

# INTERNATIONAL INVESTING: DIVERSIFICATION AND BEYOND <sup>\*</sup>

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## **Abstract**

A fundamental question in international asset pricing is whether a unified system of risks and their prices extends across national borders. Answering it requires looking beneath country indices. Returns on individual stocks from 26 markets reveal the cross-country structure of systematic risks and their prices. Risk Gaps reflect compensated systematic risks unspanned locally; Price Gaps reflect different premia for shared risks. Both are pervasive; their portfolios have positive returns and low cross-country and US-market correlations. The US market's Sharpe ratio is 0.55; combining it with foreign indices yields 0.41, whereas replacing them with both gap portfolios raises it to 1.20.

JEL classification codes: G12, G15, C58, C38

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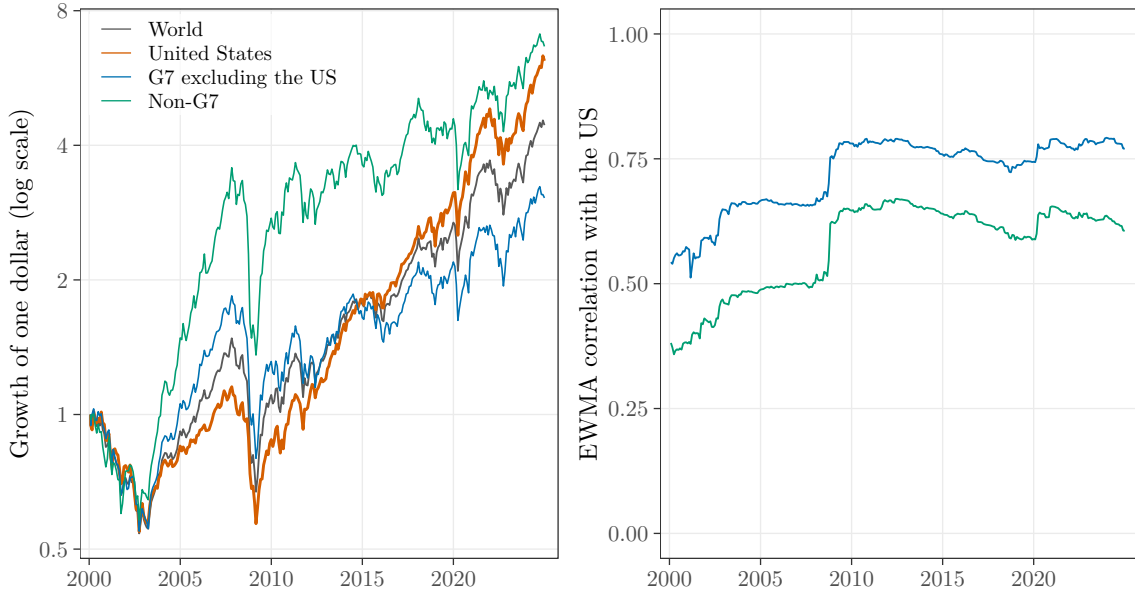
# 1 Introduction

International finance has long asked whether national borders give rise to distinct systems of asset pricing. For a local investor, this question has a precise stochastic discount factor formulation: a foreign market is integrated with the local market if the stochastic discount factor that prices local assets also prices foreign assets correctly. This cross-border pricing restriction can fail along two distinct dimensions. First, foreign assets may be exposed to priced systematic risks that are not spanned by the local market, a gap we call the *Risk Gap*. Second, systematic risks shared across the two markets may command different premia, a gap we call the *Price Gap*. Although economically distinct, the two gaps are complementary manifestations of a failure of the same cross-border pricing restriction. We therefore develop a unified framework that disentangles and measures both dimensions.

Country indices provide a natural starting point for evaluating cross-market integration, but their returns alone cannot establish whether either gap is present. Figure 1 illustrates why index-level evidence may make foreign markets appear to offer little additional value to a US investor. Its first panel shows the growth of one dollar invested in the US and foreign country indices. Over the sample as a whole, the US index has outperformed most foreign indices, although relative performance has varied considerably over time. The second panel shows a broad increase in the correlation of foreign indices with the US market. Taken together, these patterns suggest that foreign indices have delivered weaker relative performance without commensurate diversification benefits. Yet a country index represents only one particular combination of the systematic risks present in a foreign market. Identifying the Risk Gap requires determining whether the US market spans the foreign market’s systematic return space; identifying the Price Gap requires comparing the premia assigned to risks shared across the two markets. Both tasks therefore require moving beyond country indices to characterize the systematic risks present in each market and how shared risks are priced across them.

Doing so, however, is empirically challenging because the relevant systematic risks are not directly observed, nor is it known which risks are shared across markets. We address this challenge using large cross-sections of individual stocks. Under a latent-factor structure, common variation across stocks reveals each market’s systematic return space without requiring the relevant factors to be specified *ex ante*.

Figure 1: MSCI country indices and their correlation with the US market, 2000–2024. The figure shows MSCI country total-return indices in US dollars. The left panel plots the growth of one dollar invested in January 2000, on a log scale, in the US market, in a World aggregate of all countries in our sample (including the US, each country weighted by its one-month-lagged total market capitalization in US dollars), and in two equal-weighted groups of the remaining countries: the G7 excluding the US and the non-G7 markets. The right panel plots the correlation between monthly US dollar returns on the country indices and the US market, averaged equally within the same two foreign groups. Correlations are exponentially weighted with a half-life of 60 months, so each point uses only past returns. Both panels use the same colors and share a common time axis; the legend in the left panel applies to both. Country-level correlation panels appear in Figure 9.



In particular, we recover and compare these systematic return spaces in two steps. We first estimate each market’s latent factor space using Projected Principal Component Analysis and then align the estimated spaces using Canonical Correlation Analysis. This alignment decomposes foreign systematic variation into a component shared with the local market and a component orthogonal to the local factor space. The orthogonal component isolates foreign systematic risks that are not locally spanned, while the shared component allows us to compare their premia across markets. Priced risks in the orthogonal component identify a Risk Gap; differences in premia within the shared component identify a Price Gap. Both gaps thus emerge from the same comparison of the local and foreign factor spaces.

This factor-space comparison also maps the two gaps into complementary investment strategies. The Risk Gap portfolio uses the orthogonal foreign component to provide exposure to systematic risks unavailable locally. The Price Gap portfolio trades

matched foreign and local factor portfolios, exploiting cross-market differences in premia while hedging their shared innovations. The former captures classical gains from diversification, whereas the latter captures gains beyond diversification. Under global integration, the former has no priced component and the latter has zero expected return; our model accordingly shows that a well-diversified local investor obtains no additional asymptotic benefit from foreign assets.

We apply the framework to large cross-sections of individual stocks in 26 countries over 1986–2024, taking the United States as the local market and comparing its factor space with those of the other 25 markets. Using rolling 36-month estimation windows and one-month holding periods, we construct all Risk Gap and Price Gap portfolio returns out of sample. When equally weighted across foreign markets, the Risk Gap and Price Gap portfolios achieve annualized Sharpe ratios of 1.14 and 1.02, respectively, and are essentially uncorrelated with the US market. Equal-weighted combinations of the two strategies achieve Sharpe ratios of 1.21–1.62 across country groups, demonstrating that the gains from diversification and beyond diversification are both substantial and complementary.

For a US investor, the contrast with country-index investing is stark. Adding foreign indices lowers the US market’s annualized Sharpe ratio from 0.55 to 0.41. Adding the two gap portfolios as an overlay instead produces Sharpe ratios of 1.20 under market-capitalization weights and 1.60 under lagged Sharpe-based weights. These gains reflect broad allocations across foreign markets rather than the selection of a few successful countries.

Nor are the results confined to less-developed markets or earlier periods. Across the six foreign G7 markets, the Risk Gap and Price Gap portfolios achieve average annualized Sharpe ratios of 0.59 and 0.39 over the full sample and 0.67 and 0.34 over 2005–2024. The persistence of both gaps among developed markets and in recent decades reinforces that weak country-index performance should not be interpreted as evidence of integration.

Further analyses support the economic interpretation and practical relevance of both strategies. Their returns are not explained by bilateral exchange-rate changes, systematic currency factors, or conventional risk exposures, and both retain low exposure to the US market during downturns. Together, these results weigh against currency exposure and market downturns as mechanical explanations for the measured gains.

More generally, we provide a framework for analyzing the risk and price dimensions of integration across market pairs. Using large cross-sections of individual assets, our approach encompasses a broad and representative set of investment opportunities rather than focusing on selected factors or portfolios. It recovers and aligns systematic return spaces to identify risks present in one market but not spanned by another and to determine whether risks shared across markets command different premia. Each dimension maps into an investable strategy, allowing its economic importance to be measured directly. By shifting the analysis to market-wide systematic return spaces, the framework provides a more comprehensive assessment of cross-market integration.

## Related Literature

Our analysis contributes to three strands of literature. The first concerns the classical diversification motive for international investment: foreign markets can expand the domestic investment opportunity set by providing access to systematic risks that are not spanned locally. Early studies such as Grubel (1968) and Solnik (1974) document substantial gains from international diversification, attributing them to low correlations across national markets. Heston and Rouwenhorst (1994) show that there are distinct country-specific components to asset returns. As markets became more integrated, however, global risks assumed a larger role in national-market returns (Bekaert and Harvey, 1995; Carrieri et al., 2007). Pukthuanthong and Roll (2009) develop a factor-based measure of integration and document substantial increases in integration for most countries. Consistent with this broader increase, Siquefield (1996) and Quinn and Voth (2008) question the diversification benefits available across developed-market indices, whereas Berger et al. (2011) show that less-integrated frontier markets retain meaningful diversification gains. Much of this evidence, however, evaluates country indices, which represent only particular combinations of the risks present in each market. Using firm-level data, Bae et al. (2019) show that portfolios of developed-market firms with economic exposure to emerging countries provide diversification benefits unavailable through emerging-market indices. The contrast between index- and firm-level evidence shifts the relevant question from whether foreign indices are weakly correlated with the domestic market to whether the systematic risks embedded in foreign stocks can be spanned by domestic assets.

The theoretical foundation for viewing international diversification as a risk-spanning problem is Errunza and Losq (1985). Under mild segmentation, securities unavailable to constrained investors earn an additional premium for the component of their risk that cannot be spanned by accessible assets. Related work asks whether foreign-market opportunities can be replicated using domestically traded assets (Errunza et al., 1999) or recovered by selecting individual foreign stocks with low exposure to global factors (Eun et al., 2017). Closest to our market-wide analysis, Chaieb et al. (2021) use a large panel of international stocks and show that a local market factor remains necessary to capture systematic return variation, even after accounting for world, regional, style, and currency factors, and that it commands a significant premium in many countries. We share these papers’ emphasis on looking beyond country indices but shift the object of analysis from selected exposures or prespecified factors to latent systematic return spaces. By recovering the component of the foreign systematic return space orthogonal to the local factor space, the Risk Gap portfolio identifies and targets foreign systematic risks unavailable locally.

A second strand evaluates international market integration through cross-market asset-pricing restrictions, particularly whether risks shared across markets command common prices. Early studies test restrictions jointly implied by market integration and a specified factor-pricing model (Cho et al., 1986; Korajczyk and Viallet, 1989). As emphasized by Cho et al. (1986), a rejection may therefore reflect either market segmentation or misspecification of the imposed factor model. Related studies estimate the compensation attached to specified sources of international risk: Harvey (1991) studies world covariance risk, whereas Korajczyk and Viallet (1992) and Dumas and Solnik (1995) examine foreign-exchange risk. Karolyi and Stulz (2003) offer a qualified resolution to the distinction between local and global pricing: country risk premia reflect covariance with the world market, while home bias preserves local influences on asset prices. More recent work develops measures of de facto segmentation that do not require a prespecified international factor model. Bekaert et al. (2011) use differences between local and global industry earnings yields, while Sandulescu et al. (2021) and Sandulescu and Schneider (2021) use stochastic discount factor methods to document country-specific pricing variation. Such evidence identifies departures from integration but may reflect a Risk Gap, a Price Gap, or both. Patton and Weller (2022) focus more directly on the Price Gap by studying cross-country heterogeneity in factor

compensation using clustering methods. Our framework separates the two gaps within a common latent-factor framework: the Risk Gap portfolio targets foreign systematic risks orthogonal to the local factor space, while the Price Gap portfolio trades matched local and foreign factors to isolate differences in compensation.

Third, our framework contributes methodologically to the literature on large-panel latent factor models and high-dimensional asset-pricing methods. The factor-recovery step builds on Projected Principal Component Analysis developed by Fan et al. (2016) and the single-market application of Kim et al. (2021). We extend the analysis of Kim et al. (2021) to a cross-market setting and use canonical correlation methods related to the factor-grouping procedures of Andreou et al. (2019, 2020) to align the estimated factor spaces. Our analysis also contributes to work on global and national latent components in international equity markets, including covariance-matrix estimation (Choi and Kim, 2023) and dynamic return-factor models (Linton et al., 2025), and to methods for estimating systematic factors and risk premia in short panels (Connor and Korajczyk, 1988; Kim and Skoulakis, 2018; Raponi et al., 2020; Kim and Korajczyk, 2024; Zaffaroni, 2025; Fortin et al., 2023). More broadly, it connects to characteristic-based and high-dimensional methods for asset-pricing panels, including approaches that use firm characteristics to estimate latent factor structures or expected returns and methods that accommodate incomplete observations (Kelly et al., 2019; Freyberger et al., 2020, 2024). By combining and extending these econometric methods, we provide a unified framework that estimates both the Risk Gap and the Price Gap within a common short rolling-panel setting and maps each dimension into an out-of-sample investment strategy.

