Optimal Disclosure in Asset Markets with Heterogeneous Belief Formation*

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Abstract

Biases in investor expectations can affect welfare by driving asset prices away from fundamentals, giving rise to boom-bust cycles. We study the optimal design of public signals in asset markets where heterogeneous investors differ in their belief formation. In our model, a social planner chooses how to disclose the realization of a private signal to maximize social surplus. Full disclosure is optimal only when investors' belief formation is sufficiently homogeneous and close to rational. Otherwise, no disclosure is optimal, as communication can amplify disagreement and reduce risk-sharing. Moreover, disclosure can cause over- or under-production of risky assets when investors overreact to signals. With short-sale constraints, partial disclosure can be optimal. Communication is more effective in the early phase of an asset price boom, before behavioral investors' priors have become too entrenched.

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1 Introduction

Investors neglect risks during booms and are too optimistic, while they overreact and are too pessimistic during busts (see, e.g., Kindleberger and Aliber 2011; Gennaioli and Shleifer 2018). Thus far, societies take such biases in beliefs as a fact of life, trying to mitigate their consequences ex-ante or ex-post using a variety of tools from financial regulation, monetary policy, and fiscal policy. Yet, arguably the least distortive way to address the underlying friction might be to manage investors' expectations directly. This could potentially be achieved by a social planner sending informative signals about asset over- or undervaluations over the cycle. However, there is no theoretical guidance on how communication should be used when investors are heterogeneous in their expectation formation and some may deviate from rationality.

Our model considers an asset market with two types of investors: rational and behavioral investors. Rational investors assign correct prior probabilities and update their beliefs using Bayes' rule. In contrast, behavioral investors may hold incorrect priors about asset returns and do not necessarily follow standard Bayesian updating. Therefore, behavioral investors may over-react or under-react to new information relative to rational investors (see, e.g., Bordalo et al. 2021). Based on their beliefs, investors determine their optimal portfolio allocation between a risky asset and a risk-free asset in a two-period framework. We allow the supply of the risky asset to be endogenous, by introducing a competitive asset production sector. The supply of the risky asset depends on its equilibrium price, which can be distorted due to irrational beliefs held by behavioral investors. Hence, biases in beliefs can lead to over- or underproduction of the risky asset.

A social planner (SP), which is endowed with a signal informative about the risky asset's payoff, decides how to communicate its information to investors. For instance, in the context of the housing market, the information could represent insights obtained from supervisory data on the lending conditions, delinquency rates, or other factors that impact the demand and supply of the housing market. Our framework imposes minimal restrictions on the set of feasible communication strategies, including full disclosure and no disclosure as special cases. Specifically, the SP can choose any mapping between its signal realizations and a flexible set of messages, following the persuasion framework of Kamenica and Gentzkow (2011).

The SP chooses a communication strategy that maximizes the social surplus, defined as the sum of all agents' expected utility.[1] In the special case where all investors are ratio-

[1]Our baseline model considers a paternalistic SP who believes it holds the true prior beliefs. We relax this assumption in Section 4.4 and demonstrate that the results remain qualitatively the same.

nal, the optimal communication strategy is full disclosure; because, the SP's communication unambiguously improves investment efficiency in the risky asset by reducing investors' uncertainty about its payoff. In the presence of behavioral investors, two additional forces kick in, which generate trade-offs regarding the optimal degree of disclosure. First, investors' response to communication is heterogeneous. This means that communication can result in more divergent asset positions, which reduces risk sharing among investors. For example, when behavioral investors are over-reactive, positive information about the risky asset's performance leads to a concentration of risk among them, as they aggressively buy the asset from rational investors, who are less reactive. Second, the response of behavioral investors can distort the equilibrium price of the risky asset, thereby leading to its overproduction or underproduction. Thus, the SP trades off these costs of communication against the benefit of sharing information with rational investors, who can use it to improve price efficiency, and consequently, investment efficiency.

In our baseline model, we characterize the optimal communication strategy within a class of "linear" updating rules, defined by two parameters: the prior beliefs q_0 and the updating gain G relative to the rational benchmark. Intuitively, behavioral investors have a larger updating gain when they face greater subjective uncertainty about the risky asset's payoff prior to communication, or when they assign more weight to the SP's communication in updating their beliefs. For rational investors, q_0 corresponds to the true prior probabilities, and $G = 1$. These linear updating rules allow us to analyze the optimal communication strategy under a flexible framework that accommodates heterogeneity in beliefs and reactions. Additionally, they can be interpreted as linear approximations of a broader class of updating rules, such as those studied by de Clippel and Zhang (2022). With this interpretation, our results inform the first-order effects of communication in asset markets and their welfare implications when investors do not necessarily form beliefs rationally. We show that our results remain qualitatively robust to alternative model specifications, such as considering non-linear updating rules for behavioral investors and introducing ambiguity into the SP's beliefs about the true probabilities.

The optimal communication strategy depends on the updating gain of behavioral investors. We show that full disclosure is optimal only when the updating gain of behavioral investors is sufficiently close to that of rational investors; that is, their G is sufficiently close to one. In this case, investors respond to new information in a nearly homogeneous way, enhancing investment efficiency with minimal impact on disagreement. However, no disclosure is the optimal communication strategy when the updating gain of behavioral investors is either too high or too low – that is, when they are either highly responsive or minimally responsive to the SP's communication about the risky asset's payoff. For instance, following a positive communication, if behavioral investors are overly responsive, they increase their demand for the risky asset more than rational investors would, leading to both overproduction of the risky asset and an increase in the concentration of risk among behavioral investors, as rational investors sell in response to the risky asset's overvaluation.

When behavioral investors are less responsive to the SP's communication compared to rational investors, the communication has two opposing effects on social surplus. On the one hand, investment efficiency improves since the price movement following the communication is mostly driven by rational investors. On the other hand, the communication can exacerbate the divergence in beliefs between rational and behavioral investors due to their different reactions, resulting in a greater concentration of risk within one group of investors. In this case, no disclosure is optimal when the second effect dominates the first one.

Investors' responsiveness to communication varies over time. Strings of good or bad news, may lead to waves of optimism or pessimism, and may make investors' more certain about their views. Hence, the optimal communication strategy may also be time-varying. Our results suggest that communication is most effective during normal times, before investors' beliefs become deeply entrenched and less responsive to new information. Communication can be counterproductive if extreme beliefs are held by a subset of investors while others remain fairly responsive, as it may lead to excessive trading between the two groups and, consequently, increased risk concentration. During periods of high subjective uncertainty, communication may worsen disagreement and reduce investment efficiency, as investors are more likely to overreact.

Our results highlight the limitations of communication as a policy tool when some investors do not respond rationally to communication. We show that macroprudential policies, such as imposing tighter leverage constraints, as suggested by Farhi and Werning (2020), may help reduce excessive risk-taking and address the concentration of risk more effectively in this case. Another challenge in implementing optimal communication strategies is the need for strong commitment to adhere to them. Specifically, the SP may have an incentive to disclose information favorable to market conditions ex-post and to withhold unfavorable information during bad times. However, if investors anticipate this behavior, they would interpret no disclosure as an unfavorable signal, worsening the market condition. We find that ex-post incentives to disclose favorable information are particularly strong when behavioral investors have extreme views about asset returns. In fact, if the SP has limited commitment power, it may end up fully revealing its signal, even though this might be an inefficient communication strategy.

Our model is motivated by the growing literature on asset return expectations. Adam and Nagel (2023) argue that subjective expectations about future cash flows and price levels are a major contributor to price dynamics. At the same time, survey evidence on those expectations suggests their inconsistency with rational expectation models (Greenwood and Shleifer, 2014a; Adam et al., 2017; Nagel and Xu, 2021; Fuster et al., 2022). Gennaioli and Shleifer (2018) attribute the house price boom prior to the financial crisis of 2008-2009 to investors' over-optimistic return expectations, a view that received substantial empirical support (see, e.g., Piazzesi and Schneider 2009; Adelino et al. 2016; Kaplan et al. 2020).

Over-reaction of expectations to recent signals arises naturally in diagnostic expectations models (see, e.g., Bordalo et al. 2018, 2019, 2020), and recent work on the role of memories in belief formation (see, e.g., Bordalo et al. 2023; Jiang et al. 2024). Asset overvaluations are deemed a threat to financial stability (Stein, 2014), and central banks, in fact, communicate frequently about them, in particular in the context of the housing market (see appendix B for more information).[2] However, there is little guidance on whether, or under which circumstances, it is socially optimal for central banks or other entities to communicate their views on asset valuations. We fill this gap by analyzing a social planner's optimal communication strategy for managing asset return expectations.

Our paper contributes to the literature on optimal communication in asset markets by examining the role of irrationality and heterogeneity in investors' belief formation. Previous studies find that it is not always optimal for regulators to fully disclose their information about firms' fundamentals when it hurts risk-sharing opportunities (Hirshleifer, 1978), when there are strategic complementarities in investors' actions, such as incentives to run on the liabilities of a bank or a financial institution (Morris and Shin, 2002; Angeletos and Pavan, 2007; Inostroza, 2023), or in the presence of financial frictions (Goldstein and Leitner, 2018; Quigley and Walther, 2024). To the best of our knowledge, we present the first theoretical model that investigates optimal communication in asset markets with investors who exhibit heterogeneous and potentially irrational reactions to new information. We find that full disclosure is optimal only when investors' reactions have limited heterogeneity and are not too far from the rational benchmark.

Methodologically, our framework builds on the insight from Bayesian Persuasion models that

[2] An extensive empirical literature examines the effect of central banks' communications on households' macroeconomic expectations (e.g., see Binder 2017; Coibion et al. 2022) and on financial markets (e.g., see Born et al. 2014; Neuhierl and Weber 2019; Leombroni et al. 2021; Gorodnichenko et al. 2023). Communication in speeches and other reports influences households' house price expectations (Binder et al., 2023), and their risk-taking behavior (Beutel et al., 2021). See Blinder et al. (2022) for a review of this literature. Overall, there is a substantial body of evidence suggesting that subjective expectations are responsive to central banks' communication.

the communication problems can be formulated as selecting a distribution of the receivers' posterior beliefs (Kamenica and Gentzkow, 2011), subject to a Bayesian plausibility constraint. As noted by Alonso and Câmara (2016) and de Clippel and Zhang (2022), this approach can also be extended to models where receivers are non-Bayesian or have prior beliefs different from the sender's, provided there exists a one-to-one mapping between the receivers' and sender's posterior beliefs. We adapt their methodology to a framework of communication in asset markets with heterogeneous investors, and examine optimal communication with a focus on investment efficiency and risk sharing.[3]

2 A Model of Communication with Investor Heterogeneity in Belief Formation

2.1 Economic Environment

There are two periods ($t = 0, 1$). The economy is populated with measure one of atomistic investors, indexed by $i \in [0, 1]$, each endowed with e units of consumption goods in period 0, and one unit of a risky asset that pays random payoff $\tilde{D}$ in period one.[4] The payoff is either D_H or D_L, where $D_H > D_L \geq 0$. The unit price of the risky asset in period zero is denoted by P, which is determined endogenously. We assume that high and low payoffs are equally likely. This can be interpreted as a flat prior when no data on the risky asset's payoff is available. Our results do not depend on this assumption; however, it helps simplify the presentation of the model's insights.

