

Conditional Expected Returns on Individual Stocks with and without Intertemporal Hedging

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Abstract

We derive tractable lower and upper bounds on the conditional expected excess return of an individual stock in a dynamic, multi-period economy with portfolio rebalancing. The bounds depend on higher-order risk-neutral joint moments of market and stock returns and are implementable ex ante using index and single-name option prices. Our framework nests a no-intertemporal-hedging benchmark, isolating and quantifying the incremental role of hedging motives. Empirically, the bounds are tight and outperform leading one-period benchmarks in out-of-sample forecasts of stock returns. The significant gap between hedging and non-hedging bounds highlights the role of intertemporal hedging in expected returns on individual stocks.

JEL Classification: G11, G12

Keywords: conditional expected return, intertemporal hedging, option-implied moments, higher-order risk preferences, cross-sectional stock returns

1 Introduction

Expected returns on individual stocks are fundamental to asset pricing, capital allocation, and portfolio choice. Given this central role, a growing literature has sought reliable and implementable proxies. Recent advances, such as [Martin and Wagner \(2019\)](#), [Kadan and Tang \(2020\)](#), and [Chabi-Yo, Dim, and Vilkov \(2023\)](#), show that option prices can be used to bound or approximate expected returns, highlighting the value of forward-looking information embedded in option markets. These approaches, however, rest on simplifying assumptions,¹ most notably their confinement to a static, one-period setting. While analytically convenient, this restriction abstracts from time-varying economic conditions and portfolio rebalancing incentives, yielding only a partial characterization of equilibrium risk premia.

We address this limitation by introducing a dynamic multi-period economy in which investors rebalance portfolios as state variables evolve over time. [Merton \(1973\)](#) shows that expected excess returns compensate investors not only for contemporaneous market risk but also for exposure to innovations in state variables that shift the future investment opportunity set, because investors hedge these opportunity-set risks by tilting portfolios toward assets that perform well in adverse states. Subsequent work highlights the significance of these intertemporal hedging motives in economies with time-varying state variables (e.g., [Brennan, Schwartz, and Lagnado, 1997](#); [Campbell and Viceira, 1999](#); [Wachter, 2002, 2013](#); [Campbell, Chan, and Viceira, 2003](#); [Liu, 2007](#)). Incorporating these dynamics is therefore crucial for assessing expected returns. We build on this insight to derive option-implied bounds on stock-level equity premia in a multi-period setting with intertemporal hedging.

Our framework has several advantages. First, it does not impose a specific functional form for investors' utility. Under no-arbitrage and mild regularity conditions on the inverse marginal utility, the stochastic discount factor (SDF) can be characterized in terms of market returns by applying a Taylor expansion to the inverse marginal utility and invoking the economic principle that the product of risk prices and factor exposures must be positive. This design allows for broad applicability, accommodating preference specifications beyond

¹Specifically, [Martin and Wagner \(2019\)](#) rely on a log-utility benchmark; [Kadan and Tang \(2020\)](#) approximate marginal utility using a first-order expansion; and [Chabi-Yo, Dim, and Vilkov \(2023\)](#) depend on realized betas for parameter calibration, despite employing higher-order expansions of the inverse marginal utility to move beyond the first-order simplification.

constant relative risk aversion (CRRA), constant absolute risk aversion (CARA), or Epstein-Zin recursive utility.

Second, our framework delivers tractable lower and upper bounds on conditional expected returns for individual stocks using only option prices, without relying on historical realized returns or accounting-based measures. The bounds depend on higher-order risk-neutral joint moments between market and stock returns: the lower bound uses untruncated moments, while the upper bound uses truncated, tail-sensitive moments. We obtain implementable approximations from a Taylor expansion of the risk-neutral conditional expectation of stock returns given market returns, matching the expansion coefficients to higher-order risk-neutral moments of the stock and market marginal distributions. Because the inputs come from current index and single-name option prices, our bounds are forward-looking and implementable in real time.

Third, our framework nests the benchmark economy without intertemporal hedging, allowing us to quantify the incremental role of hedging motives. In the one-period benchmark, the SDF is determined entirely by the current market return and its higher-order terms. In the multi-period setting, it additionally depends on risk-neutral moments of future market returns and their interactions, capturing forward-looking opportunity-set risk. This nesting delivers a direct within-framework comparison of equity premia with and without hedging, enabling us to assess how the pure effect of hedging varies over time and across stocks.

We test the framework empirically using S&P 500 stocks at forecast horizons ranging from one month to one year. In the static one-period setting, the average annualized lower (upper) bounds range from 8.1% (13.9%) at the one-month horizon to 9.4% (16.4%) at the one-year horizon. The bounds predict future returns, and a long-short strategy formed on bound quintiles earns 12.5% to 17.2% across horizons. Relative to [Martin and Wagner \(2019\)](#), [Kadan and Tang \(2020\)](#), and [Chabi-Yo, Dim, and Vilkov \(2023\)](#), our bounds add predictive power in double sorts and reduce out-of-sample prediction errors even within the static setting.

We then extend the analysis to a dynamic multi-period setting with intertemporal hedging. Incorporating hedging raises the bounds, particularly when the investment horizon extends beyond the forecast horizon. For example, at the one-month forecast horizon, increasing the investment horizon from three months to two years raises the average annual lower (upper) bound from 9.1% (17.9%) to 15.7% (26.0%). The dynamic hedging bounds

remain strongly correlated with future returns and improve forecast accuracy relative to both existing benchmarks and our static bounds.

Figure 1 illustrates time series of the static lower and upper bounds (LB, UB) and their dynamic counterparts with hedging (HLB, HUB) for Apple and JP Morgan at the one-month forecast horizon. The hedging bounds are higher throughout, with the gap widening during stress episodes such as the global financial crisis, indicating that intertemporal hedging amplifies expected returns when risk premia are elevated. The right-hand panels show the share of the bounds attributable purely to hedging components, which is sizable and time-varying and often exceeds one-half of the total bound. Overall, ignoring intertemporal hedging understates both the level and the variation of expected returns.

[Insert Figure 1 around here]

Beyond these baseline results, we decompose the intertemporal hedging component and examine its drivers. First, we attribute hedging contributions to terms linked to forward conditional variance, skewness, and kurtosis of future market returns, and find that variance- and skewness-related components dominate, with their relative importance varying with investors' horizon structure. Second, following [Campbell et al. \(2018\)](#), we relate hedging contributions to cash-flow, discount-rate, and volatility news betas. Although these betas are significantly associated with hedging contributions, they jointly explain only a small fraction, suggesting that our option-implied hedging component reflects additional intertemporal risks beyond the conventional news decomposition.

This study contributes to the literature on option-implied equity premia. For the market portfolio, [Martin \(2017\)](#), [Schneider and Trojani \(2019\)](#), [Chabi-Yo and Loudis \(2020\)](#), and [Tetlock, McCoy, and Shah \(2024\)](#) use option-implied moments to deliver bounds or point estimates of the equity premium. At the individual stock level, [Martin and Wagner \(2019\)](#) and [Kadan and Tang \(2020\)](#) develop a variance-based bounds, and [Chabi-Yo, Dim, and Vilkov \(2023\)](#) generalize these results using the entire risk-neutral distribution. These approaches are static, one-period frameworks. We extend option-implied identification of stock-level expected returns to a dynamic multi-period setting that incorporates intertemporal hedging and quantifies its impact on risk premia.

Our study also connects to the intertemporal asset pricing literature. [Samuelson \(1969\)](#) and [Merton \(1969, 1971\)](#) develop dynamic consumption–portfolio choice under uncertainty,

and [Merton \(1973\)](#) formalizes the ICAPM, linking expected returns to state variables that govern investment opportunities. Subsequent work derives portfolio implications under time-varying expected returns, interest rates, and volatility, typically under CRRA, CARA, or Epstein–Zin preferences (e.g., [Kim and Omberg, 1996](#); [Campbell and Viceira, 1999](#); [Wachter, 2002](#); [Brennan and Xia, 2002](#); [Chacko and Viceira, 2005](#)). More recent studies emphasize disaster risk and volatility as key drivers of risk premia ([Wachter 2013](#); [Tsai and Wachter 2015](#); [Campbell et al. 2018](#)). We build on this tradition by bringing option-implied identification to a multi-period setting and allowing intertemporal risks to operate through higher moments (skewness and kurtosis), not only volatility.

For the cross-section of stock returns, this literature uses the Campbell–Shiller decomposition ([Campbell and Shiller, 1988](#)) to separate cash-flow news and discount-rate news. [Campbell and Vuolteenaho \(2004\)](#) links higher premia to cash-flow “bad beta” relative to discount-rate “good beta,” while [Campbell, Polk, and Vuolteenaho \(2010\)](#) and [Campbell, Giglio, and Polk \(2013\)](#) connect these channels to value–growth differences and recessions. [Chabi-Yo, Gonçalves, and Loudis \(2025\)](#) translate ICAPM state variables into tradable factors priced in the cross-section. We complement this line of work by making intertemporal hedging empirically tractable at the individual-stock level using option prices, allowing its role to be assessed over time and across firms.

Our empirical finding that intertemporal hedging raises expected returns contrasts with results under CRRA, CARA, or Epstein–Zin preferences, where hedging often lowers the equity premium. We therefore study the CRRA case within our framework. Under CRRA utility, we obtain an exact option-based expression for expected excess returns with and without intertemporal hedging and find a negative hedging component; the CRRA-implied proxy underperforms our utility-free bounds. [Chabi-Yo, Gourier, and Langlois \(2025\)](#) shows that, without imposing a utility form, empirically plausible conditions can imply a multi-period premium above the one-period bound when bad states combine low returns with higher future volatility. Motivated by this mechanism, we avoid functional-form restrictions and use economically reasonable calibrations to discipline the sign and magnitude of intertemporal hedging effects on stock-level expected excess returns.

The remainder of the paper is organized as follows. Section 2 develops the theoretical framework and derives lower and upper bounds on conditional expected returns for individual stocks in both one-period and multi-period settings. Section 3 details the construction of the

empirical counterparts of the bounds from option price data. Section 4 presents the empirical analysis, evaluating their predictive performance against existing benchmarks. Section 5 concludes.

2 Theory on Individual Stocks

Consider a static economy over the interval from time t to T_1 and let $R_{i,t \rightarrow T_1}$ denote the return on the risky asset over this period. We define the excess return on the stock i as $\mathcal{R}_{i,t \rightarrow T_1} = R_{i,t \rightarrow T_1} - R_{f,t \rightarrow T_1}$ and the excess market return as $\mathcal{R}_{M,t \rightarrow T_1} = R_{M,t \rightarrow T_1} - R_{f,t \rightarrow T_1}$. Under the no-arbitrage condition, it follows that

$$\mathbb{E}_t(\mathcal{R}_{i,t \rightarrow T_1}) = \text{COV}_t^* \left(\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}}, \mathcal{R}_{i,t \rightarrow T_1} \right), \quad (2.1)$$

where $m_{t \rightarrow T_1}$ is the Stochastic Discount Factor (SDF) from time t to time T_1 . $\text{COV}_t^*(\cdot, \cdot)$ denotes the conditional covariance under the risk-neutral measure. Throughout, a superscript $*$ indicates quantities evaluated under the risk-neutral measure. In the following sections, we characterize the SDF without imposing a functional form on utility. We then use both the SDF and Eq.(2.1) to derive bounds on the expected excess returns of individual stocks.

2.1 Expected Excess Return without Intertemporal Hedging

We assume that the representative agent maximizes expected utility $u[\cdot]$ at time t

$$\max_{\omega_t} \mathbb{E}_t u[W_{t \rightarrow T_1}]$$

subject to

$$W_{t \rightarrow T_1} = W_t (R_{f,t \rightarrow T_1} + \omega_t^\top (\mathbf{R}_{t \rightarrow T_1} - R_{f,t \rightarrow T_1} \mathbf{1})),$$

where $\mathbf{R}_{t \rightarrow T_1}$ is a vector of returns of risky assets. Since there is no rebalancing between the dates t and T_1 , intertemporal hedging motives are absent. The SDF implied by the first-order conditions is proportional to $u' [W_t R_{M,t \rightarrow T_1}]$, where $R_{M,t \rightarrow T_1}$ is used as a proxy for the return on aggregate wealth. The inverse SDF is proportional to the inverse marginal

utility and can be written as:

$$\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}} = \frac{v_{t \rightarrow T_1}}{\mathbb{E}_t^* v_{t \rightarrow T_1}} \quad (2.2)$$

with

$$v_{t \rightarrow T_1} = \frac{u' [W_t R_{f,t \rightarrow T_1}]}{u' [W_t R_{M,t \rightarrow T_1}]} \quad (2.3)$$

Notice that $v_{t \rightarrow T_1} > 0$. Therefore, $\mathbb{E}_t^* v_{t \rightarrow T_1} > 0$. We next specify preference parameters:

$$\begin{aligned} \tau_t &= -\frac{u' [W_t R_{f,t \rightarrow T_1}]}{W_t R_{f,t \rightarrow T_1} u'' [W_t R_{f,t \rightarrow T_1}]}, & \kappa_t &= \frac{\frac{1}{3!} u'''' [W_t R_{f,t \rightarrow T_1}] (u' [W_t R_{f,t \rightarrow T_1}])^2}{(u'' [W_t R_{f,t \rightarrow T_1}])^3}, \\ \rho_t &= \frac{\frac{1}{2} u''' [W_t R_{f,t \rightarrow T_1}] u' [W_t R_{f,t \rightarrow T_1}]}{(u'' [W_t R_{f,t \rightarrow T_1}])^2}, & \xi_t &= \frac{\frac{1}{4!} u''''' [W_t R_{f,t \rightarrow T_1}] (u' [W_t R_{f,t \rightarrow T_1}])^3}{(u'' [W_t R_{f,t \rightarrow T_1}])^4}. \end{aligned} \quad (2.4)$$

The parameter τ_t denotes risk tolerance, with $1/\tau_t$ corresponding to relative risk aversion. The skewness preference ρ_t scales the third derivative of utility and captures the agent's prudence, namely the tendency to increase precautionary savings in response to risk in future income (Kimball, 1990). When $\rho_t > 1$, the agent is prudent and increases savings when facing income risk (Gollier, 2001). The parameter κ_t measures normalized kurtosis aversion, capturing sensitivity to fourth-moment risk in wealth. Although less studied than ρ_t (prudence), κ_t enters third-order Taylor expansions of inverse marginal utility and captures temperance, i.e., the agent's aversion to combining independent risks. Temperance is widely used in asset allocation, stochastic dominance, and higher-order risk premium models. Finally, ξ_t captures the fifth derivative of utility and reflects a higher-order "hyper-temperance," i.e., sensitivity to even finer features of risk, such as variation in how risks interact (e.g., interactions between skewness and kurtosis risks). Recent evidence shows that higher-order risk preferences such as prudence and temperance are strongly related to financial choices, and that relying on risk tolerance alone can lead to misleading inferences (Schneider and Sutter, 2026).

Under no-arbitrage conditions and assuming that regularity conditions on $v_{t \rightarrow T_1}$ are satisfied, we show in Online Appendix OA.1 that the inverse of the one-period SDF is given by (2.2), where a $\mathcal{K}^{\text{th}}$ order Taylor expansion-series of $v_{t \rightarrow T_1}$ around $\mathcal{R}_{M,t \rightarrow T_1} = 0$ yields

$$v_{t \rightarrow T_1} = 1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathcal{R}_{M,t \rightarrow T_1}^j \quad (2.5)$$

Specifically, the first four coefficients are

$$a_{1,t} = \frac{1}{\tau_t}, \quad a_{2,t} = \frac{1 - \rho_t}{\tau_t^2}, \quad a_{3,t} = \frac{1 - 2\rho_t + \kappa_t}{\tau_t^3}, \quad a_{4,t} = \frac{1 - 3\rho_t + \rho_t^2 + 2\kappa_t - \xi_t}{\tau_t^4}. \quad (2.6)$$

For interpretation, applying the Carr and Madan (2001) spanning formula to $\mathcal{R}_{M,t \rightarrow T_1}^j$, $j > 1$, we show in Online Appendix (OA.2.1) that the inverse SDF (2.2) can be expressed as a portfolio payoff from holding the market return and a continuum of call and put options maturing at T_1 .

To sign $a_{j,t}$ and thus derive the implied restrictions on the preference parameters $1/\tau_t$, ρ_t , κ_t and ξ_t , we appeal to economic theory. Specifically, theory implies that

$$\underbrace{\frac{a_{j,t}}{\mathcal{R}_{f,t \rightarrow T_1}^j \mathbb{E}_t^* v_{t \rightarrow T_1}}}_{\text{Price of Risk}} \times \underbrace{\text{COV}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^j, \mathcal{R}_{M,t \rightarrow T_1})}_{\text{Quantity of Risk}} > 0. \quad (2.7)$$

To see why this inequality holds, set $i = M$ in (2.1) and substitute (2.5) into (2.1), which yields a sum of risk premium terms. We rely on the following intuition: if the quantity of risk is positive, investors require a premium to hold an asset with positive exposure to the j -th factor, so the product of the quantity and the price of risk is positive. If the quantity of risk is negative, the asset has negative exposure to the j -th factor; investors still require compensation, implying a negative price of risk so that the premium remains positive. Hence, we posit that each risk premium, defined as the price of risk times exposure to the priced factor, is positive.

Given that $v_{t \rightarrow T_1} > 0$, it follows that $\mathbb{E}_t^* v_{t \rightarrow T_1} > 0$. Empirically, the moment conditions $\mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^{2(j+1)} \geq 0$ and $\mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^{2j+1} \leq 0$ are supported by the data. As a result, inequality (2.7) implies that $a_{2j+1,t} \geq 0$ and $a_{2j,t} \leq 0$, respectively. The sign of $a_{j,t}$ imposes the following restrictions on the parameters:

$$\frac{1}{\tau_t} > 0, \quad \rho_t > 1, \quad \kappa_t > 2\rho_t - 1, \quad \xi_t > (1 - 2\rho_t + \kappa_t) - \rho_t(1 - \rho_t) + \kappa_t. \quad (2.8)$$

Next, replacing the inverse SDF (2.2) in (2.1) produces a sum of risk premia of the form

$$\underbrace{\frac{a_{j,t}}{\mathcal{R}_{f,t \rightarrow T_1}^j \mathbb{E}_t^* v_{t \rightarrow T_1}}}_{\text{Price of Risk}} \times \underbrace{\text{COV}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^j, \mathcal{R}_{i,t \rightarrow T_1})}_{\text{Quantity of Risk}} > 0. \quad (2.9)$$

This term represents exposure to the risk factor $\mathcal{R}_{M,t \rightarrow T_1}^j$. Consistent with economic theory, we assume that investors require a positive premium to hold a stock with positive exposure to this factor. Empirically, (2.9) may be violated at a given time t . We interpret such violations as measurement error in (2.9); in that case, it is natural to use 0 as a lower bound for this expression. Ultimately, the choice of preference parameters is an empirical matter. Next, we provide lower and upper bounds on the expected excess return.

Proposition 2.1. *Assume that (2.7) and (2.9) hold. Under no-arbitrage conditions, for any $\mathcal{K} > 0$, the expected excess return on stock i admits a lower bound:*

$$\mathbb{E}_t [\mathcal{R}_{i,t \rightarrow T_1}] \geq \frac{\sum_{j=1}^{\mathcal{K}} \frac{a_{j,t} \mathcal{M}_{i,M,t}^{*(j)}}{R_{f,t \rightarrow T_1}^j}}{1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^j}, \quad (2.10)$$

where

$$\mathcal{M}_{i,M,t}^{*(j)} = \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j \mathcal{R}_{i,t \rightarrow T_1}]. \quad (2.11)$$

Furthermore, up to the $\mathcal{K}^{\text{th}}$ -order expansion of the inverse marginal utility, for any threshold $k_0 \leq R_{f,t \rightarrow T_1}$, the expected excess return on stock i satisfies the upper bound:

$$\mathbb{E}_t [\mathcal{R}_{i,t \rightarrow T_1}] \leq \frac{\mathcal{M}_{i,M,t}^{*(0)} [k_0] + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathcal{M}_{i,M,t}^{*(j)} [k_0]}{1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^j}, \quad (2.12)$$

where

$$\mathcal{M}_{i,M,t}^{*(j)} [k_0] = \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j \mathcal{R}_{i,t \rightarrow T_1} 1_{\mathcal{R}_{i,t \rightarrow T_1} \geq k_0}]. \quad (2.13)$$

Proof. See Online Appendix OA.1. $\square$

Proposition 2.1 establishes static lower and upper bounds on the expected excess return

of an individual stock for a given value of $\mathcal{K}$. Both bounds are functions of risk-neutral quantities and the preference parameters $a_{j,t}$ defined in (2.6). The central challenge is to develop a methodology for computing the risk-neutral quantities in equations (2.11) and (2.13). Because there are no traded derivatives written directly on the joint payoff of the market and individual stocks, these joint moments are not directly observable. To provide practically implementable approximations, we note that the conditional risk-neutral expectations $\mathbb{E}_t^*[\mathcal{R}_{i,t \rightarrow T_1} | \mathcal{R}_{M,t \rightarrow T_1}]$ and $\mathbb{E}_t^*[\mathcal{R}_{i,t \rightarrow T_1}^c | \mathcal{R}_{M,t \rightarrow T_1}]$, where $\mathcal{R}_{i,t \rightarrow T_1}^c = \mathcal{R}_{i,t \rightarrow T_1} 1_{\mathcal{R}_{i,t \rightarrow T_1} \geq k_0}$, are unknown functions of the excess market return $\mathcal{R}_{M,t \rightarrow T_1}$. To address this, we present the next two corollaries, which provide tractable expressions for the relevant risk-neutral quantities. These expressions can be computed directly from option prices observed at time t , without relying on realized returns.

Corollary 2.2. *The conditional expectation $\mathbb{E}_t^*[\mathcal{R}_{i,t \rightarrow T_1} | \mathcal{R}_{M,t \rightarrow T_1}] = g_i[\mathcal{R}_{M,t \rightarrow T_1}]$ is an unknown function of the excess market return. A Taylor expansion-series of $g_i[\cdot]$ around $\mathcal{R}_{M,t \rightarrow T_1} = 0$ gives*

$$g_i[\mathcal{R}_{M,t \rightarrow T_1}] = \sum_{k=1}^{\mathcal{N}} \underline{\gamma}_{i,k} \mathcal{R}_{M,t \rightarrow T_1}^k, \quad (2.14)$$

where $\mathcal{N}$ is the order of the Taylor expansion-series. We decompose the excess return as

$$\mathcal{R}_{i,t \rightarrow T_1} = g_i[\mathcal{R}_{M,t \rightarrow T_1}] - \mathbb{E}_t^* g_i[\mathcal{R}_{M,t \rightarrow T_1}] + \varepsilon_{i,t \rightarrow T_1}^*.$$

Since $\mathbb{E}_t^* g_i[\mathcal{R}_{M,t \rightarrow T_1}] = 0$, $\varepsilon_{i,t \rightarrow T_1}^*$ is orthogonal to the excess market return. Thus,

$$\mathbb{E}_t^*[\mathcal{R}_{i,t \rightarrow T_1}^q] = \mathbb{E}_t^*[(g_i[\mathcal{R}_{M,t \rightarrow T_1}])^q] + \sum_{j=1}^q \frac{q!}{(q-j)!j!} \mathbb{E}_t^*[(g_i[\mathcal{R}_{M,t \rightarrow T_1}])^{q-j} \varepsilon_{i,t \rightarrow T_1}^{*j}] \quad (2.15)$$

The coefficients $\underline{\gamma}_{i,k}$ are identified at time t by matching moments

$$\mathbb{E}_t^*(g_i[\mathcal{R}_{M,t \rightarrow T_1}])^q = \mathbb{E}_t^*[\mathcal{R}_{i,t \rightarrow T_1}^q], \text{ for any } q > 1. \quad (2.16)$$

This choice ensures that the second component of (2.15) is negligible for $q > 1$. It follows

that (2.11) can be expressed as

$$\mathcal{M}_{i,M,t}^{*(j)} = \sum_{k=1}^{\mathcal{N}} \underline{\gamma}_{i,k} \mathbb{E}_t^* \left(\mathcal{R}_{M,t \rightarrow T_1}^j \left(\mathcal{R}_{M,t \rightarrow T_1}^k - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^k \right) \right). \quad (2.17)$$

This corollary shows how to approximate the conditional risk-neutral expectation $\mathcal{M}_{i,M,t}^{*(j)}$. The coefficients $\underline{\gamma}_{i,k}$ are functions of time t , but we omit the subscript for simplicity. The expansion coefficients are identified by matching higher-order moments under the risk neutral measure, allowing the computation of cross-moments between individual stock returns and the market under the risk-neutral measure at each time t without relying on realized returns.

Corollary 2.3. *Define the truncated return $\mathcal{R}_{i,t \rightarrow T_1}^c = \mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}$. Then, the conditional expectation $\mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1}^c | \mathcal{R}_{M,t \rightarrow T_1}] = h_i [\mathcal{R}_{M,t \rightarrow T_1}]$ is an unknown function of the market return. A Taylor expansion-series of $h_i [\cdot]$ around $\mathcal{R}_{M,t \rightarrow T_1} = 0$:*

$$h_i [\mathcal{R}_{M,t \rightarrow T_1}] = \sum_{k=1}^{\mathcal{N}} \bar{\gamma}_{i,k} \mathcal{R}_{M,t \rightarrow T_1}^k, \quad (2.18)$$

where $\mathcal{N}$ is the order of the Taylor expansion-series. We decompose the truncated return as

$$\mathcal{R}_{i,t \rightarrow T_1}^c = h_i [\mathcal{R}_{M,t \rightarrow T_1}] - \mathbb{E}_t^* h_i [\mathcal{R}_{M,t \rightarrow T_1}] + \varepsilon_{i,t \rightarrow T_1}^*,$$

where $\mathbb{E}_t^* [\varepsilon_{i,t \rightarrow T_1}^*] = \mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1}^c]$. Thus,

$$\begin{aligned} \mathbb{E}_t^* \left[(\mathcal{R}_{i,t \rightarrow T_1}^c)^q \right] &= \mathbb{E}_t^* \left[(h_i [\mathcal{R}_{M,t \rightarrow T_1}] - \mathbb{E}_t^* h_i [\mathcal{R}_{M,t \rightarrow T_1}])^q \right] \\ &+ \sum_{j=1}^q \frac{q!}{(q-j)!j!} \mathbb{E}_t^* \left[(h_i [\mathcal{R}_{M,t \rightarrow T_1}] - \mathbb{E}_t^* h_i [\mathcal{R}_{M,t \rightarrow T_1}])^{q-j} \varepsilon_{i,t \rightarrow T_1}^{*j} \right] \end{aligned} \quad (2.19)$$

The coefficients $\bar{\gamma}_{i,k}$ are determined by matching the moments:

$$\mathbb{E}_t^* (h_i [\mathcal{R}_{M,t \rightarrow T_1}] - \mathbb{E}_t^* h_i [\mathcal{R}_{M,t \rightarrow T_1}])^q = \mathbb{E}_t^* (\mathcal{R}_{i,t \rightarrow T_1}^c - \mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1}^c])^q, \quad q > 1. \quad (2.20)$$

This choice ensures that the second component of (2.19) are negligible for $q > 1$. It then

follows that (2.13) can be expressed as:

$$\mathcal{M}_{i,M,t}^{*(j)}[k_0] = \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_t^* \mathcal{R}_{i,t \rightarrow T_1}^c + \sum_{k=1}^{\mathcal{N}} \bar{\gamma}_{i,k} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j (\mathcal{R}_{M,t \rightarrow T_1}^k - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^k)] . \quad (2.21)$$

This corollary extends the previous result to truncated returns, allowing the computation of risk-neutral cross-moments conditional on extreme-return realizations. The coefficients $\bar{\gamma}_{i,k}$ vary over time t , but we omit the subscript for simplicity. The result yields an implementable expression for the upper bound that does not rely on realized returns. While a static one-period economy is informative, a dynamic setting is essential when investors can adjust portfolios over time and hence engage in intertemporal hedging.

In the static economy, the SDF depends solely on the market return. As we show next, with intertemporal hedging, it depends not only on the contemporaneous market return but also on the conditional distribution of future market excess returns (i.e., risk-neutral conditional moments of the market's excess return at $T_N > T_1$, such as variance, skewness, and kurtosis, given information at time T_1), implying that intertemporal hedging affects individual-stock expected excess returns. The next section develops the framework and derives bounds on expected excess returns when intertemporal hedging is central.

2.2 Expected Excess Return with Intertemporal Hedging

We assume that the representative agent maximizes expected utility recursively over N periods, beginning at time t , according to the following objective:²

$$\max_{\omega_t} \mathbb{E}_t \max_{\omega_{T_1}} \mathbb{E}_{T_1} \max_{\omega_{T_2}} \mathbb{E}_{T_2} \dots \max_{\omega_{T_{N-1}}} \mathbb{E}_{T_{N-1}} u[W_{t \rightarrow T_N}] .$$

We define the wealth evolution recursively by:

$$W_{T_j \rightarrow T_{j+1}} = W_{T_{j-1} \rightarrow T_j} \left(R_{f,T_j \rightarrow T_{j+1}} + \omega_{T_j}^\top (\mathbf{R}_{T_j \rightarrow T_{j+1}} - R_{f,T_j \rightarrow T_{j+1}} \mathbf{1}) \right) ,$$

²Allowing for intermediate consumption is feasible, but it would complicate the analysis and, in practice, preclude real-time implementation because doing so would require specifying and estimating a structural model of consumption dynamics from historical data. We forgo a fully specified consumption-based structure in exchange for parsimony and a purely option-based implementation.

for $j = 0, \dots, N - 1$. In this framework, we assume no interest rate risk and $R_{f,T_j \rightarrow T_{j+1}}$ is known at time t for all j . The SDF from time t to T_N is proportional to marginal utility $u' [W_{t \rightarrow T_N}]$. Suppose that we wish to price a two-period payoff formed by investing in asset i over $[t, T_1]$, liquidating the position at T_1 , and rolling the proceeds into the risk-free asset over $[T_1, T_N]$. The gross payoff from this rollover strategy equals $R_{i,t \rightarrow T_1} R_{f,T_1 \rightarrow T_N}$. Subtracting the risk-free gross return over the same horizon, $R_{f,t \rightarrow T_1} R_{f,T_1 \rightarrow T_N}$, yields the corresponding excess payoff:

$$R_{i,t \rightarrow T_1} R_{f,T_1 \rightarrow T_N} - R_{f,t \rightarrow T_1} R_{f,T_1 \rightarrow T_N} = \mathcal{R}_{i,t \rightarrow T_1} R_{f,T_1 \rightarrow T_N}.$$

Under no-arbitrage conditions, the expected excess return satisfies

$$\mathbb{E}_t (\mathcal{R}_{i,t \rightarrow T_1} R_{f,T_1 \rightarrow T_N}) = \mathbb{E}_t^* \left(\frac{\mathbb{E}_t m_{t \rightarrow T_N}}{m_{t \rightarrow T_N}} \mathcal{R}_{i,t \rightarrow T_1} R_{f,T_1 \rightarrow T_N} \right), \quad (2.22)$$

where the inverse SDF satisfies

$$\frac{\mathbb{E}_t m_{t \rightarrow T_N}}{m_{t \rightarrow T_N}} = \frac{\vartheta_{t \rightarrow T_N}}{\mathbb{E}_t^* \vartheta_{t \rightarrow T_N}},$$

with

$$\vartheta_{t \rightarrow T_N} = \frac{u' [W_t R_{f,t \rightarrow T_N}]}{u' [W_t R_{M,t \rightarrow T_N}]}.$$

Assuming there is no interest rate risk, $R_{f,T_1 \rightarrow T_N}$ is known at time t and the expression (2.22) simplifies to

$$\mathbb{E}_t \mathcal{R}_{i,t \rightarrow T_1} = \mathbb{E}_t^* \left(\mathcal{R}_{i,t \rightarrow T_1} \mathbb{E}_{T_1}^* \frac{\mathbb{E}_t m_{t \rightarrow T_N}}{m_{t \rightarrow T_N}} \right). \quad (2.23)$$

Since $\mathcal{R}_{i,t \rightarrow T_1}$ is an excess return for one period, the standard price relation without arbitrage (2.1) applies. Thus, comparing expression (2.1) and (2.23), and using the uniqueness of the SDF,

$$\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}} = \mathbb{E}_{T_1}^* \frac{\mathbb{E}_t m_{t \rightarrow T_N}}{m_{t \rightarrow T_N}} = \frac{v_{t \rightarrow T_1}}{\mathbb{E}_t^* v_{t \rightarrow T_1}} \quad (2.24)$$

Given that $R_{M,t \rightarrow T_N} = R_{M,t \rightarrow T_1} R_{M,T_1 \rightarrow T_N}$, $\vartheta_{t \rightarrow T_N}$ is a function of $R_{M,t \rightarrow T_1}$ and $R_{M,T_1 \rightarrow T_N}$. Assuming that the necessary regularity conditions in $\vartheta_{t \rightarrow T_N}$ are satisfied, A Taylor expansion

series of $\vartheta_{t \rightarrow T_N}$ around $(\mathcal{R}_{M,t \rightarrow T_1}, \mathcal{R}_{M,T_1 \rightarrow T_N}) = (0, 0)$ yields the following.

$$v_{t \rightarrow T_1} = 1 + \sum_{k=1}^{\infty} \sum_{j=0}^k \frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} f_{T_1}^{(j,k-j)}, \quad (2.25)$$

with

$$f_{T_1}^{(j,k-j)} = \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j}, \quad (2.26)$$

where the lower-order terms are identified as

$$a_{0,0,t} = 1, \quad a_{1,0,t} = a_{1,t}, \quad a_{2,0,t} = a_{2,t}, \quad a_{3,0,t} = a_{3,t}, \quad a_{4,0,t} = a_{4,t}, \quad (2.27)$$

and

$$a_{0,2,t} = a_{2,t}, \quad a_{0,3,t} = a_{3,t}, \quad a_{0,4,t} = a_{4,t}, \quad a_{1,2,t} = 2a_{2,t} + 3a_{3,t}, \quad (2.28)$$

and

$$a_{1,3,t} = 3a_{3,t} + 3 \left(1 - 2 \frac{a_{2,t}}{a_{1,t}^2} + a_{3,t} \right) (a_{1,t} - 1) + 4a_{4,t}, \quad (2.29)$$

$$a_{2,2,t} = a_{2,t} + 6a_{3,t} + 6a_{4,t}. \quad (2.30)$$

The coefficients $a_{j,k,t}$ are analogous to those defined in (2.6) and (2.4), with the sole difference that T_1 is replaced by T_N in (2.6) and (2.4). The inverse SDF (2.24) is a function of priced factors $f_{T_1}^{(j,k-j)}$ for all pairs $(j, k-j)$ not equal to $(0, 0)$ and for pairs where $k-j \neq 1$. When $k-j=1$, this term vanishes because $\mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}$ is zero. In the special case where $j=0$ and $k-j \geq 2$, the expression simplifies to $f_{T_1}^{(j,k-j)} = \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j}$ which represents the future risk-neutral variance (for $k-j=2$), the future risk-neutral skewness (for $k-j=3$), and the future risk-neutral kurtosis (for $k-j=4$). This result demonstrates that when intertemporal hedging is relevant, that is, when the representative agent can rebalance and is concerned about future shifts in economic environment, the inverse SDF (2.24) depends not only on the market return, but also on the higher-order moments of the entire risk-neutral distribution, as well as on interaction terms between the market return and these moments.

Applying the Carr and Madan (2001) spanning result to both $\mathcal{R}_{M,t \rightarrow T_1}^j$ and $\mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j}$, we show in Online Appendix (OA.2.2) that the inverse SDF (2.24) admits a decomposition involving the market return, a continuum of options maturing at T_1 , and a continuum of

options maturing at T_N that will be sold at T_1 .

Using (2.1), (2.2), and (2.25), the expected excess market return (setting $i = M$) can be written as a sum of risk premium terms of the form

$$\underbrace{\frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j} \mathbb{E}_t^* v_{t \rightarrow T_1}}}_{\text{Price of Risk}} \times \underbrace{\text{COV}_t^* \left(\mathcal{R}_{M,t \rightarrow T_1}, \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} \right)}_{\text{Quantity of Risk}} \geq 0. \quad (2.31)$$

The left-hand side of (2.31) is the compensation for exposure to the priced factor $f_{T_1}^{(j,k-j)}$. This implies that $a_{j,k-j,t} \geq (\leq) 0$ if $\text{COV}_t^* \left(\mathcal{R}_{M,t \rightarrow T_1}, f_{T_1}^{(j,k-j)} \right) \geq (\leq) 0$. When $k - j = 0$, inequality (2.31) coincides with (2.7). When $k - j \geq 2$ and j and $k - j$ are both odd or both even, we posit that $a_{j,k-j,t} \leq 0$. If j is odd (even) and $k - j$ is even (odd), we posit that $a_{j,k-j,t} \geq 0$.

When $i \neq M$, combining (2.1), (2.2), and (2.25) shows that the expected excess return on individual stock is a sum of risk premium terms of the form

$$\underbrace{\frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j} \mathbb{E}_t^* v_{t \rightarrow T_1}}}_{\text{Price of Risk}} \times \underbrace{\text{COV}_t^* \left(\mathcal{R}_{i,t \rightarrow T_1}, \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} \right)}_{\text{Quantity of Risk}} \geq 0. \quad (2.32)$$

As in the case without intertemporal hedging, restriction (2.32) follows from the economically intuitive assumption that the price of risk times the quantity of risk is positive.

Proposition 2.4. *Assume that for all pairs $(j, k - j)$, inequalities (2.31) and (2.32) hold. Under no-arbitrage conditions, for any $\mathcal{K} > 0$, the expected excess return on individual stock i admits the following lower bound*

$$\mathbb{E}_t [\mathcal{R}_{i,t \rightarrow T_1}] \geq \frac{\sum_{k=1}^{\mathcal{K}} \sum_{j=0}^k \frac{a_{j,k-j}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \mathcal{M}_{i,M,t}^{*(j,k-j)}}{1 + \sum_{k=1}^{\mathcal{K}} \sum_{j=0}^k \frac{a_{j,k-j}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \mathbb{E}_t^* f_{T_1}^{(j,k-j)}}, \quad (2.33)$$

with

$$\mathcal{M}_{i,M,t}^{*(j,k-j)} = \mathbb{E}_t^* \left[\mathcal{R}_{i,t \rightarrow T_1} f_{T_1}^{(j,k-j)} \right]. \quad (2.34)$$

Furthermore, up to a $\mathcal{K}^{\text{th}}$ order expansion of the inverse marginal utility, and for any thresh-

old $k_0 \leq R_{f,t \rightarrow T_1}$, the expected excess return on stock i satisfies the following upper bound

$$\mathbb{E}_t [\mathcal{R}_{i,t \rightarrow T_1}] \leq \frac{\mathcal{M}_{i,M,t}^{*(0)} [k_0] + \sum_{k=1}^{\mathcal{K}} \sum_{j=0}^k \frac{a_{j,k-j}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \mathcal{M}_{i,M,t}^{*(j,k-j)} [k_0]}{1 + \sum_{k=1}^{\mathcal{K}} \sum_{j=0}^k \frac{a_{j,k-j}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \mathbb{E}_t^* f_{T_1}^{(j,k-j)}}, \quad (2.35)$$

with

$$\mathcal{M}_{i,M,t}^{*(j,k-j)} [k_0] = \mathbb{E}_t^* \left[\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0} f_{T_1}^{(j,k-j)} \right]. \quad (2.36)$$

Proof. See Online Appendix [OA.1](#). □

Proposition 2.4 provides lower and upper bounds for the expected excess return on individual stock i when intertemporal hedging is relevant. Given the preference parameters, one needs to compute the risk-neutral quantities in (2.33) and (2.35). Since $\mathcal{R}_{M,T_1 \rightarrow T_N} = R_{M,T_1 \rightarrow T_N} - R_{f,T_1 \rightarrow T_N}$, it follows that, for $j_2 \geq 2$,

$$\mathcal{R}_{M,T_1 \rightarrow T_N}^{j_2} = \sum_{l=0}^{j_2} \frac{j_2! (-1)^{j_2-l} R_{f,T_1 \rightarrow T_N}^{j_2-l}}{(j_2-l)! l!} R_{M,T_1 \rightarrow T_N}^l. \quad (2.37)$$

Taking the conditional expectation yields

$$\mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{j_2} = \sum_{l=2}^{j_2} \phi_{j_2,l} \left(\mathbb{E}_{T_1}^* R_{M,T_1 \rightarrow T_N}^l - R_{f,T_1 \rightarrow T_N}^l \right), \quad (2.38)$$

with

$$\phi_{j_2,l} = \frac{j_2! (-1)^{j_2-l} R_{f,T_1 \rightarrow T_N}^{j_2-l}}{(j_2-l)! l!}. \quad (2.39)$$

When $l \geq 2$, closed-form expressions for $\mathbb{E}_{T_1}^* R_{M,T_1 \rightarrow T_N}^l - R_{f,T_1 \rightarrow T_N}^l$ in terms of option prices are not available. Since Jensen's inequality implies $\mathbb{E}_{T_1}^* R_{M,T_1 \rightarrow T_N}^l \geq R_{f,T_1 \rightarrow T_N}^l$, a tractable approach is to express the positive term $\mathbb{E}_{T_1}^* R_{M,T_1 \rightarrow T_N}^l - R_{f,T_1 \rightarrow T_N}^l$ using time T_1 information, such as $R_{M,t \rightarrow T_1}$, by assuming for any $l = 2, 3, 4$ that

$$\mathbb{E}_{T_1}^* R_{M,T_1 \rightarrow T_N}^l - R_{f,T_1 \rightarrow T_N}^l = \theta_{l,t}^* \left\{ \begin{aligned} & (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} \\ & + (R_{f,t \rightarrow T_1}^l - R_{M,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}} \end{aligned} \right\}. \quad (2.40)$$

The specification assumes that the future positive moment $\mathbb{E}_{T_1}^* R_{M,T_1 \rightarrow T_N}^l - R_{f,T_1 \rightarrow T_N}^l$ is solely driven by the intermediate variable $R_{M,t \rightarrow T_1}$. The right-hand side of (2.40) can be interpreted as the payoff of a static portfolio of options, and the parameter $\theta_{l,t}^*$ is chosen so that the payoffs on the left- and right-hand sides of (2.40) have identical prices. For completeness, we provide the full motivation and derivations in the Online Appendix OA.3. To determine $\theta_{l,t}^*$, we multiply (2.40) by $R_{M,t \rightarrow T_1}^l$, exploit $R_{M,t \rightarrow T_N}^l = R_{M,t \rightarrow T_1}^l R_{M,T_1 \rightarrow T_N}^l$, and take expectation, which leads to

$$\theta_{l,t}^* = \frac{\mathbb{E}_t^* R_{M,t \rightarrow T_N}^l - R_{f,T_1 \rightarrow T_N}^l \mathbb{E}_t^* R_{M,t \rightarrow T_1}^l}{\begin{pmatrix} \mathbb{E}_t^* \left[R_{M,t \rightarrow T_1}^l (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1}^l > R_{f,t \rightarrow T_1}^l} \right] \\ - \mathbb{E}_t^* \left[R_{M,t \rightarrow T_1}^l (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1}^l < R_{f,t \rightarrow T_1}^l} \right] \end{pmatrix}}. \quad (2.41)$$

We now build on (2.41) to provide expressions for (2.34) and (2.36) in terms of observable risk-neutral quantities.

