

When Can Positive Return Autocorrelation Arise in Rational Expectations Models?

Wen Chen, Albert S. Kyle, and Yajun Wang*

September 16, 2026

Abstract

In a dynamic trading model with rational expectations, we show that the interaction between public information and liquidity shocks can generate positive return autocorrelation. In the absence of anticipated future liquidity shocks, rational uninformed traders tend to follow public signals and act as momentum traders—buying after price increases and selling after price declines—and thereby reinforce return continuation. When future liquidity shocks are anticipated, informed traders may instead engage in momentum trading in response to sequential public information, generating positive return autocorrelation prior to the shocks' arrival. Using flow-induced mutual-fund trading as a proxy for non-informational liquidity pressure, we find that greater pre-announcement selling predicts higher announcement returns, while post-announcement continuation is stronger following larger prior liquidity shocks. These results provide a fully rational explanation for predictable return dynamics without relying on behavioral biases.

JEL Classification Codes: B41, D8, D53, D61, D82, G12, G14, G18

Keywords: Return autocorrelation, public information, return predictability, rational expectations.

*For helpful comments, we are grateful to Travis Johnson, Mina Lee, Jun Pan, Cameron Peng, and seminar participants at Baruch College, Texas Tech University, 12th WIM Conference, European Winter Finance Summit, and Lone Star Finance Conference 2025. Chen: Rawls College of Business at Texas Tech University, Email: Wen.Chen@ttu.edu. Kyle: Smith School of Business, University of Maryland, College Park, MD, 20742, askyle4@umd.edu. Wang: Baruch College, Email: yajun.wang@baruch.cuny.edu.

1 Introduction

Short-run return continuation, or positive return autocorrelation, is a well-documented feature of financial markets. Although extensive empirical evidence documents such return predictability, explaining it within rational expectations equilibrium (REE) models remains challenging. In this paper, we develop a dynamic trading model with rational investors and show that the interaction between public information and liquidity shocks can generate positive return autocorrelation.

Return autocorrelation is often negative in REE models because liquidity trading pushes prices away from fundamental values and prices subsequently revert. We show that public information arriving after a liquidity shock can instead generate positive return autocorrelation. As new public signals arrive, informed and uninformed traders adjust their positions, and these adjustments can generate persistent trading in the same direction and return continuation.

In our model, informed traders initially establish positions based on both private information and liquidity needs. Their liquidity-driven trading pushes prices away from fundamental values because hedging motives distort their optimal response to private information. In the absence of future liquidity shocks, uninformed traders—who initially provide liquidity to informed traders—anticipate that prices will eventually revert toward fundamentals. When public information arrives, they use the signal to infer whether prices are too high or too low relative to fundamental values and trade in the direction of the signal. Consequently, they buy following price increases and sell following price declines, behaving as momentum traders. Informed traders, by contrast, act as contrarian traders, adjusting their positions at more favorable prices by buying when prices fall and selling when prices rise. Moreover, traders who optimally bought or sold before the public information release often continue trading in the same direction afterward, generating positively autocorrelated trades and returns.

We then examine how anticipated future liquidity shocks affect trading behavior and return autocorrelation before those shocks arrive. The resulting return dynamics depend critically on the volatility of the anticipated shocks. When future shocks are much less volatile than the ini-

tial shock, their expected impact is small and their anticipated arrival has little effect on current trading. As in the case without future liquidity shocks, uninformed traders continue to trade in the direction of public signals, producing autocorrelated trades and positive return autocorrelation.

When future liquidity shocks are highly volatile, the mechanism changes. Informed traders alternate their trading direction from one period to the next in response to public information because the sign of the future liquidity shock is highly uncertain. To avoid accumulating excessively large long or short positions, they behave as contrarian traders, buying when past prices are low and selling when past prices are high. In doing so, they demand liquidity, pushing prices upward when they buy and downward when they sell. Uninformed traders, while providing liquidity to informed traders, also reverse their trading direction from period to period. These reversals in trading generate negative return autocorrelation.

A different pattern emerges when the volatility of future liquidity shocks is comparable to that of the initial shock. In this case, uninformed traders no longer behave as momentum traders. Because future liquidity shocks may move prices away from fundamentals, a rising price no longer necessarily indicates convergence toward fundamental value. Uninformed traders therefore act as contrarian traders and provide liquidity by buying when prices are low and selling when prices are high.

Informed traders behave differently. After initially trading on private information and liquidity needs, they reverse their positions following the first round of public information and then continue trading in the same direction as subsequent public signals arrive. As more public information becomes available, uncertainty about the asset payoff—and therefore about the payoff of the associated non-traded risky asset—declines. When anticipated future liquidity shocks have moderate volatility, informed traders gradually reduce the magnitude of their initial hedging positions. As a result, they behave as momentum traders and demand liquidity, generating positive return autocorrelation.

These results show that positive return autocorrelation can arise in REE models under sim-

ple assumptions. Even without information asymmetry, sequential public information following a liquidity shock can generate positive return autocorrelation as the effect of the liquidity shock on the risk premium declines over time. With information asymmetry but no future liquidity shocks, uninformed traders behave as momentum traders, reinforcing return continuation. When future liquidity shocks are anticipated and have volatility comparable to that of the initial shock, informed traders instead engage in momentum trading and generate positive return autocorrelation.

The model also delivers empirical predictions linking pre-announcement liquidity trading to subsequent returns. Using flow-induced mutual-fund trading as a proxy for non-informational liquidity pressure, we provide supporting evidence that greater liquidity selling is associated with lower pre-announcement returns and predicts higher announcement returns, while post-announcement return continuation is stronger following larger prior liquidity shocks.

More broadly, the model can generate a range of return dynamics, including short-run return continuation and long-run reversal. The framework therefore provides a fully rational explanation for predictable return dynamics arising from the interaction of sequential information and liquidity shocks. Although we assume for tractability that public information arrives sequentially, the results remain qualitatively similar when informed traders interpret public signals as private information and trade on them before future liquidity shocks arrive. The mechanism is therefore applicable to settings in which public information and liquidity-motivated trading, such as portfolio rebalancing, occur repeatedly over time.

The case without future liquidity shocks is related to [Brennan and Cao \(1996\)](#) and [Brennan and Cao \(1997\)](#). [Brennan and Cao \(1996\)](#) analyze the value of improved trading opportunities, either through more frequent trading in the underlying asset or through access to derivative securities. They show that trading in an appropriately designed derivative can achieve Pareto efficiency in a single trading round. In their model, better-informed investors also sell when prices are high and buy when prices are low. Similarly, [Brennan and Cao \(1997\)](#) show that domestic investors with a cumulative informational advantage over foreign investors tend to buy

foreign assets following high foreign returns and sell following low returns. While their focus is on trading opportunities and international portfolio flows, respectively, we study how the interaction between public information and liquidity shocks shapes return dynamics. We further examine how anticipated future liquidity shocks alter traders' behavior and return autocorrelation before those shocks arrive.

Our paper contributes to the theoretical literature on positive return autocorrelation and short-run return continuation. A large literature explains return continuation through gradual information diffusion, behavioral biases, or heterogeneous beliefs. For example, [Hong and Stein \(1999\)](#) show how slow information diffusion can generate underreaction and subsequent return continuation, while [Barberis and Shleifer \(2003\)](#) study style-based trading that can induce predictable return patterns. [Kyle, Obizhaeva, and Wang \(2023\)](#) obtain time-series momentum when rational traders agree to disagree about the precision of private information: traders speculate on predictable future disagreement, and this short-term speculation generates return predictability.

Other papers generate positive return autocorrelation within rational models through information dynamics or trading frictions. [Cespa and Vives \(2012\)](#) show that endogenous short-term speculation in a noisy dynamic information model can generate momentum and reversal, with investors switching between accommodating order flow and chasing trends depending on market conditions. [Andrei and Cuijean \(2017\)](#) develop a rational-expectations model in which accelerating private information diffusion through word-of-mouth communication generates short-run momentum and long-run reversal. [Bogousslavsky \(2016\)](#) shows that infrequent portfolio rebalancing can generate return autocorrelations that change sign and become positive at the rebalancing horizon. [Albuquerque and Miao \(2014\)](#) show that advance private information about future earnings can raise expected risk premia and generate short-run momentum followed by reversal. A related literature emphasizes higher-order beliefs and gradual price adjustment. [Allen, Morris, and Shin \(2006\)](#) show that iterated expectations can generate sluggish adjustment of prices in a fully rational model with noisy prices and heterogeneous informa-

tion. [Banerjee, Kaniel, and Kremer \(2009\)](#), however, argue that standard rational-expectations models with heterogeneous information do not by themselves generate price drift; higher-order differences of opinion are required.

Our mechanism differs from these explanations. Positive return autocorrelation arises from the interaction between an initial liquidity distortion and subsequent public information. In particular, private information is not required: even under symmetric information, public signals arriving after a liquidity shock can generate positive return autocorrelation as the effect of the liquidity shock on the risk premium declines over time.

Finally, [Berk, Green, and Naik \(1999\)](#) and [Johnson \(2002\)](#) generate positive return autocorrelation through persistence in fundamentals. Their mechanisms rely on the dynamics of dividends, growth opportunities, or expected returns. In contrast, we examine how public information released after liquidity shocks influences traders' expectations, generating positive return autocorrelation even in the absence of fundamental changes.

The paper is structured as follows. Section 2 presents the model, Section 3 characterizes the equilibrium, Section 4 examines return dynamics, Section 5 discusses empirical implications, and Section 6 concludes. All proofs are provided in the Appendix.

2 The Model

We consider a multi-period trading model with a continuum of identical informed investors (I -type) of mass α and a continuum of identical uninformed investors (U -type) of mass $1 - \alpha$. Both informed and uninformed investors are perfectly competitive. Traders can trade two assets: a risk-free asset and a risky asset (the stock) at trading periods $t = 0, 1, \dots, T$. The risk-free asset is in perfectly elastic supply, serves as the numeraire, and has its price normalized to one. The total supply of the stock is Θ . Each share of the stock pays a liquidation value of $V \sim \mathcal{N}(\bar{V}, \tau_V^{-1})$ at period $T + 1$, where $\bar{V}$ is a constant, and $\mathcal{N}$ denotes the normal distribution.

At trading period $t = 0$, informed traders may observe a signal

$$v = V + \eta, \quad (1)$$

about the liquidation value V , where $\eta \sim \mathcal{N}(0, \tau_\eta^{-1})$ is an independent noise. At period t , we assume that each informed investor is endowed with X_t total number of shares of a nontradable risky asset,¹

$$X_t = X_{t-1} + x_t, \quad (2)$$

where $X_{-1} = 0$, $x_t \sim \mathcal{N}(0, \tau_{x,t}^{-1})$, x_t is the number of shares received by each informed trader at period t and is independent of V and η . The nontradable risky asset can be liquidated only at period $T + 1$. Without loss of generality, we assume that each unit of the nontradable risky asset pays a liquidation value of $V - \bar{V}$ at the final period $T + 1$.² If $\tau_{x,t}^{-1} = 0$, then no new liquidity shock occurs at period t .

In addition to the private signal at $t = 0$, at each $t = 1, 2, \dots, T$, all traders observe a public signal

$$s_t = V + \varepsilon_t, \quad (3)$$

where $\varepsilon_t \sim \mathcal{N}(0, \tau_{\varepsilon,t}^{-1})$ is an independent noise term.³

Define $\underline{Z}_t := (Z_0, Z_1, \dots, Z_t)$ for any stochastic process $\{Z_t\}$, representing the history of Z_t up to t . On each date t , each informed investor chooses a demand schedule $\Theta_t^I(v, X_t, \underline{s}_t; \cdot)$. Each uninformed trader chooses a demand schedule $\Theta_t^U(\underline{s}_t; \cdot)$. The schedules Θ_t^I and Θ_t^U are traders' strategies. Given prices $\underline{P}_t$, the quantities demanded by informed and uninformed investors can be written as $\theta_t^I = \Theta_t^I(v, \underline{X}_t, \underline{s}_t, \underline{P}_t)$ and $\theta_t^U = \Theta_t^U(\underline{s}_t, \underline{P}_t)$. Investor i 's information set, $\mathcal{F}_t^i$ can

¹Similar to [Vayanos and Wang \(2012\)](#), [Goldstein, Li, and Yang \(2014\)](#), and [Chen and Wang \(2025\)](#), we assume that informed investors have two trading motives to prevent information from being fully revealed in equilibrium. For tractability, we assume that all informed traders share the same private information and experience the same liquidity shock.

²We can also assume that the nontradable asset has a payoff correlated with V . This correlation generates a hedging demand for the security. Such a generalization would not lead to qualitatively different results.

³If the signal s_t ($t \geq 1$), is privately observed by informed traders and there are no future liquidity shocks, uninformed traders can infer s_t from observed prices and thus s_t becomes public. We also verify that our results remain qualitatively similar even when s_t is privately observed and additional liquidity shocks occur in future periods.

be written as

$$\mathcal{F}_t^I = \{v, \underline{s}_t, \underline{X}_t, \underline{P}_t\}, \quad \mathcal{F}_t^U = \{\underline{s}_t, \underline{P}_t\}. \quad (4)$$

Given prices $\underline{P}_t$, the problem for a type $i \in \{I, U\}$ investor is to solve:

$$\max_{\theta_t^i} \mathbb{E}[-e^{-A^i W_{T+1}^i} | \mathcal{F}_t^i], \quad (5)$$

where W_{T+1}^i and A^i represent the final wealth and risk aversion coefficient of type i traders, respectively. The final wealth is given by:

$$W_{T+1}^i = W_{-1}^i - \sum_{t=0}^T (\theta_t^i - \theta_{t-1}^i) P_t + \theta_T^i V + X_t^i (V - \bar{V}), \quad (6)$$

where $W_{-1}^i = \theta_{-1}^i \bar{V}$ is the initial wealth of type i -investors, $X_t^I = X_t$, and $X_t^U = 0$.

The definition of a perfect Bayesian equilibrium is given as follows.

Definition 1. An equilibrium $(\Theta_t^I(P_t), \Theta_t^U(P_t), P_t)$ is such that

1. given all prices up to time t , $\Theta_t^i(P_t)$ solves the problem (5) for a type- i investor, $i \in \{I, U\}$, where the information sets of informed and uninformed traders are as given in equation (4);
2. the price P_t clears the risky asset market: $\Theta = \alpha \Theta_t^I(P_t) + (1 - \alpha) \Theta_t^U(P_t)$, and the risk-free asset market is also cleared;
3. for every realization of the signals v , $\{\underline{s}_t\}$ and $\{\underline{X}_t\}$, the beliefs of all investors are consistent with the joint conditional probability distribution in equilibrium.

3 The Equilibrium

In this section, we solve for the dynamic equilibrium prices and trading in closed form. We focus on a linear equilibrium in which trading prices are expressed as linear functions of the model's state variables. At $t = 0$, both informed investors' private signal v and liquidity shock X_0 affect informed investors' trading and thus the equilibrium price. From market price P_0 ,

other investors can only infer the value of the composite signal

$$s_0 := v - hx_0 = V + \varepsilon_0, \quad (7)$$

where

$$\varepsilon_0 = \eta - hx_0 \sim \mathcal{N}(0, \tau_0^{-1}), \quad h = A^I \tau_\eta^{-1}, \quad \tau_0^{-1} = \tau_\eta^{-1} + h^2 \tau_{x,0}^{-1}. \quad (8)$$

Therefore, P_0 is informationally equivalent to the composite signal s_0 , which is a linear combination of informed investors' private signal and the amount of the nontradable risky asset. Therefore, the information set at period t for uninformed investors $\mathcal{F}_t^U = \{s_0, \underline{s}_t\}$.

We obtain the following lemma on the dynamic Kalman filtering formulas.

Lemma 1. (*Kalman filtering*). At t , for $i \in \{I, U\}$, trader i 's estimate of the stock's liquidation value $\hat{V}_t^i := E[V | \mathcal{F}_t^i]$ and trader U 's estimate of the liquidity shocks $\hat{X}_t^U := E[X_t | \mathcal{F}_t^U]$ are

$$\hat{V}_t^I = \hat{V}_{t-1}^I + o_{V,t}^I \tau_{\varepsilon,t} e_t^I, \quad \hat{V}_t^U = \hat{V}_{t-1}^U + o_{V,t}^U \tau_t e_t^U, \quad \hat{X}_t^U = \hat{X}_{t-1}^U + K_{X,t}^U e_t^U, \quad (9)$$

where

$$e_t^i = s_t - \hat{V}_{t-1}^i, \text{ for } i \in \{I, U\} \text{ and } t \geq 1, \text{ and } e_0^I = v - \bar{V}, \quad e_0^U = s_0 - \bar{V}, \quad (10)$$

$\hat{X}_{-1}^U = 0$, $\hat{V}_{-1}^I = \hat{V}_{-1}^U = \bar{V}$, the coefficient $K_{X,t}^U$ is defined in (A-6) in the Appendix, and the conditional variances of V for informed and uninformed traders at t are,

$$o_{V,t}^I := \text{Var}[V | \mathcal{F}_t^I] = (\tau_V + \sum_{s=0}^t \tau_{\varepsilon,s})^{-1}, \quad o_{V,t}^U := \text{Var}[V | \mathcal{F}_t^U] = (\tau_V + \sum_{s=0}^t \tau_s)^{-1},$$

where $\tau_{\varepsilon,0} = \tau_\eta$, $\tau_0 := (\tau_\eta^{-1} + h^2 \tau_{x,0}^{-1})^{-1}$, and $\tau_s = \tau_{\varepsilon,s}$ for $s \geq 1$.

Proof. See Appendix A.1. □

Lemma 1 shows that informed traders' estimate $\hat{V}_t^I$ can be written as a recursive equation of $\hat{V}_{t-1}^I$ with an innovation term $s_t - \hat{V}_{t-1}^I$ and uninformed traders' estimate $(\hat{V}_t^U, \hat{X}_t^U)$ can be expressed as a recursive equation of $(\hat{V}_{t-1}^U, \hat{X}_{t-1}^U)$ with an innovation term $s_t - \hat{V}_{t-1}^U$. The conditional variance and mean of the security's payoff for traders have intuitive expressions. Specif-

ically, the conditional variance of the payoff corresponds to the inverse of the total precision of traders' signals, while the conditional mean of the payoff (or the estimate of V) is determined by the precision-weighted average of signal innovations.

The sufficient statistic $\hat{V}_t^I - \mu_t X_t$ is a composite signal of informed traders, capturing their dual trading motives: speculating based on private information and hedging against risks from liquidity shocks. The noise-to-signal ratio μ_t is

$$\mu_t = A^I o_{V,t}^I = A^I \left(\tau_V + \sum_{s=0}^t \tau_{\varepsilon,s} \right)^{-1}. \quad (11)$$

Since the sufficient statistic $\hat{V}_t^I - \mu_t X_t$ can be inferred by uninformed traders from prices, it follows that $\hat{V}_t^I - \mu_t X_t = \hat{V}_t^U - \mu_t \hat{X}_t^U$. Equation (11) implies that the noise-to-signal ratio for uninformed traders μ_t decreases over time, as prices progressively convey more information about informed traders' estimation of the stock value.

We provide closed-form expressions for equilibrium prices and trading quantities in the following proposition.