There are two groups of investors. A fraction $1 - b$ of the investors, where $b \in [0, 1]$, assigns correct prior probabilities and updates their beliefs rationally in response to new information. We refer to this group as "rational" investors. The remaining investors, called "behavioral investors," may have incorrect prior beliefs about the risky asset's payoff and do not necessarily follow a standard Bayes' rule when updating their beliefs. Behavioral investors assign a prior probability of $q_0^B \in (0, 1)$ to $\tilde{D} = D_H$. Therefore, $q_0^B > 0.5$ ($q_0^B < 0.5$) corresponds to irrational optimism (pessimism) about the asset's payoff. In practice, such incorrect beliefs can arise when investors make irrational inferences from past data. For instance, some investors might have extrapolative beliefs, making them optimistic about the risky asset's performance after a sequence of positive returns, and pessimistic after a

[3]Herbert (2021) examines a communication model with Bayesian agents holding heterogeneous macroeconomic beliefs; in our framework, some agents are non-Bayesian and we model the effects of communication on trading, asset prices, and asset production.

[4]This symmetric choice of the endowments isolates the model from the Hirshleifer effect.

sequence of negative returns (Afrouzi et al., 2023).

In period zero, investors choose their investment in the risky asset (z_i) and their investment in a risk-free asset (I_i), which generates a gross return of $R \geq 1$ in period one. To capture investors' risk-aversion, we consider a quadratic cost for their investment in the risky asset, $\frac{1}{2}\gamma z_i^2$. Therefore, investors' utility is defined as:

$$\tilde{u}_i = I_i R + z_i \tilde{D} - \frac{1}{2}\gamma z_i^2, \quad i \in [0,1]. \tag{1}$$

Such linear-quadratic preferences are helpful to obtain tractability, and have been utilized in previous theoretical studies of financial markets (see, e.g., Rostek and Weretka 2012).

Let q^i be the probability that investor i assigns to $\tilde{D} = D_H$ when making her portfolio choice. Investors choose a portfolio of the risky and risk-free assets that maximizes their expected utility ($\mathbb{E}^i[\tilde{u}_i]$) given their subjective beliefs, subject to the following budget constraint:

$$I_i + P z_i \leq e + P \tag{2}$$

The right-hand side in the budget constraint, $e + P$, represents the market value of the endowments in period 0, which can be allocated to the risky and risk-free assets.

With our ultimate welfare analysis in mind, we allow the supply of the risky asset to be endogenous to account for potential overinvestment or underinvestment caused by irrational beliefs. Specifically, we assume that there exist a measure-one set of infinitesimal and competitive asset producers, indexed by $j \in [0,1]$. For instance, asset producers could represent the developer sector in the housing market. Asset producer j can produce x_j units of the asset in period zero at cost $lx_j + \frac{1}{2}kx_j^2$, and sell at price P to investors. For simplicity, we assume that asset producers carry no units of the risky asset to the next period.[5]

Therefore, an asset producer's profit from producing x_j units of the risky asset is:

$$A_j = (P - l)x_j - \frac{1}{2}kx_j^2, \quad j \in [0,1]. \tag{3}$$

The first-order condition implies that the total production of the risky asset is

$$X = k^{-1}(P - l). \tag{4}$$

[5]This assumption can be microfounded by considering a larger discount rate for asset producers than investors.

We assume that $0 \leq l \leq R^{-1}(D_L - \gamma)$ to ensure that the production is non-negative. The overall supply of the risky asset is the sum of the pre-existing stock, which is one, and the amount produced (X). The marking clearing implies:

$$\int_0^1 z_i di = 1 + X. \tag{5}$$

2.2 Asset Payoff Communication

A social planner (SP) is endowed with a signal informative about the risky asset's payoff. The signal has a finite set of possible realizations, represented by S. Let $q : S \to (0, 1)$ denote the (true) conditional probabilities:

$$q(s) = Prob(\tilde{D} = D_H|s), \quad s \in S. \tag{6}$$

Furthermore, let $\pi_0(s)$ be the probability that the signal realization is s. We assume that there are at least two signals in S (i.e., $|S| > 1$), and no two signals have the same conditional probabilities (i.e., $q(s) \neq q(s')$ for $s, s' \in S$). Note that the law of iterated expectations implies that the expectation of $q(s)$ should be 0.5, the unconditional probability of $\tilde{D} = D_H$:

$$\mathbb{E}[q(s)] = \sum_{s \in S} \pi_0(s) q(s) = 0.5. \tag{7}$$

The signals can, for instance, represent the result of a government agency's (including central banks) research about the asset's future performance. For instance, in the context of the housing market, the signal could be any information implied from supervisory data on the delinquency rates, the status of inventory, macroeconomic and demographic forecasts, and lending conditions. We assume that investors cannot observe the signal realization. To simplify the exposition, we assume that the SP shares the same prior beliefs as rational investors. However, this assumption is relaxed in Section 4.4, where we introduce ambiguity to the SP's beliefs about the true probabilities.

To communicate its information, the SP has access to a set of messages, denoted by $\mathcal{M}$. We assume that $\mathcal{M}$ is finite but sufficiently large. Messages in $\mathcal{M}$ can be interpreted as the reports or summaries to be shared with investors. The SP chooses a mapping between its signals S and messages in $\mathcal{M}$, represented by conditional probabilities $\pi(m|s)$, $s \in S$, $m \in \mathcal{M}$. To simplify the notation, we let $m = s$ be the message that communicates that the realized signal is s. Note that making no communication itself can also be interpreted as a message, which we represent by "ND" (no disclosure).

Two extreme communication strategies are "no disclosure" and "full disclosure," denoted by π^{ND} and π^{FD}, respectively. When no disclosure is chosen, the SP makes no communication with investors, regardless of the realization of its signal; that is,

$$\pi^{ND}(ND|s) = 1, \quad \forall\, s \in S. \tag{8}$$

Full disclosure is a communication strategy that always shares the realized signal with probability one; that is,

$$\pi^{FD}(s|s) = 1, \qquad \forall\, s \in S. \tag{9}$$

We impose minimal restrictions on the set of feasible communication strategies in our baseline model to understand the trade-offs in communication when some investors do not form beliefs rationally. In particular, we only require $\pi(\cdot|s)$ to satisfy standard properties of probability functions for each signal realization. We call such a mapping admissible. Definition 1 specifies the admissibility condition.

Definition 1. *Mapping π is admissible if:*

$$\begin{aligned} &\pi(m|s) \in [0,1] \quad \forall\ m \in \mathcal{M}, s \in S \\ &\sum_{m \in \mathcal{M}} \pi(m|s) = 1, \quad \forall s \in S. \end{aligned} \tag{10}$$

We assume that investors know the SP's communication strategy π and update their beliefs accordingly. For instance, if the SP decides to consistently communicate a negative message about the risky asset's payoff, investors understand that the message carries no new information, so they do not update their beliefs. Investors may have learned this communication strategy from observing past communications and outcomes, or the SP may have directly communicated its strategy.

An assumption in our baseline model is that the SP can commit to its choice of communication strategies. This means the SP does not deviate from its communication strategy after any realization of s. In practice, government agencies can obtain commitment power, for instance by following a certain reporting methodology, such as regularly reporting a certain set of indicators and statistics. We relax this assumption in Section 4.2 and analyze the optimal communication strategy when the SP has limited commitment power.

2.3 Investors' Belief Formation

Let $\hat{q}(m)$ and $q^B(m)$ be the posterior probability that rational and behavioral investors assign, respectively, to $\tilde{D} = D_H$, when message m is communicated. Rational investors update their beliefs according to Bayes' rule:

$$\hat{q}(m) = \frac{\sum_{s \in S} \pi(m|s)\pi_0(s)q(s)}{\sum_{s \in S} \pi(m|s)\pi_0(s)}, \tag{11}$$

and we consider the following updating rule for behavioral investors:

$$q^B(m) = q_0^B + G_B\left(\hat{q}(m) - \frac{1}{2}\right). \tag{12}$$

In Equation 12, $\hat{q}(m) - \frac{1}{2}$ reflects the change in rational investors' belief upon receiving message m, and G_B represents behavioral investors' updating gain, relative to rational investors' reaction. We assume that $q^B(s) \in [0,1]$ for all $s \in S$ to ensure that $q^B(m)$ remains in $[0,1]$ for any communication strategy.[6] This linear updating rule simplifies the exposition. In Section 4.3, we demonstrate the robustness of our results to this assumption by considering non-linear updating processes for behavioral investors.

Behavioral investors are over-reactive when $G_B > 1$, which captures diagnostic beliefs (see, e.g., Bordalo et al. 2018), as well as other mechanisms of belief overreaction, as supported by recent empirical studies (see, e.g., Armona et al. 2019; Afrouzi et al. 2023; Beutel and Weber 2023). Behavioral investors are under-reactive when $G_B < 1$, which occurs when they have strong prior beliefs about the asset payoff, i.e., q_0^B is close to its extreme values. Additionally, this under-reactivity could result from inattention, limited information processing ability, or a lack of trust in the SP's signal.[7] Overall, different values of (q_0^B, G_B) can capture a wide variety of belief-formation processes discussed in previous research.

2.4 Market Equilibrium and Social Surplus

This section examines the connection between investors' posterior beliefs and social surplus. The relationship between the two helps us analyze the welfare implications of the SP's communication.

Given the objective function in Equation 1 and the constraint in Equation 2, the optimal

[6] Note that $q^B(s) = q_0^B + G_B(q(s) - \frac{1}{2})$.