## 2 Risk Gaps and Price Gaps in a Two-Country Model

This section develops a two-country framework that connects departures from cross-market integration to investable portfolio strategies. The analysis takes the perspective of an investor in country 1, the local market, who is evaluating investment opportunities in country 2, the foreign market. Figure 2 illustrates two gaps through which these opportunities arise. First, the foreign market may contain priced systematic risks that are not spanned by the local factor space (right, yellow region), giving rise to a Risk Gap. The Risk Gap portfolio isolates these risks and captures the resulting gains from



its premium in country  $j$ . Because this risk is common to both markets, integration requires  $\lambda_1^c = \lambda_2^c$ ; a difference between these premia gives rise to a Price Gap. The factor  $g_{jt}$  captures country-specific systematic risk, and  $\lambda_j^s$  denotes its premium. From the perspective of the country 1 investor,  $g_{2t}$  represents foreign systematic risk that is not spanned by the local factor space. A nonzero premium  $\lambda_2^s$  on this risk gives rise to a Risk Gap. The remaining term,  $e_{jit}$ , is a mean-zero idiosyncratic return. To keep the example transparent, we assume that the factor loadings  $\beta_{ji}$  and  $\delta_{ji}$  are known and that the common and country-specific loading directions are cross-sectionally orthogonal, together with a few additional simplifying conditions.<sup>1</sup> Under these assumptions, each gap maps into a distinct portfolio strategy.

We first construct the Risk Gap portfolio, which isolates foreign systematic risk that is not spanned by the local factor space. Weighting country 2 assets in proportion to their foreign-specific factor loadings gives

$$\begin{aligned} R_t^{\text{RG}} &\equiv \sum_{i=1}^N \frac{\delta_{2i}}{N} R_{2it} \\ &= \frac{1}{N} \sum_{i=1}^N \delta_{2i} \beta_{2i} (\lambda_2^c + f_t) + \frac{1}{N} \sum_{i=1}^N \delta_{2i}^2 (\lambda_2^s + g_{2t}) + \frac{1}{N} \sum_{i=1}^N \delta_{2i} e_{2it} \xrightarrow{P} \lambda_2^s + g_{2t}. \end{aligned}$$

Cross-sectional orthogonality removes the common-factor component, normalization gives unit exposure to  $\lambda_2^s + g_{2t}$ , and idiosyncratic returns diversify away. Thus, the portfolio's expected return identifies a Risk Gap.

We next construct the Price Gap portfolio. To compare the compensation for common risk across the two markets, we form a common-factor portfolio in each country by weighting assets in proportion to their common-factor loadings:

$$\begin{aligned} \sum_{i=1}^N \frac{\beta_{ji}}{N} R_{jit} &= \frac{1}{N} \sum_{i=1}^N \beta_{ji}^2 (\lambda_j^c + f_t) + \frac{1}{N} \sum_{i=1}^N \beta_{ji} \delta_{ji} (\lambda_j^s + g_{jt}) + \frac{1}{N} \sum_{i=1}^N \beta_{ji} e_{jit} \\ &\xrightarrow{P} \lambda_j^c + f_t, \quad j = 1, 2. \end{aligned}$$

Normalization, orthogonality, and diversification imply that each common-factor port-

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<sup>1</sup>Formally, for each country  $j = 1, 2$ ,  $\frac{1}{N} \sum_{i=1}^N \beta_{ji} \delta_{ji} \rightarrow 0$ . For convenience, we normalize  $\frac{1}{N} \sum_{i=1}^N \beta_{ji}^2 \rightarrow 1$  and  $\frac{1}{N} \sum_{i=1}^N \delta_{ji}^2 \rightarrow 1$ . We also impose the standard diversification conditions  $\frac{1}{N} \sum_{i=1}^N \beta_{ji} e_{jit} \xrightarrow{P} 0$  and  $\frac{1}{N} \sum_{i=1}^N \delta_{ji} e_{jit} \xrightarrow{P} 0$  for each  $t$ .

folio converges to  $\lambda_j^c + f_t$ . Suppose  $\lambda_2^c > \lambda_1^c$ . The Price Gap portfolio then buys the common-factor portfolio in Country 2 and shorts its Country 1 counterpart:

$$R_t^{\text{PG}} \equiv \sum_{i=1}^N \frac{\beta_{2i}}{N} R_{2it} - \sum_{i=1}^N \frac{\beta_{1i}}{N} R_{1it}$$

$$\xrightarrow{P} (\lambda_2^c + f_t) - (\lambda_1^c + f_t) = \lambda_2^c - \lambda_1^c > 0.$$

The shared innovation  $f_t$  cancels, leaving the difference in common-risk premia. If  $\lambda_1^c > \lambda_2^c$ , the positions are reversed.

In contrast to the simple example, asset returns may be driven by multiple latent factors whose loadings must be estimated and whose common or country-specific status must be determined from the data. Before extending the model to this setting, however, we address how the local-currency denomination of foreign returns affects the systematic factor structure. Section 2.2 explains how currency risk is incorporated within the same linear framework, and Section 2.3 then presents the general multi-factor model.

## 2.2 International Factor Structure and Currency Risk

As in the motivating example, we consider returns from the perspective of an investor in country 1 and express all equity returns in that country's currency. Exchange-rate fluctuations therefore enter the local-currency returns on country 2 equities. When these fluctuations are spanned by the risks driving equity returns, they are incorporated within the existing factor structure (Solnik, 1983). When they are not, the linear representation is preserved by expanding the factor space to include additional currency-related risks (Ikeda, 1991). Exchange-rate risk may therefore affect the composition of the systematic factor space without invalidating its linear representation.

Currency risk is therefore not a separate dimension of integration in our framework. Currency-related risks enter the systematic return spaces of the two markets and may be common or country-specific.<sup>2</sup> A priced currency-related risk that is not spanned by the local factor space may contribute to a Risk Gap, whereas a currency-related risk shared across markets may contribute to a Price Gap if it commands different premia

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<sup>2</sup>A related perspective is provided by Chernov et al. (2024), who decompose exchange-rate innovations into components driven by globally traded, locally traded, and unspanned shocks. Their globally traded component shows that exchange-rate risk can be driven by shocks shared across markets and may therefore enter the common factor space.

across them. Moreover, Chaieb et al. (2021) show that premia on local factors are not fully explained by exchange-rate exposure and that country-specific compensation remains significant after accounting for currency risk. This evidence motivates allowing country-specific latent factors beyond currency risk. With currency risk incorporated in this way, we turn to the general multi-factor model.

## 2.3 General Model

The general model extends the motivating example to multiple latent factors and allows the number of assets and country-specific factors to differ across markets. Consider two countries indexed by  $g = 1, 2$ .<sup>3</sup> Country  $g$  contains  $N_g$  securities, and its return-generating process is assumed to remain stable over a relatively short horizon,  $t = 1, \dots, T$ .

We specify the return generating process of individual securities in each country  $g$ . The excess returns of individual stocks in country  $g$  follow a  $K_g$ -factor model in which the factors are unobservable, latent factors. Among  $K_g$  factors,  $K^c$  factors are common across the two countries and  $K_g^s$  factors are country-specific. Thus, it holds that  $K_g = K^c + K_g^s$ . In particular, the excess return of the  $i$ -th asset in country  $g$  at time  $t$  is generated by the following model: for  $g = 1, 2$ ,  $i = 1, \dots, N_g$  and  $t = 1, \dots, T$ ,

$$R_{git} = \beta'_{gi} (\boldsymbol{\lambda}_g^c + \mathbf{f}_t) + \delta'_{gi} (\boldsymbol{\lambda}_g^s + \mathbf{g}_{gt}) + e_{git}, \quad (2.2)$$

where

$\beta_{gi} = [\beta_{gi1} \dots \beta_{giK^c}]'$  is the  $(K^c \times 1)$  vector of factor loadings of the  $i$ -th asset to the common factors,  $\boldsymbol{\lambda}_g^c = [\lambda_{g1}^c \dots \lambda_{gK^c}^c]'$  is the  $(K^c \times 1)$  vector of risk premia in country  $g$  for exposure to common factors,  $\mathbf{f}_t = [f_{1t} \dots f_{K^c t}]'$  is the  $(K^c \times 1)$  systematic zero-mean common factor realization in period  $t$ ,  $\delta_{gi} = [\delta_{gi1} \dots \delta_{giK_g^s}]'$  is the  $(K_g^s \times 1)$  vector of loadings of asset  $i$  on the country-specific factors,  $\boldsymbol{\lambda}_g^s = [\lambda_{g1}^s \dots \lambda_{gK_g^s}^s]'$  is the  $(K_g^s \times 1)$  vector of country-specific risk premia in country  $g$ , and  $\mathbf{g}_{gt} = [g_{g1t} \dots g_{gK_g^s t}]'$  is the  $(K_g^s \times 1)$  vector of country-specific factor realizations in period  $t$ .

Throughout, we use  $\mathbf{0}_m$ ,  $\mathbf{1}_m$ , and  $\mathbf{0}_{m \times l}$  to denote the  $(m \times 1)$  vectors of zeros and

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<sup>3</sup>The two countries represent a generic market pair. In the empirical analysis in Section 4, we construct such pairs using 26 countries.

ones and the  $(m \times l)$  matrix of zeros, respectively. The return generating process of (2.2) is expressed compactly in matrices: for  $g = 1, 2$ ,

$$\mathbf{R}_g = \mathbf{B}_g \underbrace{(\boldsymbol{\lambda}_g^c \mathbf{1}'_T + \mathbf{F}')}_{\text{common risk}} + \mathbf{D}_g \underbrace{(\boldsymbol{\lambda}_g^s \mathbf{1}'_T + \mathbf{G}'_g)}_{\text{country-specific risk}} + \mathbf{E}_g, \quad (2.3)$$

where

the  $(i, t)$  element of the  $(N_g \times T)$  matrix  $\mathbf{R}_g$  is  $R_{git}$ ,

the  $i$ -th row of the  $(N_g \times K^c)$  matrix  $\mathbf{B}_g$  is  $\boldsymbol{\beta}'_{gi}$ , the  $t$ -th row of the  $(T \times K^c)$  matrix  $\mathbf{F}$  is  $\mathbf{f}'_t$ , the  $i$ -th row of the  $(N_g \times K_g^s)$  matrix  $\mathbf{D}_g$  is  $\boldsymbol{\delta}'_{gi}$ , the  $t$ -th row of the  $(T \times K_g^s)$  matrix  $\mathbf{G}_g$  is  $\mathbf{g}'_{gt}$ , and the  $(i, t)$  element of the  $(N_g \times T)$  matrix  $\mathbf{E}_g$  is  $e_{git}$ .

The decomposition in (2.3) separates two dimensions of cross-market integration from the perspective of an investor in Country 1. The country-specific component of Country 2 consists of systematic risks that are not spanned by the local factor space. When these risks command compensation, so that  $\lambda_2^s \neq 0$ , they give rise to a Risk Gap. A Risk Gap need not reflect restrictions on international capital flows; it may arise, for example, because specialization in production exposes the two countries to different systematic risks.

The common component is present in both markets. Pricing integration requires risks shared across the two markets to command the same premia, so that  $\lambda_1^c = \lambda_2^c$ . When this restriction fails, the difference in premia gives rise to a Price Gap.

The two gaps correspond to different notions of integration. Under a pricing-based notion, equal compensation for shared risks can coexist with a Risk Gap. Under the stronger opportunity-set notion, the foreign market is integrated into the local market only when the local opportunity set dominates: the foreign market contains no compensated systematic risks that are not spanned locally, and risks shared across the two markets command the same premia. Our framework evaluates these two dimensions separately.

Implementing this comparison requires recovering the systematic factor exposures,  $[\mathbf{B}_g \ \mathbf{D}_g]$ , in both countries. Conventional time-series beta estimates are imprecise over the short windows needed to accommodate time variation, and expanding the cross-section does not eliminate this estimation error. We address these limitations by extending Fan et al. (2016) and Kim et al. (2021) to the two-country setting and using the large cross-section of firm characteristics to estimate the loading components needed to

distinguish the two gaps and construct their corresponding portfolios.

In particular, we allow the systematic risk  $[\mathbf{B}_g \ \mathbf{D}_g]$  to be functions of asset-specific characteristics in each country. Let  $\mathbf{x}_{gi} = [x_{gi1} \ \cdots \ x_{giL_g}]'$  be the  $(L_g \times 1)$  vector of the characteristics associated with stock  $i$  in country  $g$ .<sup>4</sup> Then, define the  $(N_g \times L_g)$  matrix of  $\mathbf{X}_g$ , the  $i$ -th row of which is  $\mathbf{x}_{gi}'$ . We assume the following structure for  $\mathbf{B}_g$  and  $\mathbf{D}_g$ :

$$\mathbf{B}_g = \mathbf{X}_g \Theta_g^c + \Gamma_g^c \quad (2.4)$$

and

$$\mathbf{D}_g = \mathbf{X}_g \Theta_g^s + \Gamma_g^s, \quad (2.5)$$

where  $\Theta_g^c$  is the  $(L_g \times K^c)$  matrix,  $\Theta_g^s$  is the  $(L_g \times K^s)$  matrix, and the  $(N_g \times K^c)$  matrix,  $\Gamma_g^c$  and the  $(N_g \times K^s)$  matrix,  $\Gamma_g^s$  are cross-sectionally orthogonal to the characteristic space of  $\mathbf{X}_g$ .