Proposition 1. 1. *The equilibrium price at time t is*

$$P_t = \omega_t P_t^I + (1 - \omega_t) P_t^U - \frac{\Theta}{\frac{\alpha}{\gamma_t^I} + \frac{1-\alpha}{\gamma_t^U}}, \quad (12)$$

where $\omega_t = \frac{\frac{\alpha}{\gamma_t^I}}{\frac{\alpha}{\gamma_t^I} + \frac{1-\alpha}{\gamma_t^U}}$. Parameters γ_t^i , for $i \in \{I, U\}$ and $t < T$, are positive and can be computed recursively using Equations (A-38) and (A-46) and their values at $t = T$ are $\gamma_T^i = A^i o_{V,T}^i$. The non-participation prices of informed and uninformed traders are given as,

$$\begin{aligned} P_t^I &= \bar{V} + \xi_t^I (\hat{V}_t^I - \mu_t X_t - \bar{V}) + \psi_t^I (\hat{V}_t^U - \bar{V}) - f_t^I \Theta, \\ P_t^U &= \bar{V} + \xi_t^U (\hat{V}_t^I - \mu_t X_t - \bar{V}) + \psi_t^U (\hat{V}_t^U - \bar{V}) - f_t^U \Theta. \end{aligned} \quad (13)$$

The coefficient μ_t is as defined in Equation (11) and the coefficients ξ_t^i , ψ_t^i , and f_t^i , for $i \in \{I, U\}$ and $t < T$ are all positive and can be computed recursively using Equations (A-38), (A-39), (A-46), and (A-47). Their values at $t = T$ are $\xi_T^I = 1$, $\xi_T^U = 0$, $\psi_T^I = 0$, $\psi_T^U = 1$, and for

$i \in \{I, U\}$, $f_T^i = 0$, implying that the non-participation prices at T are

$$P_T^I = \hat{V}_T^I - \mu_T X_T \quad \text{and} \quad P_T^U = \hat{V}_T^U. \quad (14)$$

2. The informed and uninformed traders' equilibrium holdings at t are

$$\theta_t^I = \frac{P_t^I - P_t}{\gamma_t^I}, \quad \theta_t^U = \frac{P_t^U - P_t}{\gamma_t^U}. \quad (15)$$

Proof. See Appendix A.2. □

Equation (14) in Proposition 1 implies that, at period T , an informed trader's non-participation price is equal to the composite signal $\hat{V}_T^I - \mu_T X_T$. An uninformed trader's reservation price is equal to his estimate of the asset's liquidation value, $\hat{V}_T^U$, following the logic of a one-period model. Since the final payoff V is realized at period $T + 1$, at period T informed traders adjust their positions based on their updated payoff estimates and the realized hedging demand. Uninformed traders adjust based on their updated payoff estimates. Neither group of traders has a dynamic speculative position related to the estimation of the next period's trading prices. Consequently, at period T , each trader's non-participation price is independent of the other type of traders' estimates.

In contrast, for $t < T$, both traders' non-participation prices increase with the informed traders' composite signal $\hat{V}_t^I - \mu_t X_t$ and the uninformed traders' estimate $\hat{V}_t^U$. This implies that each trader's non-participation price is influenced by other traders' estimates. This is because, at $t < T$, each trader's optimal demand consists of a long-term stock position and a short-term speculative position. The former is based on the trader's estimate of the asset's final payoff $\hat{V}_t^i$, while the latter is based on the trader's estimate of the next period's trading prices. Since the next period's trading prices are influenced by both traders' estimates, each trader's non-participation price at period t depends on the informed traders' composite signal $\hat{V}_t^I - \mu_t X_t$ and the uninformed traders' estimate $\hat{V}_t^U$. We show that each trader's reservation price is a function of both traders' estimates, the total supply Θ , as depicted in Equation (13).

Given that both the long-term stock position and short-term speculative position of each

trader are based on his own estimate of the asset's payoff, and only the short-term speculative position is influenced by the other trader's estimates, the coefficient for $\hat{V}_t^I - \mu_t X_t$ in the informed traders' non-participation price is typically greater than that in the uninformed traders' reservation price. This implies that $\xi_t^I > \xi_t^U$ for $t < T$. Similarly, the coefficient for $\hat{V}_t^U$ in the uninformed traders' non-participation price tends to be larger than that in the informed traders' reservation price, suggesting $\psi_t^U > \psi_t^I$ for $t < T$.⁴

4 Return Dynamics

To understand the key mechanism behind positive return autocorrelation, we examine three cases. We begin with the simplest setting: symmetric information and a one-time initial liquidity shock. Next, we include asymmetric information while still considering only an initial liquidity shock. Finally, we analyze the most general case, incorporating both information asymmetry and anticipated future liquidity shocks.

4.1 Return Dynamics with Symmetric Information and an Initial Liquidity Shock

We first consider the case without information asymmetry (i.e., $\tau_{\eta,0} = 0$), where informed traders receive only a liquidity shock at time $t = 0$. In this simplified setting, trading occurs only at $t = 0$. After establishing their optimal hedging positions, there is no heterogeneity between informed and uninformed traders. Consequently, no further trading takes place, although prices continue to update as new public information arrives over time.

The equilibrium prices and return autocorrelations are given in the following proposition.

⁴We conduct an extensive numerical analysis across a wide range of parameters. We find that the parameter values consistently demonstrate $\xi_t^I > \xi_t^U$ and $\psi_t^U > \psi_t^I$ for $t < T$.

Proposition 2. 1. The equilibrium price and return at time t are given by:

$$\begin{aligned} P_t &= \hat{V}_t - \omega \mu_t \left(x_0 + \frac{\Theta}{\alpha} \right), \\ R_t &:= P_t - P_{t-1} = \omega (\mu_{t-1} - \mu_t) \left(x_0 + \frac{\Theta}{\alpha} \right) + \tau_{\varepsilon,t} o_{V,t} (s_t - \hat{V}_{t-1}). \end{aligned} \tag{16}$$

where $\omega := \frac{\alpha/A^I}{\alpha/A^I + (1-\alpha)/A^U}$, $\mu_t := A^I o_{V,t}$, and $\hat{V}_t$ and $o_{V,t}$ denote the conditional expectation and variance of V at time t , which are identical for both types of traders in the absence of information asymmetry.

2. With the arrival of public information, μ_t decreases over time, and we have

$$\text{Cov}(R_t, R_{t-1}) = \omega^2 (\mu_{t-1} - \mu_t) (\mu_{t-2} - \mu_{t-1}) \text{Var}(x_0) > 0, \text{ for } t \geq 2.$$

For $t = 1$, we have $\text{Cov}(R_1, R_0) < 0$, where $R_0 = P_0 - \bar{V}$.

The return autocorrelation at $t = 1$ is negative because an initial liquidity shock pushes prices away from their fundamental values, as in Rational Expectations models. As public information arrives, prices gradually revert toward fundamentals.

Proposition 2 shows that even in the absence of information asymmetry, a sequence of public information releases following a liquidity shock can generate positive return autocorrelation, as prices trend back toward fundamentals while the impact of the liquidity shock on the risk premium decreases over time.

As illustrated in Figure 1, one-lag return autocorrelation is negative immediately following the liquidity shock but becomes positive with the subsequent arrival of public information. In this simple case, trading occurs only in response to the initial liquidity shock, showing that continued trading is not necessary for positive return autocorrelation. Rather, the interaction between the initial liquidity distortion and subsequent public information generates return continuation as prices gradually move back toward fundamentals.

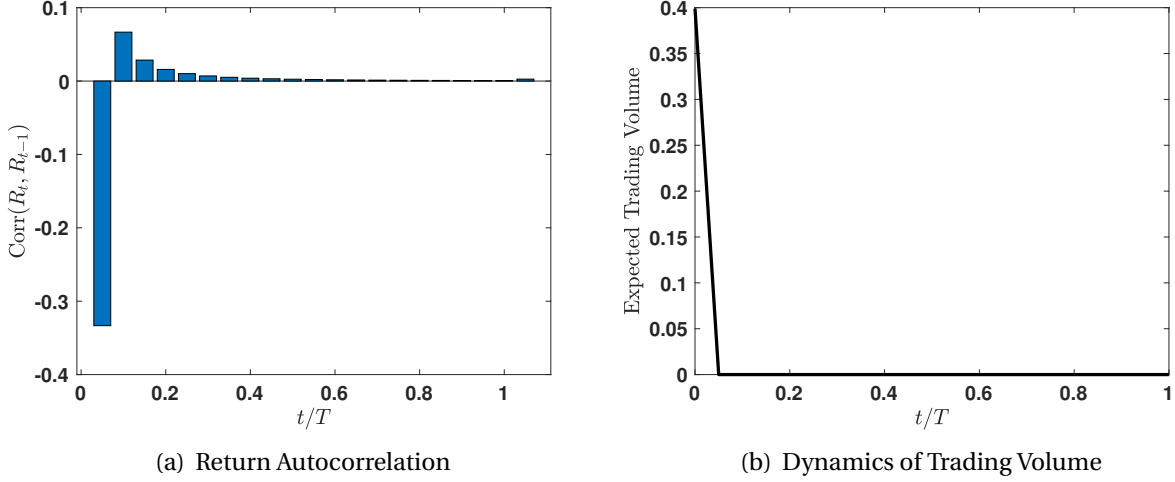


Figure 1. Symmetric Information with Initial Liquidity Shock Only. Parameters are $T = 20$, $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_{\eta,t} = 10^{-6}$, $\tau_{\varepsilon,t} = 1$, $\bar{V} = 1$, and $\tau_{x,t} = 10^6$ except for $\tau_{x,0} = 1$.

4.2 Return Dynamics with Asymmetric Information and Initial Liquidity Shocks

We next consider the case with asymmetric information and an initial liquidity shock. Since no additional liquidity shocks arrive after the initial period, any private information received after $t = 0$ is fully revealed to the market through equilibrium prices. As a result, the private signals v_t for $t = 1, 2, \dots, T$ effectively become public information. In contrast, the initial private signal v_0 and the initial liquidity shock x_0 remain unknown to uninformed traders.

The equilibrium prices and return autocorrelations are given in the following proposition.

Proposition 3. *The equilibrium stock price is*

$$\begin{aligned}
 P_t &= \omega_t(\hat{V}_t^I - \mu_t X_t) + (1 - \omega_t)\hat{V}_t^U - \omega_t \mu_t \frac{\Theta}{\alpha} \\
 &= \hat{V}_t^U - \omega_t \mu_t \left(\hat{X}_t^U + \frac{\Theta}{\alpha} \right),
 \end{aligned} \tag{17}$$

where $\omega_t = \frac{\alpha}{A^I \sigma_{V,t}^I} \left(\frac{\alpha}{A^I \sigma_{V,t}^I} + \frac{1-\alpha}{A^U \sigma_{V,t}^U} \right)^{-1}$, and $\hat{X}_t^U = E_t^U[x_0]$. The equilibrium stock holdings at t are:

$$\theta_t^I = \frac{\Theta}{\alpha} - (1 - \omega_t) \left(\hat{X}_t^U + \frac{\Theta}{\alpha} \right), \quad \theta_t^U = \frac{\alpha}{1 - \alpha} (1 - \omega_t) \left(\hat{X}_t^U + \frac{\Theta}{\alpha} \right). \tag{18}$$

Proof. See Appendix A.5. □

Proposition 3 implies that, in this simplified setting without future liquidity shocks, prices and trading quantities primarily depend on uninformed traders' expectations about the initial liquidity shock. Intuitively, if uninformed traders experienced the same liquidity shock as informed traders, their non-participation prices would be the same. Therefore, any difference in non-participation prices arises from differences in the liquidity shock, which are reflected by the difference in the estimation of the liquidity shock and the associated estimation risk premium.

We obtain the following proposition regarding the autocorrelation of returns.

Proposition 4. 1. *The stock return for $t = 1, 2, \dots, T$ is:*

$$R_t := P_t - P_{t-1} = a_t \left(\hat{X}_{t-1}^U + \frac{\Theta}{\alpha} \right) + b_t (s_t - \hat{V}_{t-1}^U), \quad (19)$$

where coefficients a_t and b_t are given as

$$a_t := \omega_{t-1} \mu_{t-1} - \omega_t \mu_t > 0, \quad b_t := \tau_{\varepsilon,t} \left(\omega_t o_{V,t}^I + (1 - \omega_t) o_{V,t}^U \right) > 0. \quad (20)$$

2. *For $t = 2, 3, \dots, T$, the covariance of returns is*

$$\text{Cov}(R_t, R_{t-1}) = a_t a_{t-1} \text{Var}(\hat{X}_{t-2}^U) + a_t b_{t-1} o_{V,t-2}^U > 0. \quad (21)$$

For $t = 1$, we have $\text{Cov}(R_1, R_0) < 0$.

Proof. See Appendix A.6. □

Proposition 4 implies that public information following a liquidity shock can generate positive return autocorrelation. To understand the trading mechanism underlying this return continuation, we next examine how public information affects traders' behavior. We obtain the following proposition.

Proposition 5. 1. *Uninformed traders' time- t demand is:*

$$\Delta \theta_t^U := \theta_t^U - \theta_{t-1}^U = \alpha_t \left(\hat{X}_{t-1}^U + \frac{\Theta}{\alpha} \right) + \beta_t (s_t - \hat{V}_{t-1}^U),$$

where $\alpha_t = \frac{\alpha}{1-\alpha} (\omega_{t-1} - \omega_t) > 0$ and $\beta_t = \frac{\alpha}{1-\alpha} \frac{\tau_{\varepsilon,t}}{\mu_t} (1 - \omega_t) (o_{V,t}^U - o_{V,t}^I) > 0$.

2. For $t = 2, 3, \dots, T$, the covariance of demands is

$$\text{Cov}(\Delta\theta_t^U, \Delta\theta_{t-1}^U) = \alpha_t \alpha_{t-1} \text{Var}(\hat{X}_{t-2}^U) + \alpha_t \beta_{t-1} o_{VX, t-2}^U > 0.$$

For $t = 1$, we also have $\text{Cov}(\Delta\theta_1^U, \Delta\theta_0^U) > 0$. Since informed traders are counterparties to uninformed traders, trades exhibit positive autocorrelation for both types of traders.

Proof. See Appendix A.7. □

Proposition 5 implies that following the arrival of public information, both informed and uninformed traders adjust their positions. Those who optimally bought (or sold) prior to the information release often continue trading in the same direction afterward, leading to autocorrelated trades and positive return autocorrelation.

We next examine which traders buy when prices rise and sell when prices fall, thereby behaving as momentum traders. Define

$$\kappa_t^i := \text{Cov}(\Delta\theta_t^i, R_{t-1}), \quad \text{for } t = 1, 2, \dots, T. \quad (22)$$

Since informed and uninformed traders are counterparties, it follows that $\kappa_t^U = -\kappa_t^I$. If $\kappa_t^i > 0$, type- i traders buy when prices rise and sell when prices fall, thereby engaging in momentum trading. In contrast, if $\kappa_t^i < 0$, they engage in contrarian trading.

It is also helpful to examine which traders act as liquidity demanders—those whose sales drive down the concurrent price and whose purchases drive it up, thereby imposing a positive price impact. Following Vayanos and Wang (2012), we define the price impact of type- i traders' demand as

$$\lambda_t^i := \frac{\text{Cov}(R_t, \Delta\theta_t^i)}{\text{Var}(\Delta\theta_t^i)}, \quad \text{for } t = 0, 1, \dots, T. \quad (23)$$

Similarly, since informed and uninformed traders are counterparties, it follows that $\lambda_t^U = -\lambda_t^I$. If $\lambda_t^i > 0$, type- i traders exert a positive price impact, indicating they are demanding liquidity. In contrast, if $\lambda_t^i < 0$, they are providing liquidity.

Proposition 6. *Following the arrival of public information:*

1. *Uninformed traders buy when prices rise and sell when prices fall, thereby behaving as momentum traders. In contrast, informed traders act as contrarian traders, i.e., $\kappa_t^I < 0$ and $\kappa_t^U > 0$, for $2 \leq t \leq T$;*
2. *Uninformed traders demand liquidity: their sales drive down the concurrent price, and their purchases drive it up. Informed traders, by contrast, provide liquidity, i.e., $\lambda_t^I < 0$, and $\lambda_t^U > 0$, for $1 \leq t \leq T$.*

Proof. See Appendix A.8. □

At $t = 0$, informed traders optimally establish positions based on both private information and liquidity needs. Their liquidity-driven trading pushes prices away from fundamental values, as hedging motives distort their optimal response to private signals. Proposition 6 implies that, in the absence of future liquidity shocks, rational uninformed traders—who initially provide liquidity to informed traders—anticipate that prices will eventually revert toward fundamentals. When public information arrives, uninformed traders use the signal to infer whether current prices are too high or too low relative to fundamental value and trade in the direction that moves prices toward fundamentals. As a result, they tend to buy following price increases and sell following price declines, thereby behaving as momentum traders. Moreover, because they demand liquidity, their purchases push prices further upward and their sales push prices further downward, reinforcing return continuation and generating positive return autocorrelation. Informed traders, by contrast, act as contrarian traders, adjusting their positions at more favorable prices by buying when prices fall and selling when prices rise.

As Figure 2 illustrates, returns exhibit positive autocorrelation for periods $t = 2, 3, \dots, T$, during which uninformed traders behave as momentum traders and demand liquidity. In general, when $\lambda_t^i \kappa_t^i > 0$, momentum traders also demand liquidity: their purchases following positive returns push prices further upward, while their sales following negative returns push prices further downward. As a result, their trading reinforces return continuation and generates positive return autocorrelation.

Figure 3 shows that both return autocorrelation and trading volume tend to increase with

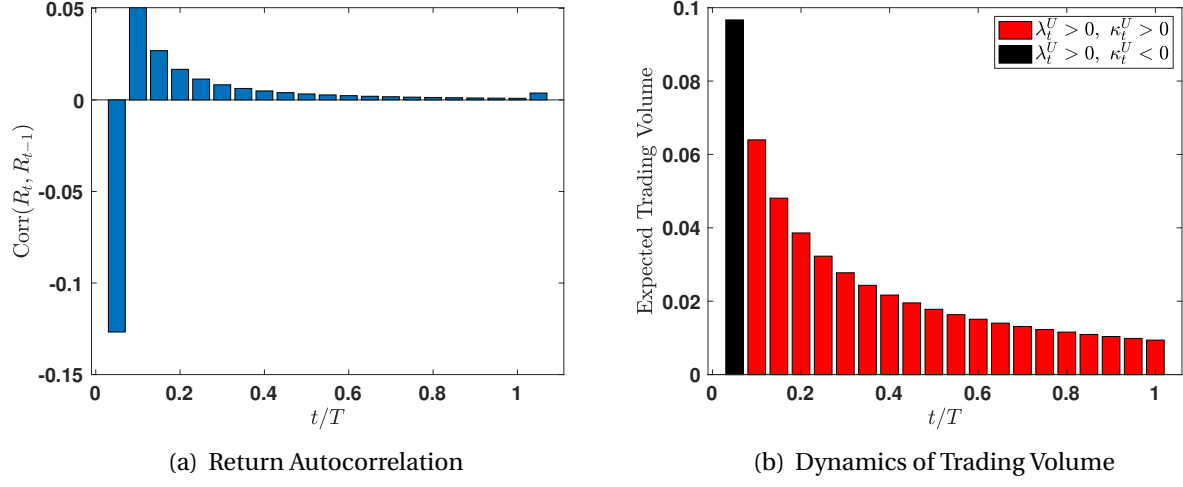


Figure 2. Parameters are $T = 20$, $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_{\eta,t} = 1$, $\tau_{\varepsilon,t} = 1$, $\bar{V} = 1$, and $\tau_{x,0} = 1$. In Panel (b), the red bars indicate that positive return autocorrelation occurs when $\lambda_t^U \kappa_t^U > 0$.

the precision of public information. This positive relationship becomes more pronounced when informed traders' private signals are more precise (i.e., when $\tau_{\eta,0}$ is higher), as traders respond more aggressively to public information, leading to greater trading volume and stronger return autocorrelation.

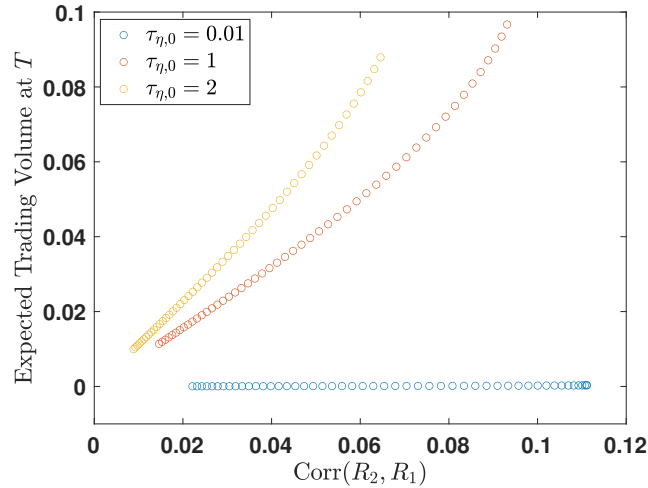


Figure 3. Return autocorrelation and trading volume as the precision of public information, $\tau_{\varepsilon,1}$ varies from 0.01 to 1. The parameter values are: $T = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\bar{V} = 1$, $\Theta = 1$, $\tau_{x,0} = 1$, and $\tau_{\eta,0}$ as indicated in the legend.

