

Portfolio Choice with Target Horizon^{*}

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Abstract

I propose a new approach to multi-period portfolio choice in high-dimensional settings that considers important aspects of the joint distribution of holding-period returns while being agnostic about the dynamics of investment opportunities. I characterize optimal final payoffs through factor principal components of holding-period returns and form tradable factor-tracking portfolios that deliver these horizon-specific payoffs. The resulting optimal portfolio is a utility-maximizing combination of the factor-tracking portfolios. Using broad samples of equity portfolios as investable test assets, I show economically large utility gains for investors compared to important benchmarks, both in-sample and out-of-sample. More importantly, the estimated portfolio rules are horizon-specific: optimal short-horizon strategies cost investors with long horizons, while optimal long-horizon strategies appear deeply unattractive when evaluated by their short-term returns. The findings provide compelling evidence for the economic value of investment horizon in portfolio decisions and challenge the conventional focus on short-term performance metrics for evaluating portfolios of long-horizon investors.

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1 Introduction

Investors have different investment horizons that demand attention to portfolio returns compounded over their target horizons. When returns are not independent and identically distributed (i.i.d.), the abundant evidence on short-run (e.g., monthly) returns may not fully describe long-horizon compounded outcomes, as investment horizon affects return distributions. Past findings confirm that multi-period compounded returns contain important information that is not readily evident in the unconditional and conditional distributions of short-horizon returns: attractive strategies based on monthly returns may lose their value when evaluated over longer holding periods (e.g., [Bessembinder, Cooper, and Zhang, 2023](#)), and popular asset pricing models fail to explain long-horizon returns (e.g., [Chernov, Lochstoer, and Lundebj, 2022](#)). The challenge lies in developing investment strategies that can effectively exploit the distinct characteristics of compounded returns across different investment horizons.

In this paper, I propose a new approach to multi-period portfolio choice that considers important properties of the distribution of holding-period returns and is suitable for realistic applications where the number of assets is large. I take the perspective of a power utility investor with no outside income who wants to maximize utility over wealth at a k -month horizon by allocating funds each month across a large number of risky assets, N , and a risk-free asset. I characterize optimal buy-and-hold portfolios as combinations of systematic factors of compounded asset returns over investors' target horizons, and form tradable factor-tracking portfolios with the same weights as the buy-and-hold factors. The resulting optimal portfolio is a utility-maximizing combination of the factor-tracking portfolios. The empirical results document economically large k -month utility gains for investors compared to important benchmarks, both in-sample and out-of-sample. What is more, the portfolio rules are horizon-specific: a k -month investor benefits from adopting a horizon-matched strategy compared to strategies optimized for most alternative horizons. The results reveal that aligning portfolios with horizon translates into robust utility gains for investors, highlighting the practical value of horizon in real-time decisions.

The study of multi-period portfolio choice has a long history in the finance literature. [Merton \(1969, 1971\)](#) formally introduced investment horizon in portfolio selection by extending [Markowitz’s \(1952\)](#) single-period framework to account for time-variation in investment opportunities. This insight led to dynamic multi-period optimization methods which rely on parametric assumptions to model particular aspects of investment opportunities (e.g., time-varying expected returns and volatility).¹ While theoretically sound, these methods are highly sensitive to misspecification, as errors in modeling the conditional dynamics accumulate over horizons, and the solutions become prohibitively complex with more than a few risky assets. As a result, the economic gains from implementing optimal dynamic strategies are often minimal, and few financial institutions implement dynamic multi-period strategies ([Brandt, 2010](#); [Cochrane, 2022](#)).²

[Cochrane \(2014\)](#) argues that characterizing final investment payoffs leads to implementing more practical multi-period strategies by separating the economic question of which payoffs to target, from the portfolio decisions that achieve those payoffs. This perspective focuses on the investment outcomes that investors ultimately care about and shifts attention away from modeling conditional return dynamics. My approach builds directly on Cochrane’s insight; I first ask what optimal investment payoffs look like—that is, what combinations of assets provide optimal investment payoffs at the desired horizon—and then develop portfolio decisions that seek to deliver those payoffs.

To characterize the investment payoffs, I work directly with compounded holding-period returns. The benefit of this choice is twofold. First, compounded returns preserve the time-series and cross-sectional relations over investors’ target horizons and implicitly reflect the effects of time-varying investment opportunities during the holding periods without requiring explicit assumptions about

¹Several studies have proposed numerical and analytical approaches to approximate the dynamic portfolio problem. Notable examples include [Brennan, Schwartz, and Lagnado \(1997\)](#) and [Barberis \(2000\)](#), who develop discrete-state numerical algorithms for finite-horizon problems. Closed-form solutions in continuous time models are derived under specific utility and market assumptions, such as [Campbell and Viceira \(1999, 2001, 2002\)](#), and [Schroder and Skiadas \(1999\)](#). Alternative approaches, such as [Brandt \(1999\)](#), [Ait-Sahalia and Brandt \(2001\)](#), and [Brandt and Santa-Clara \(2006\)](#), reformulate the problem by predicting optimal portfolio weights directly or expanding the asset space to include managed portfolios.

²The complexity of dynamic portfolio optimization, combined with estimation errors, makes these approaches impractical in real-world applications. Practitioners often argue that the utility losses from errors in solving complex dynamic problems outweigh the small gains from optimal multi-period investing.

the underlying state space.

Second, working with compounded asset returns enables me to link the differences in optimal portfolios across horizons to the horizon-dependent nature of risk and return. Recent studies document the presence of horizon-specific risks and factors in the payoff space.³ Some factors explain variation in expected returns over short horizons, while others become more relevant at longer horizons. Motivated by these findings, I assume k -month compounded excess returns admit an approximate factor structure, such that J orthogonal factors capture the systematic risks of k -month returns, and I estimate the systematic factors using principal component analysis (PCA) on the covariance matrix of k -month excess returns. Under the factor structure assumption, the optimal buy-and-hold risky payoff is a combination of payoffs to a few ($J \lll N$) systematic factors of k -month returns.

Next, I turn to constructing portfolio strategies that deliver these systematic payoffs. For each horizon k , I form J monthly factor-tracking portfolios in real-time that weight assets according to the estimated loadings on the J systematic factors of k -month returns. The factor-tracking portfolios are monthly traded managed portfolios whose compounded k -period payoffs approximate those of the target horizon factors. A k -period investor buys a fixed combination of the factor-tracking portfolios that maximizes expected utility over the k -period wealth.

In the empirical analysis, I apply this approach to a large sample of 230 equity portfolios.⁴ Investors have power utility over terminal wealth with a relative risk aversion of five and investment horizons ranging from one month to ten years. I explore a range of values for the number of factors (J) to account for potential differences in the dimensionality of return comovements across horizons. For the in-sample results, the optimal combination of factor-tracking portfolios is estimated using the full sample of k -period returns. For the out-of-sample results, I estimate this combination using only historical data available at the time of portfolio formation.

³See, for example, Kamara, Korajczyk, Lou, and Sadka (2016); Bandi, Chaudhuri, Lo, and Tamoni (2021); and Neuhierl and Varneskov (2021).

⁴The underlying test assets are the long and short legs of 115 characteristic-sorted portfolios from the Chen and Zimmermann (2022) dataset.

The first set of results evaluates the effectiveness of the managed strategies by comparing their k -period certainty-equivalent returns (CER) and statistical properties to two benchmark portfolios: an equally weighted portfolio (EW) and a portfolio that invests equally in anomaly strategies (LS).⁵ The managed strategies consistently generate higher k -period CERs than the benchmarks across horizons, both in-sample and out-of-sample. Over the short run, the long-minus-short (LS) benchmark delivers strong CERs, reflecting the performance of anomaly strategies during the sample period. However, managed strategies based on six or more factors ($J \geq 6$) generate higher CER than the LS benchmark over short horizons. With nine factor-tracking portfolios ($J = 9$), the in-sample CERs of one- and 12-period strategies are 13.78% and 12.54% per year, respectively, which are 1.19 and 1.14 times that of the LS benchmark over the respective short horizons. Over longer horizons, strategies require only a few factor-tracking portfolios to outperform benchmarks. In the simplest case, which assumes one latent systematic factor for the cross-section of k -month returns ($J = 1$), the 120-month in-sample CER of the horizon-specific strategy is 10.16% per year, which is 1.23 times that of the EW benchmark and 1.52 times that of the LS benchmark. The in-sample CERs naturally increase with a higher number of factor-tracking portfolios; with only six factors ($J = 6$), the annualized in-sample CER reaches 13.92% for the 120-month strategy, substantially higher than those of the benchmark portfolios.

The horizon-specific strategies also outperform benchmark portfolios in the out-of-sample design, showcasing the practicality of my approach for real-time multi-period investors. Over one- and 12-period, the annualized CERs of managed portfolios reach 1.30 and 1.17 times the CERs of the LS benchmark, respectively. The superior performance of the horizon-specific strategies compared to benchmarks is more prominent over longer investment horizons. Over 120 months, the CER of the horizon-specific strategy based on a single factor ($J = 1$) is 8.90% per year and reaches 13.80% with $J = 10$. Over the same sample, the annualized 120-month CERs of the EW and

⁵The equally weighted (EW) benchmark allocates equal weight to each of the 230 long and short decile portfolios. The long-minus-short (LS) benchmark invests equally in 115 anomaly strategies; each formed as the return difference between top and bottom deciles sorted on a predictor variable.

LS benchmarks are 7.23% and 5.31%, respectively. It is well-documented that naive benchmark portfolios are difficult to outperform in single-period out-of-sample settings, as estimation errors often negate the theoretical benefits of optimal allocations (e.g., [DeMiguel, Garlappi, and Uppal, 2009](#)). This challenge is amplified in multi-period settings with larger asset universes, to the extent that empirical studies rarely attempt dynamic portfolio optimization beyond a handful of risky assets.⁶ The real-time gains in this paper underscore the value of explicitly accounting for properties of compounded returns over investment target horizons in developing more practical multi-period investment strategies.

The properties of k -period payoffs further illustrate the sources of managed strategies' higher certainty-equivalent returns compared to benchmarks. Unlike the benchmark portfolios, the managed strategies benefit from increasing mean returns combined with decreasing per-period risk, as measured by k -month variance ratios. Over longer horizons, the horizon-specific strategies exhibit positive skewness and limited tail risk—an appealing return profile for long-horizon risk-averse investors.

I further examine whether the strong performance of managed strategies is driven by their alignment with investor horizons. For an investor with a horizon of k months, I compare the k -period CERs from a strategy estimated for k against the k -period CERs under strategies optimized for other horizons. This comparison isolates the role of horizon in driving multi-period performance and speaks directly to the economic value of incorporating investment horizon in portfolio decisions. The results show that k -period investors systematically benefit from the horizon-matched strategies rather than strategies optimized for alternative horizons. In-sample, a 120-month investor gains an additional 4.31% CER per year over her investment horizon by switching from the one-period (myopic) strategy to the 120-period strategy. Additionally, the differences in CERs generally increase with the gap between investment and strategy horizons: while 60-month investors lose 1.79% annualized CER over their investment horizon by following a 36-month strategy instead of their

⁶Many studies highlight the inefficiency of multi-period dynamic frameworks in achieving real-time utility gains for investors. For example, [Bianchi and Guidolin \(2014\)](#) documents that dynamic optimization fails to outperform naive equally weighted portfolios in real-time when using up to 11 equity portfolios as test assets.

own, they expect to lose 2.89% by following a myopic strategy and 5.18% by following a 120-month strategy. These horizon-specific patterns hold in the out-of-sample results. Out-of-sample, a myopic strategy costs investors with 60-, 96-, and 120-month horizons CERs of 1.92, 2.47, and 2.61 percent annually compared to their horizon strategies.

Notably, while the long-horizon strategies yield the best outcomes over their target horizons, they appear deeply unattractive when evaluated over short periods. For example, the one-month in-sample (out-of-sample) CER of the 120-period strategy is -1.52% (-15.28%) per year, despite its 120-month annualized CER of 13.09% (9.35%). These results challenge the conventional focus on short-term metrics for evaluating portfolios of long-horizon investors.

These findings indicate that the optimal policies are horizon-specific. While each managed strategy performs well relative to benchmarks, its effectiveness is closely tied to the horizon it targets, which highlights fundamental differences between short- and long-horizon strategies. The findings are robust under realistic short-sale and leverage constraints. The constrained portfolios offer superior k -period CERs compared to the benchmark portfolios, and the strategies remain horizon-specific, such that investors benefit from adopting their horizon strategies compared to those optimized for alternative horizons. Lastly, the evidence is not specific to the choice of test assets.

My paper fits within the long-standing literature on multi-period portfolio choice (e.g., [Samuelson, 1969](#); [Barberis, 2000](#); [Campbell and Viceira, 2002](#); [Wachter, 2003](#)). [Cochrane \(2014\)](#) argues that examining final payoffs provides a clearer lens for understanding optimal portfolio decisions, separating the economic question of which outcomes to target from the financial engineering problem of how to achieve them in a market structure. In line with [Cochrane \(2014\)](#), my approach begins by characterizing the outcomes and introduces a few payoffs that investors should follow during their investment horizon. [Aït-Sahalia and Brandt \(2001\)](#) and [Brandt and Santa-Clara \(2006\)](#) emphasize focusing on variables that predict optimal weights rather than those predicting return moments. Their approaches yield solutions to the conditional portfolio problem that unconditionally allocate across a set of mechanically managed portfolios. Similar to these papers, my solution takes the form

of allocating across a set of mechanically managed factor-tracking portfolios, where the variables that predict optimal k -period weights are estimated directly from the observed k -period payoffs.

This paper speaks to the broad literature examining the relationship between portfolio outcomes and investment horizon. [Fama and French \(2018\)](#) and [Anarkulova, Cederburg, and O'Doherty \(2022\)](#) examine the probability of loss from investing in stocks across different horizons. [Bessembinder, Cooper, and Zhang \(2023\)](#) show that mutual fund performance deteriorates over longer horizons. [Laarits \(2023\)](#) finds that the risk-return tradeoff of market timing strategies is less favorable for long-horizon investors. Extending this line of evidence, my paper shows that strategies designed to maximize short-term gains fail to benefit long-horizon investors. For example, a one-month strategy—despite strong monthly performance—underperforms an equally weighted benchmark over horizons longer than five years. [Campbell and Vuolteenaho \(2004\)](#) and [Campbell, Giglio, Polk, and Turley \(2018\)](#) model the riskiness of equity portfolios for long-term investors, showing that the short legs of anomaly portfolios provide valuable hedging against the deterioration of investment opportunities over the long run. Consistent with these insights, the optimal portfolios in this paper increasingly reallocate toward short-leg anomalies over longer horizons, suggesting that my approach captures their long-run hedging value without explicitly modeling return dynamics.

This paper also contributes to the literature on horizon-specific risks and returns. [Kamara, Korajczyk, Lou, and Sadka \(2016\)](#) show that equity factor risks are priced differently across investment horizons. [Bandi, Chaudhuri, Lo, and Tamoni \(2021\)](#) find that different frequency components of market risk carry distinct pricing implications in the cross-section of returns. [Neuhierl and Varneskov \(2021\)](#) show that horizon-specific risks in the stochastic discount factor (SDF) operate only through the state vector and its dynamics. [Bandi, Perron, Tamoni, and Tebaldi \(2019\)](#) document the effectiveness of past multi-period market risk in predicting future multi-period market premia of the same length. The findings in this paper confirm the existence of horizon-specific patterns in payoffs and leverage these patterns to construct portfolio strategies with target investment horizons.

2 Methodology

This section explains my approach to multi-period portfolio choice. [Section 2.1](#) states the traditional multi-period portfolio choice problem. [Section 2.2](#) discusses the characterization of k -period buy-and-hold payoffs from the large cross-section of k -period asset returns. [Section 2.3](#) introduces the portfolio decisions (i.e., monthly rebalancing rules) which seek to achieve the characterized payoffs. [Section 2.4](#) discusses the empirical details for constructing the strategies and introduces the in-sample and out-of-sample designs.

2.1 Multi-period Portfolio Problem

Consider the portfolio choice problem of a power utility investor at time t who has a k -period horizon and aims to maximize the expected utility of terminal wealth (W_{t+k}) by allocating funds across a large number of risky assets, N , and a risk-free asset at each rebalancing date, $t + s$, for $s = 0, 1, \dots, k - 1$. Assets have single-period excess returns, $R_{t+1}^{e,i}$, from date t to $t + 1$, $\mathbf{R}_{t+1}^e$ denotes the $N \times 1$ vector of excess returns, and R_{t+1}^f is the single-period risk-free rate. The investor's problem is

$$\max_{\{\mathbf{w}_{k,t+s}\}_{s=0}^{k-1}} \mathbb{E}_t [U(W_{t+k})] = \max_{\{\mathbf{w}_{k,t+s}\}_{s=0}^{k-1}} \mathbb{E}_t \left[\frac{(W_{t+k})^{1-\gamma}}{1-\gamma} \right], \quad (1)$$

subject to

$$W_{t+s+1} = W_{t+s} (1 + R_{t+s+1}^p), \quad \text{for } s = 0, 1, \dots, k - 1,$$

where

$$R_{t+s+1}^p = R_{t+s+1}^{e,p} + R_{t+s+1}^f = \mathbf{w}'_{k,t+s} \mathbf{R}_{t+s+1}^e + R_{t+s+1}^f,$$

and γ is the coefficient of relative risk aversion. This formulation assumes investors can freely lend and borrow at the risk-free rate in each period. The weight on the risk-free asset is implicitly given by $1 - \mathbf{w}'_{k,t+s} \boldsymbol{\iota}$, where $\boldsymbol{\iota}$ is an $N \times 1$ vector of ones.

The traditional solution to multi-period portfolio problems relies on the Bellman equation,

expressing each investment decision as a single-period optimization with state-dependent utility that captures the value of future investment opportunities. While theoretically sound, this procedure requires detailed modeling of state dynamics and conditional moments, which can become prohibitively complex in the presence of many assets or weak predictability.⁷ Even in the simplest cases with few risky assets, the economic gains from implementing optimal dynamic strategies are often minimal (e.g., [Brandt, 2010](#)), while the estimation burden grows exponentially with the number of assets and state variables. More, any imprecision from modeling conditional distributions accumulates through the recursive nature of the solution. Given these implementation challenges and modest benefits, practitioners typically resort to myopic portfolio strategies rather than attempting to implement theoretically optimal dynamic policies (e.g., [Brandt, 2010](#); [Cochrane, 2022](#)).

An alternative perspective is offered by [Cochrane \(2014\)](#), who argues that multi-period portfolio choice should begin with the final payoffs investors ultimately care about. By separating the economic question of which payoffs to target from the portfolio decisions that deliver those payoffs, this view shifts attention away from modeling conditional return dynamics and toward the portfolio outcomes. My approach builds directly on this insight: I first define the optimal payoffs at a given horizon—that is, what combinations of assets provide optimal investment payoffs at the desired horizon—and then develop portfolio strategies to achieve those payoffs.

2.2 Characterizing k -period Payoffs

Compounded returns contain information about optimal portfolio allocations that are not evident in the distribution of monthly asset returns. When returns are not independent and identically distributed (i.i.d.), k -month return moments, covariances, and other joint relations (i.e., investment opportunities) differ from those of the monthly returns. For example, an asset with high short-run volatility may be more attractive over longer horizons if its returns mean-revert, lowering long-run

⁷The underlying predictability in single-period returns can be unstable across time periods (e.g., [Welch and Goyal, 2008](#)) or the risk premia of predictors may switch signs (e.g., [Barroso, Boons, and Karehnke, 2021](#)), further complicating the estimation of investment opportunities purely based on predictability in single-period asset returns.

variance relative to monthly measures (e.g., [Poterba and Summers, 1988](#)). In addition, two assets that appear weakly correlated in monthly data may exhibit strong co-movements when compounded over several years, making them less useful for diversification at long horizons (e.g., [Anarkulova, Cederburg, and O'Doherty, 2023](#)). To preserve the important information in the joint distribution of compounded asset returns over the investment horizon, I characterize optimal buy-and-hold payoffs from the large cross-section of k -month asset returns. Over a k -month horizon, the buy-and-hold investment payoff is

$$\begin{aligned} 1 + R_{t,t+k}^p &= 1 + R_{t,t+k}^{e,p} + R_{t,t+k}^f \\ &= 1 + \tilde{\mathbf{w}}_k' \mathbf{R}_{t,t+k}^e + R_{t,t+k}^f, \end{aligned} \tag{2}$$

where $R_{t,t+k}^p$ is the k -period portfolio return, $R_{t,t+k}^{e,p}$ is the k -period portfolio excess return, and $R_{t,t+k}^f$ is the compounded k -period risk-free return. The k -period excess return on the portfolio is $R_{t,t+k}^{e,p} = \tilde{\mathbf{w}}_k' \mathbf{R}_{t,t+k}^e$, where $\tilde{\mathbf{w}}_k$ is the $N \times 1$ vector of portfolio weights and $\mathbf{R}_{t,t+k}^e$ is the $N \times 1$ vector of compounded excess returns.

Given the dimensionality of the asset space, it is essential to work with a much smaller set of portfolios that span the relevant investment opportunities in the large asset universe. Past evidence document the presence of horizon-specific patterns in the cross-section of returns, which suggest that short- and long-horizon returns have distinct sources of systematic variation (e.g., [Bandi, Chaudhuri, Lo, and Tamoni, 2021](#); [Kamara, Korajczyk, Lou, and Sadka, 2016](#); [Neuhierl and Varneskov, 2021](#)). To estimate a forward-looking and tractable representation of final payoffs from the observed returns of the high-dimensional asset space, I extract the systematic components of k -month excess returns.

I assume k -period excess returns admit an approximate factor structure in the sense of [Chamberlain and Rothschild \(1983\)](#),

$$\mathbf{R}_{t,t+k}^e = \mathbf{B}_k \mathbf{F}_{t,t+k} + \boldsymbol{\varepsilon}_{t,t+k}, \tag{3}$$

where $\mathbf{B}_k$ is an $N \times J$ matrix of factor loadings, $\mathbf{F}_{t,t+k}$ is the J -dimensional matrix of orthogonal systematic factors, and J is the number of pervasive factors and is assumed to be known. Assuming

that factors and residuals are uncorrelated, we can write the covariance matrix of returns as

$$\mathbf{\Sigma}_k = \mathbf{B}_k \mathbf{\Lambda}_k \mathbf{B}_k' + \mathbf{V}_k, \quad (4)$$

where $\mathbf{\Lambda}_k$ is a $J \times J$ diagonal matrix of factor variances and $\mathbf{V}_k$ is an $N \times N$ matrix of idiosyncratic variances.⁸

The factor structure assumption enables me to estimate the systematic component $(\mathbf{B}_k \mathbf{\Lambda}_k \mathbf{B}_k')$ through principal component analysis of the covariance matrix of k -period returns. The loadings $(\mathbf{B}_k)$ are estimated using the eigenvectors corresponding to the J largest eigenvalues, and the systematic factors are constructed as $\mathbf{F}_{t,t+k} = (\mathbf{B}_k' \mathbf{B}_k)^{-1} \mathbf{B}_k' \mathbf{R}_{t,t+k}^e$. Following standard convention, I normalize the eigenvectors such that $\mathbf{B}_k' \mathbf{B}_k = \mathbf{I}_J$, which simplifies the factor estimation to

$$\underbrace{\mathbf{F}_{t,t+k}}_{J \times 1} = \underbrace{\mathbf{B}_k'}_{J \times N} \underbrace{\mathbf{R}_{t,t+k}^e}_{N \times 1}, \quad (5)$$

and as a result, assets' exposures are equivalent to their contributions to the systematic factors. Under factor structure assumptions, the factors span the space of k -period expected excess returns such that

$$\mathbb{E}[\mathbf{R}_{t,t+k}^e] = \mathbf{B}_k \boldsymbol{\mu}_{F,k},$$

where $\boldsymbol{\mu}_{F,k} = \mathbb{E}[\mathbf{F}_{t,t+k}] \in \mathbb{R}^J$ is the $J \times 1$ vector of risk premia of the target horizon factors.

Remark 1: Let $\tilde{\mathbf{w}}_k \in \mathbb{R}^N$ be an arbitrary $N \times 1$ vector of portfolio weights such that the portfolio's k -period buy-and-hold excess returns are given by $R_{t,t+k}^{e,p} = \tilde{\mathbf{w}}_k' \mathbf{R}_{t,t+k}^e$. For any such portfolio, there exists a factor-only portfolio—with weights of the form $\mathbf{w}_k = \mathbf{B}_k \boldsymbol{\theta}_k$ for some $\boldsymbol{\theta}_k \in \mathbb{R}^J$ —that achieves the same k -period buy-and-hold expected return with lower k -period variance.

⁸Mathematically, an approximate factor structure implies that a finite number of factors (J) explains an increasing fraction of return variation as the number of assets grows, while the eigenvalues of $\mathbf{V}_k$ remain bounded. As a result, a diversified portfolio only contains factor variance, and the idiosyncratic risks are non-pervasive, disappearing from a well-diversified portfolio.

Proof: We can decompose any arbitrary portfolio weights $\tilde{\mathbf{w}}_k \in \mathbb{R}^N$ into a part that lies in the column subspace of $\mathbf{B}_k$ and an orthogonal component,

$$\tilde{\mathbf{w}}_k = \underbrace{\mathbf{B}_k}_{N \times J} \underbrace{\boldsymbol{\theta}_k}_{J \times 1} + \underbrace{\mathbf{u}}_{N \times 1},$$

where $\mathbf{B}_k' \mathbf{u} = \mathbf{0}_{J \times 1}$. The portfolio's expected return can then be written as

$$\begin{aligned} \mathbb{E} \left[R_{t,t+k}^{e,p} \right] &= \tilde{\mathbf{w}}_k' \mathbb{E} [\mathbf{R}_{t,t+k}^e] = (\mathbf{B}_k \boldsymbol{\theta}_k + \mathbf{u})' \mathbb{E} [\mathbf{R}_{t,t+k}^e] \\ &= (\mathbf{B}_k \boldsymbol{\theta}_k + \mathbf{u})' \mathbf{B}_k \boldsymbol{\mu}_{F,k} \\ &= \boldsymbol{\theta}_k' \mathbf{B}_k' \mathbf{B}_k \boldsymbol{\mu}_{F,k} + \mathbf{u}' \mathbf{B}_k \boldsymbol{\mu}_{F,k} \\ &= \boldsymbol{\theta}_k' \boldsymbol{\mu}_{F,k}, \end{aligned}$$

since $\mathbf{u}' \mathbf{B}_k = (\mathbf{B}_k' \mathbf{u})' = \mathbf{0}_{1 \times J}$, and $\mathbf{B}_k' \mathbf{B}_k = \mathbf{I}_J$.

