.01, 0.02, 0.04$), for $\theta = 0.2$ and $\theta = 0$ respectively. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = (1 - \pi)c = 0.05$, $\lambda = 0.02$, and $\alpha = 2.33$.

5 Complementarity of Bank and Market Debt: Evidence

A novel implication of the model is the dynamic complementarity between bank debt and market debt: holding the firm's EBIT and market debt outstanding fixed, the firm's subsequent market debt issuance intensity increases with its current bank debt balance. Since the market debt issuance intensity is scale-invariant, the model thus predicts that controlling for leverage, a greater share of bank debt in the debt structure should be associated with a higher intensity of subsequent market debt issuance. In this section, I present suggestive empirical evidence consistent with this prediction.

5.1 Firm-Level Data and Sample Construction

For my main empirical analysis, I rely on firm-level data from multiple data sources. I obtain annual firm characteristics and accounting data from Compustat for the period between 2002

and 2024. I restrict the sample to non-financial firms by excluding all firms whose NAICS codes start with 52, which correspond to the finance and insurance sector. I end up with a sample containing 144,016 firm-year observations involving 15,383 unique firms. I then supplement the Compustat data with entity-level credit ratings obtained from the S&P Credit Ratings dataset, which provides a complete history of rating changes by Standard & Poor's (S&P). In particular, I focus on the local currency LT credit rating assigned by S&P. To facilitate the analysis, I give a numeric value to each notch of the rating, with 1, 2, 3, 4, ..., denoting AAA, AA+, AA, AA−, ..., respectively.

I obtain debt structure data from the Capital Structure dataset of Capital IQ, and then merge the data into the Compustat dataset. The Capital Structure dataset categorizes firms' total debt into seven mutually exclusive debt types: commercial paper, drawn credit lines, term loans, senior bonds and notes, subordinated bonds and notes, capital leases, and other debt. The dataset allows me to compute firm-year level observations of outstanding bank debt and market debt respectively. In the baseline, I follow Becker and Ivashina (2014) and define bank debt as term loans, while market debt includes senior bonds and notes as well as subordinated bonds and notes. For robustness, I also account for short-term credit by expanding the definition of bank debt to include drawn credit lines and the definition of market debt to include commercial paper.

After removing any observation with missing debt structure information from the sample, I get 126,227 firm-year observations involving 14,592 unique firms. Following the literature, I further eliminate any firm-year observation with less than $10 million in assets. The ending sample contains 108,046 firm-year observations and 12,608 unique firms. Among the firm-year observations, 32% have both term loans and senior or subordinated bonds and notes, while 42% have both term loans or drawn credit lines and bonds/notes or commercial paper at the same time. Since I am interested in firms' debt structures and the complementarity between bank and market debt, I focus on firms that have access to both types of debt financing. Accordingly, I restrict the sample to observations with non-zero amounts of both bank debt and market debt outstanding.

Table 1: Summary Statistics of Key Financial Variables

	All Observations ($N = 34,424$)			Rated Only ($N = 17,506$)		
	Mean	Median	Std. Dev	Mean	Median	Std. Dev
Book Assets	12,000	2,362	29,414	22,317	6,731	43,065
Total Sales	7,147	1,255	17,755	13,175	3,720	26,400
Market Value	9,556	1,531	23,905	17,577	4,626	34,998
Book Leverage	0.421	0.379	0.258	0.423	0.386	0.223
Debt-to-EBITDA	3.434	2.989	9.723	4.204	3.414	6.059
Tangibility	0.337	0.275	0.259	0.361	0.316	0.249
Turnover	0.786	0.624	0.649	0.736	0.602	0.554
Net Profit Margin	-0.252	0.032	2.168	0.036	0.051	0.205
ROA	0.072	0.098	0.164	0.116	0.109	0.069
Market-to-Book	1.352	1.071	1.026	1.232	1.054	0.689
Dividend Payout	0.290	0.000	1.139	0.385	0.138	1.344
Bank Debt Share	0.399	0.347	0.309	0.305	0.238	0.266
Entity Rating				11.05	11.00	3.627

Note: The table presents summary statistics of key financial variables for all firm-year observations in the estimation sample, and those observations with available S&P credit ratings. Market value equals the product of the closing share price and the total number of shares outstanding. Book leverage is total debt divided by total book assets. Debt-to-EBITDA is the ratio of total debt to operating income before depreciation and amortization. Tangibility is net property, plant, and equipment divided by total assets. Turnover is total sales divided by total assets. Net profit margin is net income divided by total sales. ROA is operating income before depreciation and amortization divided by total assets. Market-to-book is the sum of the market value of equity, total debt, and preferred stock, divided by total assets. Dividend payout is the fraction of net income distributed as dividends. Bank debt share is the amount of bank debt divided by the sum of bank and market debt. All continuous financial variables are winsorized at the 1st and 99th percentiles. The estimation sample is constructed using data from Compustat, S&P Credit Ratings, and Capital IQ for the period between 2002 and 2024.

Table 1 reports summary statistics of key financial variables for the estimation sample (columns 2-4), and for the subsample of firm-year observations with available S&P credit ratings (columns 5-7). Market value refers to the market value of equity, and is calculated as the product of the closing share price and the total number of shares outstanding. Book leverage is the sum of current and long-term debt divided by total book assets. Debt-to-EBITDA is another measure of firm leverage, and is the ratio of total debt to operating income before depreciation and amortization. Tangibility is measured as net property, plant, and equipment scaled by total assets. Turnover is calculated as total sales divided by total assets. Net profit margin is net income divided by total sales. Return on assets (ROA) is defined as operating income before depreciation and amortization divided by total assets. Market-to-book is computed as the sum of the market value of equity, total debt, and preferred stock, divided by total assets. Dividend payout is the fraction of net income distributed to shareholders as dividends. Bank debt share is the amount of bank debt divided by the sum of bank and market debt, and captures a firm's debt structure. All continuous financial variables are winsorized at the 1st and 99th percentiles.

Among the firm-year observations with available S&P credit ratings, the mean entity rating is 11 (or BB+). Firms in the rated-only sample tend to be larger (e.g., higher book assets, total sales, and market value) and more profitable (e.g., higher net profit margin and ROA), have higher dividend payout ratio, slightly greater leverage and increased asset tangibility. The mean bank debt share is 0.305 for the rated-only sample, lower than the 0.399 in the full sample. That is, firms with available S&P credit ratings tend to have less bank debt in their debt structures on average.

5.2 Dynamic Complementarity of Bank and Market Debt

To investigate the dynamic complementarity between bank debt and market debt, I begin by plotting the relationship between bank debt share and market debt net issuance rate in the following year. Bank debt share equals a firm's bank debt outstanding divided by the total amount of bank and market debt. Market debt net issuance rate is the net increase in market debt scaled by the total amount of bank and market debt. I first sort the observations within each year into 20 equal-sized bins based on the distribution of bank debt share within the year. Each bin thus groups observations with similar values of bank debt share within the given year. Then for each bin-year, I compute the mean bank debt share and the mean market debt net issuance rate. I then average these bin-year mean values across all years. Finally, I plot the resulting binned means into a two-dimensional binscatter graph, with bank debt share on the horizontal axis and market debt net issuance rate on the vertical axis.

The plot is shown in Panel A of Figure 9. It shows a positive relationship between the market debt net issuance rate in the following year and the prevailing bank debt share. Hence, a greater amount of bank debt in the debt structure predicts higher subsequent market debt issuance, suggesting a dynamic complementarity between bank debt and market debt. Although the plot offers suggestive evidence that bank debt and market debt are dynamically complementary, a couple of main concerns need to be addressed.
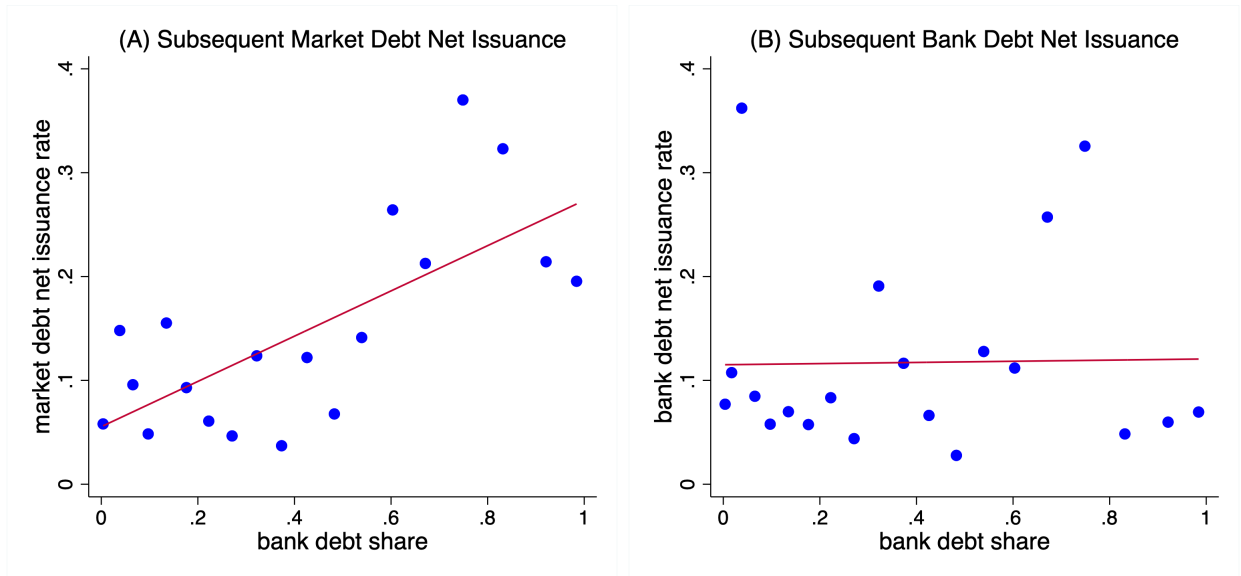
Figure 9: Binscatter Plots of Bank Debt Share and Subsequent Debt Issuances. Panel A of the figure plots market debt net issuance rate (defined as the net increase in market debt outstanding during the following year scaled by the total amount of bank and market debt) against the bank debt share (defined as the amount of bank debt divided by the sum of bank and market debt) in a binscatter graph with 20 equal-sized bins. Panel B of the figure plots bank debt net issuance rate (defined as the net increase in bank debt outstanding during the following year scaled by the total amount of bank and market debt) against the bank debt share in a binscatter graph with 20 equal-sized bins. The estimation sample is constructed using data from Compustat, S&P Credit Ratings, and Capital IQ for the period between 2002 and 2024.

First, the observed positive relationship between bank debt share and subsequent market debt net issuance may simply reflect firms rebalancing toward an optimal debt composition. That is, firms with higher-than-optimal bank debt shares accelerate market debt issuance to bring their debt structures back toward the optimal mix. However, the same logic implies that such firms should also reduce their bank debt issuance, leading to a negative relationship between bank debt share and subsequent net issuance of bank debt. Panel B of Figure 9 shows that this is not the case, suggesting that the observed positive relationship between bank debt share and subsequent market debt net issuance rate cannot be simply explained by mean-reversion to a target debt structure.

Another concern is that the bank debt reported by Capital IQ may include bridge loans, which are intended to be replaced by bond issuances in the near term. To finance investment projects or acquisitions, firms often initially turn to bank lenders for debt commitments in the form of

bridge loans. Bridge loans are short-term facilities that provide interim financing until long-term securities, such as bonds, can be issued to permanently fund the transactions. However, if the observed increase in market debt issuance following a rise in bank debt share is primarily driven by firms replacing bridge loans with bonds, one would expect to see a corresponding decrease in bank debt net issuance. This is inconsistent with the observed pattern shown in Panel B of Figure 9.

Lastly, the positive relationship between bank debt share and subsequent market debt net issuance rate could be a mechanical artifact of secular trends in debt composition over the sample period. For instance, if bank debt share were in steady decline, earlier years would naturally exhibit higher bank debt shares alongside faster market debt issuance in subsequent periods. The within-year binning and averaging procedure employed in constructing the binscatter addresses this concern. To further mitigate this concern, I plot the time series of firm-averaged bank debt shares by year in Figure 10. Bank debt shares during the sample period appear relatively stable, fluctuating around an average of about 0.4.

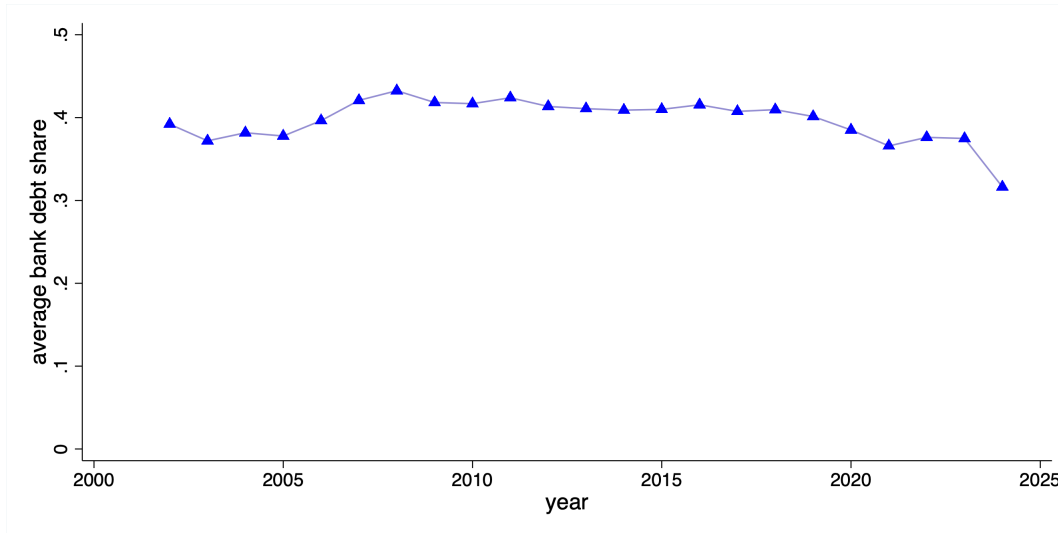**Figure 10: Average Bank Debt Share Over Time.** The figure plots the time series of firm-averaged bank debt shares by year, using data from Compustat and Capital IQ for the period between 2002 and 2024. Average bank debt share is computed by taking the average of bank debt shares across all firms within a year, where bank debt share is defined as the amount of bank debt divided by the sum of bank and market debt.

5.3 Regression Analysis

The pattern shown in Figure 9 is correlational. Several confounding factors can distort the interpretation of the positive relationship between bank debt share and subsequent market debt net issuance rate. A major concern is that firm characteristics may affect both the debt structure and market debt issuance behavior. For instance, firms with more tangible assets to pledge as collateral may enjoy better access to both bank debt financing and market debt financing. As a result, these firms may have higher proportions of bank debt in their debt structures and simultaneously issue more market debt, mechanically generating a positive correlation between the two. To address such omitted variable bias, I estimate the following regression specification

$$Market\ Debt\ Net\ Issuance\ Rate_{it} = \beta Bank\ Debt\ Share_{it} + \gamma X_{it} + \alpha_i + \alpha_t + \epsilon_{it}, \qquad (5.1)$$

where i indexes firm and t indexes year. $Market\ Debt\ Net\ Issuance\ Rate_{it}$ is the net increase in firm i's market debt outstanding from year t to $t+1$ divided by the firm's total amount of bank and market debt in year t. $Bank\ Debt\ Share_{it}$ equals firm i's bank debt outstanding in year t scaled by its total amount of bank and market debt in year t. α_i is firm fixed effect which controls for time-invariant characteristics within a firm. α_t is year fixed effect which controls for time-specific shocks common to all firms. Moreover, X_{it} is a set of controls at the firm-year level. Standard errors are double-clustered at the firm and year levels.

The coefficient of interest is β. A positive β means that an increase in the prevailing bank debt share is associated with a higher rate of market debt net issuance in the following year, after controlling for various confounding factors. I start by including the log of book assets, tangibility, ROA, book leverage, and debt-to-EBITDA in the time-varying firm-level controls (X_{it}). The regression results are shown in Table 2. Column 1 is unconditional on market debt net issuance or repurchase by a firm in a given year, while column 3 is conditional on non-zero market debt changes. In both columns, the coefficient estimates on bank debt share are positive and highly statistically significant.

Table 2: Panel Regressions: Complementarity of Bank and Market Debt

			Market Debt Net Issuance Rate			
	(1)	(2)	(3)	(4)	(5)	(6)
Bank Debt Share	0.925***	0.791***	0.998***	0.819***	0.391**	0.411**
	(0.229)	(0.163)	(0.248)	(0.174)	(0.144)	(0.152)
Log(Assets)	-0.253***	-0.210***	-0.247***	-0.218***	-0.299	-0.339
	(0.048)	(0.068)	(0.051)	(0.072)	(0.174)	(0.198)
Tangibility	0.515	0.283	0.376	0.0397	0.255	0.208
	(0.382)	(0.502)	(0.341)	(0.388)	(0.257)	(0.289)
ROA	-0.113	-0.551**	-0.205	-0.628**	-0.383	-0.384
	(0.266)	(0.233)	(0.308)	(0.269)	(0.647)	(0.783)
Book Leverage	-1.612***	-1.615***	-1.657***	-1.702***	-1.206**	-1.320**
	(0.287)	(0.408)	(0.287)	(0.392)	(0.506)	(0.578)
Debt-to-EBITDA	0.001	0.000	0.001	0.000	0.001	0.001
	(0.002)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)
Dividend Payout		-0.007		0.005	0.000	0.001
		(0.008)		(0.009)	(0.003)	(0.003)
Market-to-Book		0.205***		0.226***	0.232**	0.250*
		(0.048)		(0.054)	(0.111)	(0.122)
Stock Return		-0.011		-0.009	-0.005	-0.005
		(0.009)		(0.010)	(0.007)	(0.007)
Sales Growth		-0.022		-0.037	0.118	0.129
		(0.064)		(0.072)	(0.104)	(0.120)
Sample	All	All	All	All	Rated	Rated
Conditional on Issuance	No	No	Yes	Yes	No	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.150	0.141	0.203	0.204	0.097	0.103
Observations	28,293	21,762	25,008	19,201	12,036	10,619

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table presents results of the regression specification (5.1) using data from Compustat, S&P Credit Ratings, and Capital IQ for the period 2002-2024. The outcome variable is market debt net issuance rate defined as the net increase in market debt outstanding during the following year scaled by the total amount of bank and market debt. The coefficients of interest are the coefficients on bank debt share defined as the amount of bank debt divided by the sum of bank and market debt. Columns 1-4 use all firm-year observations from the estimation sample. Columns 5-6 restrict to observations with available S&P credit ratings. Columns 1, 2 ad 5 are unconditional on non-zero market debt changes while columns 3, 4 and 6 are conditional on non-zero market debt changes. In columns 1 and 3, the firm-year level controls include the log of book assets, tangibility, ROA, book leverage, and debt-to-EBITDA. Columns 2 and 4-6 also include dividend payout, market-to-book, stock return, and sales growth as additional control variables. Standard errors are double-clustered at the firm and year levels.