4.3 Return Dynamics with Asymmetric Information and Future Liquidity Shocks

In this section, we examine how the anticipation of future liquidity shocks influences traders' behavior and return autocorrelation prior to the shocks' arrival. We assume that there is an additional future shock $x_{t'}$ at $t = t'$. It is straightforward to see that after the additional shock, i.e., for $t \geq t'$, the return follows the same form as in the previous section, thus the return autocorrelation and dynamic of trading are the same as in Proposition 4.

For $0 \leq t < t'$, from Proposition 1, the equilibrium price can be written as

$$P_t = \delta_t(\hat{V}_t^U - \bar{V}) + \bar{V} - g_t \left(\hat{X}_t^U + \frac{\Theta}{\alpha} \right), \quad (24)$$

where $\hat{X}_t^U = E_t^U[X_t]$, $g_{t-1} > g_t > 0$, $\delta_{t-1} > \delta_t > 1$, and δ_t decreases in $\tau_{x,t'}$. It can be shown that before the additional shock, i.e., for $0 < t < t'$, the return is

$$R_t = a_t \left(\hat{X}_{t-1}^U + \frac{\Theta}{\alpha} \right) + b_t(s_t - \hat{V}_{t-1}^U) + c_t(\hat{V}_{t-1}^U - \bar{V}),$$

where a_t , b_t , and c_t are defined as

$$a_t := \omega_{t-1}\mu_{t-1} - \omega_t\mu_t > 0, \quad b_t := \tau_{\varepsilon,t} \left(\omega_t o_{V,t}^I + (1 - \omega_t) o_{V,t}^U \right) > 0, \quad c_t := \delta_t - \delta_{t-1} < 0. \quad (25)$$

It follows that, prior to the additional shock, we still have $\text{Cov}(R_1, R_0) < 0$ for $t = 1$. For $1 < t < t'$, the return covariance may be either positive or negative. More specifically, it can be shown that $\text{Cov}(R_t, R_{t-1}) = E_0 + E_1 + E_2$, where

$$\begin{aligned} E_0 &= a_t a_{t-1} \text{Var}(\hat{X}_{t-2}^U) + a_t b_{t-1} o_{VX,t-2}^U > 0, \\ E_1 &= c_t c_{t-1} \text{Var}(\hat{V}_{t-2}^U) - (a_t c_{t-1} + c_t a_{t-1}) o_{VX,t-2}^U > 0, \quad E_2 = c_t b_{t-1} o_{V,t-2}^U < 0. \end{aligned} \quad (26)$$

The terms E_0 and E_1 are positive, while E_2 is negative. Therefore, return autocorrelation prior to the arrival of the shocks can be either positive or negative. More importantly, we find that return dynamics critically depend on the volatility of these future shocks.

When future shocks are much less uncertain than the initial shock, it can be shown that

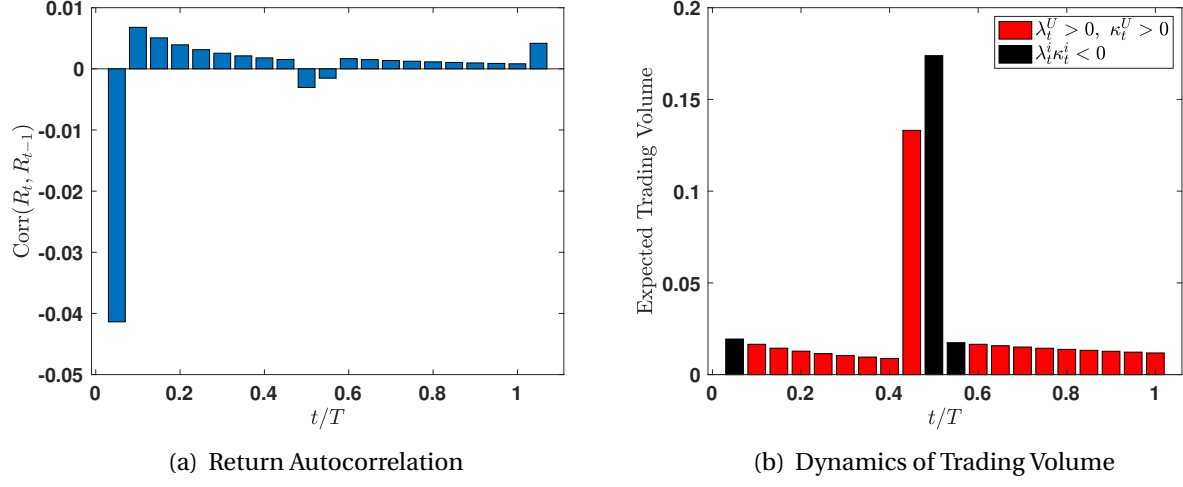


Figure 4. Informed traders receive liquidity shocks at both the beginning and midpoint of the trading duration. Parameters are $T = 20$, $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_{\eta,t} = 1$, $\tau_{\varepsilon,t} = 1$, $\bar{V} = 1$, and $\tau_{x,t} = 1e6$ except for $\tau_{x,0} = 2$, $\tau_{x,10} = 5$. In Panel (b), the red bars indicate that return continuation is driven by the trading of uninformed traders.

traders keep the same trading direction until the additional shock arrives,

$$\text{Cov}(\Delta\theta_t^U, \Delta\theta_{t-1}^U) > 0, \text{ for } 0 < t < t', \quad \text{Cov}(\Delta\theta_{t'}^U, \Delta\theta_{t'-1}^U) < 0.$$

In addition, uninformed traders are momentum traders and demanding liquidity,

$$\kappa_t^U > 0, \quad \lambda_t^U > 0, \text{ for } 0 < t \leq t',$$

and the effect of $E_0 > 0$ dominates, resulting in positive return autocorrelation prior to the arrival of future shocks: $\text{Cov}(R_t, R_{t-1}) > 0$, for $0 < t \leq t'$.

As illustrated in Figure 4, when future shocks are much less volatile than the initial shock, their expected impact is small, and their anticipated arrival has little influence on traders' behavior beforehand. As a result—similar to the case without anticipated future shocks—rational uninformed traders continue to trade in the direction of the public signal, buying in response to positive news and selling in response to negative news. This behavior leads to autocorrelated trades and positive return autocorrelation.

In contrast, when future shocks are highly volatile, traders keep changing trading direction,

$$\text{Cov}(\Delta\theta_t^U, \Delta\theta_{t-1}^U) < 0, \text{ for } 0 < t \leq t'.$$

In addition, informed traders are contrarian traders and demanding liquidity,

$$\kappa_t^I < 0, \lambda_t^I > 0, \text{ for } 0 < t \leq t'.$$

and the effect of $E_2 < 0$ dominates, resulting in negative return autocorrelation prior to the arrival of future shocks: $\text{Cov}(R_t, R_{t-1}) < 0$, for $0 < t \leq t'$.

Intuitively, when future shocks are highly volatile, informed traders alternate their trading direction from one period to the next in response to public information releases, due to uncertainty about whether the upcoming shock will be positive or negative. To avoid accumulating large long or short positions, they behave as contrarian traders—buying when past prices are low and selling when past prices are high. In doing so, they consume liquidity by pushing prices up when buying and down when selling. Rational uninformed traders, while providing liquidity to informed traders, also reverse their trading direction each period in response to public signals. This period-by-period reversal in trading direction generates negative return autocorrelation, as illustrated in Figure 5.

When the volatility of future liquidity shocks is comparable to that of the initial shock, it can be shown that traders reverse their trading direction after the initial shock and maintain that direction until the additional shock arrives,

$$\text{Cov}(\Delta\theta_1^U, \Delta\theta_0^U) < 0, \text{ Cov}(\Delta\theta_t^U, \Delta\theta_{t-1}^U) > 0, \text{ for } 1 < t < t', \text{ Cov}(\Delta\theta_{t'}^U, \Delta\theta_{t'-1}^U) < 0.$$

In this case, informed traders behave as momentum traders and demand liquidity,

$$\kappa_t^I > 0, \lambda_t^I > 0, \text{ for } 0 < t \leq t',$$

and the effect of $E_1 > 0$ dominates, resulting in positive return autocorrelation prior to the arrival of future shocks: $\text{Cov}(R_t, R_{t-1}) > 0$, for $0 < t \leq t'$.

When the volatility of future liquidity shocks is comparable to that of the initial shock, traders

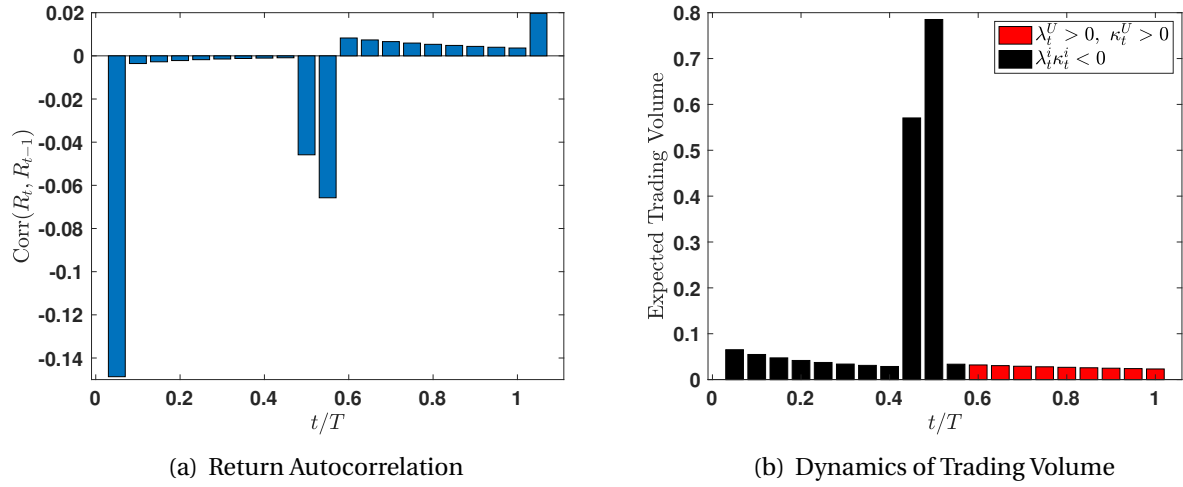


Figure 5. Informed traders receive liquidity shocks at both the beginning and midpoint of the trading duration. Parameters are $\tau_{x,0} = 2$, $\tau_{x,10} = 0.5$, $T = 20$, $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_\eta = 1$, $\tau_{\varepsilon,t} = 1$, and $\bar{V} = 1$.

behave differently from the case without anticipated future shocks. In response to public information releases, rational uninformed traders no longer act as momentum traders. They are concerned that future liquidity shocks may drive prices away from fundamental values, so a rising price does not necessarily signal convergence toward fundamentals. As a result, they refrain from buying simply because prices are increasing. Instead, they act as contrarian traders and provide liquidity by buying when prices are low and selling when prices are high.

Informed traders, who initially buy (or sell) based on private information and liquidity needs, reverse their position once following the initial round of public information. After this adjustment, they continue trading in the same direction in response to subsequent public information releases. As more public information becomes available, their uncertainty about the asset's payoff—and thus about the payoff of the associated non-traded risky asset—diminishes. When the anticipated future shocks have moderate volatility, informed traders optimally adjust their hedging positions by reducing the size of their initial long or short exposures. Thus, in contrast to the case without future liquidity shocks, informed traders exhibit momentum trading behavior and demand liquidity, which gives rise to positive return autocorrelation. This case is

illustrated in Figure 6.

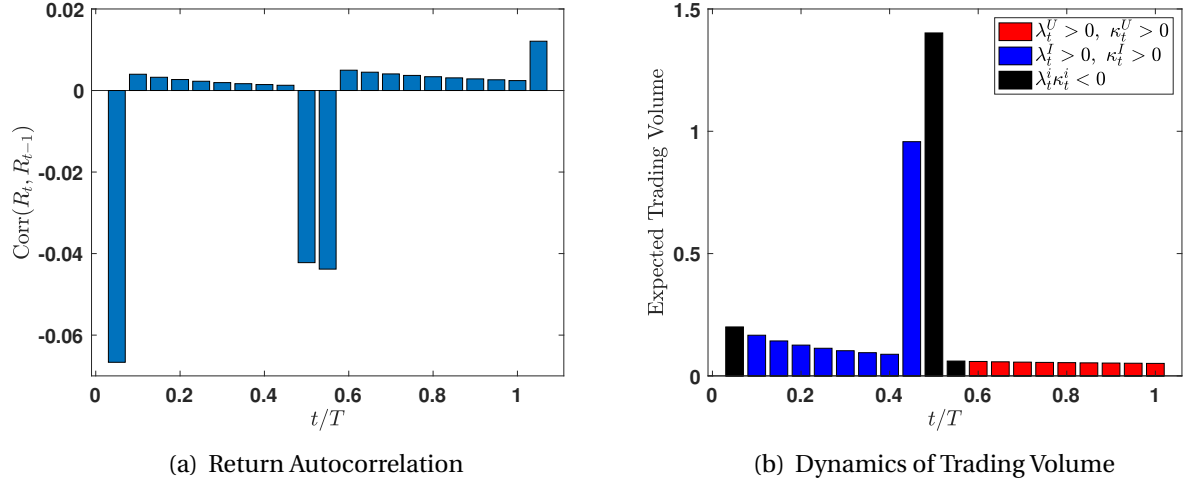


Figure 6. Informed traders experience liquidity shocks at both the beginning and midpoint of the trading duration. Parameters are $\tau_{x,0} = 2$, $\tau_{x,10} = 2$, $T = 20$, $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_\eta = 1$, $\tau_{\varepsilon,t} = 1$, and $\bar{V} = 1$.

Figure 7 shows that for moderate $\tau_{x,t'}^{-1}$ more public information leads to more positive return autocorrelation but lower trading volume. The reason is that greater public information reduces fundamental uncertainty and hence mitigates adverse selection, which lowers hedging demand and decreases trading intensity. At the same time, more public information smooths the path of the risk premium, making expected returns more persistent and thereby strengthening the predictable drift in returns.

4.4 Horizon-Dependent Return Autocorrelation

In this section, we analyze the interaction between public information and liquidity shocks and show that the model can generate rich patterns of return dynamics, including short-term return continuation followed by reversal.

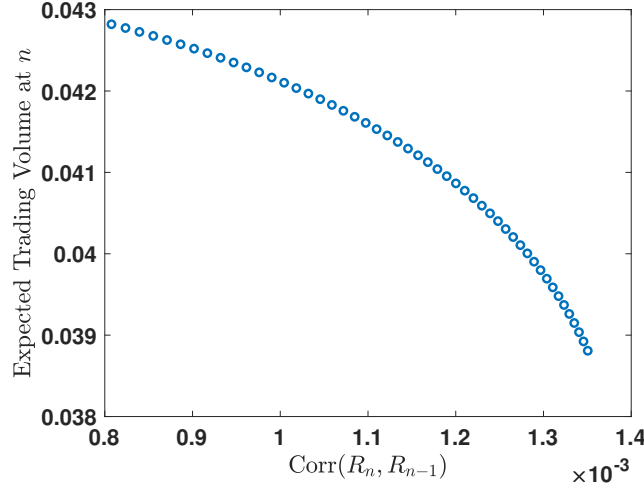


Figure 7. The parameters are: $T = 20$, $n = 5$, $\alpha = 0.5$, $A^I = A^U = 1$, $\tau_V = 1$, $\bar{V} = 1$, $\Theta = 1$, $\tau_{x,0} = 2$, $\tau_{x,10} = 2$, $\tau_\eta = 1$ and $\tau_{\varepsilon,n}$ increases from 0.01 to 1.

The covariance between R_t and its s -period lag R_{t-s} can be expressed as follows:

$$\begin{aligned} \text{Cov}(R_t, R_{t-s}) = & (g_t - g_{t-1})(g_{t-s} - g_{t-s-1})\text{Var}(\hat{X}_{t-s-1}^U) + (\delta_t - \delta_{t-1})(\delta_{t-s} - \delta_{t-s-1})\text{Var}(\hat{V}_{t-s-1}^U) \\ & + \left(\delta_{t-s} K_{V,t-s}^U - g_{t-s} K_{X,t-s}^U \right) \left[(\delta_t - \delta_{t-1}) K_{V,t-s}^U - (g_t - g_{t-1}) K_{X,t-s}^U \right] \Sigma_{t-s}^U \\ & - \left[(\delta_t - \delta_{t-1})(g_{t-s} - g_{t-s-1}) + (g_t - g_{t-1})(\delta_{t-s} - \delta_{t-s-1}) \right] \text{Cov}(\hat{V}_{t-s-1}^U, \hat{X}_{t-s-1}^U). \end{aligned} \quad (27)$$

As the lag s increases, return covariance can decline and eventually become negative, depending on the timing and magnitude of liquidity shocks and public information arrivals. Figure 8 illustrates this pattern in a numerical example in which an additional liquidity shock occurs in the middle of the sample period. The return in period n is positively correlated with subsequent returns at lags of one through seven periods, but negatively correlated with returns at longer lags. Thus, the model can generate positive return dependence at short horizons and negative return dependence at longer horizons, consistent with the empirical pattern of short-run return continuation and long-run reversal.

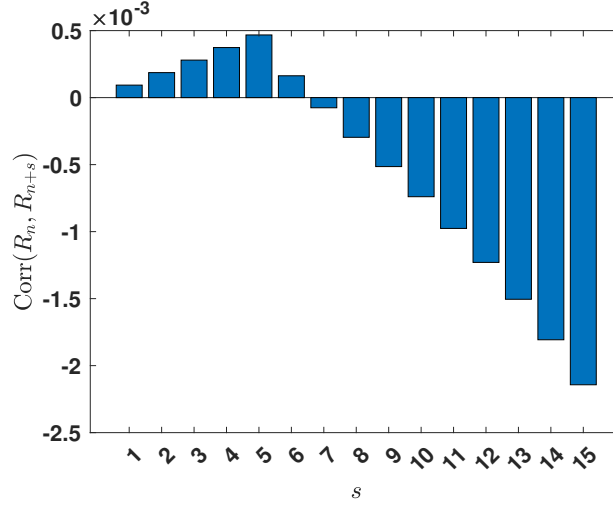


Figure 8. Liquid shocks $\tau_{x,t} = 5$ occur for $20 \leq t \leq 30$. Other parameters are $T = 50$, $\Theta = 1$, $\alpha = 0.5$, $A^I = 1$, $A^U = 1$, $\tau_V = 1$, $\tau_\eta = 2$, $\tau_{\varepsilon,t} = 1$, $\tilde{V} = 1$, and $n = 35$.

5 Empirical Analysis

Our model delivers two main empirical implications. The first links pre-announcement liquidity trading to subsequent returns and yields three related predictions. First, a liquidity shock pushes prices away from fundamental value, so the price is depressed while the shock is being absorbed. Second, when public information subsequently arrives, prices partially revert toward fundamentals, so greater pre-announcement liquidity selling predicts higher announcement returns, while greater liquidity buying predicts lower announcement returns. The prediction is about the direction of the effect rather than the sign of the return itself: expected returns are increasing in the shock x_0 , but the compensation for bearing the constant supply Θ/α means a negative x_0 need not imply a negative return. Third, post-announcement return continuation is stronger following larger pre-announcement liquidity shocks. Consistent with the announcement-return prediction, [Levi and Zhang \(2015\)](#) show that liquidity trades, particularly pre-announcement liquidity sales, are an important driver of positive announcement returns.

The second implication concerns the relationship between trading volume and return au-

to correlation. The model predicts that this relationship need not have a uniform sign because it depends on the underlying source of trading. In the baseline asymmetric-information setting with only an initial liquidity shock, stronger updating to public information induces uninformed traders to trade more aggressively in the direction of past returns. Trading volume and positive return autocorrelation therefore move together.