Following a similar approach, the variance of the arbitrary portfolio is

$$\text{Var} \left(R_{t,t+k}^{e,p} \right) = \boldsymbol{\theta}_k' \boldsymbol{\Lambda}_k \boldsymbol{\theta}_k + \mathbf{u}' \mathbf{V}_k \mathbf{u}.$$

Since $\mathbf{V}_k$ is positive semi-definite, $\mathbf{u}' \mathbf{V}_k \mathbf{u} \geq 0$, so the variance of the arbitrary portfolio is at least as large as that of the factor-only portfolio, $\boldsymbol{\theta}_k' \boldsymbol{\Lambda}_k \boldsymbol{\theta}_k$. [Appendix A](#) provides the full derivation of the portfolio variance. ■

Remark 1 shows that portfolio weights orthogonal to the factor loadings ($\mathbf{u} \in \text{col}(\mathbf{B}_k)^\perp$) increase k -period variance without contributing to expected returns. I therefore focus on the portfolio weights that lie in the column subspace of $\mathbf{B}_k$,

$$\underbrace{\mathbf{w}_k}_{N \times 1} = \underbrace{\mathbf{B}_k}_{N \times J} \underbrace{\boldsymbol{\theta}_k}_{J \times 1}. \tag{6}$$

The k -period buy-and-hold excess returns of such portfolios are

$$\begin{aligned} R_{t,t+k}^p - R_{t,t+k}^f &= R_{t,t+k}^{e,p} = \mathbf{w}_k' \mathbf{R}_{t,t+k}^e \\ &= \boldsymbol{\theta}_k' \mathbf{B}_k' \mathbf{R}_{t,t+k}^e = \boldsymbol{\theta}_k' \mathbf{F}_{t,t+k}, \end{aligned} \tag{7}$$

and represent investing in J systematic factors of k -period returns for some $J \times 1$ parameter $\boldsymbol{\theta}_k$ that determines the allocation across the systematic factors.

Note that if asset returns do not exhibit extreme higher moments (e.g., buy-and-hold returns are jointly normally distributed), a factor-only portfolio is the optimal buy-and-hold k -period portfolio for any investor with concave utility over k -period wealth, and the N -dimensional asset-by-asset optimal weights converge to $\mathbf{w}_k = \mathbf{B}_k \boldsymbol{\theta}_k$ for some utility-maximizing J -dimensional $\boldsymbol{\theta}_k$. In the presence of extreme higher moments, some portfolios with lower expected returns or higher variance may still be more desirable than the factor-only portfolio due to favorable distributional features. A cost associated with representing investment opportunities through the horizon-specific factors is that it does not explicitly capture higher-order moments such as skewness or kurtosis at the asset level. These moments are instead captured at the factor level and through the optimal combination of factors ($\boldsymbol{\theta}_k$), which accounts for the entire distribution of k -period factor returns. To the extent that the higher moments of the portfolios are captured by the factor portfolios, the information loss from accounting for these moments at the factor level is minimal, and the optimal asset-by-asset N -dimensional weights converge to $\mathbf{w}_k = \mathbf{B}_k \boldsymbol{\theta}_k$ for some utility-maximizing $\boldsymbol{\theta}_k \in \mathbb{R}^J$.

How does this payoff characterization advance the high-dimensional multi-period investor's problem? Note that the optimal policy (Eq. 6) is linear in the loadings (weights) on k -month factors, $\mathbf{B}_k$. Two assets with the same factor loadings would be weighted equally in the portfolio, which implies that loadings on the target horizon factors summarize the sufficient asset-specific information for forming these optimal portfolios. Consequently, Eq. 7 states that the optimal risky payoff is the result of investing in J ($J \ll N$) systematic factors of k -month returns. Thus, this characterization provides measurable and tractable benchmarks for investors to follow during their

holding periods.

2.3 Monthly Managed Strategies with Target Horizon

Section 2.2 characterizes optimal risky payoffs across a large number of assets through a small number of systematic factors of compounded asset returns over investors' target horizons. This section constructs trading strategies that seek to achieve these horizon-specific systematic payoffs over the investment horizon. Targeting these payoffs results in a sequence of monthly rebalancing rules, which serves as a tractable alternative to the multi-period solution from complex dynamic optimization with many assets.

The underlying assumptions in Section 2.2 implies that the loadings on the k -month systematic factors ($\mathbf{B}_k$) capture sufficient asset-specific information for achieving the horizon-specific payoffs. Let τ denote the calendar month at which portfolio decisions take place. For a k -period investor, the sequence of portfolio decisions occur over $\tau \equiv t + s$ for $s = 0, 1, \dots, k - 1$. Let $\hat{\mathbf{B}}_{k,\tau}$ be the $N \times J$ matrix of asset loadings on k -period systematic factors, estimated using PCA on the available k -period returns up to month τ . In order to achieve the optimal k -period payoffs, I model the monthly portfolio weights as a linear function of the asset loadings on the k -period factors,

$$\underbrace{\hat{\mathbf{w}}_{k,\tau}}_{N \times 1} = \underbrace{\hat{\mathbf{B}}_{k,\tau}}_{N \times J} \underbrace{\boldsymbol{\theta}_k}_{J \times 1}, \quad (8)$$

where $\boldsymbol{\theta}_k$ is a $J \times 1$ vector of coefficients that needs to be estimated.

Eq. 8 represents a monthly managed strategy that follows the characterized optimal payoffs. To see this, let

$$\tilde{\mathbf{F}}_{k,\tau+1} = \hat{\mathbf{B}}'_{k,\tau} \mathbf{R}_{\tau+1}^e \quad (9)$$

denote monthly payoffs to *factor-tracking portfolios* of k -period returns. The systematic factors of k -period returns (Eq. 5) span the investment horizon's payoff space and summarize the risks that affect the k -period outcomes. The tracking portfolios (Eq. 9) are monthly traded payoffs that proxy

for k -period investment opportunities, and their k -period returns approximate those of the target horizon factors. The subscript τ on $\hat{\mathbf{B}}_{k,\tau}$ implies that loadings are re-estimated each month with the updated return data. Additionally, asset weights naturally deviate from the target horizon exposures as they evolve with realized returns over each period; monthly rebalancing ensures that factor-tracking portfolios maintain the estimated k -period exposures. The portfolio policy (Eq. 8) implies that a k -period investor buys a managed portfolio whose monthly excess returns are

$$\begin{aligned} R_{t+s+1}^p - R_{t+s+1}^f &= R_{t+s+1}^{e,p} = \hat{\mathbf{w}}'_{k,t+s} \mathbf{R}_{t+s+1}^e \\ &= \boldsymbol{\theta}'_k \hat{\mathbf{B}}'_{k,t+s} \mathbf{R}_{t+s+1}^e = \boldsymbol{\theta}'_k \tilde{\mathbf{F}}_{k,t+s+1}, \end{aligned} \tag{10}$$

for $s = 0, 1, \dots, k-1$. Hence, the portfolio of k -period investors combines the horizon-specific factor-tracking portfolios using a vector $\boldsymbol{\theta}_k \in \mathbb{R}^J$ that maximizes expected utility over k -period wealth.⁹

The k -period wealth from holding factor-tracking portfolios is given by

$$\begin{aligned} W_{t+k} &= \prod_{s=0}^{k-1} \left(1 + \mathbf{w}'_{k,t+s} \mathbf{R}_{t+s+1}^e + R_{t+s+1}^f \right) \\ &= \prod_{s=0}^{k-1} \left(1 + \boldsymbol{\theta}'_k \tilde{\mathbf{F}}_{k,t+s+1} + R_{t+s+1}^f \right), \end{aligned} \tag{11}$$

where the initial wealth for investors with the power utility is set to one ($W_{t+s} = 1$ for $s = 0$) without loss of generality. The $J \times 1$ parameter $\boldsymbol{\theta}_k$ determines the optimal share of each factor-tracking portfolio in the optimal portfolio. The parametrization assumes that $\boldsymbol{\theta}_k$ is constant over time; that is, the optimal k -period combination of factor-tracking portfolios at a given date are the same for all dates. A constant $\boldsymbol{\theta}_k$ over time enables us to estimate the optimal share of factor-tracking portfolios from asset returns, without specifying their conditional return dynamics.

⁹Alternatively, we can examine a buy-and-hold portfolio problem, where investors choose the weights at the start of the period and do not rebalance over the entire k periods. In this case, the estimated exposures and the share of each factor in the portfolio drift freely with realized returns during the periods. [Appendix C](#) discusses this buy-and-hold design and documents the economic loss that long-horizon investors may experience compared to the monthly managed case.

Plugging the portfolio policy (Eq. 8) into the conditional utility maximization over N assets (Eq. 1) represents a static choice over J ($J \ll N$) factor-tracking portfolios,

$$\begin{aligned} \max_{\boldsymbol{\theta}_k} \mathbb{E}[U(W_{t+k})] &= \max_{\boldsymbol{\theta}_k} \mathbb{E} \left[U \left(\prod_{s=0}^{k-1} \left(1 + \hat{\mathbf{w}}'_{k,t+s} \mathbf{R}_{t+s+1}^e + R_{t+s+1}^f \right) \right) \right] \\ &= \max_{\boldsymbol{\theta}_k} \mathbb{E} \left[U \left(\prod_{s=0}^{k-1} \left(1 + \boldsymbol{\theta}'_k \tilde{\mathbf{F}}_{k,t+s+1} + R_{t+s+1}^f \right) \right) \right]. \end{aligned} \quad (12)$$

By conditioning down the maximization problem, we can solve for the optimal $\boldsymbol{\theta}_k$ by maximizing the corresponding sample analogue over the entire sample period,

$$\begin{aligned} \max_{\boldsymbol{\theta}_k} \frac{1}{M} \sum_{t=0}^{M-1} U(W_{t+k}) &= \max_{\boldsymbol{\theta}_k} \frac{1}{M} \sum_{t=0}^{M-1} \frac{(W_{t+k})^{1-\gamma}}{1-\gamma} \\ &= \max_{\boldsymbol{\theta}_k} \frac{1}{M} \sum_{t=0}^{M-1} \frac{1}{1-\gamma} \left(\prod_{s=0}^{k-1} \left(1 + \boldsymbol{\theta}'_k \tilde{\mathbf{F}}_{k,t+s+1} + R_{t+s+1}^f \right) \right)^{1-\gamma}, \end{aligned} \quad (13)$$

where M is the number of k -period observations in the sample. For example, using overlapping k -period returns from T monthly observations results in $M = T - k + 1$ k -period observations.

My approach to the multi-period problem has many conceptual advantages. First, it sidesteps the formidable task of modeling the conditional joint evolution of return distributions across many assets. Instead, I utilize the compounded asset returns which implicitly reflect the effects of various conditioning information only seen by market participants, and maintain the time-series and cross-sectional relationships over the holding periods. Second, by representing optimal payoffs through horizon-specific systematic factors, I link the differences in optimal portfolios across horizons to the horizon-dependent nature of risk and return.

Third, the multi-period optimization with a large number of assets remains computationally simple and numerically robust. The dimensionality of the optimization problem is fixed by the number of systematic factors (J), which is typically small and does not grow with the asset space. As a result, the estimation of $\boldsymbol{\theta}_k$ involves a low-dimensional nonlinear optimization problem that is stable and efficient to solve. Furthermore, optimizing the multi-period strategies with many assets

over low-dimensional parameters θ_k reduces the risk of in-sample overfitting.

2.4 Constructing Managed Strategies

This section explains the construction of managed strategies from the overlapping k -period buy-and-hold returns. I first discuss the construction of monthly factor-tracking portfolios for specific investment horizons. Next, I explain the optimal k -period combination of the horizon-specific tracking portfolios and introduce the in-sample and out-of-sample portfolio designs. Lastly, I introduce portfolio weight constraints.

2.4.1 Factor-tracking Portfolios

For each asset, the k -period overlapping excess returns are constructed as

$$R_{t,t+k}^{e,i} = R_{t,t+k}^i - R_{t,t+k}^f,$$

where $R_{t,t+k}^i = \prod_{s=0}^{k-1} (1 + R_{t+s+1}^i) - 1$ is the k -period buy-and-hold asset return and $R_{t,t+k}^f = \prod_{s=0}^{k-1} (1 + R_{t+s+1}^f) - 1$ is the k -period risk-free rate. Let $t = 1, \dots, T$ denote the total sample of monthly returns, and let $M = T - k + 1$ so that $t = 1, \dots, M$ represents the total sample of k -period overlapping returns. For each horizon k , I use the first 60 k -period excess returns as the initial training window.¹⁰ I apply principal component analysis (PCA) on the covariance matrix of these returns to estimate the first set of loadings ($\hat{\mathbf{B}}_{k,0}$). I expand the training window each month, and estimate $\hat{\mathbf{B}}_{k,t}$ to obtain the time series of horizon-specific loadings.

In all analyses, I normalize the eigenvectors so that the premia of the k -period factors are positive, which prevents sign changes in the eigenvectors across different PCA runs.¹¹ Additionally,

¹⁰For example, when $k = 36$, the initial k -period window corresponds to monthly returns from $t = 1, \dots, (36 - 1) + 60 = 95$.

¹¹PCA does not uniquely determine the sign of eigenvectors—a PCA on the same covariance matrix may yield either $B_{j,k}$ or $-B_{j,k}$. In each estimation, I project the observed k -period returns onto the eigenvectors and choose the sign so that the mean of $F_{j,t-k,t} = B'_{j,k} \mathbf{R}_{t-k,t}^e$ is positive. This is equivalent to assuming positive premia for factors capturing common variations in average returns. A similar sign normalization is used by Kelly, Pruitt, and Su (2019) and Lettau and Pelger (2020), among others.

I normalize the eigenvectors for each horizon so that the weights on the first principal component sum to one.¹² I then form the time series of monthly factor-tracking portfolios as $\tilde{\mathbf{F}}_{k,t+1} = \hat{\mathbf{B}}'_{k,t} \mathbf{R}_{t+1}^e$. This procedure ensures that the loadings at each point in time are estimated solely from historical data. Since the k -period observations begin $k - 1$ months after the start of the monthly returns and I utilize the initial 60 k -period observations for estimation of loadings, the sample of monthly factor-tracking portfolios for each investment horizon k begins $k + 59$ months after the start of sample period (month $t = k + 60$).

2.4.2 Estimating θ_k : In-Sample and Out-of-Sample Portfolios

The monthly portfolio of a k -period investor combines J factor-tracking portfolios through a $J \times 1$ parameter $\theta_k \in \mathbb{R}^J$ that maximizes utility over k -period wealth and determines the share of each tracking portfolio in the optimal portfolio. For the in-sample portfolios, I estimate the $\hat{\theta}_k$ coefficients that maximize average utility of k -period wealth using the entire sample of k -period wealth (Eq. 13). Thus, the in-sample estimates presume investors know the entire sample of k -period returns for factor-tracking portfolios. However, estimating $\hat{\theta}_k$ using the full sample of returns relies on information that is not available to investors at the time of portfolio formation. To ensure the applicability of the horizon-specific portfolios to real-time investors, in the out-of-sample results I estimate the series of $\hat{\theta}_k$ that maximize the k -period average utility only using the historical data available at the time of portfolio formation.

I estimate the out-of-sample $\hat{\theta}_k$ using an expanding window with the initial training window of 60 k -period wealth observations, making the out-of-sample period starts from $k + 59$ months after the start of the in-sample period and $2k + 118$ months after the start of the sample period.¹³ At

¹²Note that there is no inherent economic relationship between the sum of the eigenvectors of the first principal component and the investment horizon; empirically, this sum is approximately $\approx \sqrt{N}$ for all horizons. Thus, this normalization is without loss of generality.

¹³The in-sample period covers the full sample of monthly factor-tracking portfolios, which begins $k + 59$ months after the start of the sample of monthly asset returns. Since each k -period wealth observation is a compounded return over k months (i.e., k -period factor-tracking portfolio and risk-free rate), constructing 60 such observations requires an additional $k - 1 + 60 = k + 59$ months. As a result, the out-of-sample period begins $k + 59$ months after the start of the in-sample period, and $2k + 118$ months after the start of the sample of monthly asset returns. For example, for $k = 1$, the in-sample (out-of-sample) period begins 60 months (120 months) after the start of the sample period.

month t , the real-time $\hat{\boldsymbol{\theta}}_k$ is

$$\begin{aligned}\hat{\boldsymbol{\theta}}_k &= \arg \max_{\boldsymbol{\theta}_k} \left(\frac{1}{t-k+1} \right) \sum_{\tau=0}^{t-k} \frac{(W_{\tau+k})^{1-\gamma}}{1-\gamma} \\ &= \arg \max_{\boldsymbol{\theta}_k} \left(\frac{1}{t-k+1} \right) \sum_{\tau=0}^{t-k} \frac{1}{1-\gamma} \left(\prod_{s=0}^{k-1} \left(1 + \boldsymbol{\theta}'_k \tilde{\mathbf{F}}_{k,\tau+s+1} + R_{\tau+s+1}^f \right) \right)^{1-\gamma}.\end{aligned}\tag{14}$$

In both in-sample and out-of-sample estimates, I solve for $\hat{\theta}_{j,k} \in [0, 1]$ so that the contribution of each tracking portfolio to the overall portfolio lies between zero and one for $j \in \{1, \dots, J\}$. Past studies suggest that such portfolio constraints can be viewed as shrinkage estimators and improve the finite-sample properties of the estimates (e.g., [Jagannathan and Ma, 2003](#)). Additionally, since the systematic factors of k -period returns have positive k -period premia, limiting $\hat{\theta}_{j,k} \in [0, 1]$ prevents short positions in factor-tracking portfolios while ensuring no extreme allocations. Using the estimated $\hat{\boldsymbol{\theta}}_k$, I construct the time series of monthly portfolio weights, $\hat{\mathbf{w}}_{k,t} = \hat{\mathbf{B}}_{k,t} \hat{\boldsymbol{\theta}}_k$, and monthly portfolio returns, $R_{t+1}^p = \hat{\mathbf{w}}'_{k,t} \mathbf{R}_{t+1}^e + R_{t+1}^f$.

2.4.3 Portfolio Weight Constraints

In the base case analyses, I do not impose any constraints on the underlying portfolio weights ($\hat{w}_{i,k,t}$), and the only restriction applies to the $J \times 1$ parameter $\boldsymbol{\theta}_k$ in the expected utility maximization stage. Thus, the base case allows for portfolio characteristics like short-selling and leverage, and helps reveal how unconstrained optimal portfolio compositions vary across investment horizons under my approach.

Beyond the unconstrained base case, I also consider a specification that imposes leverage and short-sale constraints. Short-sale constraints are common in portfolio management, particularly among institutional investors who often face regulatory and practical restrictions, especially over longer investment horizons. Additionally, many investors face leverage constraints that limit their total risky positions. It is therefore important to examine how the managed strategies perform under these realistic constraints, as they reflect the practical investment conditions faced by many

long-horizon investors. I impose a non-negative weight restriction by setting any negative elements of the factor loadings ($\hat{\mathbf{B}}_{k,t}$) to zero.¹⁴ Intuitively, this constraint implies that while investors buy assets with positive horizon-specific factor loadings—signaling higher expected returns at their target horizon—they refrain from shorting assets with negative loadings, assigning them zero weights instead.¹⁵ Using these non-negative loadings, I construct factor-tracking portfolios and estimate $\hat{\boldsymbol{\theta}}_k$ using the procedure described in Section 2.4.2 for $\hat{\theta}_{j,k} \in [0, 1]$. To impose a leverage constraint, I estimate the time series of optimal weights ($\hat{\mathbf{w}}_{k,t} = \hat{\mathbf{B}}_{k,t} \hat{\boldsymbol{\theta}}_k$), and scale the weights to satisfy $\sum_{i=1}^N |\hat{w}_{i,k,t}| \leq c$, where c refers to the desired leverage.

3 Data and Evaluation

The empirical analysis uses the extreme decile portfolios from characteristic-sorted anomalies documented in the [Chen and Zimmermann \(2022\)](#) dataset. Specifically, I use portfolios constructed from continuous predictors in the October 2024 release, requiring non-missing returns over the period from July 1963 to December 2023. This filtering yields 230 portfolios representing the top and bottom deciles of 115 distinct anomalies.¹⁶ The market excess returns and risk-free rates are from Kenneth French’s website.¹⁷

The procedure discussed in Section 2.4 provides monthly rebalancing portfolio rules that are designed for specific investment horizons. I evaluate the performance of the portfolio rules using the k -period certainty-equivalent returns (CER) of the optimal portfolio policy under $\mathbf{w}_k$,

$$\text{CER}_k(\mathbf{w}_k) = \left[(1 - \gamma) \mathbb{E} \left[U \left(\prod_{s=0}^{k-1} \left(1 + \hat{\mathbf{w}}'_{k,t+s} \mathbf{R}_{t+s+1}^e + R_{t+s+1}^f \right) \right) \right] \right]^{\frac{1}{1-\gamma}} - 1. \quad (15)$$

¹⁴Similar to the unconstrained case, I rescale all eigenvectors so that the weights on the first PC sum to one.

¹⁵Recall that the factor-tracking portfolios represent the target horizon investment opportunities in the space of monthly returns. As a result, the natural way to remove short-selling from investment opportunity set is through constructing long-only factor-tracking portfolios.

¹⁶For robustness, I also report the main empirical findings using a broader sample from the [Chen and Zimmermann \(2022\)](#) dataset, consisting of 510 quintile portfolios corresponding to 102 anomalies with non-missing returns on all quintiles during the sample period.

¹⁷I thank Kenneth French, Andrew Y. Chen, and Tom Zimmermann for making their data available. Kenneth French’s data are accessible at mba.tuck.dartmouth.edu/pages/faculty/ken.french and Chen and Zimmermann’s data are accessible at openassetpricing.com.

Investors have power utility over k -period wealth with coefficient of relative risk aversion (γ) of five.

I measure the effectiveness of the portfolio policies in two ways. First, I compare the in-sample and out-of-sample k -period performance of the optimal policies with benchmark strategies. Second, I examine how well each horizon strategy serves investors with different investment horizons. For each investment horizon k , I calculate certainty-equivalent returns (CERs) under all horizon strategies. The difference in CERs shows the economic value of matching the portfolio rule to the actual investment horizon rather than using rules optimized for other horizons.

I use three respective benchmarks to compare with the optimal policies. The first benchmark is an equally weighted (EW) portfolio, which invests a naive $\frac{1}{N}$ weight in each risky asset each month. The k -period wealth under this strategy is given by

$$W_{t+k}^{EW} = \prod_{s=0}^{k-1} \left(1 + \frac{1}{N} \mathbf{1}'_N \mathbf{R}_{t+s+1}^e + R_{t+s+1}^f \right). \quad (16)$$

Since the underlying assets in this paper are extreme deciles of anomaly portfolios, the equally weighted benchmark sets a relatively low bar for evaluating performance, as it requires long positions in the short legs of anomalies, which have performed poorly over the sample period. I compare the results with a stronger benchmark that invests equally in the anomaly strategies (i.e., long-minus-short decile of each anomaly portfolio). The k -period wealth under this long-short (LS) benchmark is

$$W_{t+k}^{LS} = \prod_{s=0}^{k-1} \left(1 + \frac{2}{N} \mathbf{1}'_N (\mathbf{R}_{t+s+1}^{eL} - \mathbf{R}_{t+s+1}^{eS}) + R_{t+s+1}^f \right). \quad (17)$$

Lastly, the third benchmark invests in the market portfolio each month for k periods. The k -period wealth from this benchmark is $W_{t+k}^{Mkt} = \prod_{s=0}^{k-1} (1 + R_{t+s+1}^{Mkt} + R_{t+s+1}^f)$, where R_t^{Mkt} is monthly excess returns of the market portfolio. Unlike the first two benchmarks, the market portfolio is an external benchmark not constructed from the underlying test assets. Including this benchmark helps assess the economic magnitude of the k -period CERs by comparing them to the CERs of a

widely recognized and investable alternative.¹⁸

4 Results

4.1 Managed Strategies vs Benchmarks

This section evaluates the multi-period performance of the horizon-specific managed portfolios relative to benchmark portfolios. [Section 4.1.1](#) compares k -period certainty-equivalent returns (CERs) to assess whether the managed strategies deliver superior long-run utility. [Section 4.1.2](#) examines the underlying return distributions—including risk, return, and higher-order moments—to identify the payoff characteristics that support this performance advantage.

4.1.1 k -period Certainty-equivalent Returns

[Table 1](#) reports the k -period certainty-equivalent returns (CERs) for horizon-specific managed strategies alongside three benchmarks. For ease of interpretability, the k -period CERs are annualized by estimating the annual compound rate, $(1 + \text{CER}_k)^{12/k} - 1$. The first four columns report annualized k -period CERs for the horizon-specific strategies under four different J values. The choice of J indicates the assumption on the dimensionality of k -period buy-and-hold returns, and corresponds to the number of factor-tracking portfolios in the optimal portfolio. For each horizon k , the target horizon loadings ($\hat{\mathbf{B}}_{k,t}$) are estimated from historical k -period buy-and-hold returns up to month t , following the process described in [Section 2.4.1](#). Panel A documents the in-sample CERs where the optimal k -period combination of factor-tracking portfolios ($\hat{\boldsymbol{\theta}}_k$) are estimated from the entire sample of factor-tracking portfolios as per [Eq. 13](#). Panel B summarizes the out-of-sample CERs where the $\hat{\boldsymbol{\theta}}_k$ coefficients are estimated using the historical returns of factor-tracking portfolios available at the time of portfolio formation ([Eq. 14](#)). The sample period spans from July 1963 to December 2023. Due to data requirements for estimating initial $\hat{\mathbf{B}}_{k,t}$ ($\hat{\boldsymbol{\theta}}_k$), the effective in-sample (out-of-sample) window for each horizon starts $k + 59$ ($2k + 118$) months after the start of the sample.