Another potential issue concerns the timing of bank debt and market debt issuances. Firms with better investment opportunities may issue both bank debt and market debt to finance these

opportunities. If bank debt is easier or quicker to obtain than market debt (e.g., due to time required for bond roadshows), firms may initially rely more on bank debt before subsequently ramping up market debt issuance. Accordingly, I include several proxies for investment opportunities as additional control variables. These include market-to-book ratio, stock return, and sales growth. I also include dividend payout as a control variable, since distribution of dividend may also affect a firm's demand for credit. The results are shown in column 2 (unconditional on market debt net issuance or repurchase) and in column 4 (conditional on non-zero market debt issuance or repurchase).

Inclusion of the additional control variables reduces the magnitude of the coefficient estimates on bank debt share. However, the estimates remain positive and highly statistically significant. A 1% increase in the prevailing bank debt share is associated with 79-82 basis points higher net issuance rate of market debt in the subsequent year. Finally, I re-estimate the specification with the full set of control variables for the rated-only subsample. The coefficient estimates on bank debt share are positive and statistically significant, though their magnitudes are about half of those in the full estimation sample. Taken together, the regression results in Table 2 are consistent with the pattern documented in Figure 9. In sum, higher bank debt share is associated with greater subsequent net issuance of market debt, implying that bank debt and market debt are complementary.

In Appendix E.1, I provide additional robustness tests. In the first set of robustness tests, I apply alternative definitions for bank debt and market debt. In the second set of robustness tests, I use a matched-sample regression specification by including industry-year-size-leverage fixed effects as the main controls. Finally, I document complementarity between bank and market debt using time-series aggregate balance sheet data obtained from the Flow of Funds.

5.4 Heterogeneity Across Firms and Time

How does the degree of dynamic complementarity between bank and market debt vary depending on firm characteristics and time periods? To answer this question, I introduce various

dummy variables capturing firm characteristics and market conditions. For example $Large\,Size_{it}$ is a firm-year level indicator variable equal to one if the amount of book assets is above the sample median and zero otherwise. $High\,Sales_{it}$ and $High\,ROA_{it}$ are analogously defined from total sales and ROA respectively. $Div\,Payer_{it}$ is an indicator variable equal to one if firm i pays dividend in year t. $IG\,Rating_{it}$ is an indicator variable equal to one if the entity credit rating assigned by S&P is BBB- or above. $Crisis_t$ indicates crisis periods (including the GFC and COVID-19), and equals one for years 2008, 2009 and 2020. I then interact these variables with $Bank\,Debt\,Share_{it}$ and re-estimate the regression (5.1) with the full set of controls, unconditional on market debt net issuance or repurchase.

The coefficient estimates on the interaction terms are presented in Table 3. The degree of complementarity between bank and market debt tends to be lower for larger firms (e.g., firms with more assets and higher sales), for more profitable firms (e.g., firms with higher ROA), and for firms with investment-grade (IG) credit ratings. Moreover, dividend-paying firms exhibit weaker complementarity between bank and market debt. On the other hand, the degree of complementarity between bank and market debt tends to be amplified during crisis periods such as the GFC and COVID-19.

Several potential explanations may account for this heterogeneity across firms and time periods, depending on the underlying mechanisms that drive the complementarity between bank and market debt. One possible explanation is that the degree of complementarity between bank and market debt is affected by the amount of bargaining power the firm has over its bank lender. For a firm that maximizes its shareholders' value, its bargaining power affects the marginal benefit of market debt issuance to shareholders, thereby influencing its market debt issuance behavior. The results in Table 3 seem to be consistent with this explanation. Larger and more profitable firms, as well as dividend-paying firms (which are typically larger and more mature) tend to possess greater bargaining power. Firms with little default risk and firms operating under favorable economic conditions are also likely to enjoy stronger bargaining positions. The coefficient estimates in the table thus suggest that the complementarity between bank and market debt decreases as firm bargaining power increases.

Table 3: Complementarity of Bank and Market Debt: Heterogeneity Across Firms and Time

	Market Debt Net Issuance Rate					
	(1)	(2)	(3)	(4)	(5)	(6)
Bank Debt Share×Large Size	-0.667***					
	(0.221)					
Bank Debt Share×High Sales		-0.931***				
		(0.299)				
Bank Debt Share×High ROA			-0.426**			
			(0.204)			
Bank Debt Share×Div Payer				-0.402**		
				(0.162)		
Bank Debt Share×IG Rating					-0.339*	
					(0.173)	
Bank Debt Share×Crisis						0.619**
						(0.274)
FirmFE	Yes	Yes	Yes	Yes	Yes	Yes
YearFE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.141	0.141	0.141	0.141	0.141	0.141
Observations	21,762	21,762	21,762	21,762	21,762	21,762

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table regresses market debt net issuance rate in the following year (defined as the net increase in market debt scaled by the total amount of bank debt and market debt) on the interaction terms between the bank debt share variable and various dummy variables capturing firm characteristics and time periods, along with firm-year level controls and firm and year fixed effects. $Large\,Size_{it}$ is a firm-year level indicator variable equal to one if the amount of book assets is above the sample median. $High\,Sales_{it}$ and $High\,ROA_{it}$ are analogously defined from total sales and ROA respectively. $Div\,Payer_{it}$ is an indicator variable equal to one if firm i pays dividend in year t. $IG\,Rating_{it}$ is an indicator variable equal to one if the entity credit rating assigned by S&P is BBB- or above. $Crisis_t$ indicates crisis periods (including the GFC and COVID-19), and equals one for years 2008, 2009 and 2020. Data are from Compustat, S&P Credit Ratings, and Capital IQ for the period 2002-2024. Standard errors are double-clustered at the firm and year levels.

Another potential explanation is that the bank lender's monitoring capability or the contractual flexibility of bank debt generates positive spillover effects on market debt, thereby enhancing the firm's ability to raise market debt financing. To assess the validity of this explanation, I re-estimate the specification (5.1) using only observations in the bottom quintiles of book leverage and market leverage (defined as total debt divided by the sum of the market value of equity, total debt, and preferred stock). These observations have very low financial leverage, with a subsample mean book leverage ratio of 0.08 and a subsample mean market leverage ratio of 0.05. I also run the same specification for observations with entity credit ratings of A− or above, which account

for 16% of the rated-only sample and 5% of the total estimation sample. In Table 4, I present the coefficient estimates on the bank debt share variable.

Table 4: Complementarity of Bank and Market Debt for Low-Risk Firms

	Market Debt Net Issuance Rate					
	Low Book Lev		Low Mkt Lev		High Ratings	
	(1)	(2)	(3)	(4)	(5)	(6)
Bank Debt Share	1.601***	1.439***	1.460**	1.292**	0.305***	0.277***
	(0.514)	(0.457)	(0.559)	(0.507)	(0.099)	(0.097)
Conditional on Issuance	No	Yes	No	Yes	No	Yes
FirmFE	Yes	Yes	Yes	Yes	Yes	Yes
YearFE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.374	0.383	0.360	0.369	0.065	0.066
Observations	4,430	3,928	4,116	3,577	1,998	1,929

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table presents coefficient estimates on bank debt share by estimating the regression specification (5.1), using data from Compustat, S&P Credit Ratings, and Capital IQ for the period 2002-2024. The outcome variable is market debt net issuance rate defined as the net increase in market debt outstanding during the following year scaled by the total amount of bank and market debt. Bank debt share is the amount of bank debt divided by the sum of bank and market debt. Columns 1-2 use observations with book leverage in the bottom quintile. Columns 3-4 use observations with market leverage in the bottom quintile. Columns 5-6 use observations with S&P entity credit ratings of A− and above. All regressions include firm and year fixed effects, and the full set of firm-year level control variables including the log of book assets, tangibility, ROA, book leverage, debt-to-EBITDA, dividend payout, market-to-book, stock return, and sales growth. Standard errors are double-clustered at the firm and year levels.

Firms with low leverage and high credit ratings face minimal default risk. As a result, the bank lender's ability to monitor and the contractual flexibility of bank debt (such as the ability to renegotiate terms during distress) offer limited additional value. Hence, if the observed pattern of complementarity between bank and market debt were driven by these features, the pattern should be absent for firms with very low leverage and high credit ratings. However, all coefficient estimates in Table 4 are positive and statistically significant. This suggests that the pattern of complementarity between bank and market debt cannot be primarily attributed to bank monitoring or contractual flexibility, as the pattern is evident even among low-risk firms for which bank monitoring and contractual flexibility are of limited importance.

6 Conclusion

This paper develops a dynamic model of firm debt structure that simultaneously incorporates both bank debt and market debt. The bank lender and the firm form a bilateral relationship

that is inherently strategic. As the firm's earnings and leverage evolve, there is incentive by the bank lender and the firm to renegotiate and amend the bank debt. Consequently, the bank debt is periodically renegotiated. By contrast, market debt investors are arm's-length. Absent other frictions, the firm can continuously trade with the market debt investors, in the form of market debt issuances or repurchases, at competitive prices. A key insight from the model is that the firm issues or repurchases market debt to strategically position its equity holders for upcoming bank debt renegotiation, while anticipating the effect of renegotiation on the market debt price.

The model predicts a dynamic complementarity between bank debt and market debt. That is, when the firm has more bank debt in its debt structure, it increases its market debt issuance intensity. The dynamic complementarity arises from the firm's incentive to adjust market debt issuance in anticipation of bank debt renegotiation. For instance, when the bank lender has all the bargaining power, the firm captures no bargaining surplus and the equity holders receive their continuation value upon renegotiation. When there is already a high level of bank debt and the firm is close to default, issuing additional market debt does little to weaken the equity holders' already diminished continuation value (i.e., their outside option). Hence, the firm's equity holders have little to lose and are incentivized to issue more market debt to shift risk onto creditors. The dynamic complementarity is robust to how bargaining power is allocated between the firm and the bank lender. However, the degree of this complementarity decreases with the firm's bargaining power.

While the model is developed to study debt structure dynamics, the underlying framework is quite general and applicable to other contexts, such as labor contract renegotiation and sovereign debt restructuring. The model, in a nutshell, centers on the interplay among three parties: two bargaining parties and a silent non-bargaining party. Of the two bargaining party, one is a "control" bargaining party who can trade directly with the non-bargaining party, while the other is a non-control party who cannot. The central idea is that the non-bargaining party prices in the expected outcome of the bargaining game between the two bargaining parties, shaping its trade with the control bargaining party, which in turn affects the bargaining outcome by influencing the bargaining parties' outside options and joint value.

References

Admati, Anat R., Peter M. DeMarzo, Martin F. Hellwig, and Paul Pfleiderer, "The Leverage Ratchet Effect," *Journal of Finance*, 2018, *73* (1), 145–198.

Becker, Bo and Victoria Ivashina, "Cyclicality of Credit Supply: Firm Level Evidence," *Journal of Monetary Economics*, 2014, *62*, 76–93.

Bidder, Rhys, Nicolas Crouzet, Maggie M. Jacobson, and Michael Siemer, "Debt Flexibility," Working Paper 2019.

Coase, Ronald H., "Durability and Monopoly," *Journal of Law and Economics*, 1972, *15* (1), 143–149.

Crouzet, Nicolas, "Aggregate Implications of Corporate Debt Choices," *Review of Economic Studies*, 2018, *85* (3), 1635–1682.

DeMarzo, Peter M. and Zhiguo He, "Leverage Dynamics without Commitment," *Journal of Finance*, 2021, *76* (3), 1195–1250.

Greenwood, Robin M., Samuel G. Hanson, and Lawrence J. Jin, "Reflexivity in Credit Markets," *Journal of Finance*, 2023, *forthcoming.*

Hackbarth, Dirk, Christopher A. Hennessy, and Hayne E. Leland, "Can the Trade-off Theory Explain Debt Structure?," *Review of Financial Studies*, 2007, *20* (5), 1389–1428.

Park, Cheol, "Monitoring and Structure of Debt Contracts," *Journal of Finance*, 2000, *55* (5), 2157–2195.

Rauh, Joshua D. and Amir Sufi, "Capital Structure and Debt Structure," *Review of Financial Studies*, 2010, *23* (12), 4242–4280.

Roberts, Michael R. and Amir Sufi, "Renegotiation of financial contracts: Evidence from private credit agreements," *Journal of Financial Economics*, 2009, *93* (2), 159–184.

Appendix

A   Derivations and Proofs

A.1   Proof of Lemma 1

Suppose that the firm and the bank lender agree on the proposed terms $\{\bar{B}_t, \mathcal{T}_t\}$ when renegotiating the bank debt. The equity holders receive a payment $\mathcal{T}_t$ from the bank lender, and their value becomes $V^e(Y_t, M_t, \bar{B}_t)$. If the firm and the bank lender fail to agree on the proposed terms, the equity holders' value remains at $V^e(Y_t, M_t, B_t)$. Hence, the equity holders' surplus from bargaining is given by

$$V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t - V^e(Y_t, M_t, B_t). \tag{A.1}$$

Similarly, the bank lender's surplus from bargaining is

$$V^b(Y_t, M_t, \bar{B}_t) - \mathcal{T}_t - V^b(Y_t, M_t, B_t). \tag{A.2}$$

The bank debt amount post-renegotiation and the payment from the bank lender to the equity holders are both determined by Nash bargaining between the bank lender and the firm, where the firm has bargaining power θ. Let $\{\bar{B}_t, \mathcal{T}_t\}$ denote the outcomes of the Nash bargaining problem. Then

$$\{\bar{B}_t, \mathcal{T}_t\} = \underset{B_t', \mathcal{T}_t'}{\arg\max} \; \left[V^e(Y_t, M_t, B_t') + \mathcal{T}_t' - V^e(Y_t, M_t, B_t)\right]^{\theta}$$
$$\left[V^b(Y_t, M_t, B_t') - \mathcal{T}_t' - V^b(Y_t, M_t, B_t)\right]^{1-\theta}, \tag{A.3}$$

subject to $V^e(Y_t, M_t, B'_t) + \mathcal{T}'_t - V^e(Y_t, M_t, B_t) \geq 0$ and $V^b(Y_t, M_t, B'_t) - \mathcal{T}'_t - V^b(Y_t, M_t, B_t) \geq 0$. The solution to the Nash bargaining problem is characterized by

$$\bar{B}_t = \underset{B'_t}{\arg\max} \; \left[V^e(Y_t, M_t, B'_t) + V^b(Y_t, M_t, B'_t) \right], \tag{A.4}$$

and given the post-renegotiation bank debt level $\bar{B}_t$,

$$(1 - \theta)\left[V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t - V^e(Y_t, M_t, B_t) \right] \tag{A.5}$$
$$= \theta\left[V^b(Y_t, M_t, \bar{B}_t) - \mathcal{T}_t - V^b(Y_t, M_t, B_t) \right].$$

Rearranging (A.5) yields

$$\mathcal{T}_t = \theta\left[V^b(Y_t, M_t, \bar{B}_t) - V^b(Y_t, M_t, B_t) \right] - (1 - \theta)\left[V^e(Y_t, M_t, \bar{B}_t) - V^e(Y_t, M_t, B_t) \right], \tag{A.6}$$

where $\bar{B}_t$ is given by (A.4). Furthermore, substituting (A.4) and (A.6) into the equity holders' surplus (A.1) and the bank lender's surplus (A.2), we have

$$V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t - V^e(Y_t, M_t, B_t) \tag{A.7}$$
$$= \theta\left[V^e(Y_t, M_t, \bar{B}_t) + V^b(Y_t, M_t, \bar{B}_t) - V^e(Y_t, M_t, B_t) - V^b(Y_t, M_t, B_t) \right],$$

and

$$V^b(Y_t, M_t, \bar{B}_t) - \mathcal{T}_t - V^b(Y_t, M_t, B_t) \tag{A.8}$$
$$= (1 - \theta)\left[V^e(Y_t, M_t, \bar{B}_t) + V^b(Y_t, M_t, \bar{B}_t) - V^e(Y_t, M_t, B_t) - V^b(Y_t, M_t, B_t) \right].$$

In words, the firm's equity holders capture a fraction θ of the joint surplus and the bank lender captures a fraction $1 - \theta$ of the joint surplus. Let $V(Y_t, M_t, B_t) \equiv V^e(Y_t, M_t, B_t) + V^b(Y_t, M_t, B_t)$, it is immediate that

$$V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t = V^e(Y_t, M_t, B_t) + \theta\left[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t) \right], \tag{A.9}$$
$$V^b(Y_t, M_t, \bar{B}_t) - \mathcal{T}_t = V^b(Y_t, M_t, B_t) + (1 - \theta)\left[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t) \right].$$

I now verify that the solution $\{\bar{B}_t, \mathcal{T}_t\}$ satisfy $V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t - V^e(Y_t, M_t, B_t) \geq 0$ and $V^b(Y_t, M_t, \bar{B}_t) - \mathcal{T}_t - V^b(Y_t, M_t, B_t) \geq 0$. First, note that (A.4) implies that $V^e(Y_t, M_t, \bar{B}_t) + V^b(Y_t, M_t, \bar{B}_t) \geq V^e(Y_t, M_t, B_t) + V^b(Y_t, M_t, B_t)$. Substituting the inequality into (A.7) and (A.8), it is immediate that the equity holders' surplus and the bank lender's surplus are both non-negative.