By contrast, when anticipated future liquidity shocks are important, public information can strengthen return persistence while reducing liquidity- or hedging-motivated trading. In this case, trading volume and return autocorrelation can move in opposite directions. Thus, the model predicts that the sign of the volume–autocorrelation relation depends on the economic source of trading.

We focus our empirical analysis on the first implication because it is most directly tied to the model’s central mechanism. Specifically, we examine the pre-announcement return path, whether greater liquidity selling predicts higher announcement returns, and whether post-announcement return continuation is stronger following larger pre-announcement liquidity shocks. Testing the second implication directly requires separating trading volume by economic source, which observational measures do only imperfectly, and we leave it to future work.

5.1 Model Implications

Write $\ell_t := \omega(\mu_{t-1} - \mu_t)$, so that Proposition 2 gives

$$P_t = \hat{V}_t - \omega\mu_t \left(x_0 + \frac{\Theta}{\alpha} \right), \quad R_t = \ell_t \left(x_0 + \frac{\Theta}{\alpha} \right) + u_t,$$

where $u_t := \tau_{\varepsilon,t} o_{V,t}(s_t - \hat{V}_{t-1})$ is a forecast error that is serially uncorrelated and independent of x_0 , with $\sigma_{u,t}^2 := \text{Var}(u_t)$. Since $o_{V,t}$ falls whenever a public signal arrives, μ_t declines and $\ell_t > 0$ at every such date.

The first two predictions follow immediately. A positive liquidity shock x_0 depresses the

price by $\omega\mu_t x_0$, while the conditional expected return when the next public signal arrives,

$$\mathbb{E}[R_t | x_0] = \ell_t \left(x_0 + \frac{\Theta}{\alpha} \right),$$

is increasing in x_0 . The third prediction — that return continuation following the signal is stronger after a larger liquidity shock — requires conditioning on the realized shock magnitude, since this is what the empirical proxy measures.

Corollary 1. *Fix $t \geq 2$ with $\ell_{t-1}, \ell_t > 0$. Conditional on the realized shock magnitude $|x_0| = w$, the coefficient from projecting R_t on R_{t-1} is*

$$\varrho(w) = \frac{\ell_t \ell_{t-1} w^2}{\ell_{t-1}^2 w^2 + \sigma_{u,t-1}^2}, \quad \varrho(0) = 0, \quad \varrho'(w) > 0 \text{ for } w > 0.$$

Corollary 1 follows directly from Proposition 2; the proof is in Appendix A.4. The key implication is that $\varrho(w)$ increases with the magnitude of the preceding liquidity shock: larger shocks generate stronger subsequent return continuation. Because $\varrho(w)$ is nonlinear and approaches ℓ_t/ℓ_{t-1} as w becomes large, the linear interaction used below should be interpreted as a local approximation over the empirically relevant range of shock magnitudes. This dependence of continuation on *prior non-informational trading* is not a direct prediction of standard under-reaction accounts based solely on gradual processing of the earnings signal.

5.2 Empirical Results

We use 231,641 U.S. quarterly earnings announcements for 7,702 firms over 2003:Q2–2024:Q3. We proxy for non-informational selling pressure using flow-induced trading by domestic equity mutual funds, following Coval and Stafford (2007) and Lou (2012). Fund flows are inferred from share-class total net assets and returns, applied to the most recent quarter-end holdings, and aggregated across funds into a stock-month measure, denoted *SELL* and standardized within calendar quarter. Selling pressure is measured in the calendar month ending before the announcement month, so the shock window ends between 1 and 31 days before the announcement, with a median of 17 days.

Event time is measured in trading days, with day 0 defined as the first trading day on or after the Compustat announcement date. We define $CAR_i(a, b)$ as the sum of daily market-adjusted returns from event day a through event day b , where daily abnormal returns are stock returns minus the CRSP value-weighted market return. We write $ANN_i := CAR_i(0, +1)$ for the two-day announcement return. Additional details on sample construction and variable definitions are provided in Appendix A.9.

Figure 9 plots the coefficient relating subsequent returns to the return at different event-time anchors. The coefficient is negative at every non-announcement anchor but turns positive at the earnings announcement. In a stacked specification using mutually non-overlapping anchors, the non-announcement coefficient is -0.0985 , while the announcement increment is $+0.1196$, implying a coefficient of 0.0211 at the announcement anchor. (This anchor-level coefficient is a different object from φ_1 in Table 1, which happens to round to the same value.) This sign reversal is the basic empirical pattern addressed by the model: the arrival of public information can overturn liquidity-induced return reversal.

We examine the three predictions using three empirical exercises. First, we regress the return over the three months ending in the pressure month on $SELL$. Second, we compare announcement returns, $CAR(0, +1)$, across quintiles of $SELL$ formed within quarter. Third, we examine whether post-announcement return continuation is stronger following a larger liquidity shock.

For the third exercise, let L_i denote the magnitude of selling pressure, $|SELL_i|$, standardized within calendar quarter, so that $L_i = 0$ corresponds to the mean shock magnitude in that quarter and $L_i = 1$ to one standard deviation above the mean. We estimate

$$CAR_i(+2, +10) = \varphi_0 + \varphi_1 ANN_i + \varphi_2 L_i + \varphi_3 (ANN_i \times L_i) + \Lambda' Y_i + \iota_q + \epsilon_i, \quad (28)$$

where $ANN_i := CAR_i(0, +1)$ is the two-day announcement return, Y_i collects the controls, and ι_q is a calendar-quarter fixed effect. Continuation at the mean shock magnitude is φ_1 , while φ_3 measures how much the continuation slope increases for a one-standard-deviation increase in

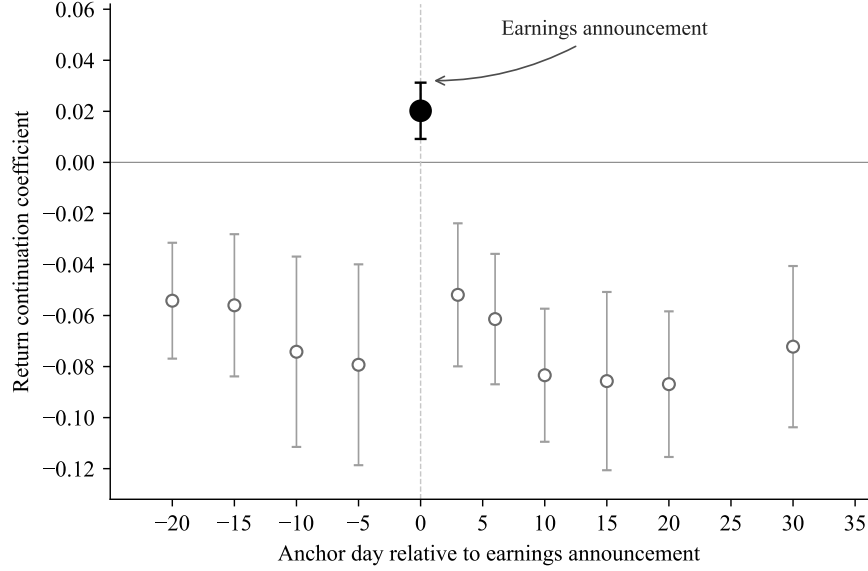


Figure 9. Return continuation around earnings announcements. Each point is the coefficient from regressing $CAR(a+2, a+10)$ on $CAR(a, a+1)$ at event-time anchor a , where anchors and horizons are in trading days. Every regression includes calendar-quarter fixed effects, and the vertical bars are 95% confidence intervals built from standard errors clustered by firm and by calendar quarter. The coefficient is negative at every non-announcement anchor and positive at the earnings announcement ($a = 0$).

shock magnitude. Thus, continuation at one standard deviation above the mean is $\varphi_1 + \varphi_3$. The prediction $\varphi_3 > 0$ is the empirical analogue of $\rho'(w) > 0$ in Corollary 1.

The regression specifications include calendar-quarter fixed effects and controls for size, book-to-market, momentum, turnover and mutual fund ownership, each measured at the pressure month, with standard errors clustered by firm and calendar quarter. The quintile comparison is an unconditional comparison of mean announcement returns across within-quarter *SELL* quintiles.

We report equation (28) in two versions. The *baseline specification* includes the firm characteristics in levels. The specification *with continuation-slope controls* additionally interacts ANN_i with size, turnover, mutual fund ownership, and pre-announcement volatility, allowing the continuation slope itself to vary with these characteristics. Pre-announcement volatility is also included in levels in this specification.

These slope controls address the possibility that the liquidity-shock interaction captures known cross-sectional variation in post-announcement drift. Prior work finds stronger drift among smaller firms (Foster, Olsen, and Shevlin, 1984; Bernard and Thomas, 1989), less liquid stocks (Sadka, 2006), stocks with lower institutional ownership (Bartov, Radhakrishnan, and Krinsky, 2000), and stocks with greater arbitrage risk (Mendenhall, 2004). Because liquidity-shock magnitude may be correlated with these characteristics, omitting the slope interactions could cause φ_3 to capture variation associated with market frictions rather than the liquidity-shock channel. We therefore treat the specification with continuation-slope controls as our preferred specification and report the baseline alongside it to show the sensitivity of the estimate.

Table 1. Evidence on pre-announcement liquidity shocks

		Estimate	<i>t</i>
<i>Panel A. Price response to selling pressure</i>			
Pre-announcement return, per 1 s.d. of <i>SELL</i>		−1.24%	−6.99
Announcement-return spread, high minus low <i>SELL</i>		17.0 bp	2.91
<i>Panel B. Post-announcement continuation, CAR(+2, +10)</i>			
Continuation at mean shock magnitude, preferred	φ_1	0.0211	3.73
Increase per 1 s.d. of shock magnitude	φ_3		
baseline specification		0.0042	1.58
with continuation-slope controls		0.0086	3.08
Implied continuation at +1 s.d., preferred	$\varphi_1 + \varphi_3$	0.0297	—

The results are consistent with the model’s mechanism and with Proposition 2 and Corollary 1. Greater selling pressure is associated with lower pre-announcement returns: a one-standard-deviation increase in *SELL* is associated with a 1.24% lower return over the three months ending in the pressure month. We use a three-month window because fund-flow episodes can extend over several months, although the stock-level measure itself is only weakly persistent from month to month. The timing evidence is therefore suggestive rather than sharp: when returns are measured over the pressure month alone, the estimate is −0.23% ($t = -1.57$). We interpret this result as describing the pre-announcement return path rather than as a clean con-

temporaneous price-impact test; both estimates are reported in Appendix A.9. Greater selling also predicts a higher announcement return: the difference between the highest- and lowest-selling-pressure quintiles is 17.0 basis points, and the underlying quintile pattern is roughly monotonic.

Most importantly for the model’s distinctive prediction, post-announcement continuation is stronger following a larger prior liquidity shock. In the preferred specification, the continuation coefficient is 0.0211 at the mean shock magnitude, and a one-standard-deviation increase in shock magnitude raises it by 0.0086, to 0.0297, an increase of about 40%.

The continuation result is sensitive to the inclusion of the continuation-slope controls. In the baseline specification, $\varphi_3 = 0.0042$ ($t = 1.58$), compared with 0.0086 ($t = 3.08$) in the preferred specification. The estimated level of continuation is nearly unchanged across the two specifications: φ_1 is 0.0203 in the baseline and 0.0211 in the preferred specification. Thus, the slope controls primarily affect the gradient of continuation with respect to liquidity-shock magnitude rather than the level of continuation. The larger estimate of φ_3 in the preferred specification comes mainly from allowing continuation to vary with mutual fund ownership. All four slope interactions enter negatively, and fund ownership is by far the most strongly correlated with shock magnitude (a correlation of 0.30, against 0.06 or less for the others), which is unsurprising given that flow-induced trading is itself scaled by fund holdings. Omitting that interaction therefore biases φ_3 downward. The four coefficients and their correlations with shock magnitude are reported in Appendix A.9.

We interpret these results as evidence consistent with the model’s mechanism rather than as a structural test. The positive relation between pre-announcement selling pressure and announcement returns is primarily cross-sectional: with firm fixed effects, the coefficient falls to 2.17 basis points ($t = 1.06$). This cross-sectional nature is not inconsistent with the model. If firms differ persistently in their exposure to liquidity shocks, or in the price impact of those shocks, firms experiencing greater liquidity pressure may also exhibit larger subsequent price recoveries. Because such cross-firm heterogeneity is not explicitly modeled, however, the cross-

sectional evidence cannot distinguish the model’s liquidity-shock mechanism from other persistent firm characteristics that may also affect announcement returns. We therefore view the announcement-return evidence as supportive rather than definitive.

The model’s more distinctive prediction concerns how the magnitude of the realized liquidity shock affects subsequent return continuation. The appendix shows that this interaction is present in within-firm variation, which more closely corresponds to the prediction in Corollary 1 that continuation increases with the realized shock magnitude. Additional between- and within-firm decompositions and results from the longer sample are reported in Appendix A.9.

6 Conclusion

In this paper, we present a dynamic trading model with rational traders and show that the interaction between public information and liquidity shocks can generate positive return autocorrelation.

In the asymmetric-information setting, the arrival of public information causes both informed and uninformed traders to adjust their positions, and these adjustments can generate persistent trading in the same direction and positive return autocorrelation. In the absence of future liquidity shocks, rational uninformed traders tend to follow the direction of public signals, buying in response to positive news and selling in response to negative news. As a result, they behave as momentum traders, generating autocorrelated trades and reinforcing return continuation. When future liquidity shocks are anticipated and their volatility is comparable to that of the initial shock, the identity of the momentum traders changes. Uninformed traders instead behave as contrarian traders and provide liquidity, while informed traders engage in momentum trading and demand liquidity, thereby generating positive return autocorrelation.

Using flow-induced mutual-fund trading as a proxy for non-informational liquidity pressure, we also provide supporting evidence that greater pre-announcement selling predicts higher announcement returns and that post-announcement return continuation is stronger

following larger prior liquidity shocks.

Overall, the paper shows that positive return autocorrelation can arise in rational expectations equilibrium models under simple and plausible assumptions. By analyzing the interaction between public information and liquidity shocks, the model can also generate horizon-dependent return dynamics, including short-run return continuation followed by longer-run reversal.

References

- Albuquerque, R., and J. Miao. 2014. Advance information and asset prices. *Journal of Economic Theory* 149:236–275. [doi:10.1016/j.jet.2013.06.001](https://doi.org/10.1016/j.jet.2013.06.001).
- Allen, F., S. Morris, and H. S. Shin. 2006. Beauty Contests and Iterated Expectations in Asset Markets. *Review of Financial Studies* 19:719–752. [doi:10.1093/rfs/hhj036](https://doi.org/10.1093/rfs/hhj036).
- Andrei, D., and J. Cujean. 2017. Information percolation, momentum and reversal. *Journal of Financial Economics* 123:617–645. [doi:10.1016/j.jfineco.2016.05.012](https://doi.org/10.1016/j.jfineco.2016.05.012).
- Banerjee, S., R. Kaniel, and I. Kremer. 2009. Price Drift as an Outcome of Differences in Higher-Order Beliefs. *Review of Financial Studies* 22:3707–3734. [doi:10.1093/rfs/hhp014](https://doi.org/10.1093/rfs/hhp014).
- Barberis, N., and A. Shleifer. 2003. Style investing. *Journal of financial Economics* 68:161–199. [doi:10.1016/S0304-405X\(03\)00064-3](https://doi.org/10.1016/S0304-405X(03)00064-3).
- Bartov, E., S. Radhakrishnan, and I. Krinsky. 2000. Investor Sophistication and Patterns in Stock Returns after Earnings Announcements. *The Accounting Review* 75:43–63. [doi:10.2308/accr.2000.75.1.43](https://doi.org/10.2308/accr.2000.75.1.43).
- Berk, J. B., R. C. Green, and V. Naik. 1999. Optimal investment, growth options, and security returns. *Journal of finance* 54:1553–1607. [doi:10.1111/0022-1082.00161](https://doi.org/10.1111/0022-1082.00161).

- Bernard, V. L., and J. K. Thomas. 1989. Post-Earnings-Announcement Drift: Delayed Price Response or Risk Premium? *Journal of Accounting Research* 27:1–36. [doi:10.2307/2491062](https://doi.org/10.2307/2491062).
- Bogousslavsky, V. 2016. Infrequent rebalancing, return autocorrelation, and seasonality. *Journal of Finance* 71:2967–3006. [doi:10.1111/jofi.12436](https://doi.org/10.1111/jofi.12436).
- Brennan, M., and H. Cao. 1996. Information, trade, and derivative securities. *Review of Financial Studies* 9:163–208. [doi:10.1093/rfs/9.1.163](https://doi.org/10.1093/rfs/9.1.163).
- Brennan, M., and H. Cao. 1997. International portfolio investment flows. *Journal of finance* 52:1851–1880. [doi:10.2307/2329467](https://doi.org/10.2307/2329467).
- Cameron, A. C., J. B. Gelbach, and D. L. Miller. 2011. Robust Inference with Multiway Clustering. *Journal of Business and Economic Statistics* 29:238–249. [doi:10.1198/jbes.2010.07136](https://doi.org/10.1198/jbes.2010.07136).
- Cespa, G., and X. Vives. 2012. Dynamic trading and asset prices: Keynes vs. Hayek. *Review of Economic Studies* 79:539–580. [doi:10.1093/restud/rdr040](https://doi.org/10.1093/restud/rdr040).
- Chen, W., and Y. Wang. 2025. Dynamic Market Making with Asymmetric Information and Market Power. *Review of Financial Studies* 38:235–293. [doi:10.1093/rfs/hhae062](https://doi.org/10.1093/rfs/hhae062).
- Coval, J., and E. Stafford. 2007. Asset Fire Sales (and Purchases) in Equity Markets. *Journal of Financial Economics* 86:479–512. [doi:10.1016/j.jfineco.2006.09.007](https://doi.org/10.1016/j.jfineco.2006.09.007).
- Foster, G., C. Olsen, and T. Shevlin. 1984. Earnings Releases, Anomalies, and the Behavior of Security Returns. *The Accounting Review* 59:574–603. [doi:10.2308/tar-4483133](https://doi.org/10.2308/tar-4483133).
- Goldstein, I., Y. Li, and L. Yang. 2014. Speculation and Hedging in Segmented Markets. *Review of Financial Studies* 27:881–922. [doi:10.1093/rfs/hht059](https://doi.org/10.1093/rfs/hht059).
- Hong, H., and J. C. Stein. 1999. A unified theory of underreaction, momentum trading, and overreaction in asset markets. *Journal of finance* 54:2143–2184. [doi:10.1111/0022-1082.00184](https://doi.org/10.1111/0022-1082.00184).

- Johnson, T. C. 2002. Rational momentum effects. *Journal of Finance* 57:585–608. [doi:10.1111/1540-6261.00435](https://doi.org/10.1111/1540-6261.00435).
- Kyle, A. S., A. A. Obizhaeva, and Y. Wang. 2023. Beliefs aggregation and return predictability. *Journal of Finance* 78:427–486. [doi:10.1111/jofi.13195](https://doi.org/10.1111/jofi.13195).
- Levi, S., and X.-j. Zhang. 2015. Asymmetric decrease in liquidity trading before earnings announcements and the announcement return premium. *Journal of Financial Economics* 118:383–398. [doi:10.1016/j.jfineco.2015.08.003](https://doi.org/10.1016/j.jfineco.2015.08.003).
- Lou, D. 2012. A Flow-Based Explanation for Return Predictability. *Review of Financial Studies* 25:3457–3489. [doi:10.1093/rfs/hhs103](https://doi.org/10.1093/rfs/hhs103).
- Mendenhall, R. R. 2004. Arbitrage Risk and Post-Earnings-Announcement Drift. *Journal of Business* 77:875–894. [doi:10.1086/422627](https://doi.org/10.1086/422627).
- Sadka, R. 2006. Momentum and Post-Earnings-Announcement Drift Anomalies: The Role of Liquidity Risk. *Journal of Financial Economics* 80:309–349. [doi:10.1016/j.jfineco.2005.04.005](https://doi.org/10.1016/j.jfineco.2005.04.005).
- Vayanos, D., and J. Wang. 2012. Liquidity and asset returns under asymmetric information and imperfect competition. *Review of Financial Studies* 25:1339–1365. [doi:10.1093/rfs/hhr128](https://doi.org/10.1093/rfs/hhr128).