¹⁸The market portfolio is the Center for Research in Securities Prices (CRSP) value-weighted index.

The last three columns document the k -period CERs of the benchmark strategies. The k -period CERs of the benchmark strategies showcase the effect of investment horizon on the attractiveness of portfolio decisions. The EW benchmark performs poorly over short-run, often showing negative or near zero CERs for horizons less than two years. In contrast, the k -period CERs of the LS portfolio indicate a stellar short-run performance for this benchmark. At the monthly interval, the EW benchmark has annualized CER of -0.76% (0.94%) over the in-sample (out-of-sample) period while the LS benchmark shows annualized CER of 11.58% (11.37%). Over longer horizons, the CERs of the EW portfolio surpass those of the LS portfolio. The annualized 120-period CERs of the EW and LS benchmarks over the in-sample (out-of-sample) period are 8.25% (7.23%) and 6.70% (5.31%), respectively. Unlike the EW and LS benchmarks, the market portfolio ('Mkt') does not systematically benefit short- or long-horizon investors. Over the in-sample period, the one-month CER of market portfolio is 4.46% and its 120-month CER is 5.79%, while over the out-of-sample period its one- and 120-month CERs are 5.19% and 4.42%, respectively.

Panel A of [Table 1](#) reports the in-sample CERs for horizon-specific strategies compared to the benchmarks. The first column ($J = 1$) represents a simple case which assumes one latent systematic factor drives common variation in k -period returns. Despite this strategy's simplicity, it outperforms all benchmark strategies over the long-run. The 96-period (120-period) CER of the horizon-specific strategy with $J = 1$ is 9.49% (10.16%) per year, which is 1.17 (1.23) times that of the EW benchmark and 1.32 (1.52) times that of the LS benchmark. The in-sample CER of the 96-period (120-period) strategy is 11.92% (13.09%) with $J = 6$ and reaches 13.92% (17.27%) when 12 factor-tracking portfolios are included.

The outperformance of horizon-specific strategies relative to benchmarks extends to all horizons. With only six factor-tracking portfolios ($J = 6$), the 60-period strategy has an in-sample CER of 12.53% per year, surpassing that of the EW benchmark (6.43%) by a factor of 1.95 and the LS benchmark (8.43%) by a factor of 1.49. The LS benchmark is by far the best performing benchmark strategy over the short-run, with annualized CER of 11.58% over one month, and 11.32% and

11.00% over six and 12 months, respectively. Nevertheless, horizon-specific strategies with six or more factor-tracking portfolios ($J \geq 6$) generate higher CERs than the LS portfolio. For example, with $J = 9$, the in-sample CERs of one-, six-, and 12-month strategies are 13.78%, 13.31%, and 12.54%, respectively.

The results in Panel A of [Table 1](#) documents strong in-sample CERs for managed strategies compared to benchmarks, showcasing the ability of horizon-specific portfolios in preserving the underlying structure of multi-period returns. One main advantage of my approach is that the multi-period portfolios with a large number of assets are optimized over low-dimensional sets of parameters. Specifically, the loadings ($\hat{\mathbf{B}}_{k,t}$) used to form monthly tracking portfolios depend solely on information available at portfolio formation, with look-ahead estimations restricted to the J -dimensional ($J \ll N$) coefficient vector $\boldsymbol{\theta}_k$. Thus, the in-sample CERs—though not fully implementable in real-time—are less likely to be driven by overfitting.

To ensure that the horizon-specific strategies are implementable in real time, Panel B of [Table 1](#) documents the CERs where $\hat{\boldsymbol{\theta}}_k$ is estimated using the historical returns available in real time. The results document the applicability of the managed strategies to real time investors. Over shorter horizons, the strategies with larger number of factor-tracking portfolios consistently outperform all benchmark strategies. In cases with 12 factor-tracking portfolios ($J = 12$), the out-of-sample annualized CERs of one-, six-, and 12-month strategies are 14.82%, 14.14%, and 12.44%, respectively—exceeding those of the LS benchmark by factors of 1.30, 1.29, and 1.17 over the corresponding horizons.

The superior performance of the horizon-specific strategies compared to the benchmarks is more prominent over longer investment horizons. Assuming six pervasive factors for the cross-section of k -period returns ($J = 6$), the real-time horizon-specific portfolios achieve CERs of 9.81% and 9.35% per year for 96- and 120-month investors, respectively. Over 96 and 120 months, the EW benchmark delivers CERs of 7.41% and 7.23%, and the LS benchmark delivers 5.96% and 5.31%, respectively. Overall, the horizon-specific strategies consistently generate higher k -period CERs

than those of the benchmark strategies across all horizons, underscoring the practicality of these managed strategies for real-time multi-period investors.

Figure 1 complements the results in Table 1 by plotting the annualized k -period CERs under various choices of J , ranging from one to 12. Each plot represents the annualized k -period CERs for the horizons shown in Table 1. Panel A plots the in-sample CERs and Panel B plots the out-of-sample CERs. The horizontal lines denote the CERs of benchmark strategies. The number of factor-tracking portfolios (J) reflects the number of pervasive factors in cross-section of k -period return. The number of systematic factors may vary across horizons, as the structure of asset comovements changes with the investment horizon.

As shown in Panel A of Figure 1, the in-sample CER of horizon-specific strategies naturally increases upon inclusion of additional factor-tracking portfolios, as the optimal portfolios maximize the k -period CERs using the entire sample of k -period tracking portfolios. However, the marginal gains from adding tracking portfolios are more pronounced at shorter horizons. For example, the in-sample CER of a 6-month strategy increases from 5.24% with one factor to 15.06% with 12 factors, representing a 187.4% improvement, while the 96-month strategy increases from 9.49% to 13.92%, representing a CER improvement of 46.7%. These patterns suggest that the underlying factor structure of long-horizon returns may be relatively sparse. Economically, this may reflect the fact that many sources of short-term risk—such as high-frequency noise or transitory components—tend to average out over time, leaving fewer sources of systematic variation at longer horizons (e.g., [Bandi, Perron, Tamoni, and Tebaldi, 2019](#); [Huang, 2023](#)).

Panel B of Figure 1 further illustrates this point by plotting the out-of-sample CERs across different values of J . Over shorter horizons, the out-of-sample CERs consistently increase with J , with the one-, six-, and 12-month strategies each attaining their peak CERs at $J = 12$, indicating that more complex factor structures are beneficial when targeting short-horizon payoffs. In contrast, over longer horizons, the highest CERs are achieved with fewer factors. The 96-month strategy reaches its maximum out-of-sample CER of 9.89% with only four factors, and the 120-month strategy

peaks at 12.90% with ten factors. Importantly, more parsimonious specifications deliver competitive performance even when they do not maximize CERs. For instance, the 60- and 120-month strategies achieve out-of-sample CERs of 8.71% and 9.35% with $J = 6$, closely matching the respective peaks.

These observations highlight the tradeoff between model complexity and out-of-sample applicability. While increasing J expands the space of target horizon outcomes, it also raises the risk of estimation error and poor out-of-sample performance—particularly at longer horizons where fewer distinct priced components may be relevant. To balance this tradeoff while avoiding the need to assign a different number of factors to each horizon, I adopt $J = 6$ as the baseline specification across all horizons in the subsequent analyses. This choice is supported by the pseudo out-of-sample exercise for determining the number of systematic factors (see [Appendix B](#)), which identifies $J = 6$ as the most frequently selected value across investment horizons. For robustness, I also report the key results under alternative values of J .

4.1.2 Properties of k -period Payoffs

This section examines the k -period return properties of the managed strategies and benchmark portfolios across investment horizons to clarify the sources of the managed strategies’ superior performance.

[Figure 2](#) displays annualized certainty-equivalent returns (Panel A) alongside five key statistics of k -period returns for the in-sample managed strategies and benchmarks for horizons spanning from one month to ten years. The statistics are annualized mean returns (Panel B), annualized standard deviations (Panel C), k -month variance ratios (Panel D), skewness (Panel E), and kurtosis (Panel F).¹⁹ Variance ratio measures the per-period risk of each portfolio strategy and is estimated as

$$\text{VR}(k) = \frac{\text{Var}(r_{t,t+k})}{k \text{Var}(r_{t+1})}, \quad (18)$$

¹⁹Except for variance ratios, all statistics are for k -period physical returns, $\prod_{s=0}^{k-1} (1 + R_{t+s+1}^{p,e} + R_{t+s+1}^f) - 1$. Standard deviations are annualized by multiplying the k -period values by $\sqrt{12/k}$. Mean returns are annualized by estimating the annual compound rates.

where $r_{t,t+k}$ and r_{t+1} represent the k -period and one-period log portfolio returns, respectively. The horizon-specific portfolios are constructed using six factor-tracking portfolios ($J = 6$).

Panels B-D of [Figure 2](#) summarize the mean-variance properties of horizon-specific portfolios and the benchmark portfolios. At shorter horizons, the EW benchmark exhibits relatively high volatility, while the LS benchmark is notably less volatile and the managed strategies lie in between; for example, at the one-month horizon, the annualized standard deviations are 21.5% for EW, 3.6% for LS, and 8.4% for the managed strategies. The managed strategies consistently deliver the highest mean returns at short horizons, with an annualized mean of 13.5% at the one-month horizon, compared to 12.2% for EW and 12.0% for LS. Both LS and horizon-specific strategies deliver high short-run CERs. As horizon increases, the per-period risk of the LS portfolio rises notably: its variance ratio reaches 12.5 at the 60-month horizon, compared to 1.02 for the managed strategies and 0.57 for the EW benchmark. Over the long run, the gap widens further: at the 120-month horizon, the variance ratio for LS approaches 20, while those of the managed strategies and EW benchmark fall just below 0.2.

Despite this increasing risk, the k -period mean return of LS remains flat, while that of the managed strategies continues to rise with horizon. Over a 60-month horizon, the mean return of the horizon-specific portfolio is 18.7% per year, noticeably higher than that of the LS benchmark with 12.4%. The EW portfolio, though initially underperforming, benefits from diversification across long and short legs of anomalies, and its CER eventually surpasses that of the LS benchmark after approximately six years.²⁰ Unlike the benchmarks, the horizon-specific strategies consistently achieve high CERs across all horizons. This strong performance is supported by the favorable risk-return tradeoffs shown in the Panels B-D. Although the annualized standard deviation naturally rises with horizon length, the managed strategies also exhibit increasing mean returns, and this higher volatility does not translate into disproportionately greater per-period risk, allowing them to

²⁰The horizon-varying patterns in CERs for LS and EW portfolios are consistent with past studies suggesting that short legs of anomaly portfolios provide valuable hedging against the deterioration of investment opportunities at longer horizons (e.g., [Campbell, Giglio, Polk, and Turley, 2018](#)).

sustain superior performance across horizons.²¹

Panels E and F of [Figure 2](#) complement the earlier analysis by highlighting higher-order features of the return distributions that are particularly relevant for long-horizon investors with power utility. Panel E plots the skewness of k -period returns. The horizon-specific portfolios exhibit moderate positive skewness over longer horizons, indicating that extreme bad outcomes are less likely relative to extreme good ones. Combined with high mean returns over the long run, this implies that most losses are modest while the potential for large gains remains substantial—an appealing profile for long-horizon investors seeking both upside potential and downside protection. In contrast, skewness of the LS benchmark turns negative over longer horizons, implying that extreme losses become more likely than extreme gains, which is unfavorable for long-run risk-averse investors. Combined with the LS portfolio’s flat mean returns, a negative skewness suggests that large losses dominate the distribution, while good outcomes lack the magnitude needed to meaningfully improve the return profile. The EW benchmark also features positive skewness, slightly higher than that of the managed strategies, reducing the likelihood of extreme losses. However, it lacks strong capital appreciation, which limits the economic value of its favorable asymmetry.

Panel F plots the kurtosis of k -period returns. The horizon-specific portfolios maintain values around three or slightly above, indicating a return distribution close to normal, with limited tail risk. The combination of limited tail risk and positive skewness suggests that extreme outcomes are more likely to reflect large gains than large losses. The EW portfolio displays stronger asymmetry and tail risk than the horizon-specific strategies, which together with a relatively modest mean returns limit its overall appeal. The LS benchmark, in contrast, displays thin tails with kurtosis well below three at longer horizons. However, combined with negative skewness, a thin tail suggests that the rare extreme events are more likely to be large losses. Overall, the horizon-specific portfolios combine steady capital appreciation, decreasing per-period risk, positive skewness, and balanced

²¹The annualized variance of compounded returns tends to increase with horizon, as the variance grows more than proportionally relative to the holding period due to the nonlinear effect of compounding. This pattern arises even under *i.i.d.* returns and is further amplified by portfolio rebalancing. See, for example, [Bessembinder \(2021\)](#).

tail risk—features that jointly support their superior long-run utility gains.²²

Lastly, Figure 3 compares the empirical quintile-quintile distributions of in-sample k -period payoffs for the horizon-specific portfolios and benchmark strategies. Panel A plots the empirical quintile pairs of 60-month payoffs and Panel B plots those of 120-month payoffs. In each subplot, the x -axis shows the sorted k -period payoffs of the horizon-specific portfolios and the y -axis shows the sorted k -period payoffs of benchmarks. The leftmost (rightmost) point in each subplot shows the worst (best) k -period payoffs from investing one dollar in each strategy pairs. Points falling below the 45-degree line indicate that the managed strategy delivers higher k -period payoffs than the benchmark at that part of the distribution.

As shown in Panel A of Figure 3, the 60-month payoffs of the horizon-specific portfolio exceed those of the LS and EW benchmarks across nearly the entire distribution. Among the 548 payoff observations, only the four worst payoffs slightly favor the LS benchmark, and only the top two payoffs slightly favor the EW benchmark. Panel B of Figure 3 shows a similar pattern for 120-month payoffs: the empirical payoff distribution from the horizon-specific strategy dominates those of both the EW and LS benchmarks.²³ These findings show that the payoff advantage of horizon-specific strategies over the long-run holds across almost the entire distribution, not just in the average or tails, reinforcing the consistency of the managed strategies’ outperformance relative to the benchmarks.

4.2 Horizon Matching: Benefits and Properties

The previous section documents the strong performance of managed strategies compared to benchmarks across various investment horizons. This section examines whether these strategies are horizon-specific—that is, whether each strategy is best suited for the investment horizon it targets. I first evaluate the economic value of matching portfolio rules to investors’ horizons. I then explore how portfolio compositions and return properties vary across different horizon strategies.

²²The out-sample version of Figure 2 is shown in Figure A1. The k -period payoff properties of the out-of-sample portfolios are qualitatively consistent with those observed in-sample.

²³Figure A2 plots the empirical quintile-quintile distribution of 60- and 120-period payoffs for the out-of-sample horizon-specific portfolios and benchmark strategies. Similar to the in-sample portfolios, the distribution of payoffs for the out-of-sample portfolios continue to dominant those of the benchmarks.

4.2.1 Economic Value of Targeting Investment Horizon

This section examines whether the managed strategies exhibit horizon-specific patterns and quantifies the economic value of aligning portfolio policies with specific investment horizons. The central question is whether portfolios designed for a specific investment horizon yield better outcomes for investors with that horizon compared to investors with alternative horizons. For an investor with a k -period investment horizon, I compute the k -period certainty-equivalent return (CER) under a strategy that was optimized for horizon h ,

$$\text{CER}_k(\mathbf{w}_h) = \left[(1 - \gamma) \mathbb{E} \left[U \left(\prod_{s=0}^{k-1} \left(1 + \hat{\mathbf{w}}'_{h,t+s} \mathbf{R}_{t+s+1}^e + R_{t+s+1}^f \right) \right) \right] \right]^{\frac{1}{1-\gamma}} - 1, \quad (19)$$

where $\hat{\mathbf{w}}_{h,t} = \hat{\mathbf{B}}_{h,t} \hat{\boldsymbol{\theta}}_h$ represents the portfolio weights at month t from a strategy with a horizon of h . This measure captures the economic value of horizon- h strategies from the perspective of a k -period investor. Formally, the difference in CERs,

$$\Delta \text{CER}_{k,h} = \text{CER}_k(\mathbf{w}_k) - \text{CER}_k(\mathbf{w}_h), \quad (20)$$

represents the economic gain (or loss) a k -period investor experiences by following the horizon-matched strategy rather than a strategy optimized for horizon h . In the subsequent analyses, I evaluate all in-sample (out-of-sample) strategies over a common period starting in June 1978 (May 1993), when the longest-horizon ($k = 120$) strategy first becomes available, which ensures consistent comparison periods across strategies with different data requirements.²⁴

Table 2 documents the economic value of matching portfolio rules to investment horizons. For each investment horizon k (columns), entries report $\Delta \text{CER}_{k,h}$ with strategy horizon h (rows). The diagonal values are trivially zero and left blank. Positive values indicate economic benefits from horizon matching. Panel A reports in-sample $\Delta \text{CER}_{k,h}$ where the optimal combination of

²⁴For $k = 120$, the in-sample period begins $k + 59 = 179$ months after the start of the sample, and the out-of-sample period begins $2k + 118 = 358$ months after the start of the sample.

factor-tracking portfolios ($\hat{\theta}_k$ and $\hat{\theta}_h$) are estimated using the full sample of tracking portfolio returns. Panel B reports out-of-sample $\Delta\text{CER}_{k,h}$ where the optimal combination of factor-tracking portfolios are estimated using the historical returns available in real time. The managed strategies are constructed using six factor-tracking portfolios (i.e., $J = 6$).

Panel A of Table 2 reveals strong horizon-specific patterns for in-sample strategies, such that strategies targeting a given horizon h , are often suboptimal for investors with different investment horizons $k \neq h$. The first row reports $\Delta\text{CER}_{k,h=1}$ —the annualized CER gain or loss for k -period investors by switching from the one-period (myopic) strategy to the strategy optimized for their own horizon. Over short horizons, the k -period CERs from horizon-specific differ only marginally from those of the one-period strategy: A 6-period investor gains an additional 0.53% annualized 6-month CER by following a 6-period strategy compared to following a one-period strategy, a 12-period investor achieves 0.07% lower CER, and a 24-period investor achieves 0.04% higher CER. The economic value of horizon-matched strategies relative to a myopic strategy excels over longer horizons. The annualized in-sample CER differences for the 36-, 60-, and 96-month horizons relative to the single-period strategy are 1.14, 2.89, and 2.76 percent, respectively. Over a 120-month horizon, an investor gains 4.31% higher CER by following the 120-period strategy rather than a one-period (myopic) one, which showcases the economic significance of horizon-specific strategies over the long-run.

The economic value of targeting investment horizon in portfolio decisions is evident across all horizons. While long-horizon investors benefit from the long-run strategies, these strategies perform poorly for those with shorter horizons. The last row of Panel A reports $\Delta\text{CER}_{k,h=120}$, which quantifies the economic gain or loss experienced by k -period investors when they follow the strategy tailored to their own investment horizon instead of the 120-period strategy. For one-, six-, and 12-month investors, using the 120-period strategy instead of the horizon-specific strategies results in lower CERs by 13.65, 20.74, and 18.77 percent, respectively. Additionally, the differences in CERs generally increase with the gap between investment and strategy horizons. For example,

the sixth column denotes that while a 60-month investor loses 1.79% annualized CER over their investment horizon by following a 36-month strategy instead of their own, they expect to lose 2.89% by following a myopic strategy and 5.18% by following a 120-period strategy.

Panel B of [Table 2](#) shows that the horizon-specific patterns hold in the out-of-sample results. For most investment horizons, the horizon-specific strategies generate higher k -period out-of-sample CERs compared to alternative horizon strategies, specifically as the gap between investment and strategy horizons widens. For mid-term investors with 36- and 60-month horizons, the myopic strategy underperforms their horizon-specific strategies by 0.35% and 1.92%, respectively. The losses from adopting a myopic strategy are more pronounced at longer horizons, reaching 2.47% and 2.61% for 96- and 120-month investors. At a one-month horizon, the one-period strategy generate 0.28% and 0.83% higher annualized CERs than the six- and 12-period strategies, respectively, while it achieves 5.03% and 24.52% higher CERs than the 96-period and 120-period strategies, respectively. While some of the out-of-sample CER differences may contribute to noise in estimating the $\hat{\theta}$ parameters, the out-of-sample results showcase strong horizon-specific patterns that are exploitable in real time.

To see the relative contribution of k -period CERs of horizon-specific versus that of mismatched strategies in driving the differences shown in [Table 2](#), [Figure 4](#) presents the k -period CERs from following different horizon strategies. Each entry shows $\text{CER}_k(\mathbf{w}_h)$, where the columns (y -axis) indicate the investment horizon (k), and the rows (x -axis) represent the strategy horizon (h). The diagonal elements (in bold) show the CERs achieved when the investment horizon matches the strategy horizon.²⁵ Panel A shows the in-sample CERs and Panel B shows the out-of-sample CERs. Subtracting each entry in a given column from the corresponding diagonal value in that column yields the $\Delta\text{CER}_{k,h}$ values reported in [Table 2](#).

Comparing across investment horizons (columns) for a given strategy shows how each strategy serves investors with different investment horizons. Despite having the best short-run performance,

²⁵The slight differences between the k -period CERs presented here and those documented in [Table 1](#) are attributed to a smaller evaluation sample used for [Figure 4](#) for horizons shorter than $k = 120$.

the k -period CERs of short-horizon strategies consistently decrease over longer horizons. The in-sample (out-of-sample) CERs of the one-and 12-period strategies are 8.78% (6.74%) and 8.72% (5.32%) over 120 months, respectively. The out-of-sample 120-month CERs of the short-horizon strategies are even smaller than that of the equally weighted (EW) benchmark, which exhibits low and negative CERs over short horizons. The deteriorating long-run performance of short-horizon strategies suggests that optimizing for short-term gains can come at the cost of long-term utility.²⁶

Notably, while the long-horizon strategies yield the best outcomes over their target horizons, they perform poorly when evaluated over short horizons. The 120-period strategy yields an annualized 120-month CER of 13.09% in-sample and 9.35% out-of-sample. Despite this long-term gain, the in-sample CERs of the 120-period strategy are -1.52%, -9.37%, and -8.44% over one-, six-, and 12-month, respectively. Out-of-sample, the 120-period strategy shows even worse short-run performance, with the CERs being -15.28%, -31.48%, and -25.83% when evaluated over the respective short horizons. This pattern highlights a fundamental tension in long-horizon investing: strategies that are optimal over long horizons can appear deeply unattractive when judged over shorter periods.

The findings in this section show that horizon-specific strategies consistently deliver higher utility than strategies optimized for other horizons. This pattern holds both in and out of sample, and becomes more pronounced as the gap between investment and strategy horizons widens. While each managed strategy performs well relative to benchmarks, its effectiveness is tightly linked to the horizon it targets, establishing investment horizon as a key dimension of optimal portfolio decisions with tangible economic value that translates directly into real-world applications. The findings show that targeting final payoffs—rather than intermediate outcomes—can lead to strategies that are not only implementable over large number of assets but also strongly aligned with investors’ long-term goals.

²⁶This finding is consistent with Bessembinder, Cooper, and Zhang (2023), who show that most mutual funds underperform the S&P 500 over long horizons, despite often generating attractive short-run returns. This pattern illustrates that strong short-run performance does not necessarily translate into long-term success—particularly when investment decisions are influenced by short-term performance metrics, as is often the case with mutual fund investments.

4.2.2 Properties of Horizon-specific Portfolios

This section explores the structural sources of the horizon-specific performance documented above by examining how portfolio compositions and statistical properties vary across investment horizons. Understanding these differences provides insights into how optimal asset allocation and portfolio characteristics fundamentally change with investment horizon.

Table 3 summarizes the weight statistics in the basis assets ($\hat{\mathbf{w}}_{k,t}$) for horizon-specific portfolios. The weight statistics reveal systematic changes in portfolio composition as the investment horizon increases. Most notably, longer-horizon portfolios take progressively fewer short positions. As shown in Panel A (Panel B), the proportion of negative weights declines from 38.00% (37.98%) for the one-period strategy to 28.85% (21.19%) for the 120-period strategy in the in-sample (out-of-sample) estimates. Additionally, in-sample (out-of-sample) estimates show that the sum of negative weights declines from -62.73% (-65.78%) under the one-period strategy to -33.73% (-25.38%) under the 120-period strategy. Furthermore, the sum of weights in risky assets increases with the investment horizon from 42.74% (40.33%) for the in-sample (out-of-sample) one-period strategy to 119.45% (155.70%) for the in-sample (out-of-sample) 120-period strategy. These findings are consistent with the conventional wisdom that investors with longer horizons should allocate a larger portion of wealth to equities.

Notably, the horizon-specific portfolio rules showcase the advantage of short-leg anomalies for long-horizon investors. Figure 6 plots the in-sample weight statistics of short- and long-leg anomalies in the optimal portfolios. Panels A–C show that short legs receive substantial negative weights at short horizons, reflecting their poor short-run performance. However, as the investment horizon increases, the optimal portfolio assigns increasingly larger positive weights to short legs: the sum of negative weights declines, the minimum weight increases, and the proportion of negative positions steadily decreases. Panel D further shows that the contribution of short legs to total risky positions grows more with horizon relative to that of long legs.²⁷ These patterns support the view that

²⁷Figure A4 plots the out-of-sample weight statistics of short- and long-leg anomalies in the k -period optimal portfolios. The patterns are consistent with those described for the in-sample weight statistics.

short-leg anomalies become increasingly valuable for long-horizon investors, consistent with their role in hedging against deteriorating investment opportunities (e.g., [Campbell and Vuolteenaho, 2004](#); [Campbell, Giglio, Polk, and Turley, 2018](#)). These patterns indicate that, without explicitly modeling return dynamics or investment opportunities, my approach captures economically meaningful shifts in portfolio composition that reflect horizon-specific investment needs. The summary of weight statistics further highlights the systematic differences in portfolio compositions across different horizon investors; the portfolio rules, while being systematically horizon-varying, serve investors with specific target horizons by providing utility over their investment horizon.