Corollary 2.5. *Denote*

$$\begin{aligned} \mathbb{M}_t^{*+}[j, q, l] &= \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j+q} (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1}^l > R_{f,t \rightarrow T_1}^l} \right], \\ \mathbb{M}_t^{*-}[j, q, l] &= \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j+q} (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1}^l < R_{f,t \rightarrow T_1}^l} \right]. \end{aligned}$$

Assume (2.40). Using results (2.14) and (2.8) in Corollaries 2.2 and 2.3, it follows that, for $k - j \neq \{0, 1\}$,

$$\mathcal{M}_{i,M,t}^{(j,k-j)} = \sum_{q=1}^{\mathcal{N}} \gamma_{i,q} \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{M}_t^{*+}[j, q, l] - \sum_{q=1}^{\mathcal{N}} \gamma_{i,q} \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{M}_t^{*-}[j, q, l], \quad (2.42)$$

and

$$\mathcal{M}_{i,M,t}^{(j,k-j)}[k_0] = \sum_{q=1}^{\mathcal{N}} \bar{\gamma}_{i,q} \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{M}_t^{*+}[j, q, l] - \sum_{q=1}^{\mathcal{N}} \bar{\gamma}_{i,q} \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{M}_t^{*-}[j, q, l], \quad (2.43)$$

and

$$\mathbb{E}_t^* f_{T_1}^{(j,k-j)} = \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{M}_t^{*+}[j, 0, l] - \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{M}_t^{*-}[j, 0, l]. \quad (2.44)$$

Here, $\phi_{j_2,l}$ is defined in (2.39) while $\bar{\gamma}_{i,k}$ and $\underline{\gamma}_{i,k}$ are defined in (2.16) and (2.20). The risk-neutral quantities $\mathcal{M}_{i,M,t}^{*(j_1)}$ and $\mathcal{M}_{i,M,t}^{*(j_1)}[k_0]$ are defined in (2.17) and (2.21). Finally, $\theta_{i,t}^*$ is defined in (2.41).

Proof. See Online Appendix OA.1. □

Corollary 2.5 provides expressions for the risk-neutral quantities needed to compute the lower and upper bounds in Proposition 2.4. Expressions for the market-return-related risk-neutral quantities appearing in (2.42), (2.43), and (2.44) are provided in Section 3.2 and Online Appendix OA.5.

3 Measuring Bounds from Option Prices

This section describes how we construct empirical counterparts of the theoretical bounds using observable option prices.

3.1 Risk Preference Parameters

Our formulas for the bounds involve four risk-preference parameters: τ (risk tolerance), ρ (prudence), κ (temperance), and ξ (hyper-temperance). To construct empirical counterparts of the bounds, we must specify values for these parameters. In theory, the parameters may vary over time and depend on economic conditions. For tractability, however, we assume constant values throughout the sample period.

We consider two parameter sets. The first is a simple default calibration with $\tau = 1$, $\rho = 2$, $\kappa = 4$, and $\xi = 8$. These values satisfy the restrictions in Eq.(2.8) and are supported by experimental evidence on higher-order risk attitudes. [Noussair, Trautmann, and Van de Kuilen \(2014\)](#) show in a large representative laboratory experiment that $\rho > 1$ holds for a substantial subset of subjects. Likewise, [Ebert and Wiesen \(2011, 2014\)](#) report jointly elicited distributions of relative risk aversion and prudence with many prudent subjects, particularly where $\rho > 1$. This choice provides a parsimonious baseline that respects theoretical restrictions on higher-order preference parameters while remaining consistent with plausible economic magnitudes.³

³The default values scale approximately as powers of two across successive higher-order preference param-

The second is a data-driven calibration, in which we choose the preference parameters to maximize the out-of-sample forecast accuracy of the average bounds (i.e., the midpoint of the lower and upper bounds) for future returns. Forecast accuracy is evaluated using out-of-sample R^2 statistics that compare predictions from the average bounds under the calibrated parameters with those obtained under the default set.⁴ Because calibration is based on the average of the bounds, the optimal parameter set for a given forecast horizon applies equally to both the lower and upper bounds. We implement the calibration by performing a grid search over the following candidate values:

$$\frac{1}{\tau} \in \{1, 1.5, 2, 2.5, 3\}, \quad \rho \in \{2, 2.5, 3, 3.5, 4\}, \quad \kappa \in \{4, 4.5, 5, 5.5, 6\}, \quad \xi \in \{8, 8.5, 9, 9.5, 10\}.$$

The search is restricted to combinations that satisfy the conditions in Eq.(2.8).

Our decision to work with calibrated, constant preference parameters rather than fully estimating a flexible preference process is motivated by both empirical and conceptual considerations. Empirically, [Chabi-Yo, Gourié, and Langlois \(2025\)](#) show, in a market-level setting, that economically reasonable constant-parameter calibrations deliver superior out-of-sample performance to specifications in which preferences are estimated from realized returns. Conceptually, estimating preferences from historical returns would run counter to our goal of deriving ex ante bounds from option prices rather than from realized returns. Accordingly, our default and data-driven parameter sets are disciplined calibrations within a narrow, economically plausible range, not unconstrained statistical estimates. This approach avoids imposing and estimating time-series dynamics for higher-order preference parameters from realized returns, which could introduce look-ahead bias in out-of-sample evaluation. Consistent with this perspective, [Chabi-Yo, Dim, and Vilkov \(2023\)](#) also adopt calibration rather than full estimation. We therefore view calibration as a sound way to bring the theory to the data, and our main conclusions are robust to either parameter set.

eters, ensuring internal consistency and avoiding implausibly high levels of prudence or temperance relative to risk aversion. This heuristic has been widely adopted in applications where higher-order preferences must be fixed rather than estimated.

⁴This out-of-sample R^2 statistic is widely used in the literature to compare predictive models (e.g., [Goyal and Welch, 2008](#); [Martin and Wagner, 2019](#); [Chabi-Yo, Dim, and Vilkov, 2023](#)).

3.2 Estimation of Risk-Neutral Moments

Once the preference parameters are specified, the next step is to compute the various types of risk-neutral moments that enter the formulas for the bounds. In total, we require seven types of moments. The first three are fundamental building blocks, while the remaining four can be expressed as combinations of these three.

Marginal risk-neutral moments. The first type consists of marginal risk-neutral moments taken separately for individual stock returns and for the market return,

$$\mathbb{E}_t^*[\mathcal{R}_{i,t \rightarrow T_1}^q] .$$

Their computation is straightforward. Following [Breedon and Litzenberger \(1978\)](#) and [Bakshi and Madan \(2000\)](#), each of these moments can be extracted from the cross-section of option prices on individual stocks and on the index, as detailed in Online Appendix [OA.5.1](#).

Truncated risk-neutral moments. The second type is the risk-neutral moments of truncated returns,

$$\mathbb{E}_t^* \left[\mathcal{R}_{i,t \rightarrow T_1} 1_{\{R_{i,t \rightarrow T_1} \geq k_0\}} \right] .$$

To compute these moments, we must choose the threshold return k_0 . Lowering k_0 tightens the bound by reducing the resulting upper bound, but it also requires using deeper out-of-the-money options, whose quotes are noisier and less reliable. We therefore set $k_0 = 0.7$, which delivers a tight bound while maintaining sufficient option-quote quality.⁵ Given this threshold, the truncated moments can again be computed from option prices, as explained in Online Appendix [OA.5.2](#).

Directional-magnitude risk-neutral co-moments. The third type consists of the interaction between powers of returns and their absolute deviations,

$$\mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j_1} |\mathcal{R}_{M,t \rightarrow T_1}|^l \right] .$$

These moments capture the joint effects of two different powers of the market return: the raw power j_1 , which preserves the sign and thus reflects the directional component of returns,

⁵We verify in robustness checks that an alternative threshold of 0.6 or 0.8 yields qualitatively similar results.

and the absolute power l , which measures the magnitude of deviations irrespective of sign. In this way, they represent interactions between directional return components and volatility-like magnitudes. The computation relies on the spanning formula and can be implemented from the cross-section of option prices, with details provided in Online Appendix [OA.5.4](#).

Other moments. The remaining four types of risk-neutral moments extend the three basic forms. For clarity, we indicate for each moment how it can be represented as a combination of the basic building blocks (marginal, truncated, and absolute-weighted moments), and we point to the corresponding appendix where the exact computational formula is provided.

1. *Central risk-neutral moments of market returns:*

$$\mathbb{E}_t^* \left(\sum_{k=1}^{\mathcal{N}} \gamma_{i,k} (\mathcal{R}_{M,t \rightarrow T_1}^k - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^k) \right)^q, \quad \mathcal{N} = 2.$$

This moment is a demeaned polynomial in market returns that can be represented as a combination of the first building block (marginal risk-neutral moments of market returns). The explicit formula is provided in Online Appendix [OA.5.5](#).

2. *Polynomial truncated risk-neutral moments:*

$$\mathbb{E}_t^* \left[(R_{i,t \rightarrow T_1} - R_{f,t \rightarrow T_1})^j 1_{\{R_{i,t \rightarrow T_1} \geq k_0\}} \right].$$

This reduces to a combination of the first building block (marginal moments of individual-stock returns) and the second building block (truncated moments). The exact reduction and implementation are given in Appendix [OA.5.6](#).

3. *Central risk-neutral moments of truncated returns:*

$$\mathbb{E}_t^* \left(\mathcal{R}_{i,t \rightarrow T_1} 1_{\{R_{i,t \rightarrow T_1} \geq k_0\}} - \mathbb{E}_t^* \left[\mathcal{R}_{i,t \rightarrow T_1} 1_{\{R_{i,t \rightarrow T_1} \geq k_0\}} \right] \right)^q.$$

By binomial expansion, this moment can be constructed from the product of a polynomial truncated risk-neutral moment and a power of the second building block (truncated risk-neutral moments), as detailed in Online Appendix [OA.5.7](#).

4. Forward-looking cross risk-neutral moments:

$$\mathbb{E}_t^*[f_{T_1}^{(j_1, j_2)}], \quad f_{T_1}^{(j_1, j_2)} = \mathcal{R}_{M, t \rightarrow T_1}^{j_1} \mathbb{E}_{T_1}^*[\mathcal{R}_{M, T_1 \rightarrow T_N}^{j_2}].$$

This can be represented as a combination of the first building block (a marginal moment of the market) and the third building block (absolute-weighted higher-order moments). The precise formula and implementation are provided in Online Appendix [OA.5.8](#).

These derived moments can be interpreted as higher-order conditional risk-neutral moments that capture interactions among return powers, truncation, and deviations from either their means or the risk-free rate. Although the notation may appear cumbersome, each moment can be expressed as a combination of the three fundamental moment types, making their computation tractable.

3.3 Data

Our sample consists of U.S. firms included in the S&P 500 index between January 1996 and August 2023. Option data on individual equities and on the S&P 500 index are obtained from OptionMetrics. Because options with exact target strikes rarely trade, we use the volatility surface data, which provides daily interpolated implied volatilities and corresponding option premiums over a standardized grid of maturities and deltas.⁶ To construct a finer grid of option prices across moneyness levels ranging from 1/3 to 3 (with 1,001 points for each maturity), we interpolate implied volatilities using a piecewise cubic Hermite polynomial and extrapolate flatly outside the boundaries, following [Chabi-Yo, Dim, and Vilkov \(2023\)](#). Option prices are then recovered from the interpolated volatilities using the Black–Scholes formula. Since the fitted implied volatilities are computed daily using only contemporaneous option quotes and require sufficient data for interpolation, the resulting option prices are free from look-ahead bias and reflect prevailing market valuations.

Two features of our option data mitigate concerns about sparse strike coverage and data quality, especially for single-name and longer-dated options. First, we restrict attention to S&P 500 constituents, whose options are actively traded and exhibit relatively dense

⁶Maturities include 30, 60, 91, 122, 152, 182, 273, 365, 547, and 730 days; Absolute deltas range from 0.1 to 0.9 in increments of 0.05.

coverage across strikes and maturities. Second, we rely on the OptionMetrics volatility surface instead of raw transaction prices at irregular strikes, which aggregates information across available strikes for each stock–maturity pair, smooths gaps, and enforces basic no-arbitrage restrictions. Consistent with recent option-based expected-return studies that use volatility surfaces for index and single-name options (e.g., [Martin and Wagner, 2019](#); [Kadan and Tang, 2020](#); [Chabi-Yo, Dim, and Vilkov, 2023](#)), these choices ensure that our empirical bounds are driven by liquid options embedded in a smooth, well-behaved volatility surface rather than by a few illiquid long-dated contracts or sparsely populated strike regions.

Underlying index levels and stock prices are obtained from CRSP, and index constituent information from Compustat. For the spot risk-free rate, we use constant-maturity Treasury yields with maturities of 1, 3, 6, and 12 months from the Federal Reserve Economic Database, matching the horizons of the equity premium. Forward risk-free rates between T_1 to T_N are computed from the spot rates at maturities T_1 and T_N .

4 Empirical Performance of the Bounds

This section evaluates the empirical performance of the bounds constructed in Section 3. We first analyze their properties and predictive power without intertemporal hedging (Section 4.1), then incorporate hedging demands (Section 4.2), and finally compare the two approaches (Section 4.3).

4.1 Bounds Without Intertemporal Hedging

4.1.1 Properties of the Bounds

Table 1 reports summary statistics of the estimates for the lower bound (LB, Eq. 2.10) and upper bound (UB, Eq. 2.12) of annualized conditional expected excess returns, as derived in Proposition 2.1. Panel A presents results under the preference parameters $(\tau, \rho, \kappa, \xi) = (1, 2, 4, 8)$ (hereafter, default parameters), while Panel B shows results obtained using preference parameters calibrated separately for each forecast horizon to maximize out-of-sample forecast accuracy (hereafter, calibrated parameters).

[Insert Table 1 around here]

In Panel A, average annualized lower bounds rise from 8.1% at the 30-day horizon to 9.4% at the one-year horizon, while the corresponding upper bounds rise from 13.9% to 16.4%. Standard deviations are sizable, indicating substantial dispersion across stocks. Skewness is uniformly positive, implying heavy right tails for both bounds. Percentile values further illustrate the wide variation: for example, at the 30-day horizon, the 10th–90th percentile ranges from 2.9% to 15.3% for the lower bound and from 3.3% to 34.8% for the upper bound. Overall, the bounds exhibit substantial cross-sectional dispersion, indicating that the admissible range of expected returns varies markedly across firms.

Panel B indicates that the bounds using the calibrated parameters, LB^* and UB^* , generally have slightly higher means than those under the default specification, with differences being economically small. A notable exception occurs at the 182-day horizon. Here, the mean LB^* increases to 14.8% (versus 8.6% under the default parameters), and the mean UB^* rises to 21.0% (versus 16.8%). This upward shift is primarily driven by a change in the risk-aversion parameter τ , which falls from 1 to $2/3$ under calibration. Because relative risk aversion equals $1/\tau$, a smaller τ implies greater risk aversion and thus higher required premia, leading both LB^* and UB^* to exhibit substantially higher means than their counterparts under the default specification.

We begin by testing the tightness of the bounds to assess whether they can serve as unbiased proxies for expected excess returns. We estimate the pooled panel regression

$$\mathcal{R}_{i,t \rightarrow T_1} = \beta_0 + \beta_1 B_{i,t \rightarrow T_1} + \varepsilon_{i,t \rightarrow T_1}, \quad (4.1)$$

where $\mathcal{R}_{i,t \rightarrow T_1}$ denotes the realized annualized excess return on stock i from t to T_1 , and $B_{i,t \rightarrow T_1}$ is the corresponding bound estimate. If the bounds are tight, we should find $\beta_0 = 0$ and $\beta_1 = 1$. We run the regression at 30-, 91-, 182-, and 365-day horizons ($t \rightarrow T_1$) using monthly data on S&P 500 stocks. Standard errors and p -values are computed via block bootstrap to account for both time-series and cross-sectional dependence.

[Insert Table 2 around here]

Table 2 presents the results. Panel A (lower bound) shows that, across all horizons and under both default and calibrated preference parameters, intercepts are indistinguishable from zero and slopes exceed one and are statistically different from zero, consistent with the lower bound understating expected excess returns by construction. Wald tests nevertheless

fail to reject the joint null hypothesis that $\beta_0 = 0$ and $\beta_1 = 1$ at any horizons; although the point estimates of β_1 exceed one, we cannot reject $\beta_1 = 1$. Taken together, these results provide preliminary evidence that the LB is an unbiased proxy for expected excess returns. Panel B (upper bound) shows intercepts generally indistinguishable from zero (except at 30 days at the 5% level) and slopes uniformly below one, as expected given that the upper bound overstates expected returns by construction. In this case, however, Wald tests reject the joint null at all horizons except 365 days, indicating that the UB is not tight in most cases. Panel C (midpoint) yields intercepts near zero and slopes close to one (except at 30 days at the 5% level). Importantly, the joint null is never rejected, implying the midpoint is an unbiased proxy for expected excess returns.

While the upper bound is empirically less tight than the lower bound, it still plays a useful complementary role. First, the UB, together with the LB, delivers an economically meaningful range for expected excess returns rather than a single point estimate, allowing investors to assess both conservative and optimistic scenarios that are consistent with option prices and our preference assumptions. Second, even when the UB is relatively loose, its combination with the LB through the midpoint produces an unbiased proxy for expected returns, while the distance between the two bounds provides a natural summary of the uncertainty surrounding those expectations. Thus, although the UB on its own is not as informative as the LB in our tightness tests, it remains valuable for bracketing expected returns and for interpreting the economic content and precision of our bound-based measures.

We next assess whether the bounds price assets consistently across aggregation levels by examining their pricing coherence between individual stocks and basket-type securities. Following [Chabi-Yo, Dim, and Vilkov \(2023\)](#), we compare the annualized 30-day expected return bounds of nine S&P 500 sector exchange-traded funds (ETFs) with the value-weighted averages of the bounds of their underlying component stocks. For each day, we compute the difference between the ETF-level and value-weighted constituent-level bounds and then take monthly averages of these daily differences. Figure 2 plots the resulting time series of these differences for our lower bound (LB), upper bound (UB), and their midpoint (LUB), alongside the benchmark bounds proposed by [Martin and Wagner \(2019, MW\)](#), [Kadan and Tang \(2020, KT\)](#), and [Chabi-Yo, Dim, and Vilkov \(2023, GLB\)](#). Grey-shaded areas indicate NBER recession periods.

[Insert Figure 2 around here]

Smaller absolute differences (i.e., values closer to zero) indicate greater pricing consistency between individual stocks and sector portfolios. Among all measures, the difference for our lower bound (solid black line) consistently lies closest to zero and remains remarkably stable over time, suggesting that it achieves stronger pricing coherence between individual stocks and ETFs than the MW and KT bounds. The differences for the GLB bound also exhibit strong pricing consistency but occasionally drift away from zero—particularly during recessions—when our LB difference remains stable and near zero. In contrast, the difference for the upper bound tends to deviate from zero to a greater extent, consistent with the earlier evidence in Table 2 that the upper bound is less tight. Overall, these results demonstrate that our lower bound not only provides a tight proxy for expected returns at the firm level but also achieves high pricing coherence between individual securities and market aggregates.

4.1.2 The Bounds and Future Returns

We next study the relation between the bounds and future returns in three ways. First, we examine whether the bounds convey predictive signals about future returns by analyzing the returns to portfolios formed based on the bounds. Second, we assess whether the bounds provide incremental information relative to benchmark models using double-sorted portfolios. Finally, we evaluate the out-of-sample forecasting performance of the bounds by comparing their accuracy with that of competing expected excess return proxies using the modified out-of-sample R^2 measure proposed in the literature.

Table 3 reports portfolio sorts on the bounds. Each month we form quintile portfolios by ranking stocks on the lower bound (Panel A), the upper bound (Panel B), or their midpoint (Panel C) as proxies for conditional expected excess returns, and compute equal-weighted realized returns over horizons of 30, 91, 182, and 365 days. Across all specifications, portfolio returns increase monotonically with the bounds, and the high–low spreads are both statistically significant and economically large. Panel A shows that sorting on the lower bound yields strong return differentials: at the 30-day horizon, average annualized returns rise from 8.3% in the lowest quintile to 21.0% in the highest, producing a high–low spread of 12.7% that is significant according to the monotonicity test of [Patton and Timmermann \(2010\)](#). The spread widens with horizon, reaching 17.2% at one year. Results are similar whether we use the default bound (LB) or the calibrated version (LB*). Panels B and C confirm that sorting on the upper bound or on the midpoint produces nearly identical

patterns, with spreads ranging from about 10% to 17% across horizons. Overall, these results indicate that both the lower and upper bounds embed strong predictive content for future stock returns and generate robust investment signals.

[Insert Table 3 around here]

Table 4 evaluates whether our bounds provide incremental predictive content relative to the measures proposed by MW, KT, and GLB. We implement a double-sort procedure: stocks are first sorted into terciles by MW, KT, or GLB bounds, and then further into terciles by our bound measures. Reported are the returns from a strategy that buys stocks in the top tercile of our bounds and shorts those in the bottom tercile within each tercile of the benchmark bounds. Since the patterns are similar across horizons, we report only the 30-day results.⁷

[Insert Table 4 around here]

Panel A uses the bounds under the default parameter set. When conditioning on MW, the high–low portfolio strategy based on the lower bound delivers a statistically significant return of 7.3% within the highest MW tercile, while the upper bound strategy generates significant returns within both the lowest and highest terciles (p -values < 0.10). The incremental predictive content is stronger when conditioning on KT and GLB: within every tercile, both lower- and upper-bound strategies yield statistically significant high–low spreads. Panel B, which uses calibrated parameters, shows broadly similar results. Taken together, these findings indicate that our bounds provide complementary predictive power beyond existing one-period benchmarks. This improvement is attributable to the fact that our model avoids the auxiliary assumptions of earlier studies, such as log utility (MW), first-order approximations of marginal utility (KT), and reliance on realized betas for calibration (GLB).

In Table 5, we compare the out-of-sample forecasting performance of our bounds with a wide range of benchmarks using the modified out-of-sample R^2 statistic (R^2_{OOS}), following Goyal and Welch (2008), Martin and Wagner (2019), and Chabi-Yo, Dim, and Vilkov (2023). Specifically, R^2_{OOS} is defined as

$$R^2_{\text{OOS}} = 1 - \frac{\sum_i \sum_t (r_{i,t} - \hat{r}_{i,t})^2}{\sum_i \sum_t (r_{i,t} - \hat{r}_{i,t}^B)^2}, \quad (4.2)$$

⁷The results for other horizons are available upon request.

where $r_{i,t}$ denotes the realized excess return of stock i at time t , $\hat{r}_{i,t}$ is the excess return forecast from our model (i.e., the bound estimates), and $\hat{r}_{i,t}^B$ is the excess return forecast from a benchmark model. A positive R_{OOS}^2 indicates that the forecasting errors from our bounds are smaller than those of the benchmark, implying higher forecasting accuracy of our bounds relative to the benchmark. We test the null hypothesis that $R_{\text{OOS}}^2 \leq 0$ using a stationary block bootstrap procedure with 10,000 replications.

We consider eight benchmarks: (1) $\overline{RX}_{i,t}$, the historical average excess return of stock i since 1990; (2) 6% p.a., a constant 6% per annum; (3) $\overline{MKT}_t$, the historical average market excess return; (4) $\overline{IV}_t$, the historical average of market implied variance; (5) $\overline{IV}_{i,t}$, the historical average of stock-level implied variance; (6) $MW_{i,t}$, the bound of [Martin and Wagner \(2019\)](#); (7) $KT_{i,t}$, the bound of [Kadan and Tang \(2020\)](#); and (8) $GLB_{i,t}$, the generalized lower bound of [Chabi-Yo, Dim, and Vilkov \(2023\)](#).

[Insert Table 5 around here]

Panel A of Table 5 reports results for the lower bound under the default parameter set. Across all horizons, R_{OOS}^2 is positive relative to every benchmark. Although statistical significance does not always follow the same pattern, the magnitude of improvement tends to increase with horizon. For instance, relative to the stock’s time-series mean ($\overline{RX}_{i,t}$), R_{OOS}^2 rises from about 1.9% at 30 days to 15.7% at one year, while relative to the MW bound ($MW_{i,t}$), it increases from about 0.7% to 8.0% over the same horizons. The gains are consistently significant against simple baselines such as historical averages, constant-return rules, and implied-variance predictors, but significance against MW and KT is limited to shorter horizons. Compared with GLB, the incremental gains are small (0.1%–1.8%) and generally not significant, except at the 182-day horizon. Overall, the lower bound delivers meaningful improvements over simple predictors and performs competitively against existing bounds, with GLB remaining as the strongest benchmark.

Panel B presents results for the upper bound under the default parameter set. In contrast to the lower bound, the upper bound often yields a negative R_{OOS}^2 , particularly at short horizons, even relative to simple benchmarks such as constant-return rules and historical market averages. This pattern is consistent with the tightness tests in Table 2, which indicated that the upper bound is looser than the lower bound. Additional results for the calibrated bounds and for the midpoint specification are reported in the Online Appendix Table OA.1; in brief,

calibration strengthens the lower-bound performance, and the midpoint results closely track the lower-bound patterns.

4.2 Bounds With Intertemporal Hedging

4.2.1 Properties of the Bounds

We now turn to the multi-period setting and incorporate intertemporal hedging demands. To differentiate the resulting bounds from the static bounds, we refer to them as hedging bounds. Unlike the one-period setting, where only the forecast horizon T_1 matters, the multi-period setting introduces an additional investment horizon T_N over which investors maximize utility. We consider forecast horizons $T_1 \in \{30, 91, 182, 365 \text{ days}\}$ and investment horizons $T_N \in \{91, 182, 365, 547, 730 \text{ days}\}$, focusing on cases with $T_N > T_1$, as $T_N = T_1$ collapses to the one-period setting. Table 6 reports summary statistics for the hedging bounds, with Panel A presenting results for the hedging lower bound (HLB) and Panel B for the hedging upper bound (HUB). In each panel, we report two sets of bounds: those based on the default preference parameters and those based on the calibrated parameters.

Panel A of Table 6 shows that incorporating hedging substantially raises the lower bounds, with the effect becoming more pronounced as the investment horizon lengthens. For example, at the 30-day forecast horizon, the average annualized HLB rises from 9.1% when $T_N = 91$ days to 15.7% when $T_N = 730$ days, compared with 8.1% in the one-period setting without hedging (Table 1). To gauge the quantitative contribution of hedging, we compute the share of the HLB attributable to intertemporal hedging by subtracting the static lower bound from HLB and scaling the difference by HLB, i.e., $(\text{HLB} - \text{LB})/\text{HLB}$. This ratio grows sharply with T_N , from 7.9% at $T_N = 91$ days to 50.1% at $T_N = 730$ days. Similar patterns hold for the data-driven calibration bounds (HLB*) and for longer forecast horizons ($T_1 = 91, 182, 365$ days), where both the mean HLB* and the hedging share ratio increase consistently with T_N .

Panel B shows analogous results for the upper bound. At the 30-day forecast horizon, the average annualized HUB increases from 17.9% at $T_N = 91$ days to 26.0% at $T_N = 730$ days. The contribution of hedging, measured by $(\text{HUB} - \text{UB})/\text{HUB}$, rises correspondingly from 4.4% to 40.2% over the same range. These patterns remain robust when using data-calibrated preference parameters and longer forecast horizons.

[Insert Table 6 around here]

Figure 3 illustrates the effect of intertemporal hedging for representative investment horizons of $T_N = 365$ and 730 days. Two patterns emerge clearly. First, for a given forecast horizon T_1 , both the mean bounds and the hedging share ratios increase with T_N . Second, for a given investment horizon T_N , both the mean bounds and the corresponding hedging share decline as T_1 increases. Together, these results indicate that intertemporal hedging plays a more prominent role when the gap between the forecast and investment horizons is larger, underscoring the economic importance of hedging motives in shaping expected returns. Additional time-series evidence on the hedging contribution is reported in the Online Appendix (Figure OA.2), showing pronounced time variation: hedging shares rise sharply during recessions—most notably around the 2008 global financial crisis—and moderate in expansions.

[Insert Figure 3 around here]

The positive contribution of intertemporal hedging to expected returns is not mechanical, but reflects the underlying preference configuration. In our theoretical framework, the bounds are governed by the four preference parameters $(\tau, \rho, \kappa, \xi)$, and Equation (2.8) specifies a set of inequality restrictions on these parameters that define economically admissible configurations. Our calibration choices satisfy these restrictions and, under such configurations, the intertemporal hedging component is positive, so that hedging raises the expected excess return on individual stocks. By contrast, if the signs of all preference parameters in Equation (2.8) except $\frac{1}{\tau}$ were reversed, the sign of the hedging component would flip, and intertemporal hedging would lower rather than raise expected excess returns.⁸ Chabi-Yo, Gourier, and Langlois (2025) provide an extensive analysis of this mechanism at the market level, showing that the link between intertemporal hedging and the expected excess market return is governed by the investor’s utility specification. In the same spirit, we do not impose a particular functional form on preferences; instead, we work with economically reasonable calibrations that determine the sign and magnitude of intertemporal hedging effects on in-

⁸For illustration, in the Online Appendix (OA.8) we apply our methodology to derive an exact option-based expression for expected excess returns under CRRA utility, both with and without intertemporal hedging. Under CRRA, intertemporal hedging reduces expected excess returns on individual stocks when risk aversion exceeds one and increases them when risk aversion is below one.

dividual expected excess returns. Thus, the increasing effect of hedging on expected returns reflects preference configurations that place greater weight on adverse future states in a way that is both theoretically admissible and consistent with empirically plausible parameter values.

We next examine the tightness of the hedging bounds using the regression specification in Eq. (4.1). For each forecast horizon T_1 , we report results for the investment horizon T_N that yields the best fit, defined as having the slope coefficient closest to one.⁹ Table 7 presents the results. Panel A reports estimates for the hedging lower bound. Across all forecast horizons, the intercepts are statistically indistinguishable from zero, and the slope coefficients are not significantly different from one. Consistently, Wald tests fail to reject the joint null hypothesis that $\beta_0 = 0$ and $\beta_1 = 1$ at any horizon. These results indicate that the hedging lower bound provides a tight bound on conditional expected excess returns and serves as a reliable proxy.

[Insert Table 7 around here]

Panel B reports the results for the hedging upper bound. When $T_1 = 30$ or 91 days, the slope coefficients are below one and statistically different from unity, and Wald tests reject the joint null, indicating that the hedging upper bound is relatively loose at short forecast horizons. At longer horizons ($T_1 = 182$ or 365 days), the slope coefficients are not significantly different from one, and the Wald tests fail to reject the joint null, suggesting improved tightness as the forecast horizon lengthens. Panel C considers the midpoint of the hedging lower and upper bounds. At the 30-day forecast horizon, the slope coefficient is significantly different from one, though the joint null cannot be rejected. For longer horizons, the slope coefficients remain close to one, and the Wald tests consistently fail to reject the joint null. These results indicate that both the hedging upper bound and the midpoint serve as reliable proxies for conditional expected returns when the forecast horizon is sufficiently long.

We now assess whether the hedging bounds price assets consistently across levels of aggregation, focusing on the coherence between individual stocks and sector ETFs. Using the same ETF–component comparison framework as in Figure 2, we calculate the daily differences between the annualized 30-day expected return bounds of nine S&P 500 sector

⁹Results for other forecast–investment horizon combinations are similar and available upon request.

ETFs and the value-weighted average bounds of their constituent stocks, and then take monthly averages of these differences. Figure 4 plots the resulting time series for the hedging lower and upper bounds (HLB and HUB) and their midpoint (HLUB), together with their static counterparts (LB, UB, and LUB), for the case with an investment horizon of $T_N = 365$ days.

[Insert Figure 4 around here]

The figure shows that the HLB series (blue solid line) closely tracks its static counterpart (LB) and often lies even closer to zero, indicating improved pricing coherence between the ETF-level and component-level bounds. Occasionally, however, the HLB series dips below zero, most notably during recessionary periods, suggesting that intertemporal hedging may temporarily weaken pricing alignment when market uncertainty intensifies. The HUB series nearly overlaps with its static counterpart (UB), implying that the incorporation of hedging adds little incremental effect on pricing consistency at the upper end of the return distribution. Overall, these results suggest that introducing intertemporal hedging preserves the consistency of our expected-return bounds in jointly pricing individual stocks and aggregated sector portfolios.

4.2.2 The Bounds and Future Returns

This subsection investigates how the inclusion of intertemporal hedging affects the predictive performance of the bounds. Parallel to the analyses in Section 4.1.2, we evaluate whether the hedging bounds contain information about future returns and how their predictive content compares with that of benchmark models. Specifically, we examine portfolio returns sorted by the hedging bounds, assess their incremental predictive power through double sorts, and evaluate their out-of-sample forecasting accuracy relative to alternative expected-return measures. This parallel analysis allows us to compare the empirical implications of incorporating hedging with those obtained under the static one-period framework.

Table 8 reports returns for portfolios formed on the hedging lower bound (Panel A) and the hedging upper bound (Panel B). Each month, stocks are ranked into quintiles based on the respective bound, and we report the realized returns of the lowest and highest quintiles as well as the high–low spreads. The exercise is conducted across multiple forecast horizons ($T_1 \in \{30, 91, 182, 365 \text{ days}\}$) and investment horizons ($T_N \in \{91, 182, 365, 547, 730 \text{ days}\}$),

considering all combinations with $T_N > T_1$. Across all specifications, portfolio returns increase monotonically with the hedging bounds, and the high–low spreads are both statistically significant and economically meaningful, confirming that incorporating intertemporal hedging preserves the strong predictive content of the bounds.

[Insert Table 8 around here]

Two noteworthy patterns emerge. First, for a given forecast horizon T_1 , the magnitude of the high–low spread tends to decline as the investment horizon T_N lengthens, with the effect most pronounced when T_1 is short. For example, in Panel A, when $T_1 = 30$ days, the spread decreases from 10.5% at $T_N = 91$ days to 8.9% at $T_N = 730$ days. Second, for a given investment horizon T_N , the spread increases with the forecast horizon T_1 . For instance, in Panel A with $T_N = 730$ days, the spread rises from 8.9% at $T_1 = 30$ days to 17.2% at $T_1 = 365$ days. Panel B shows that sorting on the hedging upper bound nearly identical patterns, with spreads generally ranging between 8% and 17% depending on the specification. Overall, these findings indicate that both the HLB and HUB contain strong predictive information for future stock returns and generate robust investment signals.¹⁰

In Table 9, we evaluate the incremental predictive content of the hedging bounds relative to the benchmarks of MW, KT, GLB, and our static model bounds using a double-sort procedure. Specifically, stocks are first sorted into terciles based on MW, KT, GLB, or our static bounds, and then further sorted into terciles by the hedging bounds. We then examine the high–low return spreads within each group. The hedging bounds continue to generate economically meaningful and statistically significant spreads in a substantial number of cases, even after conditioning on the benchmark levels.

[Insert Table 9 around here]

The table reports results for the case with $T_1 = 30$ days and $T_N = 365$ days.¹¹ Panel A presents results based on bounds constructed with the default preference parameters. When conditioning on the high MW tercile, the high–low spreads from the HLB and HUB are 6.7% ($p = 0.006$) and 6.5% ($p = 0.014$), respectively, both significantly positive. Conditioning on

¹⁰The sorting performance is robust if we use the midpoint of HLB and HUB. The results are provided in Online Appendix Table OA.3.

¹¹Results for other horizon combinations are similar and omitted for brevity, but are available upon request.

KT yields similar results: the HLB and HUB deliver spreads of 5.3% ($p = 0.024$) and 4.3% ($p = 0.048$), respectively, within the high KT tercile. When conditioning on GLB, both bounds produce significant spreads of 4–6% ($p < 0.05$) in the low and middle terciles, though not in the high tercile.

Importantly, when we condition on the midpoint of our static model’s lower and upper bounds, the incremental effect of hedging is also substantial. The HLB delivers spreads of 2.1% ($p = 0.013$) in the middle tercile and 7.4% ($p = 0.004$) in the high tercile, while the HUB yields a spread of 6.5% ($p = 0.013$) in the high tercile. Panel B, which uses bounds constructed with calibrated preference parameters, shows broadly similar results. Taken together, these findings indicate that the hedging bounds provide complementary predictive power relative to the bounds from leading one-period static models.

We next compare the out-of-sample forecasting performance of the hedging bounds with leading benchmarks using the R_{OOS}^2 measure given in Eq.(4.2). To conserve space, Table 10 reports results only for investment horizons $T_N = 365$ and 730 days.¹² Panel A reports results for the hedging lower bound under the default parameter set. Out of the 31 comparisons,¹³ R_{OOS}^2 is positive in 27 cases and significantly positive in 17. The gains become larger as the forecast horizon lengthens for a given T_N . For instance, when $T_N = 365$ days, R_{OOS}^2 relative to MW increases from about 1% ($p = 0.002$) at $T_1 = 30$ days to 12% ($p = 0.021$) at $T_1 = 365$ days. Relative to our static lower bound, the incremental gains are more modest in magnitude—ranging from 0.3% ($p = 0.071$) at $T_1 = 30$ days to 1.2% ($p = 0.014$) at $T_1 = 182$ days—but remain statistically significant at conventional levels. These findings highlight that the hedging lower bound improves upon static counterparts and delivers economically meaningful forecasting gains.

[Insert Table 10 around here]

Panel B reports results for the hedging upper bound using default parameters. In contrast to the hedging lower bound, the HUB frequently yields negative R_{OOS}^2 , especially when $T_N = 730$ days. Additional results for the calibrated and midpoint variants yield similar

¹²Results for other horizons are similar and available upon request.

¹³Across the four benchmark models (MW, KT, GLB, and our static lower/upper bounds) and four forecast horizons ($T_1 = 30, 91, 182, 365$ days), this yields 32 cases; since our multi-period hedging bounds collapse to the one-period static bounds when $T_1 = T_N$, we exclude the $T_1 = T_N$ case for the static lower and upper bounds, leaving 31 comparisons. Results for other horizons are similar and available upon request.

conclusion and are relegated to the Online Appendix Table [OA.4](#). Overall, the evidence indicates that intertemporal hedging enhances forecast accuracy in predicting stock-level expected returns, both relative to conventional benchmarks and relative to the nested static specification.

4.3 Comparison of With vs. Without Hedging

4.3.1 Prevalence and Cross-Sectional Properties of Hedging Contributions

To shed light on whether hedging is merely a marginal adjustment or a central component of expected return formation, this subsection evaluates the prevalence and magnitude of intertemporal hedging effects in the pooled sample across firms and time. While previous analyses have shown that incorporating hedging raises the overall level of the bounds, they do not reveal whether this effect is limited to a small subset of firms or pervasive across the market. To address this, we compute, for each firm and month, the proportion of the hedging bound attributable to intertemporal hedging and examine its distribution in the pooled sample.

For each forecast horizon $T_1 \in \{30, 91, 182, 365\}$ (rows) and investment horizon $T_N \in \{91, 182, 365, 547, 730\}$ (column blocks), Table [11](#) classifies firm-month observations by the share of the bound that is due to intertemporal hedging, measured as $(HLB - LB)/HLB$ and $(HUB - UB)/HUB$. The distribution is reported across five ranges: $[0\%, 20\%)$, $[20\%, 40\%)$, $[40\%, 60\%)$, $[60\%, 80\%)$, and $[80\%, 100\%]$. For a given forecast horizon T_1 , the mass shifts toward higher ranges as T_N lengthens. For example, for the HLB with $T_1 = 30$ days, when $T_N = 91$ days, almost all firms lie in the $[0\%, 20\%)$ range ($\approx 99.5\%$). When $T_N = 365$ days, about 22% of firms remain in the lowest range, while the majority fall in the $[20\%, 40\%)$ range ($\approx 67\%$). By $T_N = 730$ days, the lowest group shrinks to only 3%, while most firms move to the $[40\%, 60\%)$ range ($\approx 61\%$), with notable shares in $[20\%, 40\%)$ ($\approx 20\%$) and $[60\%, 80\%)$ ($\approx 15\%$).

[Insert Table [11](#) around here]

The qualitative pattern for the HUB is similar to that for the HLB but quantitatively more muted. For a given forecast horizon T_1 , increasing the investment horizon T_N shifts firms out of the $[0\%, 20\%)$ range, but the shift is less pronounced than for the HLB. For

example, when $T_1 = 30$ and $T_N = 730$, about 16% of firms remain in the lowest range, compared with only 3% for the HLB; when $T_1 = 91$ and $T_N = 730$, the gap is even more pronounced, with 29% versus 2%. This asymmetry suggests that the lower bound is more sensitive to state-variable hedging components, consistent with earlier evidence that the lower bound is tighter than the upper bound.

We find similar patterns for longer forecast horizons. The farther T_N exceeds T_1 , the larger the fraction of the bound attributable to hedging. This is consistent with the notion that intertemporal hedging motives matter most when investors care about future changes in investment opportunities over horizons that extend beyond the forecasting window. For short-term forecasts made by long-horizon investors, hedging accounts for a substantial share of the bounds, with the bulk lying in the [20%, 60%) range. Thus, hedging materially raises the bounds for most firms, reinforcing the conclusion that intertemporal hedging is essential for empirical assessments of expected returns.

To further characterize the cross-sectional properties of these hedging contributions, the Online Appendix extends the analysis in two directions (see [OA.6](#) and [OA.7](#)). First, portfolio sorts and Fama–MacBeth regressions show that the hedging bounds preserve standard cross-sectional patterns in expected returns, while the hedging share exhibits systematic heterogeneity across firm characteristics: it tends to be smaller for firms with greater market and idiosyncratic risk exposures and larger for firms with stronger profitability and recent performance. Second, relating hedging contributions to [Campbell et al. \(2018\)](#)’s news betas reveals that discount-rate news betas explain the largest share of the cross-sectional variation in hedging contributions, followed by volatility-news betas, with cash-flow betas accounting for a relatively small fraction. Although hedging contributions are significantly related to cash-flow, discount-rate, and volatility news betas, they jointly explain only a small portion of the overall variation, suggesting that our forward-looking bounds capture additional dimensions of intertemporal risk beyond the conventional news decomposition.

Motivated by the possibility that hedging demands vary systematically across sectors, we examine industry-level intertemporal hedging components using the Fama–French 12-industry classification (untabulated). Cross-industry dispersion is limited for the lower bound, while the upper bound shows somewhat greater variation. Industries with relatively higher average hedging shares include Consumer Nondurables, Chemicals, and Utilities. Overall, however, average hedging shares are broadly similar across industries, suggest-

ing that the prevalence of intertemporal hedging documented above is not driven by a small set of sectors.

4.3.2 Decomposition of Hedging Contributions

This subsection decomposes the incremental effect of intertemporal hedging into its underlying moment components to identify which aspects of the conditional return distribution drive the difference between the hedging and non-hedging bounds. Unlike the one-period static bounds, the hedging bounds incorporate additional components that reflect investors' concern about future changes in investment opportunities. In particular, the term $\mathcal{M}_{i,M,t}^{*(j,k-j)} = \mathbb{E}_t^* \left[\mathcal{R}_{i,t \rightarrow T_1} f_{T_1}^{(j,k-j)} \right]$ contains the factor $f_{T_1}^{(j,k-j)} = \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j}$, which captures higher-order interactions between contemporaneous and future market returns. The expectation $\mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j}$ represents the *forward moments of future market returns*, linking the current pricing kernel to the evolution of market risk beyond the forecast horizon. These forward moments embed the intertemporal hedging channel that distinguishes the multi-period framework from its static counterpart. We classify the terms in the hedging bounds by the order of the forward moments ($k - j$) and aggregate all terms sharing the same value of $k - j$. The resulting components are interpreted according to the order of the forward moments: when $k - j = 2$, the aggregated terms are associated with the future second moment of market returns (*variance-related component*); when $k - j = 3$, with the future third moment (*skewness-related component*); and when $k - j = 4$, with the future fourth moment (*kurtosis-related component*). We quantify the contribution of variance-, skewness-, and kurtosis-related terms to the total hedging effect.