A Appendix

A.1 Proof of Lemma 1

To prove Lemma 1, we first present the following lemma.

Lemma A-1. Suppose z_t is an $n \times 1$ vector and it follows the process $z_t = a_{z,t}z_{t-1} + b_{z,t}\zeta_t$, where $\zeta_t \sim \mathcal{N}(0, \Sigma_{\zeta,t})$ is a $k \times 1$ vector, $a_{z,t}$ is an $n \times n$ matrix, and $b_{z,t}$ is an $n \times k$ matrix. The signal s_t is an $m \times 1$ vector, $s_t = a_{s,t}z_t + b_{s,t}\zeta_t$, where the noise of the signal is perfectly correlated with the noise of the process z_t , $a_{s,t}$ is an $m \times n$ matrix, and $b_{s,t}$ is an $m \times k$ matrix. Write the estimates of the first and second moments as $\hat{z}_t := E_t[z_t]$, $o_t := E[(z_t - \hat{z}_t)(z_t - \hat{z}_t)^\top]$, where $E_t[\cdot] = E[\cdot | \underline{s}_t]$ and $\underline{s}_t := \{s_0, s_1, s_2, \dots, s_t\}$. Then the Kalman filtering formulas for $\hat{z}_t$ and o_t are

$$\hat{z}_t = a_{z,t}\hat{z}_{t-1} + k_t[s_t - E_{t-1}[s_t]], \quad o_t = (I_{n \times n} - k_t a_{s,t})(a_{z,t}o_{t-1}a_{z,t}^\top + b_{z,t}\Sigma_{\zeta,t}b_{z,t}^\top) - k_t b_{s,t}\Sigma_{\zeta,t}b_{z,t}^\top,$$

where k_t has been defined below in Equation (A-2).

Proof. Kalman filtering contains two parts: predict and update. The predicted (a priori) state estimate is $\hat{z}_{t|t-1} = E_{t-1}[z_t] = a_{z,t}\hat{z}_{t-1}$, and the innovation (prefit residual) is

$$s_t - E_{t-1}[s_t] = a_{s,t}z_t + b_{s,t}\zeta_t - a_{s,t}\hat{z}_{t|t-1} = a_{s,t}a_{z,t}(z_{t-1} - \hat{z}_{t-1}) + (a_{s,t}b_{z,t} + b_{s,t})\zeta_t.$$

The innovation (prefit residual) variance and covariance matrix are

$$\begin{aligned} \text{Var}_{t-1}[s_t] &= a_{s,t}a_{z,t}o_{t-1}a_{z,t}^\top a_{s,t}^\top + (a_{s,t}b_{z,t} + b_{s,t})\Sigma_{\zeta,t}(a_{s,t}b_{z,t} + b_{s,t})^\top, \\ \text{Cov}_{t-1}(z_t, s_t) &= a_{z,t}o_{t-1}a_{z,t}^\top a_{s,t}^\top + b_{z,t}\Sigma_{\zeta,t}(a_{s,t}b_{z,t} + b_{s,t})^\top, \end{aligned} \tag{A-1}$$

and the optimal Kalman gain k_t is

$$k_t = \text{Cov}_{t-1}(z_t, s_t)(\text{Var}_{t-1}[s_t])^{-1}, \tag{A-2}$$

where $\text{Cov}_{t-1}(z_t, s_t)$ and $\text{Var}_{t-1}[s_t]$ are as defined in Equation (A-1). Therefore, the updated (a posteriori) state estimate $\hat{z}_t$ and the updated (a posteriori) estimate covariance matrix o_t are

$$\hat{z}_t = a_{z,t}\hat{z}_{t-1} + k_t[s_t - E_{t-1}[s_t]], \quad o_t = E[(z_t - \hat{z}_t)(z_t - \hat{z}_t)^\top],$$

where $z_t - \hat{z}_t = (I_{n \times n} - k_t a_{s,t})a_{z,t}(z_{t-1} - \hat{z}_{t-1}) + (b_{z,t} - k_t(a_{s,t}b_{z,t} + b_{s,t}))\zeta_t$. From Equation (A-2), we

have $k_t \text{Var}_{t-1}[s_t] k_t^\top = \text{Cov}_{t-1}(z_t, s_t) k_t^\top$. Using Equations (A-1) and (A-2), the updated estimate covariance matrix can be simplified as

$$\begin{aligned} o_t &= (I_{n \times n} - k_t a_{s,t}) a_{z,t} o_{t-1} a_{z,t}^\top (I_{n \times n} - k_t a_{s,t})^\top + (b_{z,t} - k_t(a_{s,t} b_{z,t} + b_{s,t})) \Sigma_{\zeta,t} (b_{z,t} - k_t(a_{s,t} b_{z,t} + b_{s,t}))^\top \\ &= (I_{n \times n} - k_t a_{s,t}) (a_{z,t} o_{t-1} a_{z,t}^\top + b_{z,t} \Sigma_{\zeta,t} b_{z,t}^\top) - k_t b_{s,t} \Sigma_{\zeta,t} b_{z,t}^\top. \end{aligned}$$

□

We now prove Lemma 1 using Lemma A-1. Informed traders estimate $\hat{V}_t^I$ using the signals v_t , with X_t being observable. Therefore, for informed traders, $z_t = V$, implying $a_{z,t} = 1, b_{z,t} = 0$. The signals are given by $v_t = V + \varepsilon_t$, which implies $a_{s,t} = b_{s,t} = 1$ and $\zeta_t = \varepsilon_t$. It follows that $\hat{V}_t^I$ are Gaussian Markov processes under $\mathcal{F}_t^I$,

$$\begin{aligned} \hat{V}_t^I &= \hat{V}_{t-1}^I + K_{V,t}^I e_t^I, \quad \text{where} \quad e_t^I = v_t - \hat{V}_{t-1}^I = V - \hat{V}_{t-1}^I + \varepsilon_t, \\ e_t^I | \mathcal{F}_{t-1}^I &\sim \mathcal{N}(0, \sigma_t^I), \quad \sigma_t^I = \text{Var}_{t-1}^I(e_t^I) = o_{V,t-1}^I + \tau_{\varepsilon,t}^{-1}, \quad K_{V,t}^I = o_{V,t}^I \tau_{\varepsilon,t}, \end{aligned} \tag{A-3}$$

and the second moment estimation is $o_{V,t}^I = \left(\tau_V + \sum_{s=0}^t \tau_{\varepsilon,s} \right)^{-1}$.

Uninformed traders estimate both $\hat{V}_t^U$ and $\hat{X}_t^U$, the process z_t and the noise $\zeta_t \sim \mathcal{N}(0, \Sigma_{\zeta,t})$ are

$$z_t = \begin{pmatrix} V \\ X_t \end{pmatrix}, \quad \zeta_t = \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix}, \quad \Sigma_{\zeta,t} = \begin{pmatrix} \tau_{\varepsilon,t}^{-1} & 0 \\ 0 & \tau_{\eta,t}^{-1} \end{pmatrix}.$$

This implies that

$$z_t = a_{z,t} z_{t-1} + b_{z,t} \zeta_t, \quad a_{z,t} = I_{2 \times 2}, \quad b_{z,t} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Uninformed traders' signal process is $s_t = a_{s,t} z_t + b_{s,t} \zeta_t, a_{s,t} = \begin{pmatrix} 1 & 0 \end{pmatrix}, b_{s,t} = \begin{pmatrix} 1 & -h_t \end{pmatrix}$. Uninformed traders' state variables $\hat{V}_t^U$ and $\hat{X}_t^U$ are Gaussian Markov processes under $\mathcal{F}_t^U$:

$$\hat{V}_t^U = \hat{V}_{t-1}^U + K_{V,t}^U e_t^U, \quad \hat{X}_t^U = \hat{X}_{t-1}^U + K_{X,t}^U e_t^U, \quad \text{where} \tag{A-4}$$

$$\begin{aligned}
e_t^U &= v_t - h_t \eta_t - \hat{V}_{t-1}^U = V - \hat{V}_{t-1}^U - h_t \eta_t + \varepsilon_t, \quad e_t^U | \mathcal{F}_{t-1}^U \sim \mathcal{N}(0, \Sigma_t^U), \\
\Sigma_t^U &= \text{Var}_{t-1}^U(e_t^U) = o_{V,t-1}^U + \tau_{\varepsilon,t}^{-1} + h_t^2 \tau_{\eta,t}^{-1}, \quad K_{V,t}^U = o_{V,t-1}^U / \Sigma_t^U, \quad K_{X,t}^U = (o_{VX,t-1}^U - h_t \tau_{\eta,t}^{-1}) / \Sigma_t^U,
\end{aligned} \tag{A-5}$$

and the second moment estimates are

$$\begin{aligned}
o_{X,t}^U &= o_{X,t-1}^U + \tau_{\eta,t}^{-1} - (o_{VX,t-1}^U - h_t \tau_{\eta,t}^{-1})^2 / \Sigma_t^U, \quad o_{VX,t}^U = o_{VX,t-1}^U - (o_{VX,t-1}^U - h_t \tau_{\eta,t}^{-1}) o_{V,t-1}^U / \Sigma_t^U, \\
o_{V,t}^U &= (\tau_{\varepsilon,t}^{-1} + h_t^2 \tau_{\eta,t}^{-1}) o_{V,t-1}^U / \Sigma_t^U = \left(\tau_V + \sum_{s=0}^t (\tau_{\varepsilon,s}^{-1} + h_s^2 \tau_{\eta,s}^{-1})^{-1} \right)^{-1},
\end{aligned}$$

where $o_{VX,t}^U := \text{Cov}(V, X_t | \mathcal{F}_t^U)$, the priors are $o_{V,-1}^U = \tau_V^{-1}$, $o_{X,-1}^U = 0$, and $o_{VX,-1}^U = 0$. The coefficients $K_{V,t}^U$ and $K_{X,t}^U$ can be written as

$$K_{V,t}^U = o_{V,t}^U \tau_t, \quad K_{X,t}^U = o_{V,t}^U \tau_t \left(\sum_{s=0}^{t-1} (h_s \tau_{\eta,s}^{-1} - h_t \tau_{\eta,t}^{-1}) \tau_s - \tau_V h_t \tau_{\eta,t}^{-1} \right). \tag{A-6}$$

Next, we figure out the coefficient h_t in the composite signal $s_t = v_t - h_t \eta_t$. We write the sufficient statistic of all the composite signals arriving from period 0 to t as $\hat{V}_t^I - \mu_t X_t$, which can be inferred by uninformed traders from prices. It follows that $\hat{V}_t^I - \mu_t X_t = \hat{V}_t^U - \mu_t \hat{X}_t^U$. From Equations (A-3) and (A-4), we have

$$\left[K_{V,t}^I - (K_{V,t}^U - \mu_t K_{X,t}^U) \right] (v_t - \hat{V}_{t-1}^U) + \left[(K_{V,t}^U - \mu_t K_{X,t}^U) h_t - \mu_t \right] \eta_t + \left[(1 - K_{V,t}^I) \mu_{t-1} - \mu_t \right] (X_{t-1} - \hat{X}_{t-1}^U) = 0. \tag{A-7}$$

Therefore, the coefficients of $v_t - \hat{V}_{t-1}^U$, η_t , and $X_{t-1} - \hat{X}_{t-1}^U$ need to be zero. Setting the coefficients of η_t and $X_{t-1} - \hat{X}_{t-1}^U$ to be zero yields

$$\mu_t / \mu_{t-1} = \left(\tau_V + \sum_{s=0}^{t-1} \tau_{\varepsilon,s} \right) \left(\tau_V + \sum_{s=0}^t \tau_{\varepsilon,s} \right)^{-1}, \quad \mu_t / h_t = \tau_{\varepsilon,t} \left(\tau_V + \sum_{s=0}^t \tau_{\varepsilon,s} \right)^{-1}. \tag{A-8}$$

Informed traders' expected utility at $t = T$ is

$$\mathbb{E}_T^I \left[-e^{-A^I W_{T+1}^I} \right] = -e^{-A^I \left[W_T^I + \theta_T^I (\hat{V}_T^I - P_T) + X_T \hat{V}_T^I - \frac{1}{2} A^I o_{V,T}^I (\theta_T^I + X_T)^2 \right]},$$

where P_T denotes the trading price at $t = T$, taking the first-order condition with respect to θ_T^I yields

$$\theta_T^I = (\hat{V}_T^I - A^I o_{V,T}^I X_T - P_T) (A^I o_{V,T}^I)^{-1}.$$

It implies that informed traders' trading reveals the composite signal $\hat{V}_T^I - \mu_T X_T$, where

$$\mu_T = A^I o_{V,T}^I = A^I \left(\tau_V + \sum_{s=0}^T \tau_{\varepsilon,s} \right)^{-1}. \quad (\text{A-9})$$

From Equations (A-8) and (A-9), we have

$$\mu_t = A^I o_{V,t}^I = A^I \left(\tau_V + \sum_{s=0}^t \tau_{\varepsilon,s} \right)^{-1}, \quad h_t = A^I / \tau_{\varepsilon,t}. \quad (\text{A-10})$$

We then verify that the coefficient of $\nu_t - \hat{V}_{t-1}^U$ in Equation (A-7) is indeed zero.

A.2 Proof of Proposition 1

We first solve for prices and optimal trading quantities for $t = T$. The expected utility functions for informed and uninformed traders at $t = T$ are

$$\mathbb{E}_T^I \left[-e^{-A^I W_{T+1}^{I,i}} \right] = -e^{-A^I \left[W_T^I + \theta_T^I (\hat{V}_T^I - P_T) + X_T (\hat{V}_T^I - \bar{V}) - \frac{1}{2} A^I o_{V,T}^I (\theta_T^I + X_T)^2 \right]}, \quad (\text{A-11})$$

$$\mathbb{E}_T^U \left[-e^{-A^U W_{T+1}^{U,j}} \right] = -e^{-A^U \left[W_T^U + \theta_T^U (\hat{V}_T^U - P_T) - \frac{1}{2} A^U o_{V,T}^U (\theta_T^U)^2 \right]}. \quad (\text{A-12})$$

The first order condition with respect to θ_T^I and θ_T^U yields

$$\theta_T^I = \frac{P_T^I - P_T}{\gamma_T^I}, \quad \theta_T^U = \frac{P_T^U - P_T}{\gamma_T^U}, \quad (\text{A-13})$$

where the non-participating prices for informed and uninformed traders are

$$P_T^I = \hat{V}_T^I - \mu_T X_T, \quad P_T^U = \hat{V}_T^U, \quad (\text{A-14})$$

and parameters μ_T , γ_T^I , and γ_T^U are given as

$$\mu_T = A^I o_{V,T}^I, \quad \gamma_T^I = A^I o_{V,T}^I, \quad \gamma_T^U = A^U o_{V,T}^U. \quad (\text{A-15})$$

It follows that the equilibrium price at T is

$$P_T = \omega_T P_T^I + (1 - \omega_T) P_T^U - f_T \Theta, \quad (\text{A-16})$$

where coefficients f_T and ω_T are given as

$$f_T = \left(\frac{\alpha}{\gamma_T^I} + \frac{1 - \alpha}{\gamma_T^U} \right)^{-1}, \quad \omega_T = \frac{\alpha}{\gamma_T^I} f_T. \quad (\text{A-17})$$

The equilibrium holdings are

$$\theta_T^I = -(1 - \omega_T) \hat{X}_T^U + \frac{f_T}{\gamma_T^I} \Theta = (H_{\theta,T}^I)^\top \Phi_T^I, \quad \theta_T^U = \frac{\omega_T \mu_T}{\gamma_T^U} \hat{X}_T^U + \frac{f_T}{\gamma_T^U} \Theta = (H_{\theta,T}^U)^\top \Phi_T^U,$$

where $H_{\theta,T}^I$ and $H_{\theta,T}^U$ are given as

$$H_{\theta,T}^I = \left(0, 0, -(1 - \omega_T), f_T / \gamma_T^I, 0 \right)^\top, \quad H_{\theta,T}^U = \left(0, \omega_T \mu_T / \gamma_T^U, f_T / \gamma_T^U, 0 \right)^\top. \quad (\text{A-18})$$

Traders' value functions at T can be rewritten as

$$J_T^I = -\rho_T^I e^{-A^I [W_T^I + \frac{1}{2} (\Phi_T^I)^\top M_T^I \Phi_T^I]}, \quad J_T^U = -\rho_T^U e^{-A^U [W_T^U + \frac{1}{2} (\Phi_T^U)^\top M_T^U \Phi_T^U]}, \quad (\text{A-19})$$

where $\rho_T^I = 1, \rho_T^U = 1, \Phi_T^I$ and Φ_T^U are defined as

$$\Phi_T^I := \left(\hat{V}_T^I, X_T, \hat{X}_T^U, \Theta, \bar{V} \right)^\top, \quad \Phi_T^U := \left(\hat{V}_T^U, \hat{X}_T^U, \Theta, \bar{V} \right)^\top, \quad (\text{A-20})$$

and

$$M_T^I = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & -\gamma_T^I & 0 & 0 & -1 \\ 0 & 0 & \gamma_T^I (1 - \omega_T)^2 & -(1 - \omega_T) f_T & 0 \\ 0 & 0 & -(1 - \omega_T) f_T & f_T^2 / \gamma_T^I & 0 \\ 0 & -1 & 0 & 0 & 0 \end{pmatrix}, \quad M_T^U = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{(\omega_T \mu_T)^2}{\gamma_T^U} & \frac{\omega_T \mu_T f_T}{\gamma_T^U} & 0 \\ 0 & \frac{\omega_T \mu_T f_T}{\gamma_T^U} & \frac{f_T^2}{\gamma_T^U} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The equilibrium price can be expressed as

$$P_T = (H_{P,T}^I)^\top \Phi_T^I = (H_{P,T}^U)^\top \Phi_T^U, \quad (\text{A-21})$$

where $H_{P,T}^I$ and $H_{P,T}^U$ are given as

$$H_{P,T}^I = \left(1, -\mu_T, (1 - \omega_T) \mu_T, -f_T, 0 \right)^\top, \quad H_{P,T}^U = \left(1, -\omega_T \mu_T, -f_T, 0 \right)^\top. \quad (\text{A-22})$$

Next, we conjecture that traders' value functions at t are given as (A-19) where the subscript T is replaced by t . Traders' state vectors Φ_t^I and Φ_t^U are defined as (A-20) where the subscript T is replaced by t .