[Table 4](#) reports the in-sample statistical properties of various strategies across different investment horizons.²⁸ For each investment horizon k , the values are k -period statistics of the strategy horizons and are annualized for ease of interpretation. Panels A and B show that the mean and standard deviation of returns generally increase with the strategy horizon, with long-horizon strategies often delivering higher mean and standard deviation than short-horizon strategies over all investment horizons. For example, the one-month mean and standard deviation (first columns of Panel A and B) for the 120-month strategy are 21.07% and 26.25%, respectively, which are the highest among all portfolios, including the EW and LS benchmarks. However, as shown in Panel C, longer-horizon strategies display decreasing variance ratios toward their target horizons, suggesting a mean-reverting component that reduces per-period risk over longer horizons. In contrast, short-horizon strategies exhibit increasing variance ratios over longer holding periods, indicating an increase in per-period risk over time.

To illustrate these horizon-specific features more clearly, [Figure 5](#) compares the in-sample statistical properties of the myopic (one-period), 60-period, and 120-period portfolios across all investment horizons. These figures reveal distinct horizon-dependent characteristics. Although the 120-month strategy initially displays the highest short-run standard deviation, its variance ratio steadily declines with increasing holding periods, ultimately showing a lower 120-month

²⁸The out-of-sample properties of the strategies across different investment horizons are reported in [Table 5](#).

standard deviation compared to the myopic strategy. On the other hand, the k -period variance ratios of the 60-month strategy are close to one and this strategy provides a steady CER across all investment horizons, consistent with the strategy representing a medium-term portfolio and hence, its characteristics showing moderate horizon-dependent patterns. Lastly, the riskiness of the myopic strategy increases over longer horizons, and despite achieving the best short-run performance, it attains the least CER among the three strategies over horizons beyond approximately five years. Panels E and F plots skewness and kurtosis of strategies across horizons. The 120-period shows extreme higher moments when evaluated over shorter horizons. However, both skewness and kurtosis decrease over horizons, ultimately exhibiting a slight positive skewness and kurtosis around three over 120 months, a return profile that is in favor of long-horizon investors.

The properties of horizon-specific portfolios indicate that the portfolio characteristics required for achieving superior long-horizon certainty-equivalent returns (CERs)—such as consistent capital appreciation, decreasing per-period risks, and desired higher moments—emerge naturally when portfolios are explicitly optimized for final, multi-period outcomes. Additionally, the findings provide further evidence that the short-run properties of portfolios do not necessarily determine their long-run properties and performance. When portfolio decisions are judged using short-run metrics, this disconnect can lead to systematically favoring strategies that appear attractive in the near term but fail to deliver over the long run.

4.3 Constrained Portfolios

This section reexamines the main empirical analyses for portfolios that have short-sale and leverage constraints. Following the procedure described in [Section 2.4.3](#), I construct the series of optimal asset weights from investing in the long-only factor-tracking portfolios. I then impose a no-leverage constraint by scaling the weights so that sum of weights in risky positions does not exceed one (i.e., $\sum_{i=1}^N |\hat{w}_{i,k,t}| \leq 1$).

[Table 6](#) reports the k -period certainty-equivalent returns (CERs) under these constraints. Panel

A highlights that the marginal in-sample gains from increasing the number of systematic factors (J) are minimal upon imposing the constraints. For the one-period portfolio, increasing J from one to 12 improves the CER modestly, from 5.48% to 6.70%. Similarly, at the 120-month horizon, the improvement is from 10.15% (for $J = 1$) to 11.02% (for $J = 12$). Additionally, the k -period CERs are generally weaker compared to those reported in [Table 1](#) for the unconstrained portfolios. However, the decrease in CERs upon imposing the constraints are less prominent over the long-run. For instance, with 12 factor-tracking portfolios ($J = 12$), the in-sample one-period CER of the constrained portfolio is 6.70% as opposed to 15.47% for the unconstrained portfolio, while those of the 120-period constrained and unconstrained portfolios are 11.02% and 17.27%, respectively. Interestingly, the 96-period constrained portfolios with 9 and 12 factors show higher out-of-sample CERs than their unconstrained counterparts.²⁹ In contrast, the short-run CERs of the constrained portfolios remain sharply lower than the unconstrained portfolios, both in-sample and out-of-sample. These patterns suggest that the short-sale and leverage constraints are less binding for long-horizon investors and could even benefit real-time applications.

Despite the relative decline in CERs for the constrained portfolios, the horizon-specific portfolios mostly generate higher k -period CERs compared to the benchmark portfolios. The column labeled as ‘Long’ represents a constrained version of the LS portfolio that invests only in the top decile of the anomaly portfolios.³⁰ This benchmark deliver high k -period CERs, specially over the long-run. Nevertheless, the k -period CERs of the horizon-specific portfolios remain comparable to those of the ‘Long’ benchmark. With 12 tracking portfolios ($J = 12$), the out-of-sample annualized CERs of one-, six-, and 12-month strategies are 6.53%, 5.53%, and 5.56%, respectively, surpassing those of all three benchmarks. Over 96 and 120 months, the constrained horizon-specific portfolios with $J = 12$ show out-of-sample CERs of 9.08% and 9.36%, respectively. Over these respective long horizons, the EW benchmark delivers CERs of 7.41% and 7.23%, and the ‘Long’ benchmark delivers CERs of

²⁹The out-of-sample utility gains from constrained relative to unconstrained portfolios appears across many other horizons longer than six years.

³⁰With non-negative weights, the short-leg anomalies in the LS benchmark now carry zero weights, as per [Eq. 17](#). The other two benchmarks—the equally weighted benchmark and the market portfolio—remain unchanged.

9.95% and 9.69%, respectively.

Table 7 documents the economic value of matching portfolio rules to investment horizons for the constrained portfolios. The results for the constrained portfolios are consistent with those documented in Table 2 for the unconstrained portfolios, such that strategies optimized for a given horizon h are often suboptimal for investors with horizons $k \neq h$. In-sample (out-of-sample), 96- and 120-period investors gain 4.13% (2.81%) and 4.22% (3.95%) higher annualized CERs by switching from a myopic strategy to their horizons' strategies. On the other hand, investors with one, six, and 12 months horizons expect to lose 2.88 (2.85), 4.54 (5.07), and 3.64 (4.32) percent, respectively by adopting the 120-period strategy in the in-sample (out-of-sample) estimates.

Figure 7 reports the k -period CERs under different horizon strategies for the constrained portfolios. For a given strategy (row), values report the annualized CER over investment horizons (columns). Similar to the unconstrained results in Figure 4, the k -period CERs tend to peak at or around the strategies' horizons. However, imposing the constraint result in less severe k -period CERs when evaluating long-horizon strategies over short horizons. For example, the 120-month constrained strategy yields annualized CERs of 4.03% and 1.70% when evaluated over one- and six-month horizons, respectively, compared to -1.52% and -9.37% under the unconstrained version. Similar patterns hold when short-horizon strategies are evaluated over the long run: although the long-horizon outcomes of the constrained short-horizon strategies are suboptimal, they are less severe than in the unconstrained case. These patterns reflect the fact that constraints limit the extent to which portfolios adapt to their target horizons. The short-sale constraint removes short positions, which differ across horizons in both size and asset composition. In addition, leverage is no longer available to magnify portfolio differences. Together, these constraints reduce variation in portfolio compositions across horizons, which results in fewer extreme outcomes when strategies are evaluated away from their target horizons.

Lastly, Table 8 summarizes the weight statistics for the constrained horizon-specific portfolios. Upon imposing the short-sale constraint, all portfolios show zero values for minimum weight and sum

and proportion of negative weights. Similar to the unconstrained case, sum of weights in risky assets increase with investment horizons such that long-horizon investors allocate a larger portion of wealth to equities. Overall, this section provides evidence for the robustness of the horizon-specific strategies under realistic short-sale and leverage constraints. The constrained portfolios offer superior k -period CERs compared to the benchmark strategies and the strategies remain horizon-specific, such that investors benefit from using horizon-matched strategies over those optimized for alternative horizons.

5 Conclusion

I propose a novel approach to multi-period portfolio choice with many assets that does not rely on modeling the dynamics of investment opportunities. Instead of solving the complex dynamic optimization problem, I characterize final investment payoffs through the systematic factor of holding-period returns and construct tradable factor-tracking portfolios that deliver these horizon-specific payoffs. A multi-period investor buys a utility-maximizing combination of her horizons' factor-tracking portfolios. The empirical results document superior performance for the horizon-specific strategies compared to benchmark portfolios, both in-sample and out-of-sample. More notably, I find compelling evidence for the economic significance of investment horizon in portfolio decisions. Investors with specific horizons benefit from following horizon-matched portfolio rules compared to strategies optimized for alternative horizons. The findings are robust under realistic short-sale and leverage constraints.

By focusing on final payoffs rather than intermediate returns, my approach sidesteps the computational complexities and estimation errors that typically plague dynamic portfolio optimization with large asset universes. This payoff-based approach lets the data speak to the differences in optimal policies across horizons: Strategies optimized for short horizons fail to deliver over the long run, while long-horizon strategies appear deeply unattractive when judged over short periods. In practice, the investment performance of many fund managers is evaluated based on short-term returns, creating incentives that encourage short-run gains and discourage horizon-consistent strategies. The

results demonstrate that investment horizon is not merely a theoretical input but a practical and economically important dimension of portfolio design with effects that are large, measurable, and directly usable in real-time.

Table 1: k -period CERs

The table presents the k -period (annualized) certainty-equivalent returns (CERs) for horizon-specific portfolios and benchmark strategies. The underlying assets are 230 extreme decile portfolios from Chen and Zimmermann dataset. The first four columns showcase the CERs for weights estimated using J number of factors. The columns labeled ‘EW’ and ‘LS’ invest an equal portion in all assets and in long-minus-short legs, respectively. ‘Mkt’ reports CERs from holding the market portfolio. The optimal policies are following $\hat{\mathbf{w}}_{k,t} = \hat{\mathbf{B}}_{k,t} \hat{\boldsymbol{\theta}}_k$ (Eq. 8). For each horizon k , the target horizon loadings ($\hat{\mathbf{B}}_{k,t}$) are estimated using an expanding window with an initial length of 60 k -month observations. The $\hat{\boldsymbol{\theta}}_k$ coefficients are solved numerically and are restricted to be between 0 and 1. The sample period spans from July 1963 to December 2023. For each horizon k , the effective in-sample (out-of-sample) window starts from $k + 59$ months ($2k + 118$ months) after the starting of the sample, where the first $\hat{\mathbf{B}}_{k,t}$ ($\hat{\boldsymbol{\theta}}_k$) becomes available. Investors have power utility over terminal wealth with $\gamma = 5$. The CERs are annualized from k -period CERs by $(1 + \text{CER}_k(\mathbf{w}_k))^{12/k} - 1$.

Panel A: In-Sample

k	Horizon-specific Portfolios				Benchmark Strategies		
	$J = 1$	$J = 6$	$J = 9$	$J = 12$	EW	LS	Mkt
1	5.48	11.57	13.78	15.47	-0.76	11.58	4.46
6	5.24	11.70	13.31	15.06	-3.69	11.32	3.32
12	5.39	10.44	12.54	13.77	-2.21	11.00	3.24
24	5.69	10.24	11.67	12.81	-0.59	10.31	3.41
36	6.45	10.81	11.54	13.44	2.88	9.69	4.52
60	7.30	12.53	14.04	14.27	6.43	8.43	6.99
96	9.49	11.92	12.39	13.92	8.11	7.19	6.93
120	10.16	13.09	16.73	17.27	8.25	6.70	5.79

Panel B: Out-of-Sample

k	Horizon-specific Portfolios				Benchmark Strategies		
	$J = 1$	$J = 6$	$J = 9$	$J = 12$	EW	LS	Mkt
1	5.15	11.10	13.20	14.82	0.94	11.37	5.19
6	4.81	10.91	12.47	14.14	-0.91	11.00	5.24
12	4.91	9.63	11.39	12.44	0.03	10.65	4.90
24	5.07	8.27	8.79	10.82	1.09	9.81	4.79
36	5.53	9.40	9.77	11.32	4.05	8.95	5.15
60	4.05	8.71	9.39	8.43	5.30	7.36	6.11
96	8.63	9.81	7.91	8.71	7.41	5.96	5.75
120	8.90	9.35	12.90	11.87	7.23	5.31	4.42

Table 2: CER Gains of Following Horizon-Specific Portfolio Rules

The table shows differences in (annualized) certainty equivalent returns (ΔCERs) between horizon-matched and mismatched portfolio rules. For each investment horizon k (columns), entries report $\text{CER}_k(\mathbf{w}_k) - \text{CER}_k(\mathbf{w}_h)$, where h is the strategy horizon (rows). For example, the top-right entry in Panel A (4.31%) indicates that a 120-month investor earns 4.31% higher annualized in-sample CER by following the 120-period rule instead of the one-period rule. Managed strategies are constructed using six factor-tracking portfolios (i.e., $J = 6$). The sample period spans from July 1963 to December 2023, and the in-sample (out-of-sample) CERs are for a common period starting in June 1978 (May 1993), when the longest horizon strategy ($k = 120$) first becomes available. Investors have power utility over k -period wealth with $\gamma = 5$.

Panel A: In-Sample

Strategy Horizon	Investment Horizon							
	1	6	12	24	36	60	96	120
1		0.53	-0.07	0.04	1.14	2.89	2.76	4.31
6	-0.11		-0.77	-0.42	1.10	2.74	2.75	4.29
12	0.99	0.90		0.05	1.43	2.94	2.85	4.38
24	0.40	0.55	-0.22		1.56	3.19	2.96	4.43
36	-0.04	0.25	-0.77	-1.04		1.79	2.01	3.60
60	0.94	1.17	-0.45	-1.07	-0.86		-0.97	0.50
96	3.66	5.74	4.13	2.13	1.48	2.17		0.84
120	13.65	20.74	18.77	17.11	10.43	5.18	-0.71	

Panel B: Out-of-Sample

Strategy Horizon	Investment Horizon							
	1	6	12	24	36	60	96	120
1		0.10	0.01	-1.52	0.35	1.92	2.47	2.61
6	0.28		-0.25	-1.38	1.10	2.35	2.87	2.92
12	0.83	0.29		-1.40	1.14	2.85	3.87	4.03
24	0.69	0.79	0.80		2.41	3.30	3.19	3.25
36	1.26	0.82	0.20	-1.98		1.54	2.63	2.85
60	0.38	0.43	-0.32	-2.29	-1.12		-0.01	0.18
96	5.03	4.31	2.42	-1.29	-0.66	-0.12		0.06
120	24.52	39.42	33.08	26.38	17.11	7.39	0.37	

Table 3: Weight Statistics of Horizon-Specific Portfolios

This table reports the summary statistics of the asset-level weights ($\hat{\mathbf{w}}_{k,t}$) for the horizon-specific portfolios. The weight variables are time-series averages of the cross-sectional statistics (multiplied by 100), including sum of absolute weights (leverage), maximum weight, minimum weight, sum of negative weights, fraction of negative weights, and sum of weights. The sample period spans from July 1963 to December 2023, and the in-sample (out-of-sample) statistics are for a common period starting in May 1978 (April 1993), when the longest horizon portfolio weight ($k = 120$) first becomes available.

Panel A: In-Sample								
Variable	Strategy Horizon							
	1	6	12	24	36	60	96	120
$\sum w_i $	168.21	179.24	171.95	177.07	185.95	180.35	169.10	186.91
$\max w_i$	2.45	2.43	2.66	3.64	4.62	5.48	7.19	7.06
$\min w_i$	-3.85	-3.53	-3.33	-3.11	-2.71	-2.61	-2.55	-2.95
$\sum w_i I(w_i < 0)$	-62.73	-73.69	-69.09	-66.58	-66.50	-53.82	-35.71	-33.73
$\sum I(w_i < 0)/N$	38.00	43.20	42.25	42.42	43.75	40.06	34.35	28.85
$\sum w_i$	42.74	31.86	33.77	43.92	52.95	72.71	97.69	119.45
Panel B: Out-of-Sample								
	1	6	12	24	36	60	96	120
$\sum w_i $	171.88	176.01	165.56	181.92	168.44	163.87	133.18	206.46
$\max w_i$	2.45	2.28	2.64	3.78	4.57	5.90	6.45	6.46
$\min w_i$	-4.28	-4.05	-3.42	-3.13	-2.99	-1.97	-1.71	-3.15
$\sum w_i I(w_i < 0)$	-65.78	-69.54	-67.74	-66.92	-60.76	-41.20	-24.45	-25.38
$\sum I(w_i < 0)/N$	37.98	41.11	43.24	42.47	45.52	36.59	29.92	21.19
$\sum w_i$	40.33	36.92	30.08	48.09	46.91	81.47	84.28	155.70

Table 4: Statistical Properties of Strategies Across Different Investment Horizons (In-Sample)

This table shows the statistical properties of various strategies' returns across different investment horizons. The strategies include the horizon-specific portfolios and two respective benchmarks. The reported statistics are estimated over the investment horizons (columns) and include (annualized) mean return, (annualized) standard deviation, variance ratio, skewness, and kurtosis. The sample period spans from July 1963 to December 2023, and the statistics are for a common period starting in June 1978, when the in-sample portfolio of the longest horizon strategy ($k = 120$) first becomes available.

Strategies	Investment Horizon							
	1	6	12	24	36	60	96	120
Panel A: Mean								
1-month	14.01	14.20	14.32	14.45	14.52	14.55	14.54	14.67
6-month	13.88	13.99	14.13	14.39	14.51	14.36	14.17	14.19
12-month	12.69	12.76	12.89	13.01	13.03	12.84	12.70	12.75
24-month	13.30	13.37	13.50	13.67	13.83	13.91	13.94	13.97
36-month	15.06	15.26	15.44	15.55	15.62	15.66	15.50	15.65
60-month	17.90	18.06	18.12	17.87	17.72	17.30	16.86	16.79
96-month	18.61	19.18	19.41	18.97	18.69	18.56	18.49	18.75
120-month	21.07	22.20	23.06	21.49	20.06	18.96	17.40	17.30
EW	13.11	13.85	13.91	13.31	13.23	13.47	11.48	10.74
LS	11.34	11.44	11.58	11.84	12.03	12.44	12.57	12.31
Panel B: Standard Deviation								
1-month	8.27	12.08	13.86	15.79	18.18	26.63	40.01	53.21
6-month	7.71	10.41	11.87	15.03	18.27	25.85	37.47	48.63
12-month	7.38	9.80	10.92	12.53	14.45	20.07	28.43	36.41
24-month	7.41	10.01	11.39	14.01	17.26	26.04	40.90	52.49
36-month	10.11	13.40	14.86	17.27	20.52	30.44	44.13	58.00
60-month	15.11	18.69	20.22	20.59	21.22	29.33	42.59	57.15
96-month	18.47	23.56	26.53	26.91	28.98	43.49	75.54	115.63
120-month	26.25	35.15	45.56	40.22	33.14	43.86	42.82	51.84
EW	20.91	24.69	26.27	23.47	22.55	31.89	29.14	26.42
LS	3.53	5.26	7.12	10.62	13.97	20.93	34.28	43.36

Table 4: (continued)

Strategies	Investment Horizon							
	1	6	12	24	36	60	96	120
Panel C: Variance Ratio								
1-month	1.00	1.84	2.13	2.32	2.36	3.06	3.39	3.66
6-month	1.00	1.61	1.84	2.36	2.69	3.30	3.57	3.82
12-month	1.00	1.53	1.68	1.91	2.05	2.50	2.65	2.84
24-month	1.00	1.63	1.88	2.31	2.70	3.60	4.18	4.46
36-month	1.00	1.50	1.57	1.65	1.71	2.21	2.33	2.47
60-month	1.00	1.24	1.17	1.00	0.81	0.82	0.71	0.70
96-month	1.00	1.34	1.33	1.09	0.90	1.02	0.96	0.96
120-month	1.00	1.25	1.30	1.01	0.62	0.51	0.22	0.19
EW	1.00	1.20	1.13	0.81	0.57	0.60	0.31	0.21
LS	1.00	2.01	3.30	5.90	8.27	12.43	18.25	20.18
Panel D: Skewness								
1-month	0.30	0.80	0.66	0.00	0.08	0.11	-0.08	-0.07
6-month	0.29	0.62	0.51	0.21	0.22	0.16	-0.08	-0.19
12-month	-0.58	0.76	0.66	-0.13	-0.21	-0.08	-0.22	-0.26
24-month	-0.71	0.05	0.05	-0.12	0.02	0.25	0.24	0.04
36-month	-0.02	0.66	0.76	0.39	0.50	0.37	0.06	-0.03
60-month	-0.10	0.84	1.23	0.42	0.14	0.35	0.35	0.42
96-month	-0.28	0.65	1.16	0.50	0.56	0.64	0.96	1.36
120-month	0.07	2.21	3.86	1.87	0.71	1.15	0.52	0.20
EW	-0.40	0.59	0.99	0.46	0.52	1.04	0.92	0.62
LS	0.70	0.47	0.33	0.22	0.13	-0.09	-0.15	-0.13
Panel E: Kurtosis								
1-month	5.59	4.79	4.02	2.35	2.39	2.02	1.52	1.64
6-month	7.31	5.52	4.43	3.12	2.69	1.98	1.61	1.40
12-month	6.94	6.48	5.43	3.25	2.54	1.96	1.49	1.49
24-month	6.64	3.45	2.94	2.55	2.30	1.99	1.66	1.36
36-month	7.05	3.84	4.04	2.51	2.45	1.99	1.67	1.77
60-month	7.78	5.11	6.41	3.32	2.53	2.41	2.11	2.32
96-month	5.78	4.51	5.81	2.94	3.01	2.49	2.70	3.97
120-month	10.01	14.03	29.68	11.13	4.67	6.28	3.60	2.44
EW	5.46	4.71	5.31	3.67	3.27	4.17	3.35	3.46
LS	5.04	2.79	2.48	2.15	1.96	1.70	1.51	1.46

Table 5: Statistical Properties of Strategies Across Different Investment Horizons (Out-of-Sample)

This table shows the statistical properties of various strategies' returns across different investment horizons. The strategies include the horizon-specific portfolios and two respective benchmarks. The reported statistics are estimated over the investment horizons (columns) and include (annualized) mean return, (annualized) standard deviation, variance ratio, skewness, and kurtosis. The sample period spans from July 1963 to December 2023, and the statistics are for a common period starting in May 1993, when the out-of-sample portfolio of the longest horizon strategy ($k = 120$) first becomes available.

Strategies	Investment Horizon							
	1	6	12	24	36	60	96	120
Panel A: Mean								
1-month	11.37	11.52	11.67	11.91	12.03	11.73	11.83	11.68
6-month	10.80	10.90	11.01	11.18	11.12	10.57	10.54	10.27
12-month	9.94	9.97	10.14	10.39	10.41	10.06	10.09	9.81
24-month	10.51	10.54	10.67	10.87	10.98	10.39	9.91	9.83
36-month	10.07	10.10	10.22	10.47	10.69	10.76	10.97	10.88
60-month	15.96	15.99	16.12	16.11	15.84	14.69	13.97	13.86
96-month	14.73	14.90	15.26	14.83	14.39	13.19	12.41	12.60
120-month	21.62	23.36	24.32	22.38	20.42	18.76	17.00	17.13
EW	11.55	11.91	12.29	11.76	11.70	11.18	10.33	9.33
LS	8.55	8.68	8.80	9.11	9.35	9.87	9.94	8.57
Panel B: Standard Deviation								
1-month	8.95	12.95	15.07	17.28	19.90	28.56	42.99	54.50
6-month	8.23	11.06	12.55	14.95	17.23	22.50	32.82	40.05
12-month	7.51	10.17	12.03	14.29	16.70	23.44	35.78	43.47
24-month	8.37	11.22	13.12	15.86	18.78	24.67	34.44	44.32
36-month	8.75	11.31	12.46	13.92	16.23	24.28	35.85	42.65
60-month	15.93	19.55	22.50	22.73	23.48	29.50	43.59	63.40
96-month	19.44	23.61	27.94	25.70	23.82	26.73	31.61	45.83
120-month	33.19	44.11	56.64	50.75	44.19	57.31	67.78	92.27
EW	21.33	24.76	27.11	24.29	20.96	23.83	23.86	22.26
LS	3.55	4.93	6.50	9.67	12.68	18.94	29.77	32.51

Table 5: (continued)

Strategies	Investment Horizon							
	1	6	12	24	36	60	96	120
Panel C: Variance Ratio								
1-month	1.00	1.81	2.18	2.45	2.46	3.05	3.42	3.45
6-month	1.00	1.62	1.89	2.32	2.50	2.75	3.00	2.90
12-month	1.00	1.59	2.00	2.49	2.79	3.58	4.38	4.35
24-month	1.00	1.68	2.08	2.61	2.94	3.19	2.94	3.01
36-month	1.00	1.48	1.59	1.64	1.71	2.37	2.86	2.92
60-month	1.00	1.25	1.31	1.15	0.97	0.95	0.87	0.92
96-month	1.00	1.16	1.20	0.89	0.63	0.51	0.37	0.42
120-month	1.00	1.28	1.30	1.07	0.69	0.58	0.32	0.30
EW	1.00	1.20	1.20	0.88	0.55	0.45	0.25	0.18
LS	1.00	1.76	2.77	5.02	7.11	11.01	15.60	14.04
Panel D: Skewness								
1-month	0.54	1.05	0.90	0.34	0.65	0.92	0.79	1.01
6-month	0.02	0.66	0.53	0.12	0.18	0.52	0.62	0.91
12-month	-0.07	1.14	0.96	0.24	0.29	0.61	0.71	0.95
24-month	-0.63	-0.16	0.04	-0.02	0.14	0.54	1.01	1.28
36-month	-0.11	0.70	0.92	0.62	0.83	0.99	0.75	0.59
60-month	0.15	0.95	1.43	0.57	0.32	0.31	0.74	1.33
96-month	0.14	1.32	1.98	1.10	0.79	0.83	0.74	1.27
120-month	-0.12	1.74	3.12	1.52	1.01	1.00	0.80	0.89
EW	-0.16	0.70	1.10	0.63	0.41	0.56	0.58	0.49
LS	0.96	1.06	0.90	0.78	0.66	0.38	0.40	0.86
Panel E: Kurtosis								
1-month	5.14	5.27	4.27	2.29	2.88	3.02	1.99	2.61
6-month	5.95	6.25	4.59	2.95	2.42	2.39	1.72	2.39
12-month	6.10	6.75	4.88	2.32	2.02	2.20	1.78	2.42
24-month	5.35	4.31	3.40	3.06	2.59	2.28	2.41	2.99
36-month	5.36	4.40	4.85	2.96	2.88	2.90	2.09	1.79
60-month	6.60	5.32	6.41	3.28	2.41	2.04	2.26	3.69
96-month	6.48	5.99	8.31	4.08	3.10	3.15	2.36	3.86
120-month	7.58	9.70	19.74	7.74	5.46	4.73	2.48	2.56
EW	4.85	5.63	5.76	4.28	3.69	3.34	2.49	2.35
LS	5.68	4.04	3.26	2.71	2.23	1.53	1.48	2.24

Table 6: k -period CERs (Constrained Portfolios)

The table presents the k -period (annualized) certainty-equivalent returns (CERs) for horizon-specific portfolios and benchmark strategies. The underlying assets are 230 extreme decile portfolios from Chen and Zimmermann dataset. The first four columns showcase the CERs for weights estimated using J number of factors. The columns labeled ‘EW’ and ‘Long’ invest an equal portion in all assets and in the long-legs of anomaly portfolios, respectively. ‘Mkt’ reports CERs from holding the market portfolio. The optimal policies are following $\hat{\mathbf{w}}_{k,t} = \hat{\mathbf{B}}_{k,t}\hat{\boldsymbol{\theta}}_k$ (Eq. 8). For each horizon k , the target horizon loadings ($\hat{\mathbf{B}}_{k,t}$) are estimated using an expanding window with an initial length of 60 k -month observations, and are restricted to have non-negative values. The $\hat{\boldsymbol{\theta}}_k$ coefficients are solved numerically and are restricted to be between 0 and 1. Portfolio rules have non-negative constraint ($\hat{w}_{i,k,t} \geq 0$) and leverage constraint ($\sum_{i=1}^N |\hat{w}_{i,k,t}| \leq 1$). The sample period spans from July 1963 to December 2023. For each horizon k , the effective in-sample (out-of-sample) window starts from $k + 59$ months ($2k + 118$ months) after the starting of the sample, where the first $\hat{\mathbf{B}}_{k,t}$ ($\hat{\boldsymbol{\theta}}_k$) becomes available. Investors have power utility over terminal wealth with $\gamma = 5$. The CERs are annualized from k -period CERs by $(1 + \text{CER}_k(\mathbf{w}_k))^{12/k} - 1$.