We refer to  $\Theta_g^c$  and  $\Theta_g^s$  as factor loading matrices because they relate characteristics to loadings on the common and country-specific factors, respectively. The matrices  $\Gamma_g^c$  and  $\Gamma_g^s$  contain residual loading components that are orthogonal to the characteristic space of  $\mathbf{X}_g$ . In the large- $N$ /small- $T$  setting,  $\Theta_g^c$  and  $\Theta_g^s$  can be consistently estimated, whereas  $\Gamma_g^c$  and  $\Gamma_g^s$  cannot; our procedure therefore uses only their orthogonality to  $\mathbf{X}_g$ . Nonlinearities can be accommodated by expanding  $\mathbf{X}_g$  to include transformations of the underlying characteristics, as in Kim et al. (2021).

Incorporating (2.4) and (2.5) into (2.3), we have:

$$\mathbf{R}_g = (\mathbf{X}_g \Theta_g^c + \Gamma_g^c) (\boldsymbol{\lambda}_g^c \mathbf{1}_T' + \mathbf{F}') + (\mathbf{X}_g \Theta_g^s + \Gamma_g^s) (\boldsymbol{\lambda}_g^s \mathbf{1}_T' + \mathbf{G}') + \mathbf{E}_g. \quad (2.6)$$

The characteristic projection allows the relevant loading components to be estimated in short panels. Rolling estimation also permits the relation between characteristics and factor loadings to evolve over time. The residual components,  $\Gamma_g^c$  and  $\Gamma_g^s$ , absorb risks not explained by the observed characteristics, so omitted characteristics may reduce precision but do not mechanically generate either gap.

We next state the regularity and identification conditions required for estimation. We begin with standard conditions on the characteristics and residual returns.

**Assumption 1.** *In each country  $g = 1, 2$ , as  $N_g \rightarrow \infty$ , it holds that*

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<sup>4</sup>Note that the number of characteristics can differ across countries.

- (i)  $\frac{\mathbf{R}_g' \mathbf{R}_g}{N_g} \xrightarrow{p} \mathbf{V}_{R_g}$  and  $\frac{\mathbf{X}_g' \mathbf{X}_g}{N_g} \rightarrow \mathbf{V}_{X_g}$ , where  $\mathbf{V}_{R_g}, \mathbf{V}_{X_g}$  are positive definite matrices,  
(ii)  $\frac{\mathbf{X}_g' \Gamma_g^c}{N_g} \xrightarrow{p} \mathbf{0}_{L_g \times K^c}$ ,  $\frac{\mathbf{X}_g' \Gamma_g^s}{N_g} \xrightarrow{p} \mathbf{0}_{L_g \times K_g^s}$ , and  $\frac{\mathbf{X}_g' \mathbf{E}_g}{N_g} \xrightarrow{p} \mathbf{0}_{L_g \times T}$ .

Condition (i) simply states that the cross-sections of returns and characteristics are not redundant but well-spread over individual stocks in each country. Condition (ii) imposes the cross-sectional orthogonality conditions between the characteristics,  $\mathbf{X}_g$ , and factor loading residuals,  $\Gamma_g^c$  and  $\Gamma_g^s$ , and residual returns,  $\mathbf{E}_g$ , in each country.

Next, we assume mild restrictions to separately identify factor loading matrices  $\Theta_g^c$  and  $\Theta_g^s$ . We introduce the symmetric  $(T \times T)$  matrix  $\mathbf{J}_T = \mathbf{I}_T - \frac{1}{T} \mathbf{1}_T \mathbf{1}_T'$ , which corresponds to time-series demeaning. We define  $\Theta_g$  and  $\mathbf{H}_g$  as  $[\Theta_g^c \ \Theta_g^s]$  and  $[\mathbf{F} \ \mathbf{G}_g]$ , respectively.

**Assumption 2.** *In each country  $g = 1, 2$ , as  $N_g \rightarrow \infty$ , it holds that*

$$(i) \ \frac{\Theta_g' \mathbf{X}_g' \mathbf{X}_g \Theta_g}{N_g} = \begin{bmatrix} \frac{\Theta_g^{c'} \mathbf{X}_g' \mathbf{X}_g \Theta_g^c}{N_g} & \frac{\Theta_g^{c'} \mathbf{X}_g' \mathbf{X}_g \Theta_g^s}{N_g} \\ \frac{\Theta_g^{s'} \mathbf{X}_g' \mathbf{X}_g \Theta_g^c}{N_g} & \frac{\Theta_g^{s'} \mathbf{X}_g' \mathbf{X}_g \Theta_g^s}{N_g} \end{bmatrix} \rightarrow \begin{bmatrix} \Theta_g^{c'} \mathbf{V}_{X_g} \Theta_g^c & \Theta_g^{c'} \mathbf{V}_{X_g} \Theta_g^s \\ \Theta_g^{s'} \mathbf{V}_{X_g} \Theta_g^c & \Theta_g^{s'} \mathbf{V}_{X_g} \Theta_g^s \end{bmatrix} = \mathbf{V}_{\Theta_g}, \text{ where}$$

$\mathbf{V}_{\Theta_g}$  is a  $(K_g \times K_g)$  positive definite matrix,

$$(ii) \ \frac{\mathbf{H}_g' \mathbf{J}_T \mathbf{H}_g}{T} = \begin{bmatrix} \frac{\mathbf{F}' \mathbf{J}_T \mathbf{F}}{T} & \frac{\mathbf{F}' \mathbf{J}_T \mathbf{G}_g}{T} \\ \frac{\mathbf{G}_g' \mathbf{J}_T \mathbf{F}}{T} & \frac{\mathbf{G}_g' \mathbf{J}_T \mathbf{G}_g}{T} \end{bmatrix} = \begin{bmatrix} \Sigma_F & \mathbf{0}_{K^c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K^c} & \Sigma_{G_g} \end{bmatrix} = \Sigma_{H_g}, \text{ where } \Sigma_{H_g} \text{ is a}$$

$(K_g \times K_g)$  positive definite matrix,

$$(iii) \ \begin{bmatrix} \frac{\mathbf{G}_1' \mathbf{J}_T \mathbf{G}_1}{T} & \frac{\mathbf{G}_1' \mathbf{J}_T \mathbf{G}_2}{T} \\ \frac{\mathbf{G}_2' \mathbf{J}_T \mathbf{G}_1}{T} & \frac{\mathbf{G}_2' \mathbf{J}_T \mathbf{G}_2}{T} \end{bmatrix} \text{ is a } (K_1^s + K_2^s) \times (K_1^s + K_2^s) \text{ positive definite matrix.}$$

Condition (i) implies that each column of  $\mathbf{X}_g \Theta_g$  provides non-redundant information.<sup>5</sup> Condition (ii) ensures that factor realizations are not redundant in each country. Furthermore, the common factors and country-specific factors are orthogonal over the sample period. Note that these restrictions are without loss of generality because we can regress any correlated country-specific factors on the common factors and reconstruct the orthogonal country-specific factors from the residuals. Condition (iii) imposes minimal restrictions on the behavior of country-specific factors, requiring that they cannot be replicated across countries. Otherwise, the country-specific factors would lose their country-specific features and would instead be classified as common factors.

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<sup>5</sup>This assumption does not require that all characteristics contribute to the factor loadings. Any redundant characteristics will be accounted for by zeros in the corresponding entries of  $\Theta_g$ .

### 3 Estimation

We implement the framework in three steps. First, we use Projected Principal Component Analysis to recover each country's systematic factor space and Canonical Correlation Analysis to separate common from country-specific components. Second, we estimate characteristic-based factor loadings. Third, we use these loadings to construct portfolios that recover foreign country-specific factors and matched common factors, forming the Risk Gap and Price Gap portfolios.

**Step 1: Estimation of common factors and country-specific factors.** The first step of our procedure involves estimating the common factor innovation,  $\mathbf{F}'\mathbf{J}_T$ , and the country-specific factor innovation,  $\mathbf{G}'_g\mathbf{J}_T$ , from returns in the two countries. Recall that the observed returns in (2.6) are driven by the risk premiums  $\boldsymbol{\lambda}_g^c$  and  $\boldsymbol{\lambda}_g^s$ , as well as the realizations of  $\mathbf{F}$  and  $\mathbf{G}_g$ . Because our objective in this step is to estimate the demeaned factor realization rather than the risk premiums, we eliminate the effects of the risk premium by demeaning the observed returns by post-multiplying  $\mathbf{J}_T (= \mathbf{I}_T - \frac{1}{T} \mathbf{1}_T \mathbf{1}'_T)$ :

$$\begin{aligned} \mathbf{R}_g \mathbf{J}_T &= (\mathbf{X}_g \Theta_g^c + \Gamma_g^c) (\boldsymbol{\lambda}_g^c \mathbf{1}'_T + \mathbf{F}') \mathbf{J}_T + (\mathbf{X}_g \Theta_g^s + \Gamma_g^s) (\boldsymbol{\lambda}_g^s \mathbf{1}'_T + \mathbf{G}'_g) \mathbf{J}_T + \mathbf{E}_g \mathbf{J}_T \\ &= (\mathbf{X}_g \Theta_g^c + \Gamma_g^c) \mathbf{F}' \mathbf{J}_T + (\mathbf{X}_g \Theta_g^s + \Gamma_g^s) \mathbf{G}'_g \mathbf{J}_T + \mathbf{E}_g \mathbf{J}_T, \end{aligned} \quad (3.1)$$

where the last equality is from the property of  $\mathbf{1}'_T \mathbf{J}_T = \mathbf{1}'_T (\mathbf{I}_T - \frac{1}{T} \mathbf{1}_T \mathbf{1}'_T) = \mathbf{1}'_T - \frac{T}{T} \mathbf{1}'_T = \mathbf{0}'_T$ . To further isolate  $\mathbf{X}_g \Theta_g^c$  and  $\mathbf{X}_g \Theta_g^s$ , we project the demeaned returns of (3.1) on the (linear) span of  $\mathbf{X}_g$  by premultiplying the projection matrix  $\mathbf{P}_g = \mathbf{X}_g (\mathbf{X}'_g \mathbf{X}_g)^{-1} \mathbf{X}'_g$ . This gives:

$$\begin{aligned} \widehat{\mathbf{R}}_g &\equiv \mathbf{P}_g \mathbf{R}_g \mathbf{J}_T \\ &= (\mathbf{P}_g \mathbf{X}_g \Theta_g^c + \mathbf{P}_g \Gamma_g^c) \mathbf{F}' \mathbf{J}_T + (\mathbf{P}_g \mathbf{X}_g \Theta_g^s + \mathbf{P}_g \Gamma_g^s) \mathbf{G}'_g \mathbf{J}_T + \mathbf{P}_g \mathbf{E}_g \mathbf{J}_T \\ &= (\mathbf{X}_g \Theta_g^c + \mathbf{P}_g \Gamma_g^c) \mathbf{F}' \mathbf{J}_T + (\mathbf{X}_g \Theta_g^s + \mathbf{P}_g \Gamma_g^s) \mathbf{G}'_g \mathbf{J}_T + \mathbf{P}_g \mathbf{E}_g \mathbf{J}_T, \end{aligned} \quad (3.2)$$

where the last equality is from  $\mathbf{P}_g \mathbf{X}_g = \mathbf{X}_g$ . Furthermore, exploiting the orthogonality of  $\Gamma_g^c$ ,  $\Gamma_g^s$  and  $\mathbf{E}_g$  with respect to  $\mathbf{X}_g$  from Assumption 1(ii), the terms of  $\mathbf{P}_g \Gamma_g^c$ ,  $\mathbf{P}_g \Gamma_g^s$ , and  $\mathbf{P}_g \mathbf{E}_g$  in the above equation will become negligible when  $N_g$  is large. Hence, with a large  $N_g$ , it follows that  $\widehat{\mathbf{R}}_g = \mathbf{P}_g \mathbf{R}_g \mathbf{J}_T \approx [\mathbf{X}_g \Theta_g^c \ \mathbf{X}_g \Theta_g^s] [\mathbf{F}' \mathbf{J}_T \ \mathbf{G}'_g \mathbf{J}_T] = \mathbf{X}_g \Theta_g \mathbf{H}'_g \mathbf{J}_T$ , where  $\Theta_g = [\Theta_g^c \ \Theta_g^s]$  and  $\mathbf{H}_g = [\mathbf{F} \ \mathbf{G}_g]$ . Finally, we estimate  $\mathbf{J}_T \mathbf{H}_g$  by applying standard

principal component analysis to  $\widehat{\mathbf{R}}_g$ .