[7] Our solution framework can be extended to allow for asymmetric responses to positive and negative news, as is the case for motivated beliefs (e.g., see Caplin and Leahy 2019; Cassella et al. 2023).

investment strategy of investor i, with posterior belief q^i, is:

$$z_i = \gamma^{-1}\big\{\mathbb{E}^i[\tilde{D}|m] - PR\big\}. \tag{13}$$

In Equation 13, we see that investment in the risky asset increases with its expected dividend, consistent with intuition. We denote behavioral investors' asset demand by $z_B(m)$, and rational investors' asset demand by $z_R(m)$. By imposing market clearing (Equation 5), we find that

$$\begin{aligned} &bz_B(m) + (1-b)z_R(m) = 1 + X = 1 + k^{-1}(P(m) - l) \\ &\Rightarrow P(m) = \frac{1}{R + \gamma k^{-1}}\big\{\bar{\mathbb{E}}[\tilde{D}|m] - \gamma(1 - k^{-1}l)\big\}, \end{aligned} \tag{14}$$

where

$$\bar{\mathbb{E}}[\tilde{D}|m] = b\mathbb{E}^B[\tilde{D}|m] + (1-b)\mathbb{E}^R[\tilde{D}|m]. \tag{15}$$

Equations 14 and 15 indicate that the equilibrium price reflects the average expected payoff of the risky asset.

Let $\tilde{V}(m)$ be the social surplus induced by message m, defined as the sum of investors' utility and asset producers' utility:

$$\begin{aligned} \tilde{V}(m) &\equiv \int_0^1 \tilde{u}_i(m)di + R\int_0^1 A_j(m)dj \\ &= eR + \tilde{D} + (\tilde{D} - PR)X(m) - \frac{1}{2}\gamma(bz_B(m)^2 + (1-b)z_R(m)^2) + R((P-l)X(m) - \frac{1}{2}kX(m)^2) \\ &= eR + \tilde{D} + (\tilde{D} - Rl)X(m) - \frac{1}{2}RkX(m)^2 - \frac{1}{2}\gamma(bz_B(m)^2 + (1-b)z_R(m)^2). \end{aligned} \tag{16}$$

Equation 16 presents the social surplus induced by message m for each realization of $\tilde{D}$, the risky asset's payoff. The first line specifies the social surplus as the sum of the agents' utility. The weight R is assigned to asset producers' utility since their utility in Equation 3 is expressed in terms of time-0 consumption goods, while investors' utility in Equation 1 is expressed in terms of time-1 consumption goods, and R is the risk-free rate of substitution between the two periods. The second line is obtained from integration and applying the market-clearing condition in Equation 5. The last line cancels out the transactions between investors and asset suppliers.

Note that $X(m)$, $z_B(m)$, and $z_R(m)$ in Equation 16 only depend on investors' posterior beliefs. After some further simplifications, the expected total surplus can be specified in terms of the rational posterior, $\hat{q}(m)$, and the behavioral posterior, $q^B(m)$ as follows:

$$\mathbb{E}[\tilde{V}(m)|m] = c_0 + c_1\hat{q}(m) + c_2\hat{q}(m)^2 - \lambda c_2(q^B(m) - \hat{q}(m))^2, \tag{17}$$

where

$$\begin{aligned} \lambda &= (1+\gamma^{-1}Rk)b - \gamma^{-1}Rkb^2 \\ c_2 &= \frac{1}{2(\gamma+Rk)}(D_H - D_L)^2 \\ c_1 &= \frac{D_L + R(k-l)}{\gamma+Rk}(D_H - D_L) \\ c_0 &= eR - \frac{\gamma}{2} + D_L + \frac{(D_L - Rl - \gamma)^2}{2(Rk+\gamma)}. \end{aligned} \tag{18}$$

The first three terms in Equation 17 present the expected social surplus in the special case that all investors are rational. In this case, we observe that the expected surplus increases with $\hat{q}(m)$, the rational belief induced by message m, reflecting the fact that the expected surplus is positively related to the expected asset payoff. The quadratic term emerges because the overall dividends generated by the risky asset depends on its supply, which increases with investors' expectation of its payoff.

The final term shows how the presence of behavioral investors impacts the expected surplus following a message. We see that the expected surplus decreases with the difference between the posterior beliefs of behavioral and rational investors, i.e., $|q^B - \hat{q}|$. Two effects contribute to this negative effect. First, the difference reflects the degree of belief heterogeneity induced by message m. Greater heterogeneity in beliefs results in more divergent asset positions, which reduces risk sharing and thus lowers the expected surplus. For example, when behavioral investors are highly optimistic about the risky asset's payoff, the asset risk becomes concentrated among them as they take more aggressive positions.

Second, behavioral investors' beliefs distort the equilibrium price of the risky asset, leading to its overproduction or underproduction. For example, when behavioral investors are more optimistic (pessimistic) than rational investors about the risky asset's payoff, they demand more (less) of the risky asset, which results in the asset's overproduction (underproduction). Rational investors cannot fully eliminate this mispricing because of their limited risk-bearing capacity.

Note that the expectation in Equation 17 is taken with respect to the SP's beliefs, which differ from the subjective beliefs of behavioral investors. Therefore, the SP has a paternalistic view about the optimal production of the risky asset and its allocation among investors. As a result, investors might disagree with the SP about the optimal communication strategy. We relax this assumption in Section 4.4, and discuss when a communication strategy exists that is preferred by all agents.

3 Optimal Communication Strategy

3.1 Derivation of Optimal Communication Strategy

The SP's objective is to choose communication strategy π that maximizes the expected surplus. By taking the unconditional expectation of Equation 17, we find:

$$\mathbb{E}[\tilde{V}] = c_0 + \frac{1}{2}c_1 + c_2\mathbb{E}\Big[\hat{q}(m)^2 - \lambda(q^B(m) - \hat{q}(m))^2\Big]. \tag{19}$$

The second term in Equation 19 is obtained from applying the law of iterated expectations to $\hat{q}(m)$, i.e., $\mathbb{E}[\hat{q}(m)] = \frac{1}{2}$. Equation 19 reveals that communication strategy π impacts the surplus through its effect on the distribution of investors' posterior beliefs, i.e., $\hat{q}(m)$ and $q^B(m)$. Since there is a one-to-one mapping between $\hat{q}$'s and q^B's, the SP effectively selects a distribution of rational posterior beliefs by choosing π.

Let $\mathbb{E}_\pi[\cdot]$ represent unconditional expectations induced by communication strategy π. For instance, $\mathbb{E}_\pi[\hat{q}]$ is the expected rational posterior, which is $\frac{1}{2}$ according to the law of iterated expectations. For rational investors, the posterior variance $Var(\mathbb{I}_{\{\tilde{D}=D_H\}}|m)$ is given by $\hat{q}(m)(1-\hat{q}(m))$. Its expectation, $\mathbb{E}_\pi[\hat{q}(1-\hat{q})] = \frac{1}{2} - \mathbb{E}_\pi[\hat{q}^2]$, provides a measure of rational investors' uncertainty following the SP's communication. Accordingly, $\mathbb{E}_\pi[\hat{q}^2]$ can be interpreted as a measure of the informativeness of π.

Building on Equation 19, we define the SP's expected utility $V(\pi)$ as:

$$V(\pi) = \mathbb{E}_\pi[v(\hat{q})], \tag{20}$$

where

$$v(\hat{q}) = \hat{q}^2 - \lambda(q^B - \hat{q})^2. \tag{21}$$

To avoid clutter, we dropped the constant and multiplier terms in Equation 19, as they do not impact the solution to the SP's optimization problem. We can treat $v(\hat{q})$ as the SP's interim expected utility associated with inducing rational posterior belief $\hat{q}$. Since q^B is linear in $\hat{q}$ (see Equation 12), $v(\hat{q})$ is quadratic:

$$v(\hat{q}) = \left[1 - \lambda(G_B - 1)^2\right]\hat{q}^2 - \lambda(q_0^B - \frac{G_B}{2})(G_B - 1)\hat{q} - \lambda(q_0^B - \frac{G_B}{2})^2. \tag{22}$$

By substituting Equation 22 into Equation 20, and noting that $\mathbb{E}_\pi[\hat{q}] = \frac{1}{2}$, we can rewrite the expected surplus attained by π as follows:

$$V(\pi) = \left[1 - \lambda(G_B - 1)^2\right]\mathbb{E}_\pi[\hat{q}^2] + V_0, \tag{23}$$

where V_0 is a constant.

3.2 Main Result on Optimal Communication Strategy

Equation 23 reveals that more informative communication increases the expected surplus if and only if $\mathbb{E}_\pi[\hat{q}^2]$ has a positive coefficient. We use this observation to characterize the optimal communication strategy, as reported in Proposition 1.

Proposition 1. *The optimal communication strategy is full disclosure when*

$$G_B \in [1 - \frac{1}{\sqrt{\lambda}}, 1 + \frac{1}{\sqrt{\lambda}}], \tag{24}$$

where λ is defined in Equation 18. Otherwise, no disclosure is optimal.

Proposition 1 states that full disclosure is the optimal communication strategy only when the updating gain of behavioral investors, G_B, is sufficiently close to the rational benchmark of one.

In the special case where all investors are rational, increasing the informativeness of π always improves the expected surplus. This is because the efficiency of risky-asset production increases with the informativeness of the SP's communication. Moreover, risk-sharing is perfect since there is no heterogeneity in beliefs, so risk is evenly distributed among investors. This case is reminiscent of Blackwell (1951), who shows that rational decision makers always benefit from more informative communication.

By contrast, no disclosure is optimal when behavioral investors are present and are either highly responsive or minimally responsive to the SP's communications. When behavioral investors are overly responsive, following a positive message, they increase their demand for the risky asset more than rational investors, leading to the overproduction of the risky asset. Furthermore, the risky asset becomes overvalued, causing rational investors to sell to behavioral investors, and consequently, the ownership of the risky asset becomes more concentrated among behavioral investors.

When behavioral investors are less responsive to the SP's communication compared to rational investors, the communication has two opposing effects on social surplus. On the one hand, the production of the risky asset becomes more efficient since all investors revise their beliefs in the right direction without overreacting. On the other hand, the communication can exacerbate the divergence in beliefs between rational and behavioral investors, resulting in a greater concentration of risk within one group of investors.

