

Pricing News and No News with Heterogeneous Beliefs

Abstract

We study a general-equilibrium economy where a continuum of agents have heterogeneous beliefs along two dimensions: the intensity and the content of news. When intensity disagreement dominates, implied volatility appears persistent—quiet periods shift wealth toward calm-world believers, compressing risk-neutral probability of extreme events and raising asset prices. When content disagreement dominates, volatility appears mean-reverting—news shift wealth between optimists and pessimists. The information structure of news process matters for the persistence of intensity disagreement: in a Poisson limit, intensity disagreement can last but it is eliminated in a Brownian limit. The framework endogenizes implied volatility smirk and U-shaped cross-section of derivative positions across pessimists and optimists.

JEL codes: D53, D83, D84, G12, G41

Keywords: Heterogeneous beliefs, news arrival intensity, wealth dynamics, market selection, volatility persistence.

“I am an old man and have known a great many troubles,
but most of them never happened.”

1 Introduction

What moves markets is often not news itself, but the anticipation of whether news will arrive. During periods of heightened macroeconomic stress—following a geopolitical shock or a major bank failure—uncertainty spikes, and investors speculate on the likelihood of further systemic events. Similarly, around scheduled events—FOMC meetings, earnings announcements, inflation releases—some investors position for dramatic fundamental shifts while others confidently bet on a calm post-announcement market. Often, the events unfold without surprises: implied volatility falls and prices drift upward. At other times, the exact same event window detonates. These patterns hint that investors’ belief heterogeneity may be multi-dimensional: they disagree not only about whether the future is good or bad, but also about whether the future will be *eventful or calm*.

This paper formalizes this distinction. We separate belief heterogeneity about the *content* of news (what will be revealed) from heterogeneity about the *arrival intensity* of news (whether a market-moving innovation is likely to arrive over a given horizon). The distinction matters because the second dimension is naturally traded through derivatives: investors can hold similar directional views and yet take opposing positions on *convexity* if they disagree about the probability of large moves. Both the realization and non-realization of news reallocate wealth across agents with heterogeneous beliefs. These wealth shifts, in turn, reshape the risk-neutral distribution of asset prices and payoffs. In short: disagreement about *eventfulness* is not a sideshow; it is a state variable that can be priced and can feed back into prices and volatility through market selection.

We study an exchange economy in discrete time. Each period, the fundamental news process moves along a trinomial tree: *no news*, *good news*, or *bad news* regarding the terminal payoff of the market. Investors agree that these are the relevant states and how they are related to the terminal payoff. But they *agree to disagree* about the probability

of each outcome. Each investor’s belief is indexed by two parameters: $v \in (0, 1)$ governing the probability that news arrives at all (intensity), and $u \in (0, 1)$ governing the conditional probability that arriving news is good (content) for the terminal payoff. Investors trade three non-redundant assets that spans the payoff space: the market, a risk-free bond, and a derivative security.

Two modeling choices deliver tractability without shutting down the economics we care about. First, we assume logarithmic preferences throughout this paper, which imply myopic portfolio choice and allow equilibrium prices to be expressed as wealth-weighted harmonic means of future payoffs. Though, such assumption is not essential for the core mechanism.¹ Second, we model the cross-section of beliefs as a continuum with a flexible parametric distribution. These ingredients yield closed-form expressions for (i) the evolution of the wealth-weighted belief distribution (“Mr. Market”), (ii) equilibrium prices, and (iii) risk-neutral variance as a function of two sufficient statistics: the cumulative news count and the cumulative net news.

The central mechanism is simple. Because beliefs among agents differ, investors take opposite sides of trades. Optimists go long in the market and pessimists go short; those who think the market will be calm can short volatility against those who think the future would be eventful. In each period, the hindsight winner gains wealth and become more influential in pricing assets and volatility. Disagreement about content primarily reallocates wealth in response to directional outcomes; disagreement about intensity reallocates wealth in response to whether large moves arrive at all. These two channels move option-implied variance in opposite directions and imply different dynamics for volatility.

Main results. Our first result is the closed-form characterization of equilibrium price and risk-neutral variance. Prices are determined by the wealth-weighted belief distribution over terminal outcomes, and risk-neutral variance reflects the wealth-averaged belief of market risk premium.

Our second result is a dichotomy. When disagreement is primarily about *arrival inten-*

¹In the Internet Appendix, we extend our framework to general CRRA preferences and show the core mechanism persists, with the main results extending accordingly.

sity, more frequent realized news sustains dispersion in beliefs and generates persistence in risk-neutral variance; quiet periods transfer wealth toward low- v investors, compress risk-neutral tail probabilities, and support valuations. In this sense, “no news” becomes good news for risky assets precisely when markets disagree about eventfulness. When disagreement is primarily about *content*, in contrast, frequent news accelerates market selection in the content dimension and induces mean reversion in risk-neutral variance.²

Our third result establishes *when* disagreement can survive in equilibrium over a finite horizon. The wealth share of an agent with incorrect beliefs decays exponentially at a rate determined by the Kullback–Leibler divergence between the agent’s beliefs and the truth. This rate decomposes additively into a component for arrival-intensity errors and a component for content errors, weighted by the true arrival rate. In the Poisson limit—where news is rare but impactful—the contribution of the intensity component per trade tends toward zero, so agents with wrong volatility beliefs survive indefinitely and permanently influence prices. In the Brownian limit—where information flows continuously—the intensity component remains strictly positive, and market selection rapidly eliminates all disagreement about arrival rates.

Our fourth result links beliefs to derivative positions. After aggregating investors’ derivative positions by their subjective beliefs on excess returns, a U-shaped pattern emerges: investors with extreme directional views—both optimistic and pessimistic—simultaneously demand exposure to large market moves, while moderate investors become their natural counterparties.

We then extend the discrete-time environment to a continuous-time jump–diffusion limit that nests both Brownian and Poisson limits. This extension unifies continuous information flow and discrete innovations, and shows how the endogenous risk-neutral distribution generates asymmetric option prices and implied-volatility skew through state-dependent wealth transfers.

Finally, in a multi-asset economy, we show that speculation localized in one asset prop-

²This mechanism is distinct from the volatility feedback channel of [Campbell and Hentschel \(1992\)](#), in which a representative agent’s risk aversion generates a negative price response to anticipated volatility.

agates to the aggregate market through the wealth channel. This spillover compresses the market beta of non-speculative assets and can transform them into hedges—even absent fundamental linkage—generating safe-haven behavior and excess risk-neutral correlation in equilibrium.

Relevant literature. Our paper builds most directly on [Martin and Papadimitriou \(2022\)](#), who study a binary tree economy with a continuum of log-utility investors who agree to disagree about news content. Their framework generates rich dynamics for sentiment, prices, and volume through wealth redistribution. Because news in a binary tree is one-dimensional—each period is either “up” or “down”—investors can disagree about direction but not about whether something will happen. Our model introduces a “no news” state, moving beyond the two-node structure and decoupling disagreement about the mean from disagreement about the risk or distribution of outcomes. This additional dimension separates disagreement about the content of news from disagreement about its arrival intensity. This dimension is economically distinct: it maps into demand for state-contingent convexity rather than directional trading and generates volatility persistence rather than mean reversion—dynamics that are absent in the binary setting.

A parallel literature studies disagreement about volatility directly. [Atmaz and Buffa \(2023\)](#) connect volatility disagreement to variance risk premia and the leverage effect in a two-agent diffusion setting; [Cao and Ou-Yang \(2008\)](#) model options trading driven by differences in signal precision. The market selection literature ([Sandroni, 2000](#); [Blume and Easley, 2006](#); [Kogan et al., 2006](#); [Borovička, 2020](#)) characterizes long-run survival under incorrect beliefs. We show the selection pressure is affected by information structure of news process and differs across disagreement dimensions: errors about arrival intensity are always penalized, while errors about content are penalized proportionally to the true arrival rate.

More broadly, the paper contributes to the literature on speculative trading under heterogeneous beliefs ([Harrison and Kreps \(1978\)](#); [Scheinkman and Xiong \(2003\)](#); [Simsek \(2013\)](#)) by identifying a new channel—arrival-intensity disagreement—through which speculation affects equilibrium prices and volatility dynamics. We work with a contin-

uum of beliefs, as in [Geanakoplos \(2010\)](#) and [Atmaz and Basak \(2018\)](#), which allows us to separately identify the effects of wealth-weighted disagreement and the average belief. The tractability of our closed-form results benefits from log utility, which we view as a disciplined benchmark given the empirical support for the connection between risk-neutral variance and the equity premium documented by [Martin \(2017\)](#), [Kremens and Martin \(2019\)](#), [Martin and Wagner \(2019\)](#), and [Gao and Martin \(2021\)](#). [Ni, Pan and Poteshman \(2008\)](#) provide direct empirical evidence of informed volatility trading in options markets, consistent with the mechanism we formalize.

The paper proceeds as follows. Section 2 details the setup of the trinomial tree with heterogeneous beliefs. Section 3 solves the equilibrium, characterizes how disagreement shapes prices, volatility, and portfolio positions, analyses the persistence of disagreement over a finite horizon, and discusses the extensions to a setting with CRRA preferences. Section 4 extends the framework to richer environments—a general M -node tree, a jump–diffusion limit, and a multi-asset economy. Section 5 concludes. Proofs are in Appendix A.

2 Setup

We consider a finite-horizon discrete-time economy with dates t ranging from 0 to T . The state of the world evolves along a trinomial tree, where each node branches into three potential outcomes for the subsequent period: ‘no news’, ‘good news’, or ‘bad news’ regarding the terminal payoff of the underlying asset. From any single investor’s perspective, the probability distribution over these outcomes is identical and independent across periods.

Let n_{mt} , n_{gt} , n_{bt} count no, good, and bad news from 0 to t . Since $t = n_{gt} + n_{mt} + n_{bt}$, two state variables suffice to characterize the equilibrium: news count ν_t and fundamental level n_t

$$\nu_t = n_{gt} + n_{bt} , \quad n_t = n_{gt} - n_{bt} . \tag{1}$$

The news count ν_t proxies realized fundamental volatility, as each news event shifts n_t by

± 1 . Figure 1a illustrates (ν_t, n_t) along some sample paths between $t = 0$ and $t = 4$. The red path experiences one good and one bad news event before two quiet periods at $t = 3, 4$; the black path experiences two good news events and is otherwise quiet; the blue path, the most volatile of the three, experiences two good and two bad news events. At $t = 4$, the red and blue paths share the same net news $n_4 = 0$, but the blue path has a higher ν_4 ; the black and red paths share the same $\nu_4 = 2$ but the red has a lower n_4 .

Agents hold heterogeneous, dogmatic beliefs about state evolution and agree to disagree. Each agent's belief is parameterized by a vector $(v, u) \in (0, 1) \times (0, 1)$, where v is the probability of news arrival (good or bad), and u is the conditional probability that arriving news is good: v captures perceived news arrival intensity and u reflects relative optimism. The two parameters yield probabilities for no news (m), good news (g), and bad news (b):

$$m = 1 - v, \quad g = vu, \quad b = v(1 - u), \quad \text{or conversely} \quad v = g + b, \quad u = \frac{g}{g + b}. \quad (2)$$

The mass of agents with belief (v, u) follows a distribution $f_0(v, u)$, and the wealth distribution across the population evolves over time t , which we call $f_t(v, u)$.³

We assume the population (or initial wealth) distribution over (v, u) is separable. A natural parametric choice is the product of two Beta distributions.

$$f_0(v, u) = f_0(v)f_0(u) = \frac{v^{\alpha_{v0}-1}(1-v)^{\beta_{v0}-1}}{B(\alpha_{v0}, \beta_{v0})} \frac{u^{\alpha_{u0}-1}(1-u)^{\beta_{u0}-1}}{B(\alpha_{u0}, \beta_{u0})}. \quad (3)$$

The Beta family, $B(\alpha, \beta)$ with $\alpha > 0$, $\beta > 0$, is flexible enough to generate uniform, U-shaped, or highly concentrated belief distributions on $(0, 1)$. The sum $\alpha + \beta$ governs the degree of agreement—with $\alpha + \beta \rightarrow \infty$ corresponding to no disagreement—while the ratio α/β controls skewness. Figure 1b illustrates several configurations.

Asset payoffs are structured to ensure dynamic market completeness. Agents trade

³The trinomial tree setup could be linked to the recombining binary tree (as in [Martin and Papadimitriou \(2022\)](#)) by mapping two binary periods to one trinomial period—for an agent with binary up-move probability $h \in [0, 1]$, good news corresponds to up-up ($g = h^2$), bad news to down-down ($b = (1 - h)^2$), and no news to up-down or down-up ($m = 2h(1 - h)$)— $u = h^2/[1 - 2h + 2h^2]$ and $v = h^2 + (1 - h)^2 \geq 1/2$ restrict agents' beliefs to a one-dimensional subset of $(v, u) \in [0, 1] \times [0, 1]$ with $v \geq 0.5$. We discuss the link and difference between trinomial and binary trees in Internet Appendix I.A.5.

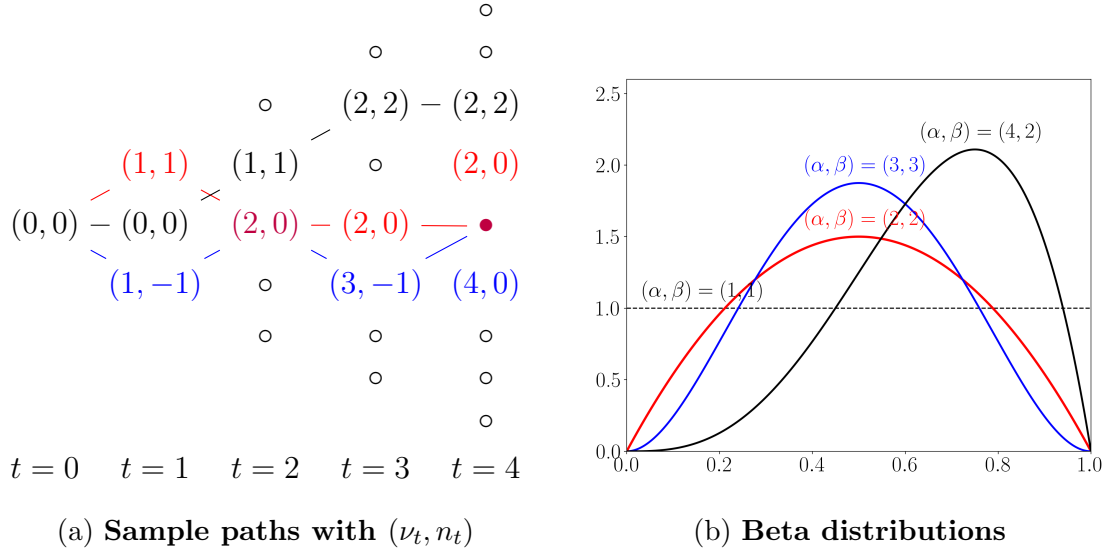


Figure 1: **Examples of news sample paths and beta distribution of beliefs**

three non-redundant assets: (1) a risk-free bond with zero net supply and gross return normalized to one, (2) a long-lived underlying asset in unit supply, and (3) a derivative on the underlying in zero net supply. All agents exhibit logarithmic preferences over terminal wealth. We use p_t to denote the time- t price of the underlying market asset, and q_t the price of the one-period derivative with payoff x_{t+1} at $t+1$. At the subsequent nodes, underlying prices are $(p_{t+1,m}, p_{t+1,g}, p_{t+1,b})$ and derivative payoffs are $(x_{t+1,m}, x_{t+1,g}, x_{t+1,b})$. The underlying's terminal payoff is $p_T(n_T)$.

In a dynamically complete market, derivative payoffs do not alter wealth allocations across states. To take advantage of this, we work with a derivative contract that has a payoff uncorrelated with the underlying under the risk-neutral measure, period by period:

$$\text{Cov}_t^*[x_{t+1}, p_{t+1}] = 0. \quad (4)$$

Although this choice is abstract, it is generic. The payoff vector satisfying (4), together with the underlying payoff vector p_{t+1} and the risk-free bond $(1, 1, 1)$, forms a basis for the payoff space.⁴ Any other derivative payoff can therefore be decomposed into positions in the

⁴The bond payoff vector trivially satisfies (4). Any non-bond derivative payoff x_{t+1} cannot be linear in p_{t+1} , since the risk-neutral covariance in (4) would then be proportional to the risk-neutral variance of

bond, the underlying, and the derivative satisfying (4). Importantly, (4) does not constrain the derivative to be uncorrelated with the market under agents' subjective beliefs—only under the risk-neutral measure.

3 Equilibrium

Logarithmic utility implies myopic optimization: an agent with belief (v, u) allocates wealth to $\theta_t^{(v,u)}$ units of the underlying, $\phi_t^{(v,u)}$ of the derivative, and the rest in the risk-free bond, solving

$$\max_{\theta_t, \phi_t} \mathbb{E}_t^{(v,u)} \log \left[w_t^{(v,u)} + \theta_t^{(v,u)}(p_{t+1} - p_t) + \phi_t^{(v,u)}(x_{t+1} - q_t) \right], \quad (5)$$

where $w_t^{(v,u)}$ represents the time t wealth of the individual agent with beliefs (v, u) .

In equilibrium, each agent's subjective probability-weighted marginal rate of substitution pins down the risk-neutral probabilities:

$$m_t^* = m \cdot \frac{w_t^{(v,u)}}{w_{t+1,m}^{(v,u)}}, \quad g_t^* = g \cdot \frac{w_t^{(v,u)}}{w_{t+1,g}^{(v,u)}}, \quad b_t^* = b \cdot \frac{w_t^{(v,u)}}{w_{t+1,b}^{(v,u)}}, \quad (6)$$

where (m_t^*, g_t^*, b_t^*) are risk-neutral probabilities; $w_t^{(v,u)}$ is the wealth of the agent with beliefs (v, u) at time t ; and $w_{t+1,m}^{(v,u)}$, $w_{t+1,g}^{(v,u)}$, and $w_{t+1,b}^{(v,u)}$ denote this agent's wealth at the respective nodes at time $t + 1$.

Optimal allocations align Arrow-Debreu weights with subjective beliefs (m, g, b) . One-period wealth growth is the ratio of subjective belief to the risk-neutral measure: in the medium state, wealth grows by m/m_t^* ; in the good state, by g/g_t^* ; and in the bad state, by b/b_t^* . Aggregate wealth growth equals the market portfolio change, so one-period state-dependent returns satisfy

$$\frac{p_{t+1,m}}{p_t} = \frac{M_t}{m_t^*}, \quad \frac{p_{t+1,g}}{p_t} = \frac{G_t}{g_t^*}, \quad \frac{p_{t+1,b}}{p_t} = \frac{B_t}{b_t^*}. \quad (7)$$

p_{t+1} , which is strictly positive.

Here, (M_t, G_t, B_t) are wealth-weighted average beliefs—investors with those beliefs at t allocate fully to the underlying, whom we call *Mr. Market*, adopting the terminology of [Martin and Papadimitriou \(2022\)](#).⁵

Equation (7) also implies p_t^{-1} is a sentiment-weighted harmonic mean of next-period prices:

$$p_t^{-1} = M_t p_{t+1,m}^{-1} + G_t p_{t+1,g}^{-1} + B_t p_{t+1,b}^{-1}. \quad (8)$$

Thus, sentiment—the wealth-weighted (m, g, b) —drives pricing intertemporally.

3.1 Asset Prices

Although aggregate beliefs are path-dependent, (8) yields tractable pricing. The market price and risk-neutral variance are determined by the wealth-weighted distribution over terminal states, $\Pr(n_T | n_t, f_t)$, which aggregates over all news trajectories from t to T :

Result 1 (Market Price and Risk-Neutral Variance). *Assuming the trinomial tree has T periods and the payoffs are $p_T(n_T)$ with $n_T \in [-T, T]$, the value of the market at time t is given by*

$$p_t^{-1} = \mathbb{E}_t [p_T^{-1}(n_T)] , \quad (9)$$

and the risk-neutral variance of the return from time t to T is given by:

$$\text{Var}_t^*[p_T/p_t - 1] = \mathbb{E}_t [p_T(n_T)] \mathbb{E}_t [p_T^{-1}(n_T)] - 1, \quad (10)$$

where the expectation is taken under the wealth-weighted belief distribution over terminal

⁵The term originates with [Graham \(1949\)](#), who personified the market as an emotional counterparty whose quotes reflect shifting sentiment rather than fundamental value. In our setting, Mr. Market’s beliefs are not irrational but instead aggregate the wealth-weighted beliefs of a heterogeneous population.

states n_T , given by:

$$\Pr(n_T|n_t, f_t) = \sum_{\substack{\forall n_g+n_m+n_b=T-t \\ n_g-n_b=n_T-n_t}} \frac{(T-t)!}{n_m!n_g!n_b!} \int v^{T-t-n_m}(1-v)^{n_m}u^{n_g}(1-u)^{n_b}f_t(v,u)dvdu, \quad (11)$$

conditional on the current fundamental n_t and the wealth distribution f_t .

In limits with no v - or u -disagreement, markets ignore arrivals (n_m) or content (n_g, n_b), reducing $f(v, u)$ to a marginal distribution. Result 1 holds generally for any initial wealth distribution.

We now specialize the initial wealth distribution to the separable Beta form in (3).⁶ Comparing (6)–(7), the time- t wealth distribution is

$$f_t(v, u) = \frac{v^{\alpha_{vt}-1}(1-v)^{\beta_{vt}-1}}{B(\alpha_{vt}, \beta_{vt})} \frac{u^{\alpha_{ut}-1}(1-u)^{\beta_{ut}-1}}{B(\alpha_{ut}, \beta_{ut})}, \quad (12)$$

where $t = n_{gt} + n_{mt} + n_{bt}$ and $(\alpha_{vt}, \beta_{vt}, \alpha_{ut}, \beta_{ut})$ evolve according to the following:

$$\alpha_{vt} = \alpha_{v0} + \nu_t, \quad \beta_{vt} = \beta_{v0} + t - \nu_t, \quad \alpha_{ut} = \alpha_{u0} + \frac{n_t + \nu_t}{2}, \quad \beta_{ut} = \beta_{u0} + \frac{\nu_t - n_t}{2}. \quad (13)$$

The derivations of (12) and (13) are in Appendix A. Under the Beta specification, (11) takes the closed form:

$$\Pr(n_T|n_t, f_t) = \sum_{\substack{\forall n_g-n_b=n_T-n_t, \\ n_g+n_m+n_b=T-t}} \frac{(T-t)!}{n_m!n_g!n_b!} \frac{B(\alpha_{vt} + T - t - n_m, \beta_{vt} + n_m)}{B(\alpha_{vt}, \beta_{vt})} \frac{B(\alpha_{ut} + n_g, \beta_{ut} + n_b)}{B(\alpha_{ut}, \beta_{ut})}. \quad (14)$$

Under the separable Beta specification, heterogeneous agents' pricing is equivalent to that of a representative log-utility agent whose prior is (3) and who updates via Bayes' rule. The representative-agent economy is not dynamically identical—without heterogeneous beliefs there is no trade—but the equivalence highlights a key feature: markets learn

⁶This setting nests the Dirichlet distribution on (m, g, b) when $\alpha_{v0} = \alpha_{u0} + \beta_{u0}$, yielding parameters $(\alpha_{u0}, \beta_{v0}, \beta_{u0})$. The Dirichlet distribution's rotational symmetry implies a weak linkage between content and arrival disagreement; our framework relaxes this.

collectively through wealth redistribution, with hindsight winners amplifying their beliefs in pricing.

Result 2 (Wisdom of the Crowd). *Pricing in the heterogeneous-agent economy is identical to pricing in an economy with a single log-utility representative agent whose prior over the probability of future news processes coincides with the initial wealth distribution of the heterogeneous economy, and who updates beliefs over time via Bayes' rule.*

The crowd's wisdom can never surpass its most informed members. If the belief distribution includes the truth $(v_{\text{truth}}, u_{\text{truth}})$, prolonged data enables Mr. Market to converge to the truth with large N . In contrast, a uniform bias in v or u precludes learning. Under Beta-distributed beliefs, the representative agent's sentiment takes a simple closed form:

$$V_t(\nu_t, n_t) = \frac{\nu_t + \alpha_{v0}}{t + \alpha_{v0} + \beta_{v0}}, \quad U_t(\nu_t, n_t) = \frac{n_{gt} + \alpha_{u0}}{\nu_t + \alpha_{u0} + \beta_{u0}}. \quad (15)$$

When disagreement about v is prominent—i.e., when $\alpha_{v0} + \beta_{v0}$ is not too large— V_t is sensitive to the ratio ν_t/t : higher realized arrival intensity elevates the wealth-weighted v . Conversely, when disagreement is nearly absent ($\alpha_{v0} + \beta_{v0} \rightarrow \infty$), finite values of ν_t and t have negligible effect on V_t , analogous to a Bayesian representative agent with a highly precise prior. When disagreement about u is high, $n_{gt} = \frac{1}{2}(\nu_t + n_t)$ —the proportion of good news among all arrived news—increases the value of U_t . Disagreement in the heterogeneous economy thus maps directly into the learning speed of the Bayesian representative agent.

Example—A trinomial tree with $T = 3$. A three-period example illustrates the core mechanism. We assume agents' beliefs are uniformly distributed in both u and v . Figure 2 reports the pricing and sentiment on several news sample paths between $t = 0$ and $t = 2$: the black path experiences good news followed by no news; the red path is the calmest outcome where no news occurs; the blue path is the most volatile and leads to the same net news as the red one—bad news at $t = 1$ followed by good news. At $t = 1$, good or bad news raises V from 0.5 to 0.67, as high- v agents outperform low- v ones; good news also shifts wealth to high- u agents, raising U , while bad news lowers it. No news preserves U

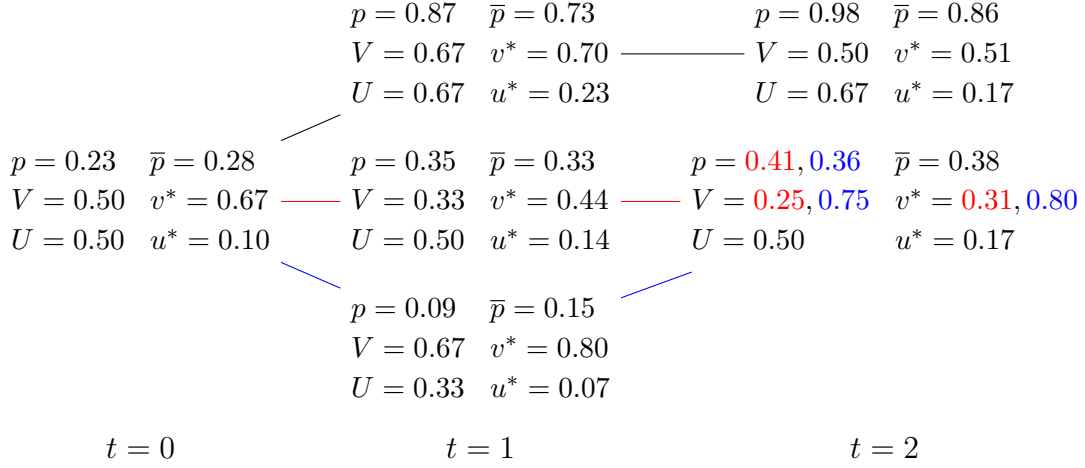


Figure 2: **An example with $T = 3$.** In this example agents’ beliefs u and v are both uniformly distributed. p is the market price, $\bar{p}$ is the price in the homogeneous economy where all agents hold the median belief $(v, u) = (1/2, 1/2)$. (v, u) represents Mr. Market’s beliefs and (v^*, u^*) is the risk-neutral measure. The nodes at $T = 3$ are omitted from the figure to make it compact: top to bottom nodes are $p = (4.95, 2.22, 1.00, 0.45, 0.20, 0.09, 0.04)$.