Panel A: In-Sample

k	Horizon-specific Portfolios				Benchmark Strategies		
	$J = 1$	$J = 6$	$J = 9$	$J = 12$	EW	Long	Mkt
1	5.48	6.70	6.70	6.70	-0.76	3.52	4.46
6	5.24	5.96	5.96	5.96	-3.69	0.56	3.32
12	5.39	6.06	6.06	6.06	-2.21	1.88	3.24
24	5.69	6.67	6.67	6.67	-0.59	2.90	3.41
36	6.45	7.81	7.81	7.81	2.88	6.24	4.52
60	7.30	9.95	9.95	9.95	6.43	9.03	6.99
96	9.49	10.72	10.69	10.57	8.11	10.93	6.93
120	10.15	10.54	11.02	11.02	8.25	10.99	5.79

Panel B: Out-of-Sample

k	Horizon-specific Portfolios				Benchmark Strategies		
	$J = 1$	$J = 6$	$J = 9$	$J = 12$	EW	Long	Mkt
1	5.15	6.53	6.54	6.53	0.94	5.21	5.19
6	4.81	5.53	5.53	5.53	-0.91	3.23	5.24
12	4.91	5.56	5.54	5.54	0.03	3.93	4.90
24	5.07	4.62	4.62	4.62	1.09	4.21	4.79
36	5.53	7.02	7.02	6.95	4.05	7.01	5.15
60	4.05	6.39	5.62	5.62	5.30	7.81	6.11
96	8.63	8.40	8.73	9.08	7.41	9.95	5.75
120	8.90	9.00	9.58	9.36	7.23	9.69	4.42

Table 7: CER Gains of Following Horizon-specific Portfolio Rules (Constrained Portfolios)

The table shows (annualized) differences in certainty equivalent returns (ΔCERs) between horizon-matched and mismatched portfolio rules. Portfolio rules have non-negative constraint ($\hat{w}_{i,k,t} \geq 0$) and leverage constraint ($\sum_{i=1}^N |\hat{w}_{i,k,t}| \leq 1$). For each investment horizon k (columns), entries report $\text{CER}_k(\mathbf{w}_k) - \text{CER}_k(\mathbf{w}_h)$, where h is the strategy horizon (rows). The sample period spans from July 1963 to December 2023, and the in-sample (out-of-sample) CERs are for a common period starting in June 1978 (May 1993), when the longest horizon strategy ($k = 120$) first becomes available.

Panel A: In-Sample								
Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.14	-0.26	0.04	1.11	2.84	4.13	4.22
6	0.33		-0.10	0.27	1.49	3.15	4.46	4.50
12	0.42	0.14		0.33	1.46	3.14	4.46	4.52
24	0.03	-0.07	-0.21		0.90	2.53	3.64	3.66
36	0.47	0.63	0.13	-0.24		1.59	2.68	2.72
60	0.92	1.30	0.39	-0.48	-1.09		0.03	-0.07
96	2.42	4.17	3.31	2.37	0.77	0.79		-0.33
120	2.88	4.54	3.64	2.76	0.82	0.97	0.25	

Panel B: Out-of-Sample								
Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.55	-0.79	-2.43	0.34	0.40	2.81	3.95
6	0.76		-0.18	-1.61	1.45	1.53	4.02	5.21
12	0.76	0.18		-1.47	1.39	1.44	3.86	5.04
24	0.75	1.34	1.30		2.17	1.65	3.55	4.72
36	0.67	0.31	0.04	-2.00		-0.01	2.38	3.53
60	1.18	2.76	2.45	0.71	1.25		0.81	1.93
96	2.97	5.96	5.52	3.34	2.66	0.23		0.89
120	2.85	5.07	4.32	1.68	0.56	-1.24	-0.86	

Table 8: Weight Statistics of Horizon-Specific Portfolios (Constrained Portfolios)

This table reports the summary weight statistics of the horizon-specific portfolios. The weight variables are time-series averages of the cross-sectional statistics (multiplied by 100), including sum of absolute weights (leverage), maximum weight, minimum weight, sum of negative weights, fraction of negative weights, and sum of weights. Portfolio rules have non-negative constraint ($\hat{w}_{i,k,t} \geq 0$) and leverage constraint ($\sum_{i=1}^N |\hat{w}_{i,k,t}| \leq 1$). The sample period spans from July 1963 to December 2023, and the in-sample (out-of-sample) statistics are for a common period starting in May 1978 (April 1993), when the longest horizon portfolio weights ($k = 120$) first becomes available.

Panel A: In-Sample								
Variable	Strategy Horizon							
	1	6	12	24	36	60	96	120
$\sum w_i $	40.48	33.44	35.88	44.23	67.09	88.95	100.00	99.96
$\max w_i$	1.45	1.19	0.75	1.66	2.15	3.53	3.98	4.15
$\min w_i$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
$\sum w_i I(w_i < 0)$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sum I(w_i < 0)/N$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sum w_i$	40.48	33.44	35.88	44.23	67.09	88.95	100.00	99.96

Panel B: Out-of-Sample								
	1	6	12	24	36	60	96	120
$\sum w_i $	39.81	31.92	36.80	54.07	54.32	86.60	97.26	100.00
$\max w_i$	1.46	0.93	1.09	1.67	2.29	3.79	4.25	4.03
$\min w_i$	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
$\sum w_i I(w_i < 0)$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sum I(w_i < 0)/N$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sum w_i$	39.81	31.92	36.80	54.07	54.32	86.60	97.26	100.00

Panel A: In-Sample

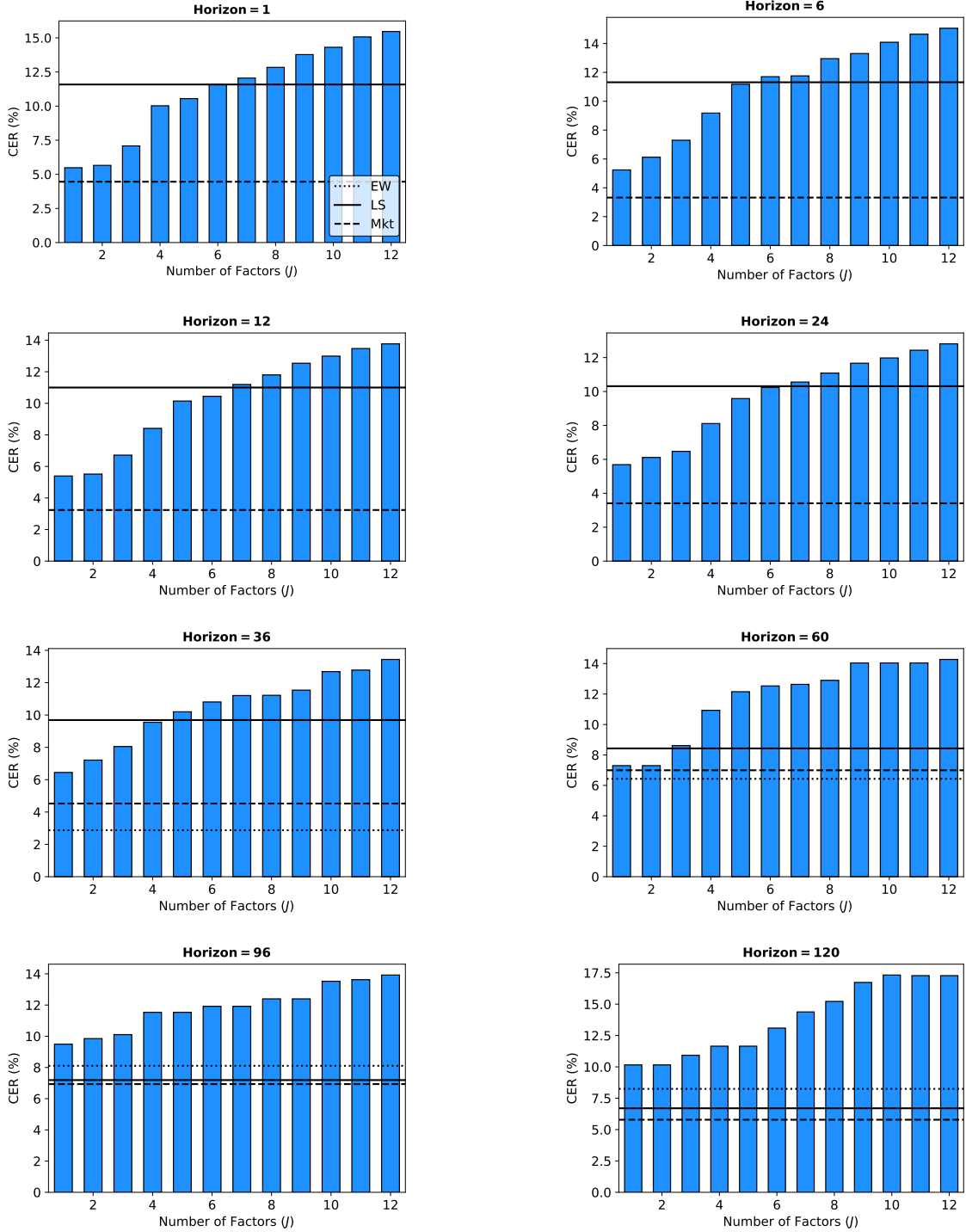


Figure 1: CERs Across Different Number of Factors

This figure presents the (annualized) k -period in-sample (Panel A) and out-of-sample (Panel B) certainty-equivalent returns (CER) under varying number of factors (J). The dotted lines show the CER of the equally weighted (EW) portfolio, the solid lines show the CER of the long-minus-short (LS) portfolio, and the dashed lines show the CER of the market portfolio.

Panel B: Out-of-Sample

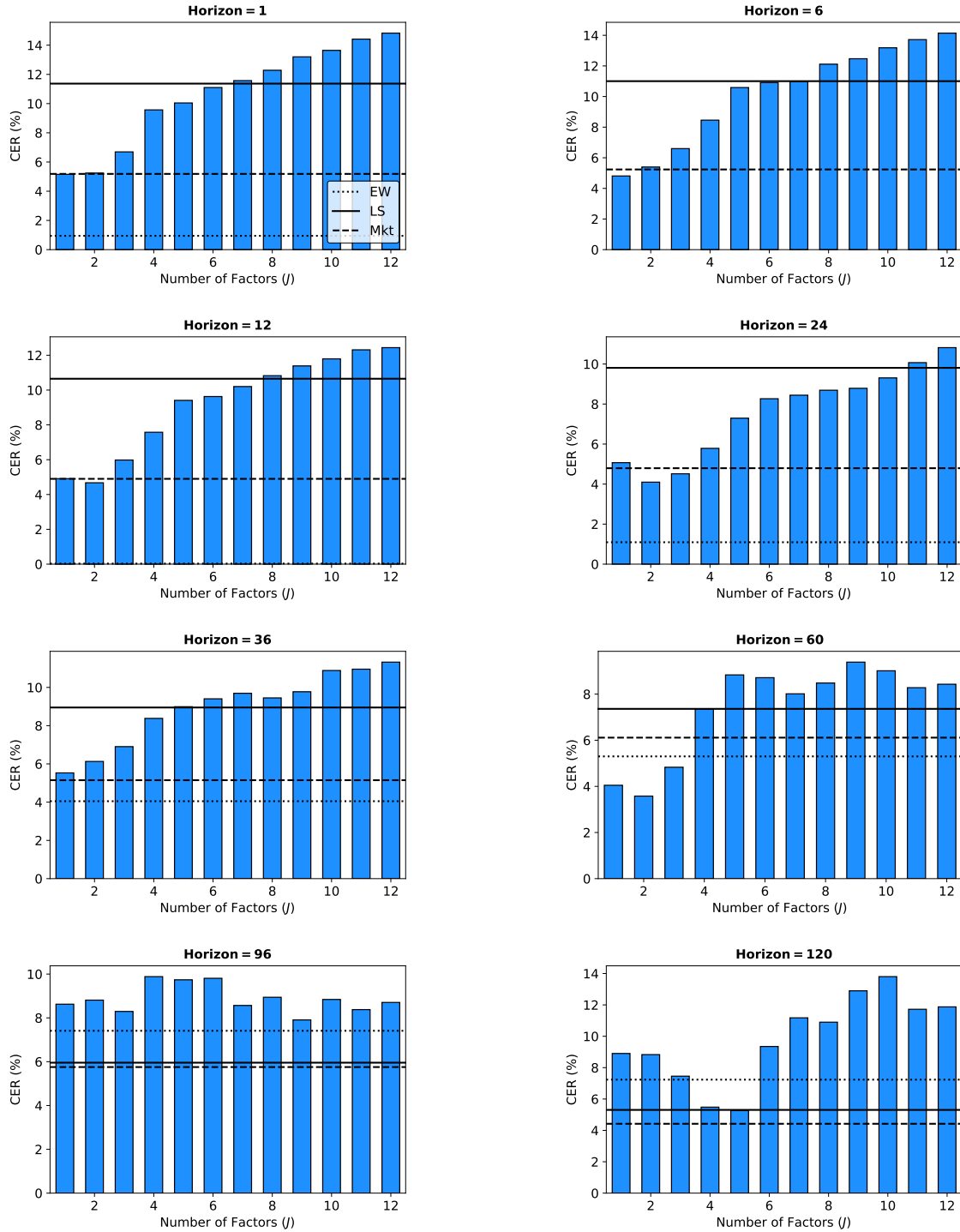


Figure 1 (continued): CERs Across Different Number of Factors

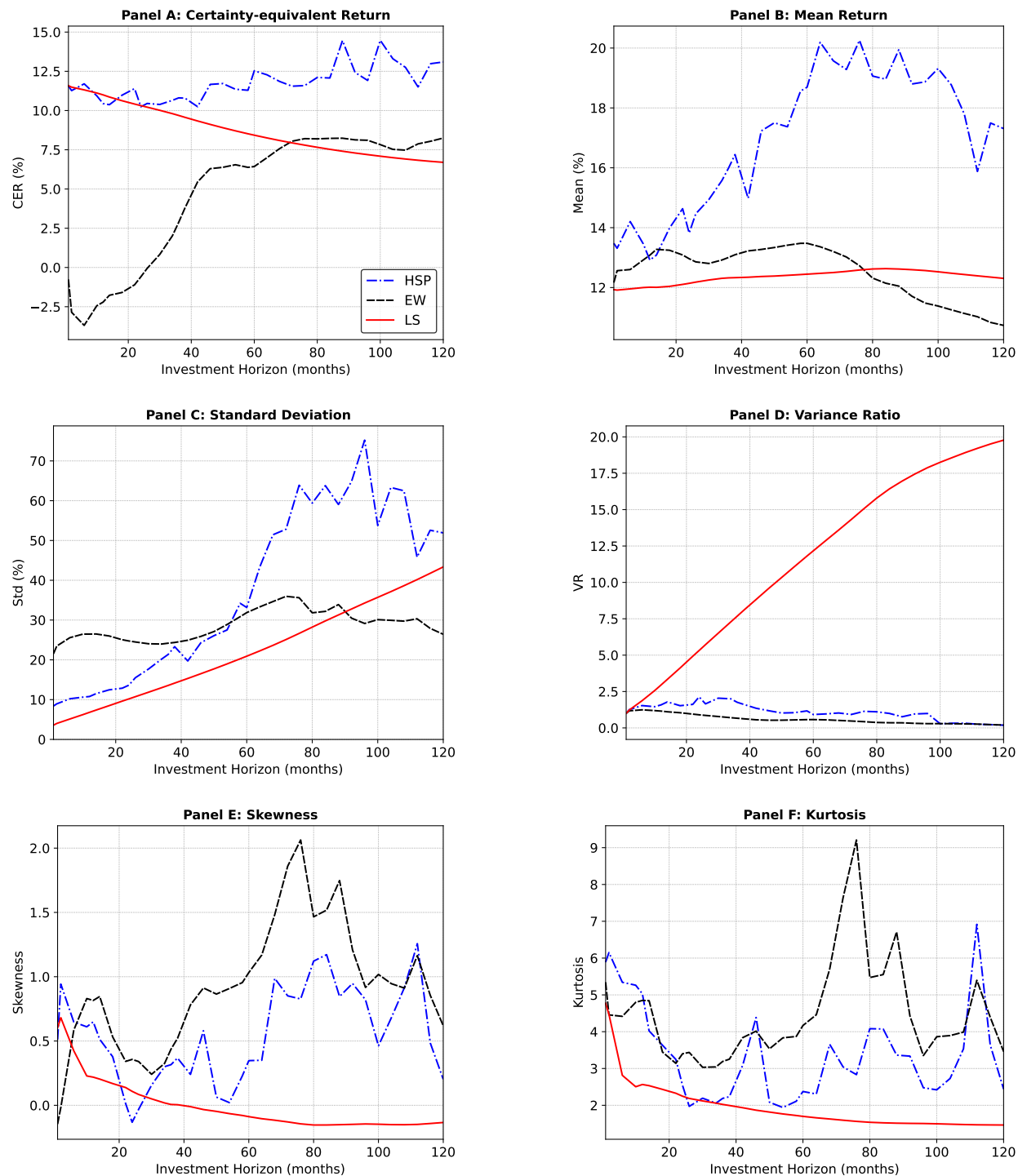
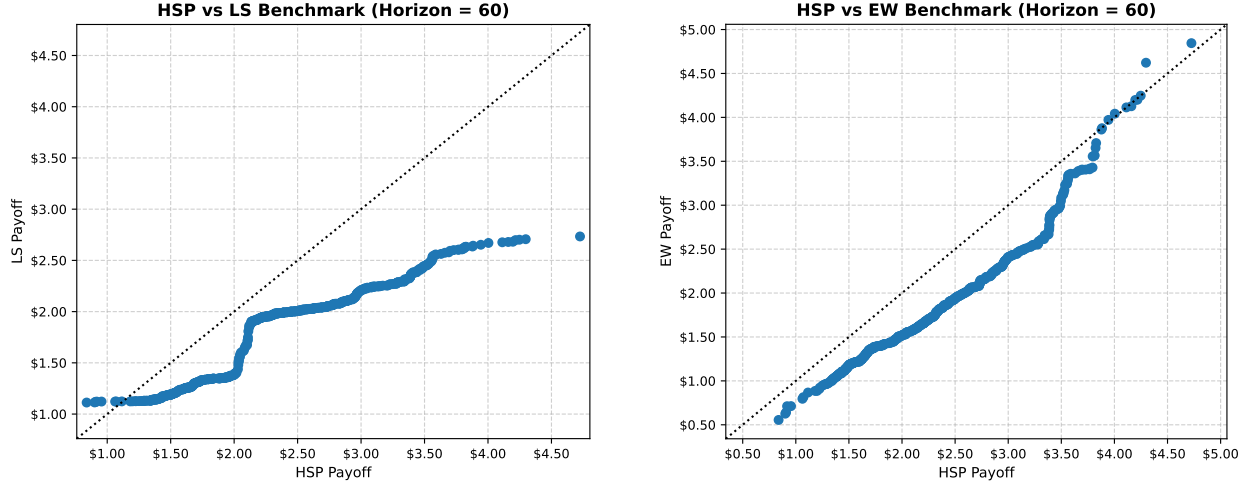


Figure 2: Horizon-specific vs. Benchmark Portfolios (In-Sample)

This figure shows the in-sample properties of the horizon-specific portfolios (HSP) and the two benchmark portfolios across different investment horizons. The reported statistics are estimated over the investment horizons and include A:(annualized) certainty-equivalent return, B: (annualized) mean return, C: (annualized) standard deviation, D: variance ratio, E: skewness, and F: Kurtosis. The sample period spans from July 1963 to December 2023 and the effective in-sample window starts from $k + 59$ months after the starting of the sample sample.

Panel A: $k = 60$



Panel B: $k = 120$

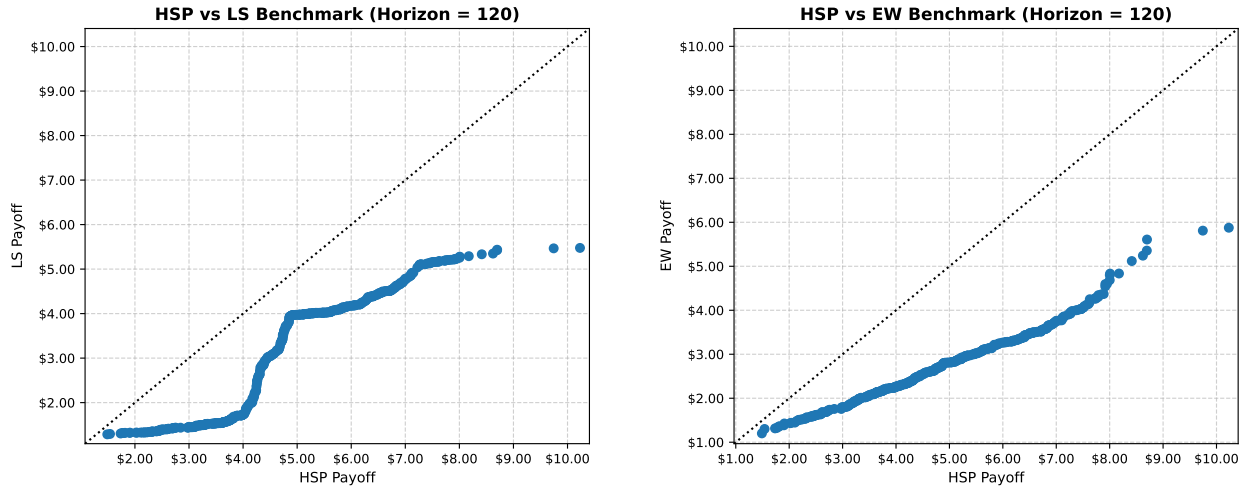


Figure 3: Q-Q Plots of Long-run Payoffs: HSP vs. Benchmarks (In-Sample)

This figure presents Q-Q plots comparing the empirical distributions of in-sample 60-period payoffs (Panel A) and 120-period payoffs (Panel B). In each subplot, the x -axis shows the sorted k -period payoffs from horizon-specific portfolio (HSP), and the y -axis shows the sorted k -period benchmark payoffs. Each point represents a matched order statistic—i.e., the i -th smallest HSP payoff against the i -th smallest benchmark payoff. Each plot is based on 548 observations for $k = 60$ and 428 observations for $k = 120$. The 45-degree reference line indicates equality in distributions. Points below (above) the line suggest HSP outperforms (underperforms) the benchmark at that part of the distribution. The sample period spans from July 1963 to December 2023 and the effective in-sample window starts from $k + 59$ months after the starting of the sample.