**Theorem 3.1.** *Let  $\widehat{\mathbf{H}}_g$  denote the  $(T \times K_g)$  matrix, the  $k$ -th column of which is the eigenvector of  $\frac{\widehat{\mathbf{R}}'_g \widehat{\mathbf{R}}_g}{N_g}$  corresponding to the  $k$ -th largest eigenvalue of  $\frac{\widehat{\mathbf{R}}'_g \widehat{\mathbf{R}}_g}{N_g}$ , where  $\widehat{\mathbf{R}}_g$  is given by (3.2). Under Assumptions 1 and 2, as  $N_g$  increases,  $\widehat{\mathbf{H}}_g \xrightarrow{p} \mathbf{J}_T \mathbf{H}_g \mathcal{O}_g$ , where the  $(K_g \times K_g)$  matrix  $\mathcal{O}_g$  is given in Lemma C.1.*

As is standard in latent factor models, the demeaned factors are identified only up to a (nonsingular) rotation. Theorem 3.1 establishes that  $\widehat{\mathbf{H}}_g$  consistently estimates the rotated factors  $\mathbf{J}_T \mathbf{H}_g \mathcal{O}_g$ . Lemma C.1 and its proof provide the normalization defining  $\mathcal{O}_g$  and the corresponding rotation details.

Then, we proceed to identify common factors and country-specific factors. The main analytical tool for this purpose is Canonical Correlation Analysis (CCA). Define the  $(K_1 \times K_1)$  matrix  $\widehat{\Sigma}_{H_1}$ , the  $(K_2 \times K_2)$  matrix  $\widehat{\Sigma}_{H_2}$ , and the  $(K_1 \times K_2)$  matrix  $\widehat{\Sigma}_{H_{12}}$  as  $\widehat{\Sigma}_{H_1} = \frac{\widehat{\mathbf{H}}'_1 \widehat{\mathbf{H}}_1}{T}$ ,  $\widehat{\Sigma}_{H_2} = \frac{\widehat{\mathbf{H}}'_2 \widehat{\mathbf{H}}_2}{T}$ , and  $\widehat{\Sigma}_{H_{12}} = \frac{\widehat{\mathbf{H}}'_1 \widehat{\mathbf{H}}_2}{T}$ , respectively.

Because the setup is symmetric, we demonstrate the identification of common factors and country-specific factors using the estimated systematic factors from country 1. We propose a common factor estimator,  $\widehat{\mathbf{F}}_1$ , and a country-specific factor estimator,  $\widehat{\mathbf{G}}_1$ , as follows. Let  $\widehat{\mathbf{W}}_1$  be the  $(K_1 \times K_1)$  matrix, the  $k$ -th column of which is the  $k$ -th canonical direction (or, the eigenvector of  $\widehat{\Sigma}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\Sigma}_{H_2}^{-1} \widehat{\Sigma}'_{H_{12}}$ , corresponding to the  $k$ -th largest eigenvalue), such that  $\widehat{\mathbf{W}}'_1 \widehat{\Sigma}_{H_1} \widehat{\mathbf{W}}_1 = \mathbf{I}_{K_1}$ . We then partition  $\widehat{\mathbf{W}}_1$  as  $\begin{bmatrix} \widehat{\mathbf{W}}_1^c & \widehat{\mathbf{W}}_1^s \end{bmatrix}$ , where  $\widehat{\mathbf{W}}_1^c$  is the  $(K_1 \times K^c)$  matrix and  $\widehat{\mathbf{W}}_1^s$  is the  $(K_1 \times K_1^s)$  matrix. Finally, we propose the estimators:

$$\widehat{\mathbf{F}}_1 = \widehat{\mathbf{H}}_1 \widehat{\mathbf{W}}_1^c, \quad \widehat{\mathbf{G}}_1 = \widehat{\mathbf{H}}_1 \widehat{\mathbf{W}}_1^s.$$

Canonical Correlation Analysis (CCA) is designed to find linear combinations of two sets of variables that maximize their correlation. In our two-country setting, because both  $\widehat{\mathbf{H}}_1$  and  $\widehat{\mathbf{H}}_2$  contain the common factors, CCA will extract the common factors, which generate correlations of one, and then proceed to the country-specific factors, which generate correlations strictly less than one. Accordingly, we recover the common factors from  $\widehat{\mathbf{F}}_1$  using the first  $K^c$  canonical directions and the country-specific factors from  $\widehat{\mathbf{G}}_1$  using the next  $K_1^s$  canonical directions. The following theorem formalizes this result.

**Theorem 3.2.** *Under Assumptions 1 and 2, it holds that  $\widehat{\mathbf{F}}_g \xrightarrow{p} \mathbf{J}_T \mathbf{F}^c$  and  $\widehat{\mathbf{G}}_g \xrightarrow{p}$*

$\mathbf{J}_T \mathbf{G}_g \mathcal{S}_g^s$  for some  $\mathcal{S}^c$  such that  $\mathcal{S}^c \mathcal{S}^{c'} = \Sigma_F^{-1}$  and some  $\mathcal{S}_g^s$  such that  $\mathcal{S}_g^s \mathcal{S}_g^{s'} = \Sigma_{G_g}^{-1}$  for  $g = 1, 2$ .

**Step 2: Estimation of factor loading matrices on common and country-specific factors.** Recall that in Step 1, we introduce  $\widehat{\mathbf{R}}_g = \mathbf{P}_g \mathbf{R}_g \mathbf{J}_T$  and show the following property of  $\widehat{\mathbf{R}}_g$ :

$$\widehat{\mathbf{R}}_g \approx \mathbf{X}_g \Theta_g^c \mathbf{F}' \mathbf{J}_T + \mathbf{X}_g \Theta_g^s \mathbf{G}'_g \mathbf{J}_T.$$

Because we already obtain consistent estimators  $\widehat{\mathbf{F}}'_g \xrightarrow{p} \mathcal{S}^{c'} \mathbf{F}' \mathbf{J}_T$  and  $\widehat{\mathbf{G}}'_g \xrightarrow{p} \mathcal{S}_g^{s'} \mathbf{G}'_g \mathbf{J}_T$  from Step 1,  $\Theta_g^c$  and  $\Theta_g^s$  can be recovered by regressing  $\widehat{\mathbf{R}}_g$  on the interactions of  $\mathbf{X}_g$  with  $\widehat{\mathbf{F}}'_g$  and  $\widehat{\mathbf{G}}'_g$ . The following theorem formalizes this procedure.

**Theorem 3.3.** *Let  $\widehat{\Theta}_g^c$  (a  $(L_g \times K^c)$  matrix) and  $\widehat{\Theta}_g^s$  (a  $(L_g \times K_g^s)$  matrix) solve the following optimization problem:*

$$\left( \widehat{\Theta}_g^c, \widehat{\Theta}_g^s \right) = \arg \min_{(\Theta_g^c, \Theta_g^s)} \left\| \widehat{\mathbf{R}}_g - (\mathbf{X}_g \Theta_g^c) \widehat{\mathbf{F}}'_g - (\mathbf{X}_g \Theta_g^s) \widehat{\mathbf{G}}'_g \right\|. \quad (3.3)$$

*Under Assumptions 1 and 2, it holds that  $\widehat{\Theta}_g^c \xrightarrow{p} \Theta_g^c \mathcal{S}^{c'-1}$  and  $\widehat{\Theta}_g^s \xrightarrow{p} \Theta_g^s \mathcal{S}_g^{s'-1}$  for some  $\mathcal{S}^c$  such that  $\mathcal{S}^c \mathcal{S}^{c'} = \Sigma_F^{-1}$  and some  $\mathcal{S}_g^s$  such that  $\mathcal{S}_g^s \mathcal{S}_g^{s'} = \Sigma_{G_g}^{-1}$  for  $g = 1, 2$ .*

The optimization problem in the above theorem can be transformed into a conventional ordinary least squares problem.

**Step 3: Construction of the Risk Gap and Price Gap Portfolios.** We next construct the Risk Gap and Price Gap portfolios. It is convenient to rewrite (2.6) as follows:

$$\mathbf{R}_g = (\mathbf{X}_g \Theta_g^c) (\boldsymbol{\lambda}_g^c \mathbf{1}'_T + \mathbf{F}') + (\mathbf{X}_g \Theta_g^s) (\boldsymbol{\lambda}_g^s \mathbf{1}'_T + \mathbf{G}'_g) + \mathbf{U}_g, \quad (3.4)$$

where  $\mathbf{U}_g = \Gamma_g^c (\boldsymbol{\lambda}_g^c \mathbf{1}'_T + \mathbf{F}') + \Gamma_g^s (\boldsymbol{\lambda}_g^s \mathbf{1}'_T + \mathbf{G}'_g) + \mathbf{E}_g$ .

Consider  $(N_g \times K_g)$  infeasible portfolios  $\mathbf{w}_g^b = \frac{\mathbf{x}_g}{N} [\Theta_g^c \ \Theta_g^s] \left( [\Theta_g^c \ \Theta_g^s]' \mathbf{V}_{X_g} [\Theta_g^c \ \Theta_g^s] \right)^{-1}$ . These portfolios deliver common factors and country-specific factors along with their

associated premium as follows:

$$\mathbf{w}_g^{b'} \mathbf{R}_g = \left( [\Theta_g^c \ \Theta_g^s]' \mathbf{V}_{X_g} [\Theta_g^c \ \Theta_g^s] \right)^{-1} [\Theta_g^c \ \Theta_g^s]' \frac{\mathbf{X}_g' \mathbf{X}_g}{N} [\Theta_g^c \ \Theta_g^s] \begin{bmatrix} \boldsymbol{\lambda}_g^c \mathbf{1}_T' + \mathbf{F}' \\ \boldsymbol{\lambda}_g^s \mathbf{1}_T' + \mathbf{G}_g' \end{bmatrix} \quad (3.5)$$

$$+ \left( [\Theta_g^c \ \Theta_g^s]' \mathbf{V}_{X_g} [\Theta_g^c \ \Theta_g^s] \right)^{-1} [\Theta_g^c \ \Theta_g^s]' \frac{\mathbf{X}_g' \mathbf{U}_g}{N} \quad (3.6)$$

$$\xrightarrow{p} \begin{bmatrix} \boldsymbol{\lambda}_g^c \mathbf{1}_T' + \mathbf{F}' \\ \boldsymbol{\lambda}_g^s \mathbf{1}_T' + \mathbf{G}_g' \end{bmatrix}, \quad (3.7)$$

where the last limit follows from Assumptions 1(ii).

The next theorem shows that the feasible analogue of

$$\widehat{\mathbf{w}}_g^b = \frac{\mathbf{X}_g}{N_g} [\widehat{\Theta}_g^c \ \widehat{\Theta}_g^s] \left( [\widehat{\Theta}_g^c \ \widehat{\Theta}_g^s]' \frac{\mathbf{X}_g' \mathbf{X}_g}{N_g} [\widehat{\Theta}_g^c \ \widehat{\Theta}_g^s] \right)^{-1}$$

constructed using the Step 2 results also converges to the same limits, up to a rotation.

**Theorem 3.4.** *Under Assumptions 1 and 2, it holds that  $\widehat{\mathbf{w}}_g^{b'} \mathbf{R}_g \xrightarrow{p} \begin{bmatrix} \mathcal{S}^c (\boldsymbol{\lambda}_g^c \mathbf{1}_T' + \mathbf{F}') \\ \mathcal{S}_g^{s'} (\boldsymbol{\lambda}_g^s \mathbf{1}_T' + \mathbf{G}_g') \end{bmatrix}$ , for some  $\mathcal{S}^c$  such that  $\mathcal{S}^c \mathcal{S}^{c'} = \Sigma_F^{-1}$  and some  $\mathcal{S}_g^s$  such that  $\mathcal{S}_g^s \mathcal{S}_g^{s'} = \Sigma_{G_g}^{-1}$  for  $g = 1, 2$ .*

The above theorem has the following implication. If we partition  $\widehat{w}_g^b$  into  $[\widehat{w}_g^c \ \widehat{w}_g^s]$ , where  $\widehat{w}_g^c$  is an  $(N_g \times K^c)$  matrix and  $\widehat{w}_g^s$  is an  $(N_g \times K_g^s)$  matrix, investors can use the portfolios  $\widehat{w}_g^s$  to gain exposure to the country-specific factors in country  $g$ . From the perspective of the country 1 investor, the portfolios  $\widehat{w}_2^s$  provide the building blocks of the Risk Gap portfolio.

Furthermore, it holds that  $\widehat{\mathbf{w}}_1^{c'} \mathbf{R}_1 \xrightarrow{p} \mathcal{S}^c (\boldsymbol{\lambda}_1^c \mathbf{1}_T' + \mathbf{F}')$  and  $\widehat{\mathbf{w}}_2^{c'} \mathbf{R}_2 \xrightarrow{p} \mathcal{S}^c (\boldsymbol{\lambda}_2^c \mathbf{1}_T' + \mathbf{F}')$ . Thus, the two sets of portfolios recover the same common-factor innovations but potentially different premia. Comparing their returns therefore allows investors to identify the Price Gap. In particular, the difference in common-factor premia can be estimated through the time-series regression formalized in the following theorem.

**Theorem 3.5.** *Regress  $\widehat{\mathbf{w}}_2^{c'} \mathbf{R}_2$  on a constant and  $\widehat{\mathbf{w}}_1^{c'} \mathbf{R}_1$ . Under Assumptions 1 and 2, the  $(K^c \times 1)$  intercept vector converges to  $\mathcal{S}^c (\boldsymbol{\lambda}_2^c - \boldsymbol{\lambda}_1^c)$  for some  $\mathcal{S}^c$  such that  $\mathcal{S}^c \mathcal{S}^{c'} = \Sigma_F^{-1}$ .*

The above theorem provides the basis for constructing the Price Gap Portfolio. If an element of the intercept vector is substantially positive, the investor buys the corre-

sponding common-factor portfolio in country 2 and takes an appropriate short position in its counterpart in country 1. The resulting long-short position forms the corresponding component of the Price Gap portfolio: it hedges the common-factor innovation and isolates the difference in premia across the two countries.