(no content update) but depresses V , elevating prices—“no news is good news”. At $t = 2$, no news on both the black path and the red path further depresses V , as volatility sellers gain more. The blue path’s good news offsets the $t = 1$ bad news ($n_2 = 0$, like red), but a higher ν_2 on the blue path yields $V_2 = 0.75$ (vs. red’s 0.25), depressing prices relative to the red path despite the good news at $t = 2$.

Bayesian Individuals. The model can be extended to allow heterogeneous individuals to update their beliefs over time according to Bayes’ rule. In this setting, (v, u) serve as indices of agents, while the actual news content and arrival beliefs are characterized by $(\tilde{v}, \tilde{u})$. The initial mass of the population across heterogeneous agents is given by (3), and agent (v, u) holds prior beliefs $\tilde{v} \sim \text{Beta}(\zeta v, \zeta(1 - v))$ and $\tilde{u} \sim \text{Beta}(\zeta u, \zeta(1 - u))$, where the Beta distribution is defined on $(0, 1)$ and its parameters determine the expected value and confidence of the belief. In this parametrization, the prior mean equals (v, u) for $(\tilde{v}, \tilde{u})$, while the concentration parameter ζ governs the dispersion of beliefs: higher ζ corresponds to more precise priors and lower perceived uncertainty. The value of the market at time t is

given by the same expression as in (9), where the expectation is taken under a generalized distribution of (14), $\Pr(n_T)$:

$$\sum_{\substack{\forall n_g - n_b = n_T - n_t, \\ n_g + n_m + n_b = T - t}} \frac{(T - t)!}{n_m! n_g! n_b!} \int_0^1 \frac{B(\zeta v + T - n_m, \zeta(1 - v) + n_m)}{B(\zeta v, \zeta(1 - v))} \frac{B(\zeta u + n_g, \zeta(1 - u) + n_b)}{B(\zeta u, \zeta(1 - u))} f_0(u, v) du dv.$$

In the limit as $\zeta \rightarrow \infty$, agents hold dogmatic beliefs, the terminal states distribution converges to (14), and the value of the market converges accordingly.

3.2 The Effects of Disagreements

News arrival exerts two distinct effects on the market. First, a high historical count of news increases the market's average belief about the future news arrival rate, V_t . This upward shift in expectations places downward pressure on asset prices. Second, frequent news arrivals lead to a concentration of wealth among agents whose beliefs are correct ex-post, and reduce disagreement over news content. The level of “agreement” among investors, $\alpha_{ut} + \beta_{ut} = \alpha_{u0} + \beta_{u0} + \nu_t$, increases with ν_t . The overall impact depends on the relative strength of these two channels, which is, in turn, determined by the degree of disagreement among investors regarding the news arrival rate.

We specialize to an exponential terminal payoff, $p_T(n_T) = e^{\sigma n_T}$, under which the dichotomy takes a sharp form.⁷

Result 3 (The Dichotomy of News Arrival Intensity Disagreement). *Consider realized histories that share the same wealth-weighted expected fundamental growth, $G_t - B_t$. The impact of realized news count, ν_t , on asset valuations and risk-neutral variance is dichotomous:*

- i) When disagreement about the news arrival rate dominates disagreement about news content ($\alpha_{v0} + \beta_{v0} \leq \alpha_{u0} + \beta_{u0}$), a higher realized news count ν_t strictly increases the risk-neutral variance $\text{Var}_t^*[p_T/p_t - 1]$ and lowers the normalized asset price $p_t/e^{\sigma n_t}$.*

⁷This specification lends itself naturally to a continuous-time limit, where the interval between trading opportunities shrinks to zero. Unlike an arithmetic payoff, the exponential (or geometric) form guarantees that the payoff remains strictly positive and cannot fall below zero.

- ii) *When all agents agree on the news arrival rate, a higher realized news count ν_t strictly reduces the risk-neutral variance $\text{Var}_t^*[p_T/p_t - 1]$ and increases the normalized asset price $p_t/e^{\sigma n_t}$.*

The intuition is direct. Under high arrival-intensity disagreement, each news event triggers large wealth transfers—particularly away from derivative sellers, whose short-convexity positions are exposed to any discrete move. Wealth flows toward agents with extreme views, dispersing the belief distribution and amplifying implied volatility. Under low arrival-intensity disagreement, derivatives are thinly traded and news instead concentrates wealth among agents with accurate directional views, compressing the belief distribution. This dichotomy provides a structural mechanism for the asymmetric response of implied volatility around scheduled events.⁸

No news becomes good news when disagreement about news arrival intensity is high. The market contains agents who assign large probability to extreme outcomes and demand substantial compensation to hold the market. These agents are marginal price-setters when they control significant wealth. A quiet period—one in which no news arrives—is more surprising to high- v types than to low- v types, so wealth flows disproportionately from volatile-world believers to calm-world believers. As calm agents gain pricing influence, the market’s effective assessment of tail risk falls, risk-neutral variance compresses, and prices rise. “No news is good news” is thus a statement about the endogenous evolution of risk-bearing capacity: quiet periods relax the aggregate demand for tail-risk compensation by shifting wealth toward agents who perceive less tail risk.

The mechanism is visible in the cross-section of wealth. Figure 3 plots the wealth distribution across investor cohorts indexed by their expected return r , comparing the volatile blue path and the quiet red path from Figure 1a under high and low arrival-intensity disagreement (closed-form expressions are in Internet Appendix I.A.2, Result IA1).

The same realized volatility generates opposite effects on the wealth distribution de-

⁸An empirical literature documents that option-implied variance reliably increases in the days prior to major announcements (such as FOMC meetings) and collapses immediately afterward, accompanied by a positive drift in the underlying index—often summarized as the “pre-FOMC announcement drift” (Lucca and Moench, 2015; Hu, Pan and Wang, 2022).

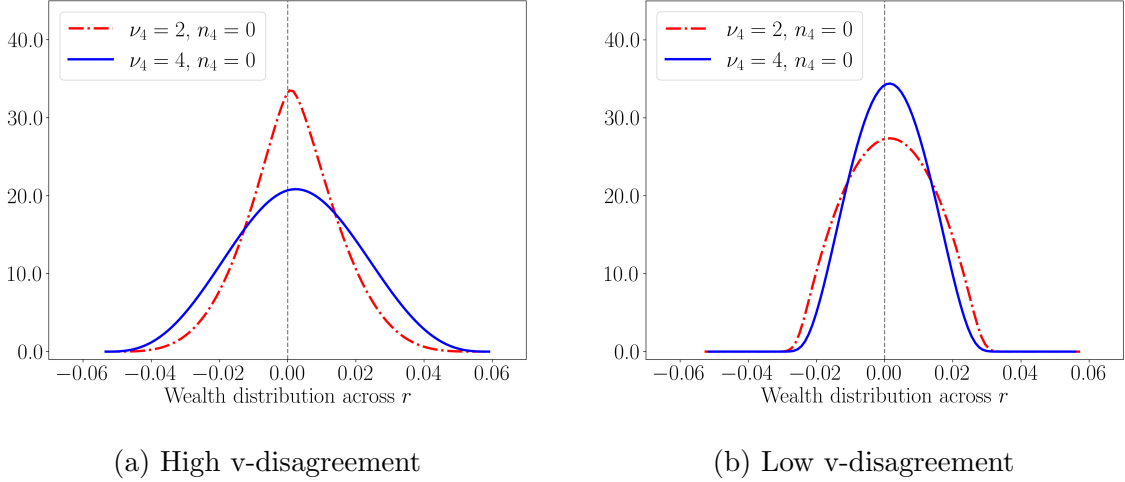


Figure 3: **Wealth distribution across investor cohorts by expected return r .** The figure illustrates the wealth distribution across investor cohorts as a function of the expected return r . The left panel (3a) assumes a uniform initial wealth distribution across agents, with $(\alpha_{v0}, \beta_{v0}, \alpha_{u0}, \beta_{u0}) = (1, 1, 1, 1)$. In contrast, the right panel (3b) corresponds to low disagreement regarding the news arrival rate, with parameters $(\alpha_{v0}, \beta_{v0}, \alpha_{u0}, \beta_{u0}) = (100, 100, 1, 1)$. The terminal payoff at $T = 6$ is given by the exponential function $e^{0.05n_T}$. See Figure 1a for an illustration of the possible sample paths leading to the corresponding wealth distributions and gamma positions at $t = 4$.

pending on the regime. Under high arrival-intensity disagreement (Figure 3a), the volatile blue path ($\nu_4 = 4$) disperses wealth toward agents with extreme views, widening the distribution relative to the quiet red path ($\nu_4 = 2$)—consistent with point *i*) of Result 3. Under low arrival-intensity disagreement (Figure 3b), the same volatile path instead concentrates wealth among agents with accurate directional beliefs, narrowing the distribution—consistent with point *ii*). This reversal is the core empirical prediction: whether news amplifies or dampens implied volatility depends entirely on the prevailing regime of arrival-intensity disagreement.

3.3 How Persistent Are The Disagreements?

Because market prices reflect wealth-weighted beliefs (Result 2), a key question is how persistent disagreement over arrival intensity can be when trading opportunities grow but the underlying information remains unchanged.

A key object governing belief selection in this environment is the Kullback–Leibler (KL) divergence. For Bernoulli distributions with parameters a and b , it is defined as $KL(a, b) = a \log \frac{a}{b} + (1-a) \log \frac{1-a}{1-b}$. The KL divergence determines the exponential rate at which agents with incorrect beliefs lose wealth in the limit of frequent trading.

Result 4 (Decay Rate of Individual Wealth Share). *Assume there exists a true data-generating process for fundamental news characterized by (v_{true}, u_{true}) . The wealth share of an agent with beliefs (v, u) decays exponentially at rate $KL(v_{true}, v) + v_{true} KL(u_{true}, u)$ per trading period.*

Result 4 characterizes wealth decay *per trading period*. Whether agents with incorrect beliefs are driven out over a finite horizon therefore depends on how this decay rate behaves as trading opportunities increase. The asymmetric structure of the decay rate has a natural interpretation. Errors in beliefs about the arrival rate of news (v) are always penalized, whereas errors about the direction of news (u) matter only when news actually arrives and are therefore weighted by v_{true} . When the true arrival rate of news is low, accurate beliefs about volatility are thus far more important for survival than accurate beliefs about direction.

A Poisson Limit. The Poisson limit both sharpens the dichotomy in Result 3 and illustrates the survival mechanism: prices and risk-neutral variance admit closed-form expressions, and the KL penalty for intensity errors tends toward zero per trading period. We partition $[0, T]$ into $N = T/\delta$ periods. Each piece of news is assumed to have a substantial impact on fundamentals, and the terminal payoff remains identical to the discrete-time case: $p_T = e^{\sigma n_T}$. With α_{v0} , α_{u0} , and β_{u0} held constant and independent of N , and β_{v0} defined as $\beta_{v0} = \alpha_{v0} (\lambda^{-1} \frac{N}{T} - 1)$, the cross-sectional averages of (v, u) are given by $\mathbb{E} v = \lambda \delta$ and $\mathbb{E} u = \alpha_{u0}/(\alpha_{u0} + \beta_{u0})$. In the limit as $N \rightarrow \infty$ and $\delta \rightarrow 0$, the counts of “good news” and “bad news” events follow Poisson distributions. Taking the limit $N \rightarrow \infty$, we obtain the market price and risk-neutral variance.

Result 5 (Poisson Limit of Trinomial Tree with Heterogeneous Beliefs). *In the Poisson limit, the market price at time t is given by*

$$p_t = e^{\sigma n_t} [1 - \Lambda_t(e^\sigma - 1)]^{\alpha_{vt}} {}_2F_1\left(\alpha_{vt}, \alpha_{ut}; \alpha_{ut} + \beta_{ut}; \frac{e^\sigma - e^{-\sigma}}{e^\sigma - 1 - \Lambda_t^{-1}}\right)^{-1}, \quad (16)$$

where $\Lambda_t = (T - t)/(\alpha_{v0}\lambda^{-1} + t)$, ${}_2F_1(\dots)$ denotes the Gauss hypergeometric function, and the expressions for α_{vt} , α_{ut} , and β_{ut} are provided in (13).

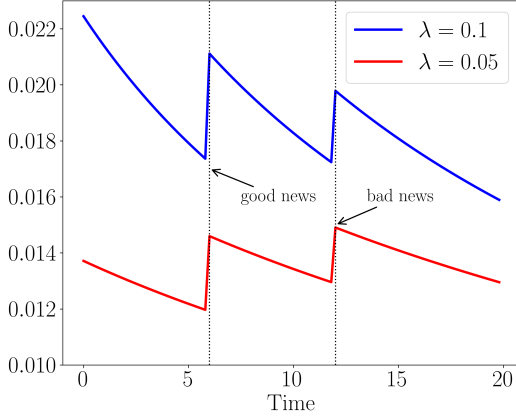
The risk-neutral variance of the return from t to T is:

$$\text{Var}_t^*[p_T/p_t] = p_t^{-1} e^{\sigma n_t} [1 + \Lambda_t(1 - e^{-\sigma})]^{-\alpha_{vt}} {}_2F_1\left(\alpha_{vt}, \alpha_{ut}; \alpha_{ut} + \beta_{ut}; \frac{e^\sigma - e^{-\sigma}}{1 + \Lambda_t^{-1} - e^{-\sigma}}\right) - 1. \quad (17)$$

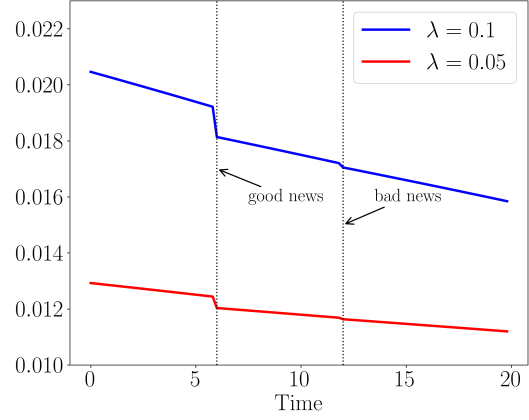
Figure 4 illustrates Result 5 for $T = 20$, $\alpha_{u0} = \beta_{u0} = 1$, and terminal payoff $e^{0.05n_T}$. It presents trajectories for the implied volatility $\sqrt{\text{Var}_t^*[p_T/p_t]/(T - t)}$ following a “good news” event at $t = 6$ and a “bad news” event at $t = 12$, under high and low disagreement about the news arrival rate v . The implied volatility also equals the strike of the simple variance swap of Martin (2011).⁹ In the left panel, implied volatility rises after news arrivals, following point *i*) from Result 3. In contrast, the right panel shows implied volatility falling after news events, reflecting point *ii*) of Result 3.

A Diffusion Limit. The interval $[0, T]$ is partitioned into $N = T/\delta$ periods, each of length δ . The terminal payoff is defined as $p_T(n_T) = e^{\sigma\sqrt{\frac{T}{N}}n_T}$, reflecting that each piece of news has a smaller effect on the fundamental, which is scaled as $\sqrt{T/N}n_T$. This is the same setting as in Cox, Ross and Rubinstein (1979). As N grows, we scale the wealth distribution parameters— $\alpha_{v,0} = \lambda\theta N$, $\beta_{v,0} = \alpha_{v,0}(\lambda^{-1} - 1)$ —to concentrate v -beliefs around the true arrival rate. We impose this not merely for analytical convenience, but as an equilibrium necessity: if v -dispersion were maintained in the continuous limit, according to Result 4, the *infinite frequency* of trading opportunities would instantly impoverish all agents except

⁹The simple variance swap pays $\mathcal{V}_t/p_t^2$, where $\mathcal{V}_t = \sum_{s=t}^{T-1} (p_{s+1} - p_s)^2$. Its time- t price equals the risk-neutral variance $\text{Var}_t^*[p_T/p_t - 1]$.



(a) High v -disagreement



(b) Low v -disagreement

Figure 4: Implied volatility trajectories. The figure plots the implied volatility from t to T under high disagreement ($\alpha_{v0} = 2$) and low disagreement ($\alpha_{v0} = 1000$) about news arrival rate, following a “good news” event at $t = 6$ and a “bad news” event at $t = 12$. The parameters are $\alpha_{u0} = \beta_{u0} = 1$, and the terminal payoff at $T = 20$ is $e^{0.05n_T}$. Blue lines correspond to a higher expected news arrival rate $\lambda = 0.1$, while red lines correspond to a lower rate $\lambda = 0.05$. We compute this per-period implied volatility as $\sqrt{\text{Var}_t^*[p_T/p_t]/(T-t)}$.

those holding the true belief v_{true} . Thus, the Brownian limit isolates an economy where only disagreement about continuous news content (θ) survives to affect risk-neutral variance.

We set $\alpha_{u0} = \frac{1}{2}\lambda\theta N + \frac{1}{2}\eta\sqrt{N}$ and $\beta_{u0} = \frac{1}{2}\lambda\theta N - \frac{1}{2}\eta\sqrt{N}$. The cross-sectional average of (v, u) becomes: $\mathbb{E} v = \lambda$, $\mathbb{E} u = \frac{1}{2} + \frac{\eta}{2\lambda\theta}N^{-\frac{1}{2}}$, where θ captures disagreement about news content, and the model reduces to the homogeneous belief case when $\theta \rightarrow \infty$. In the large- N limit, prices and risk-neutral variance take closed form.

Result 6 (Brownian Limit of Trinomial Tree with Heterogeneous Beliefs). *In the Brownian limit, the market price at time t is*

$$p_t = \exp \left[\frac{1-\phi}{\theta+\phi} \eta \sigma \sqrt{T} + \frac{\theta+1}{\theta+\phi} \eta_t \sigma \sqrt{\phi T} - \frac{1}{2} \frac{\theta+1}{\theta+\phi} (1-\phi) \lambda \sigma^2 T \right]. \quad (18)$$

where $\phi = t/T$ and $\eta_t = n_t/\sqrt{\phi N}$.

The risk-neutral variance of the return from t to T is given by:

$$\text{Var}_t^*[p_T/p_t] = \exp\left(\frac{\theta+1}{\theta+\phi}(1-\phi)\lambda\sigma^2T\right) - 1. \quad (19)$$

The Brownian limit corresponds to a binary tree model with only content disagreement: everyone agrees that news arrives with probability λ .¹⁰ Risk-neutral variance is deterministic and unaffected by news realizations—precisely because the arrival-rate disagreement has been eliminated by market selection.

These two limits yield opposite conclusions for the survival of intensity disagreement. In the Poisson limit, both v_{true} and v tend to zero, and the KL divergence governing expected wealth growth,

$$KL(v_{\text{true}}, v) = v_{\text{true}} \log\left(\frac{v_{\text{true}}}{v}\right) + (1 - v_{\text{true}}) \log\left(\frac{1 - v_{\text{true}}}{1 - v}\right), \quad (20)$$

approaches zero as $N \rightarrow \infty$. Although wealth shares decay exponentially in trading time, the per-period decay rate converges to zero. As a result, agents with misspecified beliefs can retain non-negligible wealth over any fixed calendar horizon, even as the number of trading opportunities approaches infinity. This mechanism supports persistent disagreement over news arrival intensity and allows such disagreement to materially affect prices.

By contrast, in the Brownian limit, persistent disagreement about the frequency of news arrivals cannot be sustained. The per-period KL divergence remains strictly positive whenever $v \neq v_{\text{true}}$. As trading opportunities increase, agents with incorrect beliefs repeatedly make small but systematic mistakes, driving their wealth shares to zero. Short-maturity volatility derivatives play a central role in this process: by permitting frequent bets on continuous variance, they turn volatility disagreement into repeated trading gains and losses, rapidly reallocating wealth toward agents with more accurate beliefs and rendering residual disagreements irrelevant for prices.

The contrast between the two limits has a broader implication for the role of financial

¹⁰The case when $\lambda = 1$ was studied extensively in [Martin and Papadimitriou \(2022\)](#). At time $t = 0$, equation (18) has a simple expression $p_0 = \exp\left[\frac{\eta}{\theta}\sigma\sqrt{T} - \frac{1+\theta}{2\theta}\lambda\sigma^2T\right]$.

markets in aggregating dispersed beliefs. When fundamentals evolve continuously—the Brownian limit—markets are highly effective information aggregators: frequent trading rapidly concentrates wealth on agents with accurate beliefs, and risk-neutral distributions converge toward fundamentals. When information arrives in discrete lumps—the Poisson limit—the same wealth-dynamics mechanism that disciplines content beliefs is too slow to eliminate intensity disagreement over any finite horizon. Markets fail to reach consensus on *whether an event is likely to happen*. This is not a market failure—agents trade optimally given their beliefs—but it implies that the risk-neutral distribution, and therefore the prices of all state-contingent claims, persistently reflects the wealth-weighted average of potentially incorrect beliefs about eventfulness.

3.4 Portfolio Choice and Convexity Demand

Having established that arrival-intensity disagreement persists in equilibrium, we now characterize how it manifests in observable portfolio positions. For any derivative security payoff satisfying the zero-delta assumption (4), we prove (details in appendix A) that it must be spanned by the risk-free payoff $(1, 1, 1)$ and the vector $(0, b_t^* - B_t, G_t - g_t^*)$:

$$(x_{t+1,m}, x_{t+1,g}, x_{t+1,b}) \in \text{span} \left\{ (1, 1, 1), (0, b_t^* - B_t, G_t - g_t^*) \right\}. \quad (21)$$

This means any component of the derivative payoff proportional to $(1, 1, 1)$ can be replicated independently without affecting the demand for the derivative itself. Consequently, without loss of generality, we may restrict attention to the remaining component and represent the derivative as

$$x_{t+1,m} = 0, \quad x_{t+1,g} = p_t(b_t^* - B_t), \quad x_{t+1,b} = p_t(G_t - g_t^*). \quad (22)$$

The return of this derivative is uncorrelated with the market under the risk-neutral measure—its delta is zero. Therefore, the derivative can be interpreted as a delta-neutral combination of one-period call and put options on the market portfolio, with a strike price at the “no

news” realization $p_{t+1,m}$. Since $(0, b_t^* - B_t, G_t - g_t^*) = (0, 1, 1)(b_t^* - B_t) - (0, 0, 1)(v_t^* - V_t)$, the trading of x_{t+1} at t reflects agents’ two motives in trading options: speculation on both the arrival intensity and content of news.

Because of market completeness, agents’ holdings in Arrow-Debreu securities are isomorphic to positions in the underlying asset and the derivative security. For an agent with beliefs (v, u) , the optimal positions in stock and derivative are given by:

$$\omega_{\Delta,t}^{(v,u)} = \frac{\theta_t^{(v,u)} p_t}{\omega_t^{(v,u)}} = \frac{\mathbb{E}_t^{(v,u)}[p_{t+1}/p_t] - 1}{\text{Var}_t^*[p_{t+1}/p_t]}, \quad \omega_{\Gamma,t}^{(v,u)} = \frac{\phi_t^{(v,u)} q_t}{w_t^{(v,u)}} = \frac{\mathbb{E}_t^{(v,u)}[x_{t+1}/q_t] - 1}{\text{Var}_t^*[x_{t+1}/q_t]}. \quad (23)$$

The expressions in (23) resemble the Merton (1969) formula for risky share in a homogeneous agent setting: risky share = $\mu/\gamma\sigma^2$, where μ is the expected risk premium, $\gamma > 0$ is the agent’s risk aversion, and σ^2 is the variance of return. In Merton (1969), physical and risk-neutral variances coincide. Here, agents disagree about physical variance but agree on its risk-neutral counterpart—so the risk-neutral variance, not the physical one, determines portfolio weights.

Martin (2017) showed, with log agents, the simple risk-neutral variance of the market return equalizes the representative agent’s expectation of the risk premium: $\text{Var}_t^*[p_{t+1}/p_t] = \mathbb{E}_t^{(V_t, U_t)}[p_{t+1}/p_t] - 1$. This allows us to express agents’ stock positions as:

$$\omega_{\Delta,t}^{(v,u)} = \frac{\mathbb{E}_t^{(v,u)}[p_{t+1}/p_t] - 1}{\mathbb{E}_t^{(V_t, U_t)}[p_{t+1}/p_t] - 1}. \quad (24)$$

Agents who are more bullish than Mr. Market deploy more than 100% of their wealth into the stock market, while those who believe the expected excess return is negative go short.

An agent’s derivative exposure, which is *delta-neutral* by construction,¹¹ can be related

¹¹Agents with a Gamma-neutral position have beliefs that satisfy

$$(m, g, b) = \kappa(M_t, G_t, B_t) + (1 - \kappa)(m_t^*, g_t^*, b_t^*),$$

where κ is a scalar. This can be verified by substituting subjective beliefs back into (23).

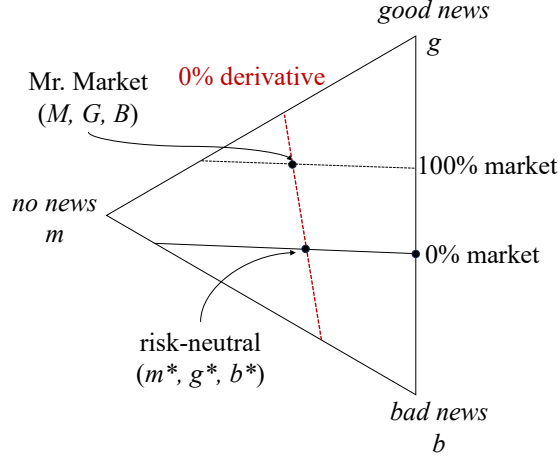


Figure 5: **Agents' range of beliefs on (m, g, b) domain.**

to their beliefs about news intensity and delta exposure:

$$\omega_{\Gamma,t}^{(v,u)} = \frac{v - v_t^*}{1 - v_t^*} + \frac{v_t^* - V_t}{1 - v_t^*} \omega_{\Delta,t}^{(v,u)}. \quad (25)$$

The first component of the gamma portfolio weight comes from the agent's speculation about news arrival intensity, and the second component reflects the agent's speculation about news content through options trading.