To illustrate the decomposition, we start from the numerator of the hedging lower bound, denoted as HLB_{num} . For the fourth-order approximation ($\mathcal{K} = 4$) based on the formulation in Eq.(2.33), HLB_{num} can be expressed as

$$\text{HLB}_{\text{num}} = \sum_{k=1}^4 \sum_{j=0}^k \frac{a_{j,k-j}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \mathcal{M}_{i,M,t}^{*(j,k-j)}.$$

We group terms with the same value of $k - j$ (denoted as m), and rewrite it as

$$\text{HLB}_{\text{num}} = \sum_{m=0}^4 \sum_{j=\max\{0, 1-m\}}^{4-m} \frac{a_{j,m}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^m} \mathcal{M}_{i,M,t}^{*(j,m)}.$$

This reformulation allows us to isolate the contribution of each m term and connect it to its economic interpretation:

- When $m = 0$, the inner summation coincides exactly with the numerator of the one-period lower bound in Eq.(2.10), denoted as LB_{num} ; that is,

$$\sum_{j=1}^4 \frac{a_{j,0}}{R_{f,t \rightarrow T_1}^j} \mathcal{M}_{i,M,t}^{*(j,0)} = \text{LB}_{\text{num}}.$$

- When $m = 1$, $\mathcal{M}_{i,M,t}^{*(j,1)} = 0$ by construction, so this term vanishes.
- When $m = 2, 3, 4$, the remaining terms correspond to components associated with the second-, third-, and fourth-order conditional moments of future market returns—namely, variance-, skewness-, and kurtosis-related components—in the intertemporal hedging effect.

Accordingly, the numerator of the hedging lower bound can be expressed as

$$\text{HLB}_{\text{num}} = \text{LB}_{\text{num}} + \underbrace{\sum_{j=0}^2 \frac{a_{j,2}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^2} \mathcal{M}_{i,M,t}^{*(j,2)}}_{m=2} + \underbrace{\sum_{j=0}^1 \frac{a_{j,3}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^3} \mathcal{M}_{i,M,t}^{*(j,3)}}_{m=3} + \underbrace{\frac{a_{0,4}}{R_{f,T_1 \rightarrow T_N}^4} \mathcal{M}_{i,M,t}^{*(0,4)}}_{m=4}.$$

Hence, the incremental contribution of intertemporal hedging relative to the static bound can be decomposed as

$$\text{HLB}_{\text{num}} - \text{LB}_{\text{num}} = \underbrace{\text{Variance}}_{m=2} + \underbrace{\text{Skewness}}_{m=3} + \underbrace{\text{Kurtosis}}_{m=4}.$$

and the proportion of each moment-related component in the hedging effect is given by

$$\begin{aligned} \text{Variance share} &= \frac{\text{Variance}}{\text{HLB}_{\text{num}} - \text{LB}_{\text{num}}}, \\ \text{Skewness share} &= \frac{\text{Skewness}}{\text{HLB}_{\text{num}} - \text{LB}_{\text{num}}}, \\ \text{Kurtosis share} &= \frac{\text{Kurtosis}}{\text{HLB}_{\text{num}} - \text{LB}_{\text{num}}}. \end{aligned}$$

The same decomposition logic applies to the hedging upper bound, where the corresponding numerator is partitioned into the numerator of the static upper bound and the variance-, skewness-, and kurtosis-related terms in an analogous manner.

Table 12 reports the decomposition results for each forecast horizon $T_1 \in \{30, 91, 182, 365 \text{ days}\}$ (rows) and investment horizon $T_N \in \{91, 182, 365, 547, 730 \text{ days}\}$ (column blocks). Two systematic patterns stand out. First, variance and skewness jointly account for the majority of the hedging contributions, whereas kurtosis plays a comparatively minor role. In the lower-bound specification, variance explains roughly 34–51% and skewness 36–45%, leaving only 6–29% for kurtosis. For the upper bound, the corresponding ranges are 41–59%, 27–41%, and 6–26%, respectively. Second, holding the investment horizon T_N constant, lengthening the forecast horizon T_1 reduces the share of variance while increasing that of kurtosis. For example, in the lower-bound specification, when $T_N = 730$ days, the variance share falls from 49.8% to 35.3%, while the kurtosis share rises from 8.7% to 28.1% as T_1 increases from 30 to 365 days. This pattern suggests that as forecast horizons lengthen, the role of variance in the hedging contribution diminishes, whereas the importance of tail risk captured by kurtosis increases.

[Insert Table 12 around here]

Figure 5 illustrates these relationships more clearly by fixing the investment horizon at representative longer values, $T_N = 365$ and 730 days, and varying the forecast horizon T_1 . For each fixed T_N , the figure plots the average share of the hedging contribution attributable to variance, skewness, and kurtosis across different T_1 values. The left panels show the results for the lower bound, and the right panels for the upper bound. For both bounds, the trade-off between variance and kurtosis is clear and monotonic: as T_1 increases, the variance share declines while the kurtosis contribution rises. The share of skewness also tends to decrease with longer horizons, though the pattern is less monotonic across intermediate horizons. Additional time-series evidence on the decomposition is reported in the Online Appendix (Figure OA.3), which tracks the cross-sectional average variance-, skewness-, and kurtosis-shares over time; the main patterns remain unchanged, with variance and skewness as the primary drivers and kurtosis becoming relatively more important as T_1 increases. As a robustness check, we also compute moment shares using the hedging bound itself (HLB or HUB) as the denominator, rather than the incremental hedging effect (HLB–LB or

HUB–UB), and obtain similar patterns (see Figures OA.4-OA.5 and Table OA.6 in Online Appendix OA.9).

[Insert Figure 5 around here]

5 Conclusion

This paper develops a framework for bounding the conditional expected excess returns of individual stocks in a dynamic multi-period economy. By nesting the one-period benchmark within a richer setting that allows for portfolio rebalancing, we show how intertemporal hedging considerations alter the stochastic discount factor and thereby shape stock-level risk premia. The resulting lower and upper bounds are expressed in terms of higher-order risk-neutral cross-moments between the market and individual stocks, quantities that can be recovered from option prices and implemented without reliance on realized returns.

Empirically, our results show that the proposed bounds are tight, strongly correlated with future returns, and generate economically large and statistically significant investment signals. They deliver incremental predictive power relative to leading benchmarks and achieve lower out-of-sample errors. Incorporating intertemporal hedging further enhances these properties, underscoring its importance for explaining individual stock return risk premia. Finally, by contrasting our utility-free bounds with the CRRA case—under which the intertemporal-hedging component turns negative and the implied expected-return proxy performs worse in the data—we highlight that the impact of hedging depends on preference restrictions. Overall, our analysis complements the option-implied equity premium and intertemporal asset pricing literature by providing tractable, forward-looking, and stock-specific measures of expected returns.

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Table 1 Summary Statistics of Static Bounds

This table reports descriptive statistics for the static bounds on conditional expected excess returns at the individual-stock level. Panel A uses the default preference parameters $[\tau, \rho, \kappa, \xi]=[1, 2, 4, 8]$ and reports the mean, standard deviation, skewness, and selected percentiles of the lower and upper bounds (LB and UB). Panel B reports the corresponding statistics for optimized bounds (LB^* and UB^*), where preference parameters are chosen to maximize out-of-sample forecast accuracy of the midpoint $(LB+UB)/2$ relative to the default parameter values $[1, 2, 4, 8]$. The optimal parameter values (common to LB^* and UB^* within each maturity) are $[1, 2, 5, 9.5]$ (30 days), $[1, 2, 4, 10]$ (91 days), $[2/3, 2, 4, 8]$ (182 days), and $[1, 2, 5, 10]$ (365 days). The sample spans January 1996 to August 2023.

Panel A. With Default Preference Parameters								
Statistics	30 Days		91 Days		182 Days		365 Days	
	LB	UB	LB	UB	LB	UB	LB	UB
Mean	0.081	0.139	0.083	0.166	0.086	0.168	0.094	0.164
Standard Deviation	0.059	0.132	0.061	0.123	0.059	0.115	0.063	0.108
Skewness	1.953	1.628	2.397	1.159	2.386	1.539	2.301	1.783
1%	0.017	0.016	0.020	0.024	0.024	0.027	0.027	0.034
10%	0.029	0.033	0.032	0.041	0.036	0.054	0.040	0.066
50%	0.064	0.086	0.065	0.131	0.069	0.140	0.076	0.134
90%	0.153	0.348	0.153	0.343	0.154	0.322	0.168	0.304
99%	0.310	0.567	0.332	0.547	0.330	0.578	0.356	0.570
Panel B. With Calibrated Preference Parameters								
Statistics	30 Days		91 Days		182 Days		365 Days	
	LB^*	UB^*	LB^*	UB^*	LB^*	UB^*	LB^*	UB^*
Mean	0.086	0.165	0.085	0.177	0.148	0.210	0.108	0.174
Standard Deviation	0.072	0.175	0.067	0.143	0.098	0.134	0.081	0.118
Skewness	3.026	1.761	3.015	1.521	1.912	1.455	2.749	1.916
1%	0.017	0.015	0.021	0.024	0.039	0.043	0.029	0.035
10%	0.029	0.033	0.032	0.041	0.059	0.076	0.043	0.068
50%	0.065	0.092	0.066	0.135	0.119	0.174	0.084	0.140
90%	0.161	0.440	0.156	0.368	0.271	0.392	0.195	0.322
99%	0.391	0.749	0.368	0.675	0.526	0.678	0.462	0.628

Table 2 Tightness Test of Static Bounds

This table presents Wald tests of $H_0 : \beta_0 = 0, \beta_1 = 1$ and $H_0 : \beta_1 = 1$ using all stocks in the sample, with observations winsorized at the 1% level. Reported p -values (in parentheses) are based on 10,000 bootstrap replications used to estimate the covariance matrix and account for overlapping observations. Tests are conducted for the lower bound (LB; Panel A), the upper bound (UB; Panel B), and their midpoint ($(LB + UB)/2$; Panel C) across maturities from 30 to 365 days. Preference parameters for the optimized bounds (LB^* and UB^*) are reported in Table 1. The sample spans January 1996 to August 2023.

Panel A. Lower Bound								
Statistics	LB				LB^*			
	30 Days	91 Days	182 Days	365 Days	30 Days	91 Days	182 Days	365 Days
β_0	−0.0058 (0.889)	0.0138 (0.745)	−0.0548 (0.249)	−0.0386 (0.613)	−0.0143 (0.704)	0.0111 (0.808)	−0.0278 (0.470)	−0.0237 (0.584)
β_1	1.4927 (0.006)	1.3702 (0.024)	1.7105 (0.000)	1.3991 (0.082)	1.6297 (0.000)	1.3966 (0.029)	0.9671 (0.000)	1.2559 (0.011)
$R^2(\%)$	0.72	1.44	4.82	2.36	0.83	1.41	5.26	6.88
$H_0 : \beta_0 = 0, \beta_1 = 1$	0.426	0.327	0.223	0.879	0.287	0.343	0.572	0.848
$H_0 : \beta_1 = 1$	0.366	0.541	0.086	0.62	0.175	0.535	0.892	0.606
Panel B. Upper Bound								
Statistics	UB				UB^*			
	30 Days	91 Days	182 Days	365 Days	30 Days	91 Days	182 Days	365 Days
β_0	0.0593 (0.022)	0.0533 (0.088)	−0.0111 (0.834)	−0.0417 (0.382)	0.0588 (0.023)	0.0522 (0.095)	−0.0207 (0.650)	−0.0369 (0.411)
β_1	0.3499 (0.001)	0.4207 (0.038)	0.7833 (0.015)	0.9285 (0.005)	0.3506 (0.001)	0.424 (0.034)	0.649 (0.002)	0.8544 (0.005)
$R^2(\%)$	0.43	0.75	2.89	5.53	0.44	0.78	3.89	5.83
$H_0 : \beta_0 = 0, \beta_1 = 1$	0.000	0.015	0.250	0.256	0.000	0.014	0.006	0.194
$H_0 : \beta_1 = 1$	0.000	0.004	0.500	0.831	0.000	0.004	0.098	0.632
Panel C. Midpoint								
Statistics	$(LB + UB)/2$				$(LB^* + UB^*)/2$			
	30 Days	91 Days	182 Days	365 Days	30 Days	91 Days	182 Days	365 Days
β_0	0.0358 (0.206)	0.0366 (0.289)	−0.0309 (0.555)	−0.045 (0.330)	0.035 (0.216)	0.0356 (0.304)	−0.0283 (0.502)	−0.035 (0.421)
β_1	0.6538 (0.000)	0.6987 (0.026)	1.1911 (0.004)	1.2033 (0.004)	0.6528 (0.000)	0.699 (0.025)	0.8005 (0.001)	1.0398 (0.005)
$R^2(\%)$	0.56	0.97	3.62	6.12	0.57	0.99	4.57	6.39
$H_0 : \beta_0 = 0, \beta_1 = 1$	0.141	0.554	0.839	0.577	0.134	0.566	0.118	0.569
$H_0 : \beta_1 = 1$	0.049	0.339	0.645	0.629	0.045	0.334	0.384	0.915

Table 3 Portfolios Sorted by Static Bounds

This table reports average annualized excess returns for portfolios sorted on the static lower bound (LB; Panel A), static upper bound (UB; Panel B), or their midpoint $(LB + UB)/2$ (Panel C), with maturities shown at the top of each column. Reported p -values (in parentheses) are based on the monotonicity test of [Patton and Timmermann \(2010\)](#) using 10,000 stationary block bootstrap replications. Preference parameters for the optimized bounds (LB^* and UB^*) are reported in Table 1. The sample spans January 1996 to August 2023.

Panel A. Sorting by Lower Bound (LB)								
Portfolio	30 Days		91 Days		182 Days		365 Days	
	LB	LB^*	LB	LB^*	LB	LB^*	LB	LB^*
Low	0.083	0.090	0.091	0.091	0.092	0.093	0.093	0.093
2	0.108	0.105	0.109	0.108	0.110	0.109	0.112	0.112
3	0.124	0.127	0.122	0.122	0.123	0.123	0.122	0.123
4	0.148	0.143	0.148	0.148	0.149	0.150	0.148	0.148
High	0.210	0.198	0.207	0.207	0.230	0.228	0.266	0.266
High – Low	0.127	0.108	0.117	0.116	0.138	0.136	0.172	0.172
p -Value	(0.013)	(0.020)	(0.011)	(0.008)	(0.009)	(0.009)	(0.005)	(0.003)
Panel B. Sorting by Upper Bound (UB)								
Portfolio	30 Days		91 Days		182 Days		365 Days	
	UB	UB^*	UB	UB^*	UB	UB^*	UB	UB^*
Low	0.082	0.094	0.092	0.092	0.092	0.091	0.093	0.093
2	0.107	0.105	0.109	0.108	0.110	0.110	0.111	0.111
3	0.123	0.125	0.123	0.123	0.124	0.124	0.124	0.124
4	0.151	0.147	0.148	0.149	0.149	0.150	0.148	0.148
High	0.207	0.199	0.201	0.203	0.227	0.226	0.264	0.264
High – Low	0.125	0.104	0.110	0.111	0.135	0.135	0.170	0.171
p -Value	(0.014)	(0.021)	(0.015)	(0.012)	(0.010)	(0.009)	(0.006)	(0.006)
Panel C. Sorting by $(LB+UB)/2$								
Portfolio	30 Days		91 Days		182 Days		365 Days	
	$\frac{(LB+UB)}{2}$	$\frac{(LB^*+UB^*)}{2}$	$\frac{(LB+UB)}{2}$	$\frac{(LB^*+UB^*)}{2}$	$\frac{(LB+UB)}{2}$	$\frac{(LB^*+UB^*)}{2}$	$\frac{(LB+UB)}{2}$	$\frac{(LB^*+UB^*)}{2}$
Low	0.093	0.093	0.091	0.091	0.091	0.092	0.093	0.094
2	0.106	0.106	0.108	0.108	0.110	0.110	0.112	0.111
3	0.120	0.120	0.124	0.124	0.123	0.123	0.123	0.123
4	0.152	0.152	0.148	0.147	0.151	0.150	0.148	0.148
High	0.202	0.200	0.203	0.204	0.226	0.227	0.265	0.265
High – Low	0.109	0.107	0.112	0.113	0.134	0.135	0.171	0.171
p -Value	(0.015)	(0.016)	(0.012)	(0.009)	(0.011)	(0.008)	(0.005)	(0.005)

Table 4 Double-Sorted Portfolios by Static Bounds and Other Expected Excess Return Proxies

This table reports high-minus-low spreads in average annualized excess returns between the top and bottom terciles of the second-stage sort—LB, UB, or their midpoint—for double-sorted portfolios with $T_1 = 30$ days. Stocks are first sorted into terciles by MW, KT, or GLB and then sorted by the second-stage measure within each first-stage group. p -values (in parentheses) are based on Newey–West (1987) standard errors. Panel A uses bounds computed under the baseline calibration [1, 2, 4, 8], while Panel B uses bounds computed under horizon-specific optimized calibrations (see Table 1). The sample spans January 1996 to August 2023.

Panel A. With Default Preference Parameters									
	LB			UB			(LB+UB)/2		
Sort by MW	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.019	0.019	0.073	0.014	0.014	0.063	0.007	0.015	0.058
p -Value	(0.180)	(0.140)	(0.005)	(0.049)	(0.104)	(0.016)	(0.585)	(0.382)	(0.017)
Sort by KT	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.017	0.015	0.060	0.012	0.014	0.044	0.012	0.006	0.048
p -Value	(0.030)	(0.082)	(0.013)	(0.077)	(0.061)	(0.063)	(0.323)	(0.646)	(0.022)
Sort by GLB	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.055	0.071	0.066	0.049	0.071	0.069	0.051	0.056	0.069
p -Value	(0.021)	(0.003)	(0.018)	(0.025)	(0.005)	(0.015)	(0.017)	(0.025)	(0.015)
Panel B. With Calibrated Preference Parameters									
	LB^*			UB^*			$(LB^* + UB^*)/2$		
Sort by MW	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.022	0.022	0.067	0.006	0.013	0.072	0.008	0.016	0.058
p -Value	(0.122)	(0.185)	(0.017)	(0.644)	(0.436)	(0.012)	(0.553)	(0.360)	(0.027)
Sort by KT	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.020	0.018	0.053	0.010	0.005	0.061	0.012	0.005	0.047
p -Value	(0.132)	(0.166)	(0.057)	(0.397)	(0.664)	(0.023)	(0.324)	(0.677)	(0.028)
Sort by GLB	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.057	0.057	0.059	0.046	0.055	0.072	0.051	0.056	0.068
p -Value	(0.010)	(0.026)	(0.020)	(0.025)	(0.026)	(0.013)	(0.016)	(0.025)	(0.155)

Table 5 Out-of-sample Forecast Accuracy of Static Bounds

This table reports out-of-sample R^2 (in percent) from predicting future realized excess returns using the lower bound (Panel A) and upper bound (Panel B) computed with the default preference parameters $[\tau, \rho, \kappa, \xi]=[1, 2, 4, 8]$. The benchmarks include the recursive historical mean excess return for stock i from 1990 ($\overline{RX}_{i,t}$), a constant 6% annual excess return (6% p.a.), the recursive historical mean market excess returns from 1996 ($\overline{MKT}_{i,t}$), the historical mean market-implied variance ($\overline{IV}_t$), the historical mean stock-level implied variance ($\overline{IV}_{i,t}$), and the option-based one-period measures $MW_{i,t}$ (Martin and Wagner (2019)), $KT_{i,t}$ (Kadan and Tang (2020)), and $GLB_{i,t}$ (Chabi-Yo, Dim, and Vilkov (2023)). Reported p -values (in parentheses) test $H_0 : R^2 \leq 0$ and are computed using a stationary block bootstrap with 10,000 replications. The sample spans January 1996 to August 2023.

Panel A. Lower Bound								
	$\overline{RX}_{i,t}$	6% p.a.	$\overline{MKT}_t$	$\overline{IV}_t$	$\overline{IV}_{i,t}$	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$
30 Days	1.900 (0.000)	0.788 (0.007)	0.758 (0.007)	0.889 (0.006)	1.923 (0.000)	0.674 (0.003)	2.026 (0.005)	0.111 (0.589)
91 Days	5.073 (0.000)	1.718 (0.017)	1.629 (0.016)	2.059 (0.015)	3.993 (0.002)	2.693 (0.001)	3.894 (0.057)	0.671 (0.168)
182 Days	10.895 (0.000)	5.032 (0.006)	4.928 (0.004)	5.62 (0.005)	8.398 (0.000)	4.559 (0.013)	5.004 (0.134)	1.819 (0.040)
365 Days	15.663 (0.000)	3.576 (0.103)	3.459 (0.049)	4.418 (0.090)	9.816 (0.000)	8.056 (0.138)	2.835 (0.398)	0.240 (0.544)
Panel B. Upper Bound								
	$\overline{RX}_{i,t}$	6% p.a.	$\overline{MKT}_t$	$\overline{IV}_t$	$\overline{IV}_{i,t}$	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$
30 Days	0.198 (0.471)	-0.859 (0.878)	-0.893 (0.884)	-0.755 (0.853)	0.213 (0.388)	-0.424 (0.865)	0.833 (0.093)	-1.777 (0.997)
91 Days	2.983 (0.020)	-0.263 (0.556)	-0.386 (0.565)	0.100 (0.509)	2.081 (0.035)	2.033 (0.076)	2.257 (0.000)	-1.388 (0.800)
182 Days	9.177 (0.000)	3.328 (0.177)	3.208 (0.176)	3.926 (0.149)	6.765 (0.000)	3.639 (0.046)	3.373 (0.015)	-0.053 (0.506)
365 Days	17.129 (0.000)	5.632 (0.140)	5.498 (0.121)	6.448 (0.124)	11.322 (0.000)	10.532 (0.015)	3.957 (0.107)	1.261 (0.414)

Table 6 Summary Statistics of Hedging Bounds

This table reports descriptive statistics for the hedging lower bound (HLB, Panel A) and the hedging upper bound (HUB, Panel B) across horizons defined by T_1 and $T_N > T_1$. The table reports mean, standard deviation, and the hedging share, defined as (HLB-LB)/HLB or (HUB-UB)/HUB. The default preference parameters are $[\tau, \rho, \kappa, \xi] = [1, 2, 4, 8]$. For the optimized bounds (HLB^* and HUB^*), preference parameters are chosen to maximize out-of-sample forecast accuracy of the midpoint relative to the default calibration [1,2,4,8]; the horizon-specific optimal values $(T_1, T_N) = [\tau, \rho, \kappa, \xi]$ are (30,91)=[1,2,4.5,8.5], (30,182)=[1,2.5,5,10], (30,356)=[1,2.5,4.5,9], (30,547)=[1,2.5,4.5,9], (30,730)=[1,2.5,4.5,9], (91,182)=[1,2,4.5,8.5], (91,365)=[1,2.5,5,10], (91,547)=[1,2.5,4.5,9], (91,730)=[1,2.5,4.5,9], (182,365)=[1,2,5,9.5], (182,547)=[1,2,4.5,8.5], (182,730)=[1,2.5,5,10], (365,547)=[1,2,5,9.5], (365,730)=[1,2.5,5,10]. The sample spans January 1996 to August 2023.

Panel A. Hedging Lower Bound											
$[T_1, T_N]$		$T_N = 91$		$T_N = 182$		$T_N = 365$		$T_N = 547$		$T_N = 730$	
		HLB	HLB^*	HLB	HLB^*	HLB	HLB^*	HLB	HLB^*	HLB	HLB^*
30	Mean	0.091	0.094	0.100	0.098	0.111	0.106	0.137	0.119	0.157	0.132
	Standard Deviation	0.077	0.080	0.083	0.080	0.079	0.083	0.102	0.088	0.112	0.094
	(Hedge-Unhedge)/Hedge	0.079	0.114	0.172	0.152	0.298	0.215	0.417	0.311	0.501	0.390
91	Mean			0.093	0.097	0.103	0.101	0.123	0.104	0.139	0.112
	Standard Deviation			0.074	0.078	0.072	0.079	0.092	0.080	0.100	0.084
	(Hedge-Unhedge)/Hedge			0.097	0.137	0.233	0.178	0.346	0.198	0.429	0.265
182	Mean					0.101	0.118	0.118	0.127	0.133	0.116
	Standard Deviation					0.074	0.089	0.088	0.092	0.094	0.085
	(Hedge-Unhedge)/Hedge					0.158	0.283	0.284	0.340	0.378	0.269
365	Mean							0.111	0.131	0.127	0.115
	Standard Deviation							0.082	0.095	0.091	0.085
	(Hedge-Unhedge)/Hedge							0.149	0.296	0.271	0.188
Panel B. Hedging Upper Bound											
$[T_1, T_N]$		$T_N = 91$		$T_N = 182$		$T_N = 365$		$T_N = 547$		$T_N = 730$	
		HUB	HUB^*	HUB	HUB^*	HUB	HUB^*	HUB	HUB^*	HUB	HUB^*
30	Mean	0.179	0.182	0.189	0.190	0.184	0.203	0.234	0.223	0.260	0.244
	Standard Deviation	0.189	0.189	0.189	0.188	0.151	0.188	0.188	0.186	0.188	0.184
	(Hedge-Unhedge)/Hedge	0.044	0.062	0.107	0.113	0.210	0.187	0.314	0.283	0.402	0.365
91	Mean			0.186	0.189	0.189	0.197	0.214	0.203	0.229	0.213
	Standard Deviation			0.151	0.153	0.138	0.157	0.161	0.160	0.166	0.164
	(Hedge-Unhedge)/Hedge			0.044	0.062	0.116	0.110	0.193	0.144	0.258	0.198
182	Mean					0.183	0.196	0.199	0.207	0.213	0.207
	Standard Deviation					0.127	0.140	0.140	0.145	0.146	0.148
	(Hedge-Unhedge)/Hedge					0.076	0.140	0.152	0.189	0.219	0.192
365	Mean							0.181	0.196	0.196	0.192
	Standard Deviation							0.127	0.137	0.136	0.136
	(Hedge-Unhedge)/Hedge							0.085	0.166	0.165	0.145

Table 7 Tightness Test of Hedging Bounds

This table presents Wald tests of $H_0 : \beta_0 = 0, \beta_1 = 1$ and $H_0 : \beta_1 = 1$. p -values (in parentheses) are based on 10,000 bootstrap replications used to estimate the covariance matrix and account for overlapping observations. Tests are conducted for the hedging lower bound (Panel A), the hedging upper bound (Panel B), and their midpoint $(HLB + HUB)/2$ (Panel C) across maturities T_1 from 30 to 365 days. For each T_1 , we report the value of T_N that minimizes $|\beta_1 - 1|$ over candidate values of T_N . Preference parameters for the optimized bounds (HLB^* and HUB^*) are reported in Table 6.

Panel A. Hedging Lower Bound								
Statistics	HLB				HLB^*			
	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$
	[30, 365]	[91, 365]	[182, 730]	[365, 730]	[30, 730]	[91, 547]	[182, 547]	[365, 730]
β_0	-0.0008 (0.984)	0.0316 (0.495)	-0.0084 (0.818)	-0.0063 (0.881)	-0.01 (0.808)	0.0201 (0.674)	-0.0234 (0.542)	-0.013 (0.764)
β_1	1.0322 (0.003)	0.8894 (0.078)	0.9207 (0.000)	0.9039 (0.028)	0.9785 (0.003)	1.0477 (0.054)	1.1033 (0.000)	1.068 (0.024)
$R^2(\%)$	0.95	1.39	5.12	6.32	0.89	1.39	5.48	6.62
$H_0 : \beta_0 = 0, \beta_1 = 1$	0.993	0.650	0.848	0.882	0.897	0.657	0.830	0.950
$H_0 : \beta_1 = 1$	0.926	0.827	0.762	0.815	0.948	0.930	0.715	0.886
Panel B. Hedging Upper Bound								
Statistics	HUB				HUB^*			
	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$
	[30, 365]	[91, 182]	[182, 365]	[365, 547]	[30, 91]	[91, 182]	[182, 365]	[365, 547]
β_0	0.0382 (0.194)	0.0501 (0.112)	-0.0171 (0.735)	-0.0341 (0.440)	0.0521 (0.053)	0.0478 (0.131)	-0.0198 (0.677)	-0.0341 (0.440)
β_1	0.3655 (0.000)	0.4178 (0.032)	0.7506 (0.006)	0.808 (0.005)	0.3619 (0.000)	0.4208 (0.028)	0.7087 (0.003)	0.808 (0.005)
$R^2(\%)$	0.60	0.81	3.42	5.77	0.50	0.87	3.90	5.77
$H_0 : \beta_0 = 0, \beta_1 = 1$	0.000	0.009	0.105	0.151	0.000	0.007	0.037	0.151
$H_0 : \beta_1 = 1$	0.000	0.003	0.362	0.507	0.000	0.003	0.216	0.507
Panel C. Hedging Midpoint								
Statistics	$(HLB + HUB)/2$				$(HLB^* + HUB^*)/2$			
	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$	$[T_1, T_N]$
	[30, 91]	[91, 182]	[182, 365]	[365, 547]	[30, 91]	[91, 182]	[182, 365]	[365, 547]
β_0	0.0299 (0.303)	0.0347 (0.319)	-0.0307 (0.517)	-0.0334 (0.440)	0.0267 (0.363)	0.033 (0.345)	-0.0271 (0.531)	-0.0334 (0.440)
β_1	0.6561 (0.000)	0.6676 (0.025)	1.0587 (0.001)	0.9931 (0.006)	0.6591 (0.000)	0.6600 (0.022)	0.9269 (0.001)	0.9931 (0.006)
$R^2(\%)$	0.62	1.04	4.18	6.22	0.67	1.11	4.66	6.22
$H_0 : \beta_0 = 0, \beta_1 = 1$	0.123	0.521	0.710	0.527	0.114	0.498	0.478	0.527
$H_0 : \beta_1 = 1$	0.042	0.264	0.858	0.985	0.04	0.239	0.783	0.985

Table 8 Portfolios Sorted by Hedging Bounds

This table reports average annualized excess returns for portfolios sorted on the hedging lower bound (HLB, Panel A) or hedging upper bound (HUB, Panel B), for each forecast horizon $T_1 \in \{30, 91, 182, 365\}$ (rows) and investment horizon $T_N \in \{91, 182, 365, 547, 730\}$ (column blocks). Reported p -values (in parentheses) are based on the monotonicity test of [Patton and Timmermann \(2010\)](#) using 10,000 stationary block bootstrap replications. Preference parameters for the optimized bounds (HLB^* and HUB^*) are reported in Table 6. The sample spans January 1996 to August 2023.

Panel A. Hedging Lower Bound											
$[T_1, T_N]$		$T_N = 91$		$T_N = 182$		$T_N = 365$		$T_N = 547$		$T_N = 730$	
		HLB	HLB^*	HLB	HLB^*	HLB	HLB^*	HLB	HLB^*	HLB	HLB^*
30	Low	0.091	0.102	0.095	0.104	0.101	0.109	0.105	0.110	0.104	0.105
	High	0.196	0.198	0.193	0.197	0.195	0.192	0.193	0.193	0.194	0.190
	High – Low	0.105	0.096	0.098	0.093	0.093	0.084	0.088	0.084	0.089	0.085
	p -Value	(0.031)	(0.038)	(0.042)	(0.043)	(0.046)	(0.057)	(0.049)	(0.048)	(0.042)	(0.048)
91	Low			0.091	0.092	0.093	0.092	0.094	0.094	0.095	0.093
	High			0.207	0.207	0.206	0.208	0.206	0.206	0.205	0.207
	High – Low			0.116	0.115	0.114	0.116	0.112	0.112	0.110	0.114
	p -Value			(0.008)	(0.008)	(0.009)	(0.008)	(0.009)	(0.009)	(0.009)	(0.007)
182	Low					0.092	0.092	0.093	0.092	0.093	0.093
	High					0.228	0.227	0.229	0.228	0.228	0.228
	High – Low					0.136	0.136	0.136	0.136	0.134	0.135
	p -Value					(0.006)	(0.006)	(0.006)	(0.006)	(0.007)	(0.007)
365	Low							0.094	0.093	0.094	0.093
	High							0.265	0.266	0.265	0.266
	High – Low							0.172	0.173	0.172	0.173
	p -Value							(0.004)	(0.003)	(0.004)	(0.003)
Panel B. Hedging Upper Bound											
$[T_1, T_N]$		$T_N = 91$		$T_N = 182$		$T_N = 365$		$T_N = 547$		$T_N = 730$	
		HUB	HUB^*	HUB	HUB^*	HUB	HUB^*	HUB	HUB^*	HUB	HUB^*
30	Low	0.094	0.094	0.096	0.097	0.100	0.103	0.107	0.111	0.112	0.115
	High	0.197	0.201	0.197	0.201	0.196	0.201	0.197	0.200	0.197	0.196
	High – Low	0.104	0.107	0.102	0.105	0.096	0.099	0.090	0.089	0.085	0.082
	p -Value	(0.025)	(0.031)	(0.033)	(0.038)	(0.040)	(0.041)	(0.045)	(0.051)	(0.054)	(0.051)
91	Low			0.092	0.091	0.093	0.092	0.093	0.093	0.094	0.094
	High			0.203	0.203	0.203	0.203	0.203	0.203	0.203	0.204
	High – Low			0.111	0.111	0.110	0.112	0.110	0.110	0.109	0.110
	p -Value			(0.008)	(0.008)	(0.009)	(0.010)	(0.010)	(0.010)	(0.011)	(0.010)
182	Low					0.092	0.092	0.092	0.092	0.092	0.092
	High					0.227	0.227	0.227	0.227	0.226	0.227
	High – Low					0.135	0.136	0.135	0.136	0.134	0.136
	p -Value					(0.006)	(0.006)	(0.006)	(0.006)	(0.007)	(0.006)
365	Low							0.093	0.093	0.093	0.093
	High							0.264	0.265	0.264	0.264
	High – Low							0.170	0.172	0.171	0.171
	p -Value							(0.004)	(0.003)	(0.004)	(0.005)

Table 9 Double-Sorted Portfolios by Hedging Bounds and Other Expected Excess Return Proxies

This table reports high-minus-low spreads in average annualized excess returns between the top and bottom terciles of the second-stage sort variable—HLB, HUB, or their midpoint—for double-sorted portfolios with $T_1 = 30$ days and $T_N = 365$ days. Stocks are first sorted into terciles by MW, KT, GLB, or the midpoint of our static bounds, and then sorted by the second-stage variable within each first-stage group. p -values (in parentheses) are based on Newey–West (1987) standard errors. Panel A uses bounds computed with the baseline preference parameters [1, 2, 4, 8], while Panel B uses bounds computed with horizon-specific optimized parameters (see Table 6). The sample spans January 1996 to August 2023.

Panel A. With Default Preference Parameters									
Sort by MW	HLB			HUB			(HLB+HUB)/2		
	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.011	0.015	0.067	0.006	0.016	0.065	0.000	0.015	0.068
p -Value	(0.157)	(0.106)	(0.006)	(0.381)	(0.069)	(0.014)	(0.965)	(0.163)	(0.010)
Sort by KT	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.010	0.008	0.053	0.006	0.015	0.043	-0.001	0.012	0.038
p -Value	(0.164)	(0.335)	(0.024)	(0.365)	(0.052)	(0.048)	(0.891)	(0.166)	(0.121)
Sort by GLB	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.046	0.061	0.052	0.038	0.052	0.071	0.035	0.063	0.062
p -Value	(0.024)	(0.010)	(0.229)	(0.040)	(0.025)	(0.133)	(0.001)	(0.001)	(0.028)
Sort by (LB+UB)/2	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.010	0.021	0.074	0.002	0.005	0.065	-0.005	0.007	0.060
p -Value	(0.173)	(0.013)	(0.004)	(0.806)	(0.455)	(0.013)	(0.467)	(0.366)	(0.020)
Panel B. With Calibrated Preference Parameters									
Sort by MW	HLB*			HUB*			(HLB* + HUB*)/2		
	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.004	0.014	0.077	-0.006	0.014	0.073	-0.004	0.015	0.078
p -Value	(0.561)	(0.194)	(0.002)	(0.337)	(0.165)	(0.004)	(0.554)	(0.127)	(0.003)
Sort by KT	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.003	0.011	0.062	-0.009	0.012	0.047	-0.004	0.010	0.051
p -Value	(0.635)	(0.190)	(0.006)	(0.121)	(0.129)	(0.044)	(0.536)	(0.230)	(0.033)
Sort by GLB	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.039	0.057	0.056	0.020	0.052	0.072	0.030	0.060	0.071
p -Value	(0.015)	(0.018)	(0.022)	(0.020)	(0.013)	(0.009)	(0.003)	(0.001)	(0.012)
Sort by (LB* + UB*)/2	Low	Middle	High	Low	Middle	High	Low	Middle	High
High–Low	0.004	0.011	0.078	-0.006	0.002	0.069	-0.002	0.009	0.075
p -Value	(0.589)	(0.172)	(0.001)	(0.374)	(0.832)	(0.008)	(0.752)	(0.212)	(0.004)

Table 10 Out-of-sample Forecast Accuracy of Hedging Bounds

The table reports out-of-sample R^2 (percent) from predicting future realized excess returns using the hedging lower bound (Panel A) and hedging upper bound (Panel B) computed with the default preference parameters $\tau = 1, \rho = 2, \kappa = 4$, and $\xi = 8$. Results are reported for each forecast horizon $T_1 \in \{30, 91, 182, 365\}$ (rows) and investment horizon $T_N \in \{365, 730\}$ (column blocks). Benchmarks, shown in the column headers, include $MW_{i,t}$, $KT_{i,t}$, $GLB_{i,t}$, and the corresponding static bounds (LB/UB). Reported p -values (in parentheses) test $H_0 : R^2 \leq 0$ and are computed using a stationary block bootstrap with 10,000 replications. The sample spans January 1996 to August 2023.

Panel A. Hedging Lower Bound								
	$T_N = 365$				$T_N = 730$			
	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$	$LB_{i,t}$	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$	$LB_{i,t}$
30 Days	1.045 (0.002)	2.667 (0.000)	0.29 (0.260)	0.346 (0.071)	0.642 (0.230)	2.271 (0.009)	-0.116 (0.645)	-0.079 (0.607)
91 Days	3.971 (0.000)	4.527 (0.007)	1.067 (0.175)	0.348 (0.267)	2.72 (0.148)	3.282 (0.132)	-0.223 (0.610)	-0.982 (0.702)
182 Days	5.918 (0.000)	5.678 (0.077)	2.383 (0.001)	1.233 (0.014)	6.497 (0.000)	6.26 (0.010)	2.986 (0.036)	1.785 (0.103)
365 Days	11.946 (0.021)	5.479 (0.224)	2.835 (0.020)	0.000 (0.000)	12.35 (0.004)	5.912 (0.151)	3.281 (0.127)	0.408 (0.393)
Panel B. Hedging Upper Bound								
	$T_N = 365$				$T_N = 730$			
	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$	$UB_{i,t}$	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$	$UB_{i,t}$
$T_1 = 30$	-1.029 (0.955)	0.236 (0.404)	-2.39 (0.997)	-0.643 (0.993)	-3.016 (0.997)	-1.726 (0.903)	-4.404 (1.000)	-2.679 (1.000)
$T_1 = 91$	1.035 (0.306)	1.261 (0.199)	-2.42 (0.858)	-1.034 (0.948)	-2.216 (0.726)	-1.983 (0.755)	-5.785 (0.931)	-4.374 (0.991)
$T_1 = 182$	3.514 (0.059)	3.248 (0.007)	-0.183 (0.512)	-0.147 (0.582)	1.072 (0.352)	0.799 (0.379)	-2.718 (0.727)	-2.722 (0.952)
$T_1 = 365$	10.532 (0.015)	3.957 (0.107)	1.261 (0.414)	0.000 (0.000)	7.221 (0.167)	0.403 (0.527)	-2.393 (0.641)	-3.73 (0.956)

Table 11 Distribution of Firms by Hedging Contributions

This table summarizes the proportion of firms whose intertemporal hedging contributions fall within each specified range. For each forecast horizon $T_1 \in \{30, 91, 182, 365\}$ (rows) and investment horizon $T_N \in \{91, 182, 365, 547, 730\}$ (column blocks), we compute the incremental contribution of intertemporal hedging relative to the static bound— $(\text{HLB} - \text{LB})/\text{HLB}$ and $(\text{HUB} - \text{UB})/\text{HUB}$ —and classify firm-month observations by these ratios. The distribution is reported across five percentage ranges: $[0\%, 20\%)$, $[20\%, 40\%)$, $[40\%, 60\%)$, $[60\%, 80\%)$, and $[80\%, 100\%]$.

	$[T_1, T_N]$	$T_N = 91$		$T_N = 182$		$T_N = 365$		$T_N = 547$		$T_N = 730$	
		HLB	HUB	HLB	HUB	HLB	HUB	HLB	HUB	HLB	HUB
30	$[0\%, 20\%)$	99.52	98.52	80.22	68.83	22.24	37.18	6.03	22.63	3.37	16.01
	$[20\%, 40\%)$	0.43	1.44	19.69	28.95	67.01	41.28	43.79	28.46	20.09	22.18
	$[40\%, 60\%)$	0.00	0.00	0.09	2.19	10.68	19.24	47.42	35.29	61.24	32.22
	$[60\%, 80\%)$	0.00	0.02	0.00	0.01	0.07	2.27	2.75	13.29	15.23	26.95
	$[80\%, 100\%]$	0.05	0.00	0.00	0.02	0.00	0.02	0.01	0.33	0.07	2.64
91	$[0\%, 20\%)$			99.32	99.91	51.64	84.48	6.24	48.13	1.89	28.75
	$[20\%, 40\%)$			0.67	0.08	46.70	15.35	78.38	47.57	47.61	52.89
	$[40\%, 60\%)$			0.00	0.00	1.66	0.16	15.24	4.20	48.02	17.85
	$[60\%, 80\%)$			0.00	0.00	0.00	0.01	0.15	0.08	2.48	0.49
	$[80\%, 100\%]$			0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01
182	$[0\%, 20\%)$					91.52	98.86	21.58	81.01	1.84	47.83
	$[20\%, 40\%)$					8.46	1.12	75.30	18.77	75.21	48.15
	$[40\%, 60\%)$					0.01	0.01	3.11	0.19	21.89	3.96
	$[60\%, 80\%)$					0.01	0.00	0.01	0.02	1.05	0.04
	$[80\%, 100\%]$					0.00	0.00	0.01	0.00	0.01	0.01
365	$[0\%, 20\%)$							94.41	99.64	24.21	83.45
	$[20\%, 40\%)$							5.56	0.34	73.42	16.47
	$[40\%, 60\%)$							0.03	0.02	2.34	0.06
	$[60\%, 80\%)$							0.00	0.00	0.03	0.02
	$[80\%, 100\%]$							0.00	0.00	0.00	0.00

Table 12 Decomposition of Hedging Contributions

This table summarizes the decomposition of the hedging component into contributions from variance, skewness, and kurtosis. We compute the variance-, skewness-, and kurtosis-related components by aggregating the forward moment terms corresponding to the coefficients $a_{j,k-j}$ in Proposition 2.4 for each $k - j$ with varying j to derive the total contribution of each moment in the intertemporal hedging case. Specifically, the component associated with $k - j = 2$ captures the variance-related contribution, $k - j = 3$ captures the skewness-related contribution, and $k - j = 4$ captures the kurtosis-related contribution. We then calculate the incremental contribution of intertemporal hedging relative to the static bound (e.g., HLB–LB). The contribution of each forward moment is then calculated as the ratio of the respective forward moment to the incremental contribution (e.g., HLB–LB or HUB–UB).