## 4 Data & Implementation

### 4.1 Data

Our dataset builds on Jensen et al. (2023) and covers 26 equity markets summarized in Table 1. Consistent with the two-country framework, we designate the United States as Country 1, the local market, and treat each of the remaining 25 markets in turn as Country 2, the foreign market. We classify the six other G7 members: Canada, France, Germany, Italy, Japan, and the United Kingdom as G7 markets and the remaining 19 foreign markets as non-G7 markets. Return data are from CRSP for the United States and Compustat for all other markets, while accounting data are from Compustat for all markets. Sample coverage varies across markets, beginning as early as January 1966 for the United States; all samples extend through December 2024. Because foreign vendor returns can contain material errors, we apply the return-cleaning procedure detailed in Appendix E to all non-US returns; US returns are exempt.

For each country, we retain only securities whose primary listing is in that market, thereby excluding cross-listed securities. We exclude the smallest 20% of firms in each country-year, ranked on company market capitalization in US dollars as of the preceding December; the same screen governs the out-of-sample month. This screen limits the influence of illiquid and micro-cap stocks without introducing look-ahead bias. We then construct up to 153 firm characteristics following Jensen et al. (2023).<sup>6</sup> We impute missing characteristics using the procedure of Freyberger et al. (2024). Within each country-month, we rank-normalize each characteristic to the  $[0, 1]$  interval, preserving its cross-sectional information while limiting the influence of outliers.

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<sup>6</sup>We thank the authors for sharing their code.

Table 1: Summary statistics.

This table reports sample coverage, the number of distinct firms, the distribution of firm-level market capitalization (in USD millions), the cross-sectional sample size within each estimation window, and summary statistics for value-weighted market excess returns across twenty-six countries. Countries are identified by their ISO three-letter codes and ordered alphabetically by code. Market capitalization columns report the 25th percentile ( $q_{25}$ ), mean (Avg.), and 75th percentile ( $q_{75}$ ). Cross-sectional columns report the minimum ( $N_{\min}$ ), mean ( $\bar{N}$ ), and maximum ( $N_{\max}$ ) number of firms across estimation windows. The final two columns report the annualized mean,  $\mu(R_M)$ , and standard deviation,  $\sigma(R_M)$ , of each country's value-weighted market excess return.

| Code | Beg.     | End.     | # Firms | Market Cap. |         |          | Cross Section |           |            | Excess Return |               |
|------|----------|----------|---------|-------------|---------|----------|---------------|-----------|------------|---------------|---------------|
|      |          |          |         | $q_{25}$    | Avg.    | $q_{75}$ | $N_{\min}$    | $\bar{N}$ | $N_{\max}$ | $\mu(R_M)$    | $\sigma(R_M)$ |
| ARG  | Jun-2002 | Dec-2024 | 108     | 53.83       | 889.17  | 710.15   | 33            | 54.24     | 69         | 0.17          | 0.40          |
| AUS  | Feb-1986 | Dec-2024 | 3822    | 18.11       | 877.17  | 238.82   | 119           | 865.96    | 1580       | 0.09          | 0.22          |
| AUT  | Jul-1995 | Dec-2024 | 181     | 112.48      | 1551.00 | 1624.20  | 46            | 61.22     | 79         | 0.05          | 0.22          |
| BEL  | Apr-1990 | Dec-2024 | 292     | 112.26      | 2417.45 | 1422.42  | 24            | 103.29    | 135        | 0.06          | 0.18          |
| BRA  | Aug-2005 | Dec-2024 | 351     | 319.69      | 2786.25 | 2424.18  | 47            | 147.52    | 222        | 0.09          | 0.31          |
| CAN  | Jan-1983 | Dec-2024 | 3241    | 31.68       | 1420.27 | 519.59   | 166           | 754.86    | 1431       | 0.07          | 0.18          |
| CHL  | Jun-1993 | Dec-2024 | 235     | 157.76      | 1346.94 | 1284.29  | 28            | 105.49    | 145        | 0.07          | 0.22          |
| DEU  | Jan-1987 | Dec-2024 | 1848    | 44.56       | 2490.90 | 777.20   | 75            | 508.48    | 832        | 0.06          | 0.20          |
| DNK  | Jan-1991 | Dec-2024 | 386     | 52.05       | 1888.88 | 679.23   | 24            | 117.49    | 167        | 0.10          | 0.18          |
| ESP  | Jan-1987 | Dec-2024 | 441     | 189.17      | 4057.08 | 2425.10  | 32            | 116.95    | 197        | 0.07          | 0.22          |
| FRA  | Jan-1987 | Dec-2024 | 1872    | 64.40       | 3136.00 | 932.07   | 95            | 486.21    | 696        | 0.07          | 0.20          |
| GBR  | Jan-1987 | Dec-2024 | 5609    | 46.40       | 1839.99 | 583.97   | 509           | 1238.00   | 1836       | 0.06          | 0.17          |
| HKG  | Jan-1987 | Dec-2024 | 3034    | 73.55       | 1860.52 | 700.46   | 39            | 863.88    | 1963       | 0.08          | 0.26          |
| IND  | Jan-1989 | Dec-2024 | 5891    | 7.85        | 743.54  | 188.61   | 29            | 1432.22   | 4073       | 0.11          | 0.31          |
| ITA  | Jan-1987 | Dec-2024 | 845     | 119.03      | 2511.55 | 1406.41  | 43            | 200.28    | 340        | 0.04          | 0.23          |
| JPN  | Jan-1987 | Dec-2024 | 5846    | 94.90       | 1607.34 | 841.68   | 1112          | 2564.67   | 3259       | 0.02          | 0.20          |
| KOR  | Aug-1988 | Dec-2024 | 3538    | 52.67       | 705.22  | 254.93   | 33            | 1047.46   | 2060       | 0.04          | 0.32          |
| MEX  | May-1995 | Dec-2024 | 233     | 424.34      | 3220.74 | 2705.67  | 33            | 78.86     | 105        | 0.09          | 0.24          |
| NOR  | Jun-1989 | Dec-2024 | 704     | 83.09       | 1249.33 | 645.48   | 26            | 146.12    | 277        | 0.08          | 0.24          |
| POL  | Oct-1995 | Dec-2024 | 1201    | 8.88        | 323.57  | 110.63   | 31            | 353.89    | 676        | 0.07          | 0.31          |
| SGP  | Jan-1987 | Dec-2024 | 1154    | 40.74       | 883.08  | 371.01   | 29            | 354.65    | 662        | 0.08          | 0.23          |
| SWE  | Jan-1992 | Dec-2024 | 1452    | 35.29       | 1355.93 | 590.45   | 49            | 326.64    | 801        | 0.10          | 0.24          |
| THA  | Aug-1988 | Dec-2024 | 1202    | 41.51       | 578.00  | 280.34   | 43            | 387.42    | 716        | 0.07          | 0.30          |
| TUR  | Dec-1993 | Dec-2024 | 704     | 44.12       | 641.90  | 382.37   | 51            | 246.22    | 452        | 0.15          | 0.46          |
| TWN  | Jan-1989 | Dec-2024 | 2714    | 70.25       | 781.58  | 388.20   | 38            | 906.41    | 1805       | 0.06          | 0.30          |
| USA  | Jan-1966 | Dec-2024 | 26153   | 48.62       | 3132.69 | 898.31   | 1548          | 4003.90   | 6421       | 0.07          | 0.16          |

Returns are measured from the perspective of a US investor: foreign returns are converted into US dollars, and all returns are expressed in excess of the US risk-free rate. Table 1 reports the sample coverage, number of distinct firms, distribution of firm-level market capitalization in USD millions, cross-sectional sample size within the estimation

windows, and annualized mean and standard deviation of each country’s value-weighted market excess return. The United States has the largest sample, with 26,153 distinct firms over the sample period and an average firm-level market capitalization of \$3,132.69 million, whereas Argentina has the smallest sample, with 108 firms and an average market capitalization of \$889.17 million. Across countries, annualized mean market excess returns range from 2% in Japan to 17% in Argentina, while annualized volatility ranges from 16% in the United States to 46% in Turkey.

## 4.2 Implementation

We implement the Risk Gap and Price Gap portfolios developed in Section 3 separately for each of the 25 US–foreign country pairs. At the end of month  $t$ , we estimate the factor spaces and portfolio weights using a rolling 36-month window ending in that month and hold the resulting portfolios in month  $t + 1$ . Repeating this procedure each month produces fully out-of-sample return series with one-month holding periods.

We set the total number of systematic factors in each market to seven,  $K_1 = K_2 = 7$ . The simulation evidence in Section 8 shows that overestimating the total number of factors is less consequential than underestimating it, supporting this relatively flexible specification. Within each country pair and estimation window, we select the number of common factors,  $K^c$ , using the bootstrap criterion of Gonçalves et al. (2025). The remaining  $K_g^s = K_g - K^c$  factors in market  $g$  are classified as country-specific.<sup>7</sup>

Figure 10 summarizes the distribution of canonical correlations between the estimated US and foreign factor spaces across estimation windows. The first two canonical correlations are generally high, typically exceeding 0.7, whereas the sixth and seventh are consistently below 0.2. This pattern appears among both G7 and non-G7 markets, although the canonical correlations for the US–Canada pair tend to be somewhat higher. The distributions therefore indicate that the estimated factor spaces contain both substantial common variation and distinct country-specific components.

Having specified the factor dimensions, we now describe how the estimated factor portfolios are combined to implement the two gap strategies. For an estimation window ending in month  $t$ , Theorem 3.4 provides the  $(N_g \times K_g^s)$  matrix of country-specific factor-portfolio weights,  $\hat{\mathbf{w}}_{g,t}^s$ , and the  $(N_g \times K^c)$  matrix of common-factor portfolio

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<sup>7</sup>As a robustness check, we also consider a fixed specification with  $K_1^c = K_2^c = 4$ . The main conclusions remain unchanged (Appendix F.3).

weights,  $\widehat{\mathbf{w}}_{g,t}^c$ .

**Risk Gap Portfolio.** From the perspective of the Country 1 investor, the columns of  $\widehat{\mathbf{w}}_{2,t}^s$  provide exposure to systematic risks in Country 2 that are not spanned by the local market. We combine these  $K_2^s$  building-block portfolios into a single Risk Gap portfolio using their estimated means and covariance matrix. Let  $\widehat{\mu}_t^s$  and  $\widehat{\Sigma}_t^s$  denote the sample mean vector and covariance matrix of the returns on these portfolios over the estimation window. The out-of-sample return on the Risk Gap portfolio in month  $t + 1$  is

$$r_{t+1}^{RG} = \widehat{\mu}_t^{s'} \left( \widehat{\Sigma}_t^s \right)^{-1} \left( \widehat{\mathbf{w}}_{2,t}^{s'} \mathbf{r}_{2,t+1} \right).$$

**Price Gap Portfolio.** We next implement Theorem 3.5 to exploit differences in the premia of common factors across Countries 1 and 2. Over the estimation window, we estimate the vector regression

$$\widehat{\mathbf{w}}_{2,t}^{c'} \mathbf{r}_{2,\tau} = \Delta_\lambda + \Psi \widehat{\mathbf{w}}_{1,t}^{c'} \mathbf{r}_{1,\tau} + \epsilon_\tau,$$

where  $\tau$  indexes months within the estimation window,  $\Delta_\lambda$  is a  $(K^c \times 1)$  vector of differences in common-factor premia, and  $\Psi$  is a  $(K^c \times K^c)$  matrix that aligns the common-factor portfolios across the two markets. Using ordinary least squares, we obtain  $\widehat{\Delta}_\lambda$ ,  $\widehat{\Psi}$ , and the covariance matrix of  $\widehat{\Delta}_\lambda$ , denoted by  $\widehat{\Sigma}_{\Delta_\lambda}$ . The out-of-sample return on the Price Gap portfolio in month  $t + 1$  is<sup>8</sup>

$$r_{t+1}^{PG} = \widehat{\Delta}_\lambda' \widehat{\Sigma}_{\Delta_\lambda}^{-1} \left( \widehat{\mathbf{w}}_{2,t}^{c'} \mathbf{r}_{2,t+1} - \widehat{\Psi} \widehat{\mathbf{w}}_{1,t}^{c'} \mathbf{r}_{1,t+1} \right).$$

Before holding either portfolio in month  $t + 1$ , we rescale its weights so that its in-sample annualized standard deviation equals 20%. Having constructed the Risk Gap and Price Gap portfolio returns for each country pair, Section 5 examines the extent to which these two gaps arise across international equity markets.

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<sup>8</sup>This construction is equivalent to mean-variance optimization using the sample moments of the matched common-factor spread returns  $\widehat{\mathbf{w}}_{2,t}^{c'} \mathbf{r}_{2,\tau} - \widehat{\Psi} \widehat{\mathbf{w}}_{1,t}^{c'} \mathbf{r}_{1,\tau}$  over the estimation window.