Figure 5 maps portfolio allocations in the (m, g, b) belief simplex. Two reference points anchor the figure: Mr. Market (who holds only the underlying) and the risk-neutral measure (who holds no risky assets), marked by the two crosses. The solid '0% market' line separates long and short positions in the market; the dashed '0% derivative' line separates long and short derivative positions. Near the right boundary of the simplex—where agents assign high probability to news—portfolios are dominated by derivative exposure with minimal directional bets through stocks.

The '0% derivative' line passes through both Mr. Market's belief and the risk-neutral measure: agents to its left—those who assign high probability to no news—sell the derivative, while those to its right buy it. A notable feature is the existence of agents with zero stock exposure but large derivative positions, reflecting pure volatility speculation with no

directional view.

Positions in the underlying scale linearly with an investor’s expected return (23), but derivative positions do not.

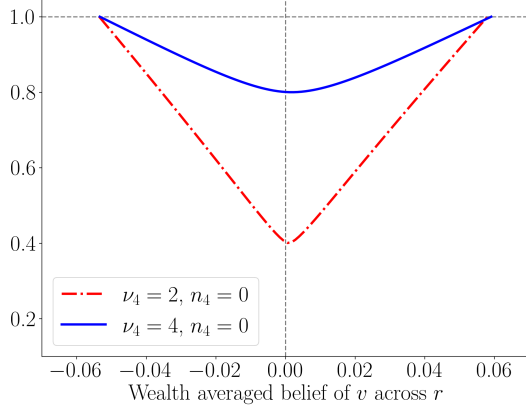
Result 7 (U-Shaped Convexity Demand). *For a cohort of investors sharing the same subjective expected market return r , the wealth-weighted average portfolio weight in the derivative security is determined by their average belief regarding the news arrival rate, $\mathbb{E}[v|r]$:*

$$\mathbb{E} \left[\omega_{\Gamma,t}^{(v,u)} | r \right] = \frac{1}{1 - v_t^*} \left(\mathbb{E}[v|r] - v_t^* + \frac{v_t^* - V_t}{\text{Var}_t^*[p_{t+1}/p_t]} r \right). \quad (26)$$

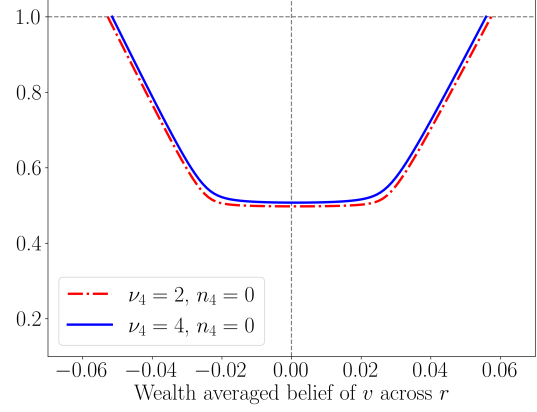
Because the cohort’s average eventfulness belief, $\mathbb{E}[v|r]$, exhibits a pronounced U-shaped relationship with its directional expectation r , derivative positioning is highly asymmetric across the population (closed-form expressions for $\mathbb{E}[v|r]$ are provided in Internet Appendix I.A.2, Result IA1). Investors with extreme directional expectations—both the hyper-optimists anticipating large rallies and the deep-pessimists anticipating crashes—simultaneously hold extreme expectations for the overall news arrival rate ($\mathbb{E}[v|r] \rightarrow 1$ at the boundaries). Investors in both tails of the directional distribution therefore hold large long-derivative positions, resembling out-of-the-money call and put buyers respectively. Market clearing requires that investors with moderate expectations—who assign lower probability to news arrivals—supply the offsetting short-derivative positions.

Figures 6a and 6b illustrate this relationship under high and low disagreement in news arrival rates, using $(\alpha_v, \beta_v) = (1, 1)$ and $(100, 100)$, respectively.¹² While both exhibit the U-shape, the low-disagreement case produces a noticeably flatter curve near the center, reflecting stronger consensus among investors. Differences in news arrival rate expectations naturally lead to variation in derivative positions. Figures 6c and 6d show the derivative holdings of investor cohorts, expressed as fractions of total wealth in the economy. These holdings are obtained by multiplying the wealth distribution in Result IA1 with the

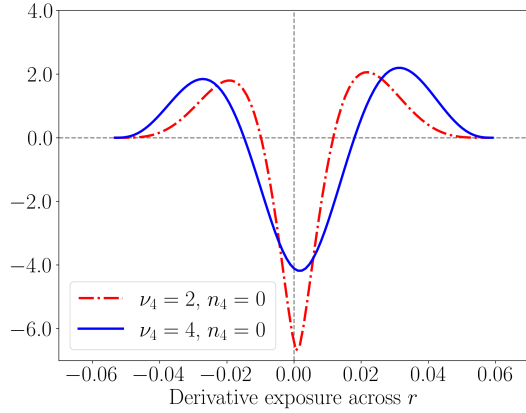
¹²In the Internet Appendix I.A.2.1, we plot the time variation of all figures at $t = 1, 2, 3, 4$ to demonstrate how different sample paths affect the wealth distribution and derivative positioning across agents’ market return expectations.



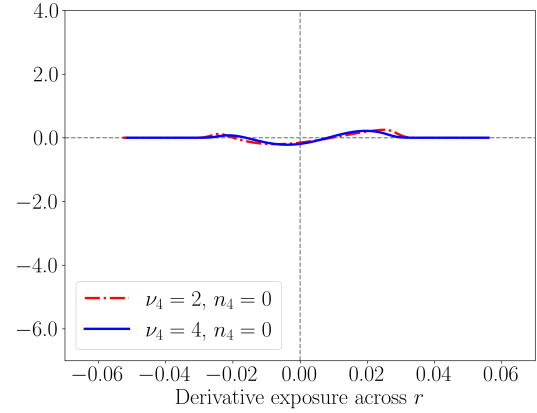
(a) High v-disagreement



(b) Low v-disagreement



(c) High v-disagreement



(d) Low v-disagreement

Figure 6: Beliefs of news intensity, market return and derivative position. The figure reports wealth-weighted average news arrival rate v and derivative exposures for the cohort of investors with expected return r . The left panels (6a and 6c) assume a uniform initial wealth distribution across agents, with $(\alpha_{v0}, \beta_{v0}, \alpha_{u0}, \beta_{u0}) = (1, 1, 1, 1)$. In contrast, in the right panels (6b and 6d), the initial wealth distribution corresponds to low disagreement in the news arrival rate dimension, characterized by $(\alpha_{v0}, \beta_{v0}, \alpha_{u0}, \beta_{u0}) = (100, 100, 1, 1)$. Terminal payoff: $e^{0.05n_T}$ with $T = 6$. See Figure 1a for the sample paths leading to these wealth distributions at $t = 4$.

corresponding portfolio weights from (26).

Since derivative contracts have zero net supply, the long-convexity positions of extreme optimists and pessimists must be absorbed by moderates who sell convexity. The equilibrium derivative price thus reflects the balance between speculative demand from the tails and liquidity supply from the center of the belief distribution.¹³

Derivative positions offer an empirically observable metric to distinguish between cases of high and low disagreement about the news arrival rate. When disagreement about the news arrival rate is high, investors take significantly larger derivative positions, as shown in Figure 6c. These sizable derivative investments, together with the resulting wealth redistribution, are the central drivers of volatility persistence in Result 3. In contrast, when disagreement is low, $\mathbb{E}[v|r]$ remains close to the market average V_t across most investor cohorts. Derivative portfolio weights are therefore small, leading to limited activity in derivative markets, as illustrated in Figure 6d.

Result 3 connects volatility dynamics to the regime of disagreement. Under high arrival-intensity disagreement, realized volatility begets implied volatility: investors who correctly anticipated turbulence gain wealth and pricing influence, so high realized volatility today elevates implied volatility going forward. Under low arrival-intensity disagreement, the opposite holds—frequent news concentrates directional beliefs, compressing implied volatility even as realized volatility rises.

3.5 Extension to CRRA Preferences

Under CRRA preferences with risk aversion γ , the competitive equilibrium is Pareto optimal and characterized by *Negishi weights* $\psi_t(v, u)$ that aggregate heterogeneous beliefs into a social welfare function. The key structural result, proved in Internet Appendix I.A.3, is

¹³Consistent with Result 7, Lakonishok et al. (2007) find that sophisticated institutional activity accounts for the bulk of structured convexity positioning, while Ni, Pan and Poteshman (2008) show that this volatility-specific demand carries predictive power distinct from directional order flow.

that Negishi weights evolve as pure likelihood ratios,

$$\frac{\psi_t(v, u)}{\psi_t(v', u')} = \frac{\psi_0(v, u)}{\psi_0(v', u')} \prod_{\tau=1}^t \frac{\Pr^{(v, u)}(s_\tau)}{\Pr^{(v', u')}(s_\tau)},$$

independent of γ , the risk-neutral measure, and all equilibrium prices. When $\psi_0(v, u)$ is initialized with the same functional form as the log-utility wealth distribution $f_0(v, u)$, the Negishi weights $\psi_t(v, u)$ track $f_t(v, u)$ exactly for all t , γ , and sample paths.

This preference invariance carries the core results directly. Because the selection rate against any agent is determined by the KL divergence of their beliefs from the truth— independently of γ —the survival dichotomy of Result 4 holds verbatim under CRRA: intensity disagreement survives in the Poisson limit and is eliminated in the Brownian limit. The CRRA consensus measure $\tilde{\pi}_t(n_T)$ inherits the same news-driven dispersion dynamics as its log-utility counterpart, so the volatility dichotomy is preserved. One qualification applies to the “no news is good news” valuation result: it requires $\gamma > 1$, the empirically relevant range for equity investors, since the additional curvature of $p_T^{-\gamma}$ relative to $p_T^{1-\gamma}$ is what translates a compressed $\tilde{\pi}_t$ into a higher price.

4 Richer Environments

The trinomial tree isolates the content–intensity distinction in its most transparent form. Three extensions demonstrate that the same economic forces—wealth-driven selection that disciplines content beliefs but not intensity beliefs, and the resulting persistence of volatility disagreement—operate in substantially richer settings. First, we generalize to an M -node tree that accommodates multidimensional fundamental news and multiple assets. Second, we develop a continuous-time jump–diffusion limit that nests both the Poisson and Brownian limits within a single framework, unifying gradual information flow and discrete innovations. Third, we study a two-asset economy in which speculation localized in one asset propagates to the broader market through the wealth channel, endogenously generating safe-haven behavior and excess risk-neutral correlation without any fundamental linkage.

4.1 The M Tree

We begin by extending the trinomial tree to M nodes per period, accommodating richer state spaces—multiple fundamental variables, asset-specific news, or combinations thereof—within the same analytical framework. An agent's subjective belief is defined as $\mathbf{h} = (h_1, h_2, \dots, h_M)$, where h_i indicates the probability of the i -th node of the tree occurring from top to bottom.¹⁴

The market is dynamically complete. There are K non-redundant risky assets with one unit of net supply, $M - K - 1$ non-redundant derivatives with zero net supply, and one risk-free bond with zero net supply. The payoff structure of all risky assets at the terminal date T is common knowledge, defined as functions of the terminal cumulative news $\mathbf{s}_T = (s_{1,T}, s_{2,T}, \dots, s_{M,T})$, where $s_{i,T} \in \{0, 1, \dots, T\}$ denotes the realization count of news i (node i) between time 0 and T .

Logarithmic utility implies myopic optimization: an agent with belief $\mathbf{h} = (h_1, h_2, \dots, h_M)$ allocates wealth to $\theta_t^{(\mathbf{h})} = (\theta_{1,t}^{(\mathbf{h})}, \theta_{2,t}^{(\mathbf{h})}, \dots, \theta_{K,t}^{(\mathbf{h})})$ units of the risky assets, $\phi_t^{(\mathbf{h})} = (\phi_{1,t}^{(\mathbf{h})}, \phi_{2,t}^{(\mathbf{h})}, \dots, \phi_{M-K-1,t}^{(\mathbf{h})})$ units of the zero-supply derivatives, and the remainder in the risk-free bond, solving:

$$\max_{\theta_t^{(\mathbf{h})}, \phi_t^{(\mathbf{h})}} \mathbb{E}_t^{(\mathbf{h})} \log \left[w_t^{(\mathbf{h})} + \sum_{i=1}^K \theta_{i,t}^{(\mathbf{h})} (p_{i,t+1} - p_{i,t}) + \sum_{i=1}^{M-K-1} \phi_{i,t}^{(\mathbf{h})} (x_{i,t+1} - q_{i,t}) \right], \quad (27)$$

where $w_t^{(\mathbf{h})}$ represents the time t wealth of individual agent $\mathbf{h}$.

The total market value $p_t = \sum_{i=1}^K p_{i,t}$, representing the sum of all assets with positive supply, satisfies:

$$p_t^{-1} = \sum_{j=1}^M H_{j,t} p_{t+1}^{-1}(j, \mathbf{s}_t), \quad (28)$$

where $H_{j,t}$ is the wealth-weighted consensus probability of node j occurring in the next period, and $p_{t+1}(j, \mathbf{s}_t)$ is the market value if node j is realized at time $t + 1$.

¹⁴Alternatively, it is possible to follow our formulation of (v, u) in the trinomial tree case and parameterize $\mathbf{h}$ as $h_1 = (1 - v_1)$, $h_2 = v_1(1 - v_2)$, $\dots$, $h_M = v_1 v_2 \cdots v_{M-1}$, where $v_i \in (0, 1)$.

Similarly to (7), the risk-neutral measures are given by:

$$h_{j,t}^* = H_{j,t} p_t(\mathbf{s}_t) p_{t+1}^{-1}(j, \mathbf{s}_t). \quad (29)$$

Given the aggregate belief $H_{j,t}$, we can solve for all asset prices backward from the terminal date T using equations (28) and (29). To obtain a tractable solution for $H_{j,t}$, we model the cross-sectional distribution of beliefs $\mathbf{h}$ using a Dirichlet distribution.

Result 8 (Sentiment and Risk-Neutral Measure). *Assuming the news path history from time 0 to time t is $\mathbf{s}_t = (s_{1,t}, \dots, s_{M,t})$ and agents' beliefs follow a Dirichlet distribution $\text{Dir}(\beta_{1,0}, \dots, \beta_{M,0})$, the aggregate belief for node j is:*

$$H_{j,t} = \beta_{j,t} \left(\sum_{j=1}^M \beta_{j,t} \right)^{-1}, \quad (30)$$

where $\beta_{j,t} = \beta_{j,0} + s_{j,t}$. The path-dependent risk-neutral measure between t and T is:

$$h_{t \rightarrow T}^*(\mathbf{s}_T) = \frac{p_t(\mathbf{s}_t)}{p_T(\mathbf{s}_T)} \frac{(T-t)!}{(s_{1,T} - s_{1,t})! \cdots (s_{M,T} - s_{M,t})!} \frac{B(\beta_{1,T}, \dots, \beta_{M,T})}{B(\beta_{1,t}, \dots, \beta_{M,t})}. \quad (31)$$

4.2 A Market with Jump and Diffusion

We previously distinguished between news and no-news outcomes, with news reflecting rare, large fundamental changes. In practice, however, fundamentals may still adjust marginally on no-news dates, even when such changes are too small to be newsworthy. We consider a model with $M = 4$ nodes and a single asset ($K = 1$), where nodes 1 and 4 correspond to rare, news-type jumps, while nodes 2 and 3 capture frequent but non-news incremental adjustments.

Taking the continuous-time limit, we partition the interval $[0, T]$ into $N = T/\delta$ periods. The terminal payoff is given by $p_T = \exp[\omega(s_{1,T} - s_{4,T}) + \sigma\sqrt{\frac{T}{N}}(s_{2,T} - s_{3,T})]$, where ω denotes the jump size and σ the scaled diffusion volatility. We scale the parameters of the Dirichlet distribution for the agent's initial beliefs as $\beta_{2,0} = \frac{1}{2}\theta N + \frac{1}{2}\eta\sqrt{N}$, $\beta_{3,0} = \frac{1}{2}\theta N - \frac{1}{2}\eta\sqrt{N}$, with $\beta_{1,0}$ and $\beta_{4,0}$ held constant and independent of N . As in Result 8

and Result 5, we express agents' time- t beliefs by $\beta_{j,t} = \beta_{j,0} + s_{j,t}$ and use $\lambda_j = \beta_j/\theta T$ to represent the corresponding expected jump intensity. Result 8 yields the market price and the risk-neutral MGF of the log return.

Result 9 (A Jump-Diffusion Limit of Market). *Let $\phi = t/T$ and $\eta_t = (s_{2,t} - s_{3,t})/\sqrt{\phi N}$. In the Jump-Diffusion limit, the market price is given by*

$$p_t = e^{\omega[(\beta_{1,t} - \beta_{4,t}) - (\beta_{1,0} - \beta_{4,0})]} \mathcal{J}(-1; \phi, \theta)^{\beta_{1,t}} \mathcal{J}(1; \phi, \theta)^{\beta_{4,t}} \exp \left[\frac{1-\phi}{\theta+\phi} \eta \sigma \sqrt{T} + \frac{\theta+1}{\theta+\phi} \eta_t \sigma \sqrt{\phi T} - \frac{1}{2} \frac{\theta+1}{\theta+\phi} (1-\phi) \sigma^2 T \right], \quad (32)$$

where $\mathcal{J}(\chi; \phi, \theta) = 1 - (1-\phi)(e^{\chi\omega} - 1)/(\theta + \phi)$.

The moment-generating function of the log return under the risk-neutral measure is

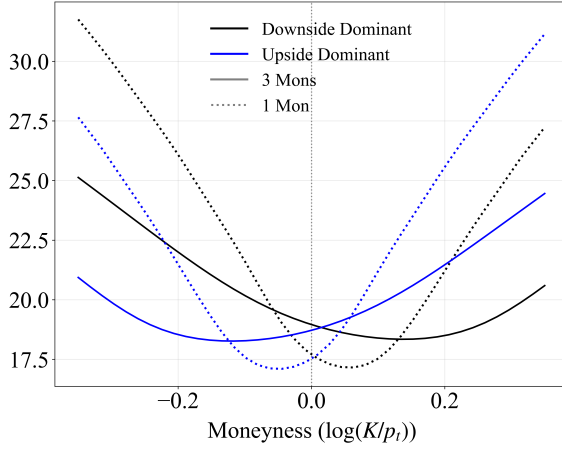
$$\mathbb{E}_t^*[e^{\chi \log(p_T/p_t)}] = \mathbb{E}_t^* \left[\left(\frac{p_T}{p_t} \right)^\chi \right] = \left[\frac{\mathcal{J}(\chi - 1; \phi, \theta)}{\mathcal{J}(-1; \phi, \theta)^{1-\chi}} \right]^{-\beta_{1,t}} \left[\frac{\mathcal{J}(1 - \chi; \phi, \theta)}{\mathcal{J}(1; \phi, \theta)^{1-\chi}} \right]^{-\beta_{4,t}} \exp \left[\frac{\chi(\chi - 1)}{2} \left(\frac{\theta+1}{\theta+\phi} \right) (1-\phi) \sigma^2 T \right]. \quad (33)$$

Evaluating (33) at $\chi = 2$ and subtracting one yields the risk-neutral variance of the simple return, $\text{Var}_t^*[p_T/p_t]$. Risk-neutral skewness and higher-order moments follow analogously from this moment-generating function. ¹⁵

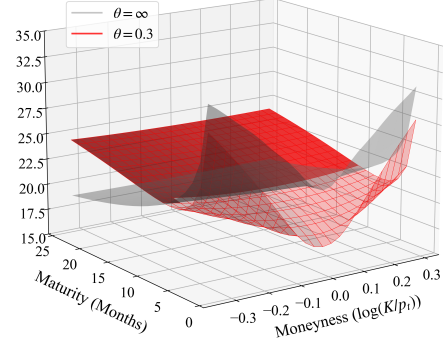
Result 9 unifies the two polar cases we have shown: the continuous Gaussian diffusion in Result 6 and the pure Poisson jump process in Result 5. Setting the jump size $\omega = 0$ yields the Brownian limit, while setting the volatility $\sigma = 0$ yields the Poisson limit, with $\alpha_{ut} = \beta_{1,t}$, $\beta_{ut} = \beta_{4,t}$, $\alpha_{vt} = \beta_{1,t} + \beta_{4,t}$, and $\beta_{vt} = \theta N$. It provides a natural extension of homogeneous-belief models, such as Cox and Ross (1976), Merton (1976), to settings with heterogeneous beliefs.

Wealth is endogenously reallocated toward the ex post correct agent. Following a jump realization, the market's wealth-weighted belief assigns a higher likelihood to future jumps,

¹⁵For the risk-neutral variance to be well defined, the jump size and the degree of disagreement must satisfy $\mathcal{J}(1; 0, \theta) = 1 - (e^\omega - 1)/\theta > 0$, i.e. $\theta > e^\omega - 1$. Too much disagreement (when θ is too small) could result in an unbounded second risk-neutral moment, or equivalently an unbounded second moment of the SDF (see Martin and Shi (2026)).



(a) Volatility Smile



(b) Implied Volatility Surface

Figure 7: Implied Volatility Dynamics. This figure illustrates the Black-Scholes implied volatility generated by the jump-diffusion model with heterogeneous agents, evaluated with continuous diffusion volatility $\sigma = 0.15$, jump size $\omega = 0.10$ and $t = 12$ months.

reflecting the same mechanism that generates volatility persistence under disagreement about news arrival rates, as characterized in Result 3.

The risk-neutral MGF (33) maps directly into an implied volatility surface via the fast Fourier transform Carr and Madan (1999). A large literature, including Campbell and Hentschel (1992), documents the asymmetric pricing of downside jump risk; Figure 7 shows how belief heterogeneity shapes these asymmetries.

Figure 7a isolates the volatility skew for short-dated options (1 and 3 months maturities) with high disagreement ($\theta = 0.3$). Holding total expected jump intensity constant ($\lambda_{total} = 4.0$), it compares two cases: bad news dominant ($\lambda_1 = 0.5, \lambda_4 = 3.5$, black) and good news dominant ($\lambda_1 = 3.5, \lambda_4 = 0.5$, blue). The left-skew observed in stock option markets (Bates (1991), Rubinstein (1994)) and the right-skew seen in VIX and commodity option markets (Mencia and Sentana (2013), Jia, Ruan and Zhang (2021)) emerge because wealth shifts toward agents whose jump predictions are realized, amplifying their influence on market prices. The 1-month dashed curves are steeper in the tails than the 3-month solid curves, reflecting the greater sensitivity of short-dated options to discrete news arrivals.¹⁶

¹⁶ Andersen, Fusari and Todorov (2015) decompose implied volatility into continuous and jump components, finding that the jump risk premium is highly episodic and heavily concentrated at short maturities. Kelly, Pástor and Veronesi (2016) and Dubinsky et al. (2019) document an inverted term structure of

Figure 7b plots the implied volatility surface against moneyness $\log(K/p_t)$ and remaining maturity (1 to 24 months) under balanced jump expectations ($\lambda_1 = \lambda_4 = 2.0$). It contrasts a homogeneous baseline with no disagreement ($\theta = \infty$, transparent gray) against high disagreement ($\theta = 0.3$, red).

While Result 9 captures the essential dynamics of asymmetric skewness and time-varying variance, it does so within a specific parametric scope. The model endogenizes disagreement over the *intensity* of the jumps ($\beta_{1,t}$ vs. $\beta_{4,t}$), but assumes the physical jump size ω is fixed and universally agreed upon. It generates realistic skewness, but the kurtosis is structurally bounded by the fixed jump magnitude. Extending the framework to allow for heterogeneous beliefs over continuous jump-size distributions (e.g., a double-exponential jump limit) represents a natural direction for future research to capture richer tail behavior.

4.3 Two Risky Assets

The M-node tree accommodates multiple assets, but rather than pursuing full generality we focus on a single question: how does concentrated speculation in one asset affect the broader market risk premium and individual market betas?

Consider a setting with $M = 4$ and $K = 2$. In each period, binary news—either ‘good news’ or ‘bad news’—arrives for both assets A and B. We construct the ‘market-wide’ news by combining these individual shocks, defining the state space as $(good, good)$, $(good, bad)$, $(bad, good)$, and (bad, bad) . To ensure dynamic completeness, agents trade assets A and B (unit net supply), a one-period risk-free bond, and a one-period derivative with zero net supply.

When good news arrives for asset A, agents who are bullish on asset A accumulate wealth. Their views regarding asset B then influence asset B’s pricing. If news frequently arrives in opposite directions for the two assets—the $(good, bad)$ and $(bad, good)$ combinations—the aggregate market generally appears less volatile since the joint news shocks are largely idiosyncratic. Conversely, when news frequently arrives in the same direction, alternating between extremely good (both assets have good news) and extremely implied volatility immediately preceding anticipated jump events like earnings or political elections.

bad (both assets have bad news), the aggregate market exhibits higher volatility. Building on this observation, we specify the one-period derivative contract as a market volatility swap: it yields positive payoffs in states $(good, good)$ and (bad, bad) , and zero otherwise.

A Diffusion Limit of Two Risky Assets. We consider the continuous-time limit to obtain tractable formulas for prices and risk-neutral return variances. The interval $[0, T]$ is partitioned into $N = T/\delta$ periods, each of length δ . The market consists of assets A and B , with terminal payoffs $p_{i,T}(n_{i,T}) = e^{\sigma_i \sqrt{\frac{T}{N}} n_{i,T}}$ for $i \in \{A, B\}$, where the composite market index is $p_T = p_{A,T} + p_{B,T}$.

To allow for distinct belief heterogeneity regarding the news of A and B (i.e., $\theta_A \neq \theta_B$), we model the population mass using two separate Beta distributions, $B(\alpha_i, \beta_i)$, joined by a *bivariate Bernoulli copula*.¹⁷ The parameters for asset $i \in \{A, B\}$ are independently scaled as $\alpha_{i,0} = \frac{1}{2}\theta_i N + \frac{1}{2}\eta_i \sqrt{N}$ and $\beta_{i,0} = \frac{1}{2}\theta_i N - \frac{1}{2}\eta_i \sqrt{N}$, where $\alpha_{i,0} + \beta_{i,0} = \theta_i N$. In the large- N limit, the wealth-weighted beliefs over log payoffs converge to a two-dimensional Gaussian distribution with mean and covariance:

$$\mu_{i,t} = \frac{1-\phi}{\theta_i + \phi} \eta_i \sigma_i \sqrt{T} + \frac{\theta_i + 1}{\theta_i + \phi} \eta_{i,t} \sigma_i \sqrt{\phi T}, \quad (34)$$

$$\sigma_{i,t}^2 = \frac{\theta_i + 1}{\theta_i + \phi} (1-\phi) \sigma_i^2 T, \quad \sigma_{AB,t} = \sqrt{\frac{\theta_A + 1}{\theta_A + \phi} \frac{\theta_B + 1}{\theta_B + \phi}} (1-\phi) \rho \sigma_A \sigma_B T. \quad (35)$$

where $\phi = t/T$ and $\eta_{i,t} = n_{i,t}/\sqrt{\phi N}$.