A.2   Proof of Proposition 2

The equity holders' value function (2.2) can be expressed recursively through the Hamilton-Jacobi-Bellman (HJB) equation

$$
\begin{aligned}
(r + \lambda)&V^e(Y_t, M_t, B_t) \\
&= \max_{G_t^m} \Big\{ (1 - \pi)\big[Y_t - c(B_t + M_t)\big] + p_t^m G_t^m + G_t^m V_M^e(Y_t, M_t, B_t) \\
&\qquad + \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t) \\
&\qquad + \lambda\big[V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t\big] \Big\}.
\end{aligned}
\tag{A.10}
$$

Taking the first-order condition of (A.10) with respect to G_t^m yields

$$
p_t^m \equiv p^m(Y_t, M_t, B_t) = -V_M^e(Y_t, M_t, B_t).
\tag{A.11}
$$

Substituting (A.11) along with (A.9) into the HJB equation (A.10), one can rewrite the HJB equation as

$$
\begin{aligned}
(r + \lambda)V^e(Y_t, M_t, B_t) =& (1 - \pi)\big[Y_t - c(B_t + M_t)\big] \\
&+ \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t) \\
&+ \lambda\Big\{V^e(Y_t, M_t, B_t) + \theta\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big]\Big\}.
\end{aligned}
\tag{A.12}
$$

Differentiating both sides of (A.12) with respect to M_t yields

$$
\begin{aligned}
(r + \lambda) & V_M^e(Y_t, M_t, B_t) \\
& = -(1 - \pi)c + \mu Y_t V_{MY}^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{MYY}^e(Y_t, M_t, B_t) \\
& \quad + \lambda \left\{ V_M^e(Y_t, M_t, B_t) + \theta \left[\frac{\partial V(Y_t, M_t, \bar{B}_t)}{\partial M_t} - V_M(Y_t, M_t, B_t) \right] \right\}.
\end{aligned} \tag{A.13}
$$

Note that

$$
\frac{\partial V(Y_t, M_t, \bar{B}_t)}{\partial M_t} = V_M(Y_t, M_t, \bar{B}_t) + V_B(Y_t, M_t, \bar{B}_t)\frac{\partial \bar{B}_t}{\partial M_t} = V_M(Y_t, M_t, \bar{B}_t), \tag{A.14}
$$

where the second equality follows the envelope theorem. Then substituting (A.14) into (A.13) yields

$$
\begin{aligned}
(r + \lambda) & V_M^e(Y_t, M_t, B_t) \\
& = -(1 - \pi)c + \mu Y_t V_{MY}^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{MYY}^e(Y_t, M_t, B_t) \\
& \quad + \lambda \left\{ V_M^e(Y_t, M_t, B_t) + \theta \left[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t) \right] \right\}.
\end{aligned} \tag{A.15}
$$

Moreover, from (A.11),

$$
p_Y^m(Y_t, M_t, B_t) = -V_{MY}^e(Y_t, M_t, B_t), \tag{A.16}
$$

$$
p_{YY}^m(Y_t, M_t, B_t) = -V_{MYY}^e(Y_t, M_t, B_t). \tag{A.17}
$$

Hence, substituting (A.11), (A.16)-(A.17) into (A.15) yields

$$
\begin{aligned}
-(r + \lambda) & p^m(Y_t, M_t, B_t) \\
& = -(1 - \pi)c - \mu Y_t p_Y^m(Y_t, M_t, B_t) - \frac{1}{2}\sigma^2 Y_t^2 p_{YY}^m(Y_t, M_t, B_t) \\
& \quad + \lambda \left\{ V_M^e(Y_t, M_t, B_t) + \theta \left[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t) \right] \right\}.
\end{aligned} \tag{A.18}
$$

Moreover from (2.4), the equilibrium market debt price $p^m(Y_t, M_t, B_t)$ satisfies the following HJB equation

$$
\begin{aligned}
(r + \lambda)p^m(Y_t, M_t, B_t) =& c + G_t^m p_M^m(Y_t, M_t, B_t) + \mu Y_t p_Y^m(Y_t, M_t, B_t) \\
&+ \frac{1}{2}\sigma^2 Y_t^2 p_{YY}^m(Y_t, M_t, B_t) + \lambda p^m(Y_t, M_t, \bar{B}_t).
\end{aligned}
\tag{A.19}
$$

Adding up (A.18) and (A.19) yields

$$
\begin{aligned}
0 =& \pi c + G_t^m p_M^m(Y_t, M_t, B_t) \\
&+ \lambda \Big\{ p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t) + \theta \big[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t) \big] \Big\}.
\end{aligned}
\tag{A.20}
$$

Rearranging yields

$$
\begin{aligned}
G_t^m =& \frac{\pi c}{-p_M^m(Y_t, M_t, B_t)} \\
&+ \lambda \frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t) + \theta \big[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t) \big]}{-p_M^m(Y_t, M_t, B_t)}.
\end{aligned}
\tag{A.21}
$$

A.3   Discrete-Time Example

Let $\{\bar{B}, \mathcal{T}\}$ be the outcome to the bargaining problem between the bank lender and the firm when renegotiating the bank debt, then as in Appendix A.1,

$$
\begin{aligned}
\{\bar{B}, \mathcal{T}\} = \underset{B', \mathcal{T}'}{\arg\max} \; & \big[V^e(M + \Delta, B') + \mathcal{T}' - V^e(M + \Delta, B) \big]^\theta \\
& \big[V^b(M + \Delta, B') - \mathcal{T}' - V^b(M + \Delta, B) \big]^{1-\theta},
\end{aligned}
\tag{A.22}
$$

subject to $V^e(M + \Delta, B') + \mathcal{T}' - V^e(M + \Delta, B) \geq 0$ and $V^b(M + \Delta, B') - \mathcal{T}' - V^b(M + \Delta, B) \geq 0$. Note that at the time of renegotiation, the face amount of market debt is $M + \Delta$ since the firm issues Δ amount of market debt at time t. Then the solution to the Nash bargaining problem is

characterized by

$$\bar{B} = \arg\max_{B'} V^e(M + \Delta, B') + V^b(M + \Delta, B'), \tag{A.23}$$

$$\mathcal{T} = \theta\big[V^b(M + \Delta, \bar{B}) - V^b(M + \Delta, B)\big] - (1 - \theta)\big[V^e(M + \Delta, \bar{B}) - V^e(M + \Delta, B)\big]. \tag{A.24}$$

Hence, the equity holders' renegotiation payoff is

$$V^e(M + \Delta, \bar{B}) + \mathcal{T} = V^e(M + \Delta, B) + \theta\big[V(M + \Delta, \bar{B}) - V(M + \Delta, B)\big]. \tag{A.25}$$

The condition (A.23) implies that $\bar{B}$ depends on $M + \Delta$ and thus can be expressed as $\bar{B} \equiv \bar{B}(M + \Delta)$. The firm's problem is

$$\max_{\Delta} V^e(M + \Delta, \bar{B}) + \mathcal{T} + \Delta p^m(M + \Delta, \bar{B}), \tag{A.26}$$

which can be rewritten as

$$\max_{\Delta} V^e(M + \Delta, B) + \theta\big[V(M + \Delta, \bar{B}(M + \Delta)) - V(M + \Delta, B)\big]$$
$$+ \Delta p^m(M + \Delta, \bar{B}(M + \Delta)). \tag{A.27}$$

Taking the first-order condition with respect to Δ and applying the envelope theorem,

$$V_M^e(M + \Delta, B) + \theta\big[V_M(M + \Delta, \bar{B}) - V_M(M + \Delta, B)\big] + p^m(M + \Delta, \bar{B})$$
$$+ \Delta p_M^m(M + \Delta, \bar{B}) + \Delta p_B^m(M + \Delta, \bar{B})\bar{B}'(M + \Delta) = 0. \tag{A.28}$$

Rearranging the first-order condition yields

$$\Delta = \frac{p^m(M + \Delta, \bar{B}) + V_M^e(M + \Delta, B) + \theta\big[V_M(M + \Delta, \bar{B}) - V_M(M + \Delta, B)\big]}{-p_M^m(M + \Delta, \bar{B})\left[1 + \frac{p_B^m(M+\Delta,\bar{B})}{p_M^m(M+\Delta,\bar{B})}\bar{B}'(M + \Delta)\right]}. \tag{A.29}$$

A.4 Proof of Proposition 4 and Corollary 4.1

In the case with a strong bank lender, that is $\theta = 0$, the equity holders' HJB equation (A.12) becomes

$$
\begin{aligned}
& rV^e(Y_t, M_t, B_t) \\
& = (1 - \pi)\big[Y_t - c(B_t + M_t)\big] + \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t).
\end{aligned}
\tag{A.30}
$$

That is, the equity holders' value can be solved as if the firm neither issues market debt nor renegotiates its bank debt.

Conditional on the face amount of market debt $M_t = M$ and the face amount of bank debt $B_t = B$, let $V^{e,0}(Y_t; M, B)$ denote the equity holders' no-issuance value, that is the equity holders' value when there is never any debt adjustment. Then $V^{e,0}(Y_t; M, B)$ must satisfy the following HJB equation

$$
\begin{aligned}
& rV^{e,0}(Y_t; M, B) \\
& = (1 - \pi)\big[Y_t - c(B + M)\big] + \mu Y_t V^{e,0\prime}(Y_t; M, B) + \frac{1}{2}\sigma^2 Y_t^2 V^{e,0\prime\prime}(Y_t; M, B).
\end{aligned}
\tag{A.31}
$$

If the firm never defaults, then the cash flows can be evaluated as growing perpetuities. Let $\tilde{V}^{e,0}(Y_t; M, B)$ denote the equity holders' no-issuance value when there is never default, then

$$
\tilde{V}^{e,0}(Y_t; M, B) = \frac{1 - \pi}{r - \mu}Y_t - \frac{(1 - \pi)c(B + M)}{r}.
\tag{A.32}
$$

The first term on the right-hand side is the no-default present value of after-tax earnings, and the second term on the right-hand side is the no-default present value of after-tax coupon payments.

The equity holders' no-issuance value $V^{e,0}(Y_t; M, B)$ must thus equal the no-default value as given by (A.32) plus the value of a default option. That is,

$$
V^{e,0}(Y_t; M, B) = \tilde{V}^{e,0}(Y_t; M, B) + \omega(Y_t; M, B)\big[0 - \tilde{V}^{e,0}(Y^\star(M, B); M, B)\big],
\tag{A.33}
$$

where $Y^\star(M, B)$ is the default boundary at which the equity holders choose to default and $\omega(Y_t; M, B)$ is the discount factor applied to the default option and follows the homogeneous version of the ODE (A.31). That is,

$$r\omega(Y_t; M, B) = \mu Y_t \omega'(Y_t; M, B) + \frac{1}{2}\sigma^2 Y_t^2 \omega''(Y_t; M, B). \tag{A.34}$$

The general solution to (A.34) is

$$\omega(Y_t; M, B) = K_\gamma Y_t^{-\gamma} + K_\eta Y_t^\eta, \tag{A.35}$$

where

$$-\gamma \equiv -\frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}, \tag{A.36}$$

$$\eta \equiv -\frac{\mu - \frac{1}{2}\sigma^2 - \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}. \tag{A.37}$$

It is easy to verify that $-\gamma < 0$ and $\eta > 1$. The constants K_γ and K_η as well as the default boundary $Y^\star(M, B)$ are determined by the boundary conditions. First, the no-bubble condition as $Y_t \to \infty$ says that $\omega(\infty; B, M) = 0$, implying that $K_\eta = 0$. The value matching condition at the default boundary requires that $\omega(Y^\star(M, B); M, B) = 1$, thus $K_\gamma = Y^\star(M, B)^\gamma$ and

$$\omega(Y_t; M, B) = \left[\frac{Y_t}{Y^\star(M, B)}\right]^{-\gamma}. \tag{A.38}$$

From (A.33), the equity holders' no-issuance value can thus be rewritten as

$$V^{e,0}(Y_t; M, B) = \frac{1 - \pi}{r - \mu}Y_t - \frac{(1 - \pi)c(B + M)}{r}$$
$$\underbrace{- \left[\frac{1 - \pi}{r - \mu}Y^\star(M, B) - \frac{(1 - \pi)c(B + M)}{r}\right]\left[\frac{Y_t}{Y^\star(M, B)}\right]^{-\gamma}}_{\text{default option value}}. \tag{A.39}$$

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The smooth pasting condition at the default boundary requires that $V^{e,0\prime}(Y_t; M, B) = 0$. From (A.39), the default boundary is thus given by

$$Y^\star(M, B) = \frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r}(B+M). \tag{A.40}$$

By inspecting (A.39), the default option as captured by the last term on the right-hand side is indeed maximized at the default boundary $Y^\star(M, B)$. Substituting the default boundary into (A.39) yields

$$\begin{aligned}
V^{e,0}(Y_t; M, B) =& \frac{1-\pi}{r-\mu}Y_t - \frac{(1-\pi)c}{r}(B+M) \\
&+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\left[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\right]^\gamma (B+M)^{1+\gamma}Y_t^{-\gamma}.
\end{aligned} \tag{A.41}$$

Notice that the default option value as captured by the last term on the right-hand side is decreasing in Y_t. The equity holders choose to default when Y_t reaches $Y^\star(M, B)$ from the above. In equilibrium, the equity holders' value equals the equity holders' no-issuance value. That is, $V^e(Y_t, M_t, B_t) = V^{e,0}(Y_t; M_t, B_t)$. From (A.41),

$$\begin{aligned}
V^e(Y_t, M_t, B_t) =& \frac{1-\pi}{r-\mu}Y_t - \frac{(1-\pi)c}{r}(B_t+M_t) \\
&+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\left[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\right]^\gamma (B_t+M_t)^{1+\gamma}Y_t^{-\gamma}.
\end{aligned} \tag{A.42}$$

Differentiating (A.42) with respect to M_t and B_t respectively, we get

$$\begin{aligned}
V_M^e(Y_t, M_t, B_t) =& V_B^e(Y_t, M_t, B_t) \\
=& -\frac{(1-\pi)c}{r} + \frac{(1-\pi)c}{r}\left[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\right]^\gamma (B_t+M_t)^\gamma Y_t^{-\gamma} \\
=& -\frac{(1-\pi)c}{r}\left\{1 - \left[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\right]^\gamma\left(\frac{B_t+M_t}{Y_t}\right)^\gamma\right\}.
\end{aligned} \tag{A.43}$$

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The equity holders choose to default when the firm's EBIT hits the default boundary (A.40) from above. Thus when the firm is not in default, $Y_t > Y^\star(M_t, B_t)$. That is

$$Y_t > \frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r}(B_t + M_t). \tag{A.44}$$

From (A.43) and (A.44), it is immediate that

$$V_M^e(Y_t, M_t, B_t) = V_B^e(Y_t, M_t, B_t) < 0. \tag{A.45}$$

Moreover, differentiating (A.43) with respect to B_t yields

$$V_{MB}^e(Y_t, M_t, B_t) = \gamma \frac{(1-\pi)c}{r} \left[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r}\right]^\gamma (B_t + M_t)^{\gamma-1} Y_t^{-\gamma} > 0. \tag{A.46}$$

A.5   Proof of Proposition 5

The equilibrium market debt price satisfies the first-order condition as given by (2.9), that is $p^m(Y_t, M_t, B_t) = -V_M^e(Y_t, M_t, B_t)$. Differentiating (A.42) with respect to M_t yields

$$V_M^e(Y_t, M_t, B_t) = -\frac{(1-\pi)c}{r} \left\{1 - \left[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r}\right]^\gamma \left(\frac{B_t + M_t}{Y_t}\right)^\gamma\right\}. \tag{A.47}$$

Thus, the equilibrium market debt price is

$$p^m(Y_t, M_t, B_t) = \frac{(1-\pi)c}{r} \left\{1 - \left[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r}\right]^\gamma \left(\frac{B_t + M_t}{Y_t}\right)^\gamma\right\}. \tag{A.48}$$

Differentiating (A.48) with respect to M_t yields

$$p_M^m(Y_t, M_t, B_t) = -\gamma \frac{(1-\pi)c}{r} \left[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r}\right]^\gamma \frac{(B_t + M_t)^{\gamma-1}}{Y_t^\gamma}. \tag{A.49}$$

Note that

$$p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t)$$

$$= \frac{(1-\pi)c}{r} \left[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r} \right]^\gamma \left[\left(\frac{B_t + M_t}{Y_t} \right)^\gamma - \left(\frac{\bar{B}_t + M_t}{Y_t} \right)^\gamma \right]. \tag{A.50}$$

The firm's optimal market debt issuance rate G_t^m is obtained from (2.10). When $\theta = 0$, it simplifies to

$$G_t^m = \frac{\pi c}{-p_M^m(Y_t, M_t, B_t)} + \lambda \frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t)}{-p_M^m(Y_t, M_t, B_t)}. \tag{A.51}$$

Using (A.49) and (A.50), we get

$$\begin{aligned} G_t^m &\equiv G^m(Y_t, M_t, B_t) \\ &= \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \left[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t + M_t} \right]^\gamma (B_t + M_t) \\ &\quad + \frac{\lambda}{\gamma} \left[1 - \left(\frac{\bar{B}_t + M_t}{B_t + M_t} \right)^\gamma \right] (B_t + M_t). \end{aligned} \tag{A.52}$$

Let $g_t^m \equiv G_t^m / (B_t + M_t)$ be market debt issuance intensity. From (A.52),

$$g_t^m = \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \left[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t + M_t} \right]^\gamma + \frac{\lambda}{\gamma} \left[1 - \left(\frac{\bar{B}_t + M_t}{B_t + M_t} \right)^\gamma \right]. \tag{A.53}$$

Since $\gamma > 0$, it is immediate that the first term on the right-hand side is decreasing in B_t. To see that the second term is increasing in B_t, note that

$$\bar{B}_t = \underset{B_t'}{\arg\max} \left[V^e(Y_t, M_t, B_t') + V^b(Y_t, M_t, B_t') \right]. \tag{A.54}$$

It is immediate that $\bar{B}_t$ depends only on Y_t and M_t. We can thus write $\bar{B}_t = \bar{B}(Y_t, M_t)$. Then (A.53) can be rewritten as

$$g_t^m = \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \left[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t + M_t} \right]^\gamma + \frac{\lambda}{\gamma} \left[1 - \left(\frac{\bar{B}(Y_t, M_t) + M_t}{B_t + M_t} \right)^\gamma \right]. \tag{A.55}$$