The bounds in (24) are tighter for larger values of λ. For example, since λ and γ are negatively related (see Equation 18), the bounds are tighter for smaller values of γ, because these correspond to more elastic investor demand. Consequently, any given level of belief dispersion generates larger divergence in asset positions. Therefore, the negative effect of communication is stronger for smaller values of γ, making no disclosure more likely to be optimal.

Furthermore, λ is larger when k, the quadratic term in the production cost of the risky asset, is larger, indicating a more inelastic supply of the risky asset. This is because when the asset supply is less elastic, the investment efficiency channel carries less weight relative to the risk-sharing channel in the social surplus, and in expectation, risk-sharing tends to worsen with more communication when investors react to it heterogeneously.[8] Therefore, the bounds in (24) become narrower as k increases. That is, full disclosure becomes optimal over a smaller range of parameters when the asset supply is less elastic.

3.3 Subjective Uncertainty and Optimal Communication

In this section, we answer the following question: Should the SP increase her communication when investors are more uncertain? Specifically, we examine how the optimal communication strategy depends on behavioral investors' subjective uncertainty. Proposition 1 characterizes the optimal communication in terms of G_B, behavioral investors' updating gain. Suppose G_B is proportional to their subjective uncertainty prior to the communication, $Var_0^B \equiv q_0^B(1-q_0^B)$; specifically,

$$G_B = 4\theta Var_0^B. \tag{25}$$

[8] In Section 4.3, we find that communication might improve risk-sharing when non-linear updating rules are considered. However, the results regarding the optimal communication strategy remain qualitatively the same.

Appendix A.1 provides a microfoundation for this relationship. In Equation 25, θ can be interpreted as the weight behavioral investors assign to the SP's signal. Corollary 1 describes how the optimal communication strategy depends on the subjective uncertainty of behavioral investors, for difference values of θ.

We impose two assumptions in Corollary 1 to better reflect periods of sentiment-driven booms and busts in asset markets. First, we assume that the asset supply is relatively inelastic compared to the demand, capturing the fact that the supply of most risky assets, such as housing, cannot dramatically change over short periods of time, whereas demand tends to be more volatile. Second, we assume that the fraction of behavioral investors, b, is large, allowing for the possibility of significant deviations of asset valuations from their fundamentals.

Corollary 1. *Suppose investors' asset demand is relatively elastic compared to the supply, i.e.,* $\gamma < Rk$, *and* b *is sufficiently large such that* $\lambda \geq 1$.

a) *If* $\theta \leq 1 - \frac{1}{\sqrt{\lambda}}$, *then the optimal communication strategy is always no disclosure.*

b) *If* $\theta \in (1 - \frac{1}{\sqrt{\lambda}}, 1 + \frac{1}{\sqrt{\lambda}})$, *then the optimal communication strategy is full disclosure when behavioral investors are sufficiently uncertain, specifically when*

$$Var_0^B \geq \frac{1}{4\theta}(1 - \frac{1}{\sqrt{\lambda}}). \tag{26}$$

c) *If* $\theta \geq 1 + \frac{1}{\sqrt{\lambda}}$, *then full disclosure is optimal when behavioral investors' uncertainty is intermediate, specifically when*

$$Var_0^B \in [\frac{1}{4\theta}(1 - \frac{1}{\sqrt{\lambda}}), \frac{1}{4\theta}(1 + \frac{1}{\sqrt{\lambda}})]. \tag{27}$$

The statements in Corollary 1 directly follow from Proposition 1. Part (a) in Corollary 1 states that when behavioral investors assign a very low weight to the SP's signal, the optimal communication strategy is no disclosure. This is because rational investors exhibit a substantially stronger response to the communication compared to behavioral investors, which impairs risk-sharing between the two groups of investors. This negative effect dominates the limited positive impact that the communication has on investment efficiency since the asset supply is relatively inelastic.

Part (b) shows that when θ is sufficiently close to one, full disclosure is optimal as long as behavioral investors do not have strong views about the risky asset's performance in either direction. The intuition is that the negative impact of communication on risk-sharing is small when behavioral and rational investors exhibit fairly similar responses to communication. Moreover, communication positively impacts investment efficiency. When behavioral investors have extreme prior beliefs, they are less responsive to the SP's communication. This increases the heterogeneity in responses among investors, amplifying the negative effect of communication on risk-sharing.

Lastly, according to Part (c), when behavioral investors assign a large weight to the SP's signal, full disclosure is optimal only for intermediate levels of their subjective uncertainty. When behavioral investors are highly uncertain, they respond aggressively to the SP's communication, which negatively impacts social surplus through both the risk-sharing and investment efficiency channels. Conversely, when they have extreme views, similar to the previous cases, they exhibit weaker reactions to communication compared to rational investors, causing the SP's communication to impair risk-sharing.

A unified message from Corollary 1 is that communication is likely ineffective in the late phase of an asset price boom or bust, when behavioral investors' priors have become deeply entrenched. Consider for instance highly optimistic behavioral investors at the late stage of a housing boom. A negative signal has minimal impact on the beliefs of behavioral investors, who contributed to the boom in the first place, while rational investors respond to the signal by reducing their asset demand. This leads to a further concentration of risk among behavioral investors, thereby reducing welfare. Therefore, communication is more effective when investors exhibit more uncertainty about assets' payoff, provided they do not assign too much or too little weight to the SP's information.

SPs can obtain this weight from surveys and empirical studies (see, e.g., Afrouzi et al. 2023; Beutel and Weber 2023). Such studies can inform not only the average θ but also the extent of heterogeneity in θ across investors. The average θ governs the impact of communication on the efficiency of aggregate investment in risky assets, while the heterogeneity in θ governs the degree of heterogeneity in investors' responses, which affects risk-sharing.

4 Additional Discussions

4.1 Optimal Communication in the Presence of Short-sale Constraints

In the baseline model, we study the optimal communication strategy in the absence of financial frictions. However, financial frictions, such as short-sale constraints, could amplify asset overvaluations and limit their undervaluation when investors have heterogeneous beliefs (Harrison and Kreps, 1978; Scheinkman and Xiong, 2003). In this section, we analyze how short-sale constraints impact optomal communication.

Specifically, suppose that investors cannot take a negative position in the risky asset, i.e., $z_i \geq 0$. The belief formation and endowments are the same as the basline model. Proposition 2 describes the optimal communication strategy.

Proposition 2. *We say that a signal $s \in S$ is extreme if message $m = s$ makes the short-sale constraint binding for some investors, i.e., $z_B = 0$ or $z_R = 0$.*

a) *If S contains extreme signals, then some disclosure is optimal.*

b) *Full disclosure is not optimal if $G_B \notin [1 - \frac{1}{\sqrt{\lambda}}, 1 + \frac{1}{\sqrt{\lambda}}]$, and there are at least two signals in S that are not extreme.*

The intuition for this result is that short-sale constraints limit the degree of heterogeneity in asset positions. Therefore, the SP is less concerned about inducing excessive disagreement among investors when disclosing extreme signals. However, full disclosure is still not optimal, because for the set of signals that are not extreme (as defined in the Proposition), the trade-offs remain the same as in the baseline. Consequently, it is not optimal to perfectly disclose the signals in this set, provided the set contains more than one signal.

This result implies that some disclosure can always be optimal in the presence of market frictions. However, full disclosure is optimal only when all investors are sufficiently close to rational belief formation.

4.2 Credibility Concerns

In our baseline model, we find that the optimal communication strategy is either full disclosure or no disclosure depending on behavioral investors' updating gain G_B. However, this result is obtained under the assumption that the SP can commit to these communication strategies. This means that if full disclosure is the optimal communication strategy, then the SP is able to commit to perfectly sharing any realization of its signal. Likewise, if no disclosure is optimal, then the SP can commit to not sharing any information ex-post, even if sharing would significantly improve the surplus.

One concern is that there may be circumstances in which the SP has an incentive to deviate from its communication strategy. For instance, during times of market turmoil, central banks might consider sharing favorable signals while withholding negative ones about asset markets in order to restore stability.

This section examines optimal communication strategies in light of such credibility constraints. Specifically, we define the following credibility constraint for communication strategies.

Definition 2. *Communication strategy π is "credible" at deviation cost $c > 0$ if:*

1. *For any signal realization $s \in S$, the gain from disclosing s does not exceed $c > 0$:*

$$\mathbb{E}[\tilde{V}(s)|s] - \mathbb{E}[\tilde{V}(m)|s] \leq c \quad if \quad \pi(m|s) > 0, \tag{28}$$

   where $\mathbb{E}[\tilde{V}(m)|s]$ is the expected social surplus when signal s is realized and message m is sent. $\mathbb{E}[\tilde{V}(s)|s]$ is the expected surplus when signal s is realized and disclosed to investors.

2. *If the SP does not communicate with a positive probability, i.e., $\pi(ND|s) > 0$ for some $s \in S$, then the gain from no disclosure does not exceed c for any signal realization $s \in S$:*

$$\mathbb{E}[\tilde{V}(ND)|s] - \mathbb{E}[\tilde{V}(m)|s] \leq c \quad \forall\, s \in S. \tag{29}$$

In our definition of credibility, we assume that the SP can deviate from following its choice of communication strategy in two ways: perfectly disclosing the realized signal, or refraining from disclosing any information. For the case of no disclosure, this assumption implies that the expected surplus increases at most by c if the SP discloses its signal realization to investors. If no disclosure occurs with a positive probability under a communication strategy, then the benefit of switching to no disclosure should not exceed c for any signal realization.

If no disclosure never occurs under π (i.e., $\pi(ND|s) = 0$, $\forall\, s \in S$), which is the case in full disclosure, Definition 2 assumes away profitable deviations to no disclosure. This assumption can be justified by noting that the SP has the strongest incentive not to communicate when sharing the message has the worst impact on the surplus. In other words, if, following no disclosure, investors consider the "worst possibility," the SP never gains from no disclosure.[9]

Note that c in our definition captures the SP's commitment power. When c is sufficiently large, any communication strategy is credible. The set of credible communication strategies shrinks as c approaches zero. Eventually, when c is very small, the credibility constraints require the SP's messages to dominate both full disclosure and no disclosure ex-post. In Propo-

[9]This argument can be formalized by employing the D1-criterion proposed by Banks and Sobel (1987).

sition 3, we describe the optimal communication strategies for different values of c.