## 5 Empirical Evidence on Risk and Price Gaps

This section examines two questions implied by our framework. First, do foreign markets contain priced systematic risks that are not spanned by the US market? Second, do systematic risks shared by the US and foreign markets command different premia across them? For each of the 25 US–foreign country pairs, we use the out-of-sample returns of the Risk Gap and Price Gap portfolios to answer these questions. We report the results for G7 and non-G7 markets in parallel to assess how pervasive the two gaps are across international equity markets.

### 5.1 Evidence on Risk Gaps

A Risk Gap arises when a foreign market contains priced systematic risks that are not spanned by the US market. The Risk Gap portfolio isolates exposure to these risks. A positive out-of-sample mean return therefore provides evidence that the foreign market contains compensated systematic risks that cannot be replicated using US assets.

Table 2 reports the out-of-sample return properties of the Risk Gap portfolios, with G7 markets in Panel A and non-G7 markets in Panel B. Among the G7 markets, annualized mean excess returns range from 5.2% for Japan to 16.8% for the United Kingdom, with four of the six mean returns statistically significant at the 1% level. Annualized volatilities remain close to the 20% in-sample target, and Sharpe ratios range from 0.26 for Japan to 0.91 for Germany. Correlations with the US market are low throughout, ranging from  $-0.08$  for Germany to  $0.11$  for France.

Panel B conveys a similar message with a wider spread. Annualized mean excess returns range from  $-1.9\%$  for Argentina to  $17.5\%$  for Hong Kong, while Sharpe ratios range from  $-0.09$  to  $0.88$ . Nine of the nineteen mean returns are statistically significant at the 1% level. Correlations with the US market remain low, ranging from  $-0.10$  for Norway to  $0.07$  for Sweden and Turkey, much as for the G7 markets.

Taken together, the results provide broad evidence of Risk Gaps across international equity markets. Mean returns are positive for 23 of the 25 country pairs and statistically significant at the 1% level for thirteen. The low correlations with the US market further indicate that the risks isolated by these portfolios are distinct from aggregate US market movements. We next examine whether risks shared by the US and foreign markets also command different premia across them.

Table 2: Properties of the Risk Gap Portfolios.

This table reports summary statistics for the out-of-sample returns of the Risk Gap Portfolio constructed for each pairing of the United States (Country 1) with a foreign market (Country 2). Panel A presents the G7 markets and Panel B presents the non-G7 markets. Foreign markets are identified by the three-letter country codes reported in Table 1. The reported statistics are the annualized mean excess return (Mean), annualized standard deviation (SD), Newey–West  $t$ -statistic for the mean return, annualized Sharpe ratio, worst and best monthly returns, correlation with the US market factor (Fama–French  $mktrf$ ,  $\rho_{US}$ ), and number of out-of-sample monthly observations ( $T_{OOS}$ ). The estimation window is 36 months.

| Country 2                      | Mean<br>(%) | SD<br>(%) | $t$   | Sharpe | Worst<br>(%) | Best<br>(%) | $\rho_{US}$ | $T_{OOS}$ |
|--------------------------------|-------------|-----------|-------|--------|--------------|-------------|-------------|-----------|
| <i>Panel A: G7 markets</i>     |             |           |       |        |              |             |             |           |
| CAN                            | 9.2         | 19.3      | 2.90  | 0.48   | −22.0        | 21.8        | 0.05        | 468       |
| DEU                            | 16.7        | 18.3      | 4.71  | 0.91   | −17.4        | 23.9        | −0.08       | 420       |
| FRA                            | 13.0        | 19.0      | 3.41  | 0.68   | −19.1        | 19.8        | 0.11        | 420       |
| GBR                            | 16.8        | 21.4      | 4.11  | 0.79   | −39.1        | 24.0        | −0.07       | 420       |
| ITA                            | 8.7         | 20.8      | 2.51  | 0.42   | −24.7        | 32.8        | 0.05        | 420       |
| JPN                            | 5.2         | 19.7      | 1.61  | 0.26   | −29.4        | 22.6        | −0.03       | 420       |
| <i>Panel B: non-G7 markets</i> |             |           |       |        |              |             |             |           |
| ARG                            | −1.9        | 20.6      | −0.42 | −0.09  | −23.4        | 21.0        | −0.03       | 235       |
| AUS                            | 16.9        | 19.2      | 4.80  | 0.88   | −23.7        | 20.8        | −0.04       | 431       |
| AUT                            | 13.1        | 18.7      | 3.35  | 0.70   | −16.9        | 23.1        | 0.05        | 318       |
| BEL                            | 7.8         | 19.3      | 2.24  | 0.40   | −33.6        | 20.4        | −0.01       | 381       |
| BRA                            | 0.7         | 19.6      | 0.15  | 0.04   | −26.0        | 13.6        | −0.03       | 197       |
| CHL                            | 5.8         | 19.4      | 1.65  | 0.30   | −14.7        | 24.4        | −0.06       | 343       |
| DNK                            | 5.4         | 21.2      | 1.16  | 0.25   | −22.6        | 26.6        | 0.06        | 372       |
| ESP                            | 10.1        | 19.1      | 3.68  | 0.53   | −21.6        | 18.8        | 0.02        | 420       |
| HKG                            | 17.5        | 22.6      | 4.75  | 0.77   | −26.1        | 33.0        | 0.02        | 420       |
| IND                            | 14.8        | 21.2      | 3.59  | 0.70   | −31.1        | 25.5        | −0.09       | 396       |
| KOR                            | −1.2        | 19.2      | −0.34 | −0.06  | −26.4        | 25.7        | −0.04       | 401       |
| MEX                            | 3.2         | 19.9      | 0.82  | 0.16   | −24.7        | 18.5        | −0.06       | 320       |
| NOR                            | 9.1         | 19.8      | 2.64  | 0.46   | −23.3        | 24.7        | −0.10       | 391       |
| POL                            | 11.9        | 18.5      | 3.05  | 0.65   | −16.0        | 23.7        | −0.05       | 315       |
| SGP                            | 15.8        | 26.2      | 3.44  | 0.60   | −20.9        | 92.3        | 0.04        | 420       |
| SWE                            | 16.0        | 20.0      | 3.88  | 0.80   | −18.4        | 22.8        | 0.07        | 360       |
| THA                            | 6.1         | 20.3      | 1.52  | 0.30   | −51.4        | 20.1        | −0.09       | 401       |
| TUR                            | 5.1         | 18.4      | 1.37  | 0.27   | −18.0        | 17.7        | 0.07        | 337       |
| TWN                            | 5.9         | 20.2      | 1.43  | 0.29   | −19.9        | 26.1        | −0.03       | 396       |

## 5.2 Evidence on Price Gaps

A Price Gap arises when a systematic risk common to the US and foreign markets commands different premia across them. The Price Gap portfolio takes offsetting positions in matched common-factor portfolios, hedging their shared innovations and isolating the difference in premia. A positive out-of-sample mean return therefore provides evidence that common systematic risks are compensated differently across the two markets.

Table 3 reports the out-of-sample performance of the Price Gap portfolios, with G7 markets in Panel A and non-G7 markets in Panel B. Among the G7 markets, annualized mean excess returns range from 0.0% for Canada to 17.3% for Japan, with two of the six mean returns statistically significant at the 1% level. Sharpe ratios range from 0.00 for Canada to 0.62 for Japan. Correlations with the US market remain low, ranging from  $-0.12$  for Italy to  $0.11$  for Canada.

Panel B conveys a similar message with a wider spread. Annualized mean excess returns range from  $-2.7\%$  for Argentina to  $25.7\%$  for Spain, while Sharpe ratios range from  $-0.10$  to  $0.96$ . Eight of the nineteen mean returns are statistically significant at the 1% level. Correlations with the US market remain low, ranging from  $-0.08$  for Mexico to  $0.13$  for Chile. Across both panels, ten of the 25 Price Gap portfolios have mean returns significant at the 1% level, compared with thirteen of the 25 Risk Gap portfolios. The evidence for the Price Gap is therefore somewhat less widespread, but it appears in both country groups, including the G7 markets where the presumption of integration is strongest.

Across both panels, realized annualized volatilities range from  $24.4\%$  to  $28.7\%$ , exceeding the 20% in-sample scaling target. Unlike the Risk Gap portfolios, the realized volatility of every Price Gap portfolio exceeds the target. This systematic difference suggests that the in-sample risk assessment is more optimistic for the Price Gap construction.<sup>9</sup>

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<sup>9</sup>Our consistency results are asymptotic and do not rule out finite-sample estimation error. The Price Gap construction depends on estimated differences in premia and cross-market hedge coefficients in addition to the portfolio covariance matrix. Finite-sample errors in these objects can make the resulting weights more sensitive to in-sample noise, causing the scaled in-sample volatility to understate subsequent realized risk.

Table 3: Properties of the Price Gap Portfolios.

This table reports summary statistics for the out-of-sample returns of the Price Gap Portfolio constructed for each pairing of the United States (Country 1) with a foreign market (Country 2). Panel A presents the G7 markets and Panel B presents the non-G7 markets. Foreign markets are identified by the three-letter country codes reported in Table 1. The reported statistics are the annualized mean excess return (Mean), annualized standard deviation (SD), Newey–West  $t$ -statistic for the mean return, annualized Sharpe ratio, worst and best monthly returns, correlation with the US market factor (Fama–French  $mktrf$ ,  $\rho_{US}$ ), and number of out-of-sample monthly observations ( $T_{OOS}$ ). The estimation window is 36 months.

| Country 2                      | Mean<br>(%) | SD<br>(%) | $t$   | Sharpe | Worst<br>(%) | Best<br>(%) | $\rho_{US}$ | $T_{OOS}$ |
|--------------------------------|-------------|-----------|-------|--------|--------------|-------------|-------------|-----------|
| <i>Panel A: G7 markets</i>     |             |           |       |        |              |             |             |           |
| CAN                            | 0.0         | 27.5      | 0.00  | 0.00   | −45.8        | 27.5        | 0.11        | 468       |
| DEU                            | 8.3         | 26.4      | 1.82  | 0.31   | −34.2        | 22.2        | −0.01       | 420       |
| FRA                            | 12.3        | 27.8      | 2.37  | 0.44   | −50.4        | 31.5        | −0.00       | 420       |
| GBR                            | 14.1        | 27.1      | 3.33  | 0.52   | −55.7        | 41.1        | 0.00        | 420       |
| ITA                            | 12.8        | 28.2      | 2.55  | 0.45   | −30.5        | 31.7        | −0.12       | 420       |
| JPN                            | 17.3        | 27.9      | 2.71  | 0.62   | −33.9        | 28.1        | −0.00       | 420       |
| <i>Panel B: non-G7 markets</i> |             |           |       |        |              |             |             |           |
| ARG                            | −2.7        | 26.5      | −0.51 | −0.10  | −32.6        | 30.2        | 0.07        | 235       |
| AUS                            | 12.7        | 27.4      | 2.38  | 0.46   | −27.6        | 26.4        | −0.05       | 431       |
| AUT                            | 20.1        | 25.8      | 4.08  | 0.78   | −25.7        | 28.7        | 0.05        | 318       |
| BEL                            | 18.6        | 25.2      | 4.58  | 0.74   | −27.1        | 24.6        | 0.03        | 381       |
| BRA                            | 17.7        | 24.5      | 3.11  | 0.72   | −16.1        | 32.2        | −0.00       | 197       |
| CHL                            | 4.0         | 28.7      | 0.77  | 0.14   | −49.2        | 33.1        | 0.13        | 343       |
| DNK                            | 13.2        | 28.1      | 2.55  | 0.47   | −37.5        | 32.8        | 0.00        | 372       |
| ESP                            | 25.7        | 26.7      | 5.44  | 0.96   | −25.1        | 34.7        | −0.01       | 420       |
| HKG                            | 8.2         | 26.0      | 1.69  | 0.32   | −21.9        | 38.3        | 0.02        | 420       |
| IND                            | 13.8        | 27.3      | 2.59  | 0.50   | −65.9        | 42.2        | 0.02        | 396       |
| KOR                            | 17.0        | 24.4      | 3.98  | 0.70   | −30.8        | 23.2        | 0.03        | 401       |
| MEX                            | −2.2        | 27.9      | −0.41 | −0.08  | −34.2        | 27.1        | −0.08       | 320       |
| NOR                            | 5.7         | 26.2      | 1.06  | 0.22   | −26.3        | 30.0        | 0.12        | 391       |
| POL                            | 9.5         | 24.4      | 2.03  | 0.39   | −29.4        | 26.6        | −0.07       | 315       |
| SGP                            | 13.6        | 26.1      | 2.96  | 0.52   | −36.5        | 42.6        | −0.02       | 420       |
| SWE                            | 8.1         | 27.0      | 1.53  | 0.30   | −44.0        | 25.0        | 0.03        | 360       |
| THA                            | 11.0        | 28.0      | 2.15  | 0.39   | −32.5        | 47.3        | 0.06        | 401       |
| TUR                            | 13.6        | 25.1      | 2.84  | 0.54   | −18.7        | 29.7        | 0.04        | 337       |
| TWN                            | 7.1         | 24.4      | 1.47  | 0.29   | −27.9        | 21.5        | −0.02       | 396       |

**Direction of the Price Gap.** The baseline results establish economically meaningful differences in the premia attached to common risks. Because these differences

may vary in direction across factors, however, the multifactor Price Gap portfolio does not reveal which market generally pays the higher premium. We therefore conduct a complementary analysis based on the first common factor.