Traders' holdings are

$$\theta_t^I = \frac{P_t^I - P_t}{\gamma_t^I}, \quad \theta_t^U = \frac{P_t^U - P_t}{\gamma_t^U}, \quad (\text{A-23})$$

where P_t^I and P_t^U are informed and uninformed traders' non-participation prices. We also conjecture that traders' non-participation prices at t are

$$\begin{aligned} P_t^I &= \delta_t^I(\hat{V}_t^I - \mu_t X_t) + (1 - \delta_t^I)\bar{V} + g_t^I \hat{X}_t^U - f_t^I \Theta, \\ P_t^U &= \delta_t^U \hat{V}_t^U + (1 - \delta_t^U)\bar{V} - g_t^U \hat{X}_t^U - f_t^U \Theta, \end{aligned} \quad (\text{A-24})$$

equation (A-14) implies that the coefficients at T , $\delta_T^I, \delta_T^U, g_T^I, g_T^U, f_T^I$, and f_T^U are

$$\delta_T^I = \delta_T^U = 1, \quad g_T^I = g_T^U = 0, \quad f_T^I = f_T^U = 0, \quad (\text{A-25})$$

The equilibrium price can be written as

$$\begin{aligned} P_t &= \omega_t P_t^I + (1 - \omega_t) P_t^U - f_t \Theta, \\ &= (H_{P,t}^I)^\top \Phi_t^I = (H_{P,t}^U)^\top \Phi_t^U, \end{aligned} \quad (\text{A-26})$$

where

$$f_t = \left(\frac{\alpha}{\gamma_t^I} + \frac{1 - \alpha}{\gamma_t^U} \right)^{-1}, \quad \omega_t = \frac{\alpha}{\gamma_t^I} f_t, \quad 1 - \omega_t = \frac{1 - \alpha}{\gamma_t^U} f_t, \quad (\text{A-27})$$

and

$$H_{P,t}^I = \begin{pmatrix} \omega_t \delta_t^I + (1 - \omega_t) \delta_t^U \\ -\mu_t [\omega_t \delta_t^I + (1 - \omega_t) \delta_t^U] \\ \omega_t g_t^I + (1 - \omega_t) (\delta_t^U \mu_t - g_t^U) \\ -(\omega_t f_t^I + (1 - \omega_t) f_t^U + f_t) \\ 1 - [\omega_t \delta_t^I + (1 - \omega_t) \delta_t^U] \end{pmatrix}, \quad H_{P,t}^U = \begin{pmatrix} \omega_t \delta_t^I + (1 - \omega_t) \delta_t^U \\ \omega_t (g_t^I - \delta_t^I \mu_t) - (1 - \omega_t) g_t^U \\ -(\omega_t f_t^I + (1 - \omega_t) f_t^U + f_t) \\ 1 - [\omega_t \delta_t^I + (1 - \omega_t) \delta_t^U] \end{pmatrix}. \quad (\text{A-28})$$

The equilibrium holdings can be written as

$$\theta_t^I = (H_{\theta,t}^I)^\top \Phi_t^I, \quad \theta_t^U = (H_{\theta,t}^U)^\top \Phi_t^U, \quad (\text{A-29})$$

where traders' state vectors Φ_t^I and Φ_t^U are defined as (A-20) where the subscript T is replaced

by t . The coefficient vectors $H_{\theta,t}^I$ and $H_{\theta,t}^U$ are given as

$$H_{\theta,t}^I := \begin{pmatrix} \frac{1-\omega_t}{\gamma_t^I}(\delta_t^I - \delta_t^U) \\ -\frac{(1-\omega_t)\mu_t}{\gamma_t^I}(\delta_t^I - \delta_t^U) \\ \frac{1-\omega_t}{\gamma_t^I}(g_t^I + g_t^U - \delta_t^U \mu_t) \\ -\frac{1-\omega_t}{\gamma_t^I}(f_t^I - f_t^U) + \frac{f_t}{\gamma_t^I} \\ -\frac{1-\omega_t}{\gamma_t^I}(\delta_t^I - \delta_t^U) \end{pmatrix}, \quad H_{\theta,t}^U := \begin{pmatrix} -\frac{\omega_t}{\gamma_t^U}(\delta_t^I - \delta_t^U) \\ -\frac{\omega_t}{\gamma_t^U}(g_t^U + g_t^I - \delta_t^I \mu_t) \\ \frac{\omega_t}{\gamma_t^U}(f_t^I - f_t^U) + \frac{f_t}{\gamma_t^U} \\ \frac{\omega_t}{\gamma_t^U}(\delta_t^I - \delta_t^U) \end{pmatrix}. \quad (\text{A-30})$$

Next, we solve the model recursively from time t to $t-1$ and verify our conjecture. We first solve the uninformed traders' optimization problem at $t-1$. Since

$$\Phi_t^U = \Phi_{t-1}^U + F_t^U e_t^U, \quad (\text{A-31})$$

where $e_t^U = v_t - h_t \eta_t - \hat{V}_{t-1}^U$, $F_t^U = (K_{V,t}^U, K_{X,t}^U, 0, 0)^T$, $K_{V,t}^U$, and $K_{X,t}^U$ are as defined in equation (A-6).

Uninformed traders' value function (A-19) at t can be rewritten as

$$\begin{aligned} J_t^U &= -\rho_t^U e^{-A^U [W_{t-1}^U - \theta_{t-1}^U P_{t-1} + \theta_{t-1}^U P_t + \frac{1}{2}(\Phi_t^U)^\top M_t^U \Phi_t^U]}, \\ &= -\rho_t^U e^{-A^U [W_{t-1}^U - \theta_{t-1}^U P_{t-1} + \theta_{t-1}^U (H_{P,t}^U)^\top \Phi_{t-1}^U + \frac{1}{2}(\Phi_{t-1}^U)^\top M_t^U \Phi_{t-1}^U]} \\ &\quad \times e^{-A^U [\left(\theta_{t-1}^U (H_{P,t}^U)^\top F_t^U + (\Phi_{t-1}^U)^\top M_t^U F_t^U\right) e_t^U + \frac{1}{2} e_t^U (F_t^U)^\top M_t^U F_t^U e_t^U]}. \end{aligned} \quad (\text{A-32})$$

The expectation at $t-1$ is

$$\begin{aligned} E_{t-1}^U [J_t^U] &= -\rho_t^U e^{-A^U [W_{t-1}^U - \theta_{t-1}^U P_{t-1} + \theta_{t-1}^U (H_{P,t}^U)^\top \Phi_{t-1}^U + \frac{1}{2}(\Phi_{t-1}^U)^\top M_t^U \Phi_{t-1}^U]} \\ &\quad \times \sqrt{\frac{\Xi_t^U}{\Sigma_t^U}} e^{-A^U \left[-\frac{1}{2} A^U \Xi_t^U \left(\theta_{t-1}^U (H_{P,t}^U)^\top F_t^U + (\Phi_{t-1}^U)^\top M_t^U F_t^U \right)^2 \right]}, \end{aligned} \quad (\text{A-33})$$

where

$$\Xi_t^U := [(\Sigma_t^U)^{-1} + A^U (F_t^U)^\top M_t^U F_t^U]^{-1}. \quad (\text{A-34})$$

The first order condition with respect to θ_{t-1}^U yields

$$-P_{t-1} + (H_{P,t}^U)^\top \Phi_{t-1}^U = A^U \Xi_t^U (H_{P,t}^U)^\top F_t^U (F_t^U)^\top \left[H_{P,t}^U \theta_{t-1}^U + M_t^U \Phi_{t-1}^U \right], \quad (\text{A-35})$$

and the second order condition requires

$$A^U \Xi_t^U \left[(H_{P,t}^U)^\top F_t^U \right]^2 > 0, \quad (\text{A-36})$$

which is satisfied. Equation (A-35) implies that the non-participation price for uninformed traders is

$$P_{t-1}^U = \left[(H_{P,t}^U)^\top - A^U \Xi_t^U (H_{P,t}^U)^\top F_t^U (F_t^U)^\top M_t^U \right] \Phi_{t-1}^U. \quad (\text{A-37})$$

Equation (A-37) implies that parameter γ_{t-1}^U , and parameters $\delta_{t-1}^U, g_{t-1}^U, f_{t-1}^U$ in

$$P_{t-1}^U = \delta_{t-1}^U \hat{V}_{t-1}^U + (1 - \delta_{t-1}^U) \bar{V} - g_{t-1}^U \hat{X}_{t-1}^U - f_{t-1}^U \Theta,$$

can be written as

$$\begin{aligned} \gamma_{t-1}^U &= A^U \Xi_t^U \left[(H_{P,t}^U)^\top F_t^U \right]^2, \\ \delta_{t-1}^U &= \left[(H_{P,t}^U)^\top - A^U \Xi_t^U (H_{P,t}^U)^\top F_t^U (F_t^U)^\top M_t^U \right]_{1,1}, \\ g_{t-1}^U &= - \left[(H_{P,t}^U)^\top - A^U \Xi_t^U (H_{P,t}^U)^\top F_t^U (F_t^U)^\top M_t^U \right]_{1,2}, \\ f_{t-1}^U &= - \left[(H_{P,t}^U)^\top - A^U \Xi_t^U (H_{P,t}^U)^\top F_t^U (F_t^U)^\top M_t^U \right]_{1,3}, \end{aligned} \quad (\text{A-38})$$

and it can be verified that

$$1 - \delta_{t-1}^U = \left[(H_{P,t}^U)^\top - A^U \Xi_t^U (H_{P,t}^U)^\top F_t^U (F_t^U)^\top M_t^U \right]_{1,4}.$$

Since $\hat{X}_{t-1}^U = \frac{1}{\mu_{t-1}} \left(\hat{V}_{t-1}^U - (\hat{V}_{t-1}^I - \mu_{t-1} X_{t-1}) \right)$, the non-participation price for uninformed traders can thus be rewritten as in Equation (13) in Proposition 1. The coefficients ξ_{t-1}^U and ψ_{t-1}^U in P_{t-1}^U are

$$\xi_{t-1}^U := g_{t-1}^U / \mu_{t-1}, \quad \psi_{t-1}^U := \delta_{t-1}^U - g_{t-1}^U / \mu_{t-1}. \quad (\text{A-39})$$

Next, we solve for the optimization problem for informed traders at $t - 1$. Since

$$\Phi_t^I = H_t^I \Phi_{t-1}^I + F_t^I u_t^I, \quad (\text{A-40})$$

where

$$H_t^I = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & K_{X,t}^U \mu_{t-1} & 1 - K_{X,t}^U \mu_{t-1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad F_t^I = \begin{pmatrix} K_{V,t}^I & 0 \\ 0 & 1 \\ K_{X,t}^U & -K_{X,t}^U h_t \\ 0 & 0 \\ 0 & 0 \end{pmatrix}. \quad (\text{A-41})$$

Informed traders' value function at t can be rewritten as

$$\begin{aligned} J_t^I &= -\rho_t^I e^{-A^I [W_{t-1}^I - \theta_{t-1}^I P_{t-1} + \theta_{t-1}^I P_t + \frac{1}{2} (\Phi_t^I)^\top M_t^I \Phi_t^I]} \\ &= -\rho_t^I e^{-A^I [W_{t-1}^I - \theta_{t-1}^I P_{t-1} + \theta_{t-1}^I (H_{P,t}^I)^\top H_t^I \Phi_{t-1}^I + \frac{1}{2} (H_t^I \Phi_{t-1}^I)^\top M_t^I H_t^I \Phi_{t-1}^I]} \\ &\quad \times e^{-A^I [\left(\theta_{t-1}^I (H_{P,t}^I)^\top F_t^I + (H_t^I \Phi_{t-1}^I)^\top M_t^I F_t^I \right) u_t^I + \frac{1}{2} u_t^I (F_t^I)^\top M_t^I F_t^I u_t^I]}. \end{aligned} \quad (\text{A-42})$$

The expectation at $t-1$ is

$$\begin{aligned} E_{t-1}^I [J_t^I] &= -\rho_t^I e^{-A^I [W_{t-1}^I - \theta_{t-1}^I P_{t-1} + \theta_{t-1}^I (H_{P,t}^I)^\top H_t^I \Phi_{t-1}^I + \frac{1}{2} (H_t^I \Phi_{t-1}^I)^\top M_t^I H_t^I \Phi_{t-1}^I]} \\ &\quad \times \sqrt{\frac{|\Xi_t^I|}{|\Sigma_t^I|}} e^{-A^I \left[-\frac{1}{2} A^I \left(\theta_{t-1}^I (H_{P,t}^I)^\top F_t^I + (H_t^I \Phi_{t-1}^I)^\top M_t^I F_t^I \right) \Xi_t^I \left(\theta_{t-1}^I (H_{P,t}^I)^\top F_t^I + (H_t^I \Phi_{t-1}^I)^\top M_t^I F_t^I \right)^\top \right]}, \end{aligned}$$

where $\Xi_t^I = [(\Sigma_t^I)^{-1} + A^I (F_t^I)^\top M_t^I F_t^I]^{-1}$.

The first order condition with respect to θ_{t-1}^I yields

$$-P_{t-1} + (H_{P,t}^I)^\top H_t^I \Phi_{t-1}^I = A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top [H_{P,t}^I \theta_{t-1}^I + M_t^I H_t^I \Phi_{t-1}^I], \quad (\text{A-43})$$

and the second order condition requires

$$A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top H_{P,t}^I > 0, \quad (\text{A-44})$$

which is satisfied under certain conditions. Equation (A-43) implies that the non-participation price for informed traders is

$$P_{t-1}^I = ((H_{P,t}^I)^\top - A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top M_t^I) H_t^I \Phi_{t-1}^I. \quad (\text{A-45})$$

Equation (A-45) implies that parameter γ_{t-1}^I , and parameters $\delta_{t-1}^I, g_{t-1}^I, f_{t-1}^I$ in

$$P_{t-1}^I = \delta_{t-1}^I (\hat{V}_{t-1}^I - \mu_{t-1} X_{t-1}) + (1 - \delta_{t-1}^I) \bar{V} + g_{t-1}^I \hat{X}_{t-1}^U - f_{t-1}^I \Theta,$$

are given as

$$\begin{aligned}
\gamma_{t-1}^I &= A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top H_{P,t}^I, \\
\delta_{t-1}^I &= [((H_{P,t}^I)^\top - A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top M_t^I) H_t^I]_{1,1}, \\
g_{t-1}^I &= [((H_{P,t}^I)^\top - A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top M_t^I) H_t^I]_{1,3}, \\
f_{t-1}^I &= -[((H_{P,t}^I)^\top - A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top M_t^I) H_t^I]_{1,4},
\end{aligned} \tag{A-46}$$

and it can be verified that

$$\begin{aligned}
1 - \delta_{t-1}^I &= [((H_{P,t}^I)^\top - A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top M_t^I) H_t^I]_{1,5}, \\
-\mu_{t-1} \delta_{t-1}^I &= [((H_{P,t}^I)^\top - A^I (H_{P,t}^I)^\top F_t^I \Xi_t^I (F_t^I)^\top M_t^I) H_t^I]_{1,2}.
\end{aligned}$$

Since $\hat{X}_{t-1}^U = \frac{1}{\mu_{t-1}} (\hat{V}_{t-1}^U - (\hat{V}_{t-1}^I - \mu_{t-1} X_{t-1}))$, the non-participation price for informed traders can thus be rewritten as in Equation (13) in Proposition 1. The coefficients ξ_{t-1}^I and ψ_{t-1}^I in P_{t-1}^I are

$$\xi_{t-1}^I := \delta_{t-1}^I - g_{t-1}^I / \mu_{t-1}, \quad \psi_{t-1}^I := g_{t-1}^I / \mu_{t-1}. \tag{A-47}$$

Define $H_{P,t-1}^I, H_{P,t-1}^U, H_{\theta,t-1}^I$, and $H_{\theta,t-1}^U$ as in equations (A-28) and (A-30), replacing t by $t-1$, we can rewrite the equilibrium price and holdings as

$$P_{t-1} = (H_{P,t-1}^I)^\top \Phi_{t-1}^I = (H_{P,t-1}^U)^\top \Phi_{t-1}^U, \tag{A-48}$$

$$\theta_{t-1}^I = (H_{\theta,t-1}^I)^\top \Phi_{t-1}^I, \quad \theta_{t-1}^U = (H_{\theta,t-1}^U)^\top \Phi_{t-1}^U. \tag{A-49}$$

Therefore, the value functions of traders at $t-1$ can be expressed as

$$J_{t-1}^I = -\rho_{t-1}^I e^{-A^I [W_{t-1}^I + \frac{1}{2} (\Phi_{t-1}^I)^\top M_{t-1}^I \Phi_{t-1}^I]}, \quad J_{t-1}^U = -\rho_{t-1}^U e^{-A^U [W_{t-1}^U + \frac{1}{2} (\Phi_{t-1}^U)^\top M_{t-1}^U \Phi_{t-1}^U]},$$

where

$$\begin{aligned}
M_{t-1}^I &= (H_t^I)^\top M_t^U H_t^I + H_{\theta,t-1}^I ((H_t^I)^\top H_{P,t}^I - H_{P,t-1}^I)^\top + \left((H_t^I)^\top H_{P,t}^U - H_{P,t-1}^I \right) (H_{\theta,t-1}^I)^\top \\
&\quad - A^I \left[H_{\theta,t-1}^I (H_{P,t}^I)^\top + (H_t^I)^\top M_t^I \right] F_t^I \Xi_t^I (F_t^I)^\top \left[H_{\theta,t-1}^I (H_{P,t}^I)^\top + (H_t^I)^\top M_t^I \right]^\top,
\end{aligned} \tag{A-50}$$

$$\begin{aligned}
M_{t-1}^U &= M_t^U + H_{\theta,t-1}^U \left(H_{P,t}^U - H_{P,t-1}^U \right)^\top + \left(H_{P,t}^U - H_{P,t-1}^U \right) (H_{\theta,t-1}^U)^\top \\
&\quad - A^U \left[H_{\theta,t-1}^U (H_{P,t}^U)^\top + M_t^U \right] F_t^U \Xi_t^U (F_t^U)^\top \left[H_{\theta,t-1}^U (H_{P,t}^U)^\top + M_t^U \right]^\top,
\end{aligned} \tag{A-51}$$

and

$$\rho_{t-1}^I = \rho_t^I \sqrt{\frac{|\Xi_t^I|}{|\Sigma_t^I|}}, \quad \rho_{t-1}^U = \rho_t^U \sqrt{\frac{|\Xi_t^U|}{|\Sigma_t^U|}}. \quad (\text{A-52})$$

The equilibrium price can be written as

$$P_t = \delta_t \hat{V}_t^U + (1 - \delta_t) \bar{V} - g_t \hat{X}_t^U - \zeta_t \Theta, \quad (\text{A-53})$$

where the coefficients δ_t , g_t , and ζ_t are given as

$$\delta_t = \omega_t \delta_t^I + (1 - \omega_t) \delta_t^U, \quad g_t = -\omega_t (g_t^I - \delta_t^I \mu_t) + (1 - \omega_t) g_t^U, \quad \zeta_t = \omega_t f_t^I + (1 - \omega_t) f_t^U + f_t. \quad (\text{A-54})$$

Thus, the informed traders' holding can be expressed as

$$N_I \theta_t^I = d_t (\hat{V}_t^U - \bar{V}) + k_t \hat{X}_t^U - l_t \Theta, \quad (\text{A-55})$$

where the coefficients d_t , k_t , and l_t are given as

$$d_t := \frac{\omega_t}{\gamma_t^U / N_U} (\delta_t^I - \delta_t^U), \quad k_t := \frac{\omega_t}{\gamma_t^U / N_U} (g_t^I + g_t^U - \delta_t^I \mu_t), \quad l_t := \frac{\omega_t}{\gamma_t^U / N_U} (f_t^I - f_t^U) - \omega_t. \quad (\text{A-56})$$

A.3 Proof of Proposition 2

For $t \geq 2$, the return covariance has two components,

$$\text{Cov}(R_t, R_{t-1}) = \text{Cov}(\hat{V}_t - \hat{V}_{t-1}, \hat{V}_{t-1} - \hat{V}_{t-2}) + \omega^2 \text{Cov}((\mu_{t-1} - \mu_t) x_0, (\mu_{t-2} - \mu_{t-1}) x_0).$$

The first component is driven by changes in the agent's belief about the fundamental value,

$$\text{Cov}(\hat{V}_t - \hat{V}_{t-1}, \hat{V}_{t-1} - \hat{V}_{t-2}) = 0, \quad (\text{A-57})$$

which is always zero because the agent's expectation about the fundamental can be iterated $E_{t-1}[\hat{V}_t] = \hat{V}_{t-1}$. The second component is driven by the reduction in the market's required risk premium due to the arrival of public information, for $t \geq 2$,

$$\text{Cov}((\mu_{t-1} - \mu_t) x_0, (\mu_{t-2} - \mu_{t-1}) x_0) = (\mu_{t-1} - \mu_t)(\mu_{t-2} - \mu_{t-1}) \tau_{x,0}^{-1}. \quad (\text{A-58})$$

With the arrival of public information at both time $t - 1$ and t , we have $\mu_{t-2} > \mu_{t-1} > \mu_t$. For $t = 1$, $\text{Cov}(R_1, R_0) = \omega^2 (\mu_1 - \mu_0) \mu_0 \text{Var}(x_0) < 0$.