A.6 Bank Debt Value When $\theta = 0$

The bank lender's value function (2.3) can be expressed through the HJB equation

$$
\begin{aligned}
(r + \lambda)V^b(Y_t, M_t, B_t) = & cB_t + G_t^m V_M^b(Y_t, M_t, B_t) \\
& + \mu Y_t V_Y^b(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^b(Y_t, M_t, B_t) \\
& + \lambda\big[V^b(Y_t, M_t, \bar{B}_t) - \mathcal{T}_t\big].
\end{aligned} \tag{A.56}
$$

Since

$$
V^b(Y_t, M_t, \bar{B}_t) - \mathcal{T}_t = V^b(Y_t, M_t, B_t) + (1 - \theta)\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big], \tag{A.57}
$$

then the bank lender's HJB equation can be written as

$$
\begin{aligned}
(r + \lambda)V^b = & cB_t + G_t^m V_M^b(Y_t, M_t, B_t) \\
& + \mu Y_t V_Y^b(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^b(Y_t, M_t, B_t) \\
& + \lambda\Big\{V^b(Y_t, M_t, B_t) + (1 - \theta)\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big]\Big\}.
\end{aligned} \tag{A.58}
$$

When $\theta = 0$, the bank lender's value $V^b(Y_t, M_t, B_t)$ must satisfy the following HJB equation

$$
\begin{aligned}
(r + \lambda)V^b(Y_t, M_t, B_t) = & cB_t + G_t^m V_M^b(Y_t, M_t, B_t) \\
& + \mu Y_t V_Y^b(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^b(Y_t, M_t, B_t) \\
& + \lambda\big[V(Y_t, M_t, \bar{B}_t) - V^e(Y_t, M_t, B_t)\big].
\end{aligned} \tag{A.59}
$$

The firm is scale invariant in the setting. Thus, the bank lender's value $V^b(Y_t, M_t, B_t)$ is homogeneous of degree one. That is,

$$
V^b(Y_t, M_t, B_t) = V^b\Big(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\Big)M_t \equiv v^b(y_t, b_t)M_t. \tag{A.60}
$$

The joint value of the equity holders and the bank lender $V(Y_t, M_t, B_t) \equiv V^e(Y_t, M_t, B_t) + V^b(Y_t, M_t, B_t)$ is also homogeneous of degree one. That is,

$$V(Y_t, M_t, B_t) = V\left(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\right) M_t \equiv v(y_t, b_t) M_t, \tag{A.61}$$

where $v(y_t, b_t) \equiv v^e(y_t, b_t) + v^b(y_t, b_t)$ with $v^e(y_t, b_t) \equiv V^e(Y_t, M_t, B_t)/M_t$. From (2.6) of Lemma 1,

$$\bar{B}_t = \arg\max_{B'_t} V(Y_t, M_t, B'_t). \tag{A.62}$$

Given the amount of market debt outstanding M_t, (A.62) is equivalent to

$$\bar{b}_t \equiv \frac{\bar{B}_t}{M_t} = \arg\max_{b'_t} v(y_t, b'_t). \tag{A.63}$$

Here, $\bar{b}_t$ is the renegotiated face amount of the bank debt scaled by the amount of market debt outstanding, that is the post-renegotiation bank debt ratio. From (A.63), it is easy to see that $\bar{b}_t$ is a function of the firm's EBIT-to-market-debt ratio y_t, that is $\bar{b}_t \equiv \bar{b}(y_t)$. Note that

$$V(Y_t, M_t, \bar{B}_t) = v(y_t, \bar{b}_t) M_t, \tag{A.64}$$

where $\bar{b}_t$ is given by (A.63). Thus, from (A.63) and (A.64),

$$V(Y_t, M_t, \bar{B}_t) = \max_{b'_t} v(y_t, b'_t) M_t. \tag{A.65}$$

From (A.60),

$$V_M^b(Y_t, M_t, B_t) = v^b(y_t, b_t) - y_t v_y^b(y_t, b_t) - b_t v_b^b(y_t, b_t), \tag{A.66}$$

$$V_Y^b(Y_t, M_t, B_t) = v_y^b(y_t, b_t), \tag{A.67}$$

$$V_{YY}^b(Y_t, M_t, B_t) = \frac{1}{M_t} v_{yy}^b(y_t, b_t). \tag{A.68}$$

Substituting (A.60), (A.61), and (A.65)-(A.68) into the bank lender's HJB equation (A.59), we obtain the following HJB equation for the case with $\theta = 0$ as

$$
\begin{aligned}
(r + \lambda)v^b(y_t, b_t)M_t =& cB_t + G_t^m \left[v^b(y_t, b_t) - y_t v_y^b(y_t, b_t) - b_t v_b^b(y_t, b_t) \right] \\
&+ \mu Y_t v_y^b(y_t, b_t) + \frac{1}{2}\sigma^2 \frac{Y_t^2}{M_t} v_{yy}^b(y_t, b_t) \\
&+ \lambda \left[\max_{b_t'} v(y_t, b_t') - v^e(y_t, b_t) \right] M_t.
\end{aligned}
\tag{A.69}
$$

Note that

$$
\frac{G_t^m}{M_t} = \frac{G_t^m}{B_t + M_t} \frac{B_t + M_t}{M_t} = g_t^m(1 + b_t).
\tag{A.70}
$$

Then dividing both sides of (A.69) by M_t and rearranging, we get

$$
\begin{aligned}
\left[r + \lambda - g_t^m(1 + b_t) \right] v^b(y_t, b_t) =& cb_t - g_t^m b_t(1 + b_t) v_b^b(y_t, b_t) + \left[\mu - g_t^m(1 + b_t) \right] y_t v_y^b(y_t, b_t) \\
&+ \frac{1}{2}\sigma^2 y_t^2 v_{yy}^b(y_t, b_t) + \lambda \left[\max_{b_t'} v(y_t, b_t') - v^e(y_t, b_t) \right],
\end{aligned}
\tag{A.71}
$$

where $v_b^b(y_t, b_t)$ denotes the first-order partial derivative with respect to b_t, while $v_y^b(y_t, b_t)$ and $v_{yy}^b(y_t, b_t)$ are the first-order and the second-order partial derivatives with respect to y_t.

The market debt issuance intensity g_t^m is given by (3.3), which can be rewritten as

$$
\begin{aligned}
g_t^m \equiv& g^m(y_t, b_t) \\
=& \frac{\pi c}{\gamma} \frac{r}{(1 - \pi)c} \left[\frac{1 + \gamma}{\gamma} \frac{r}{(r - \mu)c} \frac{y_t}{1 + b_t} \right]^\gamma + \frac{\lambda}{\gamma} \left\{ 1 - \left(\frac{1 + \bar{b}_t}{1 + b_t} \right)^\gamma \right\},
\end{aligned}
\tag{A.72}
$$

where $\bar{b}_t$ satisfies (A.63). Moreover, $v(y_t, b_t) \equiv v^e(y_t, b_t) + v^b(y_t, b_t)$. From (3.1), we have

$$
\begin{aligned}
v^e(y_t, b_t) \equiv& \frac{V^e(Y_t, M_t, B_t)}{M_t} \\
=& \frac{1 - \pi}{r - \mu} y_t - \frac{(1 - \pi)c}{r}(1 + b_t) \\
&+ \frac{1}{1 + \gamma} \frac{(1 - \pi)c}{r}(1 + b_t) \left[\frac{\gamma}{1 + \gamma} \frac{(r - \mu)c}{r} \right]^\gamma \left(\frac{y_t}{1 + b_t} \right)^{-\gamma}.
\end{aligned}
\tag{A.73}
$$

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Note that the two state variables are $y_t \equiv Y_t/M_t$ and $b_t \equiv B_t/M_t$. One can transform the state space such that the model is solved in terms of $\tilde{y}_t \equiv [Y_t - Y^\star(M_t, B_t)]/M_t = y_t - y^\star(b_t)$ and b_t. In the transformed state space, the default boundary is $\tilde{y}^\star = 0$. That is, the equity holders choose to default when the distance to default approaches zero from above. From (A.72), we can rewrite g_t^m as

$$g^m(\tilde{y}_t, b_t) = \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \left[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{\tilde{y}_t + y^\star(b_t)}{1+b_t} \right]^\gamma + \frac{\lambda}{\gamma} \left\{ 1 - \left(\frac{1+\bar{b}_t}{1+b_t} \right)^\gamma \right\}, \tag{A.74}$$

where $\bar{b}_t$ is solution to (A.63), and $y^\star(b_t)$ can be obtained from (3.2), that is

$$y^\star(b_t) = \frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r} (1+b_t). \tag{A.75}$$

Similarly, from (A.73), the equity value can be written as

$$\begin{aligned} v^e(\tilde{y}_t, b_t) =& \frac{1-\pi}{r-\mu} [\tilde{y}_t + y^\star(b_t)] - \frac{(1-\pi)c}{r}(1+b_t) \\ &+ \frac{1}{1+\gamma} \frac{(1-\pi)c}{r}(1+b_t) \left[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r} \right]^\gamma \left[\frac{\tilde{y}_t + y^\star(b_t)}{1+b_t} \right]^{-\gamma}. \end{aligned} \tag{A.76}$$

Note that

$$\frac{\partial v^b}{\partial b_t} = v_b^b(\tilde{y}_t, b_t) - v_{\tilde{y}}^b(\tilde{y}_t, b_t) y^{\star\prime}(b_t), \tag{A.77}$$

$$\frac{\partial v^b}{\partial y_t} = v_{\tilde{y}}^b(\tilde{y}_t, b_t), \tag{A.78}$$

$$\frac{\partial^2 v^b}{\partial y_t^2} = v_{\tilde{y}\tilde{y}}^b(\tilde{y}_t, b_t). \tag{A.79}$$

Substituting (A.77)-(A.79) into (A.71) yields

$$
\begin{aligned}
\left[r + \lambda - g_t^m(1 + b_t)\right] v^b(\tilde{y}_t, b_t) =& c b_t - g_t^m b_t (1 + b_t) \left[v_b^b(\tilde{y}_t, b_t) - v_{\tilde{y}}^b(\tilde{y}_t, b_t) y^{\star\prime}(b_t)\right] \\
&+ \left[\mu - g_t^m(1 + b_t)\right] \left[\tilde{y}_t + y^\star(b_t)\right] v_{\tilde{y}}^b(\tilde{y}_t, b_t) \\
&+ \frac{1}{2}\sigma^2 \left[\tilde{y}_t + y^\star(b_t)\right]^2 v_{\tilde{y}\tilde{y}}^b(\tilde{y}_t, b_t) \\
&+ \lambda \left[\bar{v}(\tilde{y}_t, b_t) - v^e(\tilde{y}_t, b_t)\right],
\end{aligned}
\tag{A.80}
$$

where

$$
\bar{v}(\tilde{y}_t, b_t) = \max_{b_t'} \ v\big(\tilde{y}_t + y^\star(b_t) - y^\star(b_t'), b_t'\big).
\tag{A.81}
$$

Rearranging (A.80) yields

$$
\begin{aligned}
\left[r + \lambda - g_t^m(1 + b_t)\right] v^b(\tilde{y}_t, b_t) =& \ c b_t - g_t^m b_t (1 + b_t) v_b^b(\tilde{y}_t, b_t) \\
&+ \left\{\left[\mu - g_t^m(1 + b_t)\right]\left[\tilde{y}_t + y^\star(b_t)\right] + g_t^m b_t(1 + b_t) y^{\star\prime}(b_t)\right\} v_{\tilde{y}}^b(\tilde{y}_t, b_t) \\
&+ \frac{1}{2}\sigma^2 \left[\tilde{y}_t + y^\star(b_t)\right]^2 v_{\tilde{y}\tilde{y}}^b(\tilde{y}_t, b_t) + \lambda\left[\bar{v}(\tilde{y}_t, b_t) - v^e(\tilde{y}_t, b_t)\right].
\end{aligned}
\tag{A.82}
$$

The boundary value of the bank debt depends on its seniority. If the bank debt is senior to the market debt, the bank lender's recovery value upon default is $\min\{\alpha Y^\star, B_{\tau^\star}\}$. Let $v^{b,\star}$ denote the bank debt value at default, then

$$
v^{b,\star} = \frac{\min\{\alpha Y^\star, B_{\tau^\star}\}}{M_{\tau^\star}} = \min\{\alpha y^\star(b_{\tau^\star}), b_{\tau^\star}\}.
\tag{A.83}
$$

If the bank debt is pari passu with the market debt or junior to the market debt, dilution ahead of default pushes the boundary value of the bank debt to zero.

A.7 General Case with $\theta > 0$

When $\theta > 0$, the equity holders' value $V^e(Y_t, M_t, B_t)$ satisfies the HJB equation

$$(r + \lambda)V^e(Y_t, M_t, B_t) = \max_{G_t^m} \left\{ (1 - \pi)\left[Y_t - c(B_t + M_t)\right] + p_t^m G_t^m + G_t^m V_M^e(Y_t, M_t, B_t) \right.$$
$$+ \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t) \tag{A.84}$$
$$\left. + \lambda\left[V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t\right] \right\}.$$

Substituting the conditions (2.7) and (2.9) into the HJB equation yields

$$(r + \theta\lambda)V^e(Y_t, M_t, B_t) = (1 - \pi)\left[Y_t - c(B_t + M_t)\right]$$
$$+ \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t) \tag{A.85}$$
$$+ \theta\lambda\left[V(Y_t, M_t, \bar{B}_t) - V^b(Y_t, M_t, B_t)\right].$$

The equity holders' value is homogeneous of degree one, that is

$$V^e(Y_t, M_t, B_t) = V^e\left(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\right)M_t \equiv v^e(y_t, b_t)M_t. \tag{A.86}$$

Similarly,

$$V(Y_t, M_t, B_t) = V\left(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\right)M_t \equiv v(y_t, b_t)M_t. \tag{A.87}$$

Then (A.85) can be rewritten as

$$(r + \theta\lambda)v^e(y_t, b_t) = (1 - \pi)\left[y_t - c(1 + b_t)\right] + \mu y_t v_y^e(y_t, b_t) + \frac{1}{2}\sigma^2 y_t^2 v_{yy}^e(y_t, b_t)$$
$$+ \theta\lambda\left[\max_{b_t'} v(y_t, b_t') - v^b(y_t, b_t)\right]. \tag{A.88}$$

Let $y^\star(b_t)$ denote the default boundary, that is the EBIT-to-market-debt ratio at which the equity holders choose to default. It is a function of the bank debt ratio b_t. The value matching

condition and the smooth pasting condition require that

$$v^e(y^\star(b_t), b_t) = 0, \tag{A.89}$$

$$v_y^e(y^\star(b_t), b_t) = 0. \tag{A.90}$$

To facilitate numerical solution, I apply the following transformation to the variable y_t by letting $\tilde{y}_t \equiv y_t - y^\star(b_t)$. Then for all b_t, the equity holders choose to default when $\tilde{y}_t$ is zero. Then,

$$
\begin{aligned}
(r + \theta\lambda)v^e(\tilde{y}_t, b_t) =&(1 - \pi)\big[\tilde{y}_t + y^\star(b_t) - c(1 + b_t)\big] + \theta\lambda\big[\bar{v}(\tilde{y}_t, b_t) - v^b(\tilde{y}_t, b_t)\big] \\
&+ \mu\big[\tilde{y}_t + y^\star(b_t)\big]v_{\tilde{y}}^e(\tilde{y}_t, b_t) + \frac{1}{2}\sigma^2\big[\tilde{y}_t + y^\star(b_t)\big]^2 v_{\tilde{y}\tilde{y}}^e(\tilde{y}_t, b_t),
\end{aligned}
\tag{A.91}
$$

where

$$\bar{v}(\tilde{y}_t, b_t) \equiv \max_{b_t'} \ v(y_t, b_t'). \tag{A.92}$$

The bank lender's value $V^b(Y_t, M_t, B_t)$ satisfies the HJB equation

$$
\begin{aligned}
\big[r + (1 - \theta)\lambda\big]V^b(Y_t, M_t, B_t) =&cB_t + G_t^m V_M^b(Y_t, M_t, B_t) \\
&+ \mu Y_t V_Y^b(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^b(Y_t, M_t, B_t) \\
&+ (1 - \theta)\lambda\big[V(Y_t, M_t, \bar{B}_t) - V^e(Y_t, M_t, B_t)\big].
\end{aligned}
\tag{A.93}
$$

Since $V^b(Y_t, M_t, B_t)$ is homogeneous of degree one, the HJB equation can thus be rewritten as

$$
\begin{aligned}
\big[r + (1 - \theta)\lambda - g_t^m\big]v^b(y_t, b_t) =&cb_t - g_t^m b_t v_b^b(y_t, b_t) \\
&+ (\mu - g_t^m)y_t v_y^b(y_t, b_t) + \frac{1}{2}\sigma^2 y_t^2 v_{yy}^b(y_t, b_t) \\
&+ (1 - \theta)\lambda\big[\max_{b_t'} \ v(y_t, b_t') - v^e(y_t, b_t)\big].
\end{aligned}
\tag{A.94}
$$

Again, one can solve the differential equation in terms of $\tilde{y}_t \equiv y_t - y^\star(b_t)$ and b_t, and thus

$$
\begin{aligned}
\left[r + (1-\theta)\lambda - g_t^m\right] v^b(\tilde{y}_t, b_t) =& cb_t - g_t^m b_t v_b^b(\tilde{y}_t, b_t) + (1-\theta)\lambda\left[\bar{v}(\tilde{y}_t, b_t) - v^e(\tilde{y}_t, b_t)\right] \\
&+ \left\{(\mu - g_t^m)[\tilde{y}_t + y^\star(b_t)] + g_t^m b_t y^{\star\prime}(b_t)\right\} v_{\tilde{y}}^b(\tilde{y}_t, b_t) \\
&+ \frac{1}{2}\sigma^2\left[\tilde{y}_t + y^\star(b_t)\right]^2 v_{\tilde{y}\tilde{y}}^b(\tilde{y}_t, b_t).
\end{aligned}
\tag{A.95}
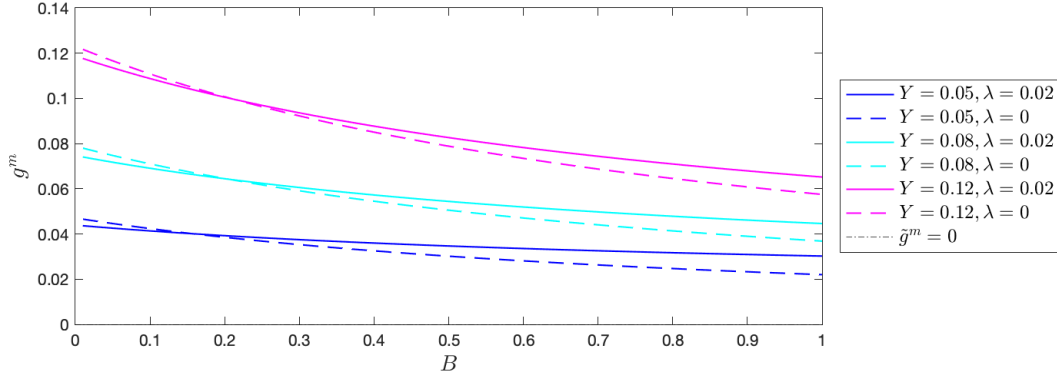$$