Figure 3 motivates this focus. The figure plots the distribution of average canonical correlations between the common-factor spaces obtained using different foreign markets as Country 2. The leading canonical correlation averages 0.91 across pairs of G7 markets, 0.89 when one market is G7, and 0.87 when neither is. The third averages 0.29, 0.27 and 0.24 across the same three groups. The leading common factor is therefore closely aligned whichever foreign market defines it, while the third is far less so, which supports using the first common factor to examine the direction of the Price Gap across country pairs.

For each US–foreign country pair, we reconstruct the Price Gap portfolio using only the first common factor and normalize its sign so that it comoves positively with the US aggregate market. With a single factor, the direction of the portfolio is transparent: it takes a long position in the common-factor portfolio of the market with the higher estimated premium and a short position in its counterpart in the other market, thereby hedging the shared factor innovation. Table 4 reports the resulting out-of-sample performance, with G7 markets in Panel A and non-G7 markets in Panel B. The final column reports the fraction of estimation windows in which the US common-factor portfolio is the long leg.

The single-factor results provide particularly strong evidence of Price Gaps. Among the G7 markets, annualized mean excess returns range from 4.1% to 9.4%, and Sharpe ratios range from 0.42 to 0.81. All six mean returns are positive and statistically significant at the 5% level. Among the non-G7 markets, annualized mean excess returns range from 2.3% to 12.7%, and Sharpe ratios range from 0.20 to 1.06. All nineteen mean returns are positive, and seventeen are statistically significant at the 5% level.

More importantly, the directional results generally point to higher common-risk premia in the United States. For every G7 market except Canada, the US common-factor portfolio is the long leg in 63% to 73% of the estimation windows. The same pattern appears among the non-G7 markets: the US common-factor portfolio is the long leg in at least half of the estimation windows for every country pair, with the fraction reaching 73% for South Korea. These results suggest that the premium associated with the leading common factor has generally been higher in the United States than abroad.

Taken together, the evidence shows that both Risk Gaps and Price Gaps appear among G7 and non-G7 markets. Foreign markets often contain compensated systematic risks that are not spanned by the US market, while risks shared with the US market frequently command different premia across the two markets. Section 6 evaluates whether the exposures associated with these gaps expand the investment opportunity set of an investor who already holds the US market.

Figure 3: Similarity of the estimated common-factor space across country pairs.

The figure shows how similar the estimated common-factor space is across country pairs. For each of the 25 US–foreign pairs and each rolling estimation window, we compute canonical correlations between the seven US factors and the seven foreign factors and retain the first three canonical variate pairs, averaging the US and the foreign side of each; these three series are that pair’s estimate of the common factors. For each of the 300 combinations of two distinct pairs, we then compute the canonical correlations between the two pairs’ three common factors in every window and average each over windows. Each violin shows the distribution of those averages across combinations. Combinations are grouped by the two foreign markets involved: both G7 (15 combinations), one G7 and one non-G7 (114), and both non-G7 (171). Wider sections indicate higher density; black diamonds mark group means. The estimation window is 36 months.

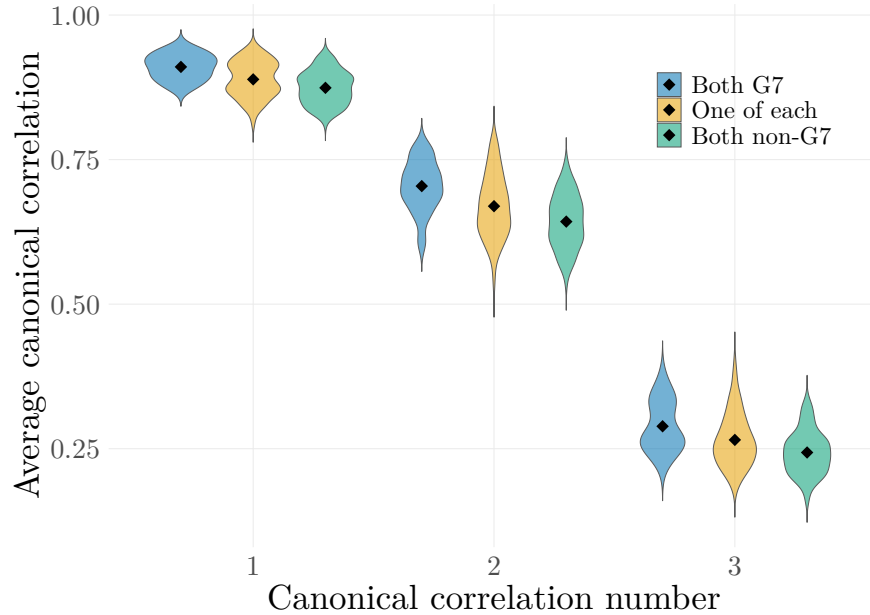


Table 4: Properties of the single-factor Price Gap Portfolios.

This table reports summary statistics for the out-of-sample returns of the Price Gap Portfolio constructed using only the first common factor for each pairing of the United States (Country 1) with a foreign market (Country 2). Panel A presents the G7 markets and Panel B presents the non-G7 markets. Foreign markets are identified by the three-letter country codes reported in Table 1. The reported statistics are the annualized mean excess return (Mean), annualized standard deviation (SD), Newey–West  $t$ -statistic for the mean return, annualized Sharpe ratio, worst and best monthly returns, correlation with the US market factor (Fama–French  $mktrf$ ,  $\rho_{US}$ ), and the fraction of estimation windows in which the US common-factor portfolio is the long leg,  $\text{Pr}(\text{US Long})$ . The estimation window is 36 months.

| Country 2                      | Mean<br>(%) | SD<br>(%) | $t$  | Sharpe | Worst<br>(%) | Best<br>(%) | $\rho_{US}$ | $\text{Pr}(\text{US Long})$ |
|--------------------------------|-------------|-----------|------|--------|--------------|-------------|-------------|-----------------------------|
| <i>Panel A: G7 markets</i>     |             |           |      |        |              |             |             |                             |
| CAN                            | 4.11        | 7.75      | 3.74 | 0.53   | −4.53        | 10.22       | −0.01       | 0.41                        |
| DEU                            | 6.16        | 11.50     | 3.15 | 0.54   | −11.55       | 13.46       | −0.06       | 0.65                        |
| FRA                            | 4.43        | 10.53     | 2.34 | 0.42   | −9.12        | 13.66       | 0.03        | 0.64                        |
| GBR                            | 7.02        | 9.71      | 4.80 | 0.72   | −8.85        | 10.40       | 0.01        | 0.63                        |
| ITA                            | 7.74        | 11.10     | 4.05 | 0.70   | −10.21       | 10.40       | 0.01        | 0.73                        |
| JPN                            | 9.42        | 11.61     | 5.10 | 0.81   | −8.37        | 11.31       | 0.00        | 0.67                        |
| <i>Panel B: non-G7 markets</i> |             |           |      |        |              |             |             |                             |
| ARG                            | 5.49        | 13.16     | 1.87 | 0.42   | −10.82       | 10.69       | −0.06       | 0.56                        |
| AUS                            | 10.35       | 11.55     | 5.33 | 0.90   | −9.48        | 13.73       | −0.09       | 0.68                        |
| AUT                            | 7.04        | 10.82     | 3.38 | 0.65   | −8.25        | 11.25       | −0.04       | 0.69                        |
| BEL                            | 9.02        | 12.47     | 4.03 | 0.72   | −9.60        | 15.42       | 0.01        | 0.65                        |
| BRA                            | 7.79        | 10.43     | 3.70 | 0.75   | −7.61        | 8.34        | −0.01       | 0.63                        |
| CHL                            | 8.76        | 12.03     | 3.76 | 0.73   | −10.89       | 9.77        | −0.01       | 0.63                        |
| DNK                            | 9.53        | 11.82     | 4.39 | 0.81   | −9.59        | 12.02       | −0.03       | 0.56                        |
| ESP                            | 12.69       | 12.01     | 5.70 | 1.06   | −10.58       | 11.93       | 0.03        | 0.70                        |
| HKG                            | 5.56        | 12.15     | 2.55 | 0.46   | −12.48       | 14.24       | −0.10       | 0.70                        |
| IND                            | 8.23        | 12.88     | 3.80 | 0.64   | −11.70       | 10.77       | 0.01        | 0.56                        |
| KOR                            | 11.07       | 12.04     | 4.74 | 0.92   | −8.61        | 14.68       | 0.00        | 0.73                        |
| MEX                            | 2.27        | 11.35     | 0.86 | 0.20   | −7.96        | 10.89       | −0.10       | 0.59                        |
| NOR                            | 4.95        | 10.60     | 2.48 | 0.47   | −9.96        | 13.02       | 0.06        | 0.51                        |
| POL                            | 5.10        | 11.74     | 2.56 | 0.43   | −7.71        | 12.25       | −0.04       | 0.55                        |
| SGP                            | 10.69       | 12.03     | 4.52 | 0.89   | −10.31       | 10.74       | −0.07       | 0.70                        |
| SWE                            | 4.50        | 11.34     | 2.51 | 0.40   | −9.77        | 12.72       | 0.02        | 0.66                        |
| THA                            | 4.96        | 12.29     | 2.07 | 0.40   | −11.86       | 8.91        | −0.07       | 0.66                        |
| TUR                            | 10.46       | 12.84     | 4.32 | 0.81   | −11.71       | 11.99       | 0.02        | 0.55                        |
| TWN                            | 4.95        | 12.08     | 2.33 | 0.41   | −10.65       | 10.17       | 0.01        | 0.50                        |

## 6 Gains for a US Investor

We consider a US investor whose benchmark risky investment is the US market portfolio. The Risk Gap and Price Gap represent two distinct channels through which foreign markets can expand this investor’s opportunity set. Exploiting a Risk Gap provides exposure to compensated foreign systematic risks that cannot be replicated using US assets, whereas exploiting a Price Gap captures differences in the compensation for risks shared across markets while hedging their common innovations. Having documented both gaps for individual US–foreign country pairs, we now evaluate the gains from combining the corresponding portfolios across foreign markets. These gains depend not only on the average returns of the individual portfolios but also on the extent to which their returns comove with the US market and with one another. We therefore begin by examining their correlation structure.

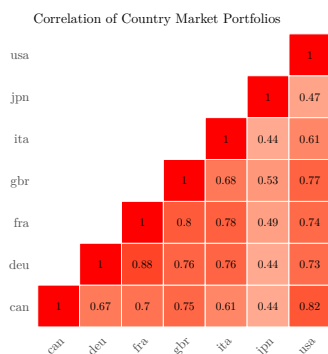
### 6.1 Correlation Structure

Tables 2 and 3 show that the returns of the individual Risk Gap and Price Gap portfolios have low correlations with the US market. This suggests that they contain return variation distinct from the US investor’s existing market exposure. Their correlations with one another are also important, however. If the portfolios constructed from different country pairs are highly correlated, combining them would provide largely redundant sources of returns. Figure 4 therefore compares correlations across conventional country market portfolios, Risk Gap Portfolios, and Price Gap portfolios.

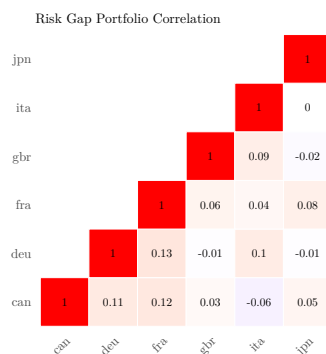
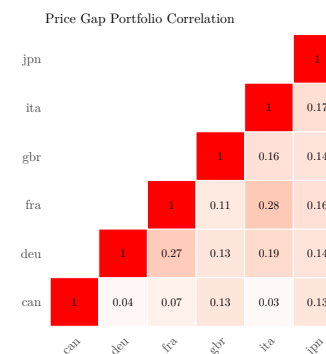
Panels A and D provide the conventional market-portfolio benchmark. Among the G7 markets, pairwise correlations range from 0.44 to 0.88. Correlations among the non-G7 market portfolios are also predominantly positive and substantial. These patterns reflect the large common component in country market returns and help explain why combining conventional foreign indices offers limited additional diversification to a US investor.

Figure 4: Correlation structure of market, Risk Gap, and Price Gap portfolios.

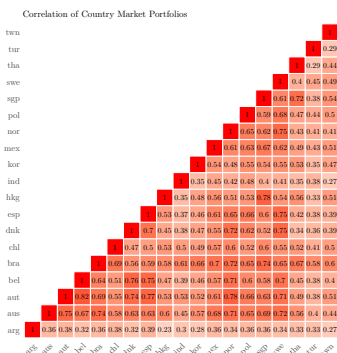
This figure presents heatmaps of pairwise correlations among conventional country market portfolios, Risk Gap Portfolios, and Price Gap Portfolios. Panels A–C present the G7 markets, and Panels D–F present the non-G7 markets. Within each row, the first panel reports correlations among value-weighted country market excess returns, including the US market as a benchmark. The second and third panels report correlations among the out-of-sample Risk Gap and Price Gap Portfolio returns documented in Tables 2 and 3, respectively. Countries are identified by the three-letter codes reported in Table 1.