Proposition 3. ***a)*** *When* $G_B \in [1 - \frac{1}{\sqrt{\lambda}}, 1 + \frac{1}{\sqrt{\lambda}}]$*, full disclosure is the optimal credible communication strategy for any value of* $c > 0$*.*

b) *When* $G_B \notin [1 - \frac{1}{\sqrt{\lambda}}, 1 + \frac{1}{\sqrt{\lambda}}]$*,*

b.1) *If* $c \geq \frac{(\lambda(G_B-1)+b)^2}{\lambda(G_B-1)^2-1}(q_0^B - \frac{1}{2})^2$*, no disclosure is the optimal credible communication strategy.*

b.2) *If* $c < \frac{(\lambda(G_B-1)+b)^2}{\lambda(G_B-1)^2-1}(q_0^B - \frac{1}{2})^2$*, the optimal credible communication strategy is to only disclose some intermediate signals; that is, there exist* $\underline{q}$ *and* $\bar{q}$*, where* $\underline{q} < \bar{q}$*, such that the SP discloses any* s *with* $q(s) \in [\underline{q}, \bar{q}]$*, and does not communicate otherwise. This might lead to unraveling of the signal when* c *is sufficiently small*

Part (a) of Proposition 3 states that full disclosure is always credible when it is the optimal communication strategy, which is straightforward from Proposition 1 and Definition 2.

Part (b.1) of Proposition 3 reveals that when G_B is sufficiently large or small, no disclosure is both optimal and credible only when c exceeds a threshold value. The threshold is larger when the prior belief of behavioral investors is closer to the extreme values. In other words, the SP is more likely to deviate from no disclosure when behavioral investors are either highly optimistic or pessimistic.

To understand the intuition, suppose behavioral investors are optimistic, i.e., $q_0^B > \frac{1}{2}$. If the SP's realized signal about the risky asset's payoff is negative, i.e., $q(s) < \frac{1}{2}$, the SP has a strong incentive to disclose the signal ex-post since doing so increases the expected surplus by partially correcting the irrational belief of behavioral investors, thereby narrowing the heterogeneity in beliefs among investors. However, if investors expect such deviations, they may interpret no disclosure as a positive signal, exacerbating both price inefficiency and concentration of risk among behavioral investors. When G_B is sufficiently large, this negative effect dominates the positive effect of disclosing favorable signals.

According to Part (b.2), when no disclosure is not credible, the SP's optimal communication strategy is to disclose only some intermediate signals, as revealing extreme signals creates greater inefficiency through both the risk concentration and investment inefficiency channels. However, the range of intermediate signals expands as the SP's commitment power declines (i.e., as c becomes smaller). This is because, in the set of non-disclosed signals, the SP always has an incentive to disclose the least extreme ones ex-post, in order to prevent non-disclosure from being misinterpreted as a more extreme message. When c becomes sufficiently small, the intermediate range may cover the entire set of signals. When the SP's commitment power is low, this can result in the perfect unraveling of its signal.

4.3 Non-linear Updating Processes

In our baseline model, we consider a linear updating rule for the investor. This section analyzes the optimal communication strategy when the following non-linear specification is considered:

$$\begin{aligned} \frac{q^B(m)}{1-q^B(m)} &= \frac{q_0^B}{1-q_0^B}\left(\frac{Prob(\tilde{D}=D_H|m)}{Prob(\tilde{D}=D_L|m)}\right)^{\theta} \\ &= \frac{q_0^B}{1-q_0^B}\left(\frac{\hat{q}(m)}{1-\hat{q}(m)}\right)^{\theta}. \end{aligned} \tag{30}$$

In Appendix A.1, we show that the linearization implies an updating gain of $G_B = 4\theta q_0^B(1-q_0^B)$.

Since the general characterization is complex, we consider the case that the SP's signal has a limited informativeness. The signal can be interpreted as the information obtained by government agencies from a new set in a series of observations. As such, the new observations are likely to have a limited impact on the agencies' overall information set. Specifically, let a_{max} represent the maximum accuracy among the realizations in S:

$$a_{max} = \max_{s\in S} \; |q(s) - \frac{1}{2}|. \tag{31}$$

We characterize the optimal communication strategies when a_{max} is sufficiently small. Proposition 4 provides the characterization.

Proposition 4. *When $a_{max} > 0$ is sufficiently small, either full disclosure or no disclosure is optimal. Full disclosure is optimal when*

$$\lambda^{-1} > (G_B - 1)^2 - 2G_B(\theta - G_B). \tag{32}$$

Otherwise, no disclosure is optimal.

We see that the optimal communication strategy depends on the updating gain of behavioral investors, in a manner similar to what Proposition 1 suggests. Specifically, full disclosure is optimal when the updating gain of behavioral investors is sufficiently close to that of rational investors, i.e., G_B is sufficiently close to one.

To see this, note that the right-hand side in Equation 32 is non-positive when $G_B = 1$; because, $G_B = 4\theta q_0^B(1-q_0^B) \le \theta$, meaning that the second-term in Equation 32 is non-negative. Therefore, full disclosure is always optimal when $G_B = 1$. Since the right-hand side in (32) is quadratic in G_B, the inequality holds for an interval including $G_B = 1$, implying full disclosure is optimal when G_B is sufficiently close to one. However, the bounds might be asymmetric.

Overall, since the second term in (32) is non-negative, Proposition 4 implies that full disclosure is optimal for a wider set of parameters compared to Proposition 1. The intuition is that the characterization in Proposition 4 accounts for non-linearities in investors' response to communication, so behavioral investors do not react as strongly as the corresponding linearized updating rule would suggests. Nonetheless, the implications remain qualitatively the same.

4.4 Communication with Ambiguity about True Probabilities

Our baseline model examines the optimal communication strategy when rational investors and the social planner have the correct prior beliefs. In practice, policymakers may hold inaccurate prior beliefs, and it can be difficult to identify which group of investors has the most accurate beliefs. This section explores the optimal design of a communication strategy in such an environment.

Specifically, suppose there are two groups of investors, who a priori assign probabilities q_0^1 and q_0^2 to $\tilde{D} = D_H$, where $q_0^1 \leq q_0^2$. They respectively comprise fractions b_1 and b_2 of the population, thus, $b_1 + b_2 = 1$. These groups respectively assign weights θ_1 and θ_2 to the SP's communication. The initial endowments and preferences are similar to the baseline model.

We analyze the optimal communication strategy for a closed set of "reasonable" prior probabilities for the event $\tilde{D} = D_H$, denoted by Q_0, rather than a single prior probability. For instance, Q_0 can include $[q_0^1, q_0^2]$, as suggested by Brunnermeier et al. (2014), who propose using the set of all linear combinations of the agents' prior beliefs for welfare analysis.

In Appendix A.6, we show that the expected utility associated with selecting communication strategy π is:

$$V(\pi; q_0) = \mathbb{E}_\pi[v(\hat{q})|q_0], \tag{33}$$

where

$$v(\hat{q}) = b_1(2\hat{q}q^1(\hat{q}) - q^1(\hat{q})^2) + b_2(2\hat{q}q^2(\hat{q}) - q^2(\hat{q})^2) - \gamma^{-1}Rkb_1b_2(q^1(\hat{q}) - q^2(\hat{q}))^2. \tag{34}$$

The interim utility function in (21) is a special case of (34), where the social planner agrees with one group of investors in their prior beliefs and belief formation. $q^i(\hat{q})$ represents the posterior belief of investors in group i when SP's message induces rational posterior $\hat{q}$. The posterior beliefs are obtained from (12):

$$\begin{gathered} q^i(\hat{q}) = q_0^i + \frac{G_i}{G(q_0)}(\hat{q} - q_0) \\ G(q_0) = 4q_0(1-q_0), \quad G_i = 4\theta_i q_0^i(1-q_0^i), \quad i = 1, 2. \end{gathered} \tag{35}$$

The details of the derivation are discussed in Appendix A.6. By substituting the posterior beliefs in (34) with the values in (35), we find that $v(\hat{q})$ is a quadratic function of $\hat{q}$. Therefore, v'' is either always positive or always negative, implying that for a given value of q_0, the optimal communication strategy is either full disclosure or no disclosure.

We are interested in the parameter values under which these communication strategies are also robust to the choice of $q_0 \in Q_0$. Definition 3 formally defines our notion of robustness. Proposition 5 specifies when a robust communication strategy exists.

Definition 3. *Communication strategy π is a "robust optimum" if it maximizes $V(\pi; q_0)$ for all values of $q_0 \in Q_0$.*

Proposition 5. *Let $\underline{G}$ and $\bar{G}$ be the smallest and largest values of $G(q_0)$ obtained by the values in Q_0.*[10] *Then,*
a) *Full disclosure is a robust optimum iff it is the optimal communication strategy when $G(q_0) = \underline{G}$. This happens when*

$$1 - b_1(1 - \frac{G_1}{\underline{G}})^2 - b_2(1 - \frac{G_2}{\underline{G}})^2 - \gamma^{-1}Rkb_1b_2(\frac{G_1}{\underline{G}} - \frac{G_2}{\underline{G}})^2 \geq 0. \tag{36}$$

b) *No disclosure is a robust optimum iff it is the optimal communication strategy when $G(q_0) = \bar{G}$. This happens when*

$$1 - b_1(1 - \frac{G_1}{\bar{G}})^2 - b_2(1 - \frac{G_2}{\bar{G}})^2 - \gamma^{-1}Rkb_1b_2(\frac{G_1}{\bar{G}} - \frac{G_2}{\bar{G}})^2 \leq 0. \tag{37}$$

Proposition 5 reveals that the optimal communication strategy depends on the range of possible rational reactions to new information (i.e., $[\underline{G}, \bar{G}]$) implied by the prior beliefs in Q_0. Note that $G(q_0)$ reflects the uncertainty associated with prior probability q_0. In practice, different values of q_0 may represent various interpretations of previous observations, leading

[10] Since Q_0 is closed, $\underline{G}$ and $\bar{G}$ are attained by some values of $q_0 \in Q_0$.

to different weightings of new information relative to old information, as captured by the reaction parameter $G(q_0)$. When $G(q_0)$ is small, past observations are very informative about the performance of the risky asset, so investors should rationally have a minimal reaction to new information. However, when $G(q_0)$ is large, new information substantially impacts rational beliefs.