B Additional Robustness Results

Figure B.1: Market debt issuance intensity $g^m(Y_t, M_t, B_t)$ (small λ). The figure plots the market debt issuance intensity as a function of the face amount of bank debt B_t, holding the firm's EBIT Y_t and market debt outstanding M_t constant. Specifically, I plot the relationship between the market debt issuance intensity and the bank debt outstanding for $Y_t = 0.05, 0.08$ and 0.12 while normalizing the amount of market debt outstanding to one. The parameters take the values: $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = (1 - \pi)c = 0.05$, $\lambda = 0.02$, and $\alpha = 2.33$.
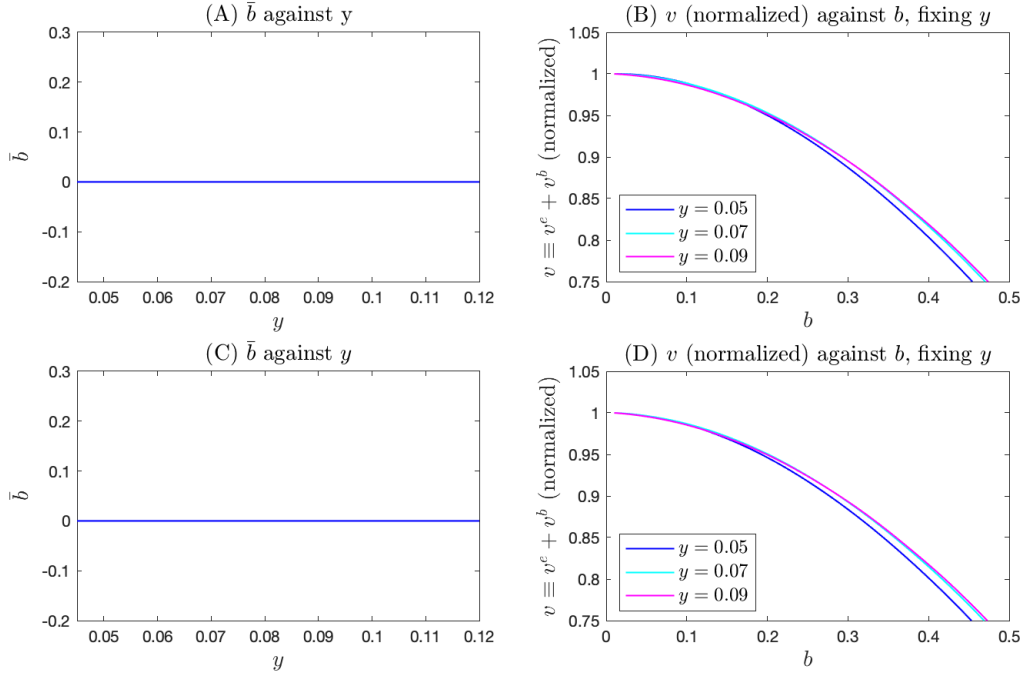

Figure B.2: Surplus-maximizing non-senior bank debt ratio $\bar{b}(y_t)$ from renegotiation. The figure plots the surplus-maximizing bank debt ratio $\bar{b}(y_t)$ as a function of the firm's EBIT-to-market-debt ratio y_t when the bank debt is pari passu with the market debt (Panel A) and when the bank debt is junior to the market debt (Panel C), when there is perfect legal protection against dilution ahead of default. Panel B and Panel D plot the normalized joint equity and bank debt value against the bank debt ratio b_t for various y_t, when the bank debt is pari passu and when it is junior. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = (1 - \pi)c = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

C   Bank Lending Shock

In this appendix, I consider the effect of a bank lending shock, modeled as a shock to the bank lender's discount rate. The economy is in one of two states, $\xi \in \{n, s\}$ with n denoting the normal state and s denoting the shock state. In the normal state, the bank lender discounts time at rate $r^n > 0$. In the shock state, the bank lender has a higher discount rate $r^s > r^n$. The economy transitions between states through a Markov switching process that is independent of the Poisson process governing bank-firm renegotiations. The economy transitions from the shock state to the normal state with intensity ζ^s, and from the normal state to the shock state with intensity ζ^n which I normalize to zero. That is, the normal state is absorbing, and my analysis examines the impact of a bank lending shock whose persistence is characterized by $1/\zeta^s$. For simplicity, I continue to focus on the case with a strong bank lender (i.e., $\theta = 0$).

Let $V^{e,\xi}(Y_t, M_t, B_t)$ denote the equity holders' value when the economy is in state ξ and the firm has EBIT Y_t, market debt outstanding M_t, and bank debt outstanding B_t. Let $p^{m,\xi}(Y_t, M_t, B_t)$ be the corresponding equilibrium market debt price. As shown in Section C.3 and analogous to before, the market debt issuance rate is a function of the state variables and is given by

$$G^{m,\xi}(Y_t, M_t, B_t) = \frac{\pi c}{-p_M^{m,\xi}(Y_t, M_t, B_t)} + \lambda \frac{p^{m,\xi}(Y_t, M_t, \bar{B}_t^{\xi}) + V_M^{e,\xi}(Y_t, M_t, B_t)}{-p_M^{m,\xi}(Y_t, M_t, B_t)}. \tag{C.1}$$

C.1   Permanent Shock

As a benchmark, I first consider a bank lending shock that is permanent. That is, $\zeta^s = 0$. For example, post-GFC banking regulations have tightened banks' balance sheet constraints, making it more costly for banks to lend to firms. I continue to use $g^{m,\xi}(Y_t, M_t, B_t) \equiv G^{m,\xi}(Y_t, M_t, B_t)/(B_t + M_t)$, the market debt issuance intensity, to capture the firm's market debt issuance behavior. As shown in the Appendix, when the bank lending shock is permanent, the firm's market debt

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issuance intensity in state $\xi \in \{n, s\}$ is given by

$$g^{m,\xi}(Y_t, M_t, B_t) = \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \left[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t + M_t} \right]^\gamma + \frac{\lambda}{\gamma} \left[1 - \left(\frac{\bar{B}_t^\xi + M_t}{B_t + M_t} \right)^\gamma \right]. \qquad \text{(C.2)}$$

Thus, the market debt issuance intensity differs in the two states of the economy, because the bank lending shock affects the surplus-maximizing level of bank debt upon renegotiation. In Figure C.1, I plot the surplus-maximizing bank debt amount post-renegotiation (Panel A) as well as the corresponding market debt issuance intensity (Panel B) in both the shock state and the normal state, for given levels of EBIT while normalizing the face amount of market debt to one. The parameters take the baseline values: $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r^n = r = c(1-\pi) = 0.05$, $r^s = 0.07$, $\lambda = 0.2$, and $\alpha = 2.33$.
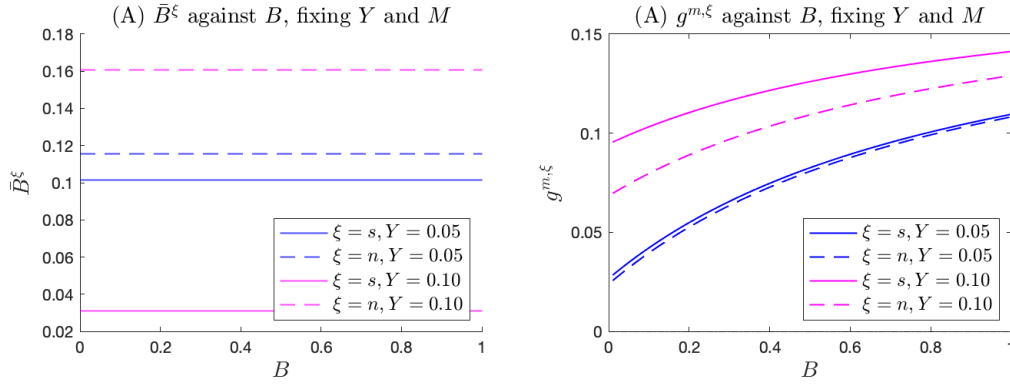

Figure C.1: Effect of permanent bank lending shock on post-renegotiation bank debt level and market debt issuance intensity. Panel A of the figure plots the surplus-maximizing bank debt level post-renegotiation in the normal state and in the shock state, $\bar{B}_t^n$ and $\bar{B}_t^s$, against the prevailing bank debt amount B_t for given levels of EBIT Y_t while normalizing the amount of market debt M_t to one. Panel B plots the corresponding market debt issuance intensity $g^{m,\xi}(Y_t, M_t, B_t)$ in both states of the economy $\xi \in \{n, s\}$. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r^n = r = c(1-\pi) = 0.05$, $r^s = 0.07$, $\lambda = 0.2$, and $\alpha = 2.33$.

The figure shows that given the prevailing states Y_t, M_t and B_t, the surplus-maximizing bank debt level post-renegotiation is lower when the economy is in the shock state than when it is in the normal state. Intuitively, the shock to the bank lender's discount rate reduces the bank lender's willingness to lend, prompting the bank lender and the firm to agree on a smaller balance of bank debt upon renegotiation. Thus, the bank lending shock can be seen as a shock to the bank credit supply. From (C.2), it is immediate that $g^{m,s}(Y_t, M_t, B_t) > g^{m,n}(Y_t, M_t, B_t)$. This is

because less bank debt means less dilution of the market debt, which increases the market debt investors' willingness to lend to the firm, incentivizing the firm to issue more market debt.

In Figure C.2, I plot the market debt issuance intensity in both states of the economy against the bank debt ratio b_t, for given levels of distance of default $\tilde{y}_t$. Again, a permanent bank lending shock results in higher market debt issuance intensity by the firm. The increase in the market debt issuance intensity is more pronounced when the firm is farther from default. The figure thus appears consistent with the empirical observation that firms increase their market debt issuance following a contraction in bank credit supply.[10]
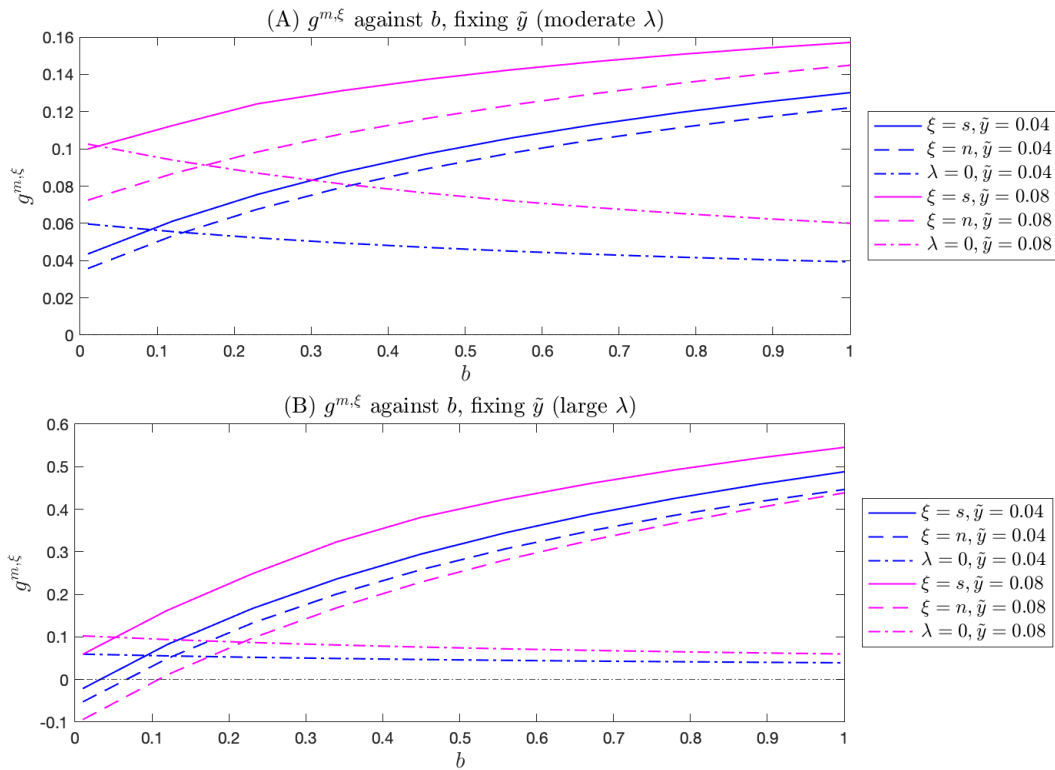

Figure C.2: State-dependent market debt issuance intensity for given distance to default. The figure plots the market debt issuance intensity against the bank debt ratio b_t for given levels of distance to default $\tilde{y}_t$, when the economy is in the normal state and when it is in the shock state. Panel A plots this relationship for $\lambda = 0.2$, while Panel B plots this relationship for $\lambda = 1$. The other parameters take the values: $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r^n = r = (1-\pi)c = 0.05$, $r^s = 0.07$, and $\alpha = 2.33$.

Importantly, the dynamic complementarity between bank debt and market debt, as captured by the second term on the right-hand side of (C.2), is still present in both the normal state and

---

[10]For example, see Becker and Ivashina (2014).

the shock state. In both states of the economy, a larger balance of bank debt in the debt structure is associated with higher market debt issuance intensity, even though the firm issues more market debt after the bank lender experiences the discount rate shock.

C.2   Transitory Shock

A bank lending shock is often transitory. I now examine how the persistence of the shock affects the firm's market debt issuance behavior. In Figure C.3, I plot the firm's market debt issuance intensity in both the shock state and the normal state, for various levels of shock persistence, ranging from highly persistent ($\zeta^s = 0.2$) to transitory ($\zeta^s = 2$). The pattern that the firm increases market debt issuance upon shock remains robust. However, the extent to which the firm ramps up its market debt issuance depends on the shock's persistence. As shown in the figure, the firm increases its market debt issuance more when the shock is more persistent.
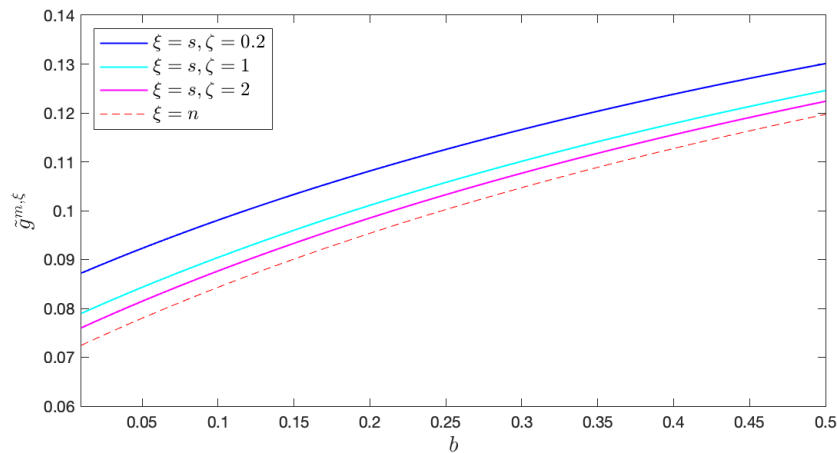

Figure C.3: Effect of shock persistence on market debt issuance intensity. This figure plots the market debt issuance intensity against the bank debt ratio b_t when the distance to default $\tilde{y}_t = 0.08$ for different levels of shock persistence. The parameters values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r^n = r = (1-\pi)c = 0.05$, $r^s = 0.07$, $\lambda = 0.2$, and $\alpha = 2.33$.