The following result gives closed-form prices and risk-neutral variances for individual assets and the aggregate market.

Result 10 (Brownian Limit of Two Risky Assets). *Let $\mu_{Z,t} = \mu_{A,t} - \mu_{B,t}$, and $\sigma_{Z,t}^2 =$*

¹⁷Specifically, at time t (or step ϕN), let $q_{A,t} = \alpha_{A,t}/(\alpha_{A,t} + \beta_{A,t})$ and $q_{B,t} = \alpha_{B,t}/(\alpha_{B,t} + \beta_{B,t})$ denote the marginal posterior probabilities of ‘good news’ for each asset. The simultaneous draws of $s_i \in \{\text{‘good’}, \text{‘bad’}\} \times \{\text{‘good’}, \text{‘bad’}\}$ have the joint distribution: $h_{1,t} = q_{A,t} q_{B,t} + \rho \varepsilon_t$, $h_{2,t} = q_{A,t} \bar{q}_{B,t} - \rho \varepsilon_t$, $h_{3,t} = \bar{q}_{A,t} q_{B,t} - \rho \varepsilon_t$, $h_{4,t} = \bar{q}_{A,t} \bar{q}_{B,t} + \rho \varepsilon_t$, with $q_{i,t} + \bar{q}_{i,t} = 1$ and $\varepsilon_t^2 = q_{A,t} \bar{q}_{A,t} q_{B,t} \bar{q}_{B,t}$. The parameter $\rho \in (-1, 1)$ represents the fundamental correlation between individual net news shocks. Summing over either asset immediately recovers its exact marginal probability.

$\sigma_{A,t}^2 + \sigma_{B,t}^2 - 2\sigma_{AB,t}$. In the Brownian limit, the market price $p_t = p_{A,t} + p_{B,t}$ is given by:

$$p_t = \exp\left(\mu_{A,t} - \frac{1}{2}\sigma_{A,t}^2\right) \mathcal{W}\left(\kappa_t(\mu_{Z,t} - \sigma_{A,t}^2 + \sigma_{AB,t})\right), \quad (36)$$

where $\mathcal{W}(x) = 1 + e^{-x}$ and the scaling factor is $\kappa_t = (1 + \frac{\pi}{8}\sigma_{Z,t}^2)^{-1/2}$. The market share of asset A is $\frac{p_{A,t}}{p_t} = \mathcal{W}(\kappa_t \mu_{Z,t})^{-1}$.

The risk-neutral variance of the market return is given by:

$$\text{Var}_t^*[p_T/p_t] = \frac{\exp(\sigma_{A,t}^2) \mathcal{W}(\mu_{Z,t} + \frac{1}{2}(\sigma_{A,t}^2 - \sigma_{B,t}^2))}{\mathcal{W}(\kappa_t(\mu_{Z,t} - \sigma_{A,t}^2 + \sigma_{AB,t}))} - 1. \quad (37)$$

The risk-neutral variance of asset A's return and its risk-neutral covariance with the market return are:

$$\text{Var}_t^*[p_{A,T}/p_{A,t}] = \frac{\exp(\sigma_{A,t}^2) \mathcal{W}(\kappa_t \mu_{Z,t})^2}{\mathcal{W}(\kappa_t(\mu_{Z,t} - \sigma_{A,t}^2 + \sigma_{AB,t})) \mathcal{W}(\kappa_t(\mu_{Z,t} + \sigma_{A,t}^2 - \sigma_{AB,t}))} - 1, \quad (38)$$

$$\text{Cov}_t^*[p_{A,T}/p_{A,t}, p_T/p_t] = \frac{\exp(\sigma_{A,t}^2) \mathcal{W}(\kappa_t \mu_{Z,t})}{\mathcal{W}(\kappa_t(\mu_{Z,t} - \sigma_{A,t}^2 + \sigma_{AB,t}))} - 1. \quad (39)$$

When assets A and B have identical volatility and perfect correlation ($\rho = 1$), every $\mathcal{W}(\cdot)$ term reduces to a constant and Result 10 collapses to Result 6. Under logarithmic utility, the no-arbitrage condition dictates that any expected risky excess return satisfies $\mathbb{E}_t[p_{i,T}/p_{i,t}] - 1 = \text{Cov}_t^*[p_{i,T}/p_{i,t}, p_T/p_t]$, as demonstrated by Martin and Wagner (2019). Consequently, we can compute the market beta of asset A by taking the ratio of (39) to (37).

To isolate these cross-sectional dynamics, we use a symmetric parameterization for Figure 8. We set the fundamental volatilities to be equal ($\sigma_A = \sigma_B = 20\%$) and, importantly, fix the fundamental correlation to zero ($\rho = 0$). This zero-correlation benchmark ensures that any observed cross-asset spillovers, volatility amplifications, or beta compressions are entirely endogenous—driven purely by the structure of heterogeneous beliefs rather than mechanical fundamental linkages. We evaluate these dynamics at an intermediate time $t/T = 0.3$, contrasting the homogeneous benchmark ($\theta_A = \theta_B = \infty$) with cases of localized

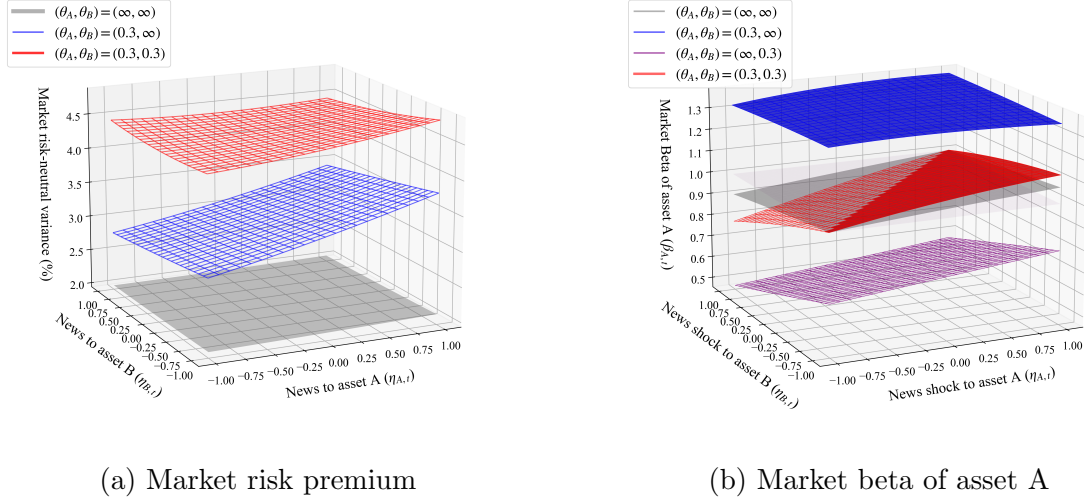


Figure 8: **The ‘Illusion’ of Safety.** We fix $\phi = t/T = 0.3$, $\sigma_A = \sigma_B = 20\%$, and $\rho = 0$ in both figures. The parameter θ_i indicates the level of agreement regarding asset i , with $\theta_i = \infty$ representing the homogeneous economy limit. Because our parameter choices for Assets A and B are symmetric, the purple surface is mathematically identical to the market beta of B when $(\theta_A, \theta_B) = (0.3, \infty)$.

and generalized disagreement where at least one $\theta_i = 0.3$ for $i \in \{A, B\}$.

Localized speculation generates cross-sectional spillovers even absent fundamental linkages. Figure 8 illustrates: when disagreement concentrates on asset A alone, its inflated risk-neutral variance dominates the market index, mechanically elevating A’s beta above one while compressing B’s beta below one (right panel, purple surface). Asset B endogenously becomes a safe-haven asset despite unchanged fundamentals. When disagreement is high for both assets (left panel), market variance peaks; when both are homogeneous, it is minimized. The betas align with the homogeneous benchmark only when both assets’ news histories are identical ($\eta_{A,t} = \eta_{B,t}$).

A Poisson Limit of Two Risky Assets. In Internet Appendix I.A.4, we develop a Poisson limit of the two-asset economy in which three types of jumps can occur: idiosyncratic jumps in asset A, idiosyncratic jumps in asset B, and simultaneous co-jumps affecting both. Investors hold heterogeneous beliefs over all three intensities, governed by a Dirichlet distribution. The resulting continuous-time limit yields closed-form expressions for mar-

ket prices, market shares, and risk-neutral variances via a single one-dimensional Fourier integral.

This environment delivers an additional prediction that the Brownian limit cannot generate: the risk-neutral correlation between asset returns strictly exceeds the physical correlation, $|\rho_t^*| > |\rho_t|$. The amplification arises because the log-utility pricing kernel disproportionately weights the co-jump state, in which the aggregate market suffers the largest loss. Each realized co-jump further increases the wealth-weighted co-jump intensity $\lambda_{AB,t}^*$, reinforcing the effect over time. Thus the same mechanism that generates volatility persistence through intensity disagreement in the single-asset economy (Result 3) produces *correlation persistence* in the multi-asset economy: a history of co-jumps elevates future risk-neutral correlation, while quiet periods gradually compress it.¹⁸

Together with the beta compression documented in Figure 8b, this result suggests that belief heterogeneity can simultaneously shape both the level and the co-movement structure of risk-neutral distributions across assets—linking the dispersion of directional and volatility views to observable quantities in index and single-name option markets.¹⁹

5 Conclusion

This paper develops a tractable general-equilibrium framework for pricing *news* and *no news* when investors disagree not only about the direction of fundamentals, but also about the likelihood of market-moving information’s arrival. The distinction between disagreement about news *content* and news *arrival intensity* is economically critical. It naturally separates directional trading from volatility speculation, driving distinct price and implied volatility dynamics through endogenous wealth redistribution.

Our central theoretical result is a dichotomy in implied volatility dynamics. When

¹⁸Driessen, Maenhout and Vilkov (2009) show that implied correlations systematically exceed realized correlations, driving a structural wedge between index and individual variance risk premia.

¹⁹Investors who speculate that dispersion will rise are effectively shorting market volatility. If they emerge as hindsight winners, the implied volatility of the market is suppressed as the market index drifts upward. For recent empirical examples of dispersion trading, see “Wall Street seems calm. A closer look shows something else.” (New York Times) and “A \$12bn dispersion trade” (Financial Times).

disagreement about eventfulness is high, realized news sustains belief dispersion, causing volatility to persist. Quiet periods, in contrast, transfer wealth toward low news intensity believers, compressing risk-neutral tail probabilities and supporting valuations—making “no news” effectively good news. Conversely, when disagreement is primarily about content, frequent news accelerates market selection, concentrates wealth among accurate directional forecasters, and induces mean reversion in implied variance. The framework explains why volatility disagreement can survive in equilibrium: because wealth transfer among agents caused by discrete jumps is episodic, errors regarding event arrival intensity are penalized at a rate that tends toward zero per trading period in the continuous-time limit. This allows heterogeneity over eventfulness to persist and shape prices over extended horizons, whereas disagreement over continuous diffusion is rapidly eliminated by continuous price observation.

These endogenous wealth dynamics generate four testable predictions for future empirical work. First, the response of implied volatility to scheduled events hinges entirely on the prevailing regime of arrival intensity disagreement. High disagreement implies volatility persistence and low disagreement implies mean reversion. Second, the cross-section of convexity demand is U-shaped: extreme directional optimists and pessimists aggressively demand convexity (long-gamma), while moderates supply it. Third, short-dated options exhibit pronounced skewness and heightened sensitivity around discrete, jump-type events compared to periods of continuous diffusion. Finally, intense localized speculation in a single asset propagates through the market portfolio, endogenously inflating risk-neutral correlations and compressing the market betas of non-speculative assets.

Our framework highlights a broader message: markets price not only beliefs about *what* will happen, but also beliefs about *whether anything will happen*.

A Mathematical Appendix

Proof for equations (12) and (13). Comparing the individual wealth growth and aggregate wealth growth equations in (6) and (7), we derive the following equation for the change

of wealth distribution:

$$\frac{f_{t+1}(m, g, b)}{f_t(m, g, b)} = \begin{cases} \frac{m}{M_t}, & \text{'no news' at } t+1 \\ \frac{g}{G_t}, & \text{'good news' at } t+1 \\ \frac{b}{B_t}, & \text{'bad news' at } t+1 \end{cases}$$

Solving this recursively backward from t to 0, the wealth distribution $f_t(m, g, b)$ must satisfy $f_t(m, g, b) = \lambda_t f_0(m, g, b) m^{n_{mt}} g^{n_{gt}} b^{n_{bt}}$, where the constant λ_t is path-dependent. Taking the integral of both sides, we have $1 = \int \lambda_t f_0(m, g, b) m^{n_{mt}} g^{n_{gt}} b^{n_{bt}} dm dg db$, which implies $\lambda_t = B(\alpha_{v0}, \beta_{v0}) B(\alpha_{u0}, \beta_{u0}) (B(\alpha_{vt}, \beta_{vt}) B(\alpha_{ut}, \beta_{ut}))^{-1}$, where $\alpha_{vt} = \alpha_{v0} + n_{gt} + n_{bt}$, $\beta_{vt} = \beta_{v0} + n_{mt}$, $\alpha_{ut} = \alpha_{u0} + n_{gt}$, and $\beta_{ut} = \beta_{u0} + n_{bt}$. Converting the notation of (n_{mt}, n_{gt}, n_{bt}) into (ν_t, n_t) , we obtain the results in equations (12) and (13). $\square$

Proof for Result 1. Due to (8), we solve from T backward to t the value $1/p_t$. It is merely the linear combination of wealth-weighted aggregate beliefs of all possible future sample paths between t and T multiplied by the inverse terminal payoff $1/p_T(n_T)$. To formally prove this statement, let (a, b, c) be the number of moves that correspond to (high, middle, low) on a sample path from t to T . They must satisfy $a - c = n_T - n_t$ and $a + b + c = T - t$. This can be solved in terms of b as $a = \frac{1}{2}(T - t + n_T - n_t - b)$ and $c = \frac{1}{2}(T - t - n_T + n_t - b)$, and (a, c) are integers only when b and $T - t - (n_T - n_t)$ are both odd or even numbers ($T - t + (n_T - n_t)$ always shares parity with $T - t - (n_T - n_t)$). Since the arguments for the positive and negative cases of $n_T - n_t$ are symmetric, we focus on $n_T - n_t > 0$. If $T - t - (n_T - n_t)$ is an odd number, b starts from 1; otherwise, it starts from 0. The size of b cannot exceed $T - t - (n_T - n_t)$ for c to stay non-negative. Once b is fixed, the values of a and c are determined. There are $\frac{(T-t)!}{a!b!c!}$ possible paths that satisfy this characteristic, i.e., a times high, b times middle, and c times low. These paths can be merged. To show this, it suffices to show the three basic cases: 1) two trips of ‘middle-low’ and ‘low-middle’ give the same risk-neutral probability given the same initial condition; 2) two trips of ‘middle-high’ and ‘high-middle’ are equivalent; 3) two trips of ‘low-high’ and ‘high-low’ are equivalent. The risk-neutral variance of the return from t to T , $p_T/p_t - 1$, is given by $p_t^{-2} \mathbb{E}_t^*[p_T^2 - p_t^2] = \mathbb{E}_t[p_T(n_T)]/p_t - 1$. Substituting into (9), we obtain (10). $\square$

Proof for Result 2. Let $f_0(v, u)$ denote the prior distribution of (v, u) at time 0. Given n_{gt} , n_{mt} , and n_{bt} realizations of good, no, and bad news up to time t , the posterior is

$$f_t(v, u) = \frac{(1-v)^{n_{mt}} (uv)^{n_{gt}} ((1-u)v)^{n_{bt}} f_0(v, u)}{\int_0^1 \int_0^1 (1-v)^{n_{mt}} (uv)^{n_{gt}} ((1-u)v)^{n_{bt}} f_0(v, u) du dv} \quad (\text{A1})$$

When the prior distribution takes the 2D Beta form in (3), the corresponding time- t posterior distribution simplifies to (12). In general, the representative agent's belief distribution over terminal states n_T follows (11). With log utility, the inverse price is a martingale $p_t^{-1} = \sum_{n_T} \Pr(n_T | n_t, f_t) p_T^{-1}(n_T)$, which is consistent with (9). $\square$

Proof for Result 3. i) Suppose news arrival rate disagreement is stronger than the news content disagreement $\alpha_{v0} + \beta_{v0} \leq \alpha_{u0} + \beta_{u0}$. Holding the average expectation of fundamental growth $G_t - B_t$ constant, a higher historical news count $\nu_t \geq \nu'_t$ suggests that: i) $g + b = v$ first-order stochastically dominates $g' + b' = v'$; ii) $g' - b' = (2u' - 1)v'$ second-order stochastically dominates $g - b = (2u - 1)v$. Thus, $n_T - n_t$ is a mean-preserving spread of $n'_T - n'_t$. As a result, $p_t^{-1} = \sum_{n_T - n_t} \Pr(n_T - n_t | n_t, f_t) e^{-\sigma(n_T - n_t)} e^{-\sigma n_t} \geq \sum_{n'_T - n'_t} \Pr(n'_T - n'_t | n'_t, f'_t) e^{-\sigma(n'_T - n'_t)} e^{-\sigma n_t} = (p'_t)^{-1} e^{-\sigma(n_t - n'_t)}$, which confirms $p_t / e^{\sigma n_t} \leq p'_t / e^{\sigma n'_t}$. Similarly, $\mathbb{E}_t[p_T | n_t, f_t] = \sum_{n_T - n_t} \Pr(n_T - n_t | n_t, f_t) e^{\sigma(n_T - n_t)} e^{\sigma n_t} \geq \sum_{n'_T - n'_t} \Pr(n'_T - n'_t | n'_t, f'_t) e^{\sigma(n'_T - n'_t)} e^{\sigma n_t} = \mathbb{E}_t[p'_T | n'_t, f'_t] e^{\sigma(n_t - n'_t)}$. Multiplying the above two inequalities, we obtain $\text{Var}_t^*[p_T / p_t - 1] \geq \mathbb{E}_t[p'_T / p'_t - 1] = \text{Var}_t^*[p'_T / p'_t - 1]$.

ii) The risk-neutral variance of the market return satisfies $\text{Var}_t^*[p_T / p_t - 1] = \mathbb{E}_t^*[p_T^2 / p_t^2] - 1 = \mathbb{E}_t[p_T | n_t, f_t] / p_t - 1$. Holding the news arrival rate v and the average expectation of fundamental growth $G_t - B_t$ constant, a higher historical news count $\nu_t \geq \nu'_t$ reduces disagreement in bullishness u , resulting in a more concentrated distribution of fundamental growth $n_T - n_t$, such that $n'_T - n'_t$ is a mean-preserving spread of $n_T - n_t$. The rest follows from the proof for part i). $\square$

Proof for Result 4. Recall the true probability measure (u_{true}, v_{true}) governs the data generating process of fundamental news. Define the wealth share of agent (v, u) at time T

with sample history (ν_T, n_T) as

$$\Omega(u, v; n_T, \nu_T) = \frac{v^{\nu_T} (1-v)^{N-\nu_T} u^{\frac{1}{2}n_T + \frac{1}{2}\nu_T} (1-u)^{\frac{1}{2}\nu_T - \frac{1}{2}n_T} f_0(v, u)}{\int v^{\nu_T} (1-v)^{N-\nu_T} u^{\frac{1}{2}n_T + \frac{1}{2}\nu_T} (1-u)^{\frac{1}{2}\nu_T - \frac{1}{2}n_T} f_0(v, u) dv du}$$

The asymptotic decay rate (exponentially) of wealth share satisfies

$$\lim_{N \rightarrow \infty} -\frac{\log \Omega(v, u; \nu_T, n_T)}{N} = \lim_{N \rightarrow \infty} -\frac{1}{N} \log \frac{v^{\nu_T} (1-v)^{N-\nu_T} u^{\frac{1}{2}n_T + \frac{1}{2}\nu_T} (1-u)^{\frac{1}{2}\nu_T - \frac{1}{2}n_T} f_0(v, u)}{\int v^{\nu_T} (1-v)^{N-\nu_T} u^{\frac{1}{2}n_T + \frac{1}{2}\nu_T} (1-u)^{\frac{1}{2}\nu_T - \frac{1}{2}n_T} f_0(v, u) dv du}$$

Note that in the large N limit, we can write $\nu_T = N v_{true}$, $n_T = n_{gT} - n_{bT} = N(u_{true} v_{true} - (1 - u_{true}) v_{true}) = N(2u_{true} v_{true} - v_{true})$. This means we can write

$$\begin{aligned} & -\frac{1}{N} \log \left(v^{\nu_T} (1-v)^{N-\nu_T} u^{\frac{1}{2}n_T + \frac{1}{2}\nu_T} (1-u)^{\frac{1}{2}\nu_T - \frac{1}{2}n_T} \right) \\ & = -v_{true} \log v - (1 - v_{true}) \log(1 - v) - v_{true} (u_{true} \log u + (1 - u_{true}) \log(1 - u)) , \end{aligned}$$

and also compute $\lim_{N \rightarrow \infty} \frac{1}{N} \log \int v^{\nu_T} (1-v)^{N-\nu_T} u^{\frac{1}{2}n_T + \frac{1}{2}\nu_T} (1-u)^{\frac{1}{2}\nu_T - \frac{1}{2}n_T} f_0(v, u) dv du = \lim_{N \rightarrow \infty} \frac{1}{N} \log \int \exp(N h(v_{true}, u_{true}; v, u) + \log f_0(v, u)) dv du = \sup_{v, u} \{h(v_{true}, u_{true}; v, u)\}$. Note that the maximum of $h(v_{true}, u_{true}; v, u)$ is achieved at (v_{true}, u_{true}) because the following quantity is always positive when $(v, u) \neq (v_{true}, u_{true})$ (this is the so-called *Gibbs's inequality* in information theory, which guarantees the relative entropy is always positive between two probability measures): $h(v_{true}, u_{true}; v_{true}, u_{true}) - h(v_{true}, u_{true}; u, v) = v_{true} \log \frac{v_{true}}{v} + (1 - v_{true}) \log \frac{1-v_{true}}{1-v} - v_{true} (u_{true} \log \frac{u_{true}}{u} + (1 - u_{true}) \log \frac{1-u_{true}}{1-u}) > 0$.

Thus we conclude $\lim_{t \rightarrow \infty} -\frac{\log \Omega(v, u; \nu_t, n_t)}{t}$ is the sum of $v_{true} \log \frac{v_{true}}{v} + (1 - v_{true}) \log \frac{1-v_{true}}{1-v}$ and $v_{true} (u_{true} \log \frac{u_{true}}{u} + (1 - u_{true}) \log \frac{1-u_{true}}{1-u})$, which is $\text{KL}(v_{true}, v) + v_{true} \text{KL}(u_{true}, u)$. $\square$

Proof for Result 5. Let $\xi = v/\delta$ denote the continuous-time Poisson arrival rate of news. As $\delta \rightarrow 0$, the distribution of ξ at time t converges to a Gamma distribution $\xi \sim \Gamma(\alpha_{vt}, \alpha_{v0}\lambda^{-1} + t)$.

For an agent with belief $(u, \xi\delta)$, the arrival rate of “good news” is ξu and bad news $\xi(1-u)$. The counts of “good news” and “bad news” for the time interval $[t, T]$ respectively

follow Poisson distributions $n_{gT} - n_{gt} \sim \text{Poisson}(\xi u(T - t))$, $n_{bT} - n_{bt} \sim \text{Poisson}(\xi(1 - u)(T - t))$.

Using the moment generating functions for $n_{gT} - n_{gt}$, $n_{bT} - n_{bt}$ and ξ , we find that

$$\begin{aligned}
p_t^{-1} &= e^{-\sigma n_t} \mathbb{E}[e^{-\sigma(n_{gT} - n_{gt})} | f_t] = e^{-\sigma n_t} \int e^{-\sigma(n_{gT} - n_{gt}) - \sigma(n_{bT} - n_{bt})} f_t(u, \xi) d\xi du \\
&= e^{-\sigma n_t} \int \exp(\xi u(T - t)(e^{-\sigma} - 1) + \xi(1 - u)(T - t)(e^{\sigma} - 1)) f_t(u, \xi) d\xi du \\
&= e^{-\sigma n_t} \int \left(1 - \frac{u(T - t)(e^{-\sigma} - 1) + (1 - u)(T - t)(e^{\sigma} - 1)}{\alpha_{v0}\lambda^{-1} + t}\right)^{-\alpha_{vt}} \frac{u^{\alpha_{ut}-1}(1 - u)^{\beta_{ut}-1}}{B(\alpha_{ut}, \beta_{ut})} du \\
&= e^{-\sigma n_t} \left(1 - \frac{(T - t)(e^{\sigma} - 1)}{\alpha_{v0}\lambda^{-1} + t}\right)^{-\alpha_{vt}} {}_2F_1\left(\alpha_{vt}, \alpha_{ut}; \alpha_{ut} + \beta_{ut}; -\frac{e^{\sigma} - e^{-\sigma}}{(\alpha_{v0}\lambda^{-1} + t)/(T - t) - (e^{\sigma} - 1)}\right).
\end{aligned}$$

The final step uses Euler's integral representation for the hypergeometric function ${}_2F_1$.

Similarly,

$$\begin{aligned}
\mathbb{E}[p_T] &= e^{\sigma n_t} \mathbb{E}[e^{\sigma(n_{gT} - n_{gt})} | f_t] \\
&= e^{\sigma n_t} \int \left(1 - \frac{u(T - t)(e^{\sigma} - 1) + (1 - u)(T - t)(e^{-\sigma} - 1)}{\alpha_{v0}\lambda^{-1} + t}\right)^{-\alpha_{vt}} \frac{u^{\alpha_{ut}-1}(1 - u)^{\beta_{ut}-1}}{B(\alpha_{ut}, \beta_{ut})} du \\
&= e^{\sigma n_t} \left(1 + \frac{(T - t)(1 - e^{-\sigma})}{\alpha_{v0}\lambda^{-1} + t}\right)^{-\alpha_{vt}} {}_2F_1\left(\alpha_{vt}, \alpha_{ut}; \alpha_{ut} + \beta_{ut}; \frac{e^{\sigma} - e^{-\sigma}}{(\alpha_{v0}\lambda^{-1} + t)/(T - t) + (1 - e^{-\sigma})}\right).
\end{aligned}$$

Substituting the above expression into (10), we obtain (17). $\square$

Proof for Result 6. By construction, from Mr. Market's point of view at time 0, the variables (n_{mT}, n_{gT}, n_{bT}) follow a Dirichlet distribution $B(\beta, \alpha, \gamma) = B(\theta N(1 - \lambda), \frac{1}{2}\theta\lambda N + \frac{1}{2}\eta\sqrt{N}, \frac{1}{2}\theta\lambda N - \frac{1}{2}\eta\sqrt{N})$. It is then straightforward to compute the reciprocal of the initial price using Result 1. To obtain a closed-form formula, we use Paul and Plackett (1978). In the limit as $N \rightarrow \infty$, the variable $n_T = n_{gT} - n_{bT}$ follows a normal distribution with the mean $\mathbb{E} n_T = N \left(\frac{\alpha - \gamma}{\alpha + \beta + \gamma} \right) = \frac{\eta}{\theta} \sqrt{N}$ and variance $\text{var}[n_T] = N \left(\lambda - \frac{\eta^2}{\theta^2} N^{-1} \right) \frac{1 + \theta}{\theta} \rightarrow \lambda \frac{1 + \theta}{\theta} N$. Thus the random variable $n_T / \sqrt{N}$ follows a normal distribution in the large- N limit: $\frac{n_T}{\sqrt{N}} \sim \mathcal{N}\left(\frac{\eta}{\theta}, \frac{\lambda(1 + \theta)}{\theta}\right)$. Consequently, the reciprocal of price is given by $p_0^{-1} = \mathbb{E} \exp\left(-\sigma \sqrt{T} \frac{n_T}{\sqrt{N}}\right)$ and this implies $p_0 = \exp\left(\frac{\eta}{\theta} \sigma \sqrt{T} - \frac{\lambda(1 + \theta)}{2\theta} \sigma^2 T\right)$.