$[T_1, T_N]$		$T_N = 91$		$T_N = 182$		$T_N = 365$		$T_N = 547$		$T_N = 730$	
		HLB-LB	HUB-UB	HLB-LB	HUB-UB	HLB-LB	HUB-UB	HLB-LB	HUB-UB	HLB-LB	HUB-UB
30	Variance (%)	49.87	57.92	50.34	58.30	51.30	58.80	49.96	56.40	49.79	55.64
	Skewness (%)	40.45	33.28	40.64	33.94	42.22	34.72	41.27	34.61	41.53	34.63
	Kurtosis (%)	9.68	8.79	9.02	7.76	6.49	6.48	8.77	8.99	8.69	9.72
91	Variance (%)			37.75	41.52	39.54	44.71	39.69	45.76	40.54	46.52
	Skewness (%)			43.22	38.32	45.34	40.76	44.49	39.79	45.43	40.89
	Kurtosis (%)			19.03	20.16	15.12	14.53	15.82	14.44	14.03	12.58
182	Variance (%)					34.32	40.77	35.05	43.60	36.03	45.41
	Skewness (%)					41.88	34.54	41.38	33.85	42.29	34.97
	Kurtosis (%)					23.81	24.69	23.57	22.55	21.68	19.62
365	Variance (%)							34.58	46.72	35.27	48.23
	Skewness (%)							36.00	26.80	36.60	27.22
	Kurtosis (%)							29.42	26.47	28.13	24.56

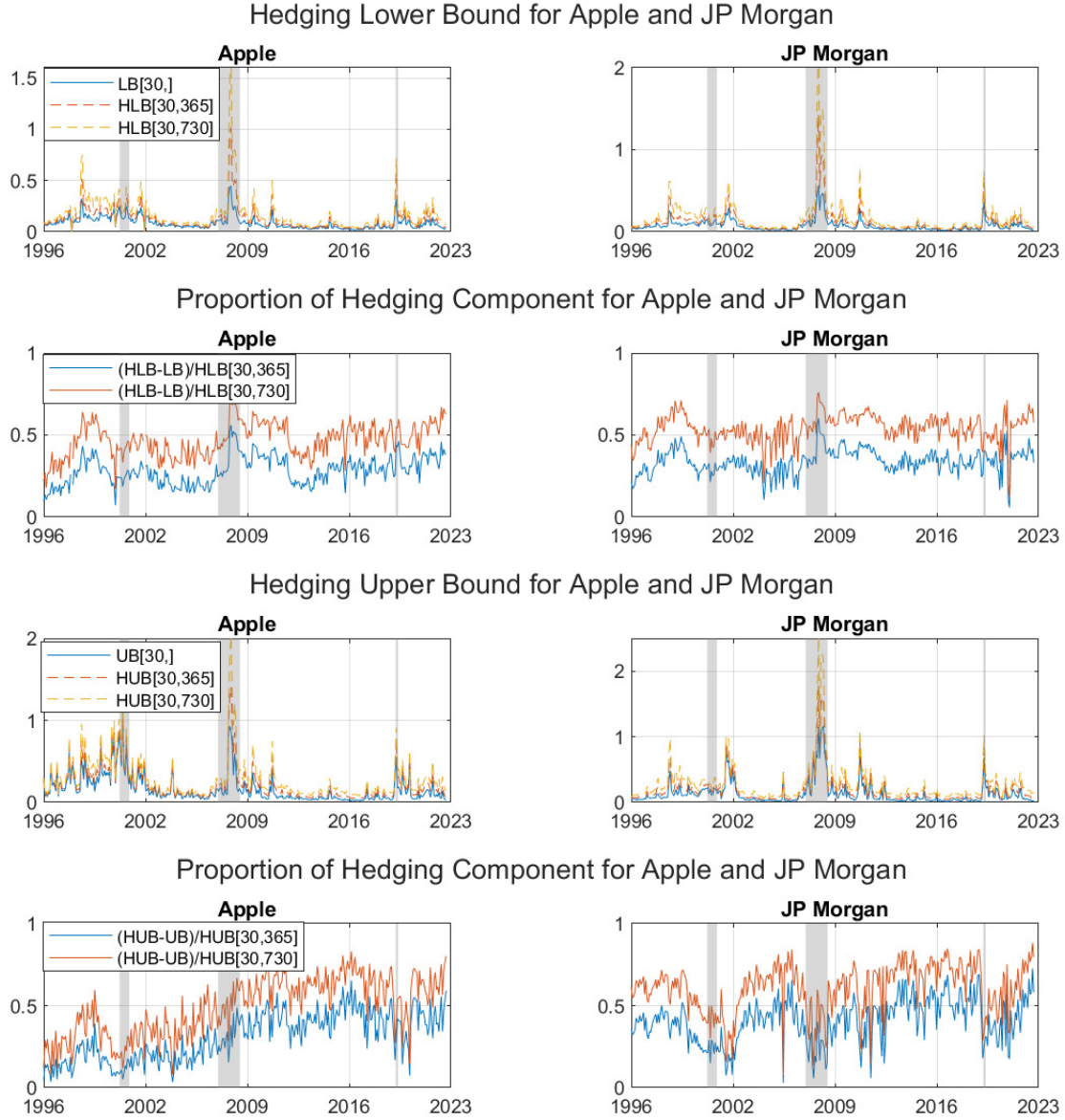


Fig. 1: Time Series of Expected Excess Return Bounds for AAPL and JPM This figure plots the time series of lower and upper bounds on expected excess returns for Apple and JP Morgan with and without hedging (LB/UB and HLB/HUB). Bounds are computed using the default preference parameters: $\tau = 1$, $\rho = 2$, $\kappa = 4$, and $\xi = 8$. The figure also reports the hedging share, defined as $(\text{HLB}-\text{LB})/\text{HLB}$ and $(\text{HUB}-\text{UB})/\text{HUB}$, with $T_1 = 30$ and $T_N = 365/730$. The sample spans January 1996 to August 2023. The grey areas indicate NBER recessions.

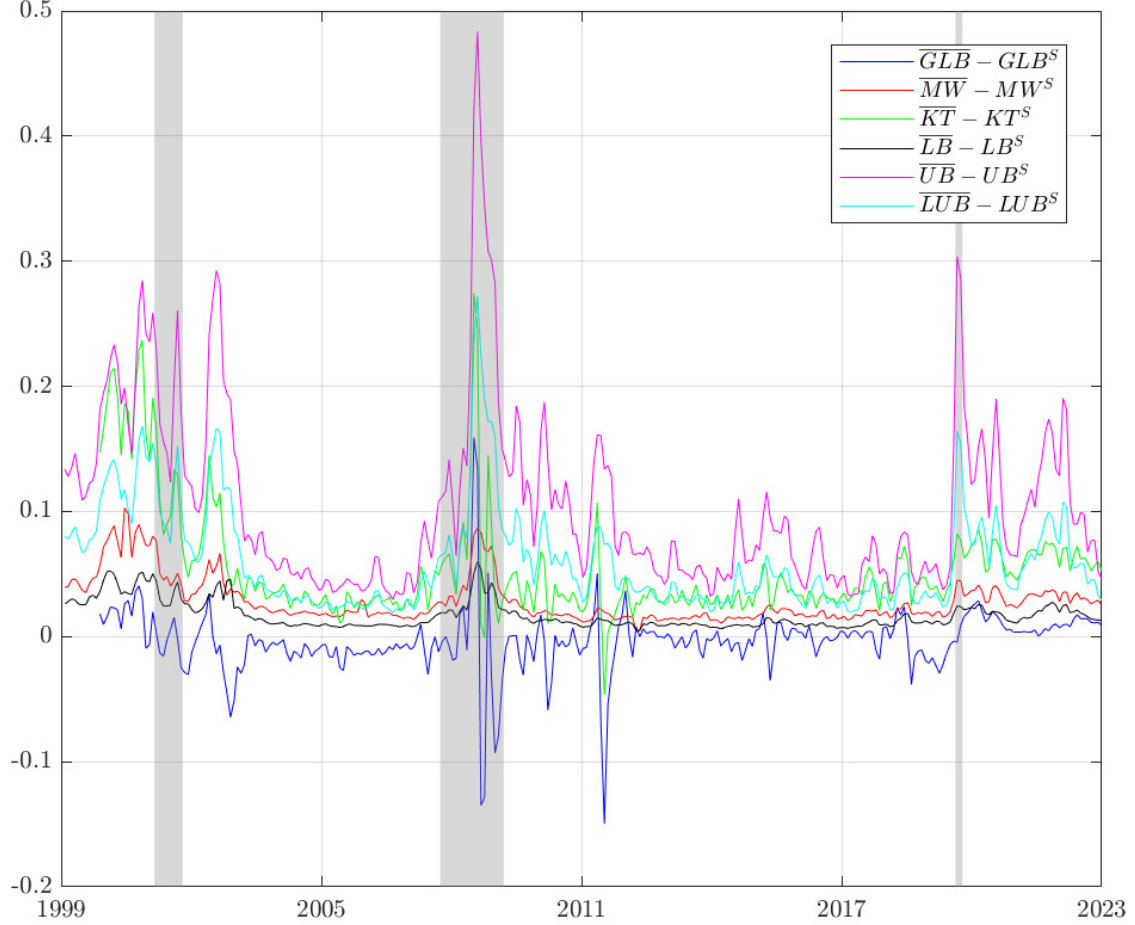
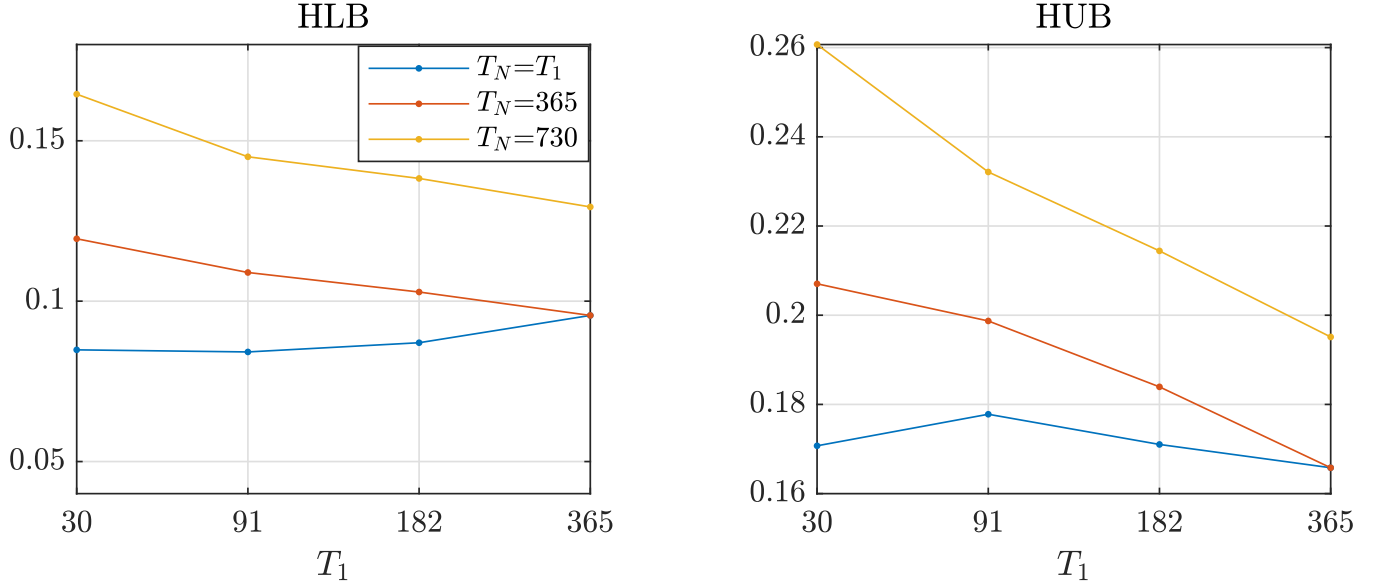


Fig. 2: Pricing Consistency of Static Bounds This figure reports average differences in 30-day annualized expected excess return bounds between nine S&P500-based SPDR ETFs (X^S) and the value-weighted bounds of their underlying constituents ($\bar{X}$). The figure includes MW (Martin and Wagner (2019)), KT (Kadan and Tang (2020)), GLB (Chabi-Yo, Dim, and Vilkov (2023)), the static bounds (LB and UB), and the midpoint (LUB=(LB+UB)/2). The static bounds are computed using the default preference parameters $\tau = 1, \rho = 2, \kappa = 4$, and $\xi = 8$. The series are monthly averages of annualized daily values from January 1999 (the start of sector-ETF option trading) to August 2023. The grey areas indicate NBER recessions.

Average HLB and HUB across Firms



Average Proportion of Hedging Component across Firms

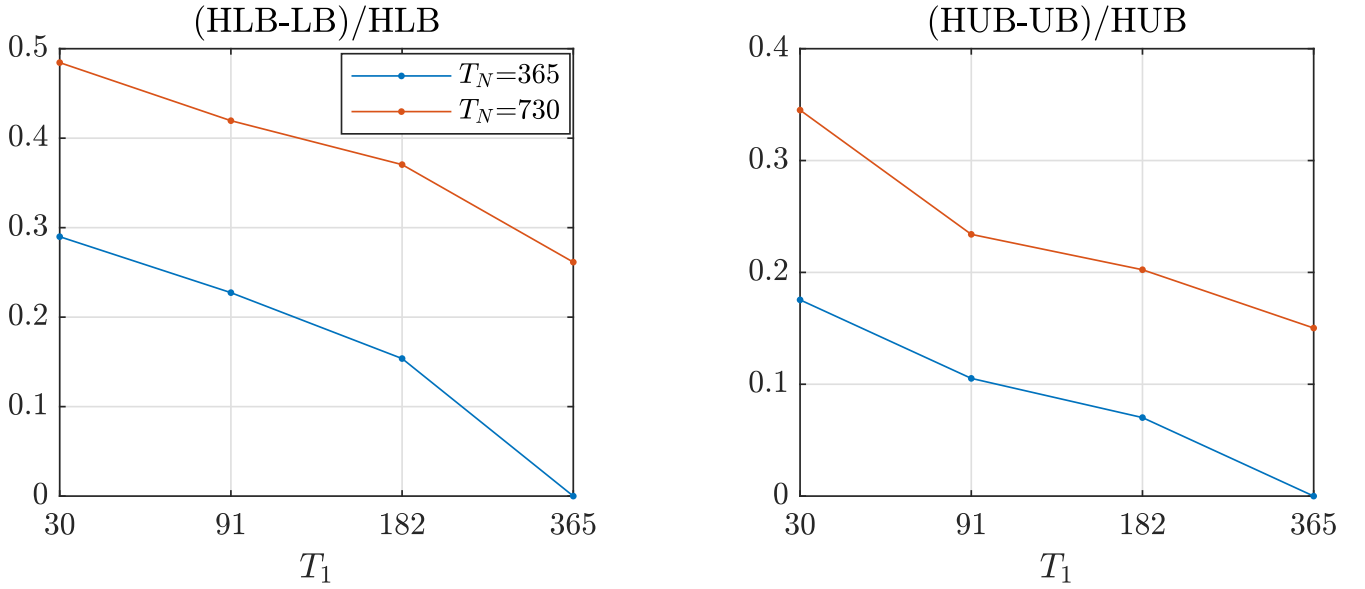


Fig. 3: Average Hedging Bounds across Forecast Horizons This figure plots averages of the hedging bounds (HLB and HUB) across all firms for maturities from 30 to 365 days. The figure also reports the hedging share, defined as $(HLB-LB)/HLB$ and $(HUB-UB)/HUB$, averaged across firms at each maturity. The sample spans January 1996 to August 2023.



Fig. 4: Pricing Consistency: Hedging vs. Static Bounds This figure reports average differences in 30-day annualized expected excess return bounds between nine S&P500-based SPDR ETFs (X^S) and the value-weighted bounds of their underlying constituents ($\bar{X}$). The figure includes the static bounds (LB and UB) and midpoint ($LUB=(LB+UB)/2$), as well as the hedging bounds (HLB and HUB) and midpoint ($HLUB=(HLB+HUB)/2$). The series are monthly averages of annualized daily values from January 1999 to August 2023. The grey areas indicate NBER recessions.

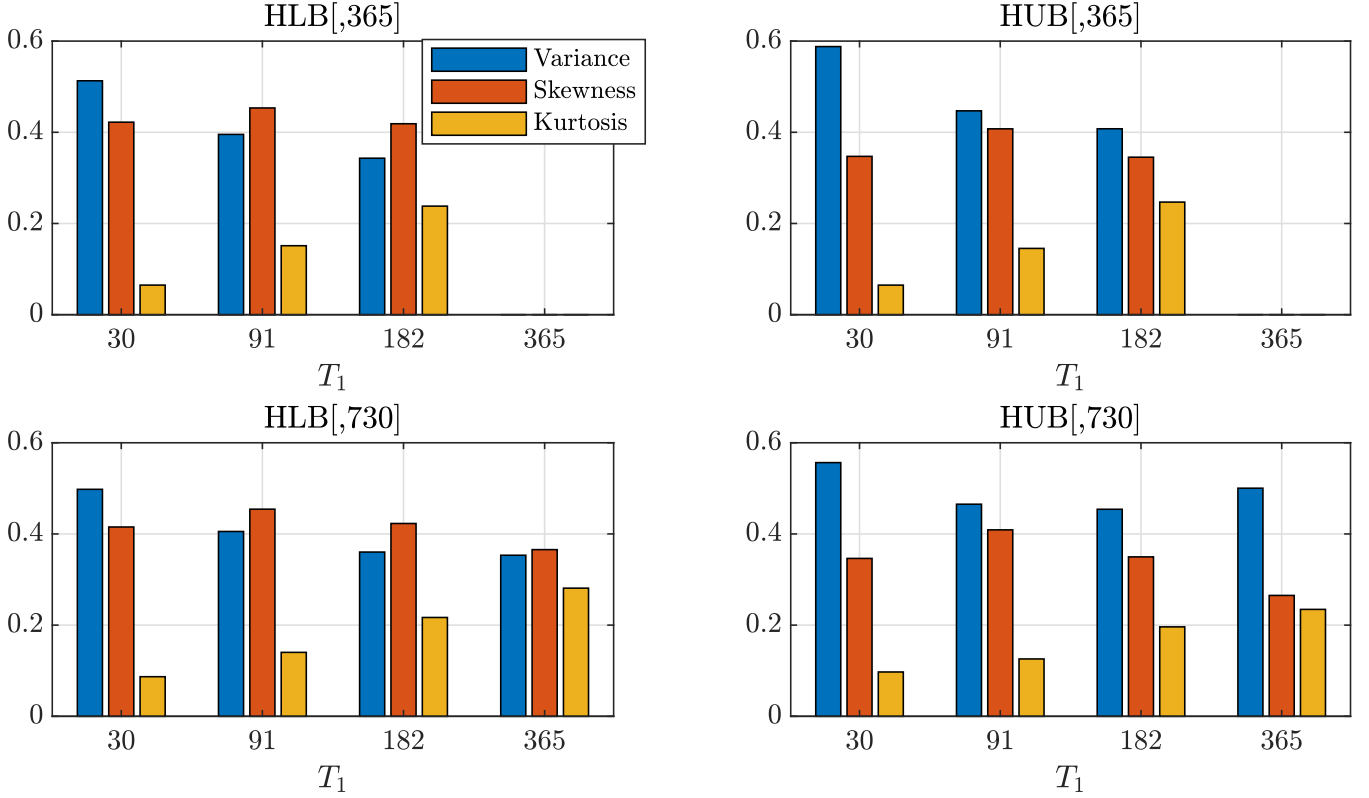


Fig. 5: Decomposition of Hedging Contributions This figure depicts the average decomposition of the hedging components of the lower bound (HLB) and upper bound (HUB) across all firms in the sample, with maturities ranging from 30 to 365 days. We compute the variance-, skewness-, and kurtosis-related components by aggregating the forward moment terms corresponding to the coefficients $a_{j,k-j}$ in Proposition 2.4 for each $k - j$ with varying j to derive the total contribution of each moment in the intertemporal hedging case. Specifically, the component associated with $k - j = 2$ captures the variance-related contribution, $k - j = 3$ captures the skewness-related contribution, and $k - j = 4$ captures the kurtosis-related contribution. We then calculate the incremental contribution of intertemporal hedging relative to the static bound (e.g., HLB–LB). The contribution of each forward moment is then calculated as the ratio of the respective forward moment to the incremental contribution (e.g., HLB–LB or HUB–UB). The sample spans January 1996 to August 2023.

Online Appendix for

“Conditional Expected Returns on Individual Stocks with and without Intertemporal Hedging”

OA.1 Proofs for Propositions and Corollaries

Proof of the SDF in Absence of Intertemporal Hedging. This section provides the proofs for all propositions and the corollary. Set $x_0 = R_{f,t \rightarrow T_1}$ and $x = R_{M,t \rightarrow T_1}$. Set $\Delta x = x - x_0$. A Taylor expansion-series of $v_{t \rightarrow T_1}$ around $\Delta x = 0$ yields

$$v_{t \rightarrow T_1} = 1 + \sum_{j=1}^{\infty} \frac{1}{j!} \left(\frac{\partial^j v_{t \rightarrow T_1}}{\partial^j x} \right)_{x=x_0} (\Delta x)^j.$$

Observe that

$$\left(\frac{\partial v_{t \rightarrow T_1}}{\partial x} \right)_{x=x_0} = \left\{ -\frac{W_t u' [W_t x_0] u'' [W_t x]}{(u' [W_t x])^2} \right\}_{x=x_0} = \frac{1}{x_0} \frac{1}{\tau_t}.$$

Next,

$$\left(\frac{\partial^2 v}{\partial^2 x} \right)_{x=x_0} = \left\{ -\frac{(W_t)^2 u' [W_t x_0] u''' [W_t x]}{(u' [W_t x])^2} + 2 \frac{(W_t)^2 u' [W_t x] u' [W_t x_0] (u'' [W_t x])^2}{(u' [W_t x])^4} \right\}_{x=x_0} = \frac{2(1 - \rho_t)}{\tau_t^2 x_0^2}.$$

Observe that

$$\left(\frac{\partial^3 v_{t \rightarrow T_1}}{\partial^3 x} \right)_{x=x_0} = \left\{ \begin{aligned} & -\frac{(W_t)^3 u' [W_t x_0] u'''' [W_t x]}{(u' [W_t x])^2} + \frac{6u'' [W_t x] (W_t)^3 u' [W_t x_0] u''' [W_t x]}{(u' [W_t x])^3} \\ & - 6 \frac{(W_t)^3 u' [W_t x_0] (u' [W_t x])^2 (u'' [W_t x])^3}{(u' [W_t x])^6} \end{aligned} \right\}_{x=x_0} = \frac{6\kappa_t + 6 - 12\rho_t}{\tau_t^3 x_0^3}.$$

Next,

$$\begin{aligned}
& \left(\frac{\partial^4 v_{t \rightarrow T_1}}{\partial^4 x} \right)_{x=x_0} \\
&= \left\{ \begin{aligned} & -\frac{(W_t)^4 u' [W_{tx_0}] u'''' [W_{tx}]}{(u' [W_{tx}])^2} + 2 \frac{(W_t)^4 u'' [W_{tx}] u' [W_{tx_0}] u'''' [W_{tx}]}{(u' [W_{tx}])^3} + \frac{6 u''' [W_{tx}] (W_t)^4 u' [W_{tx_0}] u''' [W_{tx}]}{(u' [W_{tx}])^3} \\ & + \frac{6 u'' [W_{tx}] (W_t)^4 u' [W_{tx_0}] u'''' [W_{tx}]}{(u' [W_{tx}])^3} - \frac{18 (u'' [W_{tx}])^2 (W_t)^4 u' [W_{tx_0}] u''' [W_{tx}]}{(u' [W_{tx}])^4} \\ & - 18 \frac{(W_t)^4 u' [W_{tx_0}] (u'' [W_{tx}])^2 u''' [W_{tx}]}{(u' [W_{tx}])^4} + 24 \frac{(W_t)^4 u' [W_{tx_0}] (u'' [W_{tx}])^4}{(u' [W_{tx}])^5} \end{aligned} \right\}_{x=x_0}.
\end{aligned}$$

This simplifies to

$$\left(\frac{\partial^4 v_{t \rightarrow T_1}}{\partial^4 x} \right)_{x=x_0} = \frac{24\rho_t^2 - 72\rho_t + 24 + 48\kappa_t - 24\xi_t}{\tau_t^4 x_0^4}.$$

The replacement of $v_{t \rightarrow T_1}$ in (2.2) yields the final result. $\square$

Proof of Proposition 2.1. Let us first derive the lower bound. Replace the inverse SDF obtained in Proposition 1 with expression (2.1). It follows that

$$\begin{aligned}
\mathbb{E}_t [\mathcal{R}_{i,t \rightarrow T_1}] &= \mathbb{E}_t^* \left(\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}} \mathcal{R}_{i,t \rightarrow T_1} \right) \\
&= \sum_{j=1}^{\infty} \underbrace{\frac{a_{j,t}}{\mathcal{R}_{f,t \rightarrow T_1}^j \mathbb{E}_t^* v_{t \rightarrow T_1}}}_{\text{Price of Risk}} \underbrace{\text{COV}_t^* (\mathcal{R}_{i,t \rightarrow T_1}, \mathcal{R}_{M,t \rightarrow T_1}^j)}_{\text{Quantity of Risk}}, \tag{OA.1.1}
\end{aligned}$$

where $a_{j,t}$ are defined as

$$a_{j,t} = \left\{ \frac{\partial^j v_{t \rightarrow T_1}}{\partial^j R_{M,t \rightarrow T_1}} \right\}_{\mathcal{R}_{M,t \rightarrow T_1}=0}. \tag{OA.1.2}$$

The expected excess return expands to

$$\begin{aligned}
& \mathbb{E}_t \mathcal{R}_{i,t \rightarrow T_1} \\
&= \frac{1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]}{1 + \sum_{j=1}^{\infty} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]} \times \frac{\sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j \mathcal{R}_{i,t \rightarrow T_1}]}{1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]} + \frac{\sum_{j=\mathcal{K}+1}^{\infty} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j \mathcal{R}_{i,t \rightarrow T_1}]}{1 + \sum_{j=1}^{\infty} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]}.
\end{aligned}$$

Since $\frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \text{COV}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j, R_{i,t \rightarrow T_1}] \geq 0$, it follows that

$$\frac{\sum_{j=\mathcal{K}+1}^{\infty} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j \mathcal{R}_{i,t \rightarrow T_1}]}{1 + \sum_{j=1}^{\infty} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]} \geq 0.$$

Since $a_{2j+1,t} \geq 0$, $a_{2j,t} \leq 0$, $\mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^{2j+1}$, and $\mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^{2j} \geq 0$, it follows that

$$0 \leq 1 + \sum_{j=1}^{\infty} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^j \leq 1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^j.$$

Hence,

$$\frac{1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]}{1 + \sum_{j=1}^{\infty} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]} \geq 1.$$

Thus it implies

$$\left(\frac{1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]}{1 + \sum_{j=1}^{\infty} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]} \right) \left\{ \frac{\sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j \mathcal{R}_{i,t \rightarrow T_1}]}{1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]} \right\} \geq \frac{\sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j \mathcal{R}_{i,t \rightarrow T_1}]}{1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]}.$$

and hence

$$\mathbb{E}_t \mathcal{R}_{i,t \rightarrow T_1} \geq \frac{\sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j \mathcal{R}_{i,t \rightarrow T_1}]}{1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]}.$$

Next, we derive the upper bound. For any threshold $0 \leq k_0 \leq R_{f,t \rightarrow T_1}$,

$$\mathbb{E}_t \mathcal{R}_{i,t \rightarrow T_1} \leq \mathbb{E}_t (\mathcal{R}_{i,t \rightarrow T_1} 1_{\mathcal{R}_{i,t \rightarrow T_1} \geq k_0}).$$

Up to the $\mathcal{K}^{\text{th}}$ -order expansion of the inverse marginal utility, under no-arbitrage

$$\mathbb{E}_t^* \left[\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}} \mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0} \right] = \mathbb{E}_t^* \left\{ \frac{1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathcal{R}_{M,t \rightarrow T_1}^j}{1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]} \mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0} \right\}.$$

Since, under arbitrage $\mathbb{E}_t [\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}] = \mathbb{E}_t^* \left[\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}} \mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0} \right]$, we obtain

$$\mathbb{E}_t \{ \mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0} \} = \frac{\mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}] + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j \mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}]}{1 + \sum_{j=1}^{\mathcal{K}} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^j]}.$$

This ends the proof. $\square$

Proof of the SDF in Presence of Intertemporal Hedging. Comparing expression (2.1) and (2.23), and using the uniqueness of the SDF it follows that

$$\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}} = \frac{v_{t \rightarrow T_1}}{\mathbb{E}_t^* v_{t \rightarrow T_1}},$$

where

$$v_{t \rightarrow T_1} = \mathbb{E}_{T_1}^* \vartheta_{t \rightarrow T_N} \text{ with } \vartheta_{t \rightarrow T_N} = \frac{u' [W_t x_0 y_0]}{u' [W_t x y]}. \quad (\text{OA.1.3})$$

Setting $x_0 = R_{f,t \rightarrow T_1}$, $y_0 = R_{f,T_1 \rightarrow T_N}$ and $x = R_{M,t \rightarrow T_1}$, $y = R_{M,T_1 \rightarrow T_N}$ Let's set $\Delta x = x - x_0$ and $\Delta y = y - y_0$.

$$\vartheta_{t \rightarrow T_N} = \sum_{j_1=0}^{\mathcal{K}} \sum_{j_2=0}^{\mathcal{K}} \frac{a_{j_1,j_2}}{R_{f,t \rightarrow T_1}^{j_1} R_{f,T_1 \rightarrow T_N}^{j_2}} \Delta x^{j_1} \Delta y^{j_2}. \quad (\text{OA.1.4})$$

Hence,

$$\begin{aligned}
\vartheta_{t \rightarrow T_N} = & 1 + \left(\frac{\partial \vartheta_{t \rightarrow T_N}}{\partial x} \right)_{x=x_0} \Delta x + \left(\frac{\partial \vartheta_{t \rightarrow T_N}}{\partial y} \right)_{y=y_0} \Delta y \\
& + \frac{1}{2} \left(\frac{\partial^2 \vartheta_{t \rightarrow T_N}}{\partial x^2} \right)_{x=x_0} (\Delta x)^2 + \frac{1}{2} \left(\frac{\partial^2 \vartheta_{t \rightarrow T_N}}{\partial y^2} \right)_{y=y_0} (\Delta y)^2 \\
& + \left(\frac{\partial^2 \vartheta_{t \rightarrow T_N}}{\partial x \partial y} \right)_{(x,y)=(x_0,y_0)} \Delta x \Delta y \\
& + \frac{1}{3!} \left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial x^3} \right)_{x=x_0} (\Delta x)^3 + \frac{1}{3!} \left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial y^3} \right)_{y=y_0} (\Delta y)^3 \\
& + \frac{3}{3!} \left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial y^2 \partial x} \right)_{(x,y)=(x_0,y_0)} (\Delta y)^2 \Delta x + \frac{3}{3!} \left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial x^2 \partial y} \right)_{(x,y)=(x_0,y_0)} (\Delta x)^2 \Delta y \\
& + \frac{1}{4!} \left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial x^4} \right)_{x=x_0} (\Delta x)^4 + \frac{1}{4!} \left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial y^4} \right)_{y=y_0} (\Delta y)^4 \\
& + \frac{4}{4!} \left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial x^3 \partial y} \right)_{(x,y)=(x_0,y_0)} (\Delta x)^3 (\Delta y) + \frac{4}{4!} \left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial y^3 \partial x} \right)_{(x,y)=(x_0,y_0)} (\Delta y)^3 (\Delta x) \\
& + \frac{6}{4!} \left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial x^2 \partial y^2} \right)_{(x,y)=(x_0,y_0)} (\Delta x)^2 (\Delta y)^2.
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}_{T_1}^* \frac{u'(W_t x_0 y_0)}{u'(W_t x y)} &= 1 + \left(\frac{\partial \vartheta_{t \rightarrow T_N}}{\partial x} \right)_{x=x_0} \Delta x + \frac{1}{2} \left(\frac{\partial^2 \vartheta_{t \rightarrow T_N}}{\partial x^2} \right)_{x=x_0} (\Delta x)^2 \\
&+ \frac{1}{2} \left(\frac{\partial^2 \vartheta_{t \rightarrow T_N}}{\partial y^2} \right)_{y=y_0} \mathbb{E}_{T_1}^* (\Delta y)^2 \\
&+ \frac{1}{3!} \left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial x^3} \right)_{x=x_0} (\Delta x)^3 + \frac{1}{3!} \left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial y^3} \right)_{y=y_0} \mathbb{E}_{T_1}^* (\Delta y)^3 \\
&+ \frac{3}{3!} \left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial y^2 \partial x} \right)_{(x,y)=(x_0,y_0)} \Delta x \mathbb{E}_{T_1}^* (\Delta y)^2 \\
&+ \frac{1}{4!} \left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial x^4} \right)_{x=x_0} (\Delta x)^4 + \frac{1}{4!} \left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial y^4} \right)_{y=y_0} \mathbb{E}_{T_1}^* (\Delta y)^4 \\
&+ \frac{4}{4!} \left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial y^3 \partial x} \right)_{(x,y)=(x_0,y_0)} (\Delta x) \mathbb{E}_{T_1}^* (\Delta y)^3 \\
&+ \frac{6}{4!} \left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial x^2 \partial y^2} \right)_{(x,y)=(x_0,y_0)} (\Delta x)^2 \mathbb{E}_{T_1}^* (\Delta y)^2.
\end{aligned}$$

Observe that (details are provided in the Online Appendix [OA.4](#))

$$\begin{aligned}
\left(\frac{\partial \vartheta_{t \rightarrow T_N}}{\partial x} \right)_{x=x_0} &= \frac{1}{x_0} \frac{1}{\tau_t}, \\
\left(\frac{\partial \vartheta_{t \rightarrow T_N}}{\partial y} \right)_{y=y_0} &= \frac{1}{y_0} \frac{1}{\tau_t}, \\
\left(\frac{\partial^2 \vartheta_{t \rightarrow T_N}}{\partial^2 x} \right)_{x=x_0} &= \frac{2(1-\rho_t)}{\tau_t^2 x_0^2}, \\
\left(\frac{\partial^2 \vartheta_{t \rightarrow T_N}}{\partial^2 y} \right)_{y=y_0} &= \frac{2(1-\rho_t)}{\tau_t^2 y_0^2}, \\
\left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial^3 x} \right)_{x=x_0} &= \frac{6\kappa_t + 6 - 12\rho_t}{\tau_t^3 x_0^3}, \\
\left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial^3 y} \right)_{y=y_0} &= \frac{6\kappa_t + 6 - 12\rho_t}{\tau_t^3 y_0^3},
\end{aligned}$$

$$\begin{aligned}
\left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial^2 y \partial x} \right)_{y=x_0} &= \frac{4(1-\rho_t)}{\tau_t^2 x_0 y_0^2} + \frac{6(\kappa_t + 1 - 2\rho_t)}{\tau_t^3 x_0 y_0^2}, \\
\frac{1}{24} \left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial^4 x} \right)_{x=x_0} &= \frac{\rho_t^2 - 3\rho_t + 1 + 2\kappa_t - \xi_t}{\tau_t^4 x_0^4}, \\
\left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial^2 y \partial^2 x} \right)_{(x,y)=(x_0,y_0)} &= \frac{4(1-\rho_t)}{\tau_t^2 x_0^2 y_0^2} + \frac{24(\kappa_t - 2\rho_t + 1)}{\tau_t^3 x_0^2 y_0^2} + \frac{24(2\kappa_t - \xi_t + \rho_t^2 - 3\rho_t + 1)}{\tau_t^4 x_0^2 y_0^2}, \\
\left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial^3 y \partial x} \right)_{(x,y)=(x_0,y_0)} &= \frac{18 - 36\rho_t}{\tau_t^3 x_0 y_0^3} + \frac{18\kappa_t + 48\kappa_t - 24\xi_t + 24\rho_t^2 - 72\rho_t + 24}{\tau_t^4 x_0 y_0^3} \\
&= \frac{6}{x_0 y_0^3} \left\{ 3a_{3,t} + 3 \left(1 - 2\frac{a_{2,t}}{a_{1,t}^2} + a_{3,t} \right) (a_{1,t} - 1) + 4a_{4,t} \right\}.
\end{aligned}$$

Hence

$$\begin{aligned}
v_{t \rightarrow T_1} &= \mathbb{E}_{T_1}^* \frac{u' [W_t x_0 y_0]}{u' [W_t x y]} \\
&= 1 + \frac{1}{R_{f,t \rightarrow T_1}} a_{1,t} \Delta x + \frac{a_{2,t}}{R_{f,t \rightarrow T_1}^2} (\Delta x)^2 + \frac{a_{2,t}}{R_{f,T_1 \rightarrow T_N}^2} \mathbb{E}_{T_1}^* (\Delta y)^2 \\
&\quad + \frac{a_{3,t}}{R_{f,t \rightarrow T_1}^3} (\Delta x)^3 + \frac{a_{3,t}}{R_{f,T_1 \rightarrow T_N}^3} \mathbb{E}_{T_1}^* (\Delta y)^3 + \frac{a_{5,t}}{R_{f,t \rightarrow T_1} R_{f,T_1 \rightarrow T_N}^2} \Delta x \mathbb{E}_{T_1}^* (\Delta y)^2 \\
&\quad + \frac{a_{4,t}}{R_{f,t \rightarrow T_1}^4} (\Delta x)^4 + \frac{a_{4,t}}{R_{f,T_1 \rightarrow T_N}^4} \mathbb{E}_{T_1}^* (\Delta y)^4 + \frac{a_{6,t}}{R_{f,t \rightarrow T_1} R_{f,T_1 \rightarrow T_N}^3} (\Delta x) \mathbb{E}_{T_1}^* (\Delta y)^3 \\
&\quad + \frac{a_{7,t}}{R_{f,t \rightarrow T_1}^2 R_{f,T_1 \rightarrow T_N}^2} (\Delta x)^2 \mathbb{E}_{T_1}^* (\Delta y)^2.
\end{aligned}$$

This ends the proof. $\square$

Proof of Proposition 2.4. Consider $\vartheta_{t \rightarrow T_N}$ defined in (OA.1.3). A $\mathcal{K}$ order ($\mathcal{K}$ can be finite or infinite) Taylor expansion-series of $\vartheta_{t \rightarrow T_N}$ around $(\mathcal{R}_{M,t \rightarrow T_1}, \mathcal{R}_{M,T_1 \rightarrow T_N}) = (0, 0)$ yields

$$v_{t \rightarrow T_1} = 1 + \sum_{k=1}^{\infty} \sum_{j=0}^k \frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} f_{T_1}^{(j,k-j)}, \quad (\text{OA.1.5})$$

with

$$f_{T_1}^{(j,k-j)} = \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^{k-j}.$$

Replace the inverse SDF

$$\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}} = \frac{v_{t \rightarrow T_1}}{\mathbb{E}_t^* v_{t \rightarrow T_1}} \quad (\text{OA.1.6})$$

in (2.1), it follows that the expected excess return is

$$\mathbb{E}_t (R_{M,t \rightarrow T_1} - R_{f,t \rightarrow T_1}) = \frac{\sum_{k=1}^{\infty} \sum_{j=0}^k \frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \text{COV}_t^* \left(f_{T_1}^{(j,k-j)}, R_{M,t \rightarrow T_1} \right)}{1 + \sum_{k=1}^{\infty} \sum_{j=0}^k \frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \mathbb{E}_t^* f_{T_1}^{(j,k-j)}}.$$

By definition, $v_{t \rightarrow T_1}$ is positive, thus, its expectation is also positive:

$$1 + \sum_{k=1}^{\infty} \sum_{j=0}^k \frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \mathbb{E}_t^* f_{T_1}^{(j,k-j)} \geq 0.$$

Thus,

$$\lim_{\mathcal{K} \rightarrow \infty} \sum_{\substack{j_1, j_2=0 \\ j_1+j_2 \leq \mathcal{K}}}^{\mathcal{K}} \frac{a_{j_1, j_2, t}}{R_{f,t \rightarrow T_1}^{j_1} R_{f,T_1 \rightarrow T_N}^{j_2}} \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^{j_1} \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{j_2}] \geq 0.$$

Notice that each term of the numerator above is a risk premium which by definition is compensation for an exposure to $\mathcal{R}_{M,t \rightarrow T_1}^{j_1} \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{j_2}$. It is viewed as a price of risk $\frac{a_{j_1, j_2, t}}{R_{f,t \rightarrow T_1}^{j_1} R_{f,T_1 \rightarrow T_N}^{j_2}}$ times the quantity of risk $\text{COV}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^{j_1}, \mathcal{R}_{M,T_1 \rightarrow T_N}^{j_2}]$. According to economic theory, the risk premium is positive. As a result

$$\begin{aligned} \mathbb{E}_t (R_{M,t \rightarrow T_1} - R_{f,t \rightarrow T_1}) &\geq \frac{\sum_{k=1}^{\infty} \sum_{j=0}^{\mathcal{K}} \frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \text{COV}_t^* \left(f_{T_1}^{(j,k-j)}, R_{M,t \rightarrow T_1} \right)}{1 + \sum_{k=1}^{\infty} \sum_{j=0}^{\mathcal{K}} \frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \mathbb{E}_t^* f_{T_1}^{(j,k-j)}} \\ &\geq \frac{\sum_{k=1}^{\infty} \sum_{j=0}^{\mathcal{K}} \frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \text{COV}_t^* \left(f_{T_1}^{(j,k-j)}, R_{M,t \rightarrow T_1} \right)}{1 + \sum_{k=1}^{\infty} \sum_{j=0}^{\mathcal{K}} \frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \mathbb{E}_t^* f_{T_1}^{(j,k-j)}} \\ &\quad \times \frac{1 + \sum_{k=1}^{\infty} \sum_{j=0}^{\mathcal{K}} \frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \mathbb{E}_t^* f_{T_1}^{(j,k-j)}}{1 + \sum_{k=1}^{\infty} \sum_{j=0}^{\mathcal{K}} \frac{a_{j,k-j,t}}{R_{f,t \rightarrow T_1}^j R_{f,T_1 \rightarrow T_N}^{k-j}} \mathbb{E}_t^* f_{T_1}^{(j,k-j)}}. \end{aligned}$$

Provided that

$$a_{j_1, j_2, t} \mathbb{E}_t^* \left[\mathcal{R}_{M, t \rightarrow T_1}^{j_1} \mathbb{E}_{T_1}^* \mathcal{R}_{M, T_1 \rightarrow T_N}^{j_2} \right] \leq 0,$$

it follows that

$$\frac{1 + \sum_{k=1}^{\infty} \sum_{j=0}^{\mathcal{K}} \frac{a_{j, k-j, t}}{R_{f, t \rightarrow T_1}^j R_{f, T_1 \rightarrow T_N}^{k-j}} \mathbb{E}_t^* f_{T_1}^{(j, k-j)}}{1 + \sum_{k=1}^{\infty} \sum_{j=0}^k \frac{a_{j, k-j, t}}{R_{f, t \rightarrow T_1}^j R_{f, T_1 \rightarrow T_N}^{k-j}} \mathbb{E}_t^* f_{T_1}^{(j, k-j)}} \geq 1$$

and hence

$$\mathbb{E}_t (R_{M, t \rightarrow T_1} - R_{f, t \rightarrow T_1}) \geq \frac{\sum_{k=1}^{\infty} \sum_{j=0}^{\mathcal{K}} \frac{a_{j, k-j, t}}{R_{f, t \rightarrow T_1}^j R_{f, T_1 \rightarrow T_N}^{k-j}} \mathbb{Cov}_t^* \left(f_{T_1}^{(j, k-j)}, R_{M, t \rightarrow T_1} \right)}{1 + \sum_{k=1}^{\infty} \sum_{j=0}^{\mathcal{K}} \frac{a_{j, k-j, t}}{R_{f, t \rightarrow T_1}^j R_{f, T_1 \rightarrow T_N}^{k-j}} \mathbb{E}_t^* f_{T_1}^{(j, k-j)}}.$$

We follow a similar approach as in Proposition 2.1 to derive the proof of (2.35). $\square$