C.3   Proofs

The equity holders' value $V^{e,\xi}(Y_t, M_t, B_t)$ when the economy is in state $\xi \in \{n, s\}$ and the firm has EBIT Y_t, market debt outstanding M_t, and bank debt outstanding B_t, satisfies the following

HJB equation

$$(r + \lambda)V^{e,\xi}(Y_t, M_t, B_t) = \max_{G_t^{m,\xi}} \Big\{ (1 - \pi)\big[Y_t - c(B_t + M_t)\big]$$

$$+ p_t^{m,\xi}G_t^{m,\xi} + G_t^{m,\xi}V_M^{e,\xi}(Y_t, M_t, B_t)$$

$$+ \mu Y_t V_Y^{e,\xi}(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^{e,\xi}(Y_t, M_t, B_t) \qquad \text{(C.3)}$$

$$+ \lambda\big[V^{e,\xi}(Y_t, M_t, \bar{B}_t^\xi) + \mathcal{T}_t^\xi\big]$$

$$+ \zeta^\xi\big[V^{e,-\xi}(Y_t, M_t, B_t) - V^{e,\xi}(Y_t, M_t, B_t)\big]\Big\},$$

where $-\xi \neq \xi \in \{n, s\}$. $p_t^{m,\xi}$ is the competitive market debt price in state ξ. $\bar{B}_t^\xi$ and $\mathcal{T}_t^\xi$ are the renegotiated bank debt amount and the payment from the bank lender to the equity holders respectively, which are determined by bargaining. The first-order condition with respect to the market debt issuance rate yields

$$p_t^{m,\xi} = -V_M^{e,\xi}(Y_t, M_t, B_t). \qquad \text{(C.4)}$$

Hence, the equity holders' HJB equation can be rewritten as

$$(r + \lambda)V^{e,\xi}(Y_t, M_t, B_t) = (1 - \pi)\big[Y_t - c(B_t + M_t)\big]$$

$$+ \mu Y_t V_Y^{e,\xi}(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^{e,\xi}(Y_t, M_t, B_t)$$

$$+ \lambda\big[V^{e,\xi}(Y_t, M_t, \bar{B}_t^\xi) + \mathcal{T}_t^\xi\big] \qquad \text{(C.5)}$$

$$+ \zeta^\xi\big[V^{e,-\xi}(Y_t, M_t, B_t) - V^{e,\xi}(Y_t, M_t, B_t)\big].$$

Similarly, the bank lender's value $V^{b,\xi}(Y_t, M_t, B_t)$ when the economy is in state ξ satisfies the HJB equation

$$
\begin{aligned}
(r^\xi + \lambda)V^{b,\xi}(Y_t, M_t, B_t) =& cB_t + G_t^{m,\xi}V_M^{b,\xi}(Y_t, M_t, B_t) \\
& + \mu Y_t V_Y^{b,\xi}(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^{b,\xi}(Y_t, M_t, B_t) \\
& + \lambda\big[V^{b,\xi}(Y_t, M_t, \bar{B}_t^\xi) - \mathcal{T}_t^\xi\big] \\
& + \zeta^\xi\big[V^{b,-\xi}(Y_t, M_t, B_t) - V^{b,\xi}(Y_t, M_t, B_t)\big].
\end{aligned}
\tag{C.6}
$$

Upon renegotiation of the bank debt, the terms $\{\bar{B}_t^\xi, \mathcal{T}_t^\xi\}$ are determined by Nash bargaining between the bank lender and the firm. Thus

$$
\begin{aligned}
\{\bar{B}_t^\xi, \mathcal{T}_t^\xi\} = \underset{B_t', \mathcal{T}_t'}{\arg\max}\ & \big[V^{e,\xi}(Y_t, M_t, B_t') + \mathcal{T}_t' - V^{e,\xi}(Y_t, M_t, B_t)\big]^\theta \\
& \big[V^{b,\xi}(Y_t, M_t, B_t') - \mathcal{T}_t' - V^{b,\xi}(Y_t, M_t, B_t)\big]^{1-\theta},
\end{aligned}
\tag{C.7}
$$

subject to $V^{e,\xi}(Y_t, M_t, B_t') + \mathcal{T}_t' - V^{e,\xi}(Y_t, M_t, B_t) \geq 0$ and $V^{b,\xi}(Y_t, M_t, B_t') - \mathcal{T}_t' - V^{b,\xi}(Y_t, M_t, B_t) \geq 0$. Following derivations in Appendix A.1, when $\theta = 0$ we have

$$
\bar{B}_t^\xi = \underset{B_t'}{\arg\max}\ \big[V^{e,\xi}(Y_t, M_t, B_t') + V^{b,\xi}(Y_t, M_t, B_t')\big],
\tag{C.8}
$$

and

$$
\mathcal{T}_t^\xi = V^{e,\xi}(Y_t, M_t, B_t) - V^{e,\xi}(Y_t, M_t, \bar{B}_t^\xi).
\tag{C.9}
$$

Substituting (C.9) into (C.5) yields

$$
\begin{aligned}
(r + \lambda)V^{e,\xi}(Y_t, M_t, B_t) =& (1 - \pi)\big[Y_t - c(B_t + M_t)\big] \\
& + \mu Y_t V_Y^{e,\xi}(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^{e,\xi}(Y_t, M_t, B_t) \\
& + \lambda V^{e,\xi}(Y_t, M_t, B_t) + \zeta^\xi\big[V^{e,-\xi}(Y_t, M_t, B_t) - V^{e,\xi}(Y_t, M_t, B_t)\big].
\end{aligned}
\tag{C.10}
$$

Differentiating (C.10) with respect to M_t and using the first-order condition (C.4),

$$
\begin{aligned}
-(r+\lambda)p^{m,\xi}(Y_t,M_t,B_t) =& -(1-\pi)c - \mu Y_t p_Y^{m,\xi}(Y_t,M_t,B_t) - \frac{1}{2}\sigma^2 Y_t^2 p_{YY}^{m,\xi}(Y_t,M_t,B_t) \\
& + \lambda V_M^{e,\xi}(Y_t,M_t,B_t) + \zeta^\xi\big[-p^{m,-\xi}(Y_t,M_t,B_t) + p^{m,\xi}(Y_t,M_t,B_t)\big].
\end{aligned}
\tag{C.11}
$$

Moreover, let $p^{m,\xi}(Y_t,M_t,B_t)$ denote the equilibrium market debt price in state ξ, then

$$
\begin{aligned}
(r+\lambda)p^{m,\xi}(Y_t,M_t,B_t) =& c + G_t^{m,\xi} p_M^{m,\xi}(Y_t,M_t,B_t) \\
& + \mu Y_t p_Y^{m,\xi}(Y_t,M_t,B_t) + \frac{1}{2}\sigma^2 Y_t^2 p_{YY}^{m,\xi}(Y_t,M_t,B_t) \\
& + \lambda p^{m,\xi}(Y_t,M_t,\bar{B}_t^\xi) \\
& + \zeta^\xi\big[p^{m,-\xi}(Y_t,M_t,B_t) - p^{m,\xi}(Y_t,M_t,B_t)\big].
\end{aligned}
\tag{C.12}
$$

Adding up (C.11) and (C.12) yields

$$
0 = \pi c + G_t^{m,\xi} p_M^{m,\xi}(Y_t,M_t,B_t) + \lambda\big[p^{m,\xi}(Y_t,M_t,\bar{B}_t^\xi) + V_M^{e,\xi}(Y_t,M_t,B_t)\big].
\tag{C.13}
$$

Rearranging yields

$$
G_t^{m,\xi} = \frac{\pi c}{-p_M^{m,\xi}(Y_t,M_t,B_t)} + \lambda\frac{p^{m,\xi}(Y_t,M_t,\bar{B}_t^\xi) + V_M^{e,\xi}(Y_t,M_t,B_t)}{-p_M^{m,\xi}(Y_t,M_t,B_t)}.
\tag{C.14}
$$

A. Permanent Shock

I now consider a permanent bank lending shock, that is $\zeta^s = 0$. The equity holders' value can be solved as if the firm neither issues market debt nor adjusts its bank debt. Let $V^{e,\xi,0}(Y_t;M,B)$ denote the equity holders' no-issuance value given the market debt outstanding $M_t = M$ and the bank debt outstanding $B_t = B$. Then

$$
\begin{aligned}
rV^{e,\xi,0}(Y_t;M,B) =& (1-\pi)\big[Y_t - c(B+M)\big] \\
& + \mu Y_t V^{e,\xi,0\prime}(Y_t;M,B) + \frac{1}{2}\sigma^2 Y_t^2 V^{e,\xi,0\prime\prime}(Y_t;M,B),
\end{aligned}
\tag{C.15}
$$

for $\xi \in \{n, s\}$. Following the derivations in Appendix A.4,

$$
\begin{aligned}
V^{e,\xi,0}(Y_t; M, B) = & \frac{1-\pi}{r-\mu}Y_t - \frac{(1-\pi)c}{r}(B+M) \\
& + \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\left[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\right]^\gamma (B+M)^{1+\gamma}Y_t^{-\gamma}.
\end{aligned}
\tag{C.16}
$$

Hence,

$$
\begin{aligned}
V^{e,\xi}(Y_t, M_t, B_t) = & \frac{1-\pi}{r-\mu}Y_t - \frac{(1-\pi)c}{r}(B_t+M_t) \\
& + \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\left[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\right]^\gamma (B_t+M_t)^{1+\gamma}Y_t^{-\gamma}.
\end{aligned}
\tag{C.17}
$$

Using the first-order condition with respect to the market debt issuance rate (C.4), the equilibrium market debt price in state ξ is then

$$
p^{m,\xi}(Y_t, M_t, B_t) = \frac{(1-\pi)c}{r}\left\{1 - \left[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\right]^\gamma \left(\frac{B_t+M_t}{Y_t}\right)^\gamma\right\}.
\tag{C.18}
$$

The market debt issuance intensity in state ξ is given by

$$
g_t^{m,\xi} = \frac{\pi c}{\gamma}\frac{r}{(1-\pi)c}\left[\frac{1+\gamma}{\gamma}\frac{r}{(r-\mu)c}\frac{Y_t}{B_t+M_t}\right]^\gamma + \frac{\lambda}{\gamma}\left[1 - \left(\frac{\bar{B}_t^\xi + M_t}{B_t+M_t}\right)^\gamma\right].
\tag{C.19}
$$

B. Transitory Shock

I now turn to the case with a transitory bank lending shock. Let $V^{e,\xi,0}(Y_t; M, B)$ denote the equity holders' no-issuance value given the market debt outstanding $M_t = M$ and the bank debt outstanding $B_t = B$. It must then satisfy the following HJB equation

$$
\begin{aligned}
rV^{e,\xi,0}(Y_t; M, B) = & (1-\pi)\left[Y_t - c(B+M)\right] \\
& + \mu Y_t V^{e,\xi,0\prime}(Y_t; M, B) + \frac{1}{2}\sigma^2 Y_t^2 V^{e,\xi,0\prime\prime}(Y_t; M, B) \\
& + \zeta^\xi\left[V^{e,-\xi,0}(Y_t; M, B) - V^{e,\xi,0}(Y_t; M, B)\right].
\end{aligned}
\tag{C.20}
$$

The equity holders' no-issuance value is homogeneous of degree one. Hence

$$V^{e,\xi,0}(Y_t; M, B) = v^{e,\xi,0}(y_t; b)M, \tag{C.21}$$

where $y_t \equiv Y_t/M_t$, $b \equiv B/M$, and $v^{e,\xi,0}(y_t; b)$ is the equity holders' no-issuance value scaled by the amount of market debt outstanding. I refer to $v^{e,\xi,0}(y_t; b)$ as the no-issuance equity value. The HJB equation (C.20) can be rewritten as

$$
\begin{aligned}
rv^{e,\xi,0}(y_t; b) =&(1 - \pi)\big[y_t - c(1 + b)\big] + \mu y_t v^{e,\xi,0\prime}(y_t; b) + \frac{1}{2}\sigma^2 y_t^2 v^{e,\xi,0\prime\prime}(y_t; b) \\
&+ \zeta^\xi\big[v^{e,-\xi,0}(y_t; b) - v^{e,\xi,0}(y_t; b)\big].
\end{aligned}
\tag{C.22}
$$

That is, $v^{e,n,0}(y_t; b)$ and $v^{e,s,0}(y_t; b)$ respectively satisfy

$$rv^{e,n,0}(y_t; b) =(1 - \pi)\big[y_t - c(1 + b)\big] + \mu y_t v^{e,n,0\prime}(y_t; b) + \frac{1}{2}\sigma^2 y_t^2 v^{e,n,0\prime\prime}(y_t; b), \tag{C.23}$$

and

$$
\begin{aligned}
rv^{e,s,0}(y_t; b) =&(1 - \pi)\big[y_t - c(1 + b)\big] + \mu y_t v^{e,s,0\prime}(y_t; b) + \frac{1}{2}\sigma^2 y_t^2 v^{e,s,0\prime\prime}(y_t; b) \\
&+ \zeta^s\big[v^{e,n,0}(y_t; b) - v^{e,s,0}(y_t; b)\big].
\end{aligned}
\tag{C.24}
$$

When the economy is in the normal state, at the default boundary $y^{\star,n}(b)$ the no-issuance equity value satisfies the value matching condition $v^{e,n,0}(y^{\star,n}(b); b) = 0$ and the smooth pasting condition $v^{e,n,0\prime}(y^{\star,n}(b); b) = 0$. Following the derivations in Appendix A.4, the default boundary is given by

$$y^{\star,n}(b) = \frac{\gamma}{1 + \gamma}\frac{(r - \mu)c}{r}(1 + b), \tag{C.25}$$

where

$$-\gamma \equiv -\frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2}. \tag{C.26}$$

The no-issuance equity value in the normal state is then

$$
\begin{aligned}
v^{e,n,0}(y_t; b) =& \frac{1-\pi}{r-\mu} y_t - \frac{(1-\pi)c}{r}(1+b) \\
&+ \frac{1}{1+\gamma} \frac{(1-\pi)c}{r} \left[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r} \right]^{\gamma} (1+b)^{1+\gamma} y_t^{-\gamma}.
\end{aligned}
\tag{C.27}
$$

In equilibrium, the equity holders' value equals the equity holders' no-issuance value, that is $V^{e,n}(Y_t, M_t, B_t) = V^{e,n,0}(Y_t; M_t, B_t)$. Let $v^{e,\xi}(y_t, b_t) \equiv V^{e,\xi}(Y_t, M_t, B_t)/M_t$ be the equity value in state ξ, where $b_t \equiv B_t/M_t$. Then $v^{e,n}(y_t, b_t) = v^{e,n,0}(y_t; b_t)$. Thus,

$$
\begin{aligned}
v^{e,n}(y_t, b_t) =& \frac{1-\pi}{r-\mu} y_t - \frac{(1-\pi)c}{r}(1+b_t) \\
&+ \frac{1}{1+\gamma} \frac{(1-\pi)c}{r} \left[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r} \right]^{\gamma} (1+b_t)^{1+\gamma} y_t^{-\gamma}.
\end{aligned}
\tag{C.28}
$$

Let

$$
f(y_t; b) \equiv v^{e,n,0}(y_t; b) - v^{e,s,0}(y_t; b).
\tag{C.29}
$$

Subtracting (C.24) from (C.23) yields

$$
(r + \zeta^s) f(y_t; b) = \mu y_t f'(y_t; b) + \frac{1}{2}\sigma^2 y_t^2 f''(y_t; b).
\tag{C.30}
$$

The general solution to (C.30) is

$$
f(y_t; b) = H_\phi y_t^{-\phi} + H_\psi y_t^{\psi}.
\tag{C.31}
$$

where

$$
-\phi \equiv -\frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2(r + \zeta^s)}}{\sigma^2} < 0,
\tag{C.32}
$$

$$
\psi \equiv -\frac{\mu - \frac{1}{2}\sigma^2 - \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2(r + \zeta^s)}}{\sigma^2} > 1.
\tag{C.33}
$$

Hence, from (C.27), (C.29) and (C.31)

$$
\begin{aligned}
v^{e,s,0}(y_t; b) = &- H_\phi y_t^{-\phi} - H_\psi y_t^{\psi} + \frac{1-\pi}{r-\mu} y_t - \frac{(1-\pi)c}{r}(1+b) \\
&+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(1+b)^{1+\gamma}y_t^{-\gamma}.
\end{aligned}
\tag{C.34}
$$

The no-bubble condition at $y_t \to \infty$ implies that $H_\psi = 0$. At the default boundary $y^{\star,s}(b)$, the no-issuance equity value satisfies the value matching condition $v^{e,s,0}(y^{\star,s}(b); b) = 0$ and the smooth pasting condition $v^{e,s,0\prime}(y^{\star,s}(b); b) = 0$. That is

$$
\begin{aligned}
0 = &- H_\phi y^{\star,s}(b)^{-\phi} + \frac{1-\pi}{r-\mu}y^{\star,s}(b) - \frac{(1-\pi)c}{r}(1+b) \\
&+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(1+b)^{1+\gamma}y^{\star,s}(b)^{-\gamma},
\end{aligned}
\tag{C.35}
$$

and

$$
\begin{aligned}
0 = &\phi H_\phi y^{\star,s}(b)^{-\phi-1} + \frac{1-\pi}{r-\mu} \\
&- \frac{\gamma}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(1+b)^{1+\gamma}y^{\star,s}(b)^{-\gamma-1}.
\end{aligned}
\tag{C.36}
$$

The value matching condition and the smooth pasting condition together pin down H_ϕ and $y^{\star,s}(b)$. Then

$$
\begin{aligned}
v^{e,s}(y_t, b_t) = &\frac{1-\pi}{r-\mu}y_t - \frac{(1-\pi)c}{r}(1+b_t) \\
&+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}\Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}(1+b_t)^{1+\gamma}y_t^{-\gamma} - H_\phi y_t^{-\phi}.
\end{aligned}
\tag{C.37}
$$

From (C.4),

$$
\begin{aligned}
p_t^{m,n} &= y_t v_y^{e,n}(y_t, b_t) + b_t v_b^{e,n}(y_t, b_t) - v^{e,n}(y_t, b_t) \\
&= \frac{(1-\pi)c}{r}\left\{1 - \Big[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\Big]^{\gamma}\Big(\frac{1+b_t}{y_t}\Big)^{\gamma}\right\}.
\end{aligned}
\tag{C.38}
$$

Similarly,

$$
\begin{aligned}
p_t^{m,s} &= y_t v_y^{e,s}(y_t, b_t) + b_t v_b^{e,s}(y_t, b_t) - v^{e,s}(y_t, b_t) \\
&= \frac{(1-\pi)c}{r}\left\{1 - \left[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\right]^\gamma\left(\frac{1+b_t}{y_t}\right)^\gamma\right\} + \left[(1+\phi)H_\phi - b_t\frac{dH_\phi}{db_t}\right]y_t^{-\phi}.
\end{aligned}
\tag{C.39}
$$

Substituting (C.9) into (C.6) yields

$$
\begin{aligned}
r^\xi V^{b,\xi}(Y_t, M_t, B_t) =& cB_t + G_t^{m,\xi}V_M^{b,\xi}(Y_t, M_t, B_t) \\
& + \mu Y_t V_Y^{b,\xi}(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^{b,\xi}(Y_t, M_t, B_t) \\
& + \lambda\left[V^\xi(Y_t, M_t, \bar{B}_t^\xi) - V^\xi(Y_t, M_t, B_t)\right] \\
& + \zeta^\xi\left[V^{b,-\xi}(Y_t, M_t, B_t) - V^{b,\xi}(Y_t, M_t, B_t)\right],
\end{aligned}
\tag{C.40}
$$

where $V^\xi(Y_t, M_t, B_t) \equiv V^{e,\xi}(Y_t, M_t, B_t) + V^{b,\xi}(Y_t, M_t, B_t)$, and the market debt issuance rate $G_t^{m,\xi}$ is given by (C.14).