For the market price at time t , we need to adjust the wealth distribution and the number of periods. The number of periods left is $N(1 - t/T) = N(1 - \phi)$, where $\phi = t/T$. The random variables $(n_{mT} - n_{mt}, n_{gT} - n_{gt}, n_{bT} - n_{bt})$ follow a Dirichlet distribution at time t : $B_t(\beta', \alpha', \gamma') = B_t((\theta + \phi)N(1 - \lambda), \frac{1}{2}\lambda(\theta + \phi)N + \frac{1}{2}\eta\sqrt{N} + \frac{1}{2}\eta_t\sqrt{\phi N}, \frac{1}{2}\lambda(\theta + \phi)N - \frac{1}{2}\eta\sqrt{N} - \frac{1}{2}\eta_t\sqrt{\phi N})$. The parameter η_t captures the wealth transfer between 0 and t among agents because of heterogeneous beliefs about news content $\eta_t = \frac{n_t}{\sqrt{\phi N}}$. The price at time t can then be written as $p_t^{-1} = \mathbb{E}_t p_T^{-1} = e^{-\eta_t \sigma \sqrt{\phi T}} \mathbb{E}_t \left(\exp \sigma \sqrt{T/N} (n_t - n_T) \right)$.

The large N limits for the mean and variance of $n_T - n_t$ (which is Gaussian distributed), according to [Paul and Plackett \(1978\)](#), are $\mathbb{E}_t[n_T - n_t] = (1 - \phi)N \frac{\alpha' - \gamma'}{\alpha' + \beta' + \gamma'} = \frac{1 - \phi}{\theta + \phi}(\eta + \eta_t \sqrt{\phi})\sqrt{N}$ and $\text{Var}_t[n_T - n_t] = (1 - \phi)N \left(\lambda - \left(\frac{\eta + \eta_t \sqrt{\phi}}{\theta} \frac{1 - \phi}{1 + \phi} \right)^2 N^{-1} \right) \left((1 - \phi) + \frac{1}{\theta + \phi} \right) \rightarrow \lambda \frac{\theta + 1}{\theta + \phi} (1 - \phi)N$. We can then compute p_t as:

$$p_t = \exp \left(\frac{1 - \phi}{\theta + \phi} \eta \sigma \sqrt{T} + \frac{\theta + 1}{\theta + \phi} \eta_t \sigma \sqrt{\phi T} - \frac{\lambda(\theta + 1)}{2(\theta + \phi)} (1 - \phi) \sigma^2 T \right).$$

The risk-neutral variance. Similarly,

$$\mathbb{E}_t p_T(n_T) = \lim_{N \rightarrow \infty} \mathbb{E}_t \exp \left(\sigma \sqrt{T} \frac{n_T}{\sqrt{N}} \right) = \exp \left(\frac{1 - \phi}{\theta + \phi} \eta \sigma \sqrt{T} + \frac{\theta + 1}{\theta + \phi} \eta_t \sigma \sqrt{\phi T} + \frac{\lambda(\theta + 1)}{2(\theta + \phi)} (1 - \phi) \sigma^2 T \right).$$

Thus, we have $\mathcal{V}_{t \rightarrow T} = \exp \left(\frac{\lambda(1 + \theta)}{\theta + \phi} \sigma^2 T \right)$. □

Derivation for Equation (21). Assumption (4) is mathematically equivalent to:

$$m_t^* x_{t+1,m} + g_t^* x_{t+1,g} + b_t^* x_{t+1,b} = M_t x_{t+1,m} + G_t x_{t+1,g} + B_t x_{t+1,b}. \quad (\text{A2})$$

which can be rewritten as $(g_t^* - G_t)x_{t+1,g} + (m_t^* - M_t)x_{t+1,m} + (b_t^* - B_t)x_{t+1,b} = 0$. Noting that $(m_t^* - M_t) + (g_t^* - G_t) + (b_t^* - B_t) = 0$, this implies that any vector $(x_{t+1,m}, x_{t+1,g}, x_{t+1,b})$ satisfying the zero-delta condition must lie in the span of the risk-free vector $(1, 1, 1)$, and a second vector that is linearly independent of it, which we may take as $(0, b_t^* - B_t, G_t - g_t^*)$. □

Derivation for Equation (23). The subjective expectation of the price change $p_{t+1} - p_t$, for an agent characterized by belief parameters (u, v) and represented by probabilities

(m, g, b) , is given by:

$$\mathbb{E}_t^{(v,u)}[p_{t+1} - p_t] = m(p_{t+1,m} - p_t) + g(p_{t+1,g} - p_t) + b(p_{t+1,b} - p_t). \quad (\text{A3})$$

Substituting (6) into (A3) and simplifying:

$$\mathbb{E}_t^{(v,u)}[p_{t+1} - p_t] = \frac{\theta_t^{(v,u)}}{w_t^{(v,u)}} \left[m_t^*(p_{t+1,m} - p_t)^2 + g_t^*(p_{t+1,g} - p_t)^2 + b_t^*(p_{t+1,b} - p_t)^2 \right],$$

which equalizes $\frac{\theta_t^{(v,u)}}{w_t^{(v,u)}} \text{Var}_t^*[p_{t+1} - p_t]$ and the equality follows from the fact that the risk-neutral covariance between the derivative payoff and the market price is zero. This concludes the proof for the market demand $\theta_t^{(v,u)}$. The proof for the derivative demand $\phi_t^{(v,u)}$ follows analogously. $\square$

Proof for Result 7. For an agent with belief (v, u) , the expected excess payoff of the derivative is given by:

$$\begin{aligned} \mathbb{E}_t^{(v,u)}[x_{t+1} - q_t] &= p_t(b_t^* - B_t)uv + p_t(G_t - g_t^*)(1 - u)v - p_t(b_t^*G_t - g_t^*B_t) \\ &= \frac{\text{Var}_t^*[p_{t+1}]}{(p_{t+1,g} - p_{t+1,b})} (v - v_t^*) + \frac{p_t^2(v_t^* - V_t)}{p_{t+1,g} - p_{t+1,b}} \\ &= \frac{\text{Var}_t^*[p_{t+1}]}{p_{t+1,g} - p_{t+1,b}} \left(v - v_t^* + \frac{(v_t^* - V_t)}{\mathbb{E}^{(V_t, U_t)}[r]} r \right) \\ &= \frac{\text{Var}_t^*[x_{t+1} - q_t]}{m_t^*q_t} \left(v - v_t^* + \frac{(v_t^* - V_t)}{\mathbb{E}^{(V_t, U_t)}[r]} r \right). \end{aligned} \quad (\text{A4})$$

Substituting this expression into (23), we obtain (26). $\square$

Proof for Result 8. First, we derive the aggregate belief $H_{n,t}$ for node n . As established in the proofs for the trinomial tree (e.g., Result 2), an agent with log utility and dogmatic beliefs $\mathbf{h} = (h_1, \dots, h_M)$ optimally acts as a Kelly bettor. Consequently, their wealth share in the economy at time t , denoted as $f_t(\mathbf{h})$, is proportional to their prior belief density multiplied by the likelihood of the observed news path $\mathbf{s}_t$.

Given that the prior distribution is a Dirichlet distribution $f_0(\mathbf{h}) = \frac{1}{B(\beta_1, \dots, \beta_M)} \prod_{n=1}^M h_n^{\beta_n - 1}$,

the wealth-weighted posterior distribution at time t is updated via Bayes' rule: $f_t(\mathbf{h}) \propto \prod_{n=1}^M h_n^{\beta_n-1} \prod_{n=1}^M h_n^{s_{n,t}} = \prod_{n=1}^M h_n^{\beta_{n,t}-1}$, where $\beta_{n,t} = \beta_n + s_{n,t}$. Thus, $f_t(\mathbf{h})$ is also a Dirichlet distribution with parameters $(\beta_{1,t}, \dots, \beta_{M,t})$. The aggregate sentiment $H_{n,t}$ for node n is the wealth-weighted average of the subjective probabilities, which corresponds to the mean of this updated Dirichlet distribution: $H_{n,t} = \mathbb{E}_{f_t}[h_n] = \frac{\beta_{n,t}}{\sum_{i=1}^M \beta_{i,t}} = \frac{s_{n,t} + \beta_n}{t + \sum_{i=1}^M \beta_n}$, where we used the identity $\sum_{i=1}^M s_{i,t} = t$.

Second, we derive the path-dependent risk-neutral measure $h_{t \rightarrow T}^*(\mathbf{s}_T)$. With a representative log-utility investor (whose beliefs correspond to the wealth-weighted distribution f_t), the stochastic discount factor (SDF) from t to T for a specific path ending at $\mathbf{s}_T$ is exactly the inverse of the market return: $M_{t \rightarrow T} = \frac{p_T(\mathbf{s}_T)^{-1}}{p_t} = \frac{p_t}{p_T(\mathbf{s}_T)}$. The physical probability of reaching state $\mathbf{s}_T$ from $\mathbf{s}_t$ under the aggregate belief f_t is the expectation of the multinomial likelihood over the Dirichlet posterior. This yields the standard Dirichlet-multinomial predictive distribution: $P(\mathbf{s}_T | \mathbf{s}_t) = \frac{(T-t)!}{\prod_{n=1}^M (s_{n,T} - s_{n,t})!} \frac{B(\beta_{1,T}, \dots, \beta_{M,T})}{B(\beta_{1,t}, \dots, \beta_{M,t})}$, where $B(\dots)$ is the multivariate Beta function.

The risk-neutral probability $h_{t \rightarrow T}^*(\mathbf{s}_T)$ is the physical probability multiplied by the SDF and the gross risk-free rate (which is normalized to 1). Therefore: $h_{t \rightarrow T}^*(\mathbf{s}_T) = P(\mathbf{s}_T | \mathbf{s}_t) \frac{p_t}{p_T(\mathbf{s}_T)}$. Substituting the expression for $P(\mathbf{s}_T | \mathbf{s}_t)$ yields the expression of $h_{t \rightarrow T}^*(\mathbf{s}_T)$ as in Result 8. $\square$

Proof for Result 9. The log payoff, $\log(p_T)$, decomposes into a two jump components, $\omega s_{1,T}$ and $-\omega s_{4,T}$, and a diffusion component, $\sigma \sqrt{T/N}(s_{2,T} - s_{3,T})$. In the continuous-time limit, the distribution of upward jump intensity at time t converges to a Gamma distribution $\Gamma(\beta_{1,t}, (\theta + \phi)N)$, so the wealth-weighted average beliefs of the upward jump count in $[t, T]$ converges to Negative-binomial distribution $NB(\beta_{1,t}, \frac{\theta + \phi}{\theta + 1})$. Similarly, the wealth-weighted average beliefs of the downward jump count converges to $NB(\beta_{4,t}, \frac{\theta + \phi}{\theta + 1})$. The belief corresponding to the diffusion component is derived in the proofs of Result 6.

Combining these results, we obtain the inverse of market price,

$$p_t^{-1} = \mathbb{E}[e^{-\log(p_T)} | f_t] = e^{-\omega(s_{1,t}-s_{4,t})-\sigma\sqrt{T/N}(s_{2,t}-s_{3,t})} \left(1 - \frac{(1-\phi)(e^{-\omega}-1)}{\theta+\phi}\right)^{-\beta_{1,t}} \\ \left(1 - \frac{(1-\phi)(e^{\omega}-1)}{\theta+\phi}\right)^{-\beta_{4,t}} \exp \left[-\frac{1-\phi}{\theta+\phi} \eta \sigma \sqrt{T} - \frac{1-\phi}{\theta+\phi} \eta_t \sigma \sqrt{\phi T} + \frac{1}{2} \frac{\theta+1}{\theta+\phi} (1-\phi) \sigma^2 T \right]. \quad (\text{A5})$$

Taking the reciprocal of the preceding expression gives (32). The moment-generating function of the log return is equal to $\mathbb{E}_t^* \left[\left(\frac{p_T}{p_t} \right)^\chi \right] = p_t^{1-\chi} \mathbb{E}_t [e^{(\chi-1)\log(p_T)} | f_t]$. Substituting into (A5) and using the moment-generating function of the Negative Binomial distribution yields (33). $\square$

Proof for Result 10. Let $Y_{i,T} = \log p_{i,T}$ be the terminal log-payoff of asset i . In the diffusion limit, the joint distribution is conditionally Gaussian under the measure $\mathbb{P}$, with $Y_{i,T} \sim \mathcal{N}(\mu_{i,t}, \sigma_{i,t}^2)$. This follows a similar argument as in the proof for Result 6 using Paul and Plackett (1978). We define the cross-sectional state variable $Z_T = Y_{A,T} - Y_{B,T}$, which is normally distributed as $Z_T \sim \mathcal{N}(\mu_{Z,t}, \sigma_{Z,t}^2)$.

The terminal market share of Asset A can be rewritten exactly using the $\mathscr{W}(x) = 1 + e^{-x}$ notation: $\frac{p_{A,T}}{p_T} = \frac{e^{Y_{A,T}}}{e^{Y_{A,T}} + e^{Y_{B,T}}} = \frac{1}{1 + \exp(-Z_T)} = \mathscr{W}(Z_T)^{-1}$. Taking the expectation using the MacKay Probit mapping, $\mathbb{E}[\mathscr{W}(X)^{-1}] \approx \mathscr{W}(\mu_X \kappa(\sigma_X^2))^{-1}$, immediately proves the market share identity: $\frac{p_{A,t}}{p_t} = \mathbb{E}_t[\mathscr{W}(Z_T)^{-1}] = \mathscr{W}(\kappa_t \mu_{Z,t})^{-1}$.

The inverse market price is the expectation of the terminal inverse market price $p_t^{-1} = \mathbb{E}_t[p_T^{-1}] = \mathbb{E}_t[e^{-Y_{A,T}} \mathscr{W}(Z_T)^{-1}]$. We apply a change of measure to $\tilde{\mathbb{P}}$, defined by the Radon-Nikodym derivative $\frac{d\tilde{\mathbb{P}}}{d\mathbb{P}} = \frac{e^{-Y_{A,T}}}{\mathbb{E}_t[e^{-Y_{A,T}}]}$. The expectation in the denominator is the standard log-normal moment: $\mathbb{E}_t[e^{-Y_{A,T}}] = \exp(-\mu_{A,t} + \frac{1}{2}\sigma_{A,t}^2)$. Under $\tilde{\mathbb{P}}$, the variance of Z_T is unchanged, but its mean shifts by its covariance with the tilting variable $-Y_{A,T}$: $\mu_{Z,t}^{\tilde{\mathbb{P}}} = \mu_{Z,t} + \text{Cov}_t(-Y_{A,T}, Z_T) = \mu_{Z,t} - \sigma_{A,t}^2 + \sigma_{AB,t}$. Applying the MacKay approximation under $\tilde{\mathbb{P}}$ yields: $\mathbb{E}_t^{\tilde{\mathbb{P}}}[\mathscr{W}(Z_T)^{-1}] = \mathscr{W}(\kappa_t(\mu_{Z,t} - \sigma_{A,t}^2 + \sigma_{AB,t}))^{-1}$. Combining these components, we obtain $p_t^{-1} = \exp(-\mu_{A,t} + \frac{1}{2}\sigma_{A,t}^2) \mathscr{W}(\kappa_t(\mu_{Z,t} - \sigma_{A,t}^2 + \sigma_{AB,t}))^{-1}$. Inverting this expression exactly yields equation (36).

Next, the risk-neutral variance of the market return is defined under the pricing measure

$\mathbb{Q}$: $\text{Var}_t^*[p_T/p_t] = \mathbb{E}_t^*[(p_T/p_t)^2] - 1 = \frac{1}{p_t} \mathbb{E}_t[p_T] - 1$. The expectation of the market is the sum: $\mathbb{E}_t[p_T] = F_{A,t} + F_{B,t} = F_{A,t}(1 + F_{B,t}/F_{A,t})$. We evaluate the forward ratio as $\frac{F_{B,t}}{F_{A,t}} = \exp(\mu_{B,t} + \frac{1}{2}\sigma_{B,t}^2 - \mu_{A,t} - \frac{1}{2}\sigma_{A,t}^2) = \exp(-[\mu_{Z,t} + \frac{1}{2}(\sigma_{A,t}^2 - \sigma_{B,t}^2)])$. Thus, $(1 + F_{B,t}/F_{A,t}) = \mathcal{W}(\mu_{Z,t} + \frac{1}{2}(\sigma_{A,t}^2 - \sigma_{B,t}^2))$. From our previous derivation of p_t^{-1} , multiplying by $F_{A,t} = \exp(\mu_{A,t} + \frac{1}{2}\sigma_{A,t}^2)$ yields:

$$\frac{F_{A,t}}{p_t} = \frac{\exp(\sigma_{A,t}^2)}{\mathcal{W}(\kappa_t(\mu_{Z,t} - \sigma_{A,t}^2 + \sigma_{AB,t}))}. \quad (\text{A6})$$

Multiplying (A6) by $(1 + F_{B,t}/F_{A,t})$ and subtracting 1 perfectly recovers equation (37).

Next, for the individual asset risk-neutral variance, we evaluate the second non-central moment: $\text{Var}_t^*[p_{A,T}/p_{A,t}] = \frac{p_t}{p_{A,t}^2} \mathbb{E}_t \left[\frac{p_{A,T}^2}{p_T} \right] - 1 = \frac{p_t}{p_{A,t}^2} \mathbb{E}_t [e^{Y_{A,T}} \mathcal{W}(Z_T)^{-1}] - 1$. We apply a forward change of measure $\mathbb{P}'$ defined by $\frac{d\mathbb{P}'}{d\mathbb{P}} = \frac{e^{Y_{A,T}}}{F_{A,t}}$. Under $\mathbb{P}'$, the mean of Z_T shifts by its positive covariance with $Y_{A,T}$: $\mu_{Z,t}^{\mathbb{P}'} = \mu_{Z,t} + \text{Cov}_t(Y_{A,T}, Z_T) = \mu_{Z,t} + \sigma_{A,t}^2 - \sigma_{AB,t}$. Applying the MacKay mapping under this forward measure gives: $\mathbb{E}_t [e^{Y_{A,T}} \mathcal{W}(Z_T)^{-1}] = F_{A,t} \mathcal{W}(\kappa_t(\mu_{Z,t} + \sigma_{A,t}^2 - \sigma_{AB,t}))^{-1}$. Next, we evaluate the scalar multiplier. Since $p_{A,t} = p_t \mathcal{W}(\kappa_t \mu_{Z,t})^{-1}$, we have $\frac{p_t}{p_{A,t}^2} = \frac{1}{p_t} \mathcal{W}(\kappa_t \mu_{Z,t})^2$. Combining the multiplier with the evaluated expectation yields $\frac{F_{A,t}}{p_t} \frac{\mathcal{W}(\kappa_t \mu_{Z,t})^2}{\mathcal{W}(\kappa_t(\mu_{Z,t} + \sigma_{A,t}^2 - \sigma_{AB,t}))} - 1$. Substituting equation (A6) for $F_{A,t}/p_t$ perfectly places the remaining $\mathcal{W}$ term in the denominator and attaches the $\exp(\sigma_{A,t}^2)$ scalar to the numerator, completing the proof for equation (38).

Finally, for the covariance, we expand the risk-neutral expectation of the gross returns:

$$\text{Cov}_t^*[p_{A,T}/p_{A,t}, p_T/p_t] = \frac{1}{p_{A,t}} \mathbb{E}_t \left[\left(p_{A,T} \frac{p_T}{p_t} \right) \frac{p_t}{p_T} \right] - 1 = \frac{1}{p_{A,t}} \mathbb{E}_t[p_{A,T}] - 1 = \frac{F_{A,t}}{p_{A,t}} - 1.$$

Using the identity $p_{A,t} = p_t \mathcal{W}(\kappa_t \mu_{Z,t})^{-1}$, we rewrite this as: $\frac{F_{A,t}}{p_{A,t}} = \frac{F_{A,t}}{p_t} \mathcal{W}(\kappa_t \mu_{Z,t})$. Substituting equation (A6) into this expression yields equation (39).

□

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Internet Appendix to

Pricing News and No News with Heterogeneous Beliefs

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[not for publication]

Abstract

This Internet Appendix presents supplementary material and results not included in the main body of the paper.

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I.A.1 Examples of trinomial tree with $T = 3$

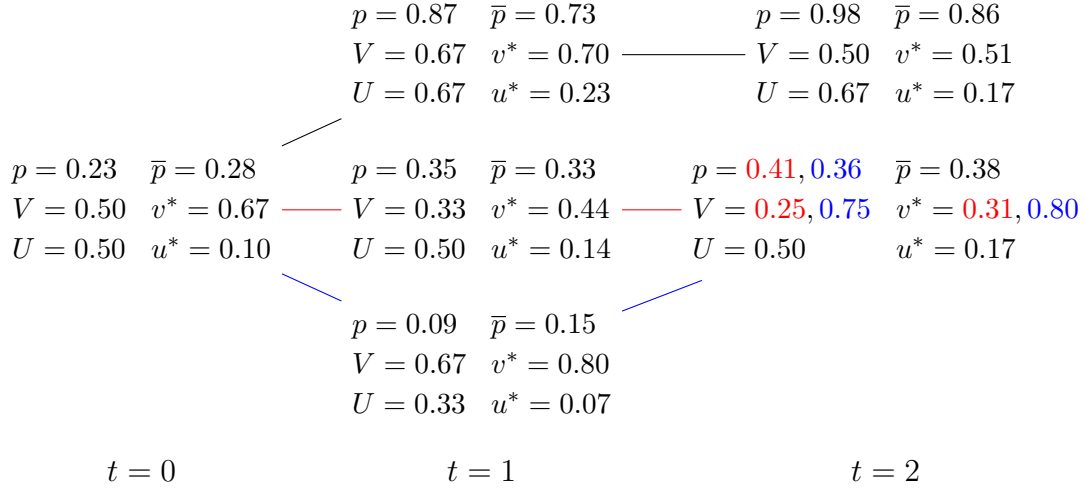


Figure I.A.1: **v -disagreement.** Agents' beliefs v are uniformly distributed and they agree on $u = 1/2$. p is the price of market, $\bar{p}$ is the market price in homogeneous economy where everyone agrees with the median agent $(v, u) = (1/2, 1/2)$. The nodes at $T = 3$ are omitted from the figure to make it compact: top to bottom nodes are $p = (4.95, 2.22, 1.00, 0.45, 0.20, 0.09, 0.04)$.

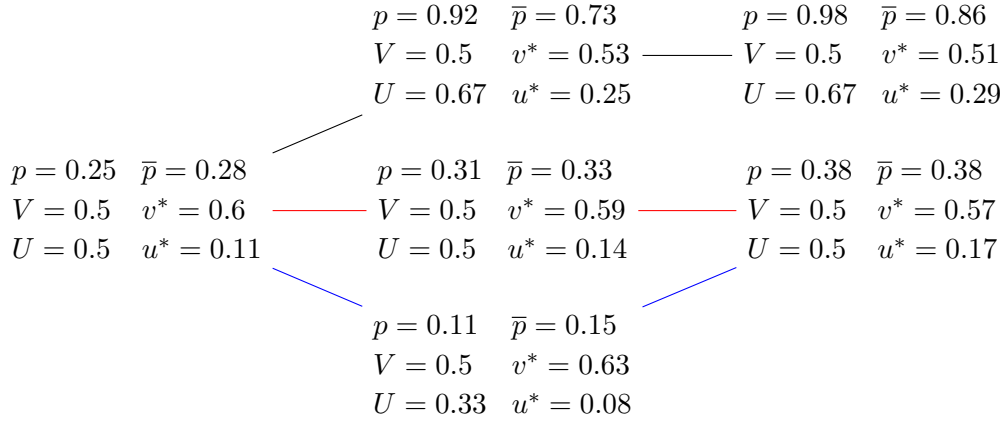


Figure I.A.2: ***u*-disagreement, neutral initial sentiment.** Agents' beliefs u are uniformly distributed and they agree on $v = 1/2$. p is the price of market, $\bar{p}$ is market price in homogeneous economy where everyone agrees with the median agent $(v, u) = (1/2, 1/2)$. The nodes at $T = 3$ are omitted from the figure to make it compact: top to bottom nodes are $p = (4.95, 2.22, 1.00, 0.45, 0.20, 0.09, 0.04)$.

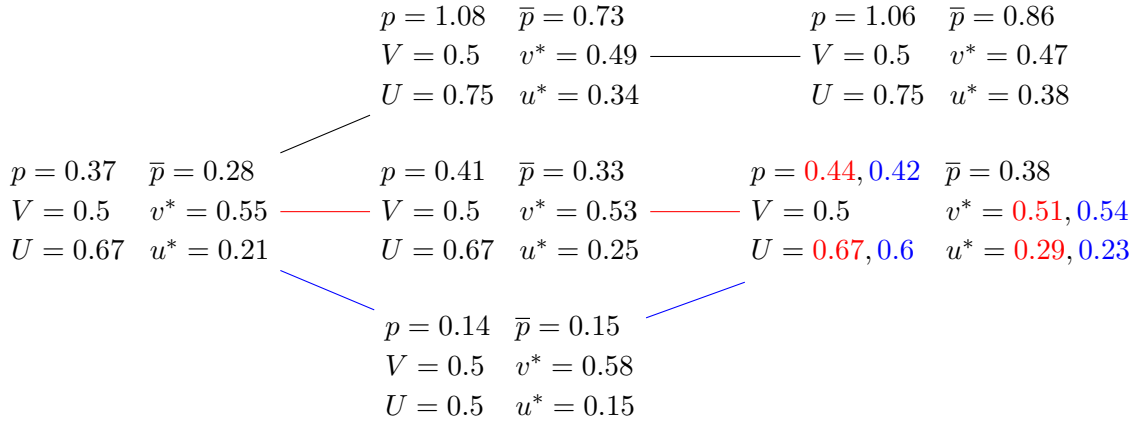


Figure I.A.3: ***u*-disagreement, bullish initial sentiment.** Agents' beliefs u follow $Beta(2, 1)$ distribution with mean $2/3$, and they agree on $v = 1/2$. p is the price of market, $\bar{p}$ is market price in homogeneous economy where everyone agrees with the agent $(v, u) = (1/2, 1/2)$. The nodes at $T = 3$ are omitted from the figure to make it compact: top to bottom nodes are $p = (4.95, 2.22, 1.00, 0.45, 0.20, 0.09, 0.04)$.