Proof of Corollary 2.5. Before we provide expressions for $\mathcal{M}_{i, M, t}^{(j, k-j)}$ and $\mathcal{M}_{i, M, t}^{(j, k-j)} [k_0]$, we provide closed-form expressions of the risk neutral quantities in $\theta_{i, t}^*$

$$\begin{aligned} H^{(j, l)} [S_{T_1}] &= R_{M, t \rightarrow T_1}^j (R_{M, t \rightarrow T_1}^l - R_{f, t \rightarrow T_1}^l) \\ &= \left(\frac{S_{T_1}}{S_t} \right)^j \left(\left(\frac{S_{T_1}}{S_t} \right)^l - R_{f, t \rightarrow T_1}^l \right) \quad l \neq \{0, 1\}, \end{aligned}$$

where

$$\begin{aligned} \mathbb{E}_t^* \left[H^{(i, j)} [S_{T_1}] 1_{S_{T_1} < S_t R_{f, t \rightarrow T_1}} \right] &= -R_{f, t \rightarrow T_1} H_S^{(i, j)} [S_t R_{f, t \rightarrow T_1}] P_t [S_t R_{f, t \rightarrow T_1}] \\ &\quad + R_{f, t \rightarrow T_1} \int_0^{S_t R_{f, t \rightarrow T_1}} H_{SS}^{(i, j)} [K] P_t [K] dK \quad (\text{OA.1.7}) \end{aligned}$$

and

$$\begin{aligned} \mathbb{E}_t^* \left[H^{(i, j)} [S_{T_1}] 1_{S_{T_1} > S_t R_{f, t \rightarrow T_1}} \right] &= R_{f, t \rightarrow T_1} H_S^{(i, j)} [S_t R_{f, t \rightarrow T_1}] C_t [S_t R_{f, t \rightarrow T_1}] \\ &\quad + R_{f, t \rightarrow T_1} \int_{S_t R_{f, t \rightarrow T_1}}^{\infty} H_{SS}^{(i, j)} [K] C_t [K] dK. \quad (\text{OA.1.8}) \end{aligned}$$

Next, we focus on providing expressions for $\mathcal{M}_{i,M,t}^{(j,k-j)}$ and $\mathcal{M}_{i,M,t}^{(j,k-j)}[k_0]$. When $k - j = 1$,

$$\mathcal{M}_{i,M,t}^{(j,1)} = \mathbb{E}_t^* \left[\mathcal{R}_{i,t \rightarrow T_1} f_{T_1}^{(j,1)} \right] = 0,$$

with

$$f_{T_1}^{(j,1)} = \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N} = 0.$$

Similarly

$$\mathcal{M}_{i,M,t}^{(j,1)}[k_0] = 0.$$

When $k - j = 0$, $\mathcal{M}_{i,M,t}^{(j,0)} = \mathcal{M}_{i,M,t}^{(j)}$:

$$\mathcal{M}_{i,M,t}^{(j,0)} = \mathbb{E}_t^* \left[\mathcal{R}_{i,t \rightarrow T_1} f_{T_1}^{(j,0)} \right],$$

with

$$f_{T_1}^{(j,0)} = \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^0 = \mathcal{R}_{M,t \rightarrow T_1}^j.$$

Thus

$$\mathcal{M}_{i,M,t}^{(j,0)} = \mathbb{E}_t^* \left[\mathcal{R}_{i,t \rightarrow T_1} \mathcal{R}_{M,t \rightarrow T_1}^j \right] = \mathcal{M}_{i,M,t}^{(j)}.$$

Similarly

$$\mathcal{M}_{i,M,t}^{(j,0)}[k_0] = \mathcal{M}_{i,M,t}^{(j)}[k_0].$$

Our goal is to compute $\mathcal{M}_{i,M,t}^{(j,k-j)}$ and $\mathcal{M}_{i,M,t}^{(j,k-j)}[k_0]$ when $k - j \neq \{0, 1\}$. Notice that

$$\mathcal{M}_{i,M,t}^{(j,k-j)} = \mathbb{E}_t^* \left[\mathcal{R}_{i,t \rightarrow T_1} f_{T_1}^{(j,k-j)} \right],$$

with

$$f_{T_1}^{(j,k-j)} = \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j}.$$

That is

$$\begin{aligned} \mathcal{M}_{i,M,t}^{(j,k-j)} &= \mathbb{E}_t^* \left[\mathcal{R}_{i,t \rightarrow T_1} \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} \right] \\ &= \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} g_i[\mathcal{R}_{M,t \rightarrow T_1}] \right], \end{aligned} \quad (\text{OA.1.9})$$

where

$$g_i[\mathcal{R}_{M,t \rightarrow T_1}] = \mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1} | \mathcal{R}_{M,t \rightarrow T_1}]$$

is given in our equation 2.13:

$$g_i [\mathcal{R}_{M,t \rightarrow T_1}] = \sum_{q=1}^{\mathcal{N}} \underline{\gamma}_{i,q} \mathcal{R}_{M,t \rightarrow T_1}^q.$$

Hence,

$$\mathcal{M}_{i,M,t}^{(j,k-j)} = \sum_{q=1}^{\mathcal{N}} \underline{\gamma}_{i,q} \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j+q} \left(\mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} \right) \right]. \quad (\text{OA.1.10})$$

We then replace $\mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j}$ by using expression

$$\mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{j_2} = \sum_{l=2}^{j_2} \phi_{j_2,l} \left(\mathbb{E}_{T_1}^* R_{M,T_1 \rightarrow T_N}^l - R_{f,T_1 \rightarrow T_N}^l \right),$$

where

$$\mathbb{E}_{T_1}^* R_{M,T_1 \rightarrow T_N}^l - R_{f,T_1 \rightarrow T_N}^l = \theta_{l,t}^* \left\{ \begin{aligned} & (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} \\ & + (R_{f,t \rightarrow T_1}^l - R_{M,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}} \end{aligned} \right\}. \quad (\text{OA.1.11})$$

Therefore,

$$\begin{aligned} \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{j_2} &= \sum_{l=2}^{j_2} \phi_{j_2,l} \theta_{l,t}^* \left\{ \begin{aligned} & (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} \\ & + (R_{f,t \rightarrow T_1}^l - R_{M,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}} \end{aligned} \right\} \\ &= \sum_{l=2}^{j_2} \phi_{j_2,l} \theta_{l,t}^* \left(R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l \right) 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} \\ &\quad + \sum_{l=2}^{j_2} \phi_{j_2,l} \theta_{l,t}^* \left(R_{f,t \rightarrow T_1}^l - R_{M,t \rightarrow T_1}^l \right) 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}}. \end{aligned} \quad (\text{OA.1.12})$$

Setting $j_2 = k - j$, it follows that

$$\begin{aligned} \mathcal{M}_{i,M,t}^{(j,k-j)} &= \sum_{q=1}^{\mathcal{N}} \underline{\gamma}_{i,q} \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j+q} \left(R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l \right) 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} \right] \\ &\quad - \sum_{q=1}^{\mathcal{N}} \underline{\gamma}_{i,q} \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j+q} \left(R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l \right) 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}} \right]. \end{aligned} \quad (\text{OA.1.13})$$

Set

$$\begin{aligned} G^{(j,l)} [S_{T_1}] &= \mathcal{R}_{M,t \rightarrow T_1}^j (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) \\ &= \left(\frac{S_{T_1}}{S_t} - R_{f,t \rightarrow T_1} \right)^j \left(\left(\frac{S_{T_1}}{S_t} \right)^l - R_{f,t \rightarrow T_1}^l \right) \quad l \neq \{0, 1\}. \end{aligned} \quad (\text{OA.1.14})$$

Thus

$$\mathcal{M}_{i,M,t}^{(j,k-j)} = \sum_{q=1}^{\mathcal{N}} \gamma_{i,q} \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{E}_t^* \left[G^{(j+q,l)} [S_{T_1}] 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} \right] \quad (\text{OA.1.15})$$

$$- \sum_{q=1}^{\mathcal{N}} \gamma_{i,q} \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{E}_t^* \left[G^{(j+q,l)} [S_{T_1}] 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}} \right], \quad (\text{OA.1.16})$$

where

$$\begin{aligned} \mathbb{E}_t^* \left[G^{(i,j)} [S_{T_1}] 1_{S_{T_1} < S_t R_{f,t \rightarrow T_1}} \right] &= -R_{f,t \rightarrow T_1} G_S^{(i,j)} [S_t R_{f,t \rightarrow T_1}] P_t [S_t R_{f,t \rightarrow T_1}] \\ &\quad + R_{f,t \rightarrow T_1} \int_0^{S_t R_{f,t \rightarrow T_1}} G_{SS}^{(i,j)} [K] P_t [K] dK \end{aligned} \quad (\text{OA.1.17})$$

and

$$\begin{aligned} \mathbb{E}_t^* \left[G^{(i,j)} [S_{T_1}] 1_{S_{T_1} > S_t R_{f,t \rightarrow T_1}} \right] &= R_{f,t \rightarrow T_1} G_S^{(i,j)} [S_t R_{f,t \rightarrow T_1}] C_t [S_t R_{f,t \rightarrow T_1}] \\ &\quad + R_{f,t \rightarrow T_1} \int_{S_t R_{f,t \rightarrow T_1}}^{\infty} G_{SS}^{(i,j)} [K] C_t [K] dK \end{aligned} \quad (\text{OA.1.18})$$

Next, notice that

$$\begin{aligned} \mathcal{M}_{i,M,t}^{(j,k-j)} [k_0] &= \mathbb{E}_t^* \left[\mathcal{R}_{i,t \rightarrow T_1}^c \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} \right] \\ &= \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} h_i [\mathcal{R}_{M,t \rightarrow T_1}] \right], \end{aligned} \quad (\text{OA.1.19})$$

with

$$h_i [\mathcal{R}_{M,t \rightarrow T_1}] = \mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1}^c | \mathcal{R}_{M,t \rightarrow T_1}]$$

given in Equation 2.16:

$$h_i [\mathcal{R}_{M,t \rightarrow T_1}] = \sum_{q=1}^{\mathcal{N}} \bar{\gamma}_{i,q} \mathcal{R}_{M,t \rightarrow T_1}^q.$$

Hence

$$\begin{aligned}
\mathcal{M}_{i,M,t}^{(j,k-j)}[k_0] &= \mathbb{E}_t^* \left[\mathcal{R}_{i,t \rightarrow T_1}^c \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} \right] \\
&= \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} h_i[\mathcal{R}_{M,t \rightarrow T_1}] \right] \\
&= \sum_{q=1}^{\mathcal{N}} \bar{\gamma}_{i,q} \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j+q} \left(\mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} \right) \right]. \tag{OA.1.20}
\end{aligned}$$

Let's replace $\mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j}$ by its expression, it follows that

$$\begin{aligned}
\mathcal{M}_{i,M,t}^{(j,k-j)}[k_0] &= \sum_{q=1}^{\mathcal{N}} \bar{\gamma}_{i,q} \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j+q} (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} \right] \\
&\quad - \sum_{q=1}^{\mathcal{N}} \bar{\gamma}_{i,q} \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j+q} (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}} \right] \tag{OA.1.21}
\end{aligned}$$

Using (OA.1.11), it follows that

$$\begin{aligned}
\mathcal{M}_{i,M,t}^{(j,k-j)}[k_0] &= \sum_{q=1}^{\mathcal{N}} \bar{\gamma}_{i,q} \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{E}_t^* \left[G^{(j+q,l)}[S_{T_1}] 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} \right] \\
&\quad - \sum_{q=1}^{\mathcal{N}} \bar{\gamma}_{i,q} \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{E}_t^* \left[G^{(j+q,l)}[S_{T_1}] 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}} \right] \tag{OA.1.22}
\end{aligned}$$

Our next goal is to compute $\mathbb{E}_t^* f_{T_1}^{(j,k-j)}$, observe that

$$f_{T_1}^{(j,k-j)} = \mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j}.$$

Computing

$$\mathbb{E}_t^* f_{T_1}^{(j,k-j)} = \mathbb{E}_t^* \left(\mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} \right)$$

- When $k - j = 0$

$$\mathbb{E}_t^* f_{T_1}^{(j,0)} = \mathbb{E}_t^* \left(\mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^0 \right) = \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^j.$$

- When $k - j = 1$

$$\begin{aligned}
\mathbb{E}_t^* f_{T_1}^{(j,1)} &= \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}) \\
&= \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^j 0) \\
&= 0.
\end{aligned}$$

Now, let's find $\mathbb{E}_t^* f_{T_1}^{(j,k-j)}$ when $k - j \neq \{0, 1\}$:

$$\begin{aligned}
\mathbb{E}_t^* f_{T_1}^{(j,k-j)} &= \mathbb{E}_t^* \left(\mathcal{R}_{M,t \rightarrow T_1}^j \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} \right) \\
&= \mathbb{E}_t^* \left(\mathcal{R}_{M,t \rightarrow T_1}^j \left(\sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} \right. \right. \\
&\quad \left. \left. + \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* (R_{f,t \rightarrow T_1}^l - R_{M,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}} \right) \right) \\
&= \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{E}_t^* \left(\mathcal{R}_{M,t \rightarrow T_1}^j (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} \right) \\
&\quad - \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{E}_t^* \left(\mathcal{R}_{M,t \rightarrow T_1}^j (R_{M,t \rightarrow T_1}^l - R_{f,t \rightarrow T_1}^l) 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}} \right) \\
&= \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{E}_t^* \left[G^{(j,l)} [S_{T_1}] 1_{S_{T_1} > S_t R_{f,t \rightarrow T_1}} \right] \\
&\quad - \sum_{l=2}^{k-j} \phi_{k-j,l} \theta_{l,t}^* \mathbb{E}_t^* \left[G^{(j,l)} [S_{T_1}] 1_{S_{T_1} < S_t R_{f,t \rightarrow T_1}} \right]. \tag{OA.1.23}
\end{aligned}$$

□

OA.2 Interpretation of the Inverse SDF

OA.2.1 Interpretation of the Inverse SDF (2.2)

We apply the Carr and Madan (2001) spanning formula to $\mathcal{R}_{M,t \rightarrow T_1}^j$, $j > 1$:

$$\mathcal{R}_{M,t \rightarrow T_1}^j = \frac{j(j-1)}{S_t^2} \left\{ \int_{S_t R_{f,t \rightarrow T_1}}^{\infty} \left(\frac{K}{S_t} - R_{f,t \rightarrow T_1} \right)^{j-2} (S_{T_1} - K)^+ dK \right. \\ \left. + \int_0^{S_t R_{f,t \rightarrow T_1}} \left(\frac{K}{S_t} - R_{f,t \rightarrow T_1} \right)^{j-2} (K - S_{T_1})^+ dK \right\}.$$

Substituting into (2.5) becomes

$$v_{t \rightarrow T_1} = 1 + \sum_{j=1}^{\kappa} \frac{a_{j,t}}{R_{f,t \rightarrow T_1}^j} \frac{j(j-1)}{S_t^2} \left\{ \int_{S_t R_{f,t \rightarrow T_1}}^{\infty} \left(\frac{K}{S_t} - R_{f,t \rightarrow T_1} \right)^{j-2} (S_{T_1} - K)^+ dK \right. \\ \left. + \int_0^{S_t R_{f,t \rightarrow T_1}} \left(\frac{K}{S_t} - R_{f,t \rightarrow T_1} \right)^{j-2} (K - S_{T_1})^+ dK \right\}.$$

Since $\mathbb{E}_t^* v_{t \rightarrow T_1}$ is constant at time t , the inverse SDF

$$\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}} = \frac{v_{t \rightarrow T_1}}{\mathbb{E}_t^* v_{t \rightarrow T_1}}$$

is the payoff of a portfolio holding the market return and a continuum of options maturing at T_1 .

OA.2.2 Interpretation of the Inverse SDF (2.24)

Our goal in this appendix is to show that the inverse SDF (2.24) admits a decomposition involving the market return, a continuum of options maturing at T_1 , and a continuum of options maturing at T_N that are unwound at T_1 . Assume that $k - j = 0$, $f^{(j,k-j)} = \mathcal{R}_{M,t \rightarrow T_1}^j$. In this case our expansion in Online Appendix (OA.2.1) applies showing that Eq. (2.26) admits a decomposition that includes the payoff of a portfolio consisting of the market return, a continuum of options maturing at T_1 . Now assume $k - j > 1$, we apply the Carr

and Madan (2001) spanning formula to $\mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} = \left(\frac{S_{T_N}}{S_{T_1}} - R_{f,T_1 \rightarrow T_N} \right)^{k-j}$:

$$\mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} = \frac{(k-j)(k-j-1)}{S_{T_1}^2} \left\{ \int_{S_{T_1} R_{f,T_1 \rightarrow T_N}}^{\infty} \left(\frac{K}{S_{T_1}} - R_{f,T_1 \rightarrow T_N} \right)^{k-j-2} (S_{T_N} - K)^+ dK \right. \\ \left. + \int_0^{S_{T_1} R_{f,T_1 \rightarrow T_N}} \left(\frac{K}{S_{T_1}} - R_{f,T_1 \rightarrow T_N} \right)^{k-j-2} (K - S_{T_N})^+ dK \right\}.$$

and

$$\mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j} = (k-j)(k-j-1) R_{f,T_1 \rightarrow T_N} \left\{ \int_{R_{f,T_1 \rightarrow T_N}}^{\infty} (\bar{k} - R_{f,T_1 \rightarrow T_N})^{k-j-2} \left(\frac{C_{T_1}[K]}{S_{T_1}} \right) d\bar{k} \right. \\ \left. + \int_0^{R_{f,T_1 \rightarrow T_N}} (\bar{k} - R_{f,T_1 \rightarrow T_N})^{k-j-2} \left(\frac{P_{T_1}[K]}{S_{T_1}} \right) d\bar{k} \right\}.$$

This shows that when $j = 0$ and $k - j \geq 2$, $f^{(j,k-j)} = \mathbb{E}_{T_1}^* \mathcal{R}_{M,T_1 \rightarrow T_N}^{k-j}$ expression (2.26) admits a decomposition that includes the payoff of a portfolio consisting of a continuum of options maturing at T_N that are unwound at T_1 .

OA.3 Motivation for Assuming that $\mathbb{E}_{T_1}^* R_{M,T_1 \rightarrow T_N}^k - R_{f,T_1 \rightarrow T_N}^k$ is a Function of $R_{M,t \rightarrow T_1}$

We begin by showing that the right-hand side of (2.40) can be represented as the payoff of a portfolio of options. Using the spanning relations of Carr and Madan (2001), we may write

$$(R_{M,t \rightarrow T_1}^k - R_{f,t \rightarrow T_1}^k) 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} = \frac{k}{S_t} R_{f,t \rightarrow T_1}^{k-1} (S_{T_1} - S_t R_{f,t \rightarrow T_1})^+ \\ + \frac{k(k-1)}{S_t^2} \int_{S_t R_{f,t \rightarrow T_1}}^{\infty} \left(\frac{K}{S_t} \right)^{k-2} (S_{T_1} - K)^+ dK$$

and

$$(R_{f,t \rightarrow T_1}^k - R_{M,t \rightarrow T_1}^k) 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}} = \frac{k}{S_t} R_{f,t \rightarrow T_1}^{k-1} (S_t R_{f,t \rightarrow T_1} - S_{T_1})^+ \\ + \frac{k(k-1)}{S_t^2} \int_0^{S_t R_{f,t \rightarrow T_1}} \left(\frac{K}{S_t} \right)^{k-2} (K - S_{T_1})^+ dK$$

Combining these expressions yields

$$\begin{aligned} \mathbb{E}_{T_1}^* R_{M,T_1 \rightarrow T_N}^k - R_{f,T_1 \rightarrow T_N}^k &= \frac{\theta_{k,t}^* k}{S_t} R_{f,t \rightarrow T_1}^{k-1} ((S_{T_1} - S_t R_{f,t \rightarrow T_1})^+ + (S_t R_{f,t \rightarrow T_1} - S_{T_1})^+) \\ &\quad + \frac{\theta_{k,t}^* k (k-1)}{S_t^2} \left(\int_{S_t R_{f,t \rightarrow T_1}}^{\infty} \left(\frac{K}{S_t} \right)^{k-2} (S_{T_1} - K)^+ dK \right. \\ &\quad \left. + \int_0^{S_t R_{f,t \rightarrow T_1}} \left(\frac{K}{S_t} \right)^{k-2} (K - S_{T_1})^+ dK \right) \quad (\text{OA.3.1}) \end{aligned}$$

Thus, the right-hand side of (OA.3.1) is the payoff of a portfolio of calls and puts with weights proportional to

$$\frac{\theta_{k,t}^* k}{S_t} R_{f,t \rightarrow T_1}^{k-1} \quad \text{and} \quad \frac{\theta_{k,t}^* k (k-1)}{S_t^2} \left(\frac{K}{S_t} \right)^{k-2},$$

where $\theta_{k,t}^*$ is determined uniquely by the option prices with maturities T_1 and T_N .

Define the payoff

$$H[S_{T_N}] = R_{M,T_1 \rightarrow T_N}^k - R_{f,T_1 \rightarrow T_N}^k = \left(\frac{S_{T_N}}{S_{T_1}} \right)^k - R_{f,T_1 \rightarrow T_N}^k, \quad k \geq 2.$$

Applying the Carr and Madan (2001) spanning representation around the reference point $S_{T_N} = S_{T_1} R_{f,T_1 \rightarrow T_N}$ yields

$$\begin{aligned} H[S_{T_N}] &= H[S_{T_1} R_{f,T_1 \rightarrow T_N}] + H_S[S_{T_1} R_{f,T_1 \rightarrow T_N}](S_{T_N} - S_{T_1} R_{f,T_1 \rightarrow T_N}) \\ &\quad + \int_{S_{T_1} R_{f,T_1 \rightarrow T_N}}^{\infty} H_{SS}[K](S_{T_N} - K)^+ dK + \int_0^{S_{T_1} R_{f,T_1 \rightarrow T_N}} H_{SS}[K](K - S_{T_N})^+ dK \end{aligned}$$

Taking risk-neutral expectations yields

$$\mathbb{E}_{T_1}^* R_{M,T_1 \rightarrow T_N}^k - R_{f,T_1 \rightarrow T_N}^k = \frac{k(k-1) R_{f,T_1 \rightarrow T_N}}{S_{T_1}^2} \left(\int_{S_{T_1} R_{f,T_1 \rightarrow T_N}}^{\infty} \left(\frac{K}{S_{T_1}} \right)^{k-2} C_{T_1}[K] dK \right. \\ \left. + \int_0^{S_{T_1} R_{f,T_1 \rightarrow T_N}} \left(\frac{K}{S_{T_1}} \right)^{k-2} P_{T_1}[K] dK \right) \quad (\text{OA.3.2})$$

At time T_1 , the volatility surface pins down

$$\mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* \log R_{M,T_1 \rightarrow T_N}, K, S_{T_1}, R_{f,T_1 \rightarrow T_N}, T_N - T_1,$$

which uniquely determine the option prices $C_{T_1}[K]$ and $P_{T_1}[K]$. Thus,

$$C_{T_1}[K] = \pi^{\text{call}}[\mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* \log R_{M,T_1 \rightarrow T_N}, K, S_{T_1}, R_{f,T_1 \rightarrow T_N}, T_N - T_1]$$

and

$$P_{T_1}[K] = \pi^{\text{put}} [\mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* \log R_{M,T_1 \rightarrow T_N}, K, S_{T_1}, R_{f,T_1 \rightarrow T_N}, T_N - T_1]$$

for known functions π^{call} and π^{put} .

For the case $k = 2$, (OA.3.2) becomes

$$\begin{aligned} \mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* R_{M,T_1 \rightarrow T_N} = \frac{R_{f,T_1 \rightarrow T_N}}{S_{T_1}^2} & \left(\int_{S_{T_1} R_{f,T_1 \rightarrow T_N}}^{\infty} \pi^{\text{call}} [\mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* \log R_{M,T_1 \rightarrow T_N}, K, S_{T_1}, R_{f,T_1 \rightarrow T_N}, T_N - T_1] dK \right. \\ & \left. + \int_0^{S_{T_1} R_{f,T_1 \rightarrow T_N}} \pi^{\text{put}} [\mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* \log R_{M,T_1 \rightarrow T_N}, K, S_{T_1}, R_{f,T_1 \rightarrow T_N}, T_N - T_1] dK \right) \end{aligned} \quad (\text{OA.3.3})$$

If $\log R_{M,T_1 \rightarrow T_N}$ is conditionally lognormal, then

$$\mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* \log R_{M,T_1 \rightarrow T_N} = \log \left(\frac{\mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* R_{M,T_1 \rightarrow T_N}}{R_{f,T_1 \rightarrow T_N}^2} + 1 \right). \quad (\text{OA.3.4})$$

Substituting (OA.3.4) into (OA.3.3) shows that

$$x = \mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* \log R_{M,T_1 \rightarrow T_N}$$

solves the fixed-point equation

$$x = \frac{R_{f,T_1 \rightarrow T_N}}{S_{T_1}^2} \left(\int_{S_{T_1} R_{f,T_1 \rightarrow T_N}}^{\infty} \pi^{\text{call}} \left[\log \left(1 + \frac{x}{R_{f,T_1 \rightarrow T_N}^2} \right), K, S_{T_1}, R_{f,T_1 \rightarrow T_N}, T_N - T_1 \right] dK \right. \\ \left. + \int_0^{S_{T_1} R_{f,T_1 \rightarrow T_N}} \pi^{\text{put}} \left[\log \left(1 + \frac{x}{R_{f,T_1 \rightarrow T_N}^2} \right), K, S_{T_1}, R_{f,T_1 \rightarrow T_N}, T_N - T_1 \right] dK \right).$$

Since the solution must be strictly positive, we may write

$$\mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* R_{M,T_1 \rightarrow T_N} = g[S_{T_1}, R_{f,T_1 \rightarrow T_N}, T_N - T_1] > 0.$$

This observation motivates specification (2.40) in the paper, where the parameter $\theta_{\ell,t}^*$ is recovered in a model-free manner. Because $\mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* R_{M,T_1 \rightarrow T_N}$ is a function of S_{T_1} , it follows that $\mathbb{V}\mathbb{A}\mathbb{R}_{T_1}^* \log R_{M,T_1 \rightarrow T_N}$ is also a function of S_{T_1} . Consequently, the right-hand side of (OA.3.2) is a function of S_{T_1} . Therefore,

$$\mathbb{E}_{T_1}^* R_{M,T_1 \rightarrow T_N}^k - R_{f,T_1 \rightarrow T_N}^k \text{ is a function of } S_{T_1}, \quad k \geq 2.$$

This observation provides the economic and technical motivation for specification (2.40).

OA.4 Detailed Proof of the SDF in Presence of Intertemporal Hedging.

Proof.

$$\vartheta_{t \rightarrow T_N} = \sum_{j_1=0}^{\kappa} \sum_{j_2=0}^{\kappa} \frac{a_{j_1, j_2}}{R_{f, t \rightarrow T_1}^{j_1} R_{f, T_1 \rightarrow T_N}^{j_2}} \Delta x^{j_1} \Delta y^{j_2}. \quad (\text{OA.4.1})$$

Observe that

$$\left(\frac{\partial \vartheta_{t \rightarrow T_N}}{\partial x} \right)_{x=x_0} = \left\{ - \frac{W_t y u' [W_t x_0 y_0] u'' [W_t x y]}{(u' [W_t x y])^2} \right\}_{x=x_0} = \frac{1}{x_0} \frac{1}{\tau_t}.$$

Similarly

$$\left(\frac{\partial \vartheta_{t \rightarrow T_N}}{\partial y} \right)_{y=y_0} = \left\{ - \frac{W_t x u' [W_t x_0 y_0] u'' [W_t x y]}{(u' [W_t x y])^2} \right\}_{y=y_0} = \frac{1}{y_0} \frac{1}{\tau_t}$$

and

$$\begin{aligned} \left(\frac{\partial^2 \vartheta_{t \rightarrow T_N}}{\partial^2 x} \right)_{x=x_0} &= \left\{ - \frac{(W_t y)^2 u' [W_t x_0 y_0] u''' [W_t x y]}{(u' [W_t x y])^2} + 2 \frac{(W_t y)^2 u' [W_t x y] u' [W_t x_0 y_0] (u'' [W_t x y])^2}{(u' [W_t x y])^4} \right\}_{x=x_0} \\ &= \frac{2(1 - \rho_t)}{\tau_t^2 x_0^2}. \end{aligned}$$

Similarly, we show

$$\left(\frac{\partial^2 \vartheta_{t \rightarrow T_N}}{\partial^2 y} \right)_{y=y_0} = \frac{2(1 - \rho_t)}{\tau_t^2 y_0^2}.$$

Observe that

$$\left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial^3 x} \right)_{x=x_0} = \left\{ \begin{aligned} & - \frac{(W_t y)^3 u' [W_t x_0 y_0] u'''' [W_t x y]}{(u' [W_t x y])^2} + \frac{6 u'' [W_t x y] (W_t y)^3 u' [W_t x_0 y_0] u''' [W_t x y]}{(u' [W_t x y])^3} \\ & - 6 \frac{(W_t y)^3 u' [W_t x_0 y_0] (u' [W_t x y])^2 (u'' [W_t x y])^3}{(u' [W_t x y])^6} \end{aligned} \right\}_{x=x_0}$$

which simplifies to

$$\left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial^3 x} \right)_{x=x_0} = \frac{6\kappa_t + 6 - 12\rho_t}{\tau_t^3 x_0^3}.$$

Similarly

$$\left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial^3 y} \right)_{y=y_0} = \frac{6\kappa_t + 6 - 12\rho_t}{\tau_t^3 y_0^3}.$$

Now, observe that

$$\left(\frac{\partial^2 \vartheta_{t \rightarrow T_N}}{\partial^2 y} \right)_{y=x_0} = \left\{ -\frac{(W_t x)^2 u' [W_t x_0 y_0] u''' [W_t x y]}{(u' [W_t x y])^2} + 2 \frac{(W_t x)^2 u' [W_t x_0 y_0] (u'' [W_t x y])^2}{(u' [W_t x y])^3} \right\}_{y=y_0}$$

and

$$\left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial^2 y \partial x} \right)_{(x,y)=(x_0,y_0)} = \left\{ \begin{aligned} & -\frac{2(W_t x) W_t u' [W_t x_0 y_0] u''' [W_t x y]}{(u' [W_t x y])^2} \\ & -\frac{(W_t x)^2 W_t y u' [W_t x_0 y_0] u'''' [W_t x y]}{(u' [W_t x y])^2} \\ & + \frac{2(u' [W_t x y]) (u'' [W_t x y]) (W_t x)^2 W_t y u' [W_t x_0 y_0] u''' [W_t x y]}{(u' [W_t x y])^4} \\ & + 4 \frac{(W_t x) W_t u' [W_t x_0 y_0] (u'' [W_t x y])^2}{(u' [W_t x y])^3} \\ & + 4 \frac{(W_t x)^2 W_t y u' [W_t x_0 y_0] u'' [W_t x y] u''' [W_t x y]}{(u' [W_t x y])^3} \\ & - 6 \frac{(u' [W_t x y])^2 u'' [W_t x y] W_t y (W_t x)^2 u' [W_t x_0 y_0] (u'' [W_t x y])^2}{(u' [W_t x y])^6} \end{aligned} \right\}_{(x,y)=(x_0,y_0)}.$$

This simplifies to

$$\left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial^2 y \partial x} \right)_{y=x_0} = \frac{4(1 - \rho_t)}{\tau_t^2 x_0 y_0^2} + \frac{6(\kappa_t + 1 - 2\rho_t)}{\tau_t^3 x_0 y_0^2}.$$

Now, observe that

$$\left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial^3 x} \right)_{x=x_0} = \left\{ \begin{aligned} & -\frac{(W_t y)^3 u' [W_t x_0 y_0] u'''' [W_t x y]}{(u' [W_t x y])^2} + \frac{6u'' [W_t x y] (W_t y)^3 u' [W_t x_0 y_0] u''' [W_t x y]}{(u' [W_t x y])^3} \\ & - 6 \frac{(W_t y)^3 u' [W_t x_0 y_0] (u' [W_t x y])^2 (u'' [W_t x y])^3}{(u' [W_t x y])^6} \end{aligned} \right\}_{x=x_0}.$$

Hence,

$$\left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial^4 x} \right)_{x=x_0} = \left\{ \begin{aligned} & - \frac{(W_t y)^4 u' [W_t x_0 y_0] u'''' [W_t x y]}{(u' [W_t x y])^2} + 2 \frac{(W_t y)^4 u'' [W_t x y] u' [W_t x_0 y_0] u'''' [W_t x y]}{(u' [W_t x y])^3} \\ & + \frac{6 u''' [W_t x y] (W_t y)^4 u' [W_t x_0 y_0] u'' [W_t x y]}{(u' [W_t x y])^3} \\ & + \frac{6 u'' [W_t x y] (W_t y)^4 u' [W_t x_0 y_0] u'''' [W_t x y]}{(u' [W_t x y])^3} \\ & - \frac{18 (u'' [W_t x y])^2 (W_t y)^4 u' [W_t x_0 y_0] u''' [W_t x y]}{(u' [W_t x y])^4} \\ & - 18 \frac{(W_t y)^4 u' [W_t x_0 y_0] (u'' [W_t x y])^2 u''' [W_t x y]}{(u' [W_t x y])^4} \\ & + 24 \frac{(W_t y)^4 u' [W_t x_0 y_0] (u'' [W_t x y])^4}{(u' [W_t x y])^5} \end{aligned} \right\}_{x=x_0}.$$

This simplifies to

$$\frac{1}{24} \left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial^4 x} \right)_{x=x_0} = \frac{\rho_t^2 - 3\rho_t + 1 + 2\kappa_t - \xi_t}{\tau_t^4 x_0^4}.$$

Finally, recall that:

$$\left(\frac{\partial^3 \vartheta_{t \rightarrow T_N}}{\partial^2 y \partial x} \right)_{(x,y)=(x_0,y_0)} = \left\{ \begin{aligned} & - \frac{2(W_t x) W_t u' [W_t x_0 y_0] u''' [W_t x y]}{(u' [W_t x y])^2} \\ & - \frac{(W_t x)^2 W_t y u' [W_t x_0 y_0] u'''' [W_t x y]}{(u' [W_t x y])^2} \\ & + \frac{6(u'' [W_t x y]) (W_t x)^2 W_t y u' [W_t x_0 y_0] u''' [W_t x y]}{(u' [W_t x y])^3} \\ & + 4 \frac{(W_t x) W_t u' [W_t x_0 y_0] (u'' [W_t x y])^2}{(u' [W_t x y])^3} \\ & - 6 \frac{W_t y (W_t x)^2 u' [W_t x_0 y_0] (u'' [W_t x y])^3}{(u' [W_t x y])^4} \end{aligned} \right\}_{(x,y)=(x_0,y_0)}.$$

Set

$$\begin{aligned} A_1 &= - \frac{2(W_t x) W_t u' [W_t x_0 y_0] u''' [W_t x y]}{(u' [W_t x y])^2}, \quad A_2 = - \frac{(W_t x)^2 W_t y u' [W_t x_0 y_0] u'''' [W_t x y]}{(u' [W_t x y])^2}, \\ A_3 &= \frac{6(u'' [W_t x y]) (W_t x)^2 W_t y u' [W_t x_0 y_0] u''' [W_t x y]}{(u' [W_t x y])^3}, \quad A_4 = 4 \frac{(W_t x) W_t u' [W_t x_0 y_0] (u'' [W_t x y])^2}{(u' [W_t x y])^3}, \\ A_5 &= -6 \frac{W_t y (W_t x)^2 u' [W_t x_0 y_0] (u'' [W_t x y])^3}{(u' [W_t x y])^4}. \end{aligned}$$

It follows that

$$\begin{aligned}\left(\frac{\partial A_1}{\partial x}\right)_{(x,y)=(x_0,y_0)} &= -\frac{4\rho_t}{\tau_t^2 x_0^2 y_0^2} + \frac{12\kappa_t}{\tau_t^3 x_0^2 y_0^2} - \frac{8\rho_t}{\tau_t^3 x_0^2 y_0^2} \\ &= -\frac{4\rho_t}{\tau_t^2 x_0^2 y_0^2} + \frac{4(3\kappa_t - 2\rho_t)}{\tau_t^3 x_0^2 y_0^2}\end{aligned}$$

and

$$\left(\frac{\partial A_2}{\partial x}\right)_{(x,y)=(x_0,y_0)} = \frac{12\kappa_t}{\tau_t^3 x_0^2 y_0^2} + \frac{12(\kappa_t - 2\xi_t)}{\tau_t^4 x_0^2 y_0^2}$$

and

$$\left(\frac{\partial A_3}{\partial x}\right)_{(x,y)=(x_0,y_0)} = \frac{12(2\rho_t^2 + 3\kappa_t - 3\rho_t)}{\tau_t^4 x_0^2 y_0^2} - \frac{24\rho_t}{\tau_t^3 x_0^2 y_0^2}$$

and

$$\left(\frac{\partial A_4}{\partial x}\right)_{(x,y)=(x_0,y_0)} = \frac{4}{\tau_t^2 x_0^2 y_0^2} + \frac{4(3 - 4\rho_t)}{\tau_t^3 x_0^2 y_0^2}$$

and

$$\begin{aligned}\left(\frac{\partial A_5}{\partial x}\right)_{(x,y)=(x_0,y_0)} &= -12 \frac{W_t y (W_t x) W_t u' [W_t x_0 y_0] (u'' [W_t x y])^3}{(u' [W_t x y])^4} \\ &\quad - 18 \frac{(W_t y)^2 (W_t x)^2 u' [W_t x_0 y_0] (u'' [W_t x y])^2 u''' [W_t x y]}{(u' [W_t x y])^4} \\ &\quad + 24 \frac{(W_t y)^2 (W_t x)^2 u' [W_t x_0 y_0] (u' [W_t x y])^3 (u'' [W_t x y])^4}{(u' [W_t x y])^8}\end{aligned}$$

and

$$\left(\frac{\partial A_5}{\partial x}\right)_{(x,y)=(x_0,y_0)} = \frac{12}{\tau_t^3 x_0^2 y_0^2} + \frac{12(2 - 3\rho_t)}{\tau_t^4 x_0^2 y_0^2}.$$

Finally

$$\begin{aligned}
\left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial^2 y \partial^2 x} \right)_{(x,y)=(x_0,y_0)} &= -\frac{4\rho_t}{\tau_t^2 x_0^2 y_0^2} + \frac{4(3\kappa_t - 2\rho_t)}{\tau_t^3 x_0^2 y_0^2} \\
&+ \frac{12\kappa_t}{\tau_t^3 x_0^2 y_0^2} + \frac{12(\kappa_t - 2\xi_t)}{\tau_t^4 x_0^2 y_0^2} \\
&+ \frac{12(2\rho_t^2 + 3\kappa_t - 2\rho_t)}{\tau_t^4 x_0^2 y_0^2} - \frac{24\rho_t}{\tau_t^3 x_0^2 y_0^2} \\
&+ \frac{4}{\tau_t^2 x_0^2 y_0^2} + \frac{4(3 - 4\rho_t)}{\tau_t^3 x_0^2 y_0^2} \\
&+ \frac{12}{\tau_t^3 x_0^2 y_0^2} + \frac{12(2 - 3\rho_t)}{\tau_t^4 x_0^2 y_0^2},
\end{aligned}$$

which simplifies to

$$\begin{aligned}
\left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial^2 y \partial^2 x} \right)_{(x,y)=(x_0,y_0)} &= \frac{4(1 - \rho_t)}{\tau_t^2 x_0^2 y_0^2} + \frac{24(\kappa_t - 2\rho_t + 1)}{\tau_t^3 x_0^2 y_0^2} \\
&+ \frac{24(2\kappa_t - \xi_t + \rho_t^2 - 3\rho_t + 1)}{\tau_t^4 x_0^2 y_0^2}.
\end{aligned}$$

Set

$$\begin{aligned}
B_1 &= -\frac{(W_t x)^3 u' [W_t x_0 y_0] u'''' [W_t x y]}{(u' [W_t x y])^2}, \\
B_2 &= \frac{6u'' [W_t x y] (W_t x)^3 u' [W_t x_0 y_0] u''' [W_t x y]}{(u' [W_t x y])^3}, \\
B_3 &= -6 \frac{(W_t x)^3 u' [W_t x_0 y_0] (u'' [W_t x y])^3}{(u' [W_t x y])^4}.
\end{aligned}$$

Then

$$\left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial^3 y \partial x} \right) = \frac{\partial B_1}{\partial x} + \frac{\partial B_2}{\partial x} + \frac{\partial B_3}{\partial x},$$

where

$$\frac{\partial B_1}{\partial x} = \left\{ \begin{aligned} &-\frac{3(W_t x)^2 W_t u' [W_t x_0 y_0] u'''' [W_t x y]}{(u' [W_t x y])^2} \\ &-\frac{W_t y (W_t x)^3 u' [W_t x_0 y_0] u'''' [W_t x y]}{(u' [W_t x y])^2} \\ &+\frac{2(W_t x)^3 W_t y u'' [W_t x y] u'''' [W_t x y]}{(u' [W_t x y])^2} \end{aligned} \right\}_{(x,y)=(x_0,y_0)}.$$

Hence

$$\left(\frac{\partial B_1}{\partial x}\right)_{(x,y)=(x_0,y_0)} = \frac{18\kappa_t}{\tau_t^4 x_0 y_0^3} + \frac{(12\kappa_t - 24\xi_t)}{\tau_t^4 x_0 y_0^3}$$

and

$$\left(\frac{\partial B_2}{\partial x}\right)_{(x,y)=(x_0,y_0)} = \left\{ \begin{array}{l} \frac{6W_t y (W_t x)^3 (u'''[W_t x y])^2}{(u'[W_t x y])^2} \\ + \frac{18u''[W_t x y](W_t x)^2 W_t u'''[W_t x y]}{(u'[W_t x y])^2} \\ + \frac{6u''[W_t x y]W_t y (W_t x)^3 u''''[W_t x y]}{(u'[W_t x y])^2} \\ - \frac{18(u''[W_t x y])^2 (W_t x)^3 W_t y u'''[W_t x y]}{(u'[W_t x y])^3} \end{array} \right\}_{(x,y)=(x_0,y_0)}$$

and

$$\left(\frac{\partial B_2}{\partial x}\right)_{(x,y)=(x_0,y_0)} = \frac{24\rho_t^2}{\tau_t^4 x_0 y_0^3} - \frac{36\rho_t}{\tau_t^3 x_0 y_0^3} + \frac{36\kappa_t}{\tau_t^4 x_0 y_0^3} - \frac{36\rho_t}{\tau_t^4 x_0 y_0^3}$$

and

$$\frac{\partial B_3}{\partial x} = \left\{ \begin{array}{l} -18 \frac{(W_t x)^2 W_t (u''[W_t x y])^3}{(u'[W_t x y])^3} \\ -18 \frac{(W_t x)^3 W_t y (u''[W_t x y])^2 u'''[W_t x y]}{(u'[W_t x y])^3} \\ +24 \frac{(W_t x)^3 W_t y (u''[W_t x y])^4}{(u'[W_t x y])^4} \end{array} \right\}_{(x,y)=(x_0,y_0)},$$

which simplifies to

$$\frac{\partial B_3}{\partial x} = \frac{18}{\tau_t^3 x_0 y_0^3} - \frac{36\rho_t}{\tau_t^4 x_0 y_0^3} + \frac{24}{\tau_t^4 x_0 y_0^3}.$$

Hence

$$\begin{aligned} \left(\frac{\partial^4 \vartheta_{t \rightarrow T_N}}{\partial^3 y \partial x}\right)_{(x,y)=(x_0,y_0)} &= \frac{18 - 36\rho_t}{\tau_t^3 x_0 y_0^3} + \frac{18\kappa_t + 48\kappa_t - 24\xi_t + 24\rho_t^2 - 72\rho_t + 24}{\tau_t^4 x_0 y_0^3} \\ &= \frac{6}{x_0 y_0^3} \left\{ \frac{\frac{3-6\rho_t}{\tau_t^3} + \frac{3\kappa_t}{\tau_t^4}}{+ \frac{4(2\kappa_t - \xi_t + \rho_t^2 - 3\rho_t + 1)}{\tau_t^4}} \right\} \\ &= \frac{6}{x_0 y_0^3} \left\{ 3a_{3,t} + 3 \left(1 - 2\frac{a_{2,t}}{a_{1,t}^2} + a_{3,t} \right) (a_{1,t} - 1) + 4a_{4,t} \right\}. \end{aligned}$$