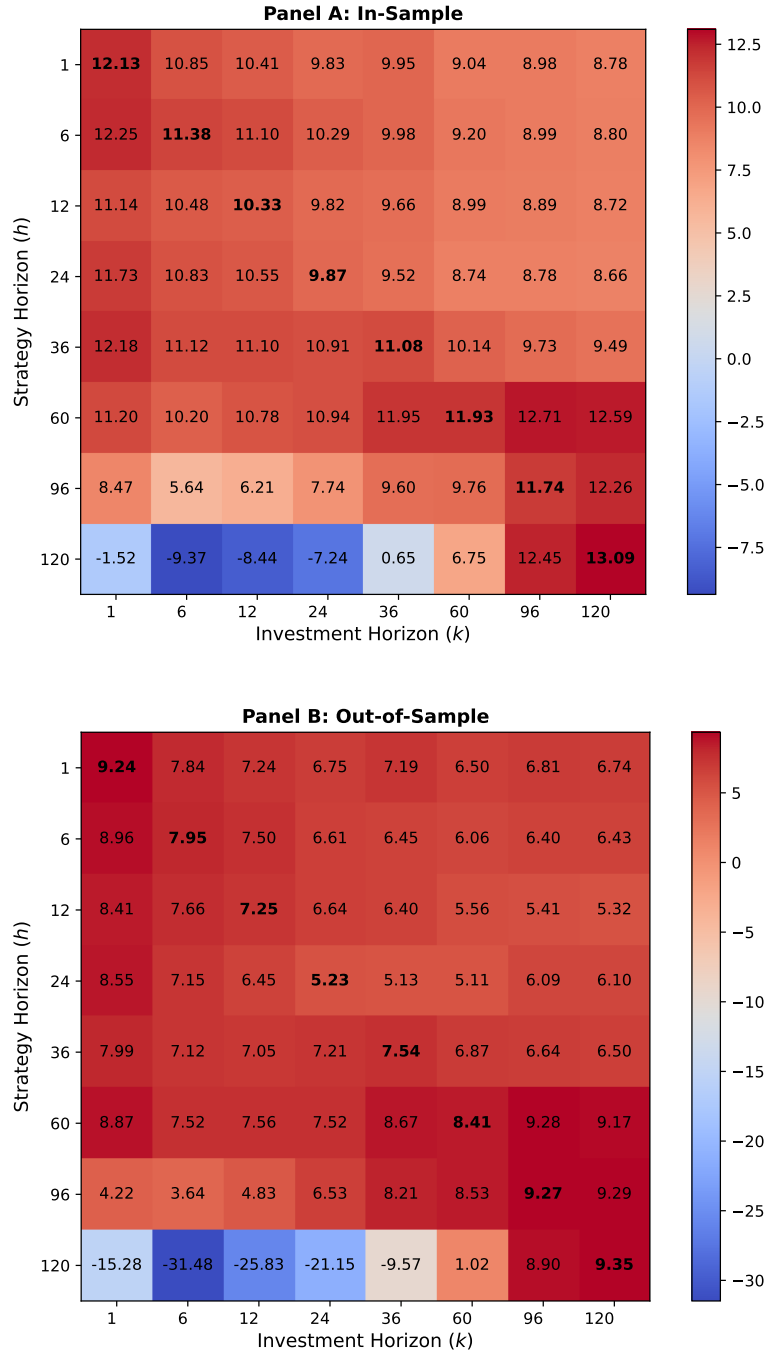


Figure 4: CERs of Horizon-Specific Portfolios over Alternative Investment Horizons

This figure shows (annualized) certainty equivalent returns (CER) from following different horizon strategies over alternative investment horizons. For each investment horizon k (columns), entries report $CER_k(\mathbf{w}_h)$, where h is the strategy horizon (rows). For example, the bottom-left entry in the in-sample panel (-1.52%) indicates that a strategy optimized for 120-period investors attains annualized CER of -1.52% over one-period (monthly) horizon. The diagonal values (in bold) showcase the (annualized) CER of the strategies at the target investment horizons. The sample period spans from July 1963 to December 2023, and the in-sample (out-of-sample) CERs are for a common period starting in June 1978 (May 1993), when the longest horizon strategy ($k = 120$) first becomes available.

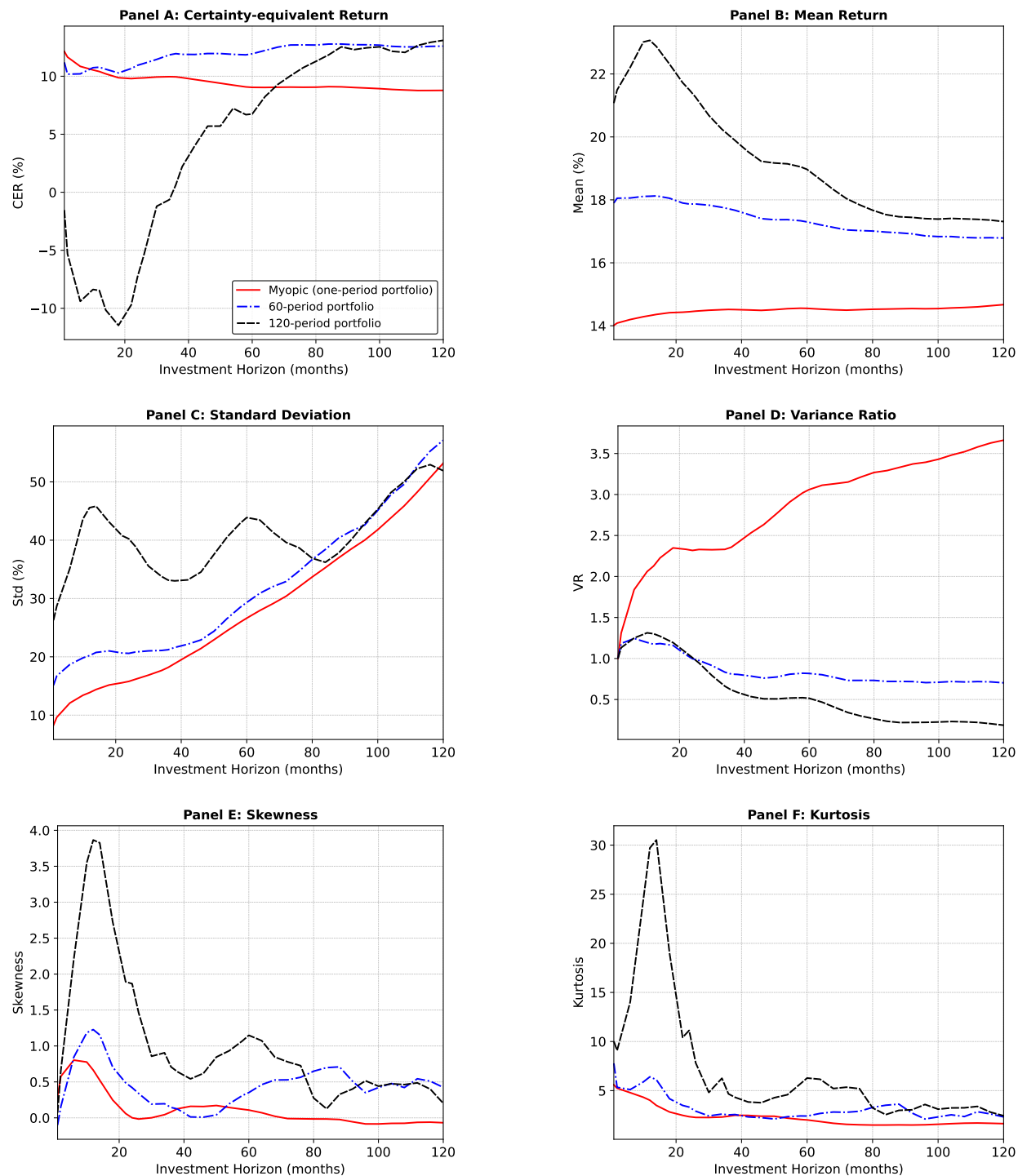


Figure 5: Myopic vs. Longer-horizon Portfolios

This figure shows the in-sample properties the one-period (myopic), 60-period, and 120-period portfolios. The reported statistics are estimated over the investment horizons and include A: (annualized) certainty-equivalent return, B: (annualized) mean return, C: (annualized) standard deviation, D: variance ratio, E: skewness, and F: Kurtosis. The sample period spans from July 1963 to December 2023, and the statistics are for a common period starting in May 1978, when the in-sample portfolio of the longest horizon strategy ($k = 120$) first becomes available.

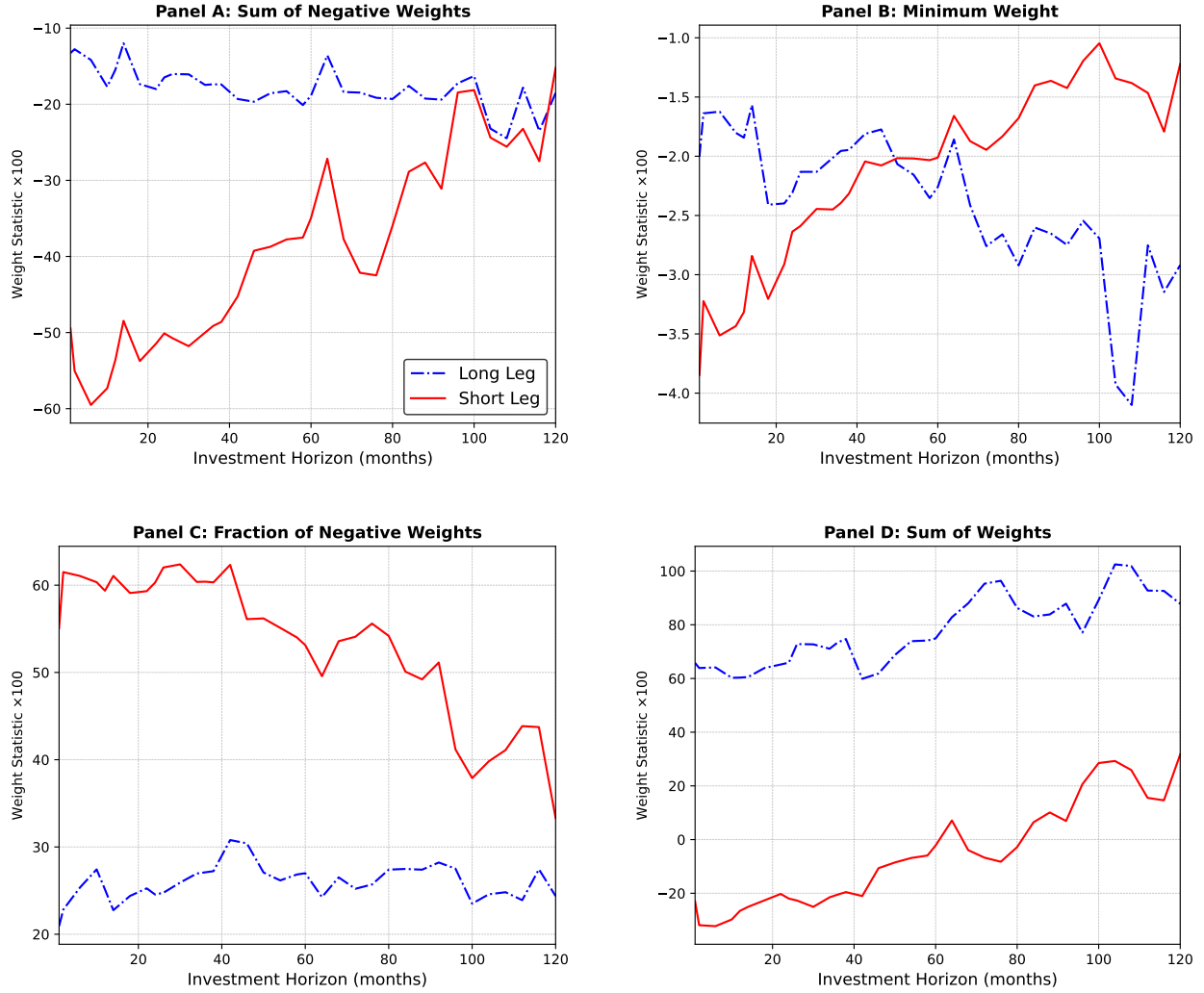


Figure 6: Long- and Short-leg Anomalies in Horizon-Specific Portfolios

This figure plots the in-sample weight statistics of long- and short-leg anomalies in optimal in-sample portfolios. The statistics are time series averages of cross-sectional variables. The weight statistics include: A: sum of negative weights, B: minimum weight, C: fraction of negative weights, and D: sum of weights. The sample period spans from July 1963 to December 2023, and the weight statistics are for a common period starting in May 1978, when the longest horizon portfolio weight ($k = 120$) first becomes available.

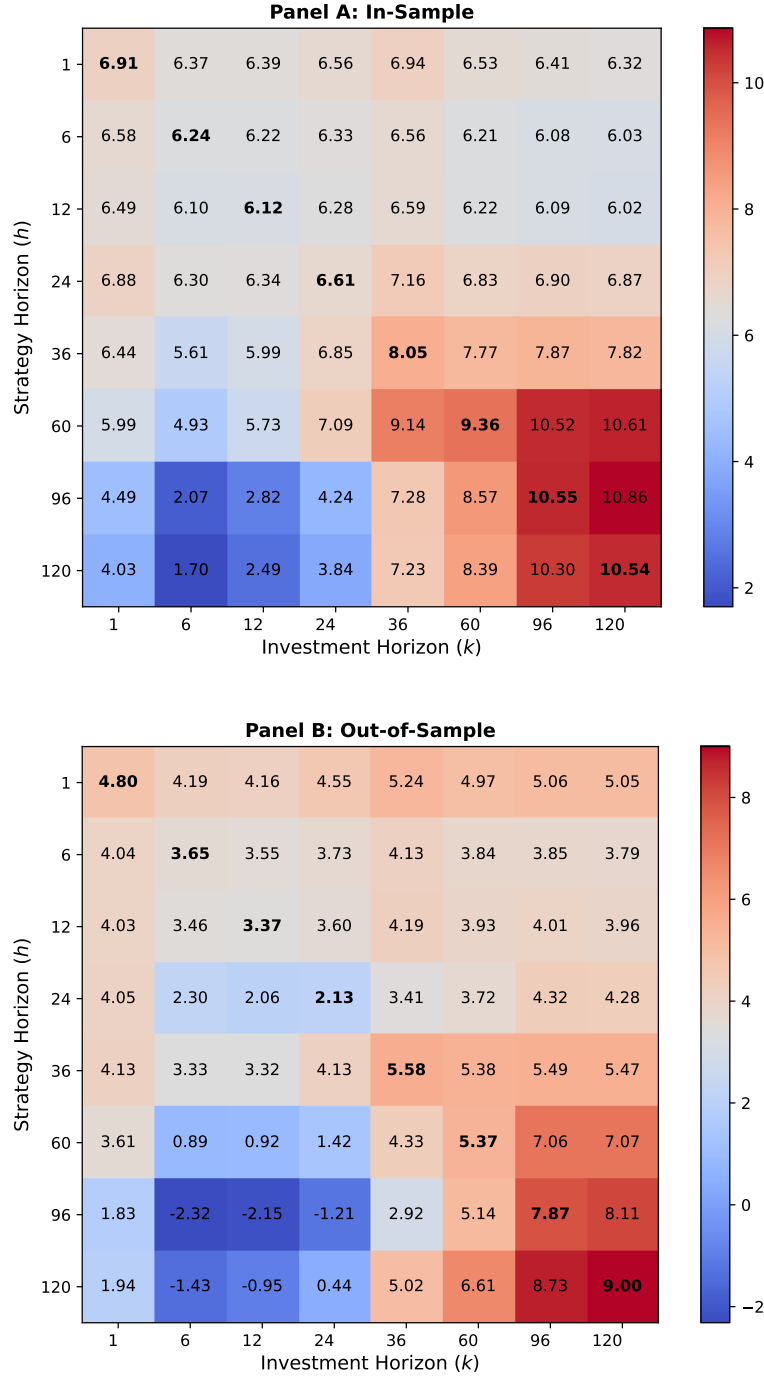


Figure 7: CERs of Horizon-specific Portfolios over Alternative Investment Horizons (Constrained Portfolios)

This figure shows (annualized) certainty equivalent returns (CER) from following different horizon strategies over alternative investment horizons. For each investment horizon k (columns), entries report $CER_k(\mathbf{w}_h)$, where h is the strategy horizon (rows). The diagonal values (in bold) showcase the (annualized) CER of the strategies at the target investment horizons. Portfolio rules have non-negative constraint ($\hat{w}_{i,k,t} \geq 0$) and leverage constraint ($\sum_{i=1}^N |\hat{w}_{i,k,t}| \leq 1$). The sample period spans from July 1963 to December 2023, and the in-sample (out-of-sample) CERs are for a common period starting in June 1978 (May 1993), when the longest horizon strategy ($k = 120$) first becomes available.

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Appendix

A Portfolio Variance

The variance of the portfolio's k -period buy-and-hold excess return is defined as:

$$\text{Var} \left(R_{t,t+k}^{e,p} \right) = \mathbb{E} \left[\left(R_{t,t+k}^{e,p} - \mathbb{E} \left[R_{t,t+k}^{e,p} \right] \right)^2 \right]. \quad (\text{A-1})$$

The portfolio's k -period excess return is $R_{t,t+k}^{e,p} = \tilde{\mathbf{w}}_k' \mathbf{R}_{t,t+k}^e$, and with the decomposition $\tilde{\mathbf{w}}_k = \mathbf{B}_k \boldsymbol{\theta}_k + \mathbf{u}$, this becomes $R_{t,t+k}^{e,p} = (\mathbf{B}_k \boldsymbol{\theta}_k + \mathbf{u})' \mathbf{R}_{t,t+k}^e = \boldsymbol{\theta}_k' \mathbf{B}_k' \mathbf{R}_{t,t+k}^e + \mathbf{u}' \mathbf{R}_{t,t+k}^e = \boldsymbol{\theta}_k' \mathbf{F}_{t,t+k} + \mathbf{u}' \mathbf{R}_{t,t+k}^e$.

From Remark 1, the expected portfolio excess return is $\mathbb{E} \left[R_{t,t+k}^{e,p} \right] = \boldsymbol{\theta}_k' \boldsymbol{\mu}_{F,k}$. Substituting these into [Eq. A-1](#):

$$\begin{aligned} \text{Var} \left(R_{t,t+k}^{e,p} \right) &= \mathbb{E} \left[\left((\boldsymbol{\theta}_k' \mathbf{F}_{t,t+k} + \mathbf{u}' \mathbf{R}_{t,t+k}^e) - \boldsymbol{\theta}_k' \boldsymbol{\mu}_{F,k} \right)^2 \right] \\ &= \mathbb{E} \left[\left(\boldsymbol{\theta}_k' (\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k}) + \mathbf{u}' \mathbf{R}_{t,t+k}^e \right)^2 \right] \\ &= \mathbb{E} \left[(\boldsymbol{\theta}_k' (\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k}))^2 + (\mathbf{u}' \mathbf{R}_{t,t+k}^e)^2 + 2(\boldsymbol{\theta}_k' (\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k}))(\mathbf{u}' \mathbf{R}_{t,t+k}^e) \right] \\ &= \underbrace{\boldsymbol{\theta}_k' \mathbb{E}[(\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k})(\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k})'] \boldsymbol{\theta}_k}_{(1)} + \underbrace{\mathbb{E}[\mathbf{R}_{t,t+k}^e \mathbf{u} \mathbf{u}' \mathbf{R}_{t,t+k}^e]}_{(2)} \\ &\quad + \underbrace{2\boldsymbol{\theta}_k' \mathbb{E}[(\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k}) \mathbf{R}_{t,t+k}^e] \mathbf{u}}_{(3)} \end{aligned} \quad (\text{A-2})$$

Evaluating each term in [Eq. A-2](#):

(1):

$$\boldsymbol{\theta}_k' \mathbb{E}[(\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k})(\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k})'] \boldsymbol{\theta}_k = \boldsymbol{\theta}_k' \text{Var}(\mathbf{F}_{t,t+k}) \boldsymbol{\theta}_k = \boldsymbol{\theta}_k' \boldsymbol{\Lambda}_k \boldsymbol{\theta}_k.$$

(2):

$$\begin{aligned}
\mathbb{E}[\mathbf{R}_{t,t+k}^e \mathbf{u} \mathbf{u}' \mathbf{R}_{t,t+k}^e] &= \mathbf{u}' \mathbb{E}[\mathbf{R}_{t,t+k}^e \mathbf{R}_{t,t+k}^{e'}] \mathbf{u} \\
&= \mathbf{u}' \mathbb{E}[(\mathbf{B}_k \mathbf{F}_{t,t+k} + \boldsymbol{\varepsilon}_{t,t+k})(\mathbf{B}_k \mathbf{F}_{t,t+k} + \boldsymbol{\varepsilon}_{t,t+k})'] \mathbf{u} \\
&= \mathbf{u}' (\mathbf{B}_k \mathbb{E}[\mathbf{F}_{t,t+k} \mathbf{F}_{t,t+k}'] \mathbf{B}_k' + \mathbf{V}_k) \mathbf{u} \quad (\text{since } \mathbb{E}[\mathbf{F} \boldsymbol{\varepsilon}'] = \mathbb{E}[\boldsymbol{\varepsilon} \mathbf{F}'] = 0) \\
&= \mathbf{u}' (\mathbf{B}_k (\boldsymbol{\Lambda}_k + \boldsymbol{\mu}_{F,k} \boldsymbol{\mu}_{F,k}') \mathbf{B}_k' + \mathbf{V}_k) \mathbf{u} \\
&= \mathbf{u}' \mathbf{B}_k (\boldsymbol{\Lambda}_k + \boldsymbol{\mu}_{F,k} \boldsymbol{\mu}_{F,k}') \mathbf{B}_k' \mathbf{u} + \mathbf{u}' \mathbf{V}_k \mathbf{u} \\
&= 0 + \mathbf{u}' \mathbf{V}_k \mathbf{u} \quad (\text{since } \mathbf{B}_k' \mathbf{u} = 0).
\end{aligned}$$

(3):

$$\begin{aligned}
2\boldsymbol{\theta}_k' \mathbb{E}[(\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k}) \mathbf{R}_{t,t+k}^e] \mathbf{u} &= 2\boldsymbol{\theta}_k' \mathbb{E}[(\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k})(\mathbf{F}_{t,t+k}' \mathbf{B}_k' + \boldsymbol{\varepsilon}_{t,t+k}')'] \mathbf{u} \\
&= 2\boldsymbol{\theta}_k' (\mathbb{E}[(\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k}) \mathbf{F}_{t,t+k}' \mathbf{B}_k'] + \mathbb{E}[(\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k}) \boldsymbol{\varepsilon}_{t,t+k}']) \mathbf{u} \\
&= 2\boldsymbol{\theta}_k' ((\mathbb{E}[\mathbf{F}_{t,t+k} \mathbf{F}_{t,t+k}'] - \boldsymbol{\mu}_{F,k} \boldsymbol{\mu}_{F,k}') \mathbf{B}_k' \\
&\quad + \mathbb{E}[(\mathbf{F}_{t,t+k} - \boldsymbol{\mu}_{F,k})(\boldsymbol{\varepsilon}_{t,t+k} - \mathbb{E}[\boldsymbol{\varepsilon}_{t,t+k}'])']) \mathbf{u} \\
&= 2\boldsymbol{\theta}_k' (\boldsymbol{\Lambda}_k \mathbf{B}_k' + \mathbf{0}) \mathbf{u} \quad (\text{since factors and residuals are uncorrelated}) \\
&= 2\boldsymbol{\theta}_k' \boldsymbol{\Lambda}_k \mathbf{B}_k' \mathbf{u} \\
&= 2\boldsymbol{\theta}_k' \boldsymbol{\Lambda}_k (\mathbf{0}) = 0 \quad (\text{since } \mathbf{B}_k' \mathbf{u} = 0).
\end{aligned}$$

Combining these three terms, the portfolio variance is:

$$\text{Var}(R_{t,t+k}^{e,p}) = \boldsymbol{\theta}_k' \boldsymbol{\Lambda}_k \boldsymbol{\theta}_k + \mathbf{u}' \mathbf{V}_k \mathbf{u}.$$

B Number of Systematic Factors

To determine the number of factors for each investment horizon, I conduct a pseudo out-of-sample analysis on k -period buy-and-hold excess returns. For each horizon, I estimate the eigenvectors using

the first half of the sample and treat them as fixed factor loadings. I then project the second-half returns onto these loadings to construct pseudo out-of-sample factors. For each candidate value of J , I compute the proportion of total variance in the second-half returns that is explained by the J factors. I then select the smallest J such that the marginal increase in explained variance is negligible, balancing model complexity with the ability to capture systematic return variation. The number of factors varies across investment horizons, as the structure of return comovement changes with the length of the horizon. To provide a consistent baseline for the main analysis, I fix $J = 6$, which is the most frequently identified value across the one- to 120-month horizons. This choice simplifies comparisons across horizons while remaining representative of the data-driven estimates.

C Buy-and-Hold Portfolios

While the main analysis focuses on monthly rebalancing strategies that track the characterized k -period optimal payoffs, this appendix examines a pure buy-and-hold implementation where investors form portfolios at the beginning of the period and hold them for the entire k -period horizon without any intermediate rebalancing. In this buy-and-hold design, an investor at time t estimates k -period systematic factors using available information through that date, determines the optimal combination of these factors, forms initial portfolio weights, and then holds this portfolio from month t to $t + k$, allowing the weights to drift naturally with realized asset returns.

Similar to the monthly managed approach, the buy-and-hold implementation constructs factors that capture the systematic variation in k -period returns. However, instead of creating monthly factor-tracking portfolios that track k -period systematic factors, the buy-and-hold approach constructs k -period factors directly at the beginning of each holding period. At time t , I estimate factor loadings $\hat{\mathbf{B}}_{k,t}$ using principal component analysis on k -period returns available through that date. These loadings are then applied to the subsequent k -period asset returns to create buy-and-hold factor excess returns,

$$\tilde{\mathbf{F}}_{t,t+k}^{bnh} = \hat{\mathbf{B}}'_{k,t} \mathbf{R}_{t,t+k}^e, \quad (\text{A-3})$$

where $\mathbf{R}_{t,t+k}^e$ is the $N \times 1$ vector of k -month buy-and-hold excess asset returns. The buy-and-hold portfolio weights at the beginning of the investment period have the form

$$\underbrace{\hat{\mathbf{w}}_{k,t}}_{N \times 1} = \underbrace{\hat{\mathbf{B}}_{k,t}}_{N \times J} \underbrace{\boldsymbol{\theta}_k}_{J \times 1}, \quad (\text{A-4})$$

where $\boldsymbol{\theta}_k$ represents the $J \times 1$ coefficient of optimal factor allocations.