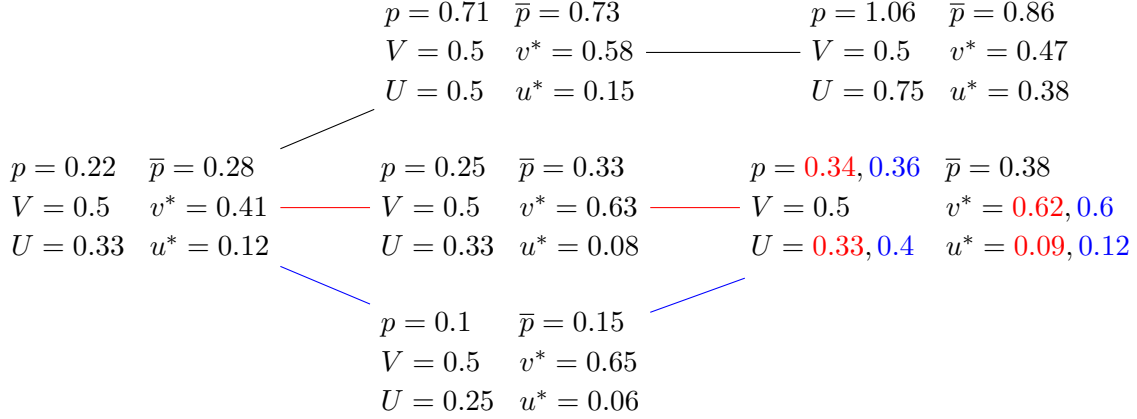


Figure I.A.4: ***u*-disagreement, bearish initial sentiment.** Agents' beliefs u follow $Beta(1, 2)$ distribution with mean $1/3$, and they agree on $v = 1/2$. p is the price of market, $\bar{p}$ is market price in homogeneous economy where everyone agrees with the agent $(v, u) = (1/2, 1/2)$. The nodes at $T = 3$ are omitted from the figure to make it compact: top to bottom nodes are $p = (4.95, 2.22, 1.00, 0.45, 0.20, 0.09, 0.04)$.

I.A.2 Wealth distribution and derivative exposure

Result IA1 (Wealth Distribution and News Intensity Beliefs Across Excess Return Expectations). *Define the variables $z(r)$, k , and γ :*

$$z(r) = \frac{(1+r)g_t^*m_t^* - g_t^*M_t}{G_tm_t^* - g_t^*M_t}, \quad k = \frac{G_tm_t^* - g_t^*M_t}{g_t^*M_t - \frac{B_t}{b_t^*}g_t^*m_t^*}, \quad \gamma = \frac{g_t^*m_t^*}{G_tm_t^* - g_t^*M_t^*}. \quad (\text{I.A.1})$$

i) The wealth share of agents who believe the expected return of the underlying is r satisfies

$$f_t(r) = \frac{\gamma k^{\beta_{ut}} z^{\alpha_{vt}-1} (1-z)^{\beta_{ut}+\beta_{vt}-1} B(\beta_{ut}, \beta_{vt})}{(k+1)^{\beta_{ut}} B(\alpha_{vt}, \beta_{vt}) B(\alpha_{ut}, \beta_{ut})} F_1 \left(\beta_{ut}; \alpha_{vt} + \beta_{vt} - 1, 1 - \alpha_{ut}; \beta_{ut} + \beta_{vt}; 1 - z, \frac{k(1-z)}{1+k} \right),$$

for $z > 0$, and

$$f_t(r) = \frac{\gamma k(-kz)^{\alpha_{vt}-1} (1+kz)^{\alpha_{ut}} B(\alpha_{ut}, \beta_{vt})}{(1+k)^{\alpha_{ut}+\beta_{vt}-1} B(\alpha_{vt}, \beta_{vt}) B(\alpha_{ut}, \beta_{ut})} F_1 \left(\alpha_{ut}; \alpha_{vt} + \beta_{vt} - 1, 1 - \beta_{ut}; \alpha_{ut} + \beta_{vt}; 1 + kz, \frac{1+kz}{1+k} \right),$$

for $z < 0$. The function $F_1(\dots)$ represents the Appell hypergeometric function, and $(\alpha_{vt}, \beta_{vt}, \alpha_{ut}, \beta_{ut})$

follows (13).

ii) The wealth-weighted average news arrival rate v among investors with expectation r , which we call $\mathbb{E}[v|r]$, satisfies, when $z \geq 0$,

$$\mathbb{E}[v|r] = z + z(1-z) \cdot \frac{\beta_{ut}}{\beta_{ut} + \beta_{vt}} \cdot \frac{F_1\left(\beta_{ut} + 1; \alpha_{vt} + \beta_{vt}, 1 - \alpha_{ut}; \beta_{ut} + \beta_{vt} + 1; 1 - z, \frac{k(1-z)}{1+k}\right)}{F_1\left(\beta_{ut}; \alpha_{vt} + \beta_{vt} - 1, 1 - \alpha_{ut}; \beta_{ut} + \beta_{vt}; 1 - z, \frac{k(1-z)}{1+k}\right)},$$

and when $z < 0$,

$$\mathbb{E}[v|r] = -kz + (-kz)(1+kz) \cdot \frac{\alpha_{ut}}{\alpha_{ut} + \beta_{vt}} \cdot \frac{F_1\left(\alpha_{ut} + 1; \alpha_{vt} + \beta_{vt}, 1 - \beta_{ut}; \alpha_{ut} + \beta_{vt} + 1; 1 + kz, \frac{1+kz}{1+k}\right)}{F_1\left(\alpha_{ut}; \alpha_{vt} + \beta_{vt} - 1, 1 - \beta_{ut}; \alpha_{ut} + \beta_{vt}; 1 + kz, \frac{1+kz}{1+k}\right)}$$

The variables $z(r)$ and k follow the same definitions as in (I.A.1), and $F_1(\dots)$ represents the Appell hypergeometric function.

Proof for Results 7 and IA1 . Consider the group of agents whose subjective expectation of the market return equals r : $\mathbb{E}_t^{(v,u)}[p_{t+1}/p_t - 1] = r$. We can express u as a function of v and r :

$$u = \frac{p_{t+1,m} - p_{t+1,b}}{p_{t+1,g} - p_{t+1,b}} + \frac{(1+r)p_t - p_{t+1,m}}{(p_{t+1,g} - p_{t+1,b})v} \quad (\text{I.A.2})$$

When $r < p_{t+1,m}/p_t - 1$, u stays in $[0, 1]$ if and only if $v \in [(p_{t+1,m} - (1+r)p_t)/(p_{t+1,m} - p_{t+1,b}), 1]$. When $r > p_{t+1,m}/p_t - 1$, u stays in $[0, 1]$ if and only if $v \in [((1+r)p_t - p_{t+1,m})/(p_{t+1,g} - p_{t+1,m}), 1]$. Substituting (I.A.2) into (3), the conditional distribution of v given r satisfies $f_t(v|r) \propto (v + kz)^{\alpha_{ut}-1} (v - z)^{\beta_{ut}-1} v^{\alpha_{vt}-\alpha_{ut}-\beta_{ut}} (1-v)^{\beta_{vt}-1}$, where z as a function of r represents the scaled return and k is a constant:

$$z = \frac{(1+r)p_t - p_{t+1,m}}{p_{t+1,g} - p_{t+1,m}} = \frac{1+r - M_t/m_t^*}{G_t/g_t^* - M_t/m_t^*}, \quad k = \frac{p_{t+1,g} - p_{t+1,m}}{p_{t+1,m} - p_{t+1,b}} = \frac{G_t/g_t^* - M_t/m_t^*}{M_t/m_t^* - B_t/b_t^*}. \quad (\text{I.A.3})$$

Integrating the joint distribution $f_t(v, z)$ over v yields the marginal distribution of z :

$$f_t(z) = \left(\frac{k}{k+1} \right)^{\beta_{ut}} \frac{B(\beta_{ut}, \beta_{vt})}{B(\alpha_{ut}, \beta_{ut})B(\alpha_{vt}, \beta_{vt})} z^{\alpha_{vt}-1} (1-z)^{\beta_{ut}+\beta_{vt}-1} \\ F_1 \left(\beta_{ut}; \alpha_{vt} + \beta_{vt} - 1, 1 - \alpha_{ut}, \beta_{ut} + \beta_{vt}; 1 - z, \frac{k(1-z)}{1+k} \right)$$

for $z > 0$ and

$$f_t(z) = \frac{k}{(k+1)^{\alpha_{ut}}} \frac{B(\alpha_{ut}, \beta_{vt})}{B(\alpha_{ut}, \beta_{ut})B(\alpha_{vt}, \beta_{vt})} (-kz)^{\alpha_{vt}-1} (1+kz)^{\alpha_{ut}+\beta_{vt}-1} \\ F_1 \left(\alpha_{ut}; \alpha_{vt} + \beta_{vt} - 1, 1 - \beta_{ut}; \alpha_{ut} + \beta_{vt}; 1 + kz, \frac{1+kz}{1+k} \right)$$

for $z < 0$. Substituting into $f_t(r) = p_t/(p_{t+1,g} - p_{t+1,m})f_t(z)$, we obtain the two equations in Result [IA1](#).

From the distribution $f_t(v|r)$, the conditional expectation $\mathbb{E}[v|r]$ can be expressed in terms of ratios of Appell hypergeometric functions. This expression allows us to compute the wealth-weighted average derivative position for this group of agents. Specifically, when $z \geq 0$, we obtain:

$$\mathbb{E}[v|r] = z + z(1-z) \cdot \frac{\beta_{ut}}{\beta_{ut} + \beta_{vt}} \cdot \frac{F_1 \left(\beta_{ut} + 1; \alpha_{vt} + \beta_{vt}, 1 - \alpha_{ut}; \beta_{ut} + \beta_{vt} + 1; 1 - z, \frac{k(1-z)}{1+k} \right)}{F_1 \left(\beta_{ut}; \alpha_{vt} + \beta_{vt} - 1, 1 - \alpha_{ut}; \beta_{ut} + \beta_{vt}; 1 - z, \frac{k(1-z)}{1+k} \right)}.$$

When $z < 0$, we obtain:

$$\mathbb{E}[v|r] = -kz + (-kz)(1+kz) \cdot \frac{\alpha_{ut}}{\alpha_{ut} + \beta_{vt}} \frac{F_1 \left(\alpha_{ut} + 1; \alpha_{vt} + \beta_{vt}, 1 - \beta_{ut}; \alpha_{ut} + \beta_{vt} + 1; 1 + kz, \frac{1+kz}{1+k} \right)}{F_1 \left(\alpha_{ut}; \alpha_{vt} + \beta_{vt} - 1, 1 - \beta_{ut}; \alpha_{ut} + \beta_{vt}; 1 + kz, \frac{1+kz}{1+k} \right)}.$$

□

I.A.2.1 Dynamic plots of Figure 3 and 6

I.A.2.1.1 High disagreement on v

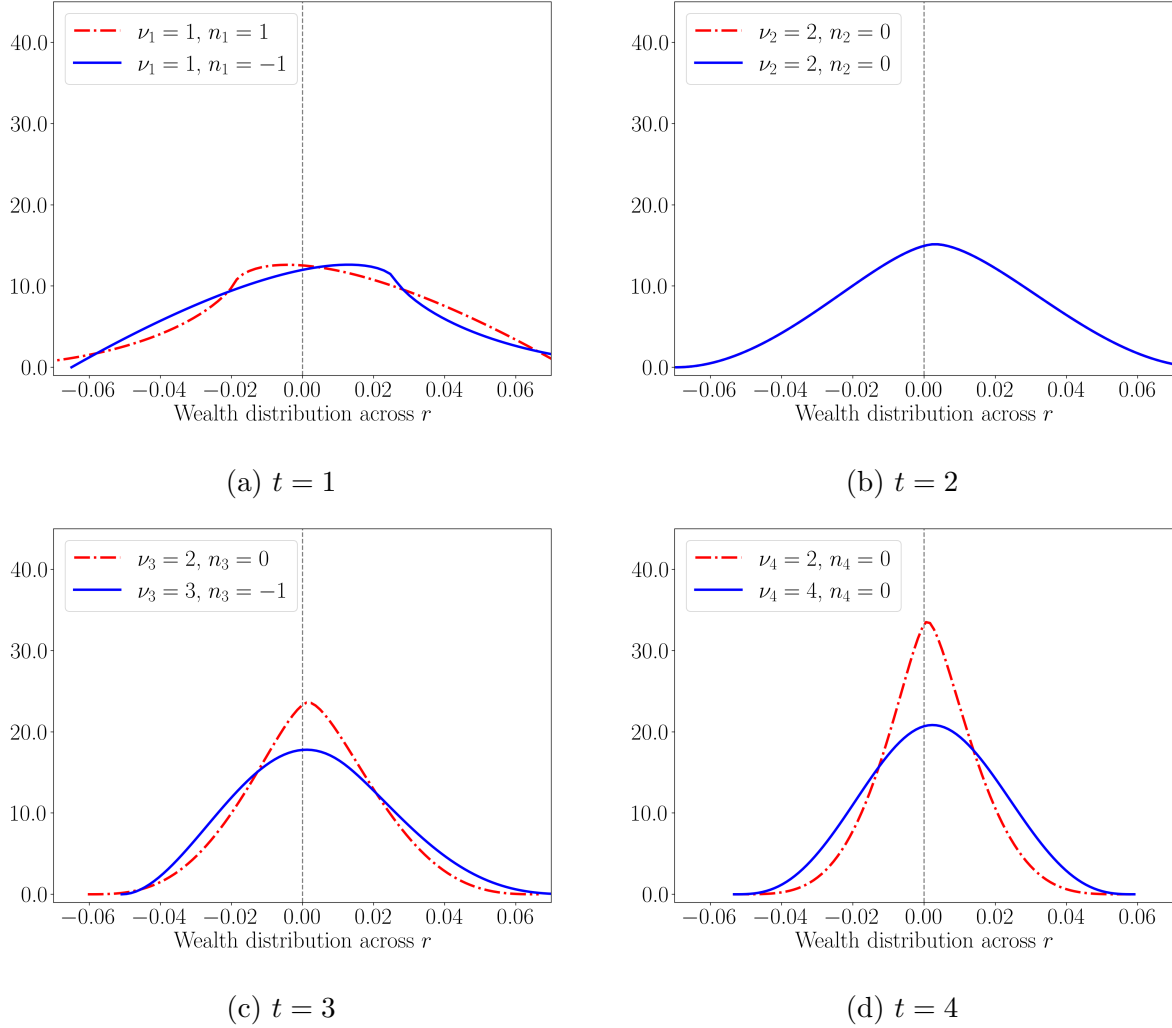
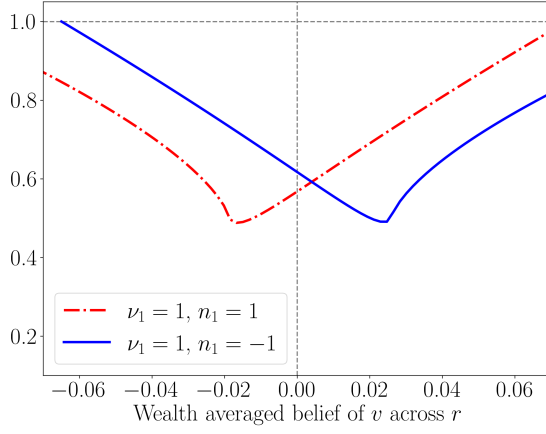
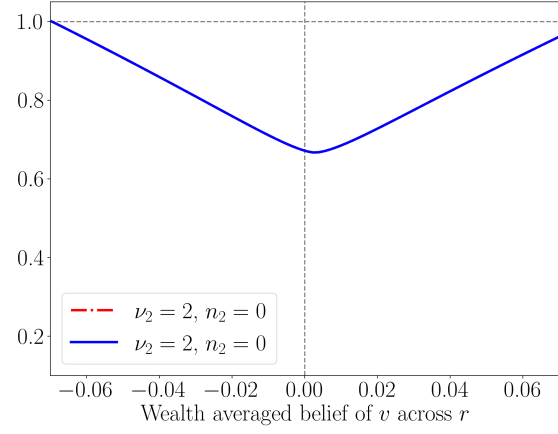


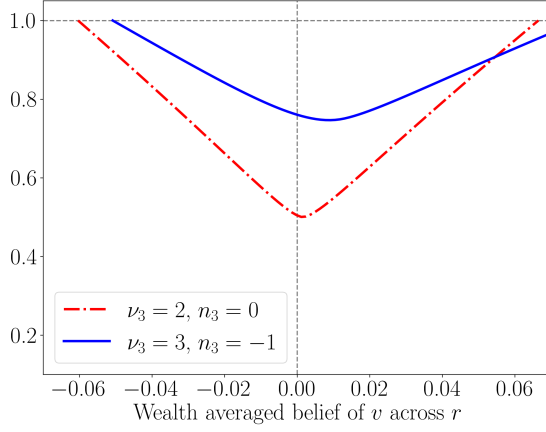
Figure I.A.5: **Wealth distribution across r .** The initial wealth distribution across agents is assumed to be uniform, i.e. $(\alpha_{v0}, \beta_{v0}, \alpha_{u0}, \beta_{u0}) = (1, 1, 1, 1)$. The two colored lines correspond to the two sample paths in Figure 1a. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05n_T}$.



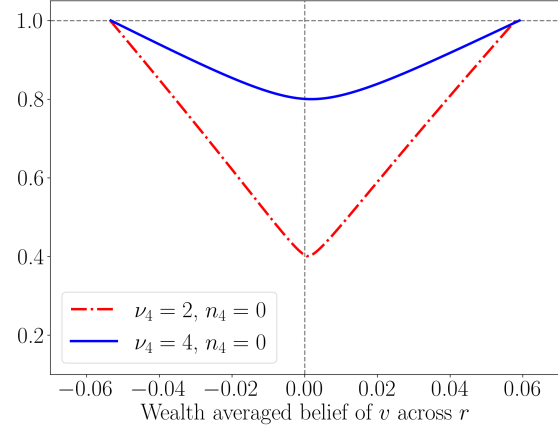
(a) $t = 1$



(b) $t = 2$

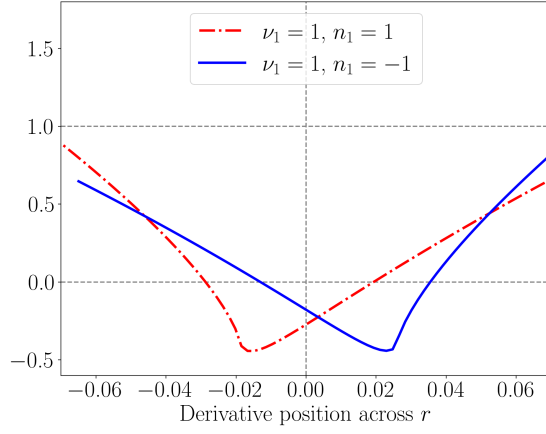


(c) $t = 3$

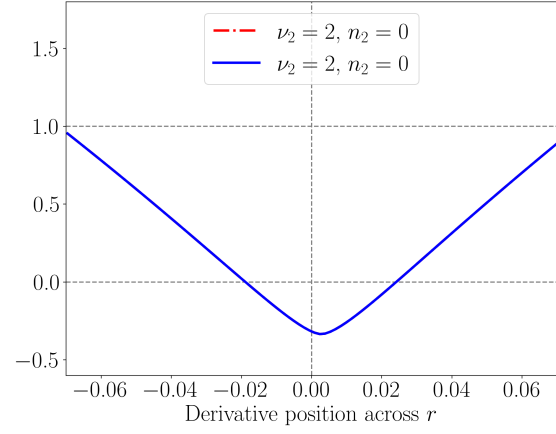


(d) $t = 4$

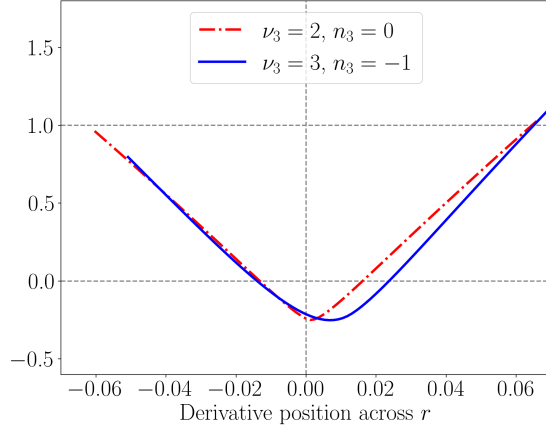
Figure I.A.6: **Wealth weighted beliefs of v across r .** The initial wealth distribution across agents is assumed to be uniform, i.e. $(\alpha_{v0}, \beta_{v0}, \alpha_{u0}, \beta_{u0}) = (1, 1, 1, 1)$. The two colored lines correspond to the two sample paths in Figure 1a. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05n_T}$.



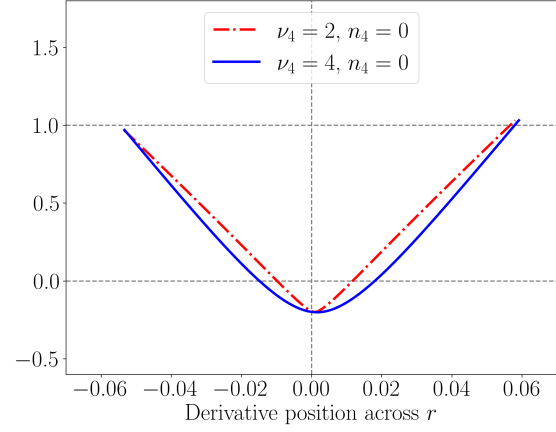
(a) $t = 1$



(b) $t = 2$

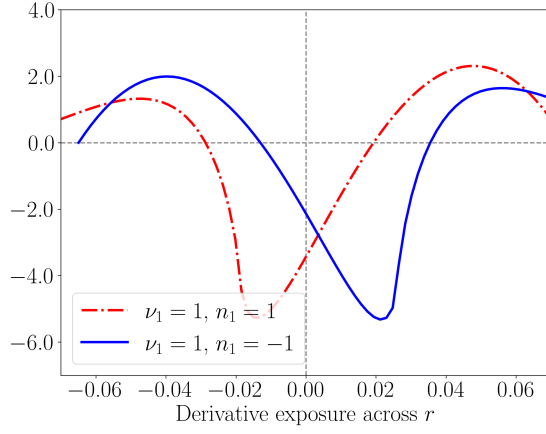


(c) $t = 3$

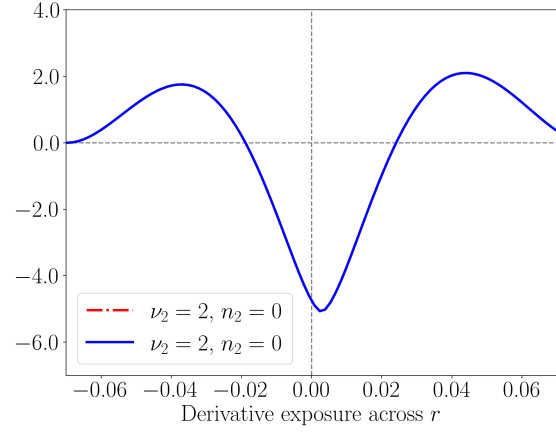


(d) $t = 4$

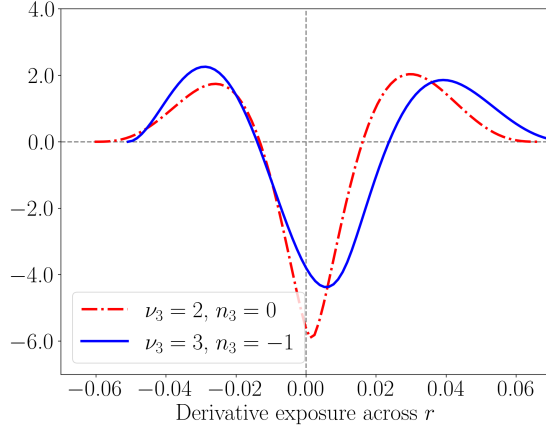
Figure I.A.7: **Portfolio weights in ‘straddle’ across r .** The initial wealth distribution across agents is assumed to be uniform, i.e. $(\alpha_{v0}, \beta_{v0}, \alpha_{u0}, \beta_{u0}) = (1, 1, 1, 1)$. The two colored lines correspond to the two sample paths in Figure 1a. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05n_T}$.



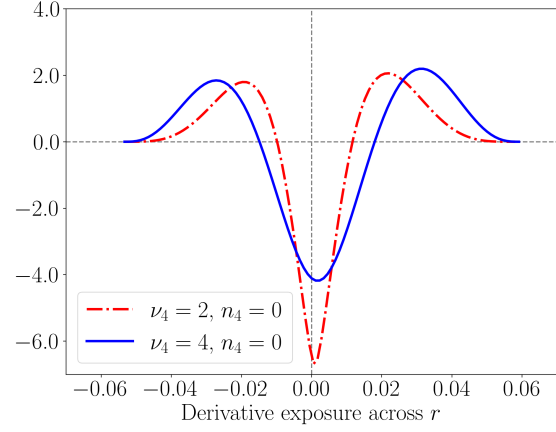
(a) $t = 1$



(b) $t = 2$



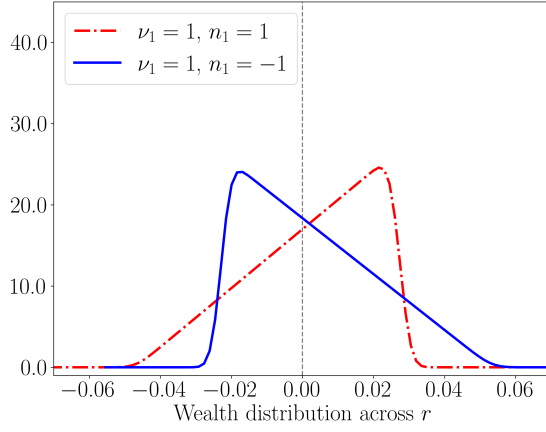
(c) $t = 3$



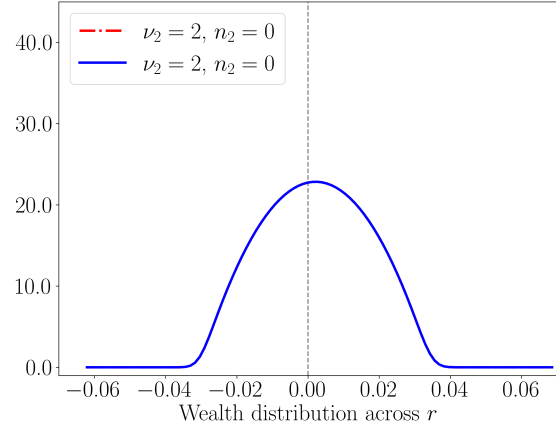
(d) $t = 4$

Figure I.A.8: **Gamma exposure across r .** The initial wealth distribution across agents is assumed to be uniform, i.e. $(\alpha_{v0}, \beta_{v0}, \alpha_{u0}, \beta_{u0}) = (1, 1, 1, 1)$. The two colored lines correspond to the two sample paths in Figure 1a. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05n_T}$.