As a result:

$$\begin{aligned}
v_{t \rightarrow T_1} &= \mathbb{E}_{T_N}^* \frac{u' [W_t x_0 y_0]}{u' [W_t x y]} \\
&= 1 + \frac{1}{R_{f,t \rightarrow T_1}} a_{1,t} \Delta x + \frac{a_{2,t}}{R_{f,t \rightarrow T_1}^2} (\Delta x)^2 + \frac{a_{2,t}}{R_{f,T_1 \rightarrow T_N}^2} \mathbb{E}_{T_1}^* (\Delta y)^2 \\
&\quad + \frac{a_{3,t}}{R_{f,t \rightarrow T_1}^3} (\Delta x)^3 + \frac{a_{3,t}}{R_{f,T_1 \rightarrow T_N}^3} \mathbb{E}_{T_1}^* (\Delta y)^3 + \frac{a_{5,t}}{R_{f,t \rightarrow T_1} R_{f,T_1 \rightarrow T_N}^2} \Delta x \mathbb{E}_{T_1}^* (\Delta y)^2 \\
&\quad + \frac{a_{4,t}}{R_{f,t \rightarrow T_1}^4} (\Delta x)^4 + \frac{a_{4,t}}{R_{f,T_1 \rightarrow T_N}^4} \mathbb{E}_{T_1}^* (\Delta y)^4 + \frac{a_{6,t}}{R_{f,t \rightarrow T_1} R_{f,T_1 \rightarrow T_N}^3} (\Delta x) \mathbb{E}_{T_1}^* (\Delta y)^3 \\
&\quad + \frac{a_{7,t}}{R_{f,t \rightarrow T_1}^2 R_{f,T_1 \rightarrow T_N}^2} (\Delta x)^2 \mathbb{E}_{T_1}^* (\Delta y)^2,
\end{aligned}$$

with

$$\begin{aligned}
a_{5,t} &= 2a_{2,t} + 3a_{3,t} \\
a_{6,t} &= 3a_{3,t} + 3 \left(1 - 2 \frac{a_{2,t}}{a_{1,t}^2} + a_{3,t} \right) (a_{1,t} - 1) + 4a_{4,t} \\
a_{7,t} &= a_{2,t} + 6a_{3,t} + 6a_{4,t}.
\end{aligned}$$

This ends the proof. □

OA.5 Computation of the Risk Neutral Moments

OA.5.1 Computation of $\mathbb{E}_t^* \mathcal{R}_{i,t \rightarrow T_1}^q$

Notice that for $q \geq 2$

$$\mathbb{E}_t^* \mathcal{R}_{i,t \rightarrow T_1}^q = \mathbb{E}_t^* \left(\frac{S_{i,T_1}}{S_{i,t}} - R_{f,t \rightarrow T_1} \right)^q \tag{OA.5.1}$$

$$= \frac{q(q-1) R_{f,t \rightarrow T_1}}{S_{i,t}^2} \left\{ \int_{S_{i,t} R_{f,t \rightarrow T_1}}^{\infty} \left(\frac{K}{S_{i,t}} - R_{f,t \rightarrow T_1} \right)^{q-2} C_{i,t} [K] dK + \int_0^{S_{i,t} R_{f,t \rightarrow T_1}} \left(\frac{K}{S_{i,t}} - R_{f,t \rightarrow T_1} \right)^{q-2} P_{i,t} [K] dK \right\} \tag{OA.5.2}$$

OA.5.2 Computation of $\mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}]$

Next,

$$\begin{aligned} \mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}] &= \frac{1}{S_{i,t}} \left\{ \mathbb{E}_t^* [(S_{i,T_1} - S_{i,t}k_0 + S_{i,t}k_0 - S_{i,t}R_{f,t \rightarrow T_1}) 1_{S_{i,T_1} \geq S_{i,t}k_0}] \right\} \\ &= \frac{R_{f,t \rightarrow T_1} C_{i,t} [S_{i,t}k_0]}{S_{i,t}} + \frac{(S_{i,t}k_0 - S_{i,t}R_{f,t \rightarrow T_1})}{S_{i,t}} \mathbb{P}_t^* [S_{i,T_1} \geq S_{i,t}k_0]. \end{aligned}$$

OA.5.3 Computation of $\mathbb{E}_t^* [R_{M,t \rightarrow T_1}^{j_1} |R_{M,t \rightarrow T_1} - R_{f,t \rightarrow T_1}|^l] = \mathbb{E}_t^* [R_{M,t \rightarrow T_1}^{j_1} |\mathcal{R}_{M,t \rightarrow T_1}|^l]$

Observe that

$$|\mathcal{R}_{M,t \rightarrow T_1}|^l = \mathcal{R}_{M,t \rightarrow T_1}^l 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} + (-1)^l \mathcal{R}_{M,t \rightarrow T_1}^l 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}}.$$

Hence,

$$\begin{aligned} \mathbb{E}_t^* (R_{M,t \rightarrow T_1}^{j_1} |\mathcal{R}_{M,t \rightarrow T_1}|^l) &= \mathbb{E}_t^* [R_{M,t \rightarrow T_1}^{j_1} \mathcal{R}_{M,t \rightarrow T_1}^l 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}}] \\ &\quad + (-1)^l \mathbb{E}_t^* [R_{M,t \rightarrow T_1}^{j_1} \mathcal{R}_{M,t \rightarrow T_1}^l 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}}]. \end{aligned}$$

Notice that

$$\mathbb{E}_t^* [R_{M,t \rightarrow T_1}^{j_1} \mathcal{R}_{M,t \rightarrow T_1}^l 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}}] = \mathbb{E}_t^* [\mathcal{G} [S_{T_1}] 1_{S_{T_1} > S_t R_{f,t \rightarrow T_1}}],$$

with

$$\mathcal{G} [S_{T_1}] = \left(\frac{S_{T_1}}{S_t} \right)^{j_1} \left(\frac{S_{T_1}}{S_t} - R_{f,t \rightarrow T_1} \right)^l.$$

Thus, if $l > 0$,

$$\begin{aligned} \mathbb{E}_t^* [\mathcal{G} [S_{T_1}] 1_{S_{T_1} > S_t R_{f,t \rightarrow T_1}}] &= R_{f,t \rightarrow T_1} \mathcal{G}_S [S_t R_{f,t \rightarrow T_1}] C_t [S_t R_{f,t \rightarrow T_1}] \\ &\quad + R_{f,t \rightarrow T_1} \int_{S_t R_{f,t \rightarrow T_1}}^{\infty} \mathcal{G}_{SS} [K] C_t [K] dK. \end{aligned}$$

Similarly,

$$\mathbb{E}_t^* [\mathcal{G} [S_{T_1}] 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}}] = \mathbb{E}_t^* [\mathcal{G} [S_{T_1}] 1_{S_{T_1} < S_t R_{f,t \rightarrow T_1}}].$$

Thus, if $l > 0$,

$$\begin{aligned}\mathbb{E}_t^* \left[\mathcal{G} [S_{T_1}] 1_{S_{T_1} < S_t R_{f,t \rightarrow T_1}} \right] &= -R_{f,t \rightarrow T_1} \mathcal{G}_S [S_t R_{f,t \rightarrow T_1}] P_t [S_t R_{f,t \rightarrow T_1}] \\ &\quad + R_{f,t \rightarrow T_1} \int_0^{S_t R_{f,t \rightarrow T_1}} \mathcal{G}_{SS} [K] P_t [K] dK.\end{aligned}$$

OA.5.4 Computation of $\mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^{j_1} |R_{M,t \rightarrow T_1} - R_{f,t \rightarrow T_1}|^l] = \mathbb{E}_t^* [\mathcal{R}_{M,t \rightarrow T_1}^{j_1} |\mathcal{R}_{M,t \rightarrow T_1}|^l]$

Observe that

$$|\mathcal{R}_{M,t \rightarrow T_1}|^l = (\mathcal{R}_{M,t \rightarrow T_1})^l 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} + (-1)^l (\mathcal{R}_{M,t \rightarrow T_1})^l 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}}.$$

Hence,

$$\begin{aligned}\mathbb{E}_t^* \left(\mathcal{R}_{M,t \rightarrow T_1}^{j_1} |\mathcal{R}_{M,t \rightarrow T_1}|^l \right) &= \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j_1+l} 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} \right] \\ &\quad + (-1)^l \mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j_1+l} 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}} \right].\end{aligned}$$

Notice that

$$\mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j_1+l} 1_{R_{M,t \rightarrow T_1} > R_{f,t \rightarrow T_1}} \right] = \mathbb{E}_t^* \left[G [S_{T_1}] 1_{S_{T_1} > S_t R_{f,t \rightarrow T_1}} \right],$$

with

$$G [S_{T_1}] = \left(\frac{S_{T_1}}{S_t} - R_{f,t \rightarrow T_1} \right)^{j_1+l}.$$

Thus, if $j_1 + l > 0$,

$$\begin{aligned}\mathbb{E}_t^* \left[G [S_{T_1}] 1_{S_{T_1} > S_t R_{f,t \rightarrow T_1}} \right] &= R_{f,t \rightarrow T_1} G_S [S_t R_{f,t \rightarrow T_1}] C_t [S_t R_{f,t \rightarrow T_1}] \\ &\quad + R_{f,t \rightarrow T_1} \int_{S_t R_{f,t \rightarrow T_1}}^{\infty} G_{SS} [K] C_t [K] dK.\end{aligned}$$

Similarly,

$$\mathbb{E}_t^* \left[\mathcal{R}_{M,t \rightarrow T_1}^{j_1+l} 1_{R_{M,t \rightarrow T_1} < R_{f,t \rightarrow T_1}} \right] = \mathbb{E}_t^* \left[H [S_{T_1}] 1_{S_{T_1} < S_t R_{f,t \rightarrow T_1}} \right],$$

with

$$H [S_{T_1}] = \left(\frac{S_{T_1}}{S_t} - R_{f,t \rightarrow T_1} \right)^{j_1+l}.$$

Thus, if $j_1 + l > 0$,

$$\begin{aligned}\mathbb{E}_t^* \left[H[S_{T_1}] 1_{S_{T_1} < S_t R_{f,t \rightarrow T_1}} \right] &= -R_{f,t \rightarrow T_1} H_S[S_t R_{f,t \rightarrow T_1}] P_t[S_t R_{f,t \rightarrow T_1}] \\ &\quad + R_{f,t \rightarrow T_1} \int_0^{S_t R_{f,t \rightarrow T_1}} H_{SS}[K] P_t[K] dK.\end{aligned}$$

OA.5.5 Computation of $\mathbb{E}_t^* \left(\sum_{k=1}^{\mathcal{N}} \underline{\gamma}_{i,k} (\mathcal{R}_{M,t \rightarrow T_1}^k - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^k) \right)^q$

The key here is to expand the term within the risk-neutral expectation and then use (OA.5.2) to compute the risk-neutral quantities.

- Assume $\mathcal{N}=2$ then $q = 2, 3$. For $q = 2$,

$$\begin{aligned}&\mathbb{E}_t^* \left(\underline{\gamma}_{i,1} \mathcal{R}_{M,t \rightarrow T_1} + \underline{\gamma}_{i,2} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2) \right)^2 \\ &= \underline{\gamma}_{i,1}^2 \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2 + \underline{\gamma}_{i,2}^2 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^2 \\ &\quad + 2\underline{\gamma}_{i,1} \underline{\gamma}_{i,2} \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)).\end{aligned}$$

For $q = 3$,

$$\begin{aligned}&\mathbb{E}_t^* \left(\underline{\gamma}_{i,1} \mathcal{R}_{M,t \rightarrow T_1} + \underline{\gamma}_{i,2} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2) \right)^3 \\ &= \underline{\gamma}_{i,1}^3 \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3 + \underline{\gamma}_{i,2}^3 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^3 \\ &\quad + 3\underline{\gamma}_{i,1}^2 \underline{\gamma}_{i,2} \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^2 (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)) \\ &\quad + 3\underline{\gamma}_{i,1} \underline{\gamma}_{i,2}^2 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^2).\end{aligned}$$

- Assume $\mathcal{N}=3$ then $q = 2, 3, 4$. For $q = 2$,

$$\begin{aligned}&\mathbb{E}_t^* \left(\underline{\gamma}_{i,1} \mathcal{R}_{M,t \rightarrow T_1} + \underline{\gamma}_{i,2} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2) + \underline{\gamma}_{i,3} (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3) \right)^2 \\ &= \underline{\gamma}_{i,1}^2 \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2 + \underline{\gamma}_{i,2}^2 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^2 \\ &\quad + \underline{\gamma}_{i,3}^2 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)^2 \\ &\quad + 2\underline{\gamma}_{i,2} \underline{\gamma}_{i,1} \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)) \\ &\quad + 2\underline{\gamma}_{i,1} \underline{\gamma}_{i,3} \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1} (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)) \\ &\quad + 2\underline{\gamma}_{i,3} \underline{\gamma}_{i,2} \mathbb{E}_t^* ((\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2) (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)),\end{aligned}$$

for $q = 3$,

$$\begin{aligned}
& \mathbb{E}_t^* \left(\underline{\gamma}_{i,1} \mathcal{R}_{M,t \rightarrow T_1} + \underline{\gamma}_{i,2} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2) + \underline{\gamma}_{i,3} (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3) \right)^3 \\
= & \underline{\gamma}_{i,1}^3 \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3 + \underline{\gamma}_{i,2}^3 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^3 + \underline{\gamma}_{i,3}^3 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)^3 \\
& + 3 \underline{\gamma}_{i,2} \underline{\gamma}_{i,1}^2 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^2 (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)) \\
& + 3 \underline{\gamma}_{i,1}^2 \underline{\gamma}_{i,3} \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^2 (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)) \\
& + 3 \underline{\gamma}_{i,1} \underline{\gamma}_{i,2}^2 \mathbb{E}_t^* \left(\mathcal{R}_{M,t \rightarrow T_1} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^2 \right) \\
& + 3 \underline{\gamma}_{i,3} \underline{\gamma}_{i,2}^2 \mathbb{E}_t^* \left((\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^2 (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2) \right) \\
& + 3 \underline{\gamma}_{i,1} \underline{\gamma}_{i,3}^2 \mathbb{E}_t^* \left(\mathcal{R}_{M,t \rightarrow T_1} (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)^2 \right) \\
& + 3 \underline{\gamma}_{i,2} \underline{\gamma}_{i,3}^2 \mathbb{E}_t^* \left((\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2) (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)^2 \right) \\
& + 6 \underline{\gamma}_{i,1} \underline{\gamma}_{i,2} \underline{\gamma}_{i,3} \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2) (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)) ,
\end{aligned}$$

and for $q = 4$,

$$\begin{aligned}
& \mathbb{E}_t^* \left(\underline{\gamma}_{i,1} \mathcal{R}_{M,t \rightarrow T_1} + \underline{\gamma}_{i,2} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2) + \underline{\gamma}_{i,3} (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3) \right)^4 \\
= & \underline{\gamma}_{i,1}^4 \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^4 + \underline{\gamma}_{i,2}^4 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^4 + \underline{\gamma}_{i,3}^4 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)^4 \\
& + 4 \underline{\gamma}_{i,1}^3 \underline{\gamma}_{i,2} \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^3 (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)) \\
& + 4 \underline{\gamma}_{i,1}^3 \underline{\gamma}_{i,3} \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^3 (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)) \\
& + 4 \underline{\gamma}_{i,1} \underline{\gamma}_{i,2}^3 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^3) \\
& + 4 \underline{\gamma}_{i,3} \underline{\gamma}_{i,2}^3 \mathbb{E}_t^* ((\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^3 (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)) \\
& + 4 \underline{\gamma}_{i,1} \underline{\gamma}_{i,3}^3 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1} (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)^3) \\
& + 4 \underline{\gamma}_{i,2} \underline{\gamma}_{i,3}^3 \mathbb{E}_t^* ((\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2) (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)^3) \\
& + 6 \underline{\gamma}_{i,1}^2 \underline{\gamma}_{i,2}^2 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^2 (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^2) \\
& + 6 \underline{\gamma}_{i,1}^2 \underline{\gamma}_{i,3}^2 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^2 (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)^2) \\
& + 6 \underline{\gamma}_{i,2}^2 \underline{\gamma}_{i,3}^2 \mathbb{E}_t^* ((\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^2 (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)^2) \\
& + 12 \underline{\gamma}_{i,1} \underline{\gamma}_{i,3} \underline{\gamma}_{i,2} \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1}^2 (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2) (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)) \\
& + 12 \underline{\gamma}_{i,1} \underline{\gamma}_{i,2}^2 \underline{\gamma}_{i,3} \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2)^2 (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)) \\
& + 12 \underline{\gamma}_{i,1} \underline{\gamma}_{i,2} \underline{\gamma}_{i,3}^2 \mathbb{E}_t^* (\mathcal{R}_{M,t \rightarrow T_1} (\mathcal{R}_{M,t \rightarrow T_1}^2 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^2) (\mathcal{R}_{M,t \rightarrow T_1}^3 - \mathbb{E}_t^* \mathcal{R}_{M,t \rightarrow T_1}^3)^2).
\end{aligned}$$

OA.5.6 Computation of $\mathbb{E}_t^* \left[(R_{i,t \rightarrow T_1} - R_{f,t \rightarrow T_1})^j 1_{R_{i,t \rightarrow T_1} \geq k_0} \right]$

Observe that

$$\mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1}^j 1_{R_{i,t \rightarrow T_1} \geq k_0}] = \mathbb{E}_t^* [(R_{i,t \rightarrow T_1} - R_{f,t \rightarrow T_1})^j 1_{R_{i,t \rightarrow T_1} \geq k_0}].$$

For $j = 0$

$$\mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1}^j 1_{R_{i,t \rightarrow T_1} \geq k_0}] = \mathbb{E}_t^* [1_{R_{i,t \rightarrow T_1} \geq k_0}] = \mathbb{P}_t^* [R_{i,t \rightarrow T_1} \geq k_0].$$

For $j = 1$

$$\mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1}^j 1_{R_{i,t \rightarrow T_1} \geq k_0}] = \mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}].$$

This is already provided. For $j > 1$

$$\begin{aligned} \mathbb{E}_t^* \left[(R_{i,t \rightarrow T_1} - R_{f,t \rightarrow T_1})^j 1_{R_{i,t \rightarrow T_1} \geq k_0} \right] &= (k_0 - R_{f,t \rightarrow T_1})^j \mathbb{P}_t^* [R_{i,t \rightarrow T_1} \geq k_0] \\ &\quad + j \frac{R_{f,t \rightarrow T_1}}{S_{i,t}} (k_0 - R_{f,t \rightarrow T_1})^{j-1} C_{i,t} [k_0 S_{i,t}] \\ &\quad + \frac{j(j-1)}{S_t^2} R_{f,t \rightarrow T_1} \int_{k_0 S_{i,t}}^{\infty} \left(\frac{K}{S_{i,t}} - R_{f,t \rightarrow T_1} \right)^{j-2} C_{i,t} [K] dK. \end{aligned} \quad (\text{OA.5.3})$$

OA.5.7 Computation of $\mathbb{E}_t^* (\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0} - \mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}])^q$

Next, how do we compute

$$\mathbb{E}_t^* (\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0} - \mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}])^q.$$

Observe that

$$\begin{aligned} &(\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0} - \mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}])^q \\ &= \sum_{j=0}^q \frac{q!}{(q-j)!j!} (\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0})^j (-1)^{q-j} (\mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}])^{q-j}. \end{aligned}$$

Thus

$$\begin{aligned} &\mathbb{E}_t^* (\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0} - \mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}])^q \\ &= \sum_{j=0}^q \frac{q! (-1)^{q-j}}{(q-j)!j!} (\mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1}^j 1_{R_{i,t \rightarrow T_1} \geq k_0}]) (\mathbb{E}_t^* [\mathcal{R}_{i,t \rightarrow T_1} 1_{R_{i,t \rightarrow T_1} \geq k_0}])^{q-j}. \end{aligned}$$

OA.5.8 Computation of $\mathbb{E}_t^* f_{T_1}^{(j_1, j_2)}$

Notice that

$$f_{T_1}^{(j_1, j_2)} = \mathcal{R}_{M, t \rightarrow T_1}^{j_1} \mathbb{E}_{T_1}^* [\mathcal{R}_{M, T_1 \rightarrow T_N}^{j_2}]$$

and

$$\mathbb{E}_{T_1}^* \mathcal{R}_{M, T_1 \rightarrow T_N}^{j_2} = \sum_{l=0}^{j_2} \phi_{j_2, l} R_{f, T_1 \rightarrow T_N}^l + \sum_{l=2}^{j_2} \phi_{j_2, l} (\mathbb{E}_{T_1}^* R_{M, T_1 \rightarrow T_N}^l - R_{f, T_1 \rightarrow T_N}^l).$$

Thus

$$\begin{aligned}
f_{T_1}^{(j_1, j_2)} &= \mathcal{R}_{M, t \rightarrow T_1}^{j_1} \left(\sum_{l=0}^{j_2} \phi_{j_2, l} R_{f, T_1 \rightarrow T_N}^l + \sum_{l=2}^{j_2} \phi_{j_2, l} (\mathbb{E}_{T_1}^* R_{M, T_1 \rightarrow T_N}^l - R_{f, T_1 \rightarrow T_N}^l) \right) \\
&= \sum_{l=0}^{j_2} \phi_{j_2, l} R_{f, T_1 \rightarrow T_N}^l \mathcal{R}_{M, t \rightarrow T_1}^{j_1} + \sum_{l=2}^{j_2} \phi_{j_2, l} \mathcal{R}_{M, t \rightarrow T_1}^{j_1} (\mathbb{E}_{T_1}^* R_{M, T_1 \rightarrow T_N}^l - R_{f, T_1 \rightarrow T_N}^l).
\end{aligned}$$

Since

$$\mathbb{E}_{T_1}^* R_{M, T_1 \rightarrow T_N}^l - R_{f, T_1 \rightarrow T_N}^l = \theta_{l, t}^* |R_{M, t \rightarrow T_1} - R_{f, t \rightarrow T_1}|^l,$$

it follows that

$$f_{T_1}^{(j_1, j_2)} = \sum_{l=0}^{j_2} \phi_{j_2, l} R_{f, T_1 \rightarrow T_N}^l \mathcal{R}_{M, t \rightarrow T_1}^{j_1} + \sum_{l=2}^{j_2} \phi_{j_2, l} \theta_{l, t}^* \mathcal{R}_{M, t \rightarrow T_1}^{j_1} |R_{M, t \rightarrow T_1} - R_{f, t \rightarrow T_1}|^l$$

and

$$\mathbb{E}_t^* f_{T_1}^{(j_1, j_2)} = \sum_{l=0}^{j_2} \phi_{j_2, l} R_{f, T_1 \rightarrow T_N}^l \mathbb{E}_t^* [\mathcal{R}_{M, t \rightarrow T_1}^{j_1}] + \sum_{l=2}^{j_2} \phi_{j_2, l} \theta_{l, t}^* \mathbb{E}_t^* [\mathcal{R}_{M, t \rightarrow T_1}^{j_1} |R_{M, t \rightarrow T_1} - R_{f, t \rightarrow T_1}|^l].$$

OA.6 Variation with Firm Characteristics

OA.6.1 Bounds Without Intertemporal Hedging

This subsection examines how the predictive performance of our bounds varies with stock characteristics. The characteristics considered include market beta, size, book-to-market ratio, momentum, profitability, investment, idiosyncratic volatility, and illiquidity, which are standard predictors for the cross-section of stock returns in the asset pricing literature (e.g., [Fama and French, 1992](#); [Cahart, 1997](#); [Hou, Xue, and Zhang, 2015](#)).

[Insert Table [OA.2](#) around here]

Table [OA.2](#) reports the [Fama and Macbeth \(1973\)](#) regressions of prediction errors—defined as the difference between realized returns and our bound estimates—on these firm characteristics. We consider both signed prediction errors and their absolute values. Results for the lower bound are shown in Panel A and for the upper bound in Panel B under the default

preference parameters.¹⁴

For the lower bound (Panel A), the signed error regressions show that most traditional characteristics—such as beta, momentum, profitability, investment, and idiosyncratic volatility—are insignificant, suggesting that the lower bound already captures much of the cross-sectional variation in expected returns typically associated with these factors. By contrast, firm size loads with significantly negative coefficients, indicating that the bound tends to overstate expected returns for larger firms. Illiquidity enters with significantly positive coefficients (up to the 182-day horizon), implying that the bound understates returns for illiquid stocks. For the absolute errors, beta and idiosyncratic volatility are positively related, whereas size, momentum, and profitability are negatively related. These results suggest that prediction errors are larger for firms with high systematic or idiosyncratic risk, but smaller for large, well-performing, and profitable firms.

For the upper bound (Panel B), the overall patterns resemble those for the lower bound, but two differences stand out in the signed error regressions. First, firm size is not significant at the 30-day horizon, though it becomes significantly negative from 91 days onward. This indicates that the upward bias of the upper bound for large firms emerges more clearly at longer horizons. Second, idiosyncratic volatility loads with significantly negative coefficients at all horizons, suggesting that the upper bound systematically overstates expected returns for firms with high firm-specific risk.

OA.6.2 Bounds With Intertemporal Hedging

This subsection examines how the performance of the hedging bounds varies with firm characteristics using Fama–MacBeth regressions of realized returns minus the hedging bounds (i.e., signed prediction errors) and of the absolute prediction errors on standard firm characteristics. For brevity, we report results only for an investment horizon of $T_N = 365$ days in Table OA.5. Panel A presents results for the hedging lower bound (HLB) and Panel B for the hedging upper bound (HUB).

[Insert Table OA.5 around here]

The results broadly mirror those obtained for the static bounds in Table OA.2. For the hedging lower bound (Panel A), most conventional predictors show little relation to the signed errors, while firm size loads with significantly negative coefficients and illiquidity with significantly positive coefficients, indicating that the bound tends to overstate returns for large firms and understate returns for illiquid firms. For the absolute errors, beta and idiosyncratic volatility are positively related, whereas size, momentum, and profitability are negatively related, implying larger errors for firms with high systematic or idiosyncratic risk,

¹⁴Results based on calibrated parameters are omitted for brevity but are very similar and available upon request.

but smaller errors for large, profitable, and past winners. For the hedging upper bound (Panel B), the patterns are broadly similar to those for Panel A but with two notable differences. In the signed error regressions, firm size is insignificant at the 30-day horizon but becomes significantly negative from 91 days onward, and idiosyncratic volatility loads negatively at all horizons, indicating that the HUB systematically overstates returns for firms with high firm-specific risk.

Overall, the hedging bounds display patterns very similar to those of their static counterparts, reinforcing the conclusion that while the hedging bounds absorb much of the variation typically attributed to traditional firm characteristics, certain sources of risk remain systematically linked to their prediction errors.

OA.6.3 Hedging Contributions

To further explore the cross-sectional sources of variation in our bounds and the role of firm characteristics in shaping hedging effects, we begin by examining how the bounds behave across portfolios sorted by key firm attributes. Figure OA.1 displays the average lower and upper bounds, with and without hedging (LB, HLB, UB, HUB), across deciles of four representative characteristics—market beta, size, book-to-market ratio, and illiquidity—using S&P 500 firms at a 30-day forecast horizon and a 365-day investment horizon.¹⁵ The figure reveals patterns that align closely with well-established cross-sectional regularities in expected returns. Both the lower and upper bounds rise monotonically with beta and book-to-market, and decline with firm size, consistent with higher expected returns for high-beta, value, and small-capitalization firms. Bounds also increase with illiquidity, suggesting that stocks facing higher trading frictions are priced with higher expected returns. Across all characteristics, the hedging versions of the bounds (HLB, HUB) lie systematically above their static counterparts (LB, UB), confirming that intertemporal hedging raises the level of expected returns in a way that is broadly consistent with the cross-sectional structure documented in the literature.

[Insert Figure OA.1 and Table OA.7 around here]

Table OA.7 provides a more formal test of which firm attributes are most closely associated with intertemporal hedging effects. We regress the hedging share ratios— $(HLB - LB)/HLB$ and $(HUB - UB)/HUB$ —on standard firm characteristics following a Fama–MacBeth procedure. For brevity, we report results for investment horizons of $T_N = 365$ days (for $T_1 = 30, 91, 182$) and $T_N = 730$ days (for $T_1 = 365$), as other combinations yield similar patterns. The results indicate that firms with higher market betas and greater idiosyncratic volatility tend to exhibit smaller hedging shares, while those that are larger or have stronger recent performance (momentum) show larger shares. Profitability also enters with a positive

¹⁵Results for other horizon combinations are very similar and omitted for brevity.

sign, although its significance weakens in some specifications. In contrast, book-to-market, investment, and illiquidity coefficients vary in sign and significance across specifications. Economically, these results suggest that intertemporal hedging plays a more important role for firms that are less exposed to market or idiosyncratic risk but have stronger fundamentals or recent price momentum—firms whose expected returns are more sensitive to gradual shifts in investment opportunities rather than immediate risk shocks.

The apparent difference between the univariate sorts in Figure OA.1 and the multivariate regressions in Table OA.7 can be reconciled by recognizing that the hedging share is a residual component of expected return formation, not a simple function of any single characteristic. While univariate sorts capture broad return patterns shared by all bounds, the regression isolates how hedging intensity covaries with firm traits after controlling for their joint effects. Together, the two analyses indicate that our hedging bounds both reflect established cross-sectional patterns in expected returns and reveal additional dimensions of heterogeneity linked to firms’ exposure to intertemporal risk.

OA.7 Relation to Cash-Flow, Discount-Rate, and Volatility News Betas

Building on Campbell et al. (2018), who decompose unexpected market returns into news about cash flows, discount rates, and volatility, this subsection investigates how the intertemporal hedging contributions in our framework relate to these distinct news components. Campbell et al. (2018) show that these three sources of news jointly explain much of the time variation in expected returns through their respective betas, capturing distinct channels of risk compensation. Integrating this perspective into our setting allows us to explore whether and how the component of expected returns arising from investors’ intertemporal hedging motives is systematically associated with any of these news-based betas.

Following Campbell et al. (2018), we estimate three factor loadings— β_{CF} , β_{DR} , and β_V —that capture a stock’s exposure to news about cash flows, discount rates, and volatility, respectively. Specifically, we estimate a vector autoregression (VAR) for the aggregate market return, realized variance, and a set of state variables such as the price–dividend ratio, short rate, default spread, and value spread. The VAR yields one-step-ahead forecasts of market returns and variance, which allow us to extract the unexpected component of the market return. Using the Campbell–Shiller log-linear approximation, this unexpected return innovation is then decomposed into three orthogonal components: innovations to expected cash flows (N_{CF}), to expected discount rates (N_{DR}), and to expected volatility (N_V). Each firm’s return innovation is then regressed on these three aggregate news series using a

60-month rolling window to obtain monthly factor loadings:

$$\beta_{CF,i} = \frac{\text{Cov}(\tilde{r}_i, N_{CF})}{\text{Var}(N_{CF})}, \quad \beta_{DR,i} = \frac{\text{Cov}(\tilde{r}_i, N_{DR})}{\text{Var}(N_{DR})}, \quad \beta_{V,i} = \frac{\text{Cov}(\tilde{r}_i, N_V)}{\text{Var}(N_V)}.$$

These betas quantify how each stock co-moves with shocks to future cash flows, discount rates, and volatility in the aggregate economy, thereby capturing the three distinct channels of risk compensation in the stochastic-volatility ICAPM.

[Insert Table [OA.8](#) around here]

Using monthly Fama–MacBeth regressions, we then relate the intertemporal hedging contributions to firm characteristics and the three news betas. Table [OA.8](#) reports the results. Across all combinations of forecast and investment horizons and for both the lower and upper bounds, three clear patterns emerge. First, the coefficients on β_{CF} are positive and highly significant throughout, indicating that stocks more exposed to favorable cash-flow news tend to exhibit larger hedging shares. This suggests that intertemporal hedging amplifies the expected-return component linked to revisions in firms’ future cash-flow prospects, consistent with the idea that cash-flow news represents a key source of priced risk. Second, the coefficients on β_{DR} are negative and significant across all horizons, consistent with a discount-rate-hedge effect: firms that covary more strongly with declines in expected discount rates (i.e., with rising valuations) require smaller intertemporal hedging adjustments. Third, β_V also enters negatively and significantly, implying a volatility-hedge channel. Stocks that move closely with increases in aggregate volatility—thus providing a hedge against worsening investment opportunities—tend to have smaller hedging contributions in expected-return terms. Taken together, these results indicate that the intertemporal hedging component in expected returns is systematically linked to the three sources of market news identified by [Campbell et al. \(2018\)](#). Cash-flow news amplifies the compensations embedded in the hedging bounds, whereas discount-rate and volatility news act as hedging forces that dampen them.

[Insert Table [OA.9](#) around here]

Table [OA.9](#) reports the average proportion of the intertemporal hedging contributions explained by the three news betas. For each firm and month, we regress the hedging contribution (HC)—defined as $(HLB - LB)/HLB$ or $(HUB - UB)/HUB$ —on the three betas as

$$HC_{i,t} = \alpha_t + \lambda_{CF,t}\beta_{i,CF} + \lambda_{DR,t}\beta_{i,DR} + \lambda_{V,t}\beta_{i,V} + \varepsilon_{i,t}.$$

The total variance of the hedging contribution can then be decomposed as

$$\text{Var}(HC_{i,t}) = \lambda_{CF,t}^2 \text{Cov}(HC_{i,t}, \beta_{i,CF}) + \lambda_{DR,t}^2 \text{Cov}(HC_{i,t}, \beta_{i,DR}) + \lambda_{V,t}^2 \text{Cov}(HC_{i,t}, \beta_{i,V}) + \text{Cov}(HC_{i,t}, \varepsilon_{i,t}).$$

The proportion of $HC_{i,t}$ attributable to each beta is thus computed as

$$\frac{\lambda_{k,t} \text{Cov}(HC_{i,t}, \beta_{i,k})}{\text{Var}(HC_{i,t})}, \quad \text{for } k \in \{CF, DR, V\},$$

which sum to one together with the residual term. Table [OA.9](#) reports the time-series averages of these proportions across firms.

Across all forecast–investment horizon combinations, discount-rate news betas (β_{DR}) explain the largest share of the hedging contributions, followed by volatility-news betas (β_V), while cash-flow betas (β_{CF}) account for a relatively small fraction. Moreover, for a given investment horizon T_N , the shares attributed to discount-rate and volatility news rise as the forecast horizon T_1 lengthens. For example, with $T_N = 365$ days and for the hedging lower bound, the fraction explained by β_{DR} increases from about 3% at $T_1 = 30$ days to about 7% at $T_1 = 182$ days, while the share explained by β_V rises from roughly 2% to 4% over the same range. Figure [OA.6](#) visualizes these patterns for representative investment horizons ($T_N = 365$ and 730 days). The panels show that discount-rate news dominates across horizons, and the shares of discount-rate and volatility news generally increase with T_1 (more pronounced for the HLB), while the cash-flow share remains small and comparatively flat.

[Insert Figure [OA.6](#) around here]

Overall, although our bounds are significantly related to cash-flow, discount-rate, and volatility news betas, only a small fraction of the hedging contributions is explained by these three sources. This finding suggests that the intertemporal hedging components captured by our bounds reflect additional dimensions of risk beyond the cash-flow, discount-rate, and volatility channels identified by [Campbell et al. \(2018\)](#). This limited explanatory power is unsurprising for two reasons. First, our measure is forward looking, whereas the [Campbell et al. \(2018\)](#) news series are constructed from past returns and are therefore essentially in-sample, backward-looking measures rather than out-of-sample forecasts of future risks. Second, these news measures are derived under the assumption of conditionally normal market returns, so that hedging demand reflects exposure to future variance only within a world governed by normal return dynamics. In contrast, our framework relaxes this normality assumption and extends the intertemporal hedging component beyond the variance implied by normality, capturing the influence of higher-order dynamics under a general, non-normal return distribution. The resulting hedging bounds incorporate multiple forward-looking elements linked to higher-order moments of the future market return distribution—specifically variance, skewness, and kurtosis—thereby capturing richer, non-Gaussian sources of intertemporal risk that the conventional ICAPM under normality cannot account for.

OA.8 Discussion: The Case of CRRA Utility

Our main results deliver bounds on individual-stock expected excess returns—with and without intertemporal hedging—when the investor’s utility function is left unspecified. However, one may be interested in characterizing expected excess returns when preferences take a specific parametric form. In particular, under CRRA preferences, existing work typically predicts that intertemporal hedging lowers expected excess returns. In this section, we apply our methodology to derive an exact option-based expression for the expected excess return under CRRA, both with and without intertemporal hedging.

This exercise serves two purposes. First, we show that under CRRA preferences our framework implies that intertemporal hedging lowers expected excess returns on individual stocks. Second, we emphasize that the relation between intertemporal hedging and expected excess returns fundamentally depends on the functional form of utility. By contrast, our utility-free approach delivers sharp statements about this relation without imposing a priori sign restrictions.

Under CRRA preferences with risk-aversion parameter α , marginal utility satisfies

$$u' [W_t R_{M,t \rightarrow T_1} R_{M,T_1 \rightarrow T_N}] \propto W_t^{-\alpha} (R_{M,t \rightarrow T_1} R_{M,T_1 \rightarrow T_N})^{-\alpha}$$

Using the representation of the inverse stochastic discount factor, we have

$$\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}} = \frac{\mathbb{E}_{T_1}^* (1/u' [W_t R_{M,t \rightarrow T_1} R_{M,T_1 \rightarrow T_N}])}{\mathbb{E}_t^* (\mathbb{E}_{T_1}^* (1/u' [W_t R_{M,t \rightarrow T_1} R_{M,T_1 \rightarrow T_N}]))}.$$

Substituting for marginal utility yields

$$\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}} = \frac{\mathbb{E}_{T_1}^* (R_{M,t \rightarrow T_1}^\alpha R_{M,T_1 \rightarrow T_N}^\alpha)}{\mathbb{E}_t^* (\mathbb{E}_{T_1}^* (R_{M,t \rightarrow T_1}^\alpha R_{M,T_1 \rightarrow T_N}^\alpha))}. \quad (\text{OA.8.1})$$

Assume α is an integer greater than 1, the binomial theorem implies

$$R_{M,t \rightarrow T_1}^\alpha = \sum_{i=0}^{\alpha} \frac{\alpha!}{(\alpha-i)! i!} (R_{M,t \rightarrow T_1} - R_{f,t \rightarrow T_1})^i R_{f,t \rightarrow T_1}^{\alpha-i},$$

and similarly

$$R_{M,T_1 \rightarrow T_N}^\alpha = \sum_{i=0}^{\alpha} \frac{\alpha!}{(\alpha-i)! i!} (R_{M,T_1 \rightarrow T_N} - R_{f,T_1 \rightarrow T_N})^i R_{f,T_1 \rightarrow T_N}^{\alpha-i}.$$

Substituting these expansions into the inverse SDF yields

$$\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}} = \frac{\sum_{i=0}^{\alpha} \sum_{k=0}^{\alpha} \frac{\alpha!}{(\alpha-i)!} \frac{\alpha!}{(\alpha-k)!} R_{f,T_1 \rightarrow T_N}^{\alpha-k} R_{f,t \rightarrow T_1}^{\alpha-i} f_{T_1}^{(i,k)}}{\sum_{i=0}^{\alpha} \sum_{k=0}^{\alpha} \frac{\alpha!}{(\alpha-i)!} \frac{\alpha!}{(\alpha-k)!} R_{f,T_1 \rightarrow T_N}^{\alpha-k} R_{f,t \rightarrow T_1}^{\alpha-i} \mathbb{E}_t^* \left[f_{T_1}^{(i,k)} \right]}, \quad (\text{OA.8.2})$$

where $f_{T_1}^{(i,k)}$ is defined in Eq.(2.26). It follows that the expected excess return on asset j admits the exact representation¹⁶

$$\mathbb{E}_t (R_{j,t \rightarrow T_1} - R_{f,t \rightarrow T_1}) = \frac{\sum_{i=0}^{\alpha} \sum_{k=0}^{\alpha} \frac{\alpha!}{(\alpha-i)!} \frac{\alpha!}{(\alpha-k)!} R_{f,T_1 \rightarrow T_N}^{\alpha-k} R_{f,t \rightarrow T_1}^{\alpha-i} \mathcal{M}_{j,M,t}^{*(i,k)}}{\sum_{i=0}^{\alpha} \sum_{k=0}^{\alpha} \frac{\alpha!}{(\alpha-i)!} \frac{\alpha!}{(\alpha-k)!} R_{f,T_1 \rightarrow T_N}^{\alpha-k} R_{f,t \rightarrow T_1}^{\alpha-i} \mathbb{E}_t^* \left[f_{T_1}^{(i,k)} \right]}, \quad (\text{OA.8.3})$$

where $\mathcal{M}_{j,M,t}^{*(i,k)}$ is defined in Eq.(2.34). Under CRRA utility, bounds are therefore unnecessary: Result (OA.8.3) provides an exact expression for expected excess return rather than an inequality.

To isolate the role of intertemporal hedging, we also consider the static one-period case. In the absence of intertemporal hedging, the inverse SDF reduces to

$$\frac{\mathbb{E}_t m_{t \rightarrow T_1}}{m_{t \rightarrow T_1}} = \frac{R_{M,t \rightarrow T_1}^{\alpha}}{\mathbb{E}_t^* (R_{M,t \rightarrow T_1}^{\alpha})} \quad (\text{OA.8.4})$$

Applying the binomial theorem again yields

$$\mathbb{E}_t (R_{j,t \rightarrow T_1} - R_{f,t \rightarrow T_1}) = \frac{\sum_{i=0}^{\alpha} \frac{\alpha!}{(\alpha-i)!} R_{f,t \rightarrow T_1}^{\alpha-i} \mathcal{M}_{j,M,t}^{*(i,0)}}{\sum_{i=0}^{\alpha} \frac{\alpha!}{(\alpha-i)!} R_{f,t \rightarrow T_1}^{\alpha-i} \mathbb{E}_t^* \left[f_{T_1}^{(i,0)} \right]} \quad (\text{OA.8.5})$$

which is clearly distinct from (OA.8.3). The primary value of (OA.8.5) lies in enabling a direct comparison of expected excess returns under CRRA preferences with and without intertemporal hedging. Together, (OA.8.3) and (OA.8.5) provide a tractable empirical framework to testing whether intertemporal hedging reduces expected excess returns on individual stocks under CRRA utility.

[Insert Figures OA.7–OA.9 around here]

Figures OA.7–OA.9 reproduce the time-series illustration for two stocks, Apple and JP

¹⁶The proof follows by substituting the inverse SDF in Eq.(OA.8.2) into Eq.(2.1).

Morgan, under CRRA utility with risk aversion $\alpha = 3, 4$, and 5. In contrast to Figure 1, incorporating intertemporal hedging now lowers the implied expected return: the hedged expected return (HER) lies systematically below its non-hedging counterpart (ER), with the gap widening in periods of elevated risk premia (e.g., around recessions). The lower panels make this particularly transparent by plotting the hedging component, $(\text{HER} - \text{ER})/\text{ER}$, which is predominantly negative and time-varying for both firms and across investment horizons. This negative hedging contribution is not an artifact of the particular (T_1, T_N) choices shown in the figure: $\text{HER} - \text{ER}$ is negative in most months across a broader set of (T_1, T_N) combinations, with average negative frequencies around 98% (reported in Table OA.10). The magnitude is economically meaningful—often a sizable fraction of ER—indicating that under CRRA preferences the intertemporal hedging term operates as an offset rather than an add-on to the risk premium. Together, Figures OA.7–OA.9 underscore that the sign and size of hedging effects depend on preference restrictions: CRRA implies a negative intertemporal hedging contribution, whereas our utility-free approach characterizes the impact of hedging without imposing an ex ante sign.