Part (a) in Proposition 5 states that full disclosure is the robust optimum when investors' reaction is sufficiently homogeneous and not too far from the most conservative reaction that may be rational (captured by $\underline{G}$). This is because the risk-sharing concern is small in this case, and investors' over-reaction to the SP's communication is the primary potential source of welfare loss. Therefore, in this case, full disclosure is the robust optimum when investors' reactions do not deviate too much from the most conservative reaction to new information, corresponding to when the over-reaction risk is most likely.

Conversely, Part (b) in Proposition 5 shows that no disclosure is the robust optimum when investors' reactions are highly heterogeneous, causing the welfare loss from reduced risk-sharing to always outweigh the benefits of disclosure, maximized when the objective uncertainty is the largest. This is when new information has the most importance for learning about the risky asset's payoff.

Note that when $q_0^1, q_0^2 \in Q_0$, both groups of investors agree on the optimality of a robust communication strategy (if it exists), provided they ex-ante understand how they will deviate from Bayes' rule in response to new information. This holds despite their disagreement about the true probabilities. For instance, when investors recognize that they react very differently to new information, they prefer no public communication to limit the concentration of risk. This result confirms that the insights developed by the baseline model regarding the optimal communication strategy are robust to the choice of welfare criterion.

4.5 Communication with Monetary and Macroprudential policy

To fulfill their mandates, central banks use various policy tools alongside communication. They manage interest rates and regulate risk-taking behavior. For example, Basel III imposes limits on exposure to different risk types. This section explores how communication can serve as a complement to these policies.

Our model identifies two forms of inefficiency that might arise from communication when some investors do not form beliefs rationally. First, investment efficiency in risky assets might decline if those investors over-react to the communication. Second, risk-sharing among investors may deteriorate if there is significant heterogeneity in their responses to the communication.

In our model, the aggregate investment in the risky asset, X, depends on investors' average belief about the asset's payoff, in addition to the interest rate and the risk-aversion parameters:

$$X = \frac{\bar{\mathbb{E}}[\tilde{D}] - \gamma - Rl}{Rk + \gamma}. \tag{38}$$

To analyze the impact of communication on risk-sharing, we can examine the differences between each group's holding of the risky asset and the average, $z_b - \bar{z}$ (behavioral investors) and $z_R - \bar{z}$ (rational investors):

$$\begin{aligned} z_B - \bar{z} &= \frac{(1-b)(D_H - D_L)}{\gamma}(q^B - \hat{q}) \\ z_R - \bar{z} &= \frac{b(D_H - D_L)}{\gamma}(\hat{q} - q^B). \end{aligned} \tag{39}$$

Equation 38 implies that central banks can manage the aggregate investment in risky assets by controlling the risk-free interest rate, R. For example, during periods of investor optimism, raising interest rates helps limit additional investment in risky assets. However, the equations in (39) indicate that interest rate policies may be ineffective in reducing risk concentration, as they suggest that the interest rate has no impact on the cross-sectional distribution of the asset holdings.

Note that when a group of investors are highly exposed to an asset class, the underperformance of those assets could spill over into the broader economy through the wealth of those exposed investors. For instance, prior to the financial crisis of 2008–2009, the overexposure of some households and institutions to the housing market dragged the economy into a recession. In the presence of significant heterogeneity in investors' reactions to communication, communication is likely ineffective at addressing this over-concentration of risk. In fact, communication might increase disagreement among investors and lead them to take more extreme positions.

An effective solution can be to impose restrictions on investors' asset demand to make them less elastic, corresponding to raising γ in (39). Such macroprudential policies could address risk concentration, in line with Farhi and Werning (2020). Yet, equation (38) suggests that these policies might cause underinvestment in risky assets when investors' belief formation deviate minimally from rationality.

In sum, when investors' responses to communication are close to rational, communication can be a useful tool for addressing inefficient investment and may potentially substitute interest rate policies to some extent. However, our results also indicate that in the presence of strong behavioral biases and heterogeneity, macroprudential policies are more effective than communication in addressing risk concentration.

5 Conclusion

Asset overvaluations are often attributed to excessive risk-taking driven by distorted beliefs. For instance, Gennaioli and Shleifer (2018) argue that the housing boom that led to the financial crisis of 2008-2009 was due to investor over-optimism. In fact, a large body of empirical research has provided evidence from households' and professionals' expectation surveys (Greenwood and Shleifer, 2014b; Adam et al., 2017; Bordalo et al., 2020), as well as experimental settings (Armona et al., 2019; Afrouzi et al., 2023), that investors' belief formation might deviate from rational benchmarks.

We develop a model to shed light on how a social planner could manage investors' expectations. Our model accounts for the heterogeneity in investors' reactions to communication, as some investors might have incorrect prior beliefs and may not follow Bayes' rule in their belief formation. We find that full disclosure is optimal only when investors' belief updating is sufficiently homogeneous and closely aligns with the rational benchmark. Otherwise, no disclosure is optimal.

The sender in our model could represent an entity of the government, the private sector, a non-profit, or a central bank. While central banks communicate frequently about asset valuations, little guidance exists on when this communication is optimal from a social welfare perspective. The asset classes on which central banks, politicians, and other senders comment include real estate, but also crypto, equity, or the bond market.

In sum, while communication can be a powerful policy tool under some circumstances, senders should be aware of its potential pitfalls and limitations. Our findings also call for more empirical evidence on the heterogeneity in investors' reactions to communication and on how these reactions vary with investors' subjective uncertainty, parameters which are needed to design the optimal communication strategy.

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A Derivations and Proofs

A.1 Linearization of the Updating Rule

We derive the linear updating rule in Equation 12 by linearizing the following generalized version of the Bayes' rule for rational prior $q_0 = 0.5$:

$$\frac{q^B}{1-q^B} = \frac{q_0^B}{1-q_0^B}\left(\frac{\hat{q}}{1-\hat{q}}/\frac{q_0}{1-q_0}\right)^\theta \Rightarrow q^B = \frac{q_0^B \hat{q}^\theta q_0^{-\theta}}{q_0^B \hat{q}^\theta q_0^{-\theta} + (1-q_0^B)(1-\hat{q})^\theta(1-q_0)^{-\theta}}. \tag{A.1}$$

We drop the message argument from the expressions of the posterior beliefs to avoid clutter. Equation A.1 describes the investor's posterior as a function of the rational posterior. Note that θ reflects the weight that the investor assigns to the SP's communication. The rational benchmark corresponds to the case with $q_0^B = q_0$ and $\theta = 1$. To linearize the expression with respect to $\hat{q}$, we compute the derivative at $\hat{q} = q_0$:

$$\frac{\mathrm{d}q^B}{\mathrm{d}\hat{q}}|_{\hat{q}=q_0} = \theta\frac{q_0^B(1-q_0^B)}{q_0(1-q_0)}. \tag{A.2}$$

Therefore, the linearized expression for q^B is:

$$q^B = q_0^B + G_B(\hat{q} - q_0), \tag{A.3}$$

where

$$G_B = \theta\frac{q_0^B(1-q_0^B)}{q_0(1-q_0)}. \tag{A.4}$$

A.2 Proof of Proposition 1

According to Equation 22, $V(\pi)$ increases (decreases) with $\mathbb{E}_\pi[\hat{q}^2]$ when $\lambda(G_B-1)^2 < 1$ ($\lambda(G_B-1)^2 > 1$). Therefore, the SP should choose the communication strategy that maximizes $\mathbb{E}_\pi[\hat{q}^2]$ when $\lambda(G_B-1)^2 < 1$, and minimizes $\mathbb{E}_\pi[\hat{q}^2]$, otherwise. According to Jensen's inequality, given $\mathbb{E}_\pi[\hat{q}] = \frac{1}{2}$, $\mathbb{E}_\pi[\hat{q}^2]$, is maximized with full disclosure and minimized with no disclosure since $\hat{q}^2$ is a convex function of $\hat{q}$.

A.3 Proof of Proposition 2

Define $v_B(\hat{q}, q^B)$ and $v_R(\hat{q}, q^B)$ as the SP's interim expected utility from inducing posterior rational belief $\hat{q}$ and posterior belief q^B for behavioral investors, in the cases where only behavioral investors or only rational investors participate in the risky asset market, respectively. With some abuse of notation, let $v(\hat{q}, q^B)$ represent the expression in Equation 21. With a procedure similar to the one in Section 3, one can show that:

$$v_B(\hat{q}, q^B) = -c_0^B + \frac{\gamma + Rk}{\gamma b^{-1} + Rk}(\hat{q}^2 - (q^B - \hat{q})^2), \tag{A.5}$$

and

$$v_R(\hat{q}, q^B) = -c_0^R + \frac{\gamma + Rk}{\gamma(1-b)^{-1} + Rk}\hat{q}^2, \tag{A.6}$$

where c_0^B and c_0^R denote the relative costs associated with restricting market access to behavioral or rational investors, respectively, when $\hat{q} = q^B = 0$, relative to the benchmark case in which all investors participate and they are all marginal.

Let $v_{SL}(\hat{q}, q^B)$ be the SP's expected utility in the presence of the short-sale constraint. We show that

$$v_{SL}(\hat{q}, q^B) = \max\{v(\hat{q}, q^B), v_B(\hat{q}, q^B), v_R(\hat{q}, q^B)\}, \tag{A.7}$$

and $v_{SL}(\hat{q}, q^B(\hat{q})) = v(\hat{q}, q^B(\hat{q}))$, only for non-extreme signals. In fact, if $G_B \notin [1-\frac{1}{\sqrt{\lambda}}, 1+\frac{1}{\sqrt{\lambda}}]$ and there are extreme signals, then $v_{SL}(\hat{q}, q^B(\hat{q}))$ is not concave in $\hat{q}$. Therefore, some disclosure can be optimal; specifically, when the concavified function of $v_{SL}(\hat{q}, q^B(\hat{q}))$ is strictly greater than $v_{SL}(q_0, q^B(q_0))$ at $q_0 = \frac{1}{2}$.

To see why Equation A.7 holds, note that $v(\hat{q}, \hat{q}) > v_B(\hat{q}, \hat{q}), v_R(\hat{q}, \hat{q})$, and for a given $\hat{q}$, these functions are quadratic in q^B, where the quadratic coefficient is the largest in absolute value for v. Specifically, the coefficient is zero for v_R, and it is straightforward to show that $\lambda > \frac{\gamma+Rk}{\gamma b^{-1}+Rk}$.