The k -period wealth from buy-and-hold strategy is

$$\begin{aligned} W_{t+k}^{bnh} &= 1 + \mathbf{w}'_{k,t} \mathbf{R}_{t,t+k}^e + R_{t,t+k}^f \\ &= 1 + \boldsymbol{\theta}'_k \tilde{\mathbf{F}}_{t,t+k}^{bnh} + R_{t,t+k}^f. \end{aligned} \quad (\text{A-5})$$

The utility-maximizing combination of buy-and-hold factors for investors with power utility over the terminal wealth is given by

$$\begin{aligned} \max_{\boldsymbol{\theta}_k} \frac{1}{M} \sum_{t=0}^{M-1} U(W_{t+k}^{bnh}) &= \max_{\boldsymbol{\theta}_k} \frac{1}{M} \sum_{t=0}^{M-1} \frac{(W_{t+k}^{bnh})^{1-\gamma}}{1-\gamma} \\ &= \max_{\boldsymbol{\theta}_k} \frac{1}{M} \sum_{t=0}^{M-1} \frac{1}{1-\gamma} \left(1 + \boldsymbol{\theta}'_k \tilde{\mathbf{F}}_{t,t+k}^{bnh} + R_{t,t+k}^f \right)^{1-\gamma}, \end{aligned} \quad (\text{A-6})$$

where M represents the number of available k -month observations in the sample, and $\gamma = 5$. For out-of-sample estimation, $\boldsymbol{\theta}_k$ is estimated using an expanding window of available k -month wealth observations up to each portfolio formation date, ensuring that the optimal combination parameter reflects only information available to investors at the time of portfolio formation.

To evaluate the economic costs of horizon misalignment in the buy-and-hold setting, I examine the performance of strategies optimized for horizon h when applied to investment periods of length k . At time t , I form portfolios using optimal weights for horizon h and evaluate the buy-and-hold performance of this portfolio over next k months. The k -period certainty-equivalent return (CER)

under a buy-and-hold strategy optimized for horizon h is

$$\text{CER}_k(\mathbf{w}_h) = \left[(1 - \gamma) \mathbb{E} \left[U \left(1 + \hat{\mathbf{w}}'_{h,t} \mathbf{R}_{t,t+k}^e + R_{t,t+k}^f \right) \right] \right]^{\frac{1}{1-\gamma}} - 1, \quad (\text{A-7})$$

where $\hat{\mathbf{w}}_{h,t} = \hat{\mathbf{B}}_{h,t} \hat{\boldsymbol{\theta}}_h$ represents the buy-and-hold portfolio weights formed at time t for a strategy optimized for horizon h .^{A1}

Table A3 report the k -period CERs for buy-and-hold horizon-specific strategies and buy-and-hold benchmark portfolios. Consistent with the monthly managed results, buy-and-hold horizon-specific strategies generate substantially higher k -period CERs than the benchmark portfolios across all investment horizons, confirming that the systematic factor approach captures economically meaningful investment opportunities even without monthly rebalancing.

Panel A presents the in-sample results, which best reveal the underlying economic relations by abstracting from parameter estimation uncertainty. Comparing these in-sample CERs to those from the monthly managed approach reported in Table 1, buy-and-hold strategies exhibit generally lower k -period CERs, particularly over longer investment horizons. This pattern highlights the advantages of the monthly managed implementation. The ability to continuously update factor loadings with new information and rebalance portfolios helps maintain exposure to systematic factors as portfolio weights naturally drift with realized returns over the holding period. Similarly, monthly rebalancing preserves the optimal combination of factors ($\boldsymbol{\theta}_k$) rather than allowing it to become distorted by realized returns. These benefits of the monthly managed approach appear more pronounced over longer horizons, where there is more time for portfolios to drift from their target factor exposures. Without rebalancing, the effective weights of assets with higher realized returns increase during holding periods, resulting a drift toward past winners and deviation from horizon-consistent allocations. Panel B presents the out-of-sample results, which show substantially smaller differences between the two approaches, likely reflecting the challenges of real-time parameter

^{A1}I evaluate the strategies over a common sample with starting periods similar to those described for the monthly managed design. Additionally, since evaluating buy-and-hold portfolios over $k = 120$ requires 120 months return observations, the series of buy-and-hold portfolio weights ends 120 months before the end of the sample period.

estimation.

Table A4 reports $\text{CER}_k(\mathbf{w}_k) - \text{CER}_k(\mathbf{w}_h)$ for buy-and-hold strategies, which show similar patterns to those documented for the monthly managed implementation in Table 2. Buy-and-hold investors still benefit from using horizon-matched strategies compared to those optimized for other horizons. However, the benefits of long-horizon strategies compared to short-horizon strategies are generally smaller than those observed under monthly managed implementation. This reduction is consistent with the monthly managed approach’s superior ability to maintain distinct factor exposures across different investment horizons. Without continuous rebalancing, portfolios optimized for different horizons may converge toward more similar allocations over time as weights drift with realized returns, reducing the economic gains of horizon-specific portfolios over the long-run.

Table A1: k -period CERs (Different Test Assets)

The table presents the k -period (annualized) certainty-equivalent returns (CERs) for horizon-specific portfolios and benchmark strategies. The underlying assets are 510 quintile portfolios corresponding to 102 characteristic-sorted portfolios from Chen and Zimmermann dataset. The first four columns showcase the CERs for weights estimated using J number of factors. The columns labeled ‘EW’ and ‘LS’ invest an equal portion in all assets and in long-minus-short legs, respectively. ‘Mkt’ reports CERs from holding the market portfolio. The optimal policies are following $\hat{\mathbf{w}}_{k,t} = \hat{\mathbf{B}}_{k,t}\boldsymbol{\theta}_k$ (Eq. 8). For each horizon k , the target horizon loadings ($\hat{\mathbf{B}}_{k,t}$) are estimated using an expanding window with an initial length of 60 k -month observations. The $\hat{\boldsymbol{\theta}}_k$ coefficients are solved numerically and are restricted to be between 0 and 1. The sample period spans from July 1963 to December 2023. For each horizon k , the effective in-sample (out-of-sample) window starts from $k + 59$ months ($2k + 118$ months) after the starting of the sample, where the first $\hat{\mathbf{B}}_{k,t}$ ($\hat{\boldsymbol{\theta}}_k$) becomes available. Investors have power utility over terminal wealth with $\gamma = 5$. The CERs are annualized from k -period CERs by $(1 + \text{CER}_k(\mathbf{w}_k))^{12/k} - 1$.

Panel A: In-Sample

k	Horizon-specific Portfolios				Benchmark Strategies		
	$J = 1$	$J = 6$	$J = 9$	$J = 12$	EW	LS	Mkt
1	6.29	10.61	11.67	12.43	2.76	9.43	4.46
6	5.92	9.23	10.60	11.38	0.48	9.21	3.32
12	6.19	9.49	10.59	11.60	1.84	8.96	3.24
24	6.57	10.46	11.16	11.62	2.99	8.42	3.41
36	7.39	9.79	10.29	11.32	5.73	7.93	4.52
60	8.87	11.68	12.25	12.44	9.02	6.90	6.99
96	10.60	12.70	14.39	15.20	10.27	5.82	6.93
120	10.90	12.93	13.68	13.70	10.36	5.35	5.79

Panel B: Out-of-Sample

k	Horizon-specific Portfolios				Benchmark Strategies		
	$J = 1$	$J = 6$	$J = 9$	$J = 12$	EW	LS	Mkt
1	6.02	10.09	11.12	11.80	4.38	9.17	5.19
6	5.53	8.37	9.82	10.50	3.26	8.88	5.24
12	5.77	8.78	9.88	10.91	4.17	8.61	4.90
24	6.00	7.73	8.14	8.47	4.96	7.95	4.79
36	6.57	7.29	7.72	8.16	7.06	7.26	5.15
60	6.44	7.84	8.46	7.86	7.87	5.89	6.11
96	9.82	11.16	12.96	13.77	9.46	4.63	5.75
120	9.82	11.66	12.00	10.76	9.30	4.00	4.42

Table A2: CER Gains of Following Horizon-Specific Portfolio Rules (Different Test Assets)

The table shows differences in (annualized) certainty equivalent returns (ΔCERs) between horizon-matched and mismatched portfolio rules. The underlying assets are 510 quintile portfolios corresponding to 102 characteristic-sorted portfolios from Chen and Zimmermann dataset. For each investment horizon k (columns), entries report $\text{CER}_k(\mathbf{w}_k) - \text{CER}_k(\mathbf{w}_h)$, where h is the strategy horizon (rows). Managed strategies are constructed using six factor-tracking portfolios (i.e., $J = 6$). The sample period spans from July 1963 to December 2023, and the in-sample (out-of-sample) CERs are for a common period starting in June 1978 (May 1993), when the longest horizon strategy ($k = 120$) first becomes available. Investors have power utility over k -period wealth with $\gamma = 5$.

Panel A: In-Sample

Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.93	-0.63	0.66	-0.07	1.09	2.20	2.54
6	1.13		0.32	1.54	0.90	2.13	3.24	3.56
12	0.91	-0.30		1.24	0.53	1.75	2.84	3.27
24	-0.29	-1.57	-1.24		-0.62	0.74	1.81	2.05
36	0.54	-0.25	0.07	1.17		0.99	1.77	2.03
60	0.86	0.82	0.45	1.67	-0.34		-0.19	-0.21
96	3.52	5.81	5.47	6.52	2.63	1.10		0.08
120	3.90	5.31	5.04	5.93	2.22	0.91	0.16	

Panel B: Out-of-Sample

Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.84	-0.12	-2.15	-2.43	-0.86	2.82	3.77
6	1.03		0.72	-1.33	-1.37	0.34	4.14	5.12
12	-0.13	-0.83		-1.71	-1.86	-0.39	3.04	3.99
24	-0.46	-0.13	1.18		-0.38	0.54	3.42	4.06
36	1.74	1.95	2.95	0.96		0.61	3.50	4.25
60	0.85	2.82	3.56	2.55	0.41		1.70	2.17
96	3.45	9.47	9.64	6.89	2.20	-0.45		0.51
120	2.86	5.08	5.25	2.50	-1.11	-2.03	-0.50	

Table A3: k -period CERs (Buy-and-Hold Portfolios)

The table presents the k -period (annualized) certainty-equivalent returns (CERs) for buy-and-hold horizon-specific portfolios and benchmark strategies. The underlying assets are 230 extreme decile portfolios from Chen and Zimmermann dataset. Investors have power utility over terminal wealth with $\gamma = 5$. The CERs are annualized from k -period CERs by $(1 + \text{CER}_k(\mathbf{w}_k))^{12/k} - 1$.

Panel A: In-Sample							
k	Horizon-specific Portfolios				Benchmark Strategies		
	$J = 1$	$J = 6$	$J = 9$	$J = 12$	EW	LS	Mkt
1	5.48	11.57	13.78	15.47	-0.76	11.58	4.46
6	5.41	11.58	13.18	14.94	-3.04	5.46	3.53
12	5.58	10.38	12.05	13.34	-1.01	4.73	3.74
24	5.95	10.04	11.24	12.24	1.70	4.22	4.36
36	6.71	10.37	11.99	13.36	5.08	3.96	5.45
60	7.44	11.29	12.12	12.87	7.24	3.54	7.09
96	9.41	12.13	12.28	13.10	8.29	2.97	6.98
120	9.95	13.83	14.12	14.47	8.32	2.67	6.22

Panel B: Out-of-Sample							
k	Horizon-specific Portfolios				Benchmark Strategies		
	$J = 1$	$J = 6$	$J = 9$	$J = 12$	EW	LS	Mkt
1	5.15	11.10	13.20	14.82	0.94	11.37	5.19
6	5.03	10.92	12.47	14.16	-0.41	5.26	5.35
12	5.16	9.21	10.62	11.73	0.90	4.52	5.15
24	5.48	9.06	10.20	11.37	2.73	3.96	5.32
36	5.59	8.28	9.00	11.01	5.56	3.46	5.85
60	4.31	8.24	8.91	8.97	6.16	2.72	6.30
96	8.55	9.09	8.01	8.40	7.54	2.00	5.94
120	8.27	10.95	11.34	10.55	7.32	1.61	4.96

Table A4: CER Gains of Following Horizon-Specific Portfolio Rules (Buy-and-Hold Portfolios)

The table shows differences in (annualized) certainty equivalent returns (ΔCERs) between horizon-matched and mismatched portfolio rules. For each investment horizon k (columns), entries report $\text{CER}_k(\mathbf{w}_k) - \text{CER}_k(\mathbf{w}_h)$, where h is the strategy horizon (rows). Buy-and-hold strategies are constructed using six factors (i.e., $J = 6$). Investors have power utility over k -period wealth with $\gamma = 5$.

Panel A: In-Sample								
Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.11	-0.64	-0.14	-0.70	0.96	1.48	2.66
6	0.38		-0.34	0.37	-0.32	1.22	1.24	2.59
12	1.81	0.92		0.56	0.24	1.95	2.04	3.19
24	0.81	0.17	-0.40		-0.37	1.38	2.39	3.26
36	1.67	1.36	0.91	0.90		1.35	1.73	2.30
60	3.91	3.27	1.42	0.31	-1.28		-0.35	0.16
96	7.14	15.90	14.27	7.30	2.33	0.74		1.69
120	16.50	112.88	104.21	96.57	65.23	9.39	-1.16	

Panel B: Out-of-Sample								
Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-1.26	-1.53	-1.08	-2.83	0.92	1.79	2.12
6	1.60		-0.19	0.52	-1.53	1.70	2.00	2.92
12	2.08	0.18		0.59	-1.18	1.83	2.94	3.57
24	1.29	0.25	-0.03		-1.86	1.36	2.35	2.59
36	1.50	-0.04	-0.43	4.36		1.36	0.88	0.99
60	1.28	1.13	0.06	-0.74	-3.59		-0.00	-0.28
96	1.89	4.40	2.24	-0.07	-3.69	2.99		-0.27
120	15.48	77.41	59.65	82.99	29.98	6.69	-0.80	

Table A5: CER Gains of Following Horizon-Specific Portfolio Rules (Different Number of Factors)

The table shows differences in (annualized) certainty equivalent returns (Δ CERs) between horizon-matched and mismatched portfolio rules. For each investment horizon k (columns), entries report $\text{CER}_k(\mathbf{w}_k) - \text{CER}_k(\mathbf{w}_h)$, where h is the strategy horizon (rows). The sample period spans from July 1963 to December 2022, and the in-sample (out-of-sample) CERs are for a common period starting in June 1978 (June 1993), when the longest horizon strategy ($k = 120$) first becomes available.

Panel A: $J = 1$

In-Sample

Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.01	0.05	0.26	0.90	1.59	4.38	5.29
6	0.06		0.09	0.37	1.10	1.81	4.65	5.57
12	-0.06	-0.07		0.21	0.85	1.55	4.35	5.26
24	-0.19	-0.09	-0.08		0.45	1.13	3.83	4.72
36	-0.08	0.28	0.17	-0.03		0.57	3.00	3.84
60	0.98	2.03	1.60	0.89	-0.13		1.71	2.43
96	4.32	6.98	5.87	4.73	1.46	0.09		0.53
120	3.78	6.49	5.50	4.51	1.05	-0.29	-0.57	

Out-of-Sample

Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.09	-0.06	-0.05	0.55	-0.45	4.65	5.46
6	0.13		0.04	0.14	0.86	-0.13	5.03	5.87
12	0.01	-0.02		0.00	0.57	-0.42	4.68	5.51
24	-0.04	0.13	0.13		0.33	-0.70	4.29	5.11
36	-0.02	0.34	0.32	0.02		-1.10	3.73	4.53
60	2.48	5.26	4.99	4.39	2.52		3.49	4.23
96	4.08	7.77	6.93	5.58	1.53	-1.95		0.49
120	3.73	7.41	6.70	5.48	1.21	-2.24	-0.51	

Table A5: (continued)

Panel B: $J = 9$ *In-Sample*

Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.08	-0.19	-0.19	0.99	3.64	2.75	7.48
6	0.59		-0.32	-0.21	1.22	3.60	2.82	7.58
12	1.20	0.47		-0.18	1.04	3.19	2.26	7.00
24	0.45	0.05	-0.16		1.56	4.07	3.25	7.96
36	0.55	-0.00	-0.60	-1.09		2.46	1.98	6.76
60	0.65	0.31	-0.89	-1.77	-1.62		-2.00	2.80
96	5.10	5.98	4.73	2.44	1.38	2.77		3.98
120	15.04	13.26	12.43	11.24	5.07	2.26	-3.85	

Out-of-Sample

Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.45	-0.28	-2.84	-0.23	1.37	0.25	5.82
6	0.93		-0.12	-2.45	0.45	1.62	0.48	6.03
12	1.19	0.12		-2.54	0.43	1.91	1.13	6.76
24	0.91	0.90	1.46		2.80	3.46	1.69	7.24
36	2.63	1.39	0.83	-2.49		1.33	0.68	6.34
60	1.25	0.45	-0.08	-3.20	-1.17		-1.83	3.88
96	6.05	5.48	4.21	-0.76	0.13	1.17		5.59
120	24.36	17.00	15.42	10.33	4.57	0.59	-5.19	

Table A5: (continued)

Panel C: $J = 12$ *In-Sample*

Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		0.34	-0.04	0.25	2.00	3.32	3.54	7.26
6	0.26		-0.75	-0.43	1.53	2.64	3.37	7.15
12	1.30	0.94		0.06	1.87	2.67	3.17	6.92
24	0.57	0.47	-0.26		2.07	3.27	3.77	7.48
36	-0.31	-0.61	-1.59	-1.72		1.34	2.23	6.01
60	1.36	1.32	-0.22	-0.83	-0.22		-0.83	2.99
96	5.65	6.36	5.34	3.67	2.43	2.21		3.00
120	17.32	14.73	12.78	11.09	5.34	1.54	-2.58	

Out-of-Sample

Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.05	-0.34	-1.15	0.37	-0.41	0.34	4.06
6	0.68		-0.72	-1.44	0.31	-0.82	0.31	4.04
12	1.45	0.70		-0.78	1.09	0.14	1.50	5.30
24	0.60	0.51	0.25		1.80	0.42	1.04	4.80
36	2.12	1.20	0.13	-1.44		-0.98	0.46	4.32
60	0.93	0.76	0.50	0.36	1.37		0.02	3.68
96	6.71	5.12	3.67	0.60	0.14	-0.63		3.64
120	37.44	18.85	16.91	13.59	5.83	1.21	-2.66	

Table A6: CER gains of following horizon-specific portfolio rules (Constrained Portfolios)

The table shows (annualized) differences in certainty equivalent returns (Δ CERs) between horizon-matched and mismatched portfolio rules. For each investment horizon k (columns), entries report $\text{CER}_k(\mathbf{w}_k) - \text{CER}_k(\mathbf{w}_h)$, where h is the strategy horizon (rows). Portfolio rules have non-negative constraint ($\hat{w}_{i,k,t} \geq 0$) and leverage constraint ($\sum_{i=1}^N |\hat{w}_{i,k,t}| \leq 1$). The sample period spans from July 1963 to December 2022, and the in-sample (out-of-sample) CERs are for a common period starting in June 1978 (June 1993), when the longest horizon strategy ($k = 120$) first becomes available.

Panel A: $J = 1$

<i>In-Sample</i>								
Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.01	0.05	0.26	0.90	1.59	4.38	5.29
6	0.06		0.09	0.37	1.10	1.81	4.65	5.57
12	-0.06	-0.07		0.21	0.85	1.55	4.35	5.26
24	-0.19	-0.09	-0.08		0.45	1.13	3.83	4.71
36	-0.08	0.28	0.17	-0.03		0.57	3.00	3.83
60	0.98	2.03	1.60	0.89	-0.13		1.71	2.43
96	4.32	6.98	5.87	4.73	1.46	0.09		0.53
120	3.78	6.49	5.51	4.51	1.05	-0.29	-0.56	

<i>Out-of-Sample</i>								
Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.09	-0.06	-0.05	0.55	-0.45	4.65	5.46
6	0.13		0.04	0.14	0.86	-0.13	5.03	5.87
12	0.01	-0.02		0.00	0.57	-0.42	4.68	5.51
24	-0.04	0.13	0.13		0.33	-0.70	4.29	5.10
36	-0.02	0.34	0.32	0.02		-1.10	3.73	4.52
60	2.48	5.26	4.99	4.39	2.52		3.49	4.23
96	4.08	7.77	6.93	5.58	1.53	-1.95		0.49
120	3.73	7.42	6.70	5.48	1.22	-2.24	-0.51	

Table A6: (continued)

Panel B: $J = 9$ *In-Sample*

Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.14	-0.26	0.04	1.11	2.84	4.10	4.70
6	0.33		-0.10	0.27	1.49	3.15	4.43	4.98
12	0.42	0.14		0.33	1.47	3.14	4.43	5.00
24	0.03	-0.07	-0.21		0.90	2.53	3.61	4.14
36	0.47	0.63	0.13	-0.24		1.59	2.65	3.20
60	0.92	1.30	0.39	-0.48	-1.09		-0.01	0.41
96	2.53	4.43	3.60	2.74	0.95	0.88		0.23
120	2.64	4.27	3.33	2.45	0.61	0.84	-0.19	

Out-of-Sample

Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.55	-0.81	-2.43	0.34	-0.60	3.14	4.53
6	0.76		-0.19	-1.60	1.45	0.54	4.36	5.80
12	0.79	0.20		-1.45	1.41	0.46	4.19	5.63
24	0.75	1.34	1.28		2.17	0.66	3.89	5.29
36	0.67	0.31	0.02	-2.00		-1.00	2.71	4.10
60	2.22	4.47	4.21	2.45	2.70		1.79	3.11
96	2.78	5.42	5.36	3.26	2.29	-1.16		1.05
120	2.16	4.62	3.87	1.27	0.39	-2.54	-1.05	

Table A6: (continued)

Panel C: $J = 12$

<i>In-Sample</i>								
Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.14	-0.26	0.04	1.11	2.84	3.98	4.70
6	0.33		-0.10	0.27	1.49	3.15	4.32	4.98
12	0.42	0.14		0.33	1.47	3.14	4.31	5.00
24	0.03	-0.07	-0.21		0.90	2.53	3.49	4.14
36	0.46	0.63	0.13	-0.24		1.59	2.53	3.20
60	0.92	1.30	0.39	-0.48	-1.09		-0.12	0.41
96	2.87	4.57	3.54	2.50	0.97	0.98		0.38
120	2.64	4.27	3.33	2.45	0.61	0.84	-0.31	
<i>Out-of-Sample</i>								
Strategy	Investment Horizon							
Horizon	1	6	12	24	36	60	96	120
1		-0.56	-0.81	-2.43	0.25	-0.60	3.49	4.31
6	0.77		-0.19	-1.60	1.37	0.54	4.72	5.58
12	0.79	0.20		-1.45	1.32	0.46	4.55	5.41
24	0.75	1.34	1.28		2.08	0.66	4.24	5.07
36	0.83	0.45	0.15	-1.91		-0.94	3.10	3.91
60	2.22	4.47	4.21	2.45	2.61		2.14	2.89
96	2.99	5.86	5.22	2.80	1.80	-1.51		0.52
120	2.32	4.76	4.01	1.46	0.51	-2.33	-0.50	

Table A7: Statistical Properties of Strategies Across Different Investment Horizons: Constrained Portfolios (In-Sample)

This table shows the statistical properties of various strategies' returns across different investment horizons. Portfolio rules have non-negative constraint ($\hat{w}_{i,k,t} \geq 0$) and leverage constraint ($\sum_{i=1}^N |\hat{w}_{i,k,t}| \leq 1$). The strategies include the horizon-specific portfolios and two respective benchmarks. The reported statistics are estimated over the investment horizons (columns) and include (annualized) mean return, (annualized) standard deviation, variance ratio, skewness, and kurtosis. The sample period spans from July 1963 to December 2022, and the statistics are for a common period starting in June 1978, when the in-sample portfolio of the longest horizon strategy ($k = 120$) first becomes available.