Given the scale invariance in the setting, the bank lender's value $V^{b,\xi}(Y_t, M_t, B_t)$ is homogeneous of degree one. That is

$$
V^{b,\xi}(Y_t, M_t, B_t) = V^{b,\xi}\left(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\right)M_t \equiv v^{b,\xi}(y_t, b_t)M_t.
\tag{C.41}
$$

Similarly,

$$
V^\xi(Y_t, M_t, B_t) = V^\xi\left(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\right)M_t \equiv v^\xi(y_t, b_t)M_t,
\tag{C.42}
$$

where $v^\xi(y_t, b_t) \equiv v^{e,\xi}(y_t, b_t) + v^{b,\xi}(y_t, b_t)$ with $v^{e,\xi}(y_t, b_t) \equiv V^{e,\xi}(Y_t, M_t, B_t)/M_t$. From (C.8),

$$
\bar{b}_t^\xi \equiv \frac{\bar{B}_t^\xi}{M_t} = \arg\max_{b_t'} v^\xi(y_t, b_t').
\tag{C.43}
$$

Hence, one can rewrite (C.6) as

$$
\begin{aligned}
\left[r^\xi + \zeta^\xi - g_t^{m,\xi}(1+b_t)\right] v^{b,\xi}(y_t, b_t) &= cb_t - g_t^{m,\xi} b_t (1+b_t) v_b^{b,\xi}(y_t, b_t) + \zeta^\xi v^{b,-\xi}(y_t, b_t) \\
&+ \left[\mu - g_t^{m,\xi}(1+b_t)\right] y_t v_y^{b,\xi}(y_t, b_t) + \frac{1}{2}\sigma^2 y_t^2 v_{yy}^{b,\xi}(y_t, b_t) + \lambda\Big[\max_{b_t'} v^\xi(y_t, b_t') - v^\xi(y_t, b_t)\Big].
\end{aligned}
\tag{C.44}
$$

D State-Dependent λ

In the main analysis, I have treated the Poisson intensity of bank debt renegotiation λ as a constant. More generally, the intensity parameter is state-dependent. That is, the bank lender and the firm can renegotiate the bank debt at Poisson arrival times with intensity $\lambda(Y_t, M_t, B_t)$. As shown in Section D.3, the optimal market debt issuance rate is written as

$$
\begin{aligned}
G_t^m = & \frac{\pi c}{-p_M^m(Y_t, M_t, B_t)} + \lambda(Y_t, M_t, B_t) \frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t)}{-p_M^m(Y_t, M_t, B_t)} \\
& + \lambda(Y_t, M_t, B_t) \frac{\theta [V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t)]}{-p_M^m(Y_t, M_t, B_t)} \\
& + \lambda_M(Y_t, M_t, B_t) \frac{\theta [V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)]}{-p_M^m(Y_t, M_t, B_t)}.
\end{aligned}
\tag{D.1}
$$

Compared to (2.10) in the main analysis, (D.14) contains an additional term on the right-hand side. This term arises because the Poisson intensity of bank debt renegotiation depends on the level of market debt. As a result, the firm's market debt issuance policy must also take into account its impact on the renegotiation intensity and thus the likelihood of realizing the bargaining surplus in a given time interval. For example, if $\lambda_M(Y_t, M_t, B_t) > 0$, issuing more market debt increases the the frequency of bank debt renegotiation, thereby raising the equity holders' expected payoff from renegotiation by allowing them to capture the bargaining surplus more often. This creates a strategic benefit of market debt issuance, leading the firm to issue more market debt than in the main analysis.

D.1 Strong Bank Lender ($\theta = 0$)

In the case with a strong bank lender who holds all the bargaining power, that is $\theta = 0$, the firm's optimal market debt issuance rate (D.14) simplifies to

$$
G_t^m = \frac{\pi c}{-p_M^m(Y_t, M_t, B_t)} + \lambda(Y_t, M_t, B_t) \frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t)}{-p_M^m(Y_t, M_t, B_t)}.
\tag{D.2}
$$

Moreover, the equity holders' value satisfies the HJB equation

$$
\begin{aligned}
rV^e&(Y_t, M_t, B_t) \\
&= (1 - \pi)\big[Y_t - c(B_t + M_t)\big] + \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t).
\end{aligned}
\tag{D.3}
$$

Notice that this is the same HJB equation as (A.30). Just as before, the equity holders' value can be solved as if the firm neither issues market debt nor renegotiates its bank debt. This is because when the bank lender has all the bargaining power, it captures the entire joint surplus from the bank debt renegotiation, and the equity holders receive only their outside option (or current continuation value). As a result, the renegotiation does not affect the equity holders' value.

As shown in Section D.3, the firm's optimal market debt issuance intensity is

$$
\begin{aligned}
g^m(Y_t, M_t, B_t) &\equiv \frac{G^m(Y_t, M_t, B_t)}{B_t + M_t} \\
&= \frac{\pi c}{\gamma}\frac{r}{(1 - \pi)c}\Big[\frac{1 + \gamma}{\gamma}\frac{r}{(r - \mu)c}\frac{Y_t}{B_t + M_t}\Big]^\gamma + \frac{\lambda(Y_t, M_t, B_t)}{\gamma}\Big[1 - \Big(\frac{\bar{B}(Y_t, M_t) + M_t}{B_t + M_t}\Big)^\gamma\Big],
\end{aligned}
\tag{D.4}
$$

which is the same as in the main analysis when $\theta = 0$, except that the Poisson intensity of bank debt renegotiation is now state-dependent. Intuitively, when the firm cannot capture any bargaining surplus, increasing the frequency of bank debt renegotiation by raising the Poisson intensity does not generate additional payoff from renegotiation to the firm's equity holders. In this case, the firm's market debt issuance policy does not account for its impact on the renegotiation intensity.

D.2 Endogenized λ

I now endogenize the Poisson intensity of bank debt renegotiation λ. Specifically, I assume that the bank lender can choose a renegotiation intensity from the interval $\lambda \in [0, \bar{\lambda}]$, subject to a proportional renegotiation cost $\kappa \lambda B_t$ with $\kappa > 0$. For simplicity, I focus on the case with a strong bank lender, that is $\theta = 0$.

The bank lender's value $V^b(Y_t, M_t, B_t)$ satisfies the HJB equation

$$rV^b(Y_t, M_t, B_t) = \max_{\lambda \in [0, \bar{\lambda}]} \Big\{ cB_t - \kappa \lambda B_t + G_t^m V_M^b(Y_t, M_t, B_t)$$

$$+ \mu Y_t V_Y^b(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^b(Y_t, M_t, B_t) \qquad \text{(D.5)}$$

$$+ \lambda \big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t) \big] \Big\}.$$

As shown in Section D.3, the optimal renegotiation intensity is

$$\lambda(Y_t, M_t, B_t) = \bar{\lambda} \mathbb{I}\{ V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t) \geq \kappa B_t \}. \qquad \text{(D.6)}$$

As in the main analysis, the scale invariance in the setting reduces the state space. One can solve the model in terms of the firm's distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ and the bank debt ratio b_t. In Figure D.1, I plot the optimal renegotiation intensity λ as a function of the firm's distance to default $\tilde{y}_t$ for given levels of bank debt ratio b_t. The figure shows that bank debt renegotiations occur more frequently when the firm is closer to default, since renegotiations generate greater gains when the firm is closer to default.
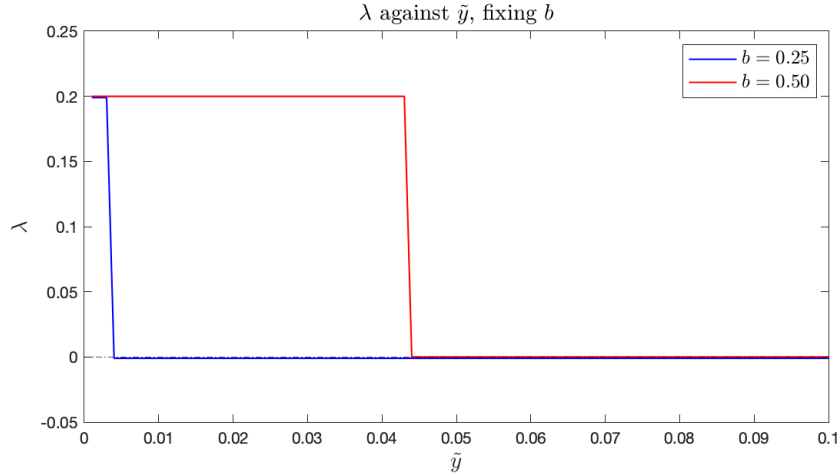

Figure D.1: Endogenous renegotiation intensity. The figure plots the optimal Poisson intensity of bank debt renegotiation λ as a function of the firm's distance to default $\tilde{y}_t$ for fixed levels of bank debt ratio b_t. The parameter values are $\mu = 0.02$, $\sigma = 0.25$, $\pi = 0.3$, $r = c(1-\pi) = 0.05$, $\lambda = 0.2$, and $\alpha = 2.33$.

D.3 Proofs

Following the derivations before, the equity holders' HJB equation can be written as

$$
\begin{aligned}
\big[r + \lambda(Y_t, M_t, B_t)\big] & V^e(Y_t, M_t, B_t) \\
= \max_{G_t^m} \Big\{ & (1 - \pi)\big[Y_t - c(B_t + M_t)\big] + p_t^m G_t^m + G_t^m V_M^e(Y_t, M_t, B_t) \\
& \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t) \\
& + \lambda(Y_t, M_t, B_t)\big[V^e(Y_t, M_t, \bar{B}_t) + \mathcal{T}_t\big] \Big\}.
\end{aligned}
\tag{D.7}
$$

Taking the first-order condition with respect to G_t^m yields

$$
p_t^m \equiv p^m(Y_t, M_t, B_t) = -V_M^e(Y_t, M_t, B_t).
\tag{D.8}
$$

Substituting (D.8) into the HJB equation (D.7) yields

$$
\begin{aligned}
\big[r + \lambda(Y_t, M_t, B_t)\big] & V^e(Y_t, M_t, B_t) \\
= & (1 - \pi)\big[Y_t - c(B_t + M_t)\big] + \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t) \\
& + \lambda(Y_t, M_t, B_t)\Big\{ V^e(Y_t, M_t, B_t) + \theta\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big] \Big\}.
\end{aligned}
\tag{D.9}
$$

Differentiating both sides of (D.9) with respect to M_t and using the envelope theorem, we get

$$
\begin{aligned}
\lambda_M(Y_t, M_t, B_t) & V^e(Y_t, M_t, B_t) + \big[r + \lambda(Y_t, M_t, B_t)\big] V_M^e(Y_t, M_t, B_t) \\
= & - (1 - \pi)c + \mu Y_t V_{MY}^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{MYY}^e(Y_t, M_t, B_t) \\
& + \lambda_M(Y_t, M_t, B_t)\Big\{ V^e(Y_t, M_t, B_t) + \theta\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big] \Big\} \\
& + \lambda(Y_t, M_t, B_t)\Big\{ V_M^e(Y_t, M_t, B_t) + \theta\big[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t)\big] \Big\}.
\end{aligned}
\tag{D.10}
$$

Moreover, applying (D.8) to (D.10) yields

$$
\begin{aligned}
&-\big[r + \lambda(Y_t, M_t, B_t)\big]p^m(Y_t, M_t, B_t) \\
&= -(1-\pi)c - \mu Y_t p_Y^m(Y_t, M_t, B_t) - \frac{1}{2}\sigma^2 Y_t^2 p_{YY}^m(Y_t, M_t, B_t) \\
&\quad + \lambda(Y_t, M_t, B_t)\Big\{V_M^e(Y_t, M_t, B_t) + \theta\big[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t)\big]\Big\} \\
&\quad + \theta\lambda_M(Y_t, M_t, B_t)\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big].
\end{aligned}
\tag{D.11}
$$

Moreover, the equilibrium market debt price $p^m(Y_t, M_t, B_t)$ satisfies the following HJB equation

$$
\begin{aligned}
&\big[r + \lambda(Y_t, M_t, B_t)\big]p^m(Y_t, M_t, B_t) \\
&= c + G_t^m p_M^m(Y_t, M_t, B_t) + \mu Y_t p_Y^m(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 p_{YY}^m(Y_t, M_t, B_t) \\
&\quad + \lambda(Y_t, M_t, B_t)p^m(Y_t, M_t, \bar{B}_t).
\end{aligned}
\tag{D.12}
$$

Adding up (D.11) and (D.12) yields

$$
\begin{aligned}
0 &= \pi c + G_t^m p_M^m(Y_t, M_t, B_t) \\
&\quad + \lambda(Y_t, M_t, B_t)\Big\{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t) + \theta\big[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t)\big]\Big\} \\
&\quad + \theta\lambda_M(Y_t, M_t, B_t)\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big].
\end{aligned}
\tag{D.13}
$$

Rearranging yields

$$
\begin{aligned}
G_t^m &= \frac{\pi c}{-p_M^m(Y_t, M_t, B_t)} + \lambda(Y_t, M_t, B_t)\frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t)}{-p_M^m(Y_t, M_t, B_t)} \\
&\quad + \lambda(Y_t, M_t, B_t)\frac{\theta\big[V_M(Y_t, M_t, \bar{B}_t) - V_M(Y_t, M_t, B_t)\big]}{-p_M^m(Y_t, M_t, B_t)} \\
&\quad + \lambda_M(Y_t, M_t, B_t)\frac{\theta\big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t)\big]}{-p_M^m(Y_t, M_t, B_t)}.
\end{aligned}
\tag{D.14}
$$

A. State-Dependent λ with Strong Bank Lender

When $\theta = 0$, the equity holders' HJB equation (D.9) becomes

$$
\begin{aligned}
rV^e(Y_t, M_t, B_t) \\
= (1 - \pi)\big[Y_t - c(B_t + M_t)\big] + \mu Y_t V_Y^e(Y_t, M_t, B_t) + \frac{1}{2}\sigma^2 Y_t^2 V_{YY}^e(Y_t, M_t, B_t).
\end{aligned}
\tag{D.15}
$$

This is the same HJB equation as (A.30). Following the derivations in Appendix A.4, the equity holders' value is given by

$$
\begin{aligned}
V^e(Y_t, M_t, B_t) = & \frac{1 - \pi}{r - \mu}Y_t - \frac{(1 - \pi)c}{r}(B_t + M_t) \\
& + \frac{1}{1 + \gamma}\frac{(1 - \pi)c}{r}\left[\frac{\gamma}{1 + \gamma}\frac{(r - \mu)c}{r}\right]^\gamma (B_t + M_t)^{1+\gamma}Y_t^{-\gamma},
\end{aligned}
\tag{D.16}
$$

with

$$
\gamma = \frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2} > 0,
\tag{D.17}
$$

and

$$
Y^\star(M_t, B_t) = \frac{\gamma}{1 + \gamma}\frac{(r - \mu)c}{r}(B_t + M_t).
\tag{D.18}
$$

Since $p^m(Y_t, M_t, B_t) = -V_M^e(Y_t, M_t, B_t)$, then the equilibrium market debt price is given by

$$
p^m(Y_t, M_t, B_t) = \frac{(1 - \pi)c}{r}\left\{1 - \left[\frac{\gamma}{1 + \gamma}\frac{(r - \mu)c}{r}\right]^\gamma \left(\frac{B_t + M_t}{Y_t}\right)^\gamma\right\}.
\tag{D.19}
$$

Furthermore, substituting $\theta = 0$ into (D.14) yields the firm's optimal market debt issuance rate when the bank lender has all the bargaining power and the Poisson intensity of bank debt renegotiation is state-dependent. That is,

$$
\begin{aligned}
G_t^m \equiv G^m(Y_t, M_t, B_t) \\
= \frac{\pi c}{-p_M^m(Y_t, M_t, B_t)} + \lambda(Y_t, M_t, B_t)\frac{p^m(Y_t, M_t, \bar{B}_t) + V_M^e(Y_t, M_t, B_t)}{-p_M^m(Y_t, M_t, B_t)}.
\end{aligned}
\tag{D.20}
$$

From (D.16) and (D.19), the market debt issuance rate can be written as

$$
\begin{aligned}
G^m(Y_t, M_t, B_t) = &\frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \Big[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t+M_t} \Big]^\gamma (B_t + M_t) \\
&+ \frac{\lambda(Y_t, M_t, B_t)}{\gamma} \Big[1 - \Big(\frac{\bar{B}_t + M_t}{B_t + M_t} \Big)^\gamma \Big] (B_t + M_t).
\end{aligned}
\tag{D.21}
$$

Thus, the market debt issuance intensity $g^m(Y_t, M_t, B_t) \equiv G^m(Y_t, M_t, B_t)/(B_t + M_t)$ is given by

$$
\begin{aligned}
&g^m(Y_t, M_t, B_t) \\
&= \frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \Big[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t+M_t} \Big]^\gamma + \frac{\lambda(Y_t, M_t, B_t)}{\gamma} \Big[1 - \Big(\frac{\bar{B}_t + M_t}{B_t + M_t} \Big)^\gamma \Big].
\end{aligned}
\tag{D.22}
$$

B. Endogenized λ

If $\theta = 0$, then following the preceding derivations, the equity holders' value is

$$
\begin{aligned}
V^e(Y_t, M_t, B_t) = &\frac{1-\pi}{r-\mu} Y_t - \frac{(1-\pi)c}{r} (B_t + M_t) \\
&+ \frac{1}{1+\gamma} \frac{(1-\pi)c}{r} \Big[\frac{\gamma}{1+\gamma} \frac{(r-\mu)c}{r} \Big]^\gamma (B_t + M_t)^{1+\gamma} Y_t^{-\gamma},
\end{aligned}
\tag{D.23}
$$

and the firm's market debt issuance rate is given by

$$
\begin{aligned}
G^m(Y_t, M_t, B_t) = &\frac{\pi c}{\gamma} \frac{r}{(1-\pi)c} \Big[\frac{1+\gamma}{\gamma} \frac{r}{(r-\mu)c} \frac{Y_t}{B_t+M_t} \Big]^\gamma (B_t + M_t) \\
&+ \frac{\lambda(Y_t, M_t, B_t)}{\gamma} \Big[1 - \Big(\frac{\bar{B}_t + M_t}{B_t + M_t} \Big)^\gamma \Big] (B_t + M_t).
\end{aligned}
\tag{D.24}
$$

The Poisson intensity parameter $\lambda(Y_t, M_t, B_t)$ is endogenously determined.