These observations imply that for a given $\hat{q}$, $v(\hat{q}, q^B) \geq v_B(\hat{q}, q^B)$ for a convex set of values of q^B. The equality holds when the belief of behavioral investors is some $\bar{q}^B > \hat{q}$ such that rational investors choose $z_R = 0$ in the unconstrained model; because, both constrained and unconstrained models imply the same market outcome. In other words, when $q^B > \bar{q}^B$, namely the short-sale constraint binds for rational investors in the constrained model, then $v_{SR}(\hat{q}, q^B) = v_B(\hat{q}, q^B) > v(\hat{q}, q^B)$. With a similar argument, when q^B is so small that the short-sale constraint binds for behavioral investors, then $v_{SR}(\hat{q}, q^B) = v_R(\hat{q}, q^B) > v(\hat{q}, q^B)$.

Overall, we find that $v_{SR} > v$ iff the short-sale constraint binds for either behavioral or rational investors. Therefore, in the presence of extreme signals, v_{SR} is not necessarily concave, even if $G_B \notin [1 - \frac{1}{\sqrt{\lambda}}, 1 + \frac{1}{\sqrt{\lambda}}]$. However, in this case, v_{SR} is still concave in the range induced only by non-extreme signals; meaning that full disclosure of these signals is not optimal if there is more than one non-extreme signal.

A.4 Proof of Proposition 3

For Part (a), note that the credibility of full disclosure directly follows from Definition 2. Full disclosure satisfies the first condition since it always discloses s, and satisfies the second condition since ND is not communicated with a positive probability. Therefore, by appealing to Proposition 1, we find that full disclosure is optimal for any value of deviation cost c when $G_B \in [1 - \frac{1}{\sqrt{\lambda}}, 1 + \frac{1}{\sqrt{\lambda}}]$.

For Part (b), we first find how large the deviation cost should be so that the SP does not disclose the signal realization ex-post. In other words, we find the maximum gain from deviation to disclosing, i.e.,

$$\bar{c} = \max_{s \in S} \ \mathbb{E}[\tilde{V}(s)|s] - \mathbb{E}[\tilde{V}(ND)|s] \tag{A.8}$$

By taking expectation of Equation 16 with respect to signal s, and after some simplifications, we find that disclosing the signal increases the expected surplus by:

$$\mathbb{E}[\tilde{V}(s) - \tilde{V}(m)|s] = (q(s) - \hat{q}(m))^2 - \lambda[\Delta q(s)^2 - \Delta q(m)^2] - 2b\Delta q(m)(q(s) - \hat{q}(m)), \tag{A.9}$$

where

$$\Delta q(s) = q^B(s) - q(s), \qquad \Delta q(m) = q^B(m) - \hat{q}(m). \tag{A.10}$$

When the SP follows no disclosure, the communication will be uninformative as it always sends ND. Therefore,

$$\begin{aligned}\mathbb{E}[\tilde{V}(s) - \tilde{V}(ND)|s] &= (q(s) - \frac{1}{2})^2 - \lambda[(q_0^B - \frac{1}{2} + (G_B - 1)(q(s) - \frac{1}{2}))^2 - (q_0^B - \frac{1}{2})^2] \\ &\quad - 2b(q_0^B - \frac{1}{2})(q(s) - \frac{1}{2}) \\ &= (1 - \lambda(G_B - 1)^2)(q(s) - \frac{1}{2})^2 - 2(\lambda(G_B - 1) + b)(q_0^B - \frac{1}{2})(q(s) - \frac{1}{2}).\end{aligned} \tag{A.11}$$

Note that following the assumption in Part b, $1 - \lambda(G_B - 1)^2$ is negative. Therefore, the expression in (A.11) obtains its maximum when $q(s) = q^* = \frac{1}{2} - \frac{\lambda(G_B-1)+b}{\lambda(G_B-1)^2-1}(q_0^B - \frac{1}{2})$, which is

$$\bar{c} = \frac{(\lambda(G_B - 1) + b)^2}{\lambda(G_B - 1)^2 - 1})(q_0^B - \frac{1}{2})^2. \tag{A.12}$$

Therefore, no disclosure is optimal when $c \geq \bar{c}$.

For Part (b.2), we guess and verify that the SP discloses an intermediate range of signals. In fact, suppose the SP discloses signals with $q(s) \in [\underline{q}, \bar{q}]$, and no disclose induces rational posterior $\hat{q}(ND)$. By employing Equation A.9, and after some simplifications, we find that the expected surplus gained from disclosing the signal is:

$$\begin{aligned}\mathbb{E}[\tilde{V}(s) - \tilde{V}(ND)|s] =&(\lambda(G_B - 1)^2 - 1)[(\hat{q}(ND) - \frac{1}{2})^2 - (q(s) - \frac{1}{2})^2] \\ &+2\big\{(1 + b(G_B - 1))(\hat{q}(ND) - \frac{1}{2}) + (b + \lambda(G_B - 1)(q_0^B - \frac{1}{2}))\big\}(\hat{q}(ND) - q(s)).\end{aligned} \tag{A.13}$$

Given $\hat{q}(ND)$, $\mathbb{E}[\tilde{V}(s) - \tilde{V}(ND)|s] - c$ is positive for an intermediate range of values of $q(s)$. In particular, if $\underline{q} < \bar{q}$ are the solutions to this quadratic equation, then the SP can commit not to disclose only signals s such that $q(s) \notin [\underline{q}, \bar{q}]$. This proves the structure of the optimal communication strategy described in Part (b.2).

Furthermore, note that we should have $\hat{q}(ND) = \mathbb{E}[q(s)|q(s) \notin [\underline{q}, \bar{q}]]$. This implies a fixed-point problem. One can construct examples of signal structures that this fixed-point problem has no solution when c is sufficiently close to zero.

A.5 Proof of Proposition 4

The goal is to find the communication strategy that maximizes $V(\pi) = \mathbb{E}_\pi[v(\hat{q})]$, as defined in (20). The main difference from the baseline model is that q^B is obtained from the non-linear specification in (30). When a_{max} is sufficiently small, the convexity or concavity of the $v(\cdot)$ around $\hat{q} = \frac{1}{2}$ informs the optimal of full disclosure or no disclosure.

One can show that:

$$\frac{1}{2}v''(\frac{1}{2}) = 1 - \lambda(q^{B\prime}(\frac{1}{2}) - 1)^2 - \lambda q^{B\prime\prime}(\frac{1}{2})(q_0^B - \frac{1}{2}). \tag{A.14}$$

Note that $q^{B\prime}(\frac{1}{2}) = G_B$. One can also show that $\lambda q^{B\prime\prime}(\frac{1}{2})(q_0^B - \frac{1}{2}) = 2G_B(\theta - G_B)$. Therefore, $v''(\frac{1}{2})$ is positive iff:

$$1 - \lambda(G_B - 1)^2 - 2G_B(\theta - G_B) > 0. \tag{A.15}$$

A.6 Derivations in Section 4.4

We can obtain the total surplus induced by message m similar to Equation 16:

$$\begin{aligned}\tilde{V}(m) = &\, c_0 + c_1\mathbb{I}_{\{\tilde{D}=D_H\}} \\ &+ c_2\Big[2b_1(\mathbb{I}_{\{\tilde{D}=D_H\}}q^1(m) - q^1(m)^2) + 2b_2(\mathbb{I}_{\{\tilde{D}=D_H\}}q^2(m) - q^2(m)^2) \\ &- \gamma^{-1}Rkb_1b_2(q^1(m) - q^2(m))^2\Big],\end{aligned} \tag{A.16}$$

where c_0, c_1, and c_2 are defined in (18). $q^1(m)$ and $q^2(m)$ are the posterior beliefs induced by message m. By taking expectation of the total surplus with respect to a candidate rational prior $q_0 \in Q_0$, we find:

$$\begin{aligned}\mathbb{E}[\tilde{V}] = &\, c_0 + c_1 q_0 \\ &+ c_2\mathbb{E}[2b_1(\hat{q}(m)q^1(\hat{q}) - q^1(m)^2) + 2b_2(\hat{q}(m)q^2(\hat{q}) - q^2(m)^2) - \gamma^{-1}Rkb_1b_2(q^1(m) - q^2(m))^2].\end{aligned} \tag{A.17}$$

Therefore, by dropping the constant and multiplier terms, we derive the interim utility presented in Equation 34.

To obtain the posterior beliefs, we employ the linear updating rule in (12). Note that (12) is a linearization of the generalized updating rule (30) with respect to changes in the beliefs of a rational investor with a flat prior beliefs about the risky asset's payoff. Let $\hat{q}_{null}(m)$ be the posterior belief of this investor in response to message m. Therefore, investors' posteriors beliefs are:

$$q^i(m) = q_0^i + G_i(\hat{q}_{null}(m) - \frac{1}{2}), \quad i = 1, 2. \tag{A.18}$$

Likewise, using the linear updating rule, the posterior belief of a rational investor with a prior belief $q_0 \in Q_0$ is:

$$\hat{q}(m) = q_0 + G(q_0)(\hat{q}_{null}(m) - \frac{1}{2}), \quad i = 1, 2. \tag{A.19}$$

The updating rule in (35) is derived from combining Equations A.18 and A.19. Note that a similar updating would have been obtained if we linearized Equation 30 with respect to changes in the belief of a rational investor with a prior belief of q_0, instead.

A.7 Proof of Proposition 5

a) For given rational prior q_0, full disclosure is optimal if the corresponding interim utility function, $v(\hat{q})$, is convex. We show what $v(\cdot)$ is convex for all values of $q_0 \in Q_0$ when the condition in (36) holds.