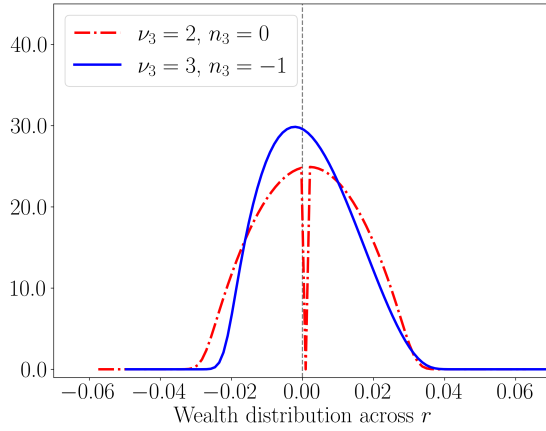
I.A.2.1.2 Low disagreement on $\mathbf{v}$



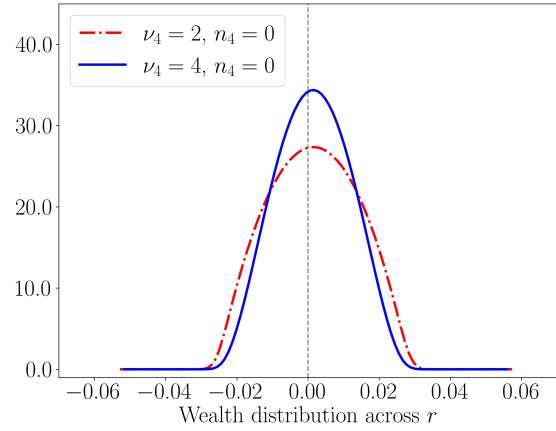
(a) $t = 1$



(b) $t = 2$

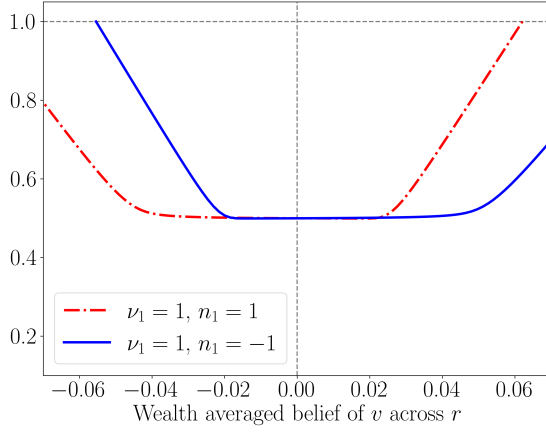


(c) $t = 3$

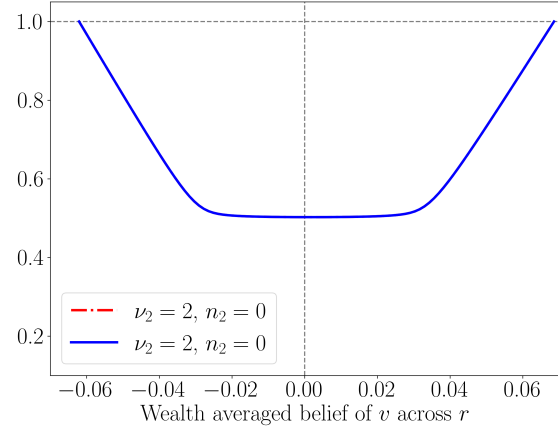


(d) $t = 4$

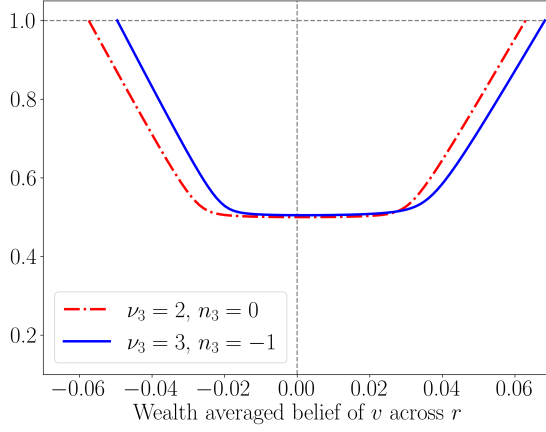
Figure I.A.9: **Wealth distribution across r .** The initial wealth distribution across agents is assumed to be uniform, i.e. $(\alpha_{v0}, \beta_{v0}, \alpha_{u0}, \beta_{u0}) = (100, 100, 1, 1)$. The two colored lines correspond to the two sample paths in Figure 1a. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05n_T}$.



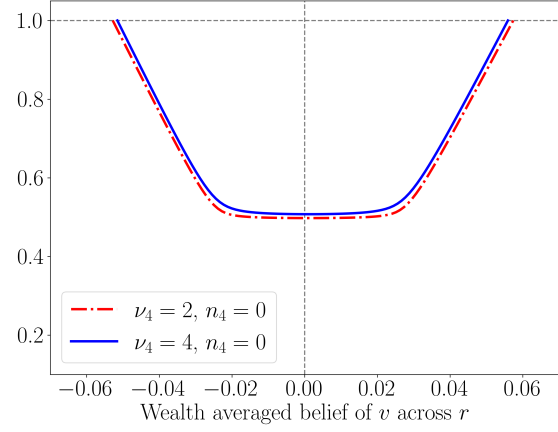
(a) $t = 1$



(b) $t = 2$

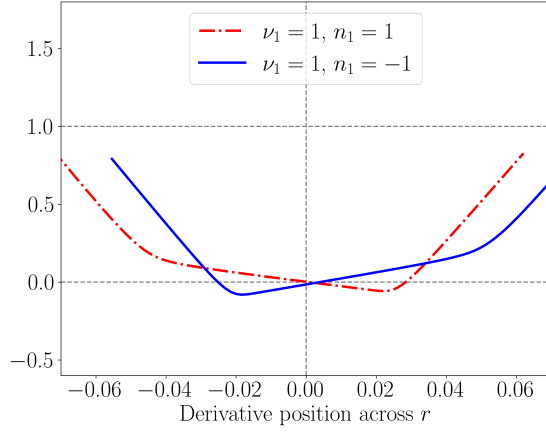


(c) $t = 3$

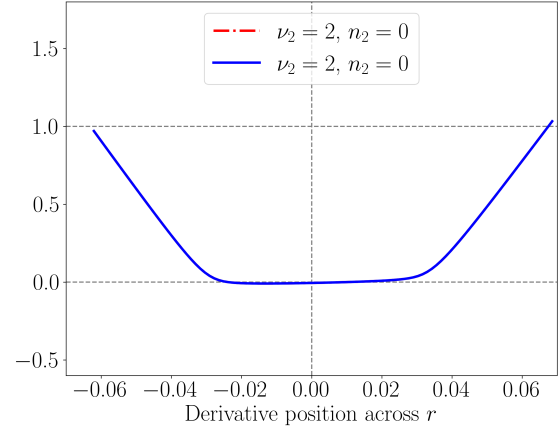


(d) $t = 4$

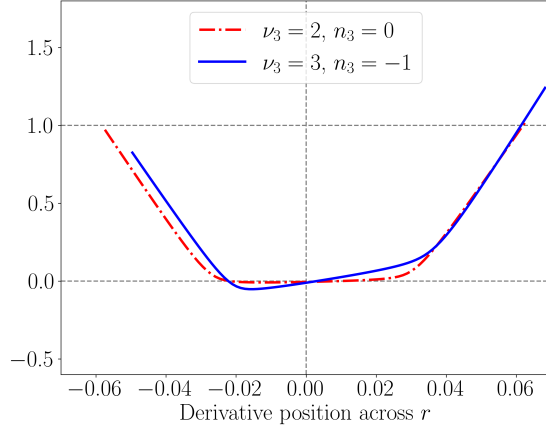
Figure I.A.10: **Wealth weighted beliefs of v across r .** The initial wealth distribution across agents is assumed to be uniform, i.e. $(\alpha_{v0}, \beta_{v0}, \alpha_{u0}, \beta_{u0}) = (100, 100, 1, 1)$. The two colored lines correspond to the two sample paths in Figure 1a. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05n_T}$.



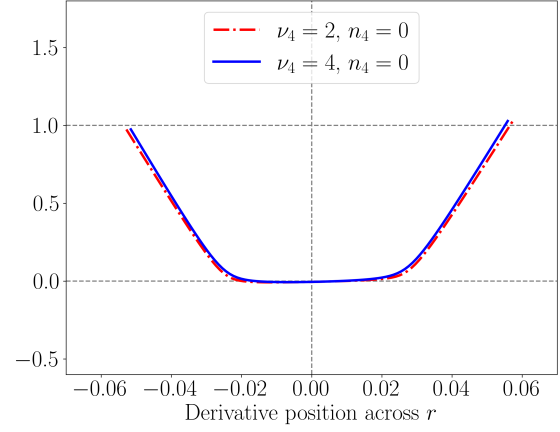
(a) $t = 1$



(b) $t = 2$



(c) $t = 3$



(d) $t = 4$

Figure I.A.11: **Portfolio weights in ‘straddle’ across r .** The initial wealth distribution across agents is assumed to be uniform, i.e. $(\alpha_{v0}, \beta_{v0}, \alpha_{u0}, \beta_{u0}) = (100, 100, 1, 1)$. The two colored lines correspond to the two sample paths in Figure 1a. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05n_T}$.

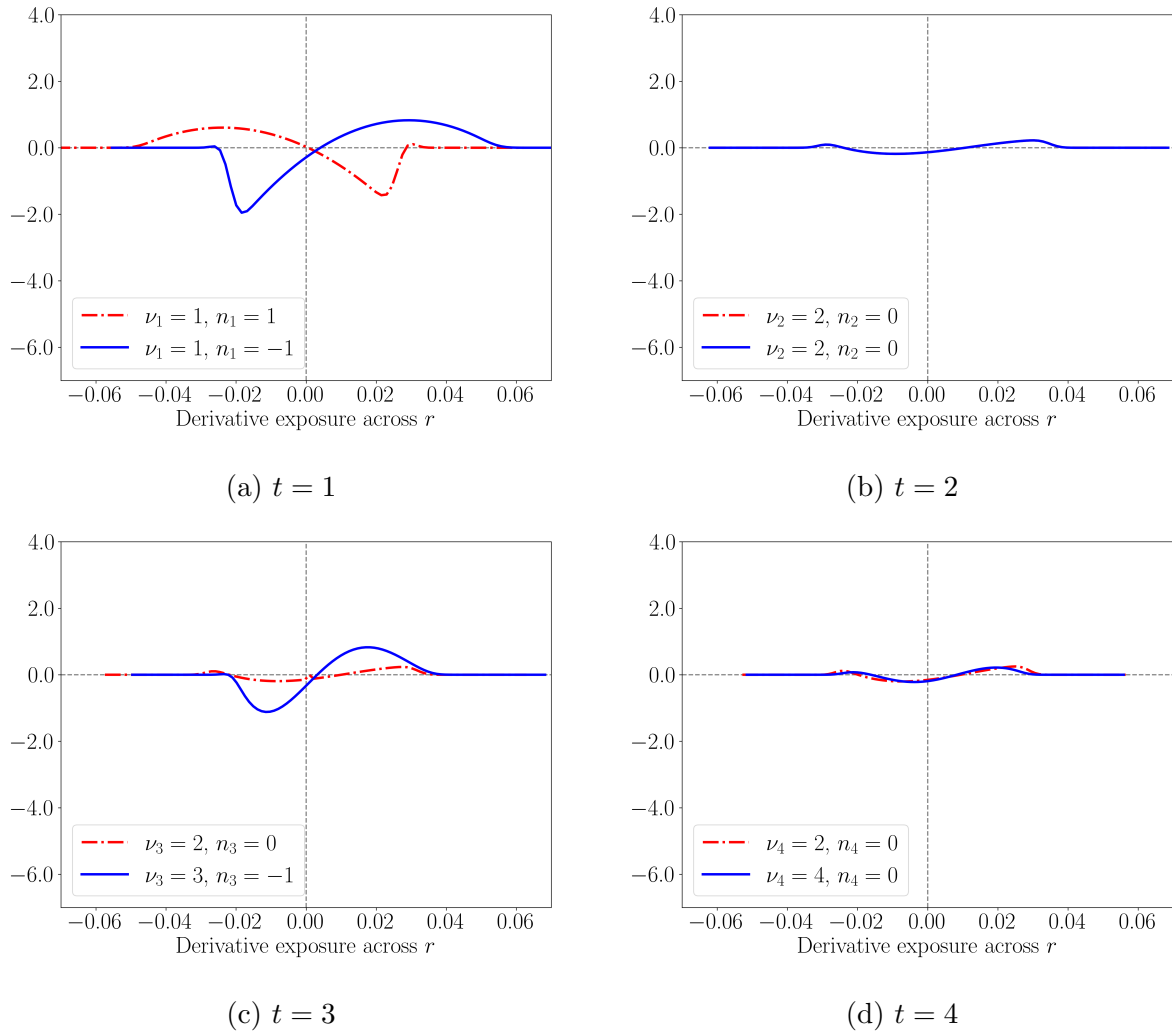


Figure I.A.12: **Gamma exposure across r .** The initial wealth distribution across agents is assumed to be uniform, i.e. $(\alpha_{v0}, \beta_{v0}, \alpha_{u0}, \beta_{u0}) = (100, 100, 1, 1)$. The two colored lines correspond to the two sample paths in Figure 1a. Terminal payoff at $T = 6$ is chosen to follow exponential function $e^{0.05n_T}$.

I.A.3 Extension to CRRA Preferences

Assume that all agents have CRRA preferences with risk aversion parameter γ over terminal wealth. In this case, the representative agent (Mr. Market) differs from the simple wealth-weighted average of agents. Because markets are complete, the competitive equilibrium allocation is Pareto optimal. We therefore characterize the equilibrium by constructing a

social welfare function with Negishi weights $\psi_t(v, u)$ (Negishi (1960)) at time t :

$$\max_{w_T^{(v,u)}} \int \psi_t(v, u) \mathbb{E}_t^{(u,v)} \left[\frac{(w_T^{(v,u)})^{1-\gamma}}{1-\gamma} \right] dv du \quad \text{subject to} \quad \int w_T^{(v,u)}(n_T) dv du = p_T(n_T). \quad (\text{I.A.4})$$

where the Negishi weight is inversely proportional to subjective expected marginal utility of terminal wealth $\mathbb{E}_t^{(u,v)} \left[(w_T^{(v,u)})^{-\gamma} \right]$.

Up to a normalization constant, the Lagrange multiplier associated with the state-by-state feasibility constraint corresponds to the Arrow–Debreu state price. Applying the first-order condition and substituting $w_T^{(v,u)}$ into the feasibility constraint implies that the Arrow–Debreu price of state n_T at time t , $q_t(n_T)$, is proportional to the marginal utility of the terminal payoff multiplied by a consensus probability of that state.

$$q_t(n_T) = p_T^{-\gamma}(n_T) \left[\int \left(\psi_t(v, u) \Pr^{(v,u)}(n_T) \right)^{\frac{1}{\gamma}} dudv \right]^{\gamma}. \quad (\text{I.A.5})$$

Using the zero interest rate condition $\sum_{n_T} q_t(n_T) = 1$, we obtain the market price

$$p_t = \sum_{n_T} q_t(n_T) p_T(n_T) = \frac{\sum_{n_T} p_T^{1-\gamma}(n_T) \left[\int \left(\psi_t(v, u) \Pr^{(v,u)}(n_T) \right)^{\frac{1}{\gamma}} dudv \right]^{\gamma}}{\sum_{n_T} p_T^{-\gamma}(n_T) \left[\int \left(\psi_t(v, u) \Pr^{(v,u)}(n_T) \right)^{\frac{1}{\gamma}} dudv \right]^{\gamma}}. \quad (\text{I.A.6})$$

$\psi_t(v, u)$ plays a role analogous to the wealth distribution $f_t(v, u)$ under logarithmic preferences in the main text, aggregating agents' heterogeneous beliefs into the equilibrium price. Under logarithmic preference, $\psi_t(v, u)$ coincides exactly with $f_t(v, u)$. Setting $\gamma = 1$ in (I.A.6) therefore recovers Result 1.

Because the allocation $w_T^{(v,u)}$ implied by the Negishi weights must be identical across periods, the Negishi weights satisfy the following evolution equation, under which the weight increases more for the agent with more ex-post correct beliefs. This is summarized by the following result.

Result IA2 (Preference-Invariant Selection). *In any complete-market economy with CRRA*

preferences, the Negishi weight of agent (v, u) relative to agent (v', u') satisfies

$$\frac{\psi_t(v, u)}{\psi_t(v', u')} = \frac{\psi_0(v, u)}{\psi_0(v', u')} \prod_{\tau=1}^t \frac{\Pr^{(v, u)}(s_\tau)}{\Pr^{(v', u')}(s_\tau)}, \quad (\text{I.A.7})$$

which is independent of γ , the risk-neutral measure, and all equilibrium prices. In particular, if $\psi_0(v, u)$ is specified by the same functional form as $f_0(v, u)$ in (3), then $\psi_t(v, u)$ has the same functional form as $f_t(v, u)$ in (12) for all t , γ , and sample paths.

Proof. In a complete market, the first-order condition for agent (v, u) in state n_T is:

$$\psi_t(v, u) \Pr^{(v, u)}(n_T) (w_T^{(v, u)}(n_T))^{-\gamma} = \ell_t(n_T)$$

where $\ell_t(n_T)$ is the Lagrange multiplier on the feasibility constraint. This gives $w_T^{(v, u)}(n_T) = (\psi_t(v, u) \Pr^{(v, u)}(n_T) / \ell_t(n_T))^{1/\gamma}$. For this allocation to be consistent across periods (i.e., $w_T^{(v, u)}(n_T)$ does not depend on t for the same equilibrium), the Negishi weight evolution must satisfy $\psi_{t+1}/\psi_t = \Pr^{(v, u)}(s_{t+1}) \ell_{t+1}(n_T) / \ell_t(n_T)$. But $\ell_{t+1}(n_T) / \ell_t(n_T)$ cancels in the cross-sectional ratio (I.A.7), leaving a pure likelihood ratio. $\square$

The market selection mechanism and the survival dynamics of heterogeneous beliefs are therefore invariant to risk aversion γ : the central planner's welfare weights shift toward the most accurate forecasters at the same rate as wealth reallocation in the log-utility economy.

Under CRRA preferences with any $\gamma > 0$, the Negishi weight of an agent with beliefs (v, u) decays exponentially at rate $KL(v_{true}, v) + v_{true} KL(u_{true}, u)$ per trading period, as in Result 4. Consequently, the Poisson-limit survival of intensity disagreement and the Brownian-limit elimination of intensity disagreement hold exactly as stated in the main text. Every step in the proof of Result 4 carries over verbatim once the wealth share $f(v, u)$ is replaced with the Negishi weight $\psi(v, u)$. The Poisson and Brownian limit results follow from properties of the KL divergence and therefore do not depend on the pricing model.

The CRRA equilibrium admits a representation in terms of an effective consensus probability measure.

Result IA3 (CRRA Consensus Representation). *Define the CRRA consensus probability of terminal state n_T as*

$$\tilde{\pi}_t(n_T) = \frac{\left[\int \left(\psi_t(v, u) \Pr^{(v, u)}(n_T) \right)^{\frac{1}{\gamma}} dv du \right]^{\gamma}}{\sum_n \left[\int \left(\psi_t(v, u) \Pr^{(v, u)}(n) \right)^{\frac{1}{\gamma}} dv du \right]^{\gamma}}. \quad (\text{I.A.8})$$

Let $\tilde{\mathbb{P}}_t$ denote the corresponding consensus probability measure. Then the equilibrium price and the risk-neutral variance satisfy

$$p_t = \frac{\tilde{\mathbb{E}}_t[p_T^{1-\gamma}]}{\tilde{\mathbb{E}}_t[p_T^{-\gamma}]}, \quad \text{Var}_t^*[p_T/p_t] = \frac{\tilde{\mathbb{E}}_t[p_T^{2-\gamma}] \tilde{\mathbb{E}}_t[p_T^{-\gamma}]}{\left(\tilde{\mathbb{E}}_t[p_T^{1-\gamma}] \right)^2} - 1. \quad (\text{I.A.9})$$

Proof. Equation (I.A.6) establishes the equilibrium price in a form equivalent to the proposition. For the risk-neutral variance formula,

$$\begin{aligned} \mathbb{E}_t^* \left[\left(\frac{p_T}{p_t} \right)^2 \right] &= \frac{\sum_{n_T} q_t(n_T) p_T^2(n_T) / p_t^2}{\sum_{n_T} q_t(n_T)} = \frac{\sum_{n_T} p_T^{2-\gamma} \tilde{\pi}_t(n_T)}{\sum_{n_T} p_T^{-\gamma} \tilde{\pi}_t(n_T)} \cdot \frac{(\sum_{n_T} p_T^{-\gamma} \tilde{\pi}_t(n_T))^2}{(\sum_{n_T} p_T^{1-\gamma} \tilde{\pi}_t(n_T))^2} \\ &= \frac{\tilde{\mathbb{E}}_t[p_T^{2-\gamma}] \tilde{\mathbb{E}}_t[p_T^{-\gamma}]}{(\tilde{\mathbb{E}}_t[p_T^{1-\gamma}])^2}. \end{aligned}$$

Subtracting 1 gives $\text{Var}_t^*[p_T/p_t]$. □

The first expression shows that the market price equals the ratio of two consensus expectations of different moments of the terminal payoff, while the second shows that the risk-neutral variance depends on other moments of p_T under the same measure.

Because the Negishi weights $\psi_t(v, u)$ follow the exact same Beta-distribution updating rule as the log-utility wealth shares, the dispersion of the CRRA consensus measure $\tilde{\pi}_t(n_T)$ responds to realized news in a similar manner. Using the expression for $\psi_t(v, u)$ in (12)

and substituting it into (I.A.8) gives

$$\tilde{\pi}_t(n_T) \propto \sum_{\substack{\forall n_g - n_b = n_T - n_t, \\ n_g + n_m + n_b = T - t}} \frac{(T - t)!}{n_m! n_g! n_b!} \frac{\left[B\left(\frac{\alpha_{vt} + T - t - n_m + \gamma - 1}{\gamma}, \frac{\beta_{vt} + n_m + \gamma - 1}{\gamma}\right) \right]^\gamma}{B(\alpha_{vt}, \beta_{vt})} \frac{\left[B\left(\frac{\alpha_{ut} + n_g + \gamma - 1}{\gamma}, \frac{\beta_{ut} + n_b + \gamma - 1}{\gamma}\right) \right]^\gamma}{B(\alpha_{ut}, \beta_{ut})}. \quad (\text{I.A.10})$$

When disagreement over the news arrival rate (v) is high, periods without news concentrate Negishi weights among low- v agents. This shift increases the likelihood of paths with high n_m and consequently compresses the variance of n_T under $\tilde{\mathbb{P}}_t$. In contrast, when there is no disagreement about the news arrival rate, periods without news leave wealth broadly distributed across agents with heterogeneous u beliefs. This increases the likelihood of paths with large net news $n_g - n_b$ and thus raises the variance of $n_T = n_t + n_g - n_b$. These forces carry over to the risk-neutral variance in (I.A.9) for exponential terminal payoffs $p_T = e^{\sigma n_T}$. Consequently, the dichotomy of volatility persistence remains a robust qualitative feature of the equilibrium.

The impact of realized news frequency on asset valuations may instead depend on the risk aversion. For typical equity investors with relative risk aversion $\gamma > 1$, a less dispersed distribution $\tilde{\pi}_t(n_T)$ implies a higher price because $p_T^{-\gamma}$ exhibits greater curvature than $p_T^{1-\gamma}$. Consequently, when disagreement about the news arrival rate is high, the “no news is good news” finding remains robust. This finding need not hold when $\gamma < 1$. In particular, as $\gamma \rightarrow 0$, $p_t \approx \tilde{\mathbb{E}}_t[p_T]$ can increase when $\tilde{\pi}_t(n_T)$ becomes more dispersed, since investors who assign greater weight to high-volatility outcomes bid up prices.

I.A.4 Poisson limit of two risky assets

We consider the continuous-time Poisson limit to obtain tractable formulas for prices and risk-neutral jump intensity in a market with 2 risky assets.

The interval $[0, T]$ is partitioned into $N = T/\delta$ periods, each of length δ . The market consists of assets A and B. In any interval δ , four mutually exclusive states can occur: no

jumps, an idiosyncratic jump in A only, an idiosyncratic jump in B only, or a simultaneous co-jump in both. When a jump occurs in asset i , its fundamental value drops by a fractional amount $\Delta_i = 1 - e^{-\omega_i}$. The terminal payoffs are

$$p_{i,T}(s_{i,T}) = e^{-\omega_i s_{i,T}},$$

where $s_{i,T}$ is the total count of jumps experienced by asset i .

To model heterogeneous beliefs, we scale the Dirichlet parameters of the 4-node tree. Let $\theta > 0$ represent the disagreement among agents. At $t = 0$, the three jump states are initialized as $\beta_{j,0} = \lambda_{j,0}\theta$ for $j \in \{A, B, AB\}$, while the no-jump state holds the remainder: $\beta_{0,0} = \frac{\theta}{\delta} - (\beta_{A,0} + \beta_{B,0} + \beta_{AB,0})$.

As the discrete step size $\delta \rightarrow 0$, the probability of the no-jump state converges to 1. This completely decouples the variance of the jump branches, meaning the aggregate, wealth-weighted physical probabilities structurally converge from a joint Dirichlet distribution to three strictly independent continuous-time Gamma-Poisson processes.

If $n_{j,t}$ events of type j accumulate over time t , the updated Dirichlet parameters (convictions) evolve exactly as $\beta_{j,t} = \beta_{j,0} + n_{j,t}$. The wealth-weighted physical jump intensities are therefore continuously updated based on the accumulated events:

$$\lambda_{j,t} = \frac{\beta_{j,t}}{\theta + t}, \quad \text{for } j \in \{A, B, AB\}. \quad (\text{I.A.11})$$

At time T , $Y_A = \ln p_{A,T}$ and $Y_B = \ln p_{B,T}$ denote the terminal log-prices. At time t , we have $Y_A = y_{A,t} - \omega_A n_{A,\tau} - \omega_A n_{AB,\tau}$ and $Y_B = y_{B,t} - \omega_B n_{B,\tau} - \omega_B n_{AB,\tau}$, where $y_{i,t}$ is the known log-fundamental at time t , $\tau = T - t$ is the remaining time to maturity, and $n_{j,\tau}$ are the unknown future jump counts. We define $\kappa_t = \frac{\tau}{\theta + t}$ for convenience.