Strategies	Investment Horizon							
	1	6	12	24	36	60	96	120
Panel A: Mean								
1-month	8.76	8.79	8.86	8.78	8.74	8.60	8.38	8.37
6-month	7.86	7.86	7.90	7.82	7.79	7.62	7.43	7.44
12-month	8.01	8.02	8.06	7.99	7.95	7.81	7.65	7.65
24-month	9.02	9.06	9.12	9.04	8.99	8.84	8.59	8.54
36-month	11.46	11.61	11.68	11.41	11.30	11.15	10.84	10.84
60-month	15.48	15.73	15.84	15.23	14.88	14.55	14.03	14.01
96-month	16.12	16.46	16.54	15.77	15.26	14.88	14.23	14.20
120-month	16.09	16.48	16.64	15.91	15.39	15.01	14.35	14.34
EW	13.11	13.85	13.91	13.31	13.23	13.47	11.48	10.74
LS	16.91	17.77	17.90	17.37	17.34	17.68	15.87	15.15
Panel B: Standard Deviation								
1-month	8.22	9.80	10.50	10.26	10.45	13.88	17.15	19.72
6-month	6.90	8.07	8.65	8.52	8.61	10.99	13.00	14.96
12-month	7.50	8.76	9.23	8.93	8.86	11.48	14.10	16.25
24-month	8.88	10.45	11.06	10.52	10.25	13.34	15.88	17.65
36-month	13.29	15.68	16.53	15.31	14.95	19.43	23.92	28.04
60-month	18.06	21.64	23.64	21.61	20.25	26.72	34.19	42.96
96-month	19.75	23.80	25.84	23.73	22.05	29.26	34.72	42.79
120-month	20.33	24.41	26.82	24.72	23.24	30.78	37.16	46.47
EW	20.91	24.69	26.27	23.47	22.55	31.89	29.14	26.42
LS	20.29	24.89	27.14	25.17	25.51	38.97	45.51	50.59

Table A7: (continued)

Strategies	Investment Horizon							
	1	6	12	24	36	60	96	120
Panel C: Variance Ratio								
1-month	1.00	1.31	1.36	1.16	0.98	1.20	1.18	1.21
6-month	1.00	1.27	1.33	1.15	0.97	1.16	1.13	1.17
12-month	1.00	1.27	1.29	1.09	0.89	1.09	1.10	1.15
24-month	1.00	1.28	1.30	1.06	0.82	0.96	0.85	0.81
36-month	1.00	1.23	1.18	0.89	0.66	0.73	0.65	0.64
60-month	1.00	1.19	1.16	0.83	0.57	0.57	0.43	0.41
96-month	1.00	1.22	1.17	0.85	0.57	0.55	0.36	0.33
120-month	1.00	1.22	1.19	0.88	0.58	0.57	0.39	0.36
EW	1.00	1.20	1.13	0.81	0.57	0.60	0.31	0.21
LS	1.00	1.25	1.19	0.89	0.66	0.73	0.48	0.39
Panel D: Skewness								
1-month	-0.51	0.21	0.47	-0.00	0.38	0.67	0.55	0.25
6-month	-0.39	0.20	0.43	0.10	0.51	0.71	0.35	0.14
12-month	-0.33	0.20	0.37	-0.09	0.21	0.46	0.32	0.09
24-month	-0.23	0.25	0.46	-0.10	0.18	0.50	0.55	0.20
36-month	-0.48	0.40	0.72	0.18	0.46	0.53	0.34	0.12
60-month	-0.26	0.68	1.13	0.43	0.40	0.51	0.58	0.67
96-month	-0.47	0.61	1.01	0.46	0.41	0.66	0.57	0.64
120-month	-0.25	0.64	1.06	0.41	0.52	0.68	0.62	0.71
EW	-0.40	0.59	0.99	0.46	0.52	1.04	0.92	0.62
LS	-0.37	0.66	1.06	0.34	0.37	0.76	0.60	0.57
Panel E: Kurtosis								
1-month	5.33	4.21	4.06	2.89	3.32	3.46	2.97	2.11
6-month	5.05	4.02	3.72	2.85	3.48	3.73	2.39	1.81
12-month	5.38	4.04	3.73	2.81	3.00	3.03	2.42	1.82
24-month	5.64	4.58	4.31	3.10	3.23	3.12	3.15	2.24
36-month	5.81	3.84	4.26	2.73	3.16	2.90	2.47	2.10
60-month	5.91	4.28	5.49	2.98	3.05	2.67	2.52	3.10
96-month	5.44	4.74	5.34	3.66	3.46	3.32	2.52	3.03
120-month	5.75	4.53	5.43	3.20	3.64	3.28	2.63	3.33
EW	5.46	4.71	5.31	3.67	3.27	4.17	3.35	3.46
LS	5.57	4.71	5.43	3.24	3.00	3.48	2.55	2.89

Table A8: Statistical Properties of Strategies Across Different Investment Horizons: Constrained Portfolios (Out-of-Sample)

This table shows the statistical properties of various strategies' returns across different investment horizons. Portfolio rules have non-negative constraint ($\hat{w}_{i,k,t} \geq 0$) and leverage constraint ($\sum_{i=1}^N |\hat{w}_{i,k,t}| \leq 1$). The strategies include the horizon-specific portfolios and two respective benchmarks. The reported statistics are estimated over the investment horizons (columns) and include (annualized) mean return, (annualized) standard deviation, variance ratio, skewness, and kurtosis. The sample period spans from July 1963 to December 2022, and the statistics are for a common period starting in June 1993, when the in-sample portfolio of the longest horizon strategy ($k = 120$) first becomes available.

Strategies	Investment Horizon							
	1	6	12	24	36	60	96	120
Panel A: Mean								
1-month	6.60	6.56	6.59	6.64	6.65	6.35	6.07	6.10
6-month	5.21	5.15	5.16	5.17	5.20	4.85	4.56	4.59
12-month	5.64	5.60	5.63	5.67	5.68	5.31	4.98	5.04
24-month	7.39	7.46	7.53	7.60	7.64	7.24	6.80	6.82
36-month	7.61	7.62	7.75	7.75	7.80	7.48	7.22	7.32
60-month	12.90	13.07	13.29	13.10	12.80	11.87	10.94	11.03
96-month	13.47	13.75	13.91	13.53	13.02	12.10	11.16	11.29
120-month	13.88	14.18	14.44	14.15	13.58	12.74	11.84	12.00
EW	11.55	11.91	12.29	11.76	11.70	11.18	10.33	9.33
LS	14.89	15.42	15.88	15.48	15.52	15.15	14.44	12.94
Panel B: Standard Deviation								
1-month	8.24	9.66	10.33	9.61	8.52	9.98	10.85	13.04
6-month	6.71	7.71	8.35	7.95	7.38	7.97	7.79	9.58
12-month	7.86	9.17	9.83	9.44	8.81	9.69	9.72	12.11
24-month	11.00	12.86	13.86	13.79	13.64	16.01	18.40	22.88
36-month	11.43	13.24	14.45	13.30	11.56	13.29	15.19	18.13
60-month	18.32	21.69	24.71	23.54	22.00	26.12	30.49	42.82
96-month	20.37	24.46	27.35	25.65	22.43	26.73	27.88	37.76
120-month	20.70	24.75	27.72	26.10	21.95	27.17	28.30	38.24
EW	21.33	24.76	27.11	24.29	20.96	23.83	23.86	22.26
LS	20.60	24.92	27.98	25.93	24.03	30.72	41.09	44.39

Table A8: (continued)

Strategies	Investment Horizon							
	1	6	12	24	36	60	96	120
Panel C: Variance Ratio								
1-month	1.00	1.30	1.36	1.11	0.76	0.80	0.64	0.68
6-month	1.00	1.27	1.39	1.19	0.91	0.89	0.65	0.77
12-month	1.00	1.31	1.40	1.22	0.92	0.90	0.67	0.79
24-month	1.00	1.37	1.45	1.34	1.11	1.11	0.90	0.97
36-month	1.00	1.24	1.31	1.02	0.65	0.65	0.56	0.60
60-month	1.00	1.24	1.31	1.09	0.78	0.73	0.51	0.56
96-month	1.00	1.27	1.31	1.04	0.68	0.59	0.34	0.35
120-month	1.00	1.24	1.27	1.01	0.59	0.53	0.30	0.31
EW	1.00	1.20	1.20	0.88	0.55	0.45	0.25	0.18
LS	1.00	1.25	1.27	0.97	0.65	0.61	0.46	0.35
Panel D: Skewness								
1-month	-0.23	0.21	0.49	-0.11	-0.05	0.25	0.55	0.84
6-month	-0.04	0.11	0.47	-0.01	0.16	0.17	0.36	0.76
12-month	0.08	0.14	0.41	-0.06	0.22	0.31	0.51	0.89
24-month	-0.63	-0.41	0.06	-0.28	-0.10	0.11	0.58	0.93
36-month	-0.11	0.46	0.88	0.33	0.45	0.38	0.44	0.48
60-month	-0.06	0.59	1.18	0.44	0.27	0.26	0.55	1.10
96-month	-0.20	0.63	1.09	0.51	0.23	0.45	0.50	1.00
120-month	-0.11	0.78	1.17	0.55	0.22	0.62	0.49	0.94
EW	-0.16	0.70	1.10	0.63	0.41	0.56	0.58	0.49
LS	-0.14	0.78	1.17	0.50	0.30	0.30	0.50	1.01
Panel E: Kurtosis								
1-month	4.44	4.73	4.16	2.72	2.59	2.01	1.78	2.47
6-month	5.60	4.77	4.44	2.91	2.57	1.84	1.80	2.51
12-month	6.02	4.75	4.20	2.84	2.68	1.94	1.83	2.63
24-month	6.37	6.22	4.90	3.56	3.23	1.84	1.93	2.52
36-month	4.58	4.59	5.07	3.16	3.12	2.07	1.82	2.02
60-month	5.46	4.96	5.98	3.20	2.86	2.04	2.00	3.09
96-month	4.78	5.19	5.62	3.64	3.16	2.72	1.93	2.98
120-month	4.95	5.79	6.18	3.87	2.98	3.15	1.88	2.86
EW	4.85	5.63	5.76	4.28	3.69	3.34	2.49	2.35
LS	4.89	5.64	5.86	3.77	3.25	2.46	2.00	2.93

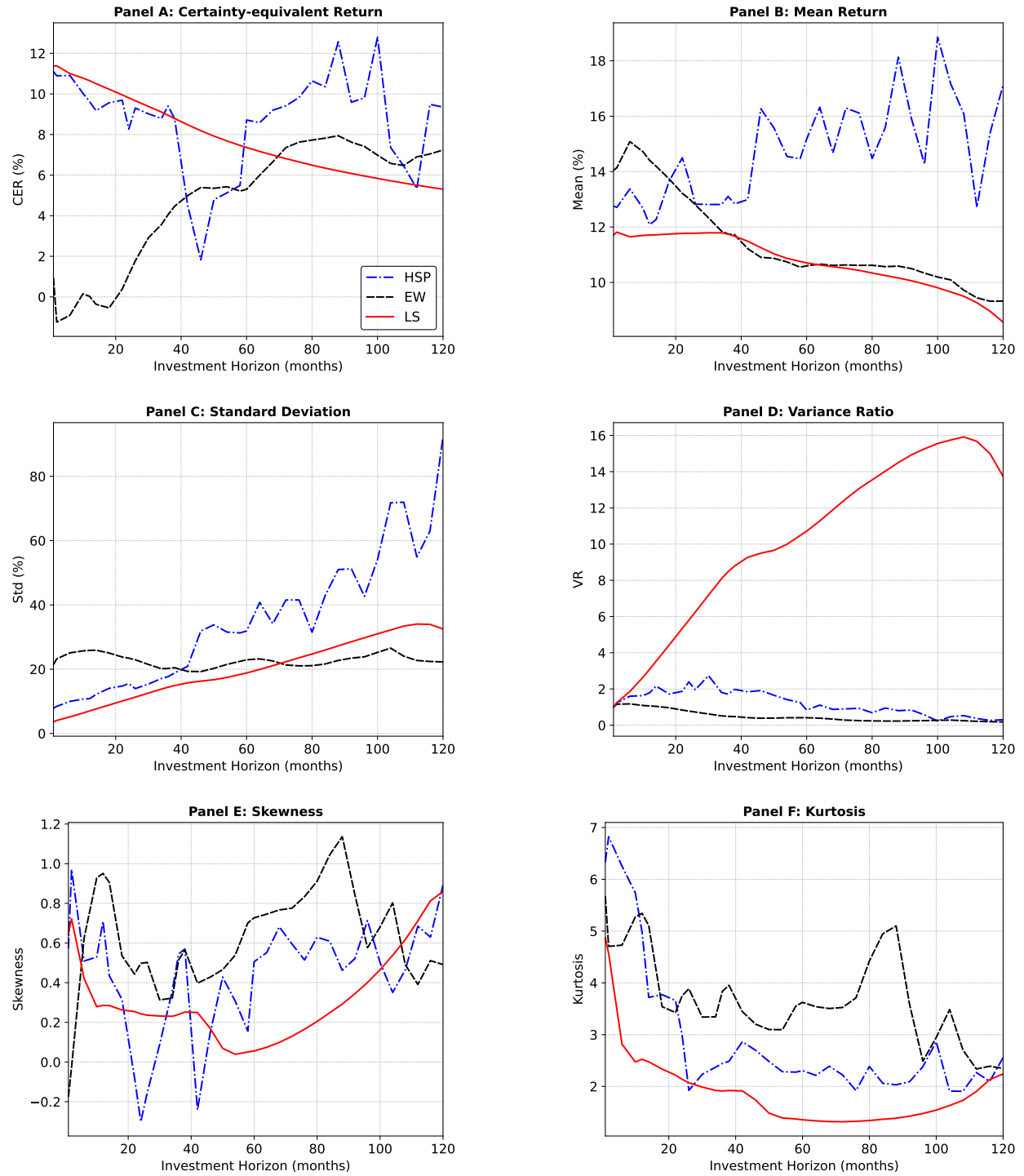
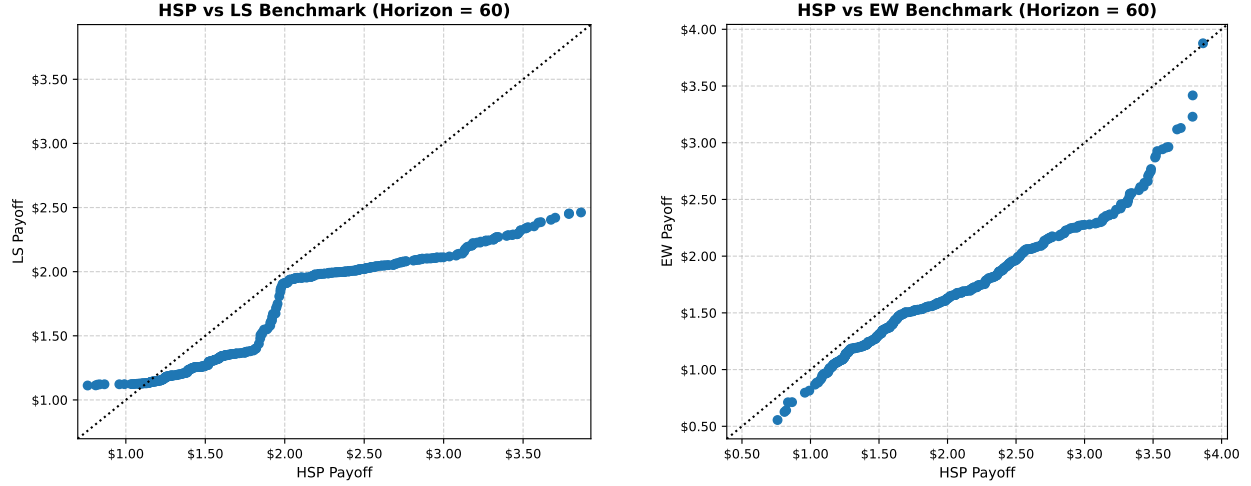


Figure A1: Horizon-specific vs. Benchmark Portfolios (Out-of-sample)

This figure shows the out-of-sample properties of the horizon-specific portfolios (HSP) and the two benchmark portfolios across different investment horizons. The reported statistics are estimated over the investment horizons and include A:(annualized) certainty-equivalent return, B: (annualized) mean return, C: (annualized) standard deviation, D: variance ratio, E: skewness, and F: Kurtosis. The sample period spans from July 1963 to December 2023 and the effective out-of-sample window starts from $2k + 118$ months after the starting of the sample sample.

Panel A: $k = 60$



Panel B: $k = 120$

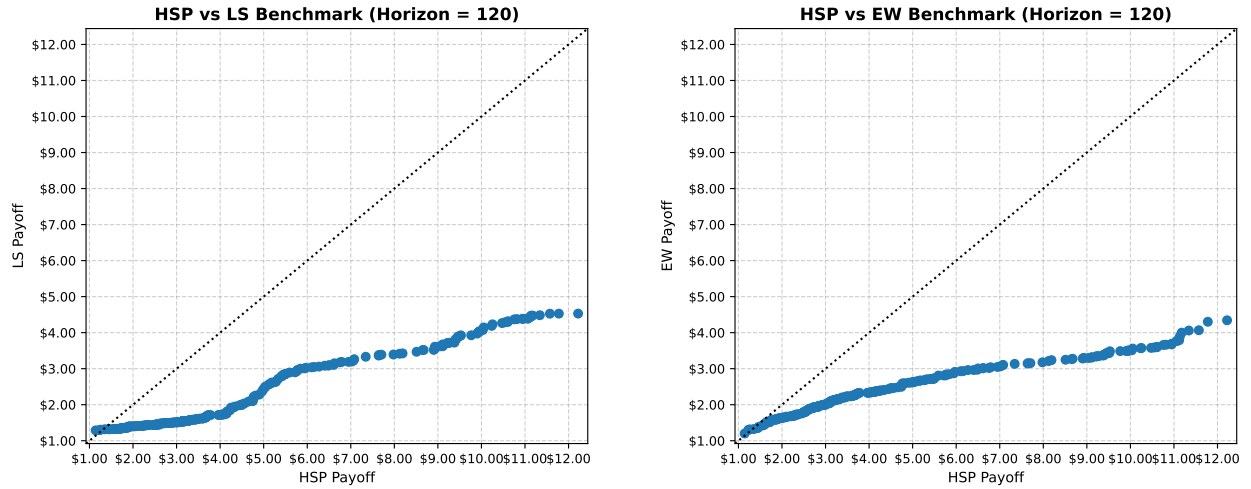


Figure A2: Q-Q Plots of Long-run Payoffs: HSP vs. Benchmarks (Out-of-Sample)

This figure presents Q-Q plots comparing the empirical distributions of out-of-sample 60-period payoffs (Panel A) and 120-period payoffs (Panel B). In each subplot, the x -axis shows the sorted k -period payoffs from horizon-specific portfolio (HSP), and the y -axis shows the sorted k -period benchmark payoffs. Each point represents a matched order statistic—i.e., the i -th smallest HSP payoff against the i -th smallest benchmark payoff. Each plot is based on 429 observations for $k = 60$ and 249 observations for $k = 120$. The 45-degree reference line indicates equality in distributions. Points below (above) the line suggest HSP outperforms (underperforms) the benchmark at that part of the distribution. The sample period spans from July 1963 to December 2023 and the effective out-of-sample window starts from $2k + 118$ months after the starting of the sample sample.

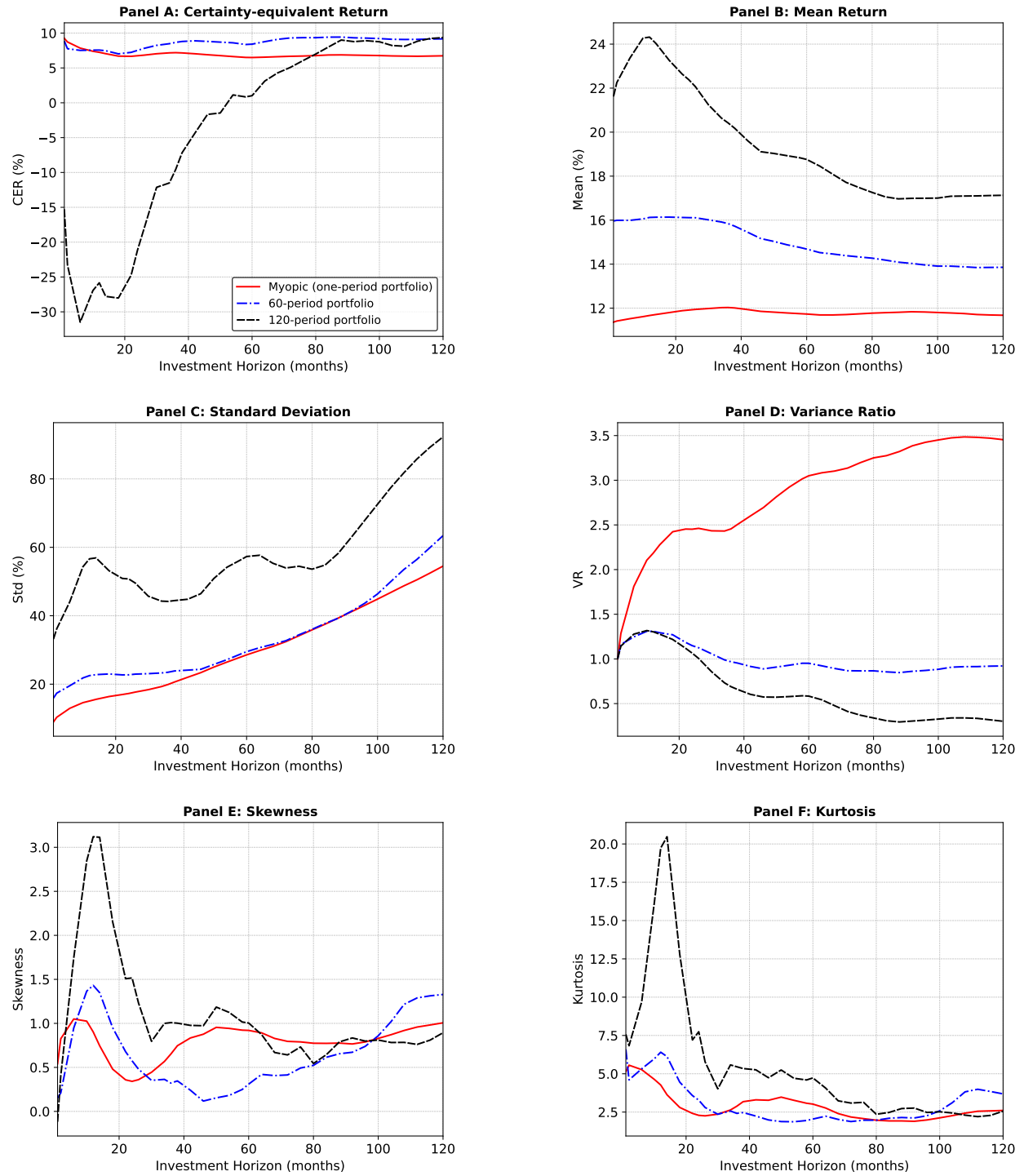


Figure A3: Myopic vs. Longer-horizon Portfolios (Out-of-Sample)

This figure shows the out-of-sample properties the one-period (myopic), 60-period, and 120-period portfolios. The reported statistics are estimated over the investment horizons and include A:(annualized) certainty-equivalent return, B: (annualized) mean return, C: (annualized) standard deviation, D: variance ratio, E: skewness, and F: Kurtosis. The sample period spans from July 1963 to December 2023, and the statistics are for a common period starting in June 1978, when the in-sample portfolio of the longest horizon strategy ($k = 120$) first becomes available.

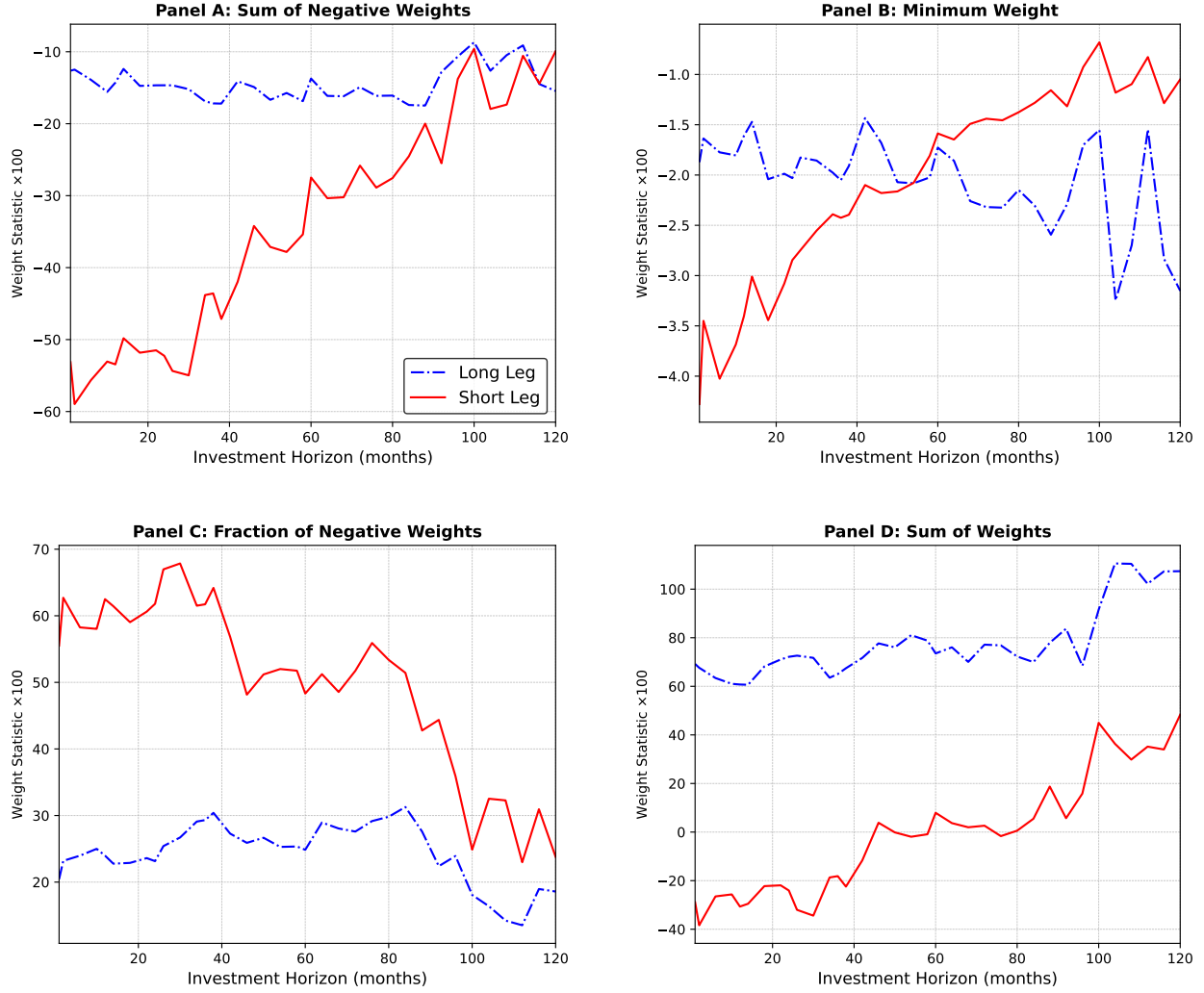


Figure A4: Long- and Short-leg Anomalies in Horizon-Specific Portfolios (Out-of-Sample)

This figure plots the out-of-sample weight statistics of long- and short-leg anomalies in optimal in-sample portfolios. The statistics are time series averages of cross-sectional variables. The weight statistics include: A: sum of negative weights, B: minimum weight, C: fraction of negative weights, and D: sum of weights. The sample period spans from July 1963 to December 2023, and the weight statistics are for a common period starting in May 1978, when the longest horizon portfolio weight ($k = 120$) first becomes available.