Note that $v(\cdot)$ is quadratic in $\hat{q}$ because q^1 and q^2 are linear in $\hat{q}$. Therefore, to demonstrate the convexity of $v(\cdot)$, we only need to show that the coefficient in front of the quadratic term, which we denote by $h(q_0)$, is positive. That is,

$$\begin{aligned} h(q_0) &\equiv b_1[\frac{2G_1}{G(q_0)} - (\frac{G_1}{G(q_0)})^2] + b_2[\frac{2G_2}{G(q_0)} - (\frac{G_2}{G(q_0)})^2] - \gamma^{-1}Rkb_1b_2(\frac{G_1 - G_2}{G(q_0)})^2 \\ &= b_1(1 - \frac{G_1}{G(q_0)})^2 + b_2(1 - \frac{G_2}{G(q_0)})^2 - \gamma^{-1}Rkb_1b_2(\frac{G_1 - G_2}{G(q_0)})^2 \geq 0. \end{aligned} \tag{A.20}$$

Note that

$$h(q_0)G(q_0)^2 = 2(b_1G_1 + b_2G_2)G(q_0) - b_1G_1^2 - b_2G_2^2 - \gamma^{-1}Rkb_1b_2(G_1 - G_2)^2. \tag{A.21}$$

We see that $h(q_0)G(q_0)^2$ is increasing in $G(q_0)$. Suppose q_{G-min} is a value in Q_0 such that $G(q_{G-min}) = \underline{G}$. Condition 36 implies that $h(q_{G-min}) \geq 0$. Therefore, $h(q_0)G(q_0)^2$, and consequently $h(q_0)$, are also positive for all the other values of $q_0 \in Q_0$. It is clear that the condition is necessary as well; because if $h(q_{G-min}) < 0$, then $v(\cdot)$ is concave when $q_0 = q_{G-min}$, meaning that full disclosure is not optimal for this value of q_0.

b) Similar to the previous part, we only need to show that $v(\cdot)$ is concave for all values of $q_0 \in Q_0$. This is equivalent to $h(q_0)G(q_0)^2$ to be negative for all values of $q_0 \in Q_0$. Let q_{G-max} be a value in Q_0 such that $G(q_{G-max}) = \bar{G}$. Condition 37 implies that $h(q_{G-max}) < 0$. Therefore, the monotonicity of $h(q_0)G(q_0)^2$ implies that $h(q_0)G(q_0)^2$, and correspondingly $h(q_0)$, are negative for all values of $q_0 \in Q_0$.

B Central Bank Communication in Practice

Central banks closely monitor asset valuations due to their connection to the real economy, and their price stability mandate. Many central banks also have a financial stability mandate, under which they monitor and communicate about house prices and financial markets.[11] They share their views about asset prices in various forms, as we review in this section.

B.1 Speeches of the Federal Reserve System

Central banks frequently comment on the status of in particular real estate markets in their speeches, especially during turbulent times.

For instance, on February 11, 2011, Governor Sarah Bloom Raskin shared her negative outlook by expressing, "With a pipeline full of distressed properties, the unfortunate consensus is that we should expect even more downward pressure on house prices. Potential buyers seem inclined to wait and see if they can get a better buy in the future."[12] Or, on March 8, 2013, Governor Elizabeth Duke shared a positive outlook about the recovery in the housing market: "In conclusion, I am optimistic that the housing recovery will continue to take root and expand. While low mortgage rates are helping support the recovery, I believe it will be the pent-up demand of household formation unleashed by improving economic conditions that will provide real momentum."[13]

Figure B.1 plots the number of speeches by a chairman, vice chairman, or governor of the Federal Reserve System that contained communication about future real estate prices. The figure suggests that the frequency of such communication varies with time.

B.2 Financial Stability Reports Globally

Central bank communication about asset prices is frequently found in financial stability reports (FSRs) which are disseminated to the general public typically once or twice per year. The reports aim to identify vulnerabilities in financial markets, such as overvaluation in asset markets, indicating the possibility of excessive risk-taking by market participants. Overvaluations are viewed as particularly concerning when they are accompanied by excessive leverage, and real estate markets are monitored closely under financial stability mandates (Adrian et al., 2015).

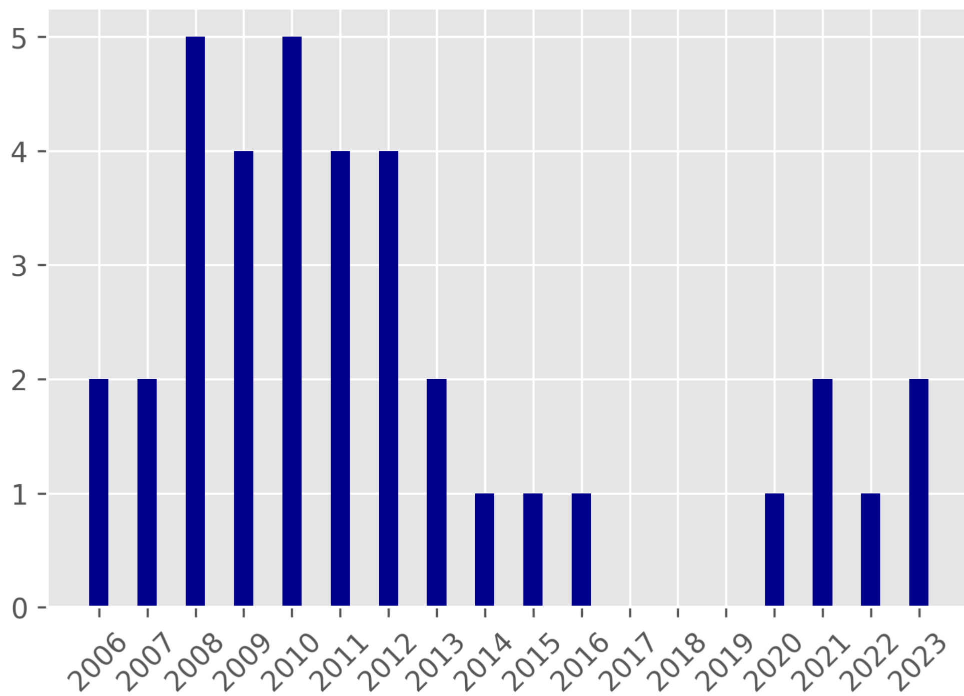
Figure B.2 provides an overview of real estate communication in financial stability reports (FSRs) across major central banks. The main take-away from Figure B.2 is that central banks exert discretion about when to communicate an outlook for real estate prices. Central banks issue FSRs at regular time intervals (semi-annually, or annually), but exert discretion on whether the FSR contains an outlook on real estate prices. This discretion can take three

[11] Our analysis applies not only to central banks, but, in general, to any entity that is aiming to manage asset return expectations through communication. For instance, the financial stability mandate is often under the shared responsibility of the central bank and the financial supervisory authority who are represented in an oversight body, to which our findings can be applied as well.

[12] `https://www.federalreserve.gov/newsevents/speech/raskin20110211a.htm`

[13] https://www.federalreserve.gov/newsevents/speech/duke20130308a.htm

Figure B.1: Real Estate Communication in Federal Reserve Speeches



Note: This figure displays the number of speeches by year in which a top official at the Federal Reserve System (chairmen, vice chairmen, or governors) shared their outlook on the real estate market. The figure is based on manual readings of all speeches over the given time interval.

forms. In some cases, central banks choose not to mention real estate markets in a given FSR. In other cases, real estate markets are discussed, but no directional outlook is given. Central banks can be vague for instance by only describing past developments without offering an assessment of current overvaluations or guidance about the future, or by emphasizing the two-sided nature of the observed developments (e.g. rising house prices, but justified by fundamentals). In rare cases, central banks choose not to issue an FSR at all for a given time period. For instance, during the global financial crisis in 2009, Deutsche Bundesbank and Banco Central do Brasil did not release an FSR.

B.3 Media Coverage of Central Bank Communication about Asset Prices

While some investors may follow central banks' communication directly, others may be exposed indirectly via media reports. These reports tend to condense the communication into crisp messages. In Appendix B, we present recent examples of these media reports, quoting their title and a key statement within the reports. Most of these reports focus on the housing market, but they also cover statements about "stretched valuations" in financial markets, and about crypto assets.

We present a simple model to understand how communication about asset prices should

Figure B.2: Real Estate Communication in Financial Stability Reports

Note: This figure provides an overview of real estate communication in financial stability reports (FSRs) across major central banks. The figure is based on manual readings of all FSRs over the the given time interval. The central banks are ordered by when they started publishing FSRs. Our figure covers Bank of England, Banco Central do Brasil, Deutsche Bundesbank, European Central Bank, Bank of Japan, People's Bank of China, Reserve Bank of India, and Federal Reserve Board. All publication dates of FSRs are marked with large grey dots. Rectangles indicate omissions where no FSR was issued at the regular interval. The colored crosses indicate FSRs which contain directional communication about real estate markets. We categorize communication as directional if the outlook is either positive or negative. By contrast, neutral communication, or communication emphasizing both positive and negative aspects of the outlook is categorized as not directional. FSRs which contain communication about real estate prices, which is however not directional, are marked with small colored dots.

be conducted optimally. While our model is general, real estate prices play a particularly prominent role in central banks' communication practice.

B.4 Examples of central bank communication captured by media

The Fed admits a sharp home price decline is possible Fed Chair Jerome Powell was asked at the FOMC press conference in September to clarify what he meant when he said a few months earlier the U.S. housing market would "reset." His response? We've entered into a "difficult [housing] correction" that will see the U.S. housing market transition to a more "balanced" market for buyers and sellers alike. *Fortune, October 7, 2022*

Federal Reserve issues warning about brewing U.S. housing bubble Those dynamics have caused some observers to question whether the U.S. is repeating the housing bubble of the early 2000s, which led to a painful housing crash in 2006 and the Great Recession the following year. The answer, warns the Federal Reserve Bank of Dallas, is that the property market is showing "signs of a brewing U.S. housing bubble." *CBS News, April 5, 2022*

ECB warns a house price correction is looming as interest rates rise Predicting that asset prices could fall further if economic growth continues to weaken or inflation rises faster than expected, the ECB said a sharp increase in rates could cause a "reversal" in eurozone house prices, which it estimated were already about 15 per cent overvalued, when weighed against overall economic output and rents. *Financial Times, May 25, 2022*

European Central Bank warns of bubbles in property and financial markets The European Central Bank warned of stretched valuations in many asset markets, as the region continues to recover from the coronavirus pandemic on the back of ultra-low interest rates and massive stimulus measures. *CNBC, November 17, 2021*

German banks vulnerable to housing bubble; should build buffers: Bundesbank "Price exaggerations in the residential real estate market have tended to increase further," the bank said. "Bundesbank estimates put them at between 10% and 30% in Germany in 2020." *Reuters, November 25, 2021*

Crypto assets are 'worth nothing,' says ECB's Christine Lagarde "I have said all along the crypto assets are highly speculative, very risky assets," Lagarde told Dutch television show College Tour in an interview to be aired on Sunday. "My very humble assessment is that it is worth nothing. It is based on nothing, there is no underlying assets to act as an anchor of safety." *Politico, May 21, 2022*