[Insert Figures OA.10–OA.12 and Table OA.10 around here]

We next revisit the pricing-consistency exercise under the CRRA benchmark. Specifically, Figures OA.10–OA.12 plot, for the same nine S&P 500 sector ETFs, the monthly averages of daily differences between the ETF-level 30-day annualized expected return and the value-weighted average of constituent-level expected returns implied by CRRA utility with risk aversion $\alpha = 3, 4$, and 5. For comparison, we also report the corresponding differences for our static lower and upper bounds (LB/UB) and their midpoint (LUB) from the one-period economy. If a proxy prices assets coherently across aggregation levels, these differences should remain close to zero. The CRRA-implied expected return exhibits noticeably weaker pricing consistency than our bounds. In particular, the $\overline{ER} - \text{ER}^S$, CRRA series displays larger and more persistent deviations from zero, with pronounced spikes during stress episodes, indicating that the preference restriction embedded in CRRA amplifies aggregation inconsistencies. By contrast, our LB remains tightly centered near zero and is substantially more stable over time, while UB and LUB deviate more, consistent with their relative looseness. In general, imposing CRRA preferences delivers an expected-return proxy that performs worse in the data than our utility-free bounds.

OA.9 Supplementary Tables and Figures

Table OA.1 Out-of-sample Forecast Accuracy of Static Bounds

This table reports out-of-sample (OOS) R^2 (expressed as a percentage) from predicting future realized excess returns using the lower bound (LB) and upper bound (UB), based on the default preference parameters: $\tau = 1, \rho = 2, \kappa = 4$, and $\xi = 8$. The benchmarks include the recursive historical average excess return for stock i from 1990 ($\overline{RX}_{i,t}$), a constant annual return of 6% for all stocks (6% p.a.), the recursive average market excess returns from 1996 ($\overline{MKT}_{i,t}$), the recursive average market implied variance from 1996 ($\overline{IV}_t$), the recursive average implied variance of stock i from 1996 ($\overline{IV}_{i,t}$), the expected excess return measure of [Martin and Wagner \(2019\)](#) ($MW_{i,t}$), the lower bound by [Kadan and Tang \(2020\)](#) ($KT_{i,t}$), and the lower bound by [Chabi-Yo, Dim, and Vilkov \(2023\)](#) ($GLB_{i,t}$). p -values (in parentheses) are for the hypothesis $H_0 : R^2 \leq 0$, computed using the stationary block bootstrap procedure with 10,000 replications. For the optimized lower bound (LB^*) and upper bound (UB^*), the preference parameters are selected to maximize OOS forecast accuracy of $(LB+UB)/2$, with the original parameters [1, 2, 4, 8] being the benchmark. The optimal parameters are: $[\tau, \rho, \kappa, \xi] = [1, 2, 5, 9.5]$ for 30 days, [1, 2, 4, 10] for 91 days, [2/3, 2, 4, 8] for 182 days, and [1, 2, 5, 10] for 365 days. The sample period spans from January 1996 to August 2023.

Panel A. Out-of-sample Forecast Accuracy for LB^*								
	$\overline{RX}_{i,t}$	6% $p.a.$	$\overline{MKT}_t$	$\overline{IV}_t$	$\overline{IV}_{i,t}$	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$
30 Days	1.966 (0.000)	0.892 (0.005)	0.864 (0.005)	0.996 (0.004)	1.909 (0.000)	0.713 (0.016)	2.340 (0.005)	-0.045 (0.719)
91 Days	5.017 (0.000)	1.813 (0.017)	1.693 (0.016)	2.170 (0.013)	4.026 (0.002)	3.634 (0.000)	4.192 (0.041)	0.719 (0.167)
182 Days	11.926 (0.000)	6.242 (0.026)	6.125 (0.026)	6.822 (0.021)	9.560 (0.000)	6.473 (0.002)	6.236 (0.001)	2.961 (0.066)
365 Days	19.262 (0.000)	8.068 (0.007)	7.936 (0.005)	8.863 (0.006)	13.609 (0.000)	12.832 (0.005)	6.429 (0.154)	3.812 (0.018)
Panel B. Out-of-sample Forecast Accuracy for UB^*								
	$\overline{RX}_{i,t}$	6% $p.a.$	$\overline{MKT}_t$	$\overline{IV}_t$	$\overline{IV}_{i,t}$	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$
30 Days	0.185 (0.482)	-0.873 (0.881)	-0.906 (0.886)	-0.768 (0.855)	0.205 (0.397)	-0.435 (0.867)	0.824 (0.098)	-1.788 (0.997)
91 Days	2.979 (0.023)	-0.267 (0.556)	-0.39 (0.566)	0.096 (0.510)	2.077 (0.039)	2.032 (0.081)	2.253 (0.000)	-1.392 (0.794)
182 Days	7.236 (0.003)	1.26 (0.359)	1.138 (0.361)	1.871 (0.327)	4.772 (0.022)	1.584 (0.286)	1.309 (0.264)	-2.192 (0.690)
365 Days	16.803 (0.000)	5.262 (0.169)	5.127 (0.149)	6.08 (0.152)	10.974 (0.000)	10.18 (0.026)	3.579 (0.128)	0.873 (0.449)

Panel C. Out-of-sample Forecast Accuracy for $(LB + UB)/2$								
	$\overline{RX}_{i,t}$	6% <i>p.a.</i>	$\overline{MKT}_t$	$\overline{IV}_t$	$\overline{IV}_{i,t}$	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$
30 Days	1.764 (0.001)	0.723 (0.124)	0.690 (0.119)	0.826 (0.111)	1.780 (0.001)	1.153 (0.001)	2.393 (0.001)	-0.178 (0.836)
91 Days	4.829 (0.001)	1.644 (0.149)	1.523 (0.141)	2.000 (0.131)	3.944 (0.001)	3.900 (0.001)	4.117 (0.002)	0.540 (0.391)
182 Days	10.591 (0.001)	4.835 (0.037)	4.717 (0.036)	5.424 (0.029)	8.215 (0.001)	5.139 (0.005)	4.877 (0.054)	1.507 (0.177)
365 Days	18.606 (0.001)	7.315 (0.029)	7.183 (0.021)	8.116 (0.025)	12.903 (0.001)	12.127 (0.001)	5.669 (0.029)	3.021 (0.076)
Panel D. Out-of-sample Forecast Accuracy for $(LB^* + UB^*)/2$								
	$\overline{RX}_{i,t}$	6% <i>p.a.</i>	$\overline{MKT}_t$	$\overline{IV}_t$	$\overline{IV}_{i,t}$	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$
30 Days	1.768 (0.001)	0.729 (0.125)	0.695 (0.120)	0.832 (0.112)	1.788 (0.001)	1.165 (0.001)	2.326 (0.001)	-0.175 (0.830)
91 Days	4.847 (0.001)	1.660 (0.151)	1.541 (0.142)	2.014 (0.132)	3.976 (0.001)	3.954 (0.001)	4.127 (0.001)	0.57 (0.384)
182 Days	10.132 (0.001)	4.343 (0.126)	4.226 (0.123)	4.933 (0.106)	7.758 (0.001)	4.842 (0.031)	4.410 (0.007)	1.010 (0.374)
365 Days	18.740 (0.001)	7.473 (0.039)	7.341 (0.029)	8.273 (0.035)	13.044 (0.001)	12.265 (0.001)	5.814 (0.023)	3.188 (0.098)

Table OA.2 Cross-Sectional Regressions of Static Bound Prediction Errors

This table presents the results of Fama-MacBeth regressions analyzing the prediction errors of the lower bound (LB) and upper bound (UB). We regress the pricing errors (indicated at the top of each column) on a set of commonly used stock characteristics, including CAPM beta, size, book-to-market ratio, momentum, profitability, investment growth, idiosyncratic volatility, and Amihud illiquidity. The analysis is conducted using all available stocks, with all variables winsorized at the 1% level on a monthly basis. Reported p -values (in parentheses) are based on the standard errors corrected using the [Newey and West \(1987\)](#) method. The sample period spans from January 1996 to August 2023.

Panel A. Lower Bound (LB)								
	30 Days		91 Days		182 Days		365 Days	
	Ret-LB	Ret-LB	Ret-LB	Ret-LB	Ret-LB	Ret-LB	Ret-LB	Ret-LB
Constant	0.2355 (0.011)	0.419 (0.000)	0.2025 (0.015)	0.2931 (0.000)	0.1646 (0.041)	0.2717 (0.000)	0.2148 (0.025)	0.3105 (0.000)
β	-0.0134 (0.698)	0.2283 (0.000)	-0.0222 (0.491)	0.1153 (0.000)	-0.0245 (0.412)	0.071 (0.000)	0.0007 (0.984)	0.0597 (0.002)
Size	-0.0242 (0.007)	-0.0272 (0.000)	-0.022 (0.007)	-0.0217 (0.000)	-0.0184 (0.014)	-0.0205 (0.000)	-0.0227 (0.008)	-0.025 (0.002)
B/M	-0.0208 (0.071)	-0.0014 (0.850)	-0.0139 (0.180)	0.0045 (0.353)	-0.011 (0.272)	0.005 (0.260)	-0.0125 (0.319)	0.0006 (0.946)
Momentum	0.0322 (0.473)	-0.1992 (0.000)	0.0488 (0.226)	-0.0906 (0.000)	0.0464 (0.256)	-0.0498 (0.042)	0.0402 (0.227)	-0.0122 (0.504)
Profitability	0.0507 (0.168)	-0.0524 (0.009)	0.0466 (0.169)	-0.0268 (0.028)	0.0409 (0.236)	-0.0254 (0.083)	-0.0156 (0.670)	-0.0335 (0.193)
Investment	-0.0051 (0.727)	0.0165 (0.113)	-0.0138 (0.257)	0.013 (0.041)	-0.0156 (0.208)	0.0153 (0.026)	-0.0293 (0.060)	0.0072 (0.416)
Idio. Vol.	-2.0513 (0.183)	27.323 (0.000)	-0.978 (0.527)	17.627 (0.000)	-0.7842 (0.614)	12.6377 (0.000)	-0.8872 (0.592)	9.0643 (0.000)
Illiquidity	0.243 (0.034)	-0.0306 (0.596)	0.2654 (0.029)	-0.092 (0.057)	0.3351 (0.017)	-0.1025 (0.033)	0.5278 (0.133)	-0.0213 (0.880)
R^2 (%)	14.02	18.95	14.19	19.93	15.22	20.04	18.06	21.06

Panel B. Upper Bound (UB)								
	30 Days		91 Days		182 Days		365 Days	
	Ret-UB	Ret-UB	Ret-UB	Ret-UB	Ret-UB	Ret-UB	Ret-UB	Ret-UB
Constant	0.0736	0.4607	0.198	0.268	0.1703	0.2363	0.1958	0.2688
	(0.510)	(0.000)	(0.019)	(0.000)	(0.031)	(0.000)	(0.035)	(0.000)
β	-0.048	0.2278	-0.0629	0.1204	-0.0589	0.0783	-0.0225	0.0667
	(0.219)	(0.000)	(0.069)	(0.000)	(0.060)	(0.000)	(0.529)	(0.000)
Size	-0.0018	-0.0323	-0.0152	-0.02	-0.0152	-0.0177	-0.0197	-0.0222
	(0.862)	(0.000)	(0.061)	(0.000)	(0.039)	(0.000)	(0.018)	(0.004)
B/M	-0.0306	0.0013	-0.0151	0.0064	-0.0107	0.0071	-0.0113	0.0000
	(0.013)	(0.857)	(0.162)	(0.190)	(0.301)	(0.117)	(0.372)	(0.996)
Momentum	0.0719	-0.217	0.0628	-0.099	0.0566	-0.054	0.0494	-0.0169
	(0.102)	(0.000)	(0.110)	(0.000)	(0.155)	(0.010)	(0.127)	(0.306)
Profitability	0.0744	-0.0649	0.0632	-0.0431	0.05	-0.0403	-0.0081	-0.0402
	(0.049)	(0.001)	(0.068)	(0.002)	(0.151)	(0.005)	(0.828)	(0.071)
Investment	0.0093	0.0115	-0.0115	0.0113	-0.017	0.0171	-0.0318	0.0134
	(0.571)	(0.279)	(0.361)	(0.074)	(0.175)	(0.013)	(0.042)	(0.077)
Idio. Vol.	-8.5388	28.8279	-6.9741	18.792	-5.2557	13.8086	-3.7961	10.2284
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.021)	(0.000)
Illiquidity	0.3372	-0.048	0.3178	-0.1034	0.3677	-0.1496	0.5646	-0.1202
	(0.017)	(0.384)	(0.017)	(0.038)	(0.015)	(0.017)	(0.114)	(0.375)
R^2 (%)	14.79	20.16	15.11	21.56	16.41	22.31	19.36	24.32

Table OA.3 Portfolios Sorted by Average Hedging Bounds

This table reports average annualized excess returns for portfolios sorted on the average of hedging lower bound (HLB) and hedging upper bound (HUB), for each forecast horizon $T_1 \in \{30, 91, 182, 365\}$ (rows) and investment horizon $T_N \in \{91, 182, 365, 547, 730\}$ (column blocks). Reported p -values (in parentheses) are based on the monotonicity test of [Patton and Timmermann \(2010\)](#) using 10,000 stationary block bootstrap replications. For the optimized bounds (HLB^* and HUB^*), preference parameters are chosen to maximize out-of-sample forecast accuracy of the midpoint relative to the default calibration [1,2,4,8]; the horizon-specific optimal values $(T_1, T_N) = [\tau, \rho, \kappa, \xi]$ are (30,91)=[1,2,4.5,8.5], (30,182)=[1,2.5,5,10], (30,365)=[1,2.5,4.5,9], (30,547)=[1,2.5,4.5,9], (30,730)=[1,2.5,4.5,9], (91,182)=[1,2,4.5,8.5], (91,365)=[1,2.5,5,10], (91,547)=[1,2.5,4.5,9], (91,730)=[1,2.5,4.5,9], (182,365)=[1,2,5,9.5], (182,547)=[1,2,4.5,8.5], (182,730)=[1,2.5,5,10], (365,547)=[1,2,5,9.5], (365,730)=[1,2.5,5,10]. The sample spans January 1996 to August 2023.

		$T_N = 91$		$T_N = 182$		$T_N = 365$		$T_N = 547$		$T_N = 730$	
		(HLB+ HUB)/2	(HLB*+ HUB*)/2	(HLB+ HUB)/2	(HLB*+ HUB*)/2	(HLB+ HUB)/2	(HLB*+ HUB*)/2	(HLB+ HUB)/2	(HLB*+ HUB*)/2	(HLB+ HUB)/2	(HLB*+ HUB*)/2
$[T_1, T_N]$											
30	Low	0.094	0.093	0.095	0.095	0.099	0.098	0.106	0.102	0.109	0.108
	High	0.198	0.203	0.195	0.202	0.194	0.200	0.195	0.203	0.195	0.202
	High – Low	0.104	0.110	0.100	0.107	0.094	0.102	0.090	0.101	0.086	0.094
	p -Value	(0.025)	(0.022)	(0.032)	(0.040)	(0.041)	(0.039)	(0.045)	(0.055)	(0.054)	(0.040)
91	Low			0.092	0.091	0.092	0.091	0.093	0.091	0.094	0.092
	High			0.204	0.205	0.204	0.205	0.204	0.204	0.203	0.204
	High – Low			0.112	0.114	0.111	0.114	0.111	0.112	0.109	0.112
	p -Value			(0.008)	(0.008)	(0.008)	(0.010)	(0.008)	(0.007)	(0.013)	(0.007)
182	Low					0.092	0.092	0.092	0.092	0.093	0.092
	High					0.226	0.227	0.226	0.227	0.227	0.226
	High – Low					0.134	0.135	0.134	0.135	0.134	0.134
	p -Value					(0.006)	(0.006)	(0.006)	(0.006)	(0.007)	(0.007)
365	Low							0.093	0.093	0.093	0.093
	High							0.265	0.265	0.265	0.264
	High – Low							0.171	0.172	0.171	0.171
	p -Value							(0.004)	(0.003)	(0.004)	(0.004)

Table OA.4 Out-of-sample Forecast Accuracy of Hedging Bounds

The table reports out-of-sample R^2 (expressed as a percentage) from predicting future realized excess returns using the hedging lower bound (HLB) and hedging upper bound (HUB), based on the default preference set: $\tau = 1, \rho = 2, \kappa = 4$, and $\xi = 8$. The benchmarks, indicated in the column headers, include the expected excess return measure of [Martin and Wagner \(2019\)](#) ($MW_{i,t}$), the expected excess return lower bound by [Kadan and Tang \(2020\)](#) ($KT_{i,t}$), the expected excess return lower bound proposed by [Chabi-Yo, Dim, and Vilkov \(2023\)](#) ($GLB_{i,t}$), and LB/UB without hedging. Reported p -values (in parentheses) are for the hypothesis $H_0 : R^2 \leq 0$, computed using the stationary block bootstrap procedure with 10,000 replications. For the optimized HLB^* and HUB^* , the preference parameters are selected to maximize the out-of-sample forecast accuracy using (HLB+HUB)/2, using the original parameters [1, 2, 4, 8] as the benchmark. HLB^* and HUB^* are identical for each maturity. Those optimal parameter values are specified in Table 6. The sample period spans from January 1996 to August 2023.

Panel A. Out-of-sample Forecast Accuracy for HLB^*								
	$T_N = 365$				$T_N = 730$			
	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$	$LB_{i,t}^*$	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$	$LB_{i,t}^*$
$T_1 = 30$	1.506 (0.000)	2.741 (0.001)	0.168 (0.351)	0.201 (0.039)	1.596 (0.000)	2.83 (0.000)	0.259 (0.316)	0.871 (0.036)
$T_1 = 91$	4.429 (0.000)	4.642 (0.011)	1.082 (0.088)	0.667 (0.118)	4.386 (0.000)	4.599 (0.005)	1.037 (0.182)	0.512 (0.188)
$T_1 = 182$	7.058 (0.000)	6.649 (0.013)	3.337 (0.002)	1.43 (0.021)	6.913 (0.000)	6.504 (0.024)	3.186 (0.001)	1.277 (0.009)
$T_1 = 365$	12.832 (0.005)	6.429 (0.154)	3.812 (0.018)	0.000 (0.000)	12.841 (0.005)	6.435 (0.147)	3.823 (0.035)	1.001 (0.016)
Panel B. Out-of-sample Forecast Accuracy for HUB^*								
	$T_N = 365$				$T_N = 730$			
	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$	$UB_{i,t}^*$	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$	$UB_{i,t}^*$
$T_1 = 30$	-1.095 (0.966)	0.173 (0.438)	-2.457 (0.998)	-0.719 (0.999)	-2.768 (0.997)	-1.479 (0.889)	-4.152 (1.000)	-2.463 (1.000)
$T_1 = 91$	1.112 (0.290)	1.335 (0.162)	-2.344 (0.850)	-0.96 (0.959)	-0.525 (0.575)	-0.298 (0.615)	-4.038 (0.909)	-2.642 (0.991)
$T_1 = 182$	3.101 (0.099)	2.831 (0.027)	-0.617 (0.554)	1.552 (0.018)	1.99 (0.228)	1.717 (0.155)	-1.77 (0.655)	0.399 (0.177)
$T_1 = 365$	10.18 (0.026)	3.579 (0.128)	0.873 (0.449)	0.000 (0.000)	8.059 (0.117)	1.302 (0.402)	-1.468 (0.593)	-2.389 (0.970)

Panel C. Out-of-sample Forecast Accuracy for $(HLB + HUB)/2$								
	$T_N = 365$				$T_N = 730$			
	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$	$\frac{(LB_{i,t}+UB_{i,t})}{2}$	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$	$\frac{(LB_{i,t}+UB_{i,t})}{2}$
$T_1 = 30$	1.057 (0.013)	2.298 (0.000)	0.274 (0.207)	0.316 (0.660)	-0.008 (0.533)	1.249 (0.141)	-1.357 (0.939)	-1.224 (0.963)
$T_1 = 91$	3.548 (0.009)	3.766 (0.000)	0.176 (0.530)	0.383 (0.155)	1.329 (0.332)	1.552 (0.331)	-2.12 (0.785)	-2.715 (0.897)
$T_1 = 182$	5.649 (0.001)	5.389 (0.006)	2.037 (0.129)	1.514 (0.045)	4.691 (0.028)	4.428 (0.013)	1.042 (0.360)	0.545 (0.634)
$T_1 = 365$	12.127 (0.001)	5.669 (0.129)	3.021 (0.176)	0.000 (0.000)	10.637 (0.025)	4.066 (0.193)	1.392 (0.209)	1.721 (0.125)
Panel D. Out-of-sample Forecast Accuracy for $(HLB^* + HUB^*)/2$								
	$T_N = 365$				$T_N = 730$			
	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$	$\frac{(LB_{i,t}^*+UB_{i,t}^*)}{2}$	$MW_{i,t}$	$KT_{i,t}$	$GLB_{i,t}$	$\frac{(LB_{i,t}^*+UB_{i,t}^*)}{2}$
$T_1 = 30$	1.093 (0.005)	2.333 (0.000)	0.239 (0.808)	1.099 (0.192)	0.547 (0.227)	1.793 (0.018)	0.793 (0.109)	-0.691 (0.954)
$T_1 = 91$	3.759 (0.002)	3.976 (0.000)	0.395 (0.471)	0.192 (0.301)	3.173 (0.036)	3.392 (0.002)	0.212 (0.614)	-0.812 (0.844)
$T_1 = 182$	5.809 (0.001)	5.546 (0.000)	2.198 (0.162)	1.22 (0.051)	5.63 (0.002)	5.367 (0.000)	2.013 (0.201)	1.018 (0.107)
$T_1 = 365$	12.265 (0.001)	5.814 (0.093)	3.188 (0.198)	0.000 (0.000)	11.52 (0.005)	5.02 (0.106)	2.372 (0.306)	0.866 (0.382)

**Table OA.5 Cross-Sectional Regressions of Hedging Bound
Prediction Errors**

This table presents the results of Fama–MacBeth regressions analyzing the prediction errors of the hedging lower bound (HLB) and hedging upper bound (HUB). Specifically, we regress the pricing errors (indicated at the top of each column) on a set of commonly used stock characteristics, including CAPM beta, size, book-to-market ratio, momentum, profitability, investment growth, idiosyncratic volatility, and Amihud illiquidity. The analysis is conducted using all available stocks, with all variables winsorized at the 1% level on a monthly basis. The reported p -values (in parentheses) are based on standard errors corrected using the [Newey and West \(1987\)](#) method. The sample period spans from January 1996 to August 2023.

Panel A. Lower Bound with Hedging (HLB)								
	$[T_1, T_N]=[30, 365]$		$[T_1, T_N]=[91, 365]$		$[T_1, T_N]=[182, 365]$		$[T_1, T_N]=[365, 365]$	
	Ret-HLB	Ret-HLB	Ret-HLB	Ret-HLB	Ret-HLB	Ret-HLB	Ret-HLB	Ret-HLB
Constant	0.209 (0.022)	0.4145 (0.000)	0.1889 (0.023)	0.2852 (0.000)	0.156 (0.051)	0.2646 (0.000)	0.2148 (0.025)	0.3105 (0.000)
β	−0.0206 (0.568)	0.228 (0.000)	−0.0281 (0.395)	0.116 (0.000)	−0.0279 (0.356)	0.0711 (0.000)	0.0007 (0.984)	0.0597 (0.002)
Size	−0.0237 (0.008)	−0.0269 (0.000)	−0.022 (0.007)	−0.0209 (0.000)	−0.0183 (0.014)	−0.0198 (0.000)	−0.0227 (0.008)	−0.025 (0.002)
B/M	−0.0213 (0.066)	−0.0008 (0.917)	−0.0139 (0.178)	0.0052 (0.286)	−0.0111 (0.272)	0.0057 (0.209)	−0.0125 (0.319)	0.0006 (0.946)
Momentum	0.0251 (0.576)	−0.1987 (0.000)	0.0473 (0.237)	−0.0892 (0.000)	0.0467 (0.250)	−0.048 (0.035)	0.0402 (0.227)	−0.0122 (0.504)
Profitability	0.0509 (0.166)	−0.0535 (0.009)	0.0467 (0.168)	−0.0284 (0.021)	0.0412 (0.232)	−0.026 (0.076)	−0.0156 (0.670)	−0.0335 (0.193)
Investment	−0.0054 (0.713)	0.0151 (0.169)	−0.0141 (0.246)	0.0124 (0.064)	−0.0157 (0.205)	0.0154 (0.029)	−0.0293 (0.060)	0.0072 (0.416)
Idio. Vol.	−2.3826 (0.122)	27.4817 (0.000)	−1.273 (0.409)	17.7657 (0.000)	−1.0128 (0.514)	12.7593 (0.000)	−0.8872 (0.592)	9.0643 (0.000)
Illiquidity	0.2408 (0.035)	−0.0326 (0.557)	0.2692 (0.028)	−0.0953 (0.052)	0.3381 (0.017)	−0.1082 (0.027)	0.5278 (0.133)	−0.0213 (0.880)
R^2 (%)	14.01	19.02	14.17	20.06	15.19	20.16	18.06	21.06

Panel B. Upper Bound with Hedging (HUB)								
	$[T_1, T_N]=[30, 365]$		$[T_1, T_N]=[91, 365]$		$[T_1, T_N]=[182, 365]$		$[T_1, T_N]=[365, 365]$	
	Ret-HUB	Ret-HUB	Ret-HUB	Ret-HUB	Ret-HUB	Ret-HUB	Ret-HUB	Ret-HUB
Constant	0.0596 (0.591)	0.4606 (0.000)	0.1869 (0.026)	0.2618 (0.000)	0.1626 (0.038)	0.2305 (0.000)	0.1958 (0.035)	0.2688 (0.000)
β	-0.0504 (0.209)	0.2285 (0.000)	-0.0668 (0.058)	0.1223 (0.000)	-0.0616 (0.051)	0.0799 (0.000)	-0.0225 (0.529)	0.0667 (0.000)
Size	-0.0047 (0.652)	-0.0324 (0.000)	-0.0156 (0.054)	-0.0194 (0.000)	-0.0152 (0.038)	-0.0172 (0.000)	-0.0197 (0.018)	-0.0222 (0.004)
B/M	-0.0299 (0.016)	0.0021 (0.770)	-0.0149 (0.170)	0.0071 (0.152)	-0.0105 (0.307)	0.0076 (0.100)	-0.0113 (0.372)	0.0000 (0.996)
Momentum	0.0656 (0.136)	-0.2159 (0.000)	0.062 (0.112)	-0.0974 (0.000)	0.0572 (0.147)	-0.0521 (0.008)	0.0494 (0.127)	-0.0169 (0.306)
Profitability	0.0751 (0.047)	-0.0667 (0.001)	0.0636 (0.067)	-0.0446 (0.001)	0.0503 (0.148)	-0.0408 (0.005)	-0.0081 (0.828)	-0.0402 (0.071)
Investment	0.0096 (0.558)	0.0103 (0.358)	-0.0117 (0.351)	0.0109 (0.112)	-0.0171 (0.171)	0.0174 (0.015)	-0.0318 (0.042)	0.0134 (0.077)
Idio. Vol.	-8.5947 (0.000)	29.084 (0.000)	-7.2092 (0.000)	19.0046 (0.000)	-5.4462 (0.000)	13.9563 (0.000)	-3.7961 (0.021)	10.2284 (0.000)
Illiquidity	0.3205 (0.018)	-0.0498 (0.376)	0.3155 (0.018)	-0.1084 (0.034)	0.3678 (0.015)	-0.1573 (0.015)	0.5646 (0.114)	-0.1202 (0.375)
R^2 (%)	14.75	20.29	15.14	21.75	16.44	22.51	19.36	24.32

Table OA.6 Contribution of Forward Market Moments to Hedging Bounds

This table summarizes the decomposition of the hedging component into contributions from variance, skewness, and kurtosis. We compute the variance-, skewness-, and kurtosis-related components by aggregating the forward moment terms corresponding to the coefficients $a_{j,k-j}$ in Proposition 2.4 for each $k - j$ with varying j to derive the total contribution of each moment in the intertemporal hedging case. Specifically, the component associated with $k - j = 2$ captures the variance-related contribution, $k - j = 3$ captures the skewness-related contribution, and $k - j = 4$ captures the kurtosis-related contribution. The contribution of each forward moment is then calculated as the ratio of the respective forward moment to the total corresponding hedging bound (e.g., HLB or HUB).

$[T_1, T_N]$		$T_N = 91$		$T_N = 182$		$T_N = 365$		$T_N = 547$		$T_N = 730$	
		HLB	HUB	HLB	HUB	HLB	HUB	HLB	HUB	HLB	HUB
30	Variance (%)	2.92	5.24	6.83	11.60	12.57	19.70	17.31	25.25	20.84	29.01
	Skewness (%)	2.34	2.61	5.46	6.04	9.97	10.22	14.14	14.37	17.11	16.78
	Kurtosis (%)	0.59	0.65	1.26	1.37	1.78	2.34	3.31	4.35	4.05	5.66
91	Variance (%)			2.54	2.81	6.53	7.22	10.08	11.12	12.96	14.25
	Skewness (%)			2.96	2.74	7.65	7.13	11.68	10.84	14.97	13.92
	Kurtosis (%)			1.32	1.29	2.67	2.52	4.19	3.93	4.70	4.45
182	Variance (%)					3.58	3.64	6.68	6.90	9.28	9.72
	Skewness (%)					4.44	3.51	8.15	6.42	11.31	9.05
	Kurtosis (%)					2.59	2.38	4.63	4.06	5.78	4.95
365	Variance (%)							3.12	3.38	5.82	6.39
	Skewness (%)							3.34	2.24	6.22	4.18
	Kurtosis (%)							2.68	2.09	4.70	3.61

Table OA.7 Cross-Sectional Determinants of Hedging Contributions

This table presents the results of Fama–MacBeth regressions analyzing the determinants of the hedging contributions. Specifically, we regress the hedging contributions (indicated at the top of each column) on a set of commonly used stock characteristics, including CAPM beta, size, book-to-market ratio, momentum, profitability, investment growth, idiosyncratic volatility, and Amihud illiquidity. The analysis is conducted using all available stocks, with all variables winsorized at the 1% level on a monthly basis. The reported p -values are based on standard errors corrected using the [Newey and West \(1987\)](#) method. The sample period spans from January 1996 to August 2023.

	$[T_1, T_N]=[30, 365]$ $\frac{(HLB-LB)}{HLB}$ $\frac{(HUB-UB)}{HUB}$		$[T_1, T_N]=[91, 365]$ $\frac{(HLB-LB)}{HLB}$ $\frac{(HUB-UB)}{HUB}$		$[T_1, T_N]=[182, 365]$ $\frac{(HLB-LB)}{HLB}$ $\frac{(HUB-UB)}{HUB}$		$[T_1, T_N]=[365, 730]$ $\frac{(HLB-LB)}{HLB}$ $\frac{(HUB-UB)}{HUB}$	
Constant	0.4152 (0.000)	0.1722 (0.016)	0.3944 (0.000)	0.3027 (0.000)	0.3472 (0.000)	0.2635 (0.000)	0.2497 (0.000)	0.1593 (0.000)
β	-0.008 (0.002)	-0.0577 (0.000)	-0.009 (0.000)	-0.0572 (0.000)	-0.0075 (0.000)	-0.0351 (0.000)	-0.006 (0.000)	-0.0124 (0.000)
Size	0.0117 (0.000)	0.0521 (0.000)	0.0064 (0.000)	0.0157 (0.000)	0.0040 (0.000)	0.0051 (0.000)	0.0020 (0.000)	0.0022 (0.000)
B/M	-0.0019 (0.019)	-0.0123 (0.000)	0.0001 (0.871)	-0.0037 (0.000)	0.0002 (0.358)	-0.0012 (0.011)	0.0002 (0.094)	0.0000 (0.838)
Momentum	0.0915 (0.000)	0.0698 (0.000)	0.0394 (0.000)	0.0154 (0.000)	0.0144 (0.000)	0.0063 (0.000)	0.0052 (0.000)	0.0028 (0.002)
Profitability	0.0099 (0.000)	0.0180 (0.000)	0.0086 (0.000)	0.0016 (0.345)	0.0058 (0.000)	0.0010 (0.336)	0.0022 (0.000)	0.0020 (0.000)
Investment	0.0030 (0.053)	0.0050 (0.270)	-0.0003 (0.822)	-0.0057 (0.002)	-0.0003 (0.576)	-0.0024 (0.005)	-0.0006 (0.067)	-0.0018 (0.001)
Idio. Vol.	-3.0258 (0.000)	-8.5199 (0.000)	-2.791 (0.000)	-6.4224 (0.000)	-1.8266 (0.000)	-3.5891 (0.000)	-1.0132 (0.000)	-1.2894 (0.000)
Illiquidity	0.0558 (0.011)	0.2527 (0.002)	-0.0059 (0.372)	0.1081 (0.003)	-0.0057 (0.155)	0.0490 (0.006)	0.0201 (0.002)	0.0402 (0.001)
R^2 (%)	24.87	51.81	42.23	55.96	54.52	54.75	59.59	52.37

**Table OA.8 Cross-Sectional Regressions of Hedging Contributions
with Cash-Flow, Discount-Rate, and Volatilit News Betas**

This table presents the results of Fama–MacBeth regressions analyzing the determinants of the hedging contributions. Specifically, we regress the hedging contributions (indicated at the top of each column) on a set of commonly used stock characteristics, including three betas (i.e., β_{CF} , β_{DR} , β_V) computed following the approach of [Campbell et al. \(2018\)](#), as well as size, book-to-market ratio, momentum, profitability, investment growth, idiosyncratic volatility, and Amihud illiquidity. The analysis is conducted using all available stocks, with all variables winsorized at the 1% level on a monthly basis. The reported p -values are based on standard errors corrected using the [Newey and West \(1987\)](#) method. The sample period spans from January 1996 to August 2023.

	$[T_1, T_N]=[30, 365]$		$[T_1, T_N]=[91, 365]$		$[T_1, T_N]=[182, 365]$		$[T_1, T_N]=[365, 730]$	
	$(HLB-LB)$	$(HUB-UB)$	$(HLB-LB)$	$(HUB-UB)$	$(HLB-LB)$	$(HUB-UB)$	$(HLB-LB)$	$(HUB-UB)$
	HLB	HUB	HLB	HUB	HLB	HUB	HLB	HUB
Constant	0.2430 (0.000)	0.0644 (0.239)	0.1933 (0.000)	0.1370 (0.000)	0.1312 (0.000)	0.0929 (0.000)	0.2487 (0.000)	0.1613 (0.000)
β_{CF}	0.0031 (0.000)	0.0149 (0.000)	0.0024 (0.000)	0.0106 (0.000)	0.0010 (0.000)	0.0050 (0.000)	0.0012 (0.000)	0.0033 (0.000)
β_{DR}	-0.0175 (0.000)	-0.0982 (0.000)	-0.0122 (0.000)	-0.0569 (0.000)	-0.0052 (0.000)	-0.0257 (0.000)	-0.0089 (0.000)	-0.0217 (0.000)
β_V	-0.0560 (0.000)	-0.2811 (0.000)	-0.0295 (0.000)	-0.1415 (0.000)	-0.0128 (0.000)	-0.0665 (0.000)	-0.0202 (0.000)	-0.0603 (0.000)
Size	0.0073 (0.000)	0.0393 (0.000)	0.0037 (0.000)	0.0084 (0.000)	0.0019 (0.000)	0.0019 (0.000)	0.0019 (0.000)	0.0021 (0.000)
B/M	-0.0008 (0.060)	-0.0091 (0.000)	0.0002 (0.421)	-0.0017 (0.001)	0.0002 (0.202)	-0.0005 (0.035)	0.0000 (0.853)	-0.0006 (0.007)
Momentum	0.0573 (0.000)	0.0394 (0.000)	0.0207 (0.000)	0.0064 (0.007)	0.0061 (0.000)	0.0023 (0.026)	0.0058 (0.000)	0.0020 (0.095)
Profitability	0.0011 (0.022)	-0.0007 (0.255)	0.0010 (0.000)	-0.0006 (0.102)	0.0006 (0.000)	-0.0001 (0.665)	0.0008 (0.000)	0.0005 (0.011)
Investment	0.0001 (0.941)	-0.0030 (0.230)	-0.0007 (0.231)	-0.0031 (0.001)	-0.0005 (0.057)	-0.0012 (0.001)	-0.0008 (0.026)	-0.0018 (0.000)
Idio. Vol.	-2.3386 (0.000)	-7.5584 (0.000)	-1.5039 (0.000)	-3.9565 (0.000)	-0.7135 (0.000)	-1.6732 (0.000)	-1.0562 (0.000)	-1.6037 (0.000)
Illiquidity	0.0151 (0.022)	0.0895 (0.001)	-0.0057 (0.069)	0.0211 (0.000)	-0.0057 (0.092)	0.0043 (0.003)	0.0037 (0.514)	0.0101 (0.003)
R^2 (%)	24.32	51.32	34.91	49.67	41.43	44.91	47.25	43.84

Table OA.9 Proportion of Hedging Contributions Explained by Cash-Flow, Discount-Rate, and Volatility News Betas

This table summarizes the decomposition of the hedging component based on the three betas proposed by [Campbell et al. \(2018\)](#). Following their approach, we first decompose market excess returns into three components: cash-flows news (CF), discount-rate news (DR), and volatility news (V). For each stock in our sample, we then compute the stock's beta with respect to each of these market excess return components using a 60-month rolling window. Next, we estimate a cross-sectional regression of the hedging component (e.g., (HLB-LB)/LB) on the three betas (i.e., $\beta_{CF}, \beta_{DR}, \beta_V$) for each month. The proportion of the hedging component attributable to each beta is calculated as the corresponding regression coefficient multiplied by the covariance between the beta and the hedging component, and then scaled by the variance of the hedging component.

$[T_1, T_N]$		$T_N = 91$		$T_N = 182$		$T_N = 365$		$T_N = 547$		$T_N = 730$	
		HLB-LB	HUB-UB	HLB-LB	HUB-UB	HLB-LB	HUB-UB	HLB-LB	HUB-UB	HLB-LB	HUB-UB
30	β_{CF} (%)	0.77	1.48	0.83	1.52	0.84	1.55	0.86	1.57	0.86	1.57
	β_{DR} (%)	3.05	7.79	3.49	8.01	3.44	8.13	3.55	8.17	3.52	8.16
	β_V (%)	1.43	4.86	1.74	5.26	1.78	5.59	1.78	5.77	1.62	5.82
91	β_{CF} (%)			1.21	1.68	1.25	1.74	1.32	1.77	1.34	1.79
	β_{DR} (%)			5.61	8.99	5.82	9.33	6.10	9.50	6.20	9.60
	β_V (%)			2.57	5.81	2.74	6.16	3.08	6.45	3.22	6.59
182	β_{CF} (%)					1.44	1.54	1.53	1.60	1.56	1.64
	β_{DR} (%)					7.06	8.86	7.50	9.18	7.57	9.40
	β_V (%)					3.72	5.97	4.33	6.30	4.45	6.53
365	β_{CF} (%)							1.45	1.12	1.48	1.17
	β_{DR} (%)							6.47	6.71	6.77	7.04
	β_V (%)							4.20	5.15	4.40	5.43

Table OA.10 Percentage of Negative Hedging Contributions Under CRRA Utility

This table summarizes the percentage of negative proportion of the hedging component by CRRA utility, measured as the difference between hedging expected excess return (HER) and non-hedging expected excess return (ER) scaled by ER with various T_1 and T_N . The risk aversions for CRRA utility are 3, 4, and 5. The negative percentage is first computed by calculating the percentage of negative (HER-ER) out of all available firm in our sample by the end of each month, and then taking the average over months. The sample period spans from January 1996 to August 2023.

	$[T_1, T_N]$ (%)	$T_N = 91$	$T_N = 182$	$T_N = 365$	$T_N = 547$	$T_N = 730$
30	Risk Aversion=3	98.95	98.86	98.64	98.38	98.07
	Risk Aversion=4	98.97	98.74	98.18	97.39	96.54
	Risk Aversion=5	98.99	98.58	97.51	96.14	94.30
91	Risk Aversion=3		99.63	99.57	98.68	99.41
	Risk Aversion=4		99.67	99.53	98.66	99.11
	Risk Aversion=5		99.71	99.48	97.77	98.63
182	Risk Aversion=3			99.84	99.82	99.80
	Risk Aversion=4			99.86	99.82	99.76
	Risk Aversion=5			99.88	99.81	99.71
365	Risk Aversion=3				99.90	99.90
	Risk Aversion=4				99.90	99.90
	Risk Aversion=5				99.90	99.90

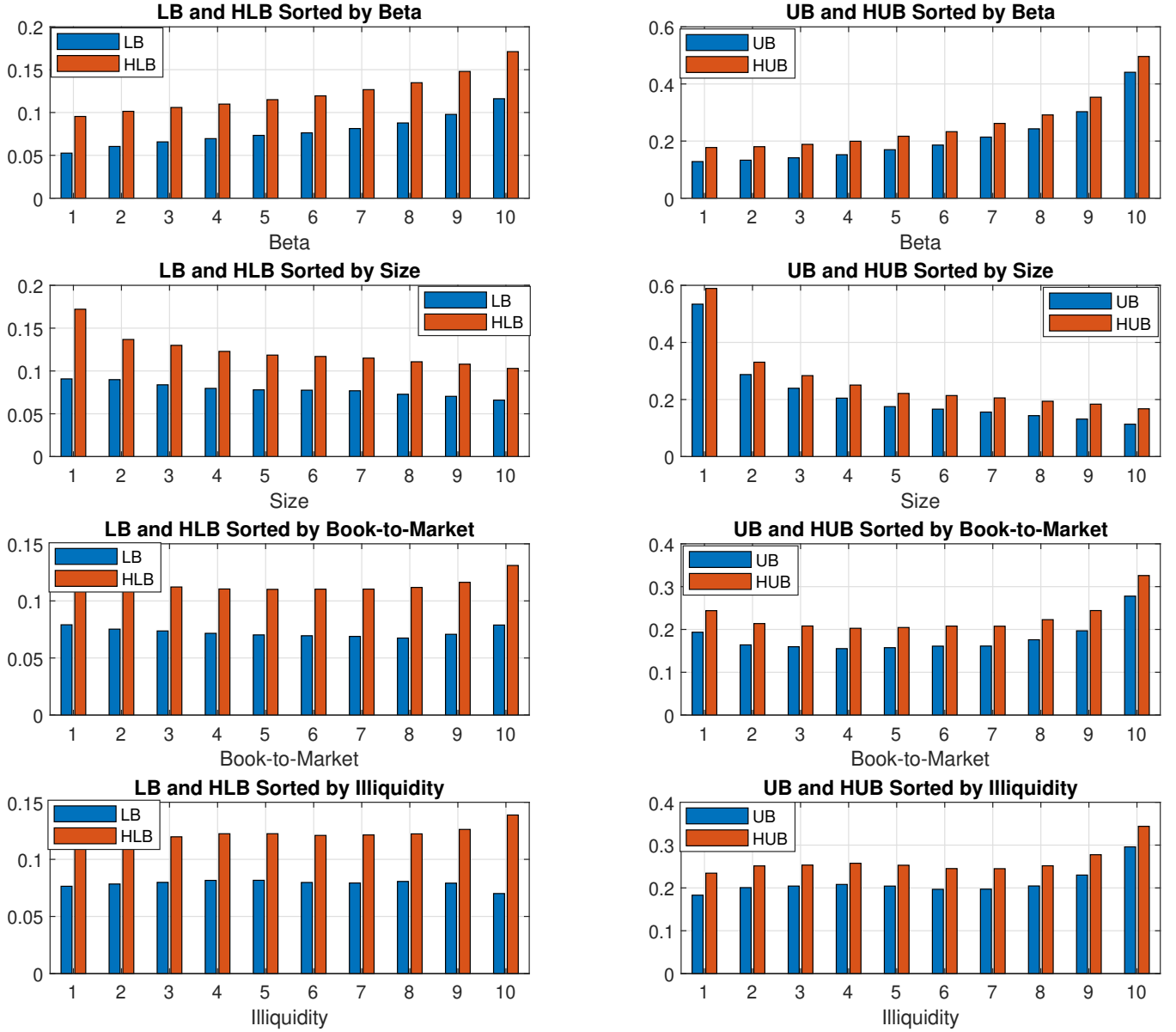


Fig. OA.1: Static vs. Hedging Bounds for Portfolios Sorted by Firm Characteristics This figure displays the average portfolio-level hedging lower bound (HLB) and hedging upper bound (HUB), as well as the unhedged lower bound (LB) and upper bound (UB), sorted by various stock characteristics, including CAPM beta, size, book-to-market ratio, and Amihud illiquidity. We set $T_1 = 30$ and $T_N = 365$. At the end of each month, we first sort all available stocks' LB, UB, HLB, and HUB into deciles ranging from 1 (smallest) to 10 (largest), and then compute the average across all firms' bounds within each decile. The sample period spans from January 1996 to August 2023.