A.4 Proof of Corollary 1

Write $R_t = \ell_t(x_0 + \frac{\Theta}{\alpha}) + u_t$ with $\ell_t = \omega(\mu_{t-1} - \mu_t)$, as in Proposition 2. The u_t are Kalman forecast errors, so $E[u_t] = 0$, $E[u_t u_{t-1}] = 0$, and u_t is independent of x_0 . Conditional on $|x_0| = w$, symmetry of x_0 gives $E[x_0 \mid |x_0| = w] = 0$ and $E[x_0^2 \mid |x_0| = w] = w^2$. Expanding the product and applying these,

$$E[R_t R_{t-1} \mid |x_0| = w] = \ell_t \ell_{t-1} E[(x_0 + \frac{\Theta}{\alpha})^2 \mid |x_0| = w] = \ell_t \ell_{t-1} (w^2 + (\frac{\Theta}{\alpha})^2) > 0.$$

Since $E[R_t \mid |x_0| = w] = \ell_t \frac{\Theta}{\alpha}$, the conditional covariance is $\ell_t \ell_{t-1} w^2$, while $\text{Var}(R_{t-1} \mid |x_0| = w) = \ell_{t-1}^2 w^2 + \sigma_{u,t-1}^2$. Their ratio is $\rho(w)$. Differentiating,

$$\rho'(w) = \frac{2\ell_t \ell_{t-1} \sigma_{u,t-1}^2 w}{(\ell_{t-1}^2 w^2 + \sigma_{u,t-1}^2)^2} > 0 \quad \text{for } w > 0,$$

and, since $\ell_{t-1} > 0$ implies $\sigma_{u,t-1}^2 > 0$, $\rho(0) = 0$.

A.5 Proof of Proposition 3

As in the proof of Proposition 1 in Section A.2, we first solve for prices and optimal trading quantities for $t = T$. The expressions of prices and optimal holdings are the same as in the general case in Proposition 1.

$$\theta_T^I = \frac{P_T^I - P_T}{\gamma_T^I}, \quad \theta_T^U = \frac{P_T^U - P_T}{\gamma_T^U}, \quad (\text{A-59})$$

where the non-participating prices for informed and uninformed traders are

$$P_T^I = \hat{V}_T^I - \mu_T X_T, \quad P_T^U = \hat{V}_T^U, \quad (\text{A-60})$$

and parameters μ_T , γ_T^I , and γ_T^U are given as

$$\mu_T = A^I o_{V,T}^I, \quad \gamma_T^I = A^I o_{V,T}^I, \quad \gamma_T^U = A^U o_{V,T}^U. \quad (\text{A-61})$$

It follows that the equilibrium price at T is

$$P_T = \omega_T P_T^I + (1 - \omega_T) P_T^U - \left(\frac{\alpha}{\gamma_T^I} + \frac{1 - \alpha}{\gamma_T^U} \right)^{-1} \Theta, \quad (\text{A-62})$$

where ω_T is given as

$$\omega_T = \frac{\frac{\alpha}{\gamma_T^I}}{\frac{\alpha}{\gamma_T^I} + \frac{1-\alpha}{\gamma_T^U}}. \quad (\text{A-63})$$

The equilibrium holdings are

$$\theta_T^I = -(1 - \omega_T) \hat{X}_T^U + \frac{\omega_T}{\alpha} \Theta, \quad \theta_T^U = \frac{\alpha}{1 - \alpha} (1 - \omega_T) \left(\hat{X}_T^U + \frac{\Theta}{\alpha} \right).$$

Since $\hat{V}_T^I - \mu_T X = \hat{V}_T^U - \mu_T \hat{X}_T^U$, the price P_T can be rewritten as

$$P_T = \hat{V}_T^U - \omega_T \mu_T \left(\hat{X}_T^U + \frac{\Theta}{\alpha} \right). \quad (\text{A-64})$$

Plug the equilibrium price into the expected utility functions, we have informed traders' value function at $t = T$ as a function of state variable Φ_T

$$J_T^{I,i} = -e^{-A^I \left[W_T^{I,i} + \hat{V}_T^I X + \frac{1}{2} \mu_T (\theta_T^{I,i})^2 - \bar{V} X - \frac{1}{2} \mu_T X^2 \right]}, \quad (\text{A-65})$$

which can be rewritten as

$$J_T^{I,i} = -\rho_T^I e^{-A^I \left[W_T^{I,i} + \hat{V}_T^I X + \frac{1}{2} m_T^I \phi_T^2 + \frac{1}{2} \mu_T (1 - \omega_T) (\hat{X}_T^U)^2 \right]}, \quad (\text{A-66})$$

where

$$m_T^I = -\mu_T \omega_T (1 - \omega_T) = -\mu_T \omega_T \frac{f_T}{N_I}, \quad (\text{A-67})$$

$$\rho_T^I = e^{A^I \left[\bar{V} X + \frac{1}{2} \mu_T X^2 - \frac{1}{2} \mu_T \omega_T \left(\frac{\Theta}{N_I} \right)^2 \right]}. \quad (\text{A-68})$$

Similarly, uninformed traders' value function at $t = T$ can be written as:

$$J_T^{U,j} = -e^{-A^U \left[W_T^{U,j} + \frac{1}{2} \gamma_T^U (\theta_T^{U,j})^2 \right]} = -\rho_T^U e^{-A^U \left[W_T^{U,j} + \frac{1}{2} m_T^U \phi_T^2 \right]}, \quad (\text{A-69})$$

where

$$m_T^U = \frac{N_I}{N_U} \mu_T \omega_T (1 - \omega_T) = \mu_T \omega_T \frac{f_T}{N_U}, \quad \rho_T^U = 1. \quad (\text{A-70})$$

Note that $N_I m_T^I + N_U m_T^U = 0$. We need to prove whether these forms of value functions are valid for time $t - 1$ if they are held for time t . For uninformed traders,

$$\hat{V}_t^U = \hat{V}_{t-1}^U + K_{V,t}^U e_t^U, \quad \phi_t = \phi_{t-1} + K_{X,t}^U e_t^U, \quad (\text{A-71})$$

$$P_t - P_{t-1} = a_t \phi_{t-1} + b_t e_t^U, \quad (\text{A-72})$$

where

$$a_t = \omega_{t-1} \mu_{t-1} - \omega_t \mu_t, \quad b_t = K_{V,t}^U - \omega_t \mu_t K_{X,t}^U, \quad (\text{A-73})$$

and

$$J_t^{U,j} = -\rho_t^U e^{-A^U} \left[W_{t-1}^{U,j} + \theta_{t-1}^{U,j} (P_t - P_{t-1}) + \frac{1}{2} m_t^U \phi_t^2 \right] \quad (\text{A-74})$$

the expected value function can be rewritten as

$$E_{t-1}[J_t^{U,j}] = -\rho_t^U \sqrt{\Xi_t^U / \Sigma_t^U} e^{-A^U} \left[W_{t-1}^{U,j} + a_t \phi_{t-1} \theta_{t-1}^{U,j} + \frac{1}{2} m_t^U \phi_{t-1}^2 - \frac{1}{2} A^U \Xi_t^U \left(m_t^U K_{X,t}^U \phi_{t-1} + b_t \theta_{t-1}^{U,j} \right)^2 \right], \quad (\text{A-75})$$

where

$$\Sigma_t^U = o_{V,t-1}^U + \tau_{\varepsilon,t}^{-1}, \quad \Xi_t^U = \left((\Sigma_t^U)^{-1} + A^U m_t^U (K_{X,t}^U)^2 \right)^{-1}. \quad (\text{A-76})$$

The first order condition with respect to $\theta_{t-1}^{U,j}$ gives

$$\theta_{t-1}^{U,j} = \frac{f_{t-1}}{N_U} \phi_{t-1}, \quad (\text{A-77})$$

where

$$f_{t-1} = N_U \left(\frac{a_t}{A^U \Xi_t^U b_t^2} - \frac{m_t^U K_{X,t}^U}{b_t} \right). \quad (\text{A-78})$$

The value function at time $t - 1$ can be written as

$$J_{t-1}^{U,j} = -\rho_{t-1}^U e^{-A^U} \left[W_{t-1}^{U,j} + \frac{1}{2} m_{t-1}^U \phi_{t-1}^2 \right], \quad (\text{A-79})$$

where

$$\rho_{t-1}^U = \rho_t^U \sqrt{\Xi_t^U / \Sigma_t^U}, \quad m_{t-1}^U = m_t^U \left(1 - \frac{a_t K_{X,t}^U}{b_t} \right) + \frac{f_{t-1}}{N_U} a_t. \quad (\text{A-80})$$

The second order condition requires that $A^U \Xi_t^U b_t > 0$.

For informed traders, since

$$\hat{V}_t^I - \mu_t X = \hat{V}_t^U - \mu_t \hat{X}_t^U, \quad (\text{A-81})$$

and

$$\hat{V}^I = \hat{V}_{t-1}^I + K_t^I e_t^I, \quad (\text{A-82})$$

$$\phi_t = \phi_{t-1} + K_{X,t}^U \mu_{t-1} (X - \hat{X}_{t-1}^U) + K_{X,t}^U e_t^I, \quad (\text{A-83})$$

thus,

$$P_t - P_{t-1} = a_t \phi_{t-1} + b_t \mu_{t-1} (X - \hat{X}_{t-1}^U) + b_t e_t^I. \quad (\text{A-84})$$

Since

$$J_t^{I,i} = -\rho_t^I e^{-A^I \left[W_{t-1}^{I,i} + \theta_{t-1}^{I,i} (P_t - P_{t-1}) + \hat{V}_t^I X + \frac{1}{2} m_t^I \phi_t^2 + \frac{1}{2} \mu_t (1 - \omega_t) (\hat{X}_t^U)^2 \right]} \quad (\text{A-85})$$

the expected value function can be rewritten as

$$\begin{aligned} E_{t-1}^I [J_t^{I,i}] &= -\rho_t^I \sqrt{\Xi_t^I / \Sigma_t^I} e^{-A^I \left[W_{t-1}^{I,i} + \hat{V}_{t-1}^I X + \theta_{t-1}^{I,i} (a_t \phi_{t-1} + b_t \mu_{t-1} (X - \hat{X}_{t-1}^U)) \right]} \\ &\quad \times e^{-A^I \left[\frac{1}{2} m_t^I (\phi_{t-1} + K_{X,t}^U \mu_{t-1} (X - \hat{X}_{t-1}^U))^2 + \frac{1}{2} \mu_t (1 - \omega_t) (\hat{X}_{t-1}^U + K_{X,t}^U \mu_{t-1} (X - \hat{X}_{t-1}^U))^2 \right]} \\ &\quad \times e^{-A^I \left[-\frac{1}{2} A^I \Xi_t^I \left[\theta_{t-1}^{I,i} b_t + K_t^I X + m_t^I K_{X,t}^U (\phi_{t-1} + K_{X,t}^U \mu_{t-1} (X - \hat{X}_{t-1}^U)) + \mu_t (1 - \omega_t) (\hat{X}_{t-1}^U + K_{X,t}^U \mu_{t-1} (X - \hat{X}_{t-1}^U)) \right]^2 \right]}, \end{aligned} \quad (\text{A-86})$$

where

$$\Sigma_t^I = o_{V,t-1}^I + \tau_{\varepsilon,t}^{-1}, \quad \Xi_t^I = \left((\Sigma_t^I)^{-1} + A^I (K_{X,t}^U)^2 (m_t^I + \mu_t (1 - \omega_t)) \right)^{-1}. \quad (\text{A-87})$$

The first order condition with respect to $\theta_{t-1}^{I,i}$ gives

$$\theta_{t-1}^{I,i} = \left(\frac{a_t}{A^I \Xi_t^I b_t^2} - \frac{m_t^I K_{X,t}^U}{b_t} \right) \phi_{t-1} - \hat{X}_{t-1}^U = \frac{\Theta}{N_I} - \frac{f_{t-1}}{N_I} \phi_{t-1}, \quad (\text{A-88})$$

which is due to the fact that

$$\mu_{t-1} = \frac{A^I \Xi_t^I K_t^I}{1 - A^I \Xi_t^I (K_{X,t}^U)^2 (m_t^I + \mu_t (1 - \omega_t))} = A^I \Sigma_t^I K_t^I = A^I o_{V,t-1}^I, \quad (\text{A-89})$$

and

$$K_t^I = K_{V,t}^U - \mu_t K_{X,t}^U, \quad K_t^I \mu_{t-1} = \mu_{t-1} - \mu_t. \quad (\text{A-90})$$

The second order condition requires that $A^I \Xi_t^I b_t > 0$. In equilibrium, the market clearing con-

dition gives

$$1 - \frac{f_{t-1}}{N_I} = 1 - \frac{N_U}{N_I} \left(\frac{a_t}{A^U \Xi_t^U b_t^2} - \frac{m_t^U K_{X,t}^U}{b_t} \right) = \frac{a_t}{A^I \Xi_t^I b_t^2} - \frac{m_t^I K_{X,t}^U}{b_t}, \quad (\text{A-91})$$

which determines a_t , so ω_{t-1} can be determined.

After plugging in the equilibrium holdings, the informed traders' value function at time $t-1$ and the parameter m_{t-1}^I can be expressed as

$$J_{t-1}^{I,i} = -\rho_{t-1}^I e^{-A^I \left[W_{t-1}^{I,i} + \hat{V}_{t-1}^I X + \frac{1}{2} m_{t-1}^I \phi_{t-1}^2 + \frac{1}{2} \mu_{t-1} (1 - \omega_{t-1}) (\hat{X}_{t-1}^U)^2 \right]}, \quad (\text{A-92})$$

where

$$\rho_{t-1}^I = \rho_t^I \sqrt{\Xi_t^I / \Sigma_t^I} e^{A^I \left[\frac{1}{2} (\mu_{t-1} - \mu_t) X^2 - \frac{1}{2} (\mu_{t-1} \omega_{t-1} - \mu_t \omega_t) \left(\frac{\theta}{N_I} \right)^2 \right]}, \quad (\text{A-93})$$

$$m_{t-1}^I = m_t^I \left(1 - \frac{a_t K_{X,t}^U}{b_t} \right) - \frac{f_{t-1}}{N_I} a_t. \quad (\text{A-94})$$

We have

$$N_I m_{t-1}^I + N_U m_{t-1}^U = (N_I m_t^I + N_U m_t^U) \left(1 - \frac{a_t K_{X,t}^U}{b_t} \right) = 0, \quad (\text{A-95})$$

since $N_I m_t^I + N_U m_t^U = 0$. By using the fact that $N_I m_t^I + N_U m_t^U = 0$, we have

$$a_t = \frac{N_I b_t^2}{\frac{N_I}{A^I \Xi_t^I} + \frac{N_U}{A^U \Xi_t^U}} = \frac{N_I \left(\omega_t K_{V,t}^I + (1 - \omega_t) K_{V,t}^U \right)^2}{\frac{N_I}{A^I \Sigma_t^I} + \frac{N_U}{A^U \Sigma_t^U} + \frac{N_I (1 - \omega_t)}{\mu_t} (K_{V,t}^U - K_t^I)^2}. \quad (\text{A-96})$$

Plug in the expression of ω_t , the recursively we have

$$\omega_t = \frac{N_I}{A^I o_{V,t}^I} \left(\frac{N_I}{A^I o_{V,t}^I} + \frac{N_U}{A^U o_{V,t}^U} \right)^{-1}. \quad (\text{A-97})$$

Plug ω_t into the recursive expressions of m_t^I and m_t^U , the we can obtain

$$m_t^I = -\mu_t \omega_t (1 - \omega_t), \quad m_t^U = \frac{N_I}{N_U} \mu_t \omega_t (1 - \omega_t). \quad (\text{A-98})$$

Plug m_t back to the formula of f_t , then we have $f_t = N_I (1 - \omega_t)$.

A.6 Proof of Proposition 4

Proof. Some parameters can be rewritten as

$$a_t = \omega_{t-1}\mu_{t-1} - \omega_t\mu_t = \frac{N_I}{\frac{N_I}{A^I} + \frac{N_U}{A^U}} \frac{\tau_{\varepsilon,t}}{(\tau_V + \bar{\tau} + \tau_{t-1})(\tau_V + \bar{\tau} + \tau_t)} > 0, \quad (\text{A-99})$$

for $t = 2, 3, \dots, T$, and where

$$\bar{\tau} = \frac{\frac{N_I}{A^I}\tau_\eta + \frac{N_U}{A^U}\tau_0}{\frac{N_I}{A^I} + \frac{N_U}{A^U}}, \quad \tau_t = \sum_{i=1}^t \tau_{\varepsilon,i}. \quad (\text{A-100})$$

And

$$\begin{aligned} b_t &= K_{V,t}^U - \omega_t\mu_t K_{X,t}^U = \omega_t K_t^I + (1 - \omega_t) K_{V,t}^U \\ &= \tau_{\varepsilon,t} \left(\omega_t o_{V,t}^I + (1 - \omega_t) o_{V,t}^U \right) = \frac{\tau_{\varepsilon,t}}{\tau_V + \tau_t + \bar{\tau}} > 0, \end{aligned} \quad (\text{A-101})$$

$$K_{X,t}^U = \frac{1}{\mu_t} \left(K_{V,t}^U - K_t^I \right) = \frac{\tau_{\varepsilon,t}}{\mu_t} \left(o_{V,t}^U - o_{V,t}^I \right) > 0, \quad (\text{A-102})$$

$$\Sigma_t^U = \frac{\tau_V + \tau_0 + \tau_t}{(\tau_V + \tau_0 + \tau_{t-1})\tau_{\varepsilon,t}} > 0, \quad (\text{A-103})$$

for $t = 1, 2, \dots, T$. Thus, price change can be rewritten as

$$\begin{aligned} P_{t+1} - P_t &= a_{t+1} \left(\hat{X}_t^U + \frac{\Theta}{N_I} \right) + b_{t+1} e_{t+1}^U \\ &= a_{t+1} \left(\hat{X}_{t-s}^U + \frac{\Theta}{N_I} \right) + a_{t+1} \sum_{i=0}^{s-1} K_{X,t-s}^U e_{t-i}^U + b_{t+1} e_{t+1}^U, \end{aligned} \quad (\text{A-104})$$

and with the lag $s = 1, 2, \dots, T-1$,

$$P_{t+1-s} - P_{t-s} = a_{t+1-s} \left(\hat{X}_{t-s}^U + \frac{\Theta}{N_I} \right) + b_{t+1-s} e_{t+1-s}^U. \quad (\text{A-105})$$

Therefore, by the law of total variance for $t = 1, 2, \dots, T$

$$\text{Var}(P_t - P_{t-1}) = a_t^2 \text{Var}(\hat{X}_{t-1}^U) + b_t^2 \Sigma_t^U. \quad (\text{A-106})$$

Similarly, by the law of total covariance,

$$\text{Cov}(P_{t+1} - P_t, P_{t+1-s} - P_{t-s}) = a_{t+1} \left(a_{t+1-s} \text{Var}(\hat{X}_{t-s}^U) + b_{t+1-s} K_{X,t+1-s}^U \Sigma_{t+1-s}^U \right) > 0, \quad (\text{A-107})$$

for $t = s, s+1, \dots, T-1$, and where

$$\begin{aligned}\text{Var}(\hat{X}_t^U) &= \sum_{s=1}^t \left(K_{X,s}^U\right)^2 \Sigma_s^U + \text{Var}(\hat{X}_0^U) \\ &= \left(\frac{\tau_\eta - \tau_0}{A^I}\right)^2 \frac{\tau_t}{(\tau_V + \tau_0)(\tau_V + \tau_0 + \tau_t)} + \text{Var}(\hat{X}_0^U).\end{aligned}\quad (\text{A-108})$$

Since

$$\hat{V}_0^U = \frac{\tau_0}{\tau_V + \tau_0}(V + \eta - hX) + \frac{\tau_V}{\tau_V + \tau_0} \bar{V}, \quad \hat{X}_0^U = -\frac{\tau_h/h}{\tau_X + \tau_h}(V + \eta - hX) + \frac{\tau_h/h}{\tau_X + \tau_h} \bar{V}, \quad (\text{A-109})$$

where

$$\tau_h = \frac{h^2}{\tau_V^{-1} + \tau_\eta^{-1}}, \quad (\text{A-110})$$

for $t = s-1$,

$$\begin{aligned}\text{Cov}(P_s - P_{s-1}, P_0) &= a_s \text{Cov}(\hat{X}_0^U, \hat{V}_0^U - \omega_0 \mu_0 \hat{X}_0^U) \\ &= -\frac{a_s \tau_h/h}{\tau_X + \tau_h} \left(\frac{\tau_0}{\tau_V + \tau_0} + \frac{\omega_0 \mu_0 \tau_h/h}{\tau_X + \tau_h} \right) \left(\frac{1}{\tau_V} + \frac{1}{\tau_\eta} + \frac{h^2}{\tau_X} \right) < 0.\end{aligned}\quad (\text{A-111})$$