The joint characteristic function $\Phi_t(z_1, z_2) = \mathbb{E}_t[\exp(iz_1 Y_A + iz_2 Y_B)]$ factors into the product of the respective moment generating functions of the three independent jumps. For complex arguments $z_1, z_2 \in \mathbb{C}$, the exact analytical closed form of the joint characteristic

function at time t is:

$$\begin{aligned} \Phi_t(z_1, z_2) = & \exp(i z_1 y_{A,t} + i z_2 y_{B,t}) \times [1 - \kappa_t(e^{-i z_1 \omega_A} - 1)]^{-\beta_{A,t}} \\ & \times [1 - \kappa_t(e^{-i z_2 \omega_B} - 1)]^{-\beta_{B,t}} \times [1 - \kappa_t(e^{-i z_1 \omega_A - i z_2 \omega_B} - 1)]^{-\beta_{AB,t}}, \end{aligned} \quad (\text{I.A.12})$$

where $\beta_{j,t}$ are the updated Dirichlet parameters tracking the accumulation of realized jumps.

The market weight of asset i at time t is $w_{i,t} = p_{i,t}/p_t$. The market price p_t and individual asset prices $p_{i,t}$ can be solved via Fourier inversion in the complex plane.

Result IA4 (Market price and market share). *The inverse market price $p_t^{-1} = \mathbb{E}_t[(e^{Y_A} + e^{Y_B})^{-1}]$ is given by a single one-dimensional Fourier integral:*

$$p_t^{-1} = \frac{1}{2} \int_{-\infty}^{\infty} \frac{\Phi_t(-u + i\alpha, u - i(\alpha - 1))}{\sin(\pi(iu + \alpha))} du, \quad (\text{I.A.13})$$

where $u \in \mathbb{R}$, and $\alpha \in (0, 1)$ is an arbitrary number. The market share of asset A at time t satisfies:

$$w_{A,t} = \frac{p_{A,t}}{p_t} = \frac{1}{2} \int_{-\infty}^{\infty} \frac{\Phi_t(u - i\alpha, -u + i\alpha)}{\sin(\pi(iu + \alpha))} du. \quad (\text{I.A.14})$$

The risk-neutral variance of the aggregate market return p_T/p_t is:

$$\text{Var}_t^*(p_T/p_t) = \frac{e^{y_{A,t}}}{p_t} \mathcal{J}_A(\kappa_t)^{-(\beta_{A,t} + \beta_{AB,t})} + \frac{e^{y_{B,t}}}{p_t} \mathcal{J}_B(\kappa_t)^{-(\beta_{B,t} + \beta_{AB,t})} - 1, \quad (\text{I.A.15})$$

where the function $\mathcal{J}_i(x) = (1 - x(e^{-\omega_i} - 1))$.

The risk-neutral variance of Asset A and its covariance with market satisfy:

$$\text{Var}_t^*(p_{A,T}/p_{A,t}) = \frac{p_t}{p_{A,t}^2} \left[\frac{1}{2} \int_{-\infty}^{\infty} \frac{\Phi_t(u - i(1 + \alpha), -u + i\alpha)}{\sin(\pi(iu + \alpha))} du \right] - 1, \quad (\text{I.A.16})$$

$$\text{Cov}_t^*(p_{A,T}/p_{A,t}, p_T/p_t) = \frac{e^{y_{A,t}}}{p_{A,t}} \mathcal{J}_A(\kappa_t)^{-(\beta_{A,t} + \beta_{AB,t})} - 1. \quad (\text{I.A.17})$$

Result IA5 (Jump intensity and correlation). *Let the physical intensities be $\lambda_{A,t}$, $\lambda_{B,t}$,*

and $\lambda_{AB,t}$. The corresponding risk-neutral jump intensities are:

$$\lambda_{A,t}^* = \lambda_{A,t} \mathcal{D}_{A,t}^{-1}, \quad \lambda_{B,t}^* = \lambda_{B,t} \mathcal{D}_{B,t}^{-1}, \quad \lambda_{AB,t}^* = \lambda_{AB,t} (\mathcal{D}_{A,t} + \mathcal{D}_{B,t} - 1)^{-1}, \quad (\text{I.A.18})$$

where $\mathcal{D}_{i,t} = 1 - w_{i,t} \Delta_i$.

The instantaneous risk-neutral variance of Asset i 's return, and its risk-neutral covariance with Asset j 's return are given by:

$$\text{Var}_t^*(dR_i)/dt = (\lambda_{i,t}^* + \lambda_{ij,t}^*) \Delta_i^2, \quad \text{Cov}_t^*(dR_i, dR_j)/dt = \lambda_{ij,t}^* \Delta_i \Delta_j. \quad (\text{I.A.19})$$

Let ρ_t denote the physical correlation of jump arrivals, and ρ_t^* denote the risk-neutral correlation. We have $|\rho_t^*| > |\rho_t|$, and specifically,

$$\rho_t^* = \rho_t \times \frac{\sqrt{(\lambda_{A,t} + \lambda_{AB,t})(\lambda_{B,t} + \lambda_{AB,t})}}{\sqrt{(\lambda_{A,t}(1 - \mathcal{D}_{A,t}^{-1}(1 - \mathcal{D}_{B,t})) + \lambda_{AB,t})(\lambda_{B,t}(1 - \mathcal{D}_{B,t}^{-1}(1 - \mathcal{D}_{A,t})) + \lambda_{AB,t})}}. \quad (\text{I.A.20})$$

I.A.4.1 Proofs for results

Proof for Result IA4. The inverse market price is the expectation of the inverse terminal market payoff:

$$p_t^{-1} = \mathbb{E}_t \left[\frac{1}{p_{A,T} + p_{B,T}} \right] = \mathbb{E}_t \left[e^{-Y_B} \frac{1}{1 + e^{Y_A - Y_B}} \right] \quad (\text{I.A.21})$$

Let $Z = Y_A - Y_B$. We define the complex variable $v = u - i\alpha$, where $u \in \mathbb{R}$ and $\alpha \in \mathbb{R}$ is a free parameter. The Fourier transform of $g(Z)$ along this contour is: $\hat{g}(v) = \int_{-\infty}^{\infty} e^{ivZ} \frac{1}{1+e^Z} dZ$. We apply the substitution $x = e^Z$, which implies $dZ = dx/x$. The limits of integration map from $(-\infty, \infty)$ to $(0, \infty)$. The integral becomes:

$$\hat{g}(v) = \int_0^{\infty} \frac{x^{iv}}{1+x} \frac{dx}{x} = \int_0^{\infty} \frac{x^{iv-1}}{1+x} dx \quad (\text{I.A.22})$$

This is the standard integral representation of the Euler Beta function, $B(s, 1-s) = \int_0^{\infty} \frac{x^{s-1}}{1+x} dx = \Gamma(s)\Gamma(1-s) = \frac{\pi}{\sin(\pi s)}$. For the integral to converge, the real part of the

exponent $s = iv$ must strictly lie in the interval $(0, 1)$. Substituting $v = u - i\alpha$, we have $iv = i(u - i\alpha) = iu + \alpha$. Therefore, the strip of analyticity is defined by $0 < \alpha < 1$. Within this strip, the Fourier transform is exact $\hat{g}(v) = \frac{\pi}{\sin(\pi iv)} = \frac{\pi}{\sin(\pi(iu + \alpha))}$.

By the Fourier inversion theorem, we can express the original function as:

$$g(Z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ivZ} \hat{g}(v) du.$$

Substituting this inverse representation back into the expectation for the market price in equation (I.A.21):

$$p_t^{-1} = \mathbb{E}_t \left[e^{-Y_B} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iv(Y_A - Y_B)} \frac{\pi}{\sin(\pi iv)} du \right] = \frac{1}{2} \int_{-\infty}^{\infty} \frac{\mathbb{E}_t \left[\exp \left(-ivY_A + (iv - 1)Y_B \right) \right]}{\sin(\pi iv)} du$$

The interchange of the expectation and the integral is justified by Fubini's Theorem due to the absolute convergence guaranteed by the damping parameter α .

We now map the terms inside the expectation to the joint characteristic function, $\Phi_t(z_1, z_2) = \mathbb{E}_t[\exp(iz_1Y_A + iz_2Y_B)]$. For Asset A, the exponent is $-iv = -iu - \alpha$. Setting $iz_1 = -iu - \alpha$ yields $z_1 = -u + i\alpha$. For Asset B, the exponent is $iv - 1 = iu + \alpha - 1$. Setting $iz_2 = iu + \alpha - 1$ yields $z_2 = u - i(1 - \alpha)$. Substituting these arguments into the characteristic function yields the exact formula in equation (I.A.13).

Next, we derive the individual price of Asset A. Under the logarithmic pricing kernel $M_T = p_t/p_T$, the market share of Asset A satisfies:

$$\frac{p_{A,t}}{p_t} = \mathbb{E}_t \left[\frac{1}{1 + e^{Y_B - Y_A}} \right]$$

Let $W = Y_B - Y_A$. This is identically $g(W) = \frac{1}{1 + e^W}$. Applying the exact same Fourier inversion derived above:

$$\frac{p_{A,t}}{p_t} = \mathbb{E}_t \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iv(Y_B - Y_A)} \frac{\pi}{\sin(\pi iv)} du \right] = \frac{1}{2} \int_{-\infty}^{\infty} \frac{\mathbb{E}_t \left[\exp \left(ivY_A - ivY_B \right) \right]}{\sin(\pi iv)} du \quad (\text{I.A.23})$$

Mapping the exponents to the joint characteristic function: For Asset A, the exponent

is $iv = iu + \alpha$. Setting $iz_1 = iu + \alpha$ yields $z_1 = u - i\alpha$. For Asset B, the exponent is $-iv = -iu - \alpha$. Setting $iz_2 = -iu - \alpha$ yields $z_2 = -u + i\alpha$. Substituting these arguments gives $\Phi_t(u - i\alpha, -u + i\alpha)$, establishing equation (I.A.14) and completing the proof.

For the risk-neutral variance of market, we need to evaluate $\mathbb{E}_t[p_T] = \mathbb{E}_t[e^{Y_A}] + \mathbb{E}_t[e^{Y_B}]$, which corresponds to evaluating the joint characteristic function at the purely imaginary arguments ($z_1 = -i, z_2 = 0$) and ($z_1 = 0, z_2 = -i$). This gives the closed-form:

$$\mathbb{E}_t[p_T] = e^{y_{A,t}} \left[1 - \kappa_t(e^{-\omega_A} - 1) \right]^{-(\beta_{A,t} + \beta_{AB,t})} + e^{y_{B,t}} \left[1 - \kappa_t(e^{-\omega_B} - 1) \right]^{-(\beta_{B,t} + \beta_{AB,t})}. \quad (\text{I.A.24})$$

For the risk-neutral variance of Asset A:

$$\text{Var}_t^*(R_A) = \mathbb{E}_t^*[R_A^2] - (\mathbb{E}_t^*[R_A])^2 = \mathbb{E}_t \left[\left(\frac{p_t}{p_T} \right) \left(\frac{p_{A,T}}{p_{A,t}} \right)^2 \right] - 1 = \frac{p_t}{p_{A,t}^2} \mathbb{E}_t \left[\frac{p_{A,T}^2}{p_{A,T} + p_{B,T}} \right] - 1.$$

We rewrite the term inside the expectation using terminal log-prices Y_A and Y_B :

$$\frac{e^{2Y_A}}{e^{Y_A} + e^{Y_B}} = e^{Y_A} \frac{1}{1 + e^{Y_B - Y_A}} = e^{Y_A} g(W),$$

where $W = Y_B - Y_A$ and $g(W) = (1 + e^W)^{-1}$. Using the exact Fourier inversion derived in Theorem IA4, we substitute the integral representation for $g(W)$ with the complex variable $v = u - i\alpha$:

$$\begin{aligned} \mathbb{E}_t [e^{Y_A} g(W)] &= \mathbb{E}_t \left[e^{Y_A} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-iv(Y_B - Y_A)} \frac{\pi}{\sin(\pi iv)} du \right] \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbb{E}_t \left[\exp \left((1 + iv)Y_A - ivY_B \right) \right] \frac{\pi}{\sin(\pi iv)} du. \end{aligned}$$

We now map the exponents to the joint characteristic function $\Phi_t(z_1, z_2) = \mathbb{E}_t[\exp(iz_1 Y_A + iz_2 Y_B)]$: For Asset A, the exponent is $1 + iv = 1 + i(u - i\alpha) = 1 + iu + \alpha$. Setting $iz_1 = 1 + iu + \alpha$ yields $z_1 = u - i(1 + \alpha)$. For Asset B, the exponent is $-iv = -i(u - i\alpha) = -iu - \alpha$. Setting $iz_2 = -iu - \alpha$ yields $z_2 = -u + i\alpha$. Substituting these arguments gives $\Phi_t(u - i(1 + \alpha), -u + i\alpha)$, establishing equation (I.A.16) and completing the proof.

Last, we evaluate the risk-neutral covariance:

$$\text{Cov}_t^*(R_A, R_M) = \mathbb{E}_t^*[R_A R_M] - \mathbb{E}_t^*[R_A] \mathbb{E}_t^*[R_M] = \mathbb{E}_t[M_T R_A R_M] - 1$$

Substituting the logarithmic pricing kernel $M_T = p_t/p_T$ and the gross returns:

$$\mathbb{E}_t \left[\left(\frac{p_t}{p_T} \right) \left(\frac{p_{A,T}}{p_{A,t}} \right) \left(\frac{p_T}{p_t} \right) \right] - 1 = \mathbb{E}_t \left[\frac{p_{A,T}}{p_{A,t}} \right] - 1 = \frac{\mathbb{E}_t[p_{A,T}]}{p_{A,t}} - 1.$$

The term $\mathbb{E}_t[p_{A,T}]$ is the physical expectation of Asset A's terminal fundamental value, which isolates the idiosyncratic and co-jump components affecting A. Evaluating $\mathbb{E}_t[\exp(Y_A)]$ yields $e^{y_{A,t}} \mathcal{J}_A(\kappa_t)^{-(\beta_{A,t} + \beta_{AB,t})}$, completing all the proofs. $\square$

Proof for Result IA5. Let the continuous-time interval dt be represented by a single period of length $\delta \rightarrow 0$ in the discrete $M = 4$ node tree. The four states are $j \in \{0, A, B, AB\}$, representing no jump, an idiosyncratic jump in A, an idiosyncratic jump in B, and a co-jump. The wealth-weighted physical probabilities of these states are $h_{00} = 1 - (\lambda_{A,t} + \lambda_{B,t} + \lambda_{AB,t})\delta$, $h_A = \lambda_{A,t}\delta$, $h_B = \lambda_{B,t}\delta$, and $h_{AB} = \lambda_{AB,t}\delta$. Let $p_t = p_{A,t} + p_{B,t}$ be the aggregate market value, and $w_{i,t} = p_{i,t}/p_t$. A jump in asset i reduces its value by the fraction $\Delta_i = 1 - e^{-\omega_i}$. The value of the aggregate market portfolio in the next period, $p_{t+1}(j)$, depends on the realized state: $p_{t+1}(0) = p_t$, $p_{t+1}(i) = p_t - p_{i,t}\Delta_i = p_t \mathcal{D}_{i,t}$, and $p_{t+1}(AB) = p_t - p_{A,t}\Delta_A - p_{B,t}\Delta_B = p_t(\mathcal{D}_{A,t} + \mathcal{D}_{B,t} - 1)$.

Using the generalized change of measure derived in equation (29) under logarithmic utility, the risk-neutral probability of state j is $h_j^* = h_j \frac{p_t}{p_{t+1}(j)}$. Substituting the state-dependent market prices yields (IA.18).

The instantaneous risk-neutral variance of asset A's return is the sum of the squared jump sizes multiplied by their risk-neutral intensities: $\text{Var}_t^*(dR_A)/dt = \lambda_{A,t}^* \Delta_A^2 + \lambda_{AB,t}^* \Delta_A^2 = (\lambda_{A,t}^* + \lambda_{AB,t}^*) \Delta_A^2$. The covariance is driven entirely by the co-jump state: $\text{Cov}_t^*(dR_A, dR_B)/dt = \lambda_{AB,t}^* \Delta_A \Delta_B$.

To prove the correlation amplification, define the physical correlation of the jump pro-

cesses as $\rho_t = \frac{\lambda_{AB,t}}{\sqrt{(\lambda_{A,t} + \lambda_{AB,t})(\lambda_{B,t} + \lambda_{AB,t})}}$. Set $\mathcal{D}_{A,t} = 1 - w_{A,t}\Delta_A$, $\mathcal{D}_{B,t} = 1 - w_{B,t}\Delta_B$, and $\mathcal{D}_{AB,t} = 1 - w_{A,t}\Delta_A - w_{B,t}\Delta_B$. The risk-neutral correlation is $\rho_t^* = \frac{\lambda_{AB,t}^*}{\sqrt{(\lambda_{A,t}^* + \lambda_{AB,t}^*)(\lambda_{B,t}^* + \lambda_{AB,t}^*)}}$. Substituting the risk-neutral intensities gives:

$$\rho_t^* = \frac{\lambda_{AB,t}/\mathcal{D}_{AB,t}}{\sqrt{(\frac{\lambda_{A,t}}{\mathcal{D}_{A,t}} + \frac{\lambda_{AB,t}}{\mathcal{D}_{AB,t}})(\frac{\lambda_{B,t}}{\mathcal{D}_{B,t}} + \frac{\lambda_{AB,t}}{\mathcal{D}_{AB,t}})}} = \frac{\lambda_{AB,t}}{\sqrt{(\lambda_{A,t}\frac{\mathcal{D}_{AB,t}}{\mathcal{D}_{A,t}} + \lambda_{AB,t})(\lambda_{B,t}\frac{\mathcal{D}_{AB,t}}{\mathcal{D}_{B,t}} + \lambda_{AB,t})}}.$$

Multiplying and dividing by the physical denominator $\sqrt{(\lambda_{A,t} + \lambda_{AB,t})(\lambda_{B,t} + \lambda_{AB,t})}$ immediately yields the multiplier structure in equation (I.A.20). Since $\mathcal{D}_{AB,t} < \mathcal{D}_{A,t}, \mathcal{D}_{B,t}$, the ratios $\frac{\mathcal{D}_{AB,t}}{\mathcal{D}_{A,t}}$ and $\frac{\mathcal{D}_{AB,t}}{\mathcal{D}_{B,t}}$ are strictly less than 1. This strictly reduces the denominator of ρ_t^* relative to ρ_t , proving that $|\rho_t^*| > |\rho_t|$. □

I.A.5 Speculation on autocorrelation

An equivalent version of our trinomial tree model, using a periodic recombining binary tree model where agents' beliefs about the binary distribution of news (being good or bad) switch periodically, can translate heterogeneous beliefs about volatility into heterogeneous views on return auto-correlation. To be precise, we consider the following parameterization of subjective beliefs on the trinomial tree (g, m, b) using (h, ε) :

$$m = 2h(1 - h) - \varepsilon, \quad g = h(h + \varepsilon), \quad b = (1 - h)(1 - h + \varepsilon), \quad (\text{I.A.25})$$

where $h \in (0, 1)$ and ε is a small number around zero. The technical condition for (m, g, b) to be a probability measure is when ε satisfies

$$\max(-h, h - 1) \leq \varepsilon \leq \min(h, 1 - h, 2h(1 - h)).$$

The more extreme h is, the narrower the range of ε is around 0.

Although equivalent to some of the trinomial tree settings, the recombining binary tree

model has a rather different interpretation: h captures agents' overall bullishness about the market going up; ε captures agents' beliefs about the inter-temporal correlation of news between two sub-periods ($t \rightarrow t + 0.5$ and $t + 0.5 \rightarrow t + 1$).

The expression of m from (I.A.25) indicates two channels that can drive volatility disagreement. The first channel is through disagreement in ε : agents who believe in a positive correlation between sub-periods perceive a higher volatility of fundamentals between t and $t + 1$. Figure I.A.13 shows a numerical example with three periods where $h = 0.5$ across all agents and $\varepsilon \in [-1/2, 1/2]$ follows a uniform distribution. Under this setting, individual agents' beliefs in (I.A.25) become

$$m = 1/2 - \varepsilon, \quad g = 1/4 + 1/2\varepsilon, \quad b = 1/4 + 1/2\varepsilon.$$

Let $g = 1/2 v$, $m = 1 - v$, and $b = 1/2 v$ by replacing $v = 1/2 + \varepsilon$. This is equivalent to a trinomial tree model when disagreement about news content is absent and $u = 0.5$ for all agents.

The red path's sub-period history on the binary tree is (up, down, up, down) and the blue path history is (down, down, up, up). The disagreement regarding ε breaks the permutation symmetry of the binary tree model and results in different pricing and sentiment regarding volatility. On the red path, those who believe in a negative ε become hindsight winners, and Mr. Market's belief of ε is negative at $t = 2$. On the contrary, the blue path results in a positive sentiment $\mathcal{E}$ because of the positive correlations of news between $0 \rightarrow 0.5$ and $0.5 \rightarrow 1$ and between $1 \rightarrow 1.5$ and $1.5 \rightarrow 2$. The lower $\mathcal{E}$ (recall that $\mathcal{E} = V - 0.5$) means a lower sentiment of news intensity, which pushes prices higher: 'no news is good news'.

The second channel is through disagreement in h ; agents with extreme bullishness (in terms of h) or bearishness (in terms of $1 - h$) think the market is more volatile across multiple sub-periods. In Figure I.A.14, we show another numerical example with $\varepsilon = 0$ and

h varying across agents with a uniform distribution. Under this setting, (I.A.25) becomes

$$m = 2h(1 - h), \quad g = h^2, \quad b = (1 - h)^2,$$

and this is essentially a recombining binary tree where agents disagree about the probability h of an up-move. The pricing of the market asset in this special case is equivalent to the model in [Martin and Papadimitriou \(2022\)](#) upon recombining odd and even periods. The permutation symmetry of the binary tree is preserved, and pricing is not different after experiencing the red or blue news sample paths.

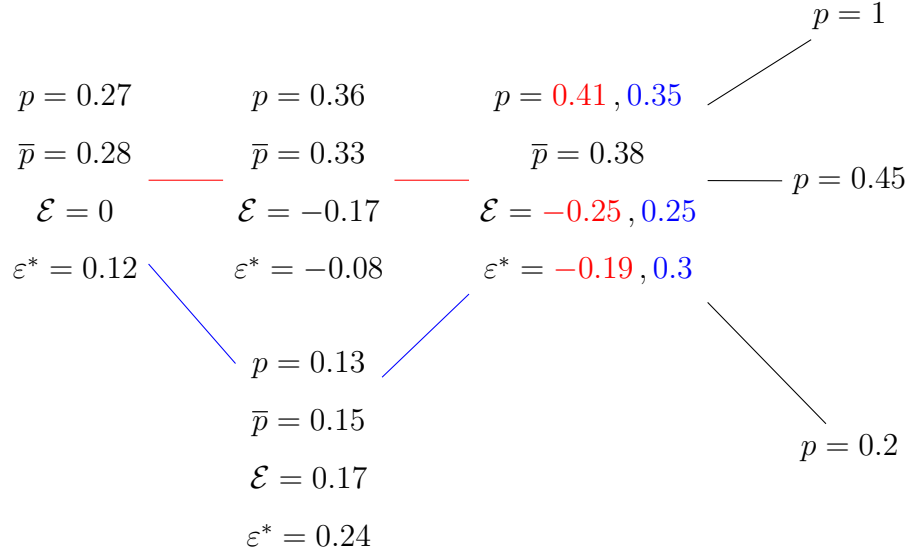


Figure I.A.13: **A numerical example of ε -disagreement.** Agents' beliefs ε are uniformly distributed and they agree on $h = 1/2$. We only report pricing at $t = 0, 1, 2, 3$.

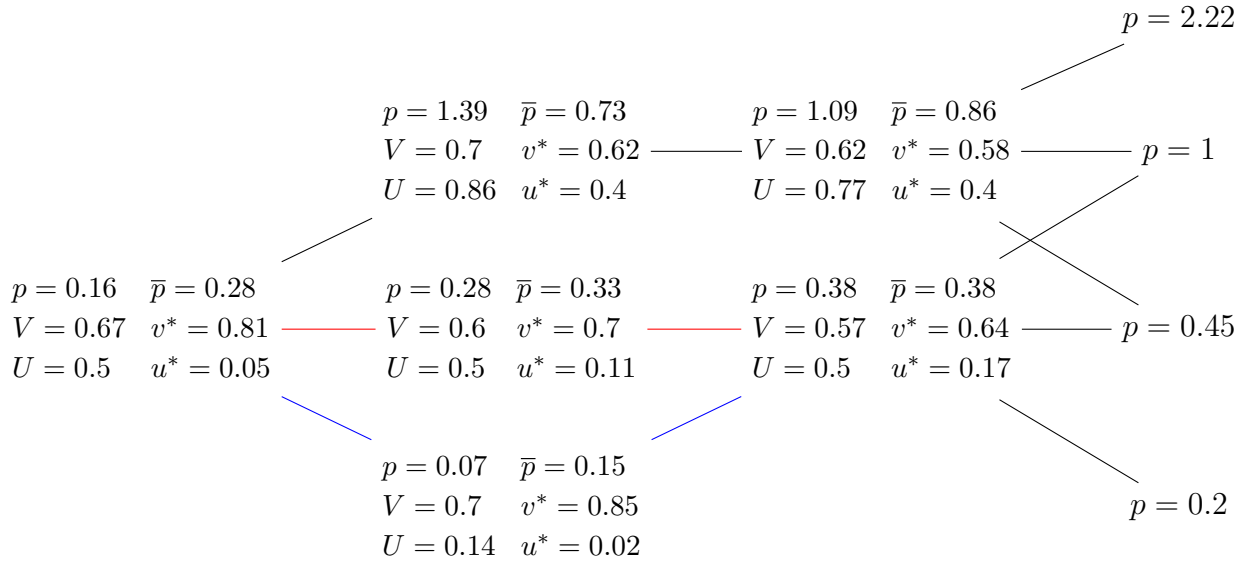


Figure I.A.14: **Recombining binary tree: h -disagreement.** This example uses the binary tree from [Martin and Papadimitriou \(2022\)](#) where h is uniformly distributed. We only report pricing at $t = 0, 1, 2, 3$ (while binary tree also have nodes at $t = 0.5, 1.5, 2.5$). p is price of market, $\bar{p}$ is price in homogeneous economy where everyone agrees with the median agent $h = 1/2$. (v, u) represents Mr. Market's beliefs and (v^*, u^*) is the risk-neutral measure.