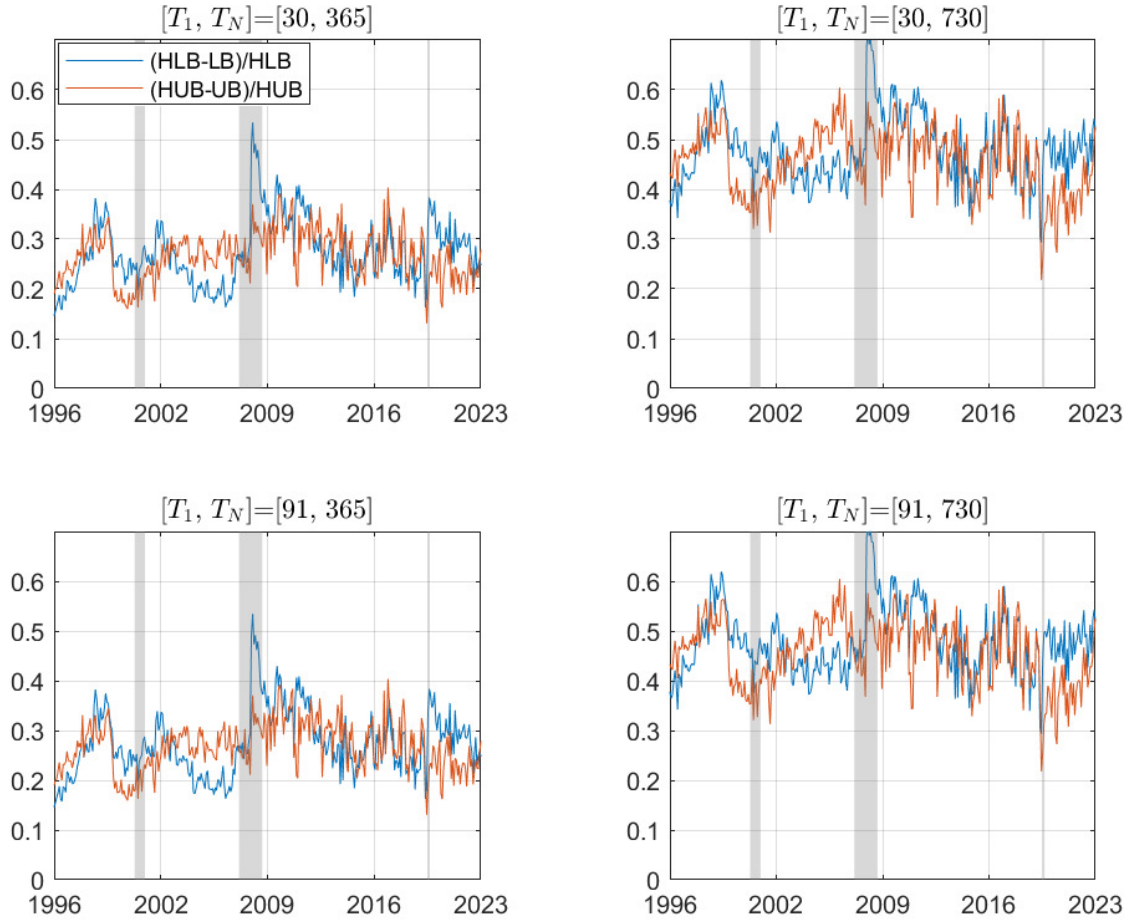


Fig. OA.2: Time Series of Hedging Contributions This figure displays the time series of average proportion of the hedging component, measured as $(HLB-LB)/HLB$ or $(HUB-UB)/UB$ over time. Each subfigure corresponds to a specific maturity pair, with T_1 set to either 30 or 91 days and T_N set to either 365 or 730 days, as indicated in the respective title. The sample period spans from January 1996 to August 2023. The grey areas indicate the National Bureau of Economic Research (NBER) recession periods.

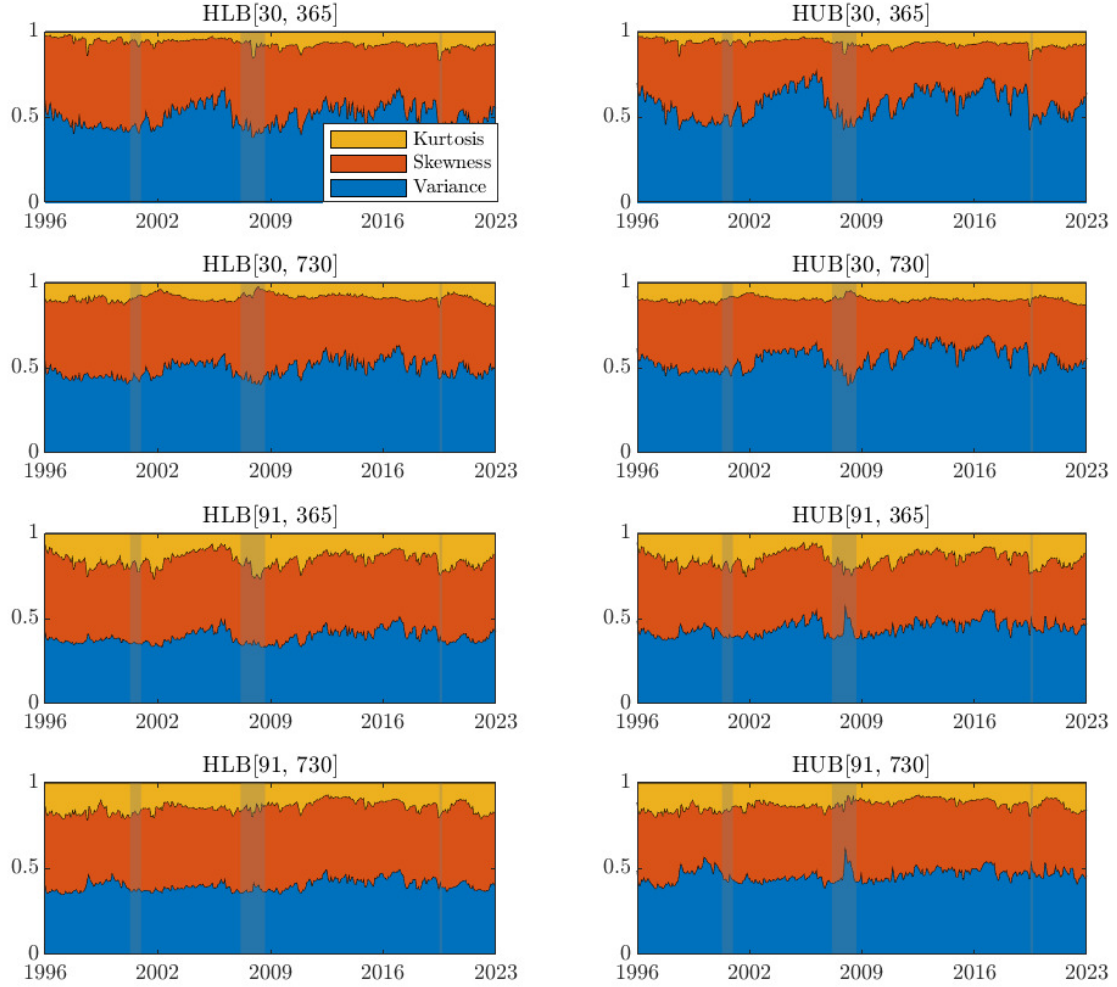


Fig. OA.3: Time Series of Decomposed Hedging Contributions This figure depicts the time-series decomposition of the hedging lower bound (HLB) and hedging upper bound (HUB) across all firms in the sample. The detailed decomposition process is described under Figure 5. Each subfigure corresponds to a specific maturity pair, with T_1 set to either 30 or 91 days and T_N set to either 365 or 730 days, as indicated in the respective title. The sample period spans from January 1996 to August 2023. The grey areas indicate the National Bureau of Economic Research (NBER) recession periods.

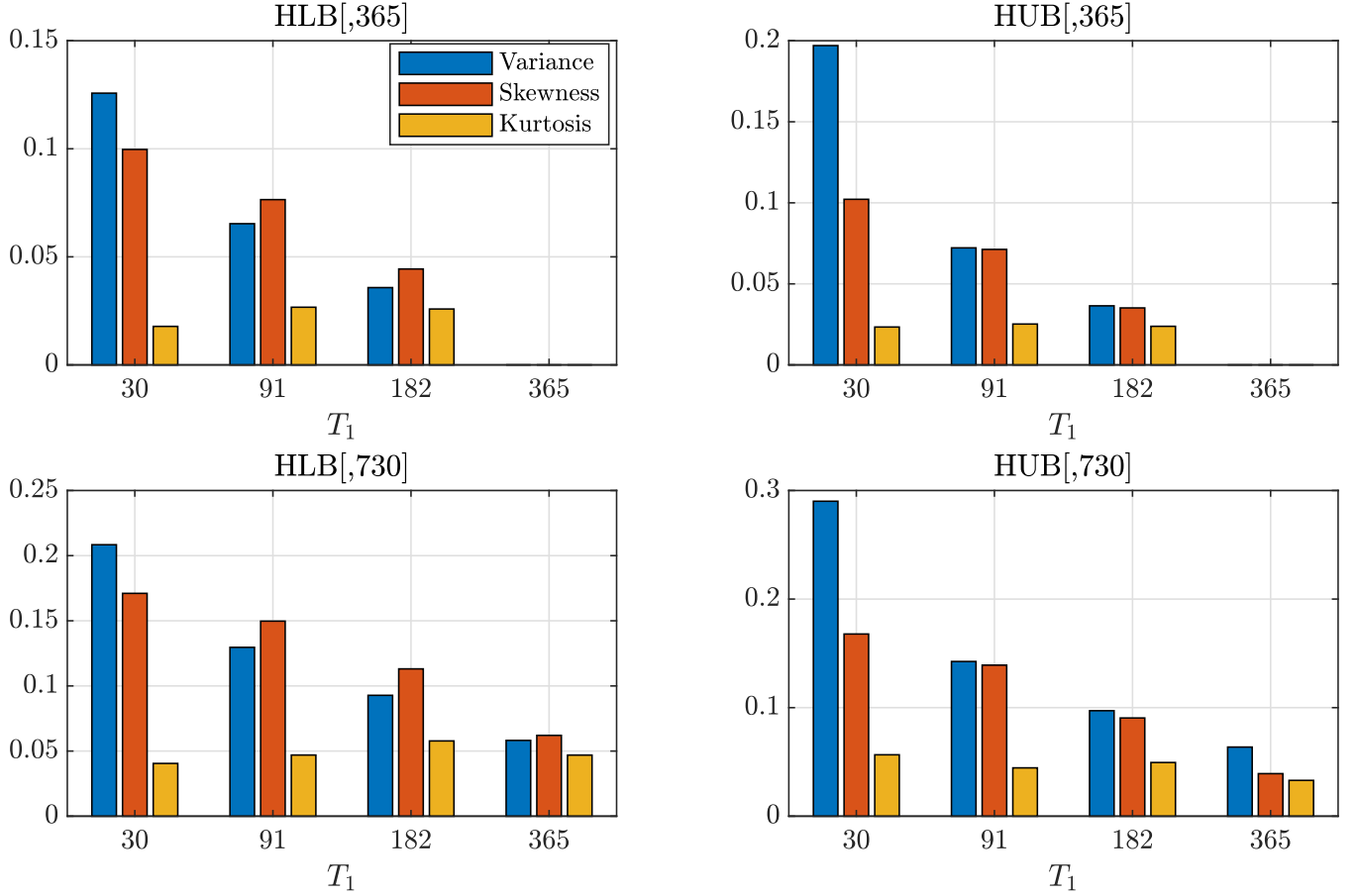


Fig. OA.4: Contribution of Forward Market Moments to Hedging Bounds This figure depicts the average decomposition of the hedging lower bound (HLB) and upper bound (HUB) across all firms in the sample, with maturities ranging from 30 to 365 days. We compute the variance-, skewness-, and kurtosis-related components by aggregating the forward moment terms corresponding to the coefficients $a_{j,k-j}$ in Proposition 2.4 for each $k-j$ with varying j to derive the total contribution of each moment in the intertemporal hedging case. Specifically, the component associated with $k-j=2$ captures the variance-related contribution, $k-j=3$ captures the skewness-related contribution, and $k-j=4$ captures the kurtosis-related contribution. The contribution of each forward moment is then calculated as the ratio of the respective forward moment to the total corresponding hedging bound (e.g., HLB or HUB). The sample period is from January 1996 to August 2023.

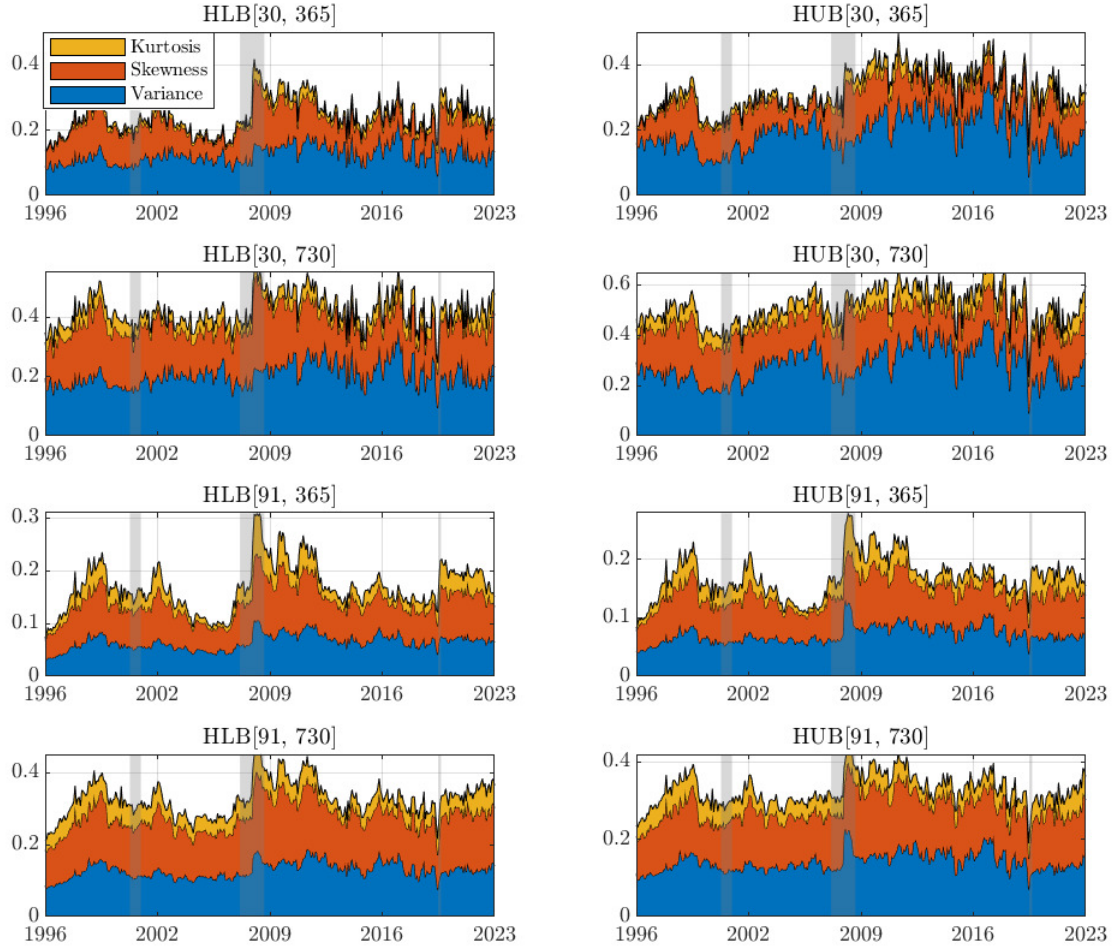


Fig. OA.5: Time Series of Forward Market Moments' Contribution to Hedging Bounds This figure depicts the time-series decomposition of the hedging lower bound (HLB) and hedging upper bound (HUB) across all firms in the sample. The detailed decomposition process is described under Figure OA.4. Each subfigure corresponds to a specific maturity pair, with T_1 set to either 30 or 91 days and T_N set to either 365 or 730 days, as indicated in the respective title. The sample period spans from January 1996 to August 2023. The grey areas indicate the National Bureau of Economic Research (NBER) recession periods.

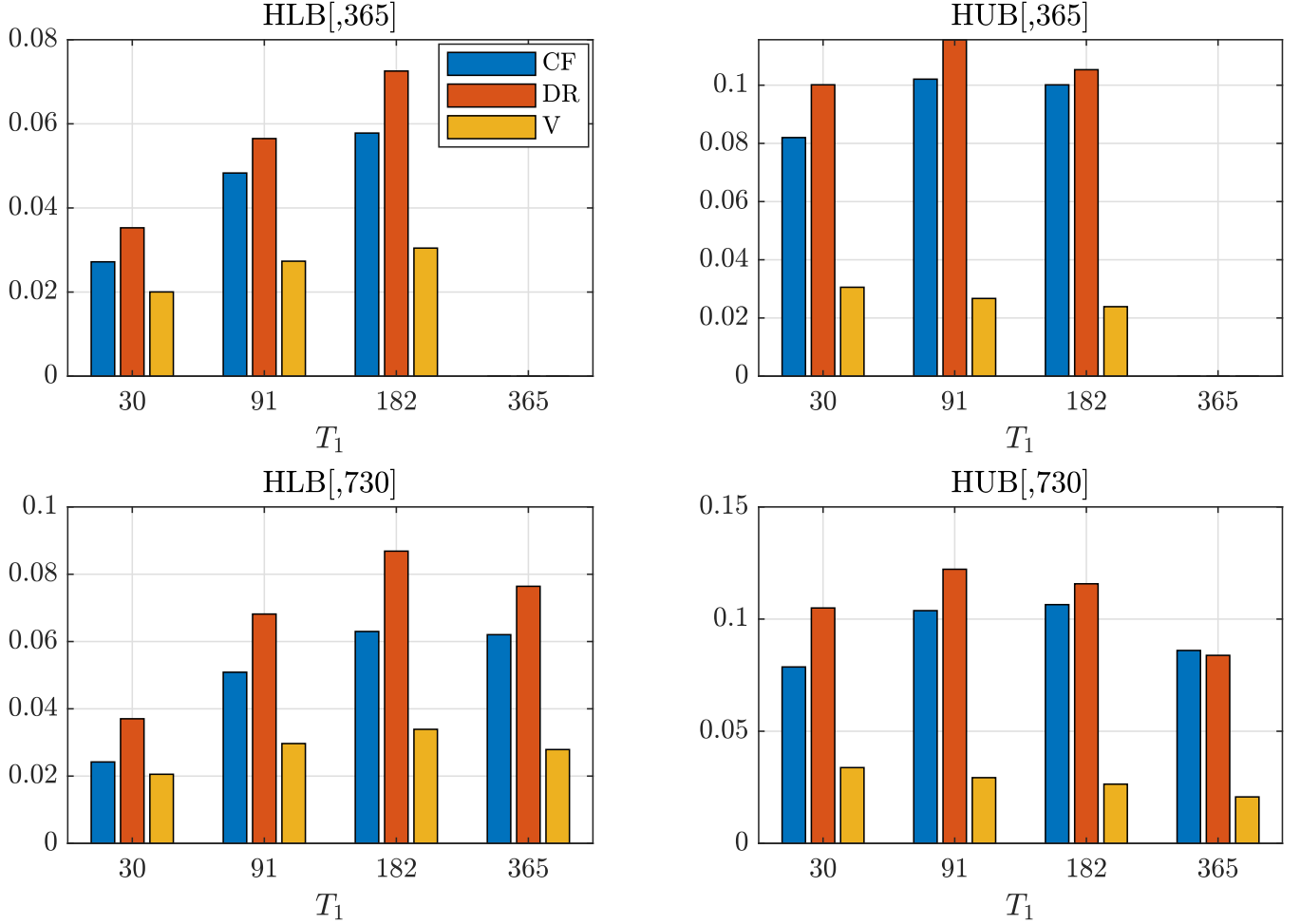


Fig. OA.6: Proportion of Hedging Contributions Explained by Cash-Flow, Discount-Rate, and Volatility News Betas This figure depicts the average decomposition of the hedging components of the hedging lower bound (HLB) and hedging upper bound (HUB) with respect to the framework of [Campbell et al. \(2018\)](#), across all firms in the sample, using maturities ranging from 30 to 365 days. Following their approach, we first decompose market excess returns into three components: cash-flows news (CF), discount-rate news (DR), and volatility news (V). For each stock in our sample, we then compute the stock's beta with respect to each of these market excess return components using a 60-month rolling window. Next, we estimate a cross-sectional regression of the hedging component (e.g., $(HLB-LB)/LB$) on the three betas (i.e., $\beta_{CF}, \beta_{DR}, \beta_V$) for each month. The proportion of the hedging component attributable to each beta is calculated as the corresponding regression coefficient multiplied by the covariance between the beta and the hedging component, and then scaled by the variance of the hedging component. We then average these proportions across all firms. The sample period spans from January 1996 to August 2023.

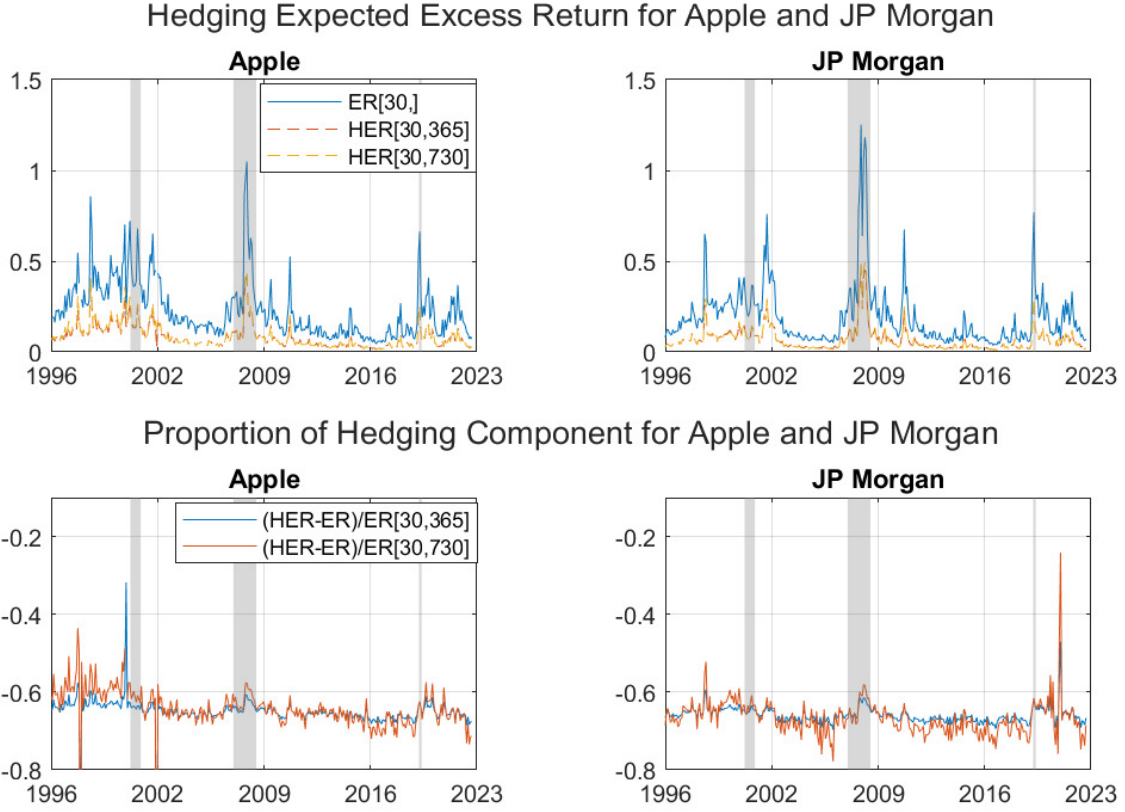


Fig. OA.7: Time Series of Expected Excess Return for AAPL and JPM Under the CRRA Utility (Risk Aversion=3) This figure depicts the time series of expected excess returns (ER) for two stocks, Apple and JP Morgan, with and without hedging, under the case of CRRA utility. In particular, we set the preference parameters with risk aversion equal to 3. The figure also plots time-series proportion of the hedging component, measured as the difference between hedging expected excess return (HER) and ER scaled by ER with $T_1 = 30$ and $T_N = 365/730$. The sample period spans from January 1996 to August 2023. The grey areas indicate the National Bureau of Economic Research (NBER) recession periods.

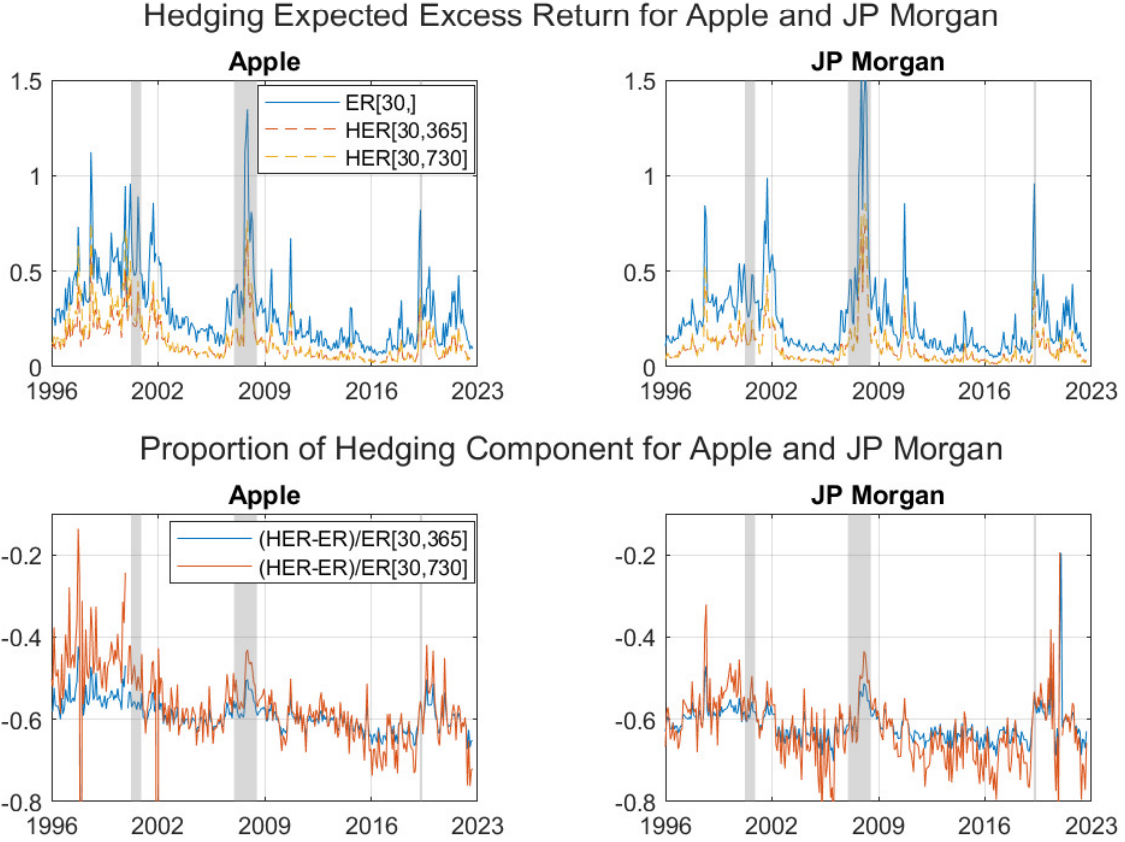


Fig. OA.8: Time Series of Expected Excess Return for AAPL and JPM Under the CRRA Utility (Risk Aversion=4) This figure depicts the time series of expected excess returns (ER) for two stocks, Apple and JP Morgan, with and without hedging, under the case of CRRA utility. In particular, we set the preference parameters with risk aversion equal to 3. The figure also plots time-series proportion of the hedging component, measured as the difference between hedging expected excess return (HER) and ER scaled by ER with $T_1 = 30$ and $T_N = 365/730$. The sample period spans from January 1996 to August 2023. The grey areas indicate the National Bureau of Economic Research (NBER) recession periods.

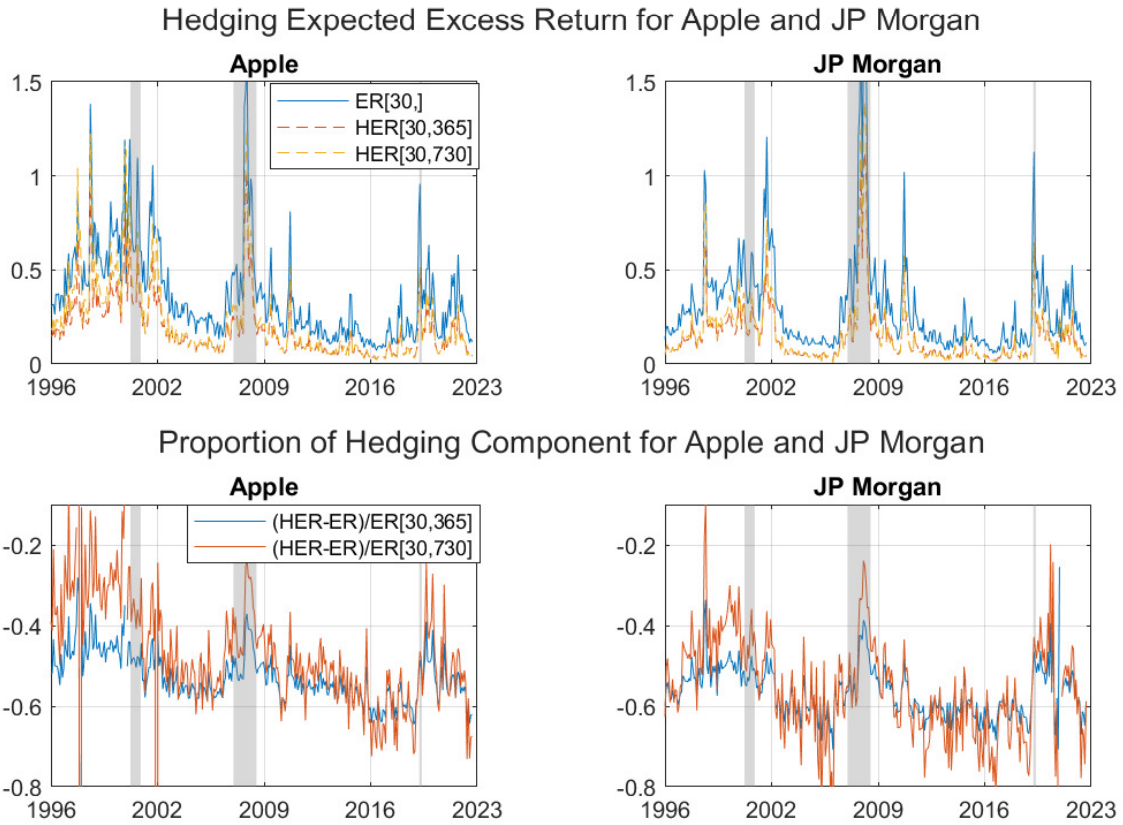


Fig. OA.9: Time Series of Expected Excess Return for AAPL and JPM Under the CRRA Utility (Risk Aversion=5) This figure depicts the time series of expected excess returns (ER) for two stocks, Apple and JP Morgan, with and without hedging, under the case of CRRA utility. In particular, we set the preference parameters with risk aversion equal to 5. The figure also plots time-series proportion of the hedging component, measured as the difference between hedging expected excess return (HER) and ER scaled by ER with $T_1 = 30$ and $T_N = 365/730$. The sample period spans from January 1996 to August 2023. The grey areas indicate the National Bureau of Economic Research (NBER) recession periods.

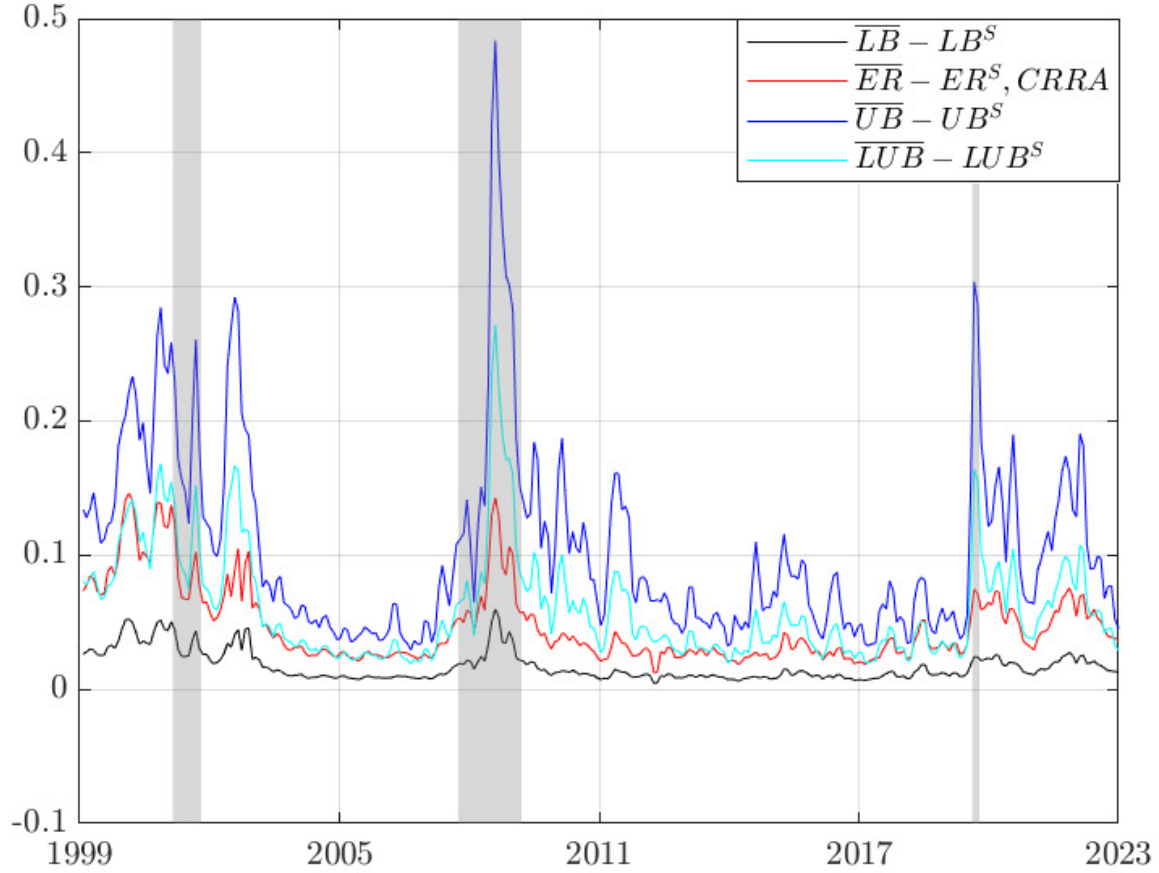


Fig. OA.10: Pricing Consistency of Static Expected Excess Return with CRRA Utility (Risk Aversion=3) This figure displays the average difference in 30-day annualized expected excess return (ER) for nine S&P500-based SPDR ETFs (X^S) and the value-weighted ERs of the ETF components ($\bar{X}$) under the case of CRRA Utility with a risk aversion equal to 3. As a comparison, we also plot the lower and upper bounds (LB/UB), and the average of them $((LB+UB)/2)$ based on our baseline setting $([\tau, \rho, \kappa, \xi]=[1, 2, 4, 8])$. The series represents monthly averages of annualized daily values, covering the period from the start of option trading in sector ETFs in January 1999 to August 2023. The grey areas indicate the National Bureau of Economic Research (NBER) recession periods.

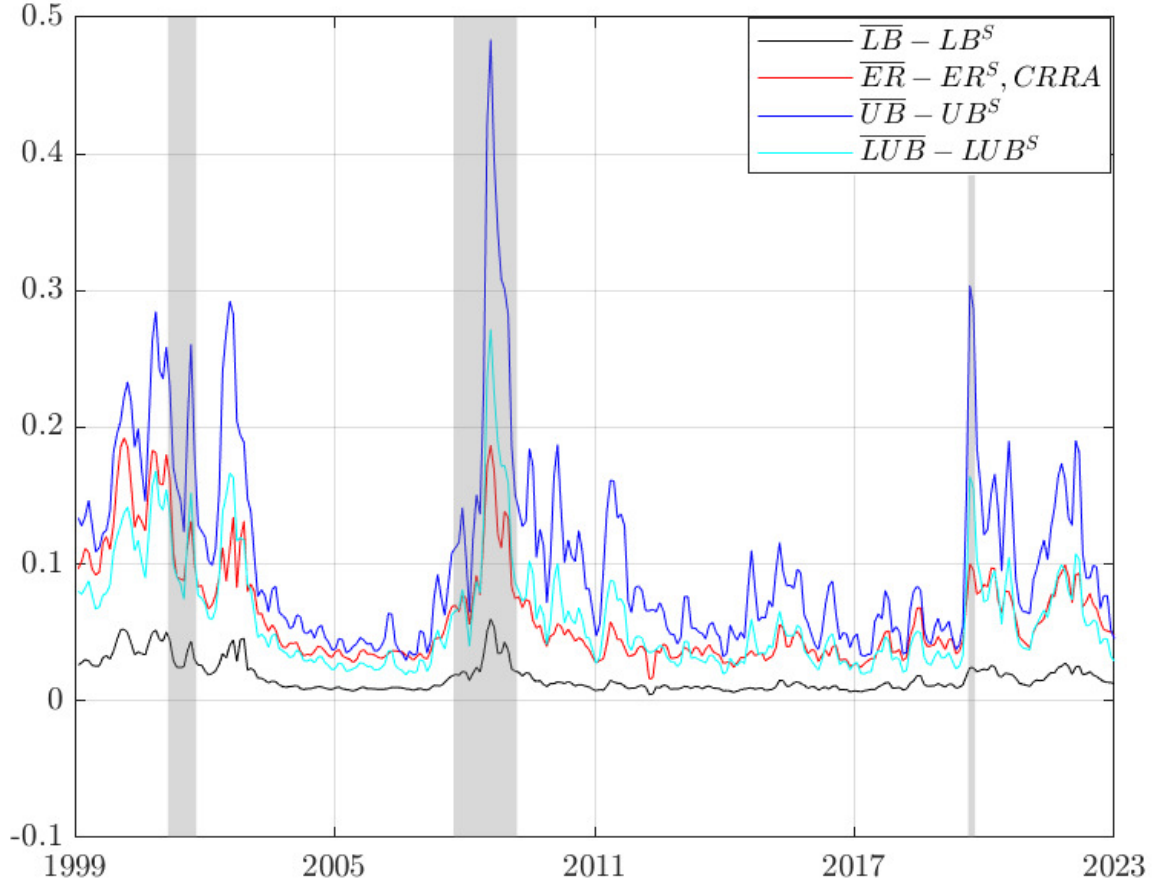


Fig. OA.11: Pricing Consistency of Static Expected Excess Return with CRRA Utility (Risk Aversion=4) This figure displays the average difference in 30-day annualized expected excess return (ER) for nine S&P500-based SPDR ETFs (X^S) and the value-weighted ERs of the ETF components ($\bar{X}$) under the case of CRRA Utility with a risk aversion equal to 4. As a comparison, we also plot the lower and upper bounds (LB/UB), and the average of them $((LB+UB)/2)$ based on our baseline setting $([\tau, \rho, \kappa, \xi]=[1, 2, 4, 8])$. The series represents monthly averages of annualized daily values, covering the period from the start of option trading in sector ETFs in January 1999 to August 2023. The grey areas indicate the National Bureau of Economic Research (NBER) recession periods.

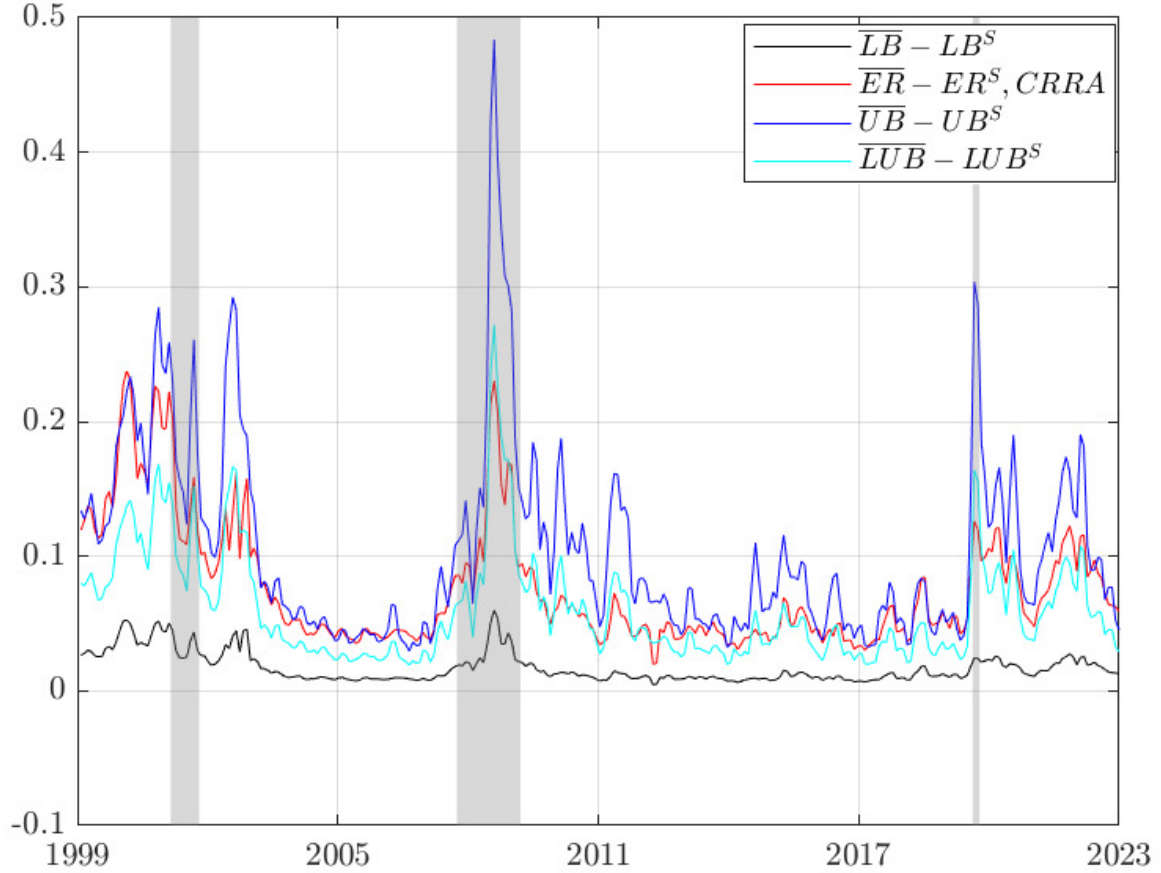


Fig. OA.12: Pricing Consistency of Static Expected Excess Return with CRRA Utility (Risk Aversion=5) This figure displays the average difference in 30-day annualized expected excess return (ER) for nine S&P500-based SPDR ETFs (X^S) and the value-weighted ERs of the ETF components ($\bar{X}$) under the case of CRRA Utility with a risk aversion equal to 5. As a comparison, we also plot the lower and upper bounds (LB/UB), and the average of them $((LB+UB)/2)$ based on our baseline setting $([\tau, \rho, \kappa, \xi]=[1, 2, 4, 8])$. The series represents monthly averages of annualized daily values, covering the period from the start of option trading in sector ETFs in January 1999 to August 2023. The grey areas indicate the National Bureau of Economic Research (NBER) recession periods.