On the other hand, the bank lender's value $V^b(Y_t, M_t, B_t)$ satisfies the following HJB equation

$$
\begin{aligned}
rV^b(Y_t, M_t, B_t) = \max_{\lambda \in [0, \bar{\lambda}]} \Big\{ &cB_t - \kappa \lambda B_t + G_t^m V_M^b(Y_t, M_t, B_t) \\
&+ \mu Y_t V_Y^b(Y_t, M_t, B_t) + \frac{1}{2} \sigma^2 Y_t^2 V_{YY}^b(Y_t, M_t, B_t) \\
&+ \lambda \big[V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t) \big] \Big\}.
\end{aligned}
\tag{D.25}
$$

Taking the first-order condition with respect to λ and applying the envelope theorem, the endogenous renegotiation intensity is derived as

$$\lambda(Y_t, M_t, B_t) = \begin{cases} \bar{\lambda} & \text{if } V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t) \geq \kappa B_t \\ 0 & \text{if } V(Y_t, M_t, \bar{B}_t) - V(Y_t, M_t, B_t) < \kappa B_t \end{cases}. \quad (\text{D.26})$$

The firm is scale invariant in the setting. Thus, the bank lender's value, the equity holders' value, and the joint value of the equity holders and the bank lender are all homogeneous of degree one. That is,

$$V^b(Y_t, M_t, B_t) = V^b\left(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\right) M_t \equiv v^b(y_t, b_t) M_t, \quad (\text{D.27})$$

$$V(Y_t, M_t, B_t) = V\left(\frac{Y_t}{M_t}, 1, \frac{B_t}{M_t}\right) M_t \equiv v(y_t, b_t) M_t, \quad (\text{D.28})$$

where $y_t \equiv Y_t/M_t$ and $b_t \equiv B_t/M_t$. Then from (D.26), the endogenous renegotiation intensity can be rewritten as

$$\lambda(Y_t, M_t, B_t) \equiv \lambda(y_t, b_t) = \bar{\lambda}\mathbb{I}\{v(y_t, \bar{b}_t) - v(y_t, b_t) \geq \kappa b_t\}. \quad (\text{D.29})$$

Substituting into the bank lender's HJB equation (D.25) yields

$$\begin{aligned}
& \left[r + \lambda(y_t, b_t) - g_t^m(1 + b_t)\right] v^b(y_t, b_t) \\
&= cb_t - g_t^m b_t(1 + b_t) v_b^b(y_t, b_t) + \left[\mu - g_t^m(1 + b_t)\right] y_t v_y^n(y_t, b_t) \\
&\quad + \frac{1}{2}\sigma^2 y_t^2 v_{yy}^b(y_t, b_t) + \lambda(y_t, b_t)\left[\max_{b_t'} v(y_t, b_t') - v^e(y_t, b_t)\right].
\end{aligned} \quad (\text{D.30})$$

Following the derivations before, we can write the optimal market debt issuance intensity as

$$g_t^m \equiv g^m(y_t, b_t) = \frac{\pi c}{\gamma} \frac{r}{(1 - \pi)c}\left[\frac{1 + \gamma}{\gamma} \frac{r}{(r - \mu)c} \frac{y_t}{1 + b_t}\right]^\gamma + \frac{\lambda(y_t, b_t)}{\gamma}\left[1 - \left(\frac{1 + \bar{b}_t}{1 + b_t}\right)^\gamma\right], \quad (\text{D.31})$$

where

$$\bar{b}_t = \arg\max_{b'_t} \ v(y_t, b'_t).$$ (D.32)

The equity value is given by

$$
\begin{aligned}
v^e(y_t, b_t) =& \frac{1-\pi}{r-\mu} y_t - \frac{(1-\pi)c}{r}(1+b_t) \\
&+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}(1+b_t)\left[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\right]^\gamma \left(\frac{y_t}{1+b_t}\right)^{-\gamma}.
\end{aligned}
$$ (D.33)

As in Appendix A.6, it is more transparent to transform the state space such that the model is solved in terms of the distance to default $\tilde{y}_t \equiv y_t - y^\star(b_t)$ and the bank debt ratio b_t. From (D.29), the endogenous renegotiation intensity can be rewritten as a function of $\tilde{y}_t$ and b_t, that is

$$\lambda(\tilde{y}_t, b_t) = \bar{\lambda}\mathbb{I}\{\bar{v}(\tilde{y}_t, b_t) - v(\tilde{y}_t, b_t) \geq \kappa b_t\},$$ (D.34)

where

$$\bar{v}(\tilde{y}_t, b_t) = \max_{b'_t} \ v(\tilde{y}_t + y^\star(b_t) - y^\star(b'_t), b'_t).$$ (D.35)

Moreover, (D.31) and (D.33) become

$$g^m(\tilde{y}_t, b_t) = \frac{\pi c}{\gamma}\frac{r}{(1-\pi)c}\left[\frac{1+\gamma}{\gamma}\frac{r}{(r-\mu)c}\frac{\tilde{y}_t + y^\star(b_t)}{1+b_t}\right]^\gamma + \frac{\lambda(\tilde{y}_t, b_t)}{\gamma}\left[1 - \left(\frac{1+\bar{b}_t}{1+b_t}\right)^\gamma\right],$$ (D.36)

and

$$
\begin{aligned}
v^e(\tilde{y}_t, b_t) =& \frac{1-\pi}{r-\mu}\left[\tilde{y}_t + y^\star(b_t)\right] - \frac{(1-\pi)c}{r}(1+b_t) \\
&+ \frac{1}{1+\gamma}\frac{(1-\pi)c}{r}(1+b_t)\left[\frac{\gamma}{1+\gamma}\frac{(r-\mu)c}{r}\right]^\gamma \left[\frac{\tilde{y}_t + y^\star(b_t)}{1+b_t}\right]^{-\gamma}.
\end{aligned}
$$ (D.37)

The HJB equation for the bank debt value (D.30) is then

$$
\begin{aligned}
\big[r &+ \lambda(\tilde{y}_t, b_t) - g_t^m(1 + b_t)\big]v^b(\tilde{y}_t, b_t) \\
&= cb_t - g_t^m b_t(1 + b_t)v_b^b(\tilde{y}_t, b_t) \\
&\quad + \big\{\big[\mu - g_t^m(1 + b_t)\big]\big[\tilde{y}_t + y^\star(b_t)\big] + g_t^m b_t(1 + b_t)y^{\star\prime}(b_t)\big\}v_{\tilde{y}}^b(\tilde{y}_t, b_t) \\
&\quad + \frac{1}{2}\sigma^2\big[\tilde{y}_t + y^\star(b_t)\big]^2 v_{\tilde{y}\tilde{y}}^b(\tilde{y}_t, b_t) + \lambda(\tilde{y}_t, b_t)\big[\bar{v}(\tilde{y}_t, b_t) - v^e(\tilde{y}_t, b_t)\big].
\end{aligned}
\tag{D.38}
$$

E   Additional Empirical Results

E.1   Further Robustness Tests

In this appendix, I present a number of robustness tests to support the empirical results in Section 5. In the first set of robustness tests, I use alternative definitions for bank debt and market debt. In the second set of robustness tests, I resort to a matched-sample regression specification by including industry-year-size-leverage fixed effects as the main controls. Finally, I show the pattern of complementarity between bank and market debt using time-series aggregate balance sheet data from the Flow of Funds.

A. Alternative Definitions

In the baseline, I define bank debt as term loans, and define market debt as the sum of senior bonds and notes and subordinated bonds and notes. For robustness, I also account for short-term credit by expanding the definition of bank debt to include drawn credit lines and the definition of market debt to include commercial paper. Credit lines and commercial paper are both short-term financing instruments that differ in nature from either term loans or bonds and notes. However, excluding credit lines and commercial paper may impact my results if firms shift between credit lines and term loans or between commercial paper and bonds and notes. I re-estimate the specification (5.1) using the alternative definitions of bank and market debt, and report the results in Table E.1.

The coefficient of interest is the coefficient on the bank debt share variable. In all columns of Table E.1, the coefficient estimates on bank debt share are all positive and highly statistically significant. The magnitudes are also similar to the baseline results reported in Table 2. The results thus show that the baseline findings are robust to alternative definitions of bank debt and market debt.

B. Matched-Sample Analysis

Table E.1: Panel Regressions: Robustness Using Alternative Definitions

	Market Debt Net Issuance Rate					
	(1)	(2)	(3)	(4)	(5)	(6)
Bank Debt Share	0.857***	0.599***	0.931***	0.607***	0.374***	0.405***
	(0.214)	(0.100)	(0.235)	(0.101)	(0.090)	(0.102)
Log(Assets)	-0.199***	-0.155***	-0.202***	-0.163***	-0.252**	-0.285**
	(0.045)	(0.040)	(0.051)	(0.041)	(0.112)	(0.127)
Tangibility	0.268	0.025	0.203	-0.125	0.027	-0.002
	(0.280)	(0.361)	(0.249)	(0.280)	(0.325)	(0.358)
ROA	-0.040	-0.320*	-0.095	-0.351*	-0.167	-0.205
	(0.181)	(0.159)	(0.209)	(0.185)	(0.355)	(0.428)
Book Leverage	-1.367***	-1.285***	-1.445***	-1.368***	-1.064***	-1.184***
	(0.216)	(0.258)	(0.216)	(0.241)	(0.323)	(0.370)
Debt-to-EBITDA	-0.000	0.000	0.000	0.000	-0.000	-0.000
	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)
Dividend Payout		-0.002		0.007	0.003	0.005*
		(0.005)		(0.006)	(0.002)	(0.002)
Market-to-Book		0.169***		0.190***	0.191***	0.212***
		(0.035)		(0.041)	(0.063)	(0.074)
Stock Return		-0.005		-0.004	-0.005	-0.007
		(0.007)		(0.008)	(0.007)	(0.007)
Sales Growth		-0.032		-0.048	0.044	0.056
		(0.057)		(0.065)	(0.078)	(0.091)
Sample	All	All	All	All	Rated	Rated
Conditional on Issuance	No	No	Yes	Yes	No	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.212	0.140	0.251	0.206	0.099	0.104
Observations	38,502	29,840	34,028	26,289	16,239	14,321

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table presents results of the regression specification (5.1) using data from Compustat, S&P Credit Ratings, and Capital IQ for the period 2002-2024. The outcome variable is market debt net issuance rate defined as the net increase in market debt outstanding during the following year scaled by the total amount of bank and market debt. The coefficients of interest are the coefficients on bank debt share defined as the amount of bank debt divided by the sum of bank and market debt. Columns 1-4 use all firm-year observations from the estimation sample. Columns 5-6 restrict to observations with available S&P credit ratings. Columns 1, 2 ad 5 are unconditional on non-zero market debt changes while columns 3, 4 and 6 are conditional on non-zero market debt changes. In columns 1 and 3, the firm-year level controls include the log of book assets, tangibility, ROA, book leverage, and debt-to-EBITDA. Columns 2 and 4-6 also include dividend payout, market-to-book, stock return, and sales growth as additional control variables. Standard errors are double-clustered at the firm and year levels.

A potential concern is that firms' debt issuance decisions are affected by time-varying credit market conditions. For example, the observed increase in subsequent market debt net issuance

following a higher bank debt share could be due to a decline in the relative credit spreads of bonds compared to bank loans. This could arise from increased investor demand for corporate bonds. Moreover, the magnitude of this credit spread differential between bonds and bank loans may vary across firms with different characteristics and across industries, further complicating identification. To address this concern, I tighten the regression specification by including the interacted industry-year-size-leverage fixed effects.

Table E.2: Panel Regressions: Robustness Using Alternative Definitions

	Market Debt Net Issuance Rate			
	(1)	(2)	(3)	(4)
Bank Debt Share	0.154**	0.159**	0.135**	0.133*
	(0.063)	(0.065)	(0.061)	(0.065)
Debt Definition	Baseline	Baseline	Robustness	Robustness
Conditional on Issuance	No	Yes	No	Yes
Industry-Year-Size-Leverage FEs	Yes	Yes	Yes	Yes
R^2	0.211	0.214	0.221	0.224
Observations	29,353	25,876	40,109	35,358

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Note: The table presents results from regressions of market debt net issuance rate (defined as the net increase in market debt outstanding during the following year scaled by the total amount of bank and market debt) on bank debt share (defined as the amount of bank debt divided by the sum of bank and market debt), while controlling for the interacted industry-year-size-leverage fixed effects. Specifically size corresponds to quintiles of firm size as measured by total assets, and leverage corresponds to quintiles of book leverage. Columns 1-2 follow baseline definitions of bank debt (term loans) and market debt (senior and subordinated bonds and notes), while columns 3-4 follow definitions of bank debt (term loans and drawn credit lines) and market debt (senior and subordinated bonds and notes and commercial paper) in the robustness tests. Columns 1 and 3 are unconditional on non-zero market debt changes while columns 2 and 4 are conditional on non-zero market debt changes. Data are from Compustat and Capital IQ for the period 2002-2024. Standard errors are double-clustered at the firm and year levels.

In particular, industry fixed effects are constructed using the first two digits of the NAICS code, capturing broad sectoral classifications. Size fixed effects are defined by quintiles of firm size as measured by book assets. Leverage fixed effects are defined by quintiles of book leverage computed as the total debt outstanding divided by book assets. The coefficient estimates on bank debt share are reported in Table E.2. In columns 1 and 2, I follow the baseline definitions of bank debt and market debt. That is, bank debt is defined as bank loans and market debt includes senior and subordinated bonds and notes. In columns 3 and 4, I use the alternative definitions used in the robustness tests above. That is, bank debt is comprised of bank loans and drawn credit lines, while market debt includes commercial paper as well as senior and subordinated

bonds and notes. Columns 1 and 3 are unconditional on market debt net issuance or repurchase, while columns 2 and 4 are conditional on non-zero market debt changes.

In all columns of Table E.2, the coefficient estimates on bank debt share are lower in magnitude compared to those reported in the baseline regressions. However, the coefficient estimates remain positive and statistically significant at conventional levels. The results suggest that the baseline findings are also robust to alternative specifications with tighter controls. These robustness tests thus provide further evidence supporting the complementarity between bank debt and market debt.

C. Aggregate-Level Evidence

I now document the complementarity of bank and market debt using aggregate time-series data. The aggregate data are obtained from the Flow of Funds, which contains quarterly balance sheet information of the U.S. non-financial corporate sector and is accessible via the FRED database of the St. Louis Fed. In Panel A of Figure E.1, I plot the relationship between bank debt share and net issuance rate of market debt in the following year, over the sample period from Q1 1953 to Q4 2024. The bank debt share is the ratio of depository institution loans to corporate bonds (ex. eREITs). The market debt net issuance rate is the net increase in corporate bonds scaled by the combined total of depository institution loans and corporate bonds. The figure shows a positive relationship between the subsequent market debt net issuance rate and the prevailing bank debt share. In other words, firms tend to issue more market debt following periods in which their debt structures contain a greater share of bank debt.

The positive relationship between bank debt share and the subsequent market debt net issuance rate may be a mechanical artifact of the secular decline in the bank-to-market-debt ratio over the sample period. To address this concern, I focus on the period between Q1 2012 and Q4 2024 when the bank debt share remained relatively stable. I exclude the first two quarters of 2020 (COVID-19 shock period) from the sample to remove any impact due to unconventional Federal Reserve policies targeting bank lending and the corporate bond market. The positive relationship

Figure E.1: Bank-to-Market-Debt Ratio and Subsequent Debt Issuances

(A) Subsequent Market Debt Net Issuance

(B) Subsequent Market Debt Net Issuance (Post-2012)
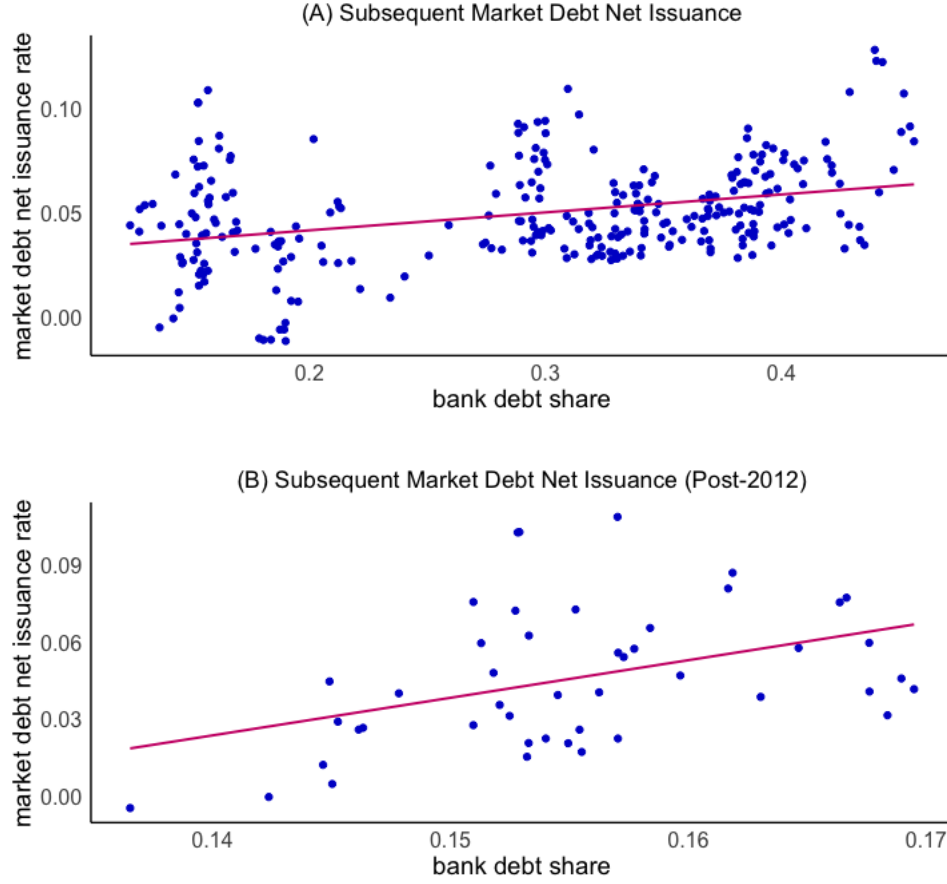
Note: The figure plots the relationship between bank debt share and market debt net issuance rate in the following year, for the full sample period (Panel A) and for the post-2012 subsample excluding the COVID-19 crisis period (Panel B). The full sample contains quarterly data from Q1 1965 to Q3 2024. The post-2012 subsample contains quarterly data from Q1 2012 to Q3 2024, while the COVID shock period is defined as the first two quarters of 2020. The bank debt share is defined as the ratio of depository institution loans to corporate bonds (ex. eREITs) obtained from the Flow of Funds. The market debt net issuance rate is the net increase in corporate bonds (ex. eREITs) scaled by the sum of depository institution loans and corporate bonds.

between bank debt share and market debt net issuance rate remains robust in this subsample period, as seen from Panel B of Figure E.1.