Since

$$o_{V,T-s}^U - b_{T-s+1} \Sigma_{T-s+1}^U = \frac{\bar{\tau} - \tau_0}{(\tau_V + \tau_0 + \tau_{T-s})(\tau_V + \bar{\tau} + \tau_{T-s+1})} > 0, \quad (\text{A-112})$$

for $t = T$,

$$\text{Cov}(V - P_T, P_{T+1-s} - P_{T-s}) = a_{T+1-s} \omega_T \mu_T \text{Var}(\hat{X}_{T-s}^U) + b_{T+1-s} \left(o_{V,T-s}^U - b_{T+1-s} \Sigma_{T+1-s}^U \right) > 0. \quad (\text{A-113})$$

Therefore, we obtain the proposition $\kappa_{t,s} < 0$ for $t = s-1$ and $\kappa_{t,s} > 0$ for $t = s, s+1, \dots, T$. For $s = 1$, i.e., with one period lag

$$\begin{aligned}\kappa_{0,1} &= -\frac{a_1 \tau_h/h}{\tau_X + \tau_h} \sqrt{\frac{\tau_V^{-1} + \tau_\eta^{-1} + h^2 \tau_X^{-1}}{a_1^2 \text{Var}(\hat{X}_0^U) + b_1^2 \Sigma_1^U}} < 0, \\ \kappa_{t,1} &= \frac{a_{t+1} \left(a_t \text{Var}(\hat{X}_{t-1}^U) + b_t K_{X,t}^U \Sigma_t^U \right)}{\sqrt{a_{t+1}^2 \text{Var}(\hat{X}_t^U) + b_{t+1}^2 \Sigma_{t+1}^U} \sqrt{a_t^2 \text{Var}(\hat{X}_{t-1}^U) + b_t^2 \Sigma_t^U}} > 0, \quad \text{for } t = 1, 2, \dots, T-1 \\ \kappa_{T,1} &= \frac{a_T \omega_T \mu_T \text{Var}(\hat{X}_{T-1}^U) + b_T (o_{V,T-1}^U - b_T \Sigma_T^U)}{\sqrt{\omega_T^2 \mu_T^2 \text{Var}(\hat{X}_T^U) + o_{V,T}^U} \sqrt{a_T^2 \text{Var}(\hat{X}_{T-1}^U) + b_T^2 \Sigma_T^U}} > 0.\end{aligned}\quad (\text{A-114})$$

□

A.7 Proof of Proposition 5

The trading volume is a normally distributed,

$$N_U(\theta_t^U - \theta_{t-1}^U) = (f_t - f_{t-1}) \left(\hat{X}_{t-1}^U + \frac{\Theta}{N_I} \right) + f_t K_{X,t}^U e_t^U, \quad (\text{A-115})$$

with mean

$$\mu_{vol} = E[N_U(\theta_t^U - \theta_{t-1}^U)] = (f_t - f_{t-1}) \frac{\Theta}{N_I}, \quad (\text{A-116})$$

and variance for $t = 1, 2, \dots, T$ can be computed by the law of total variance

$$\begin{aligned} \text{Var}(N_U(\theta_t^U - \theta_{t-1}^U)) &= E \left[\text{Var}_{t-1}^U \left((f_t - f_{t-1}) \hat{X}_{t-1}^U + f_t K_{X,t}^U e_t^U \right) \right] \\ &\quad + \text{Var} \left(E_{t-1}^U \left[(f_t - f_{t-1}) \hat{X}_{t-1}^U + f_t K_{X,t}^U e_t^U \right] \right) \\ &= \left(f_t K_{X,t}^U \right)^2 \Sigma_t^U + (f_t - f_{t-1})^2 \text{Var}(\hat{X}_{t-1}^U), \end{aligned} \quad (\text{A-117})$$

where

$$\text{Var}(\hat{X}_{t-1}^U) = \sum_{s=1}^{t-1} \left(K_{X,s}^U \right)^2 \Sigma_s^U + \text{Var}(\hat{X}_0^U), \quad (\text{A-118})$$

can also be computed by the law of total variance. For $t = 0$,

$$\text{Var}(N_U(\theta_0^U - \theta_{-1}^U)) = f_0^2 \text{Var}(\hat{X}_0^U). \quad (\text{A-119})$$

Thus, the variance of the trading volume is

$$\sigma_{vol}^2 = \text{Var}(N_U(\theta_t^U - \theta_{t-1}^U)) = \left(f_t K_{X,t}^U \right)^2 \Sigma_t^U + (f_t - f_{t-1})^2 \left[\sum_{s=1}^{t-1} \left(K_{X,s}^U \right)^2 \Sigma_s^U + \text{Var}(\hat{X}_0^U) \right]. \quad (\text{A-120})$$

Therefore,

$$\text{Vol}_t = \sqrt{\frac{2}{\pi}} \sigma_{vol} e^{-\frac{\mu_{vol}^2}{2\sigma_{vol}^2}} + \mu_{vol} \text{erf} \left(\frac{\mu_{vol}}{\sqrt{2}\sigma_{vol}} \right), \quad (\text{A-121})$$

where $\text{erf}(\cdot)$ is the error function.

Therefore, the covariance of demands is

$$\begin{aligned} \text{Cov}(N_U(\theta_t^U - \theta_{t-1}^U), N_U(\theta_{t-1}^U - \theta_{t-2}^U)) \\ = f_{t-1}(f_t - f_{t-1})(K_{X,t-1}^U)^2 \Sigma_{t-1}^U + (f_t - f_{t-1})(f_{t-1} - f_{t-2}) \text{Var}(\hat{X}_{t-2}^U) > 0, \end{aligned} \quad (\text{A-122})$$

for $t = 2, 3, \dots, T$, and for $t = 1$,

$$\text{Cov}(N_U(\theta_1^U - \theta_0^U), N_U(\theta_0^U - \theta_{-1}^U)) = f_0(f_1 - f_0)\text{Var}(\hat{X}_0^U) > 0. \quad (\text{A-123})$$

In this simple information structure, the demand autocorrelation is positive, since $f_t > f_{t-1} > 0$.

A.8 Proof of Proposition 6

Proof. Since

$$P_t - P_{t-1} = a_t \left(\hat{X}_{t-1}^U + \frac{\Theta}{N_I} \right) + b_t e_t^U, \quad (\text{A-124})$$

and

$$\alpha(\theta_t^I - \theta_{t-1}^I) = -f_t K_{X,t}^U e_t^U - (f_t - f_{t-1}) \left(\hat{X}_{t-1}^U + \frac{\Theta}{N_I} \right), \quad (\text{A-125})$$

then by the law of total covariance,

$$\text{Cov}(P_t - P_{t-1}, \theta_t^I - \theta_{t-1}^I) = -(f_t - f_{t-1}) a_t \text{Var}(\hat{X}_{t-1}^U) / \alpha - f_t K_{X,t}^U b_t \Sigma_t^U / \alpha, \quad (\text{A-126})$$

and from Appendix A.7, we have,

$$\text{Var}(\alpha(\theta_t^I - \theta_{t-1}^I)) = (f_t - f_{t-1})^2 \text{Var}(\hat{X}_{t-1}^U) + \left(f_t K_{X,t}^U \right)^2 \Sigma_t^U. \quad (\text{A-127})$$

Since all the information is public after time zero,

$$\frac{1}{o_{V,t}^I} = \frac{1}{o_{V,t-1}^I} + \tau_{\varepsilon,t}, \quad \frac{1}{o_{V,t}^U} = \frac{1}{o_{V,t-1}^U} + \tau_{\varepsilon,t}, \quad K_t^I = o_{V,t}^I \tau_{\varepsilon,t}, \quad K_{V,t}^U = o_{V,t}^U \tau_{\varepsilon,t}, \quad (\text{A-128})$$

and $f_t = \alpha(1 - \omega_t)$, then we have

$$\frac{b_t}{a_t} = \frac{f_t K_{X,t}^U}{f_t - f_{t-1}}, \quad \frac{a_t}{f_t - f_{t-1}} = \left(\frac{A^I}{N_I} + \frac{A^U}{N_U} \right) \frac{1}{\tau_\eta - \tau_0} > 0. \quad (\text{A-129})$$

Therefore, $f_t - f_{t-1} > 0$, and

$$\lambda_t^I = \begin{cases} \frac{\frac{h\tau_0}{\tau_V + \tau_0} + \frac{\omega_0 \mu_0 \tau_h}{\tau_X + \tau_h}}{\frac{f_0 \tau_h}{\tau_X + \tau_h}} > 0, & t = 0 \\ -\left(\frac{A^I}{N_I} + \frac{A^U}{N_U} \right) \frac{1}{\tau_\eta - \tau_0} < 0, & t = 1, 2, \dots, T \end{cases}$$

Also by the law of total covariance,

$$\kappa_t^I = \text{Cov}(\alpha(\theta_t^I - \theta_{t-1}^I), R_{t-1}) = -(f_t - f_{t-1}) \left[a_{t-1} \text{Var}(\hat{X}_{t-2}^U) + b_{t-1} K_{X,t-1}^U \Sigma_{t-1}^U \right]. \quad (\text{A-130})$$

Since

$$f_t - f_{t-1} > 0, \quad a_t > 0, \quad b_t > 0, \quad K_{X,t}^U > 0, \quad (\text{A-131})$$

we have

$$\kappa_t^I < 0, \quad \kappa_t^U > 0, \quad \text{for } t = 2, 3, \dots, T. \quad (\text{A-132})$$

For $t = 1$,

$$\begin{aligned} \kappa_1^I &= \text{Cov}(N_I(\theta_1^I - \theta_0^I), P_0) \\ &= (f_1 - f_0) \frac{\omega_0 \mu_0 \tau_h / h}{\tau_X + \tau_h} \left(\frac{\tau_0}{\tau_V + \tau_0} + \frac{\tau_h / h}{\tau_X + \tau_h} \right) \left(\frac{1}{\tau_V} + \frac{1}{\tau_\eta} + \frac{h^2}{\tau_X} \right) > 0, \end{aligned} \quad (\text{A-133})$$

thus $\kappa_1^U < 0$.

□

A.9 Empirical Appendix

This appendix collects the data construction behind Section 5.1 and additional specifications. Throughout, i indexes stocks (equivalently, in the regressions, announcements of that stock), p fund portfolios, and t calendar months.

A.9.1 Data and sample construction

All data are from WRDS. Daily and monthly returns, prices, shares outstanding, share volume and split adjustment factors are from CRSP; earnings announcement dates and book equity are from Compustat quarterly, linked to CRSP through the CCM link history using primary links only. Mutual fund data are from the CRSP Survivor-Bias-Free Mutual Fund database: share-class monthly total net assets and returns, the share-class-to-portfolio map, style codes, and portfolio holdings.

For fund portfolio p in month t , flows are inferred as

$$Flow_{p,t} = \frac{TNA_{p,t} - TNA_{p,t-1}(1 + ret_{p,t})}{TNA_{p,t-1}}, \quad FIT_{i,t} = \sum_p Flow_{p,t} \frac{shr_{i,p,r(t)}}{shr_{out_{i,t-1}}},$$

where TNA is total net assets — the market value of the fund's holdings less its liabilities, that is, the money it has under management, reported monthly by share class and here aggregated to the portfolio — and $ret_{p,t}$ is the TNA-weighted class return. The numerator compares the fund's actual end-of-month size with what it would have been had no investor money moved in or out, namely its beginning size grown at its own return; the difference is net purchases by investors, and dividing by beginning-of-month size expresses it as a growth rate. Here $shr_{i,p,r(t)}$ is the holding of stock i by portfolio p at the most recent quarter-end report preceding month t . We set $SELL = -FIT$, winsorize at the 1st and 99th percentiles, and standardize within calendar quarter. Funds are restricted to domestic equity portfolios (CRSP objective codes beginning ED), excluding ETFs, with beginning-of-month TNA of at least \$5 million, at least ten CRSP-matched positions, flows trimmed at $\pm 100\%$, and holdings no more than six months stale. Share counts and shares outstanding are both placed on the CRSP split-invariant basis before any ratio is formed.

The stock sample comprises ordinary common shares (CRSP share codes 10 and 11) on the NYSE, AMEX and Nasdaq with a price of at least \$5 and positive market equity. The start date is set by CRSP fund holdings coverage. All characteristics and screens are measured at the pressure month rather than at a holdings report date. Controls are log market capitalization, book-to-market, prior return momentum, share turnover and mutual fund ownership; returns spanning the event window are excluded from the control set. All other continuous variables are winsorized at 0.5% within each regression, and standard errors are clustered by firm and by calendar quarter following [Cameron, Gelbach, and Miller \(2011\)](#), with a degrees-of-freedom correction for absorbed fixed effects.

A.9.2 The day-by-day response to selling pressure

Figure A-1 plots the coefficient on *SELL* in separate regressions of each event day's abnormal return, from twenty-five trading days before the announcement to forty-five days after, with 95% confidence intervals. The coefficients are small in magnitude, ranging from -1.8 to $+2.3$ basis points, and only seven of the seventy-one daily coefficients are individually significant at the 5% level. The figure is therefore informative about the time profile rather than about individual daily estimates, and a positive point estimate should not be read as a positive effect where the interval spans zero.

Coefficients are predominantly negative during the pressure window, ten of thirteen, including four consecutive individually significant days at -22 through -19 ; small and mixed in sign over the intervening gap, where none is individually significant; and largest and positive at the announcement, although neither announcement-day coefficient is individually significant ($+2.3$ basis points with $t = 1.75$ at day 0 and $+2.3$ with $t = 1.27$ at day +1). Subsequent daily coefficients are generally small and imprecisely estimated, and roughly half are positive, which is what the mechanism allows: the distortion may unwind over several days rather than instantaneously. Two exceptions should be noted. The coefficients at days -15 and -14 are positive and individually significant ($+1.6$, $t = 2.54$; $+1.4$, $t = 2.02$), at the end of the window that can fall inside the pressure month. A recovery beginning before any public information arrives is not what the mechanism implies, although the daily estimates are individually imprecise and the seventy-one regressions are not independent of one another. Related cumulative-return results, which are estimated far more precisely than any single day, are reported in Table 1.

A.9.3 Additional specifications

The level regression of $CAR(0, +1)$ on *SELL* gives 7.01 basis points ($t = 3.20$) without controls and 4.43 basis points ($t = 2.08$) with the full control set. Adding industry fixed effects leaves the estimate similar at 4.22 basis points ($t = 1.98$). The quintile means underlying the sort in Table 1 are -0.8 , -0.7 , 10.0 , 10.3 and 16.2 basis points from Q1 to Q5. With firm fixed effects,

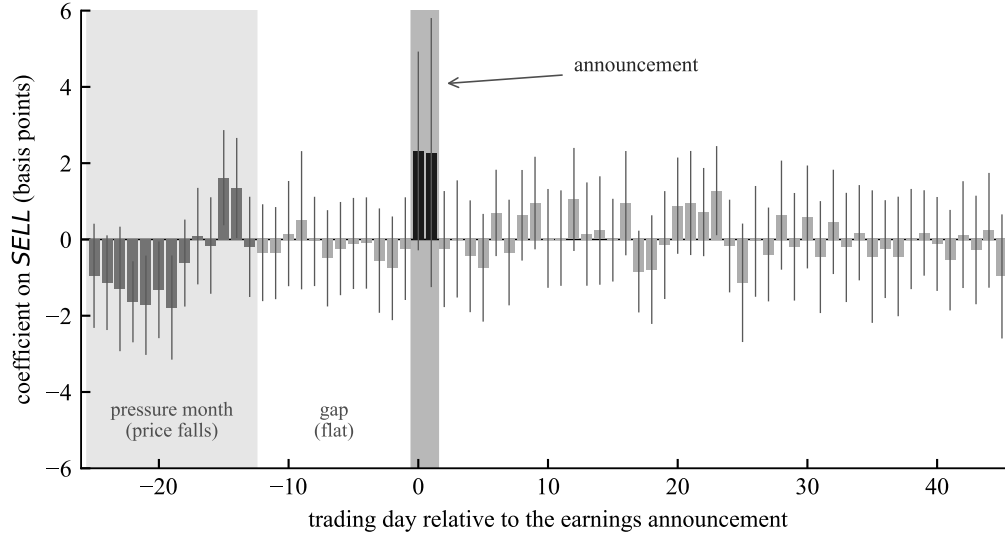


Figure A-1. Coefficient on pre-announcement selling pressure, *SELL*, in separate regressions of each event day's abnormal return, with 95% confidence intervals. Day 0 is the earnings announcement and event time is measured in trading days. The shaded region on the left marks days that can fall within the pressure window, and the darker region marks the announcement. Controls, fixed effects, and clustering are as described in Section 5.2.

the coefficient falls to 2.17 basis points ($t = 1.06$).

The price-depression regression is reported over two windows. Over the three months ending in the pressure month the coefficient on *SELL* is -0.0124 ($t = -6.99$); over the pressure month alone it is -0.0023 ($t = -1.57$), negative but not significant. The stock-level measure is only weakly persistent from one month to the next — the average within-firm autocorrelation of *SELL* across consecutive announcements is 0.13 — so the three-month window should be understood as capturing the price path over the accumulation period rather than the impact of the measured month's trading alone.

The four continuation-slope interactions in the preferred specification all enter negatively: $ANN \times \text{size}$ is -0.0159 ($t = -3.39$), $ANN \times \text{turnover}$ is -0.0069 ($t = -2.24$), $ANN \times \text{fund ownership}$ is -0.0128 ($t = -3.19$) and $ANN \times \text{pre-announcement volatility}$ is -0.0075 ($t = -1.81$). Their correlations with shock magnitude L_i are -0.03 , 0.06 , 0.30 and 0.02 respectively. Fund ownership is thus the interaction that matters for φ_3 : it is the one strongly correlated with L_i , which fol-

lows from flow-induced trading being scaled by fund holdings, and omitting it leaves φ_3 biased toward zero.

To distinguish persistent cross-firm differences from within-firm variation, we decompose *SELL* into the firm's sample mean and its deviation from that mean, standardize the two components, and enter them jointly. For the announcement return, a one-standard-deviation increase in the between-firm component is associated with a 20.07 basis point higher return ($t = 6.64$), whereas the within-firm component is 0.08 basis points ($t = 0.04$). This difference is not due to a lack of within-firm variation: 81% of the variance of *SELL* lies within firms, and the result is similar when firms with fewer than four, eight or twelve announcements are excluded.

The announcement-return result is therefore driven primarily by persistent cross-firm variation in exposure to flow-induced selling. This pattern is not inconsistent with the model: firms may differ persistently in their exposure to liquidity shocks or in the price impact of those shocks. Because such cross-firm heterogeneity is not explicitly modelled, however, the level result does not by itself distinguish the model's mechanism from other persistent firm characteristics that affect announcement returns. Within-firm variation more closely corresponds to the model's comparative static with respect to the realized liquidity shock.

The continuation interaction behaves differently. Decomposing shock magnitude in the same way, the between-firm component gives an interaction coefficient of 0.0067 ($t = 1.80$), whereas the within-firm component gives 0.0075 ($t = 2.83$). The prediction of Corollary 1, that return continuation strengthens with the realized liquidity-shock magnitude, is thus supported by within-firm variation rather than being driven solely by persistent cross-sectional differences.

Over a longer sample constructed from Thomson s12 holdings linked through MFLINKS (364,008 announcements, 1989–2024), the *announcement-return effect of pre-announcement selling pressure* is absent in the two subperiods before 2005 (−1.39 and −0.97 basis points, both insignificant), rises to 4.93 basis points ($t = 1.34$) over 2005–2012, and reaches 9.78 basis points ($t = 2.82$) over 2013–2024. This design cannot identify why the relation changes over time.