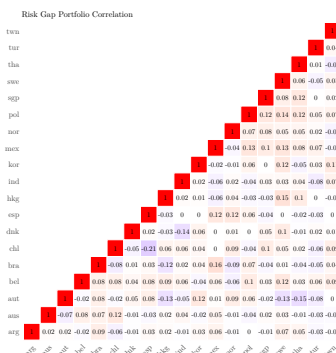
*Panel B: Risk Gap Portfolios,  $G7$*

Panel C: Price Gap Portfolios,  $G7$ 

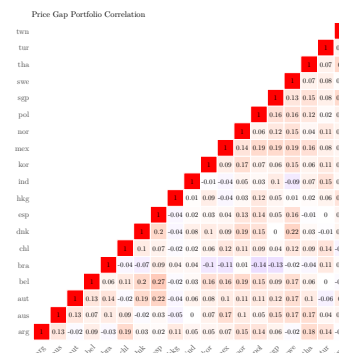
Panel D: Market Portfolios, non-G7



Panel E: Risk Gap Portfolios, non-G7



Panel F: Price Gap Portfolios, non-G7



The Risk Gap portfolios display a sharply different correlation structure. In Panel B, correlations among the G7 portfolios range from  $-0.06$  to  $0.13$ , while Panel E shows similarly low correlations among the non-G7 portfolios. Because each Risk Gap portfolio isolates compensated foreign systematic risks that are not spanned by the US market, these low correlations indicate that the unspanned risks differ substantially across foreign markets.

Correlations among the Price Gap portfolios are also considerably lower than those among conventional market portfolios. In Panel C, correlations among the G7 portfolios range from  $0.03$  to  $0.28$ , and Panel F documents a similar pattern among the non-G7 portfolios. Each Price Gap portfolio hedges innovations in risks shared with the US market and isolates the difference in their compensation across the two markets. The low correlations indicate that the resulting return opportunities remain largely distinct across country pairs.

The gap portfolios thus contain substantially less common return variation than conventional foreign market indices.

## 6.2 Mean–Variance Analysis

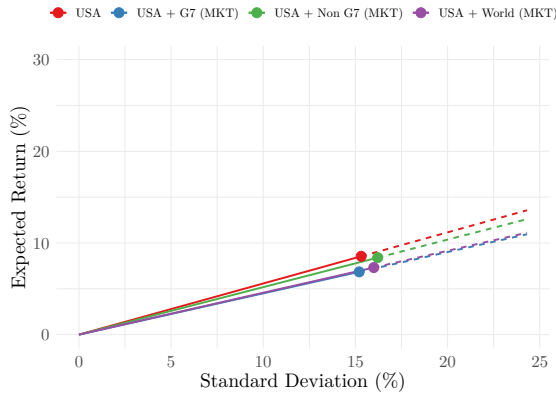
Next, we examine how this difference changes the US investor’s mean–variance tradeoff. Figure 5 compares the capital allocation lines associated with the US market and with equal-weighted combinations of the US market, conventional foreign market portfolios, Risk Gap portfolios, and Price Gap portfolios. A steeper line indicates a higher Sharpe ratio.

The upper-left panel provides the conventional international-diversification benchmark. Adding conventional foreign market portfolios does not improve the mean–variance tradeoff relative to the US market, whether the investor uses the G7 markets, the non-G7 markets, or both groups. This result reflects the substantial common component in country market returns documented in Figure 4 and is consistent with the evidence underlying the view that the gains from conventional international diversification have diminished (Carrieri et al., 2007; Quinn and Voth, 2008; Pukthuanthong and Roll, 2009).

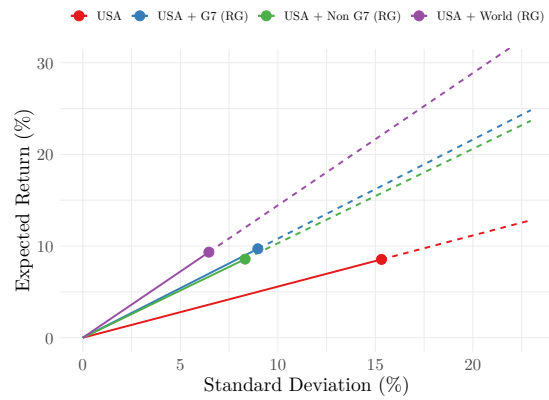
Figure 5: Diversification benefits.

This figure illustrates the mean-variance expansion of the US market portfolio through investments in international assets. The top-left panel shows the effect on the mean-variance efficiency from including country-level market portfolios: The blue line, USA + G7 (MKT), represents an equal-weighted investment in the US market portfolio and the average of the market portfolios of G7 countries. The green line, USA + Non-G7 (MKT), represents an equal-weighted investment in the US market portfolio and the average of the market portfolios of non-G7 countries. The purple line, USA + World (MKT), represents an equal-weighted investment in three assets: the US market portfolio, the average G7 market portfolio, and the average non-G7 market portfolio. The top-right panel repeats the exercise with the foreign countries' Risk Gap Portfolios in place of their market portfolios; the bottom-left panel uses the Price Gap Portfolios; the bottom-right panel uses both. Returns are excess returns relative to the risk-free rate and are annualized.

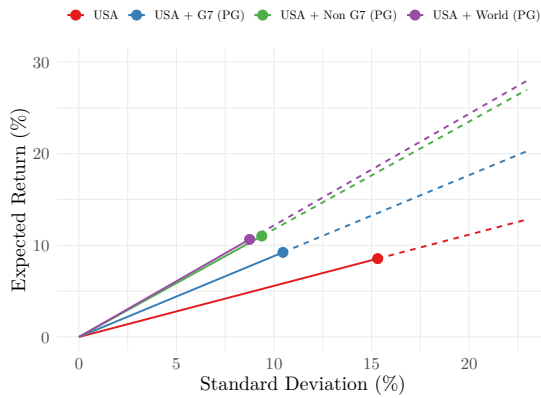
### Market Portfolios



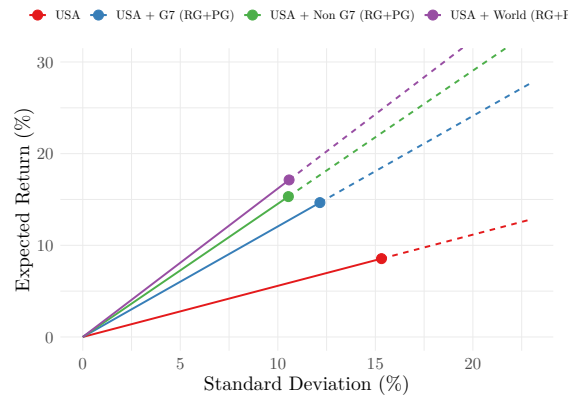
### Risk Gap Portfolios



### Price Gap Portfolios



### Risk Gap and Price Gap Portfolios



The remaining panels reveal considerably larger expansions. The upper-right panel replaces foreign market indices with Risk Gap portfolios, which give the investor access to compensated foreign systematic risks that cannot be replicated using US assets. The

lower-left panel uses Price Gap portfolios, which capture differences in the compensation for shared risks while hedging their common innovations. Both types of portfolios shift the opportunity set outward, and combining them in the lower-right panel produces the largest expansion. The improvement is especially pronounced along the risk dimension, reflecting the low correlations of the gap portfolios with the US market and with one another.

Table 5 reports the summary statistics underlying Figure 5. Equally weighted across all 25 foreign markets, the Risk Gap and Price Gap portfolios earn annualized Sharpe ratios of 1.14 and 1.02; their correlations with the US market are 0.00 and 0.04. Added to the US market, the Risk Gap portfolios raise its Sharpe ratio from 0.56 to 1.44 and the Price Gap portfolios raise it to 1.22; adding both produces Sharpe ratios of 1.21 to 1.62 across country groups.

Figure 5 uses equal-weighted combinations of the US market and the G7 and non-G7 portfolio aggregates to illustrate the mean–variance effects of exploiting the two gaps. Table 6 examines whether the improvement survives under market-capitalization weighting. In each month, the US market and the available foreign countries receive weights proportional to their one-month-lagged country market capitalizations, normalized across all markets available that month. For each foreign country, its market-capitalization weight is applied to the corresponding Risk Gap or Price Gap portfolio rather than to its market portfolio; when both gaps are included, the full country-level weight is applied separately to each gap portfolio.

Over the full sample, the US market earns an annualized mean excess return of 8.8% with a standard deviation of 15.9%, yielding a Sharpe ratio of 0.55. The market-capitalization-weighted world portfolio earns 6.4% with a standard deviation of 15.6%, reducing the Sharpe ratio to 0.41. Conventional international diversification therefore does not improve the mean–variance tradeoff relative to the US market over this sample.

Table 5: Properties of the equal-weighted Risk Gap and Price Gap Portfolios.

This table reports summary statistics for the out-of-sample returns of equal-weighted gap portfolios and of their combinations with the US market. Panel A reports the equal-weighted portfolios of the Risk Gap Portfolios (RG) and the Price Gap Portfolios (PG), averaged each month across the country pairs available at that date, for the G7 markets, the non-G7 markets, and all 25 foreign markets; the RG + PG rows sum the two gap portfolios. Panel B reports the corresponding statistics for combinations with the US market: for a single country group, an equal-weighted investment in the US market portfolio and the group’s gap portfolio; for all countries, an equal-weighted investment in three assets — the US market portfolio, the G7 gap portfolio, and the non-G7 gap portfolio. Panel B uses the months in which the US market factor and all group portfolios are available. The reported statistics are the annualized mean excess return (Mean), annualized standard deviation (SD), Newey–West  $t$ -statistic for the mean return, annualized Sharpe ratio, correlation with the US market factor (Fama–French  $mktrf$ ,  $\rho_{US}$ ), and number of monthly observations ( $T$ ). The estimation window is 36 months.

| Portfolio                                       | Mean<br>(%) | SD<br>(%) | $t$  | Sharpe | $\rho_{US}$ | $T$ |
|-------------------------------------------------|-------------|-----------|------|--------|-------------|-----|
| <i>Panel A: Equal-weighted gap portfolios</i>   |             |           |      |        |             |     |
| RG (G7)                                         | 10.5        | 10.3      | 5.58 | 1.02   | 0.03        | 468 |
| RG (non-G7)                                     | 8.7         | 7.1       | 6.70 | 1.21   | −0.04       | 431 |
| RG (all countries)                              | 8.7         | 7.6       | 5.76 | 1.14   | 0.00        | 468 |
| PG (G7)                                         | 10.5        | 15.8      | 4.06 | 0.66   | 0.01        | 468 |
| PG (non-G7)                                     | 13.6        | 10.4      | 6.00 | 1.31   | 0.03        | 431 |
| PG (all countries)                              | 12.1        | 11.8      | 5.90 | 1.02   | 0.04        | 468 |
| RG + PG (G7)                                    | 21.0        | 21.0      | 5.97 | 1.00   | 0.02        | 468 |
| RG + PG (non-G7)                                | 22.3        | 14.4      | 7.17 | 1.54   | 0.00        | 431 |
| RG + PG (all countries)                         | 20.7        | 16.0      | 7.19 | 1.30   | 0.03        | 468 |
| <i>Panel B: Combinations with the US market</i> |             |           |      |        |             |     |
| US market                                       | 8.5         | 15.3      | 3.29 | 0.56   | 1.00        | 419 |
| US + RG (G7)                                    | 9.7         | 9.0       | 6.27 | 1.08   | 0.86        | 419 |
| US + RG (non-G7)                                | 8.6         | 8.3       | 5.70 | 1.03   | 0.90        | 419 |
| US + RG (all)                                   | 9.3         | 6.5       | 7.83 | 1.44   | 0.78        | 419 |
| US + PG (G7)                                    | 9.2         | 10.5      | 4.79 | 0.88   | 0.71        | 419 |
| US + PG (non-G7)                                | 11.0        | 9.4       | 5.91 | 1.17   | 0.83        | 419 |
| US + PG (all)                                   | 10.6        | 8.7       | 6.26 | 1.22   | 0.57        | 419 |
| US + RG + PG (G7)                               | 14.7        | 12.2      | 6.60 | 1.21   | 0.61        | 419 |
| US + RG + PG (non-G7)                           | 15.3        | 10.5      | 6.95 | 1.45   | 0.72        | 419 |
| US + RG + PG (all)                              | 17.1        | 10.6      | 8.18 | 1.62   | 0.47        | 419 |

Table 6: Diversification benefits — market-cap-weighted combinations.

Each portfolio combines the US value-weighted market with the foreign components shown, every country weighted by its share of total (one-month-lagged) market capitalization that month, renormalized over the countries live at that date. Within a foreign country the Risk Gap and Price Gap Portfolios enter at weight one, and are summed in the RG+PG row.  $\bar{w}_{US}$  is the time-average US weight;  $\bar{n}_{ctry}$  the average number of live foreign components, which grows from a handful of countries in the late 1980s to all 25 from 2010-08. Mean returns, standard deviations and Sharpe ratios are annualized. The estimation window is 36 months.

| Portfolio                                           | Mean<br>(%) | SD<br>(%) | Sharpe | $N$ | $\bar{w}_{US}$<br>(%) | $\bar{n}_{ctry}$ |
|-----------------------------------------------------|-------------|-----------|--------|-----|-----------------------|------------------|
| <i>Panel A: Full sample (1986-01–2024-12)</i>       |             |           |        |     |                       |                  |
| US market only                                      | 8.8         | 15.9      | 0.55   | 468 | 100.0                 | 0.0              |
| World market (mcap-weighted)                        | 6.4         | 15.6      | 0.41   | 468 | 49.0                  | 22.0             |
| US + RG                                             | 10.1        | 10.1      | 1.00   | 468 | 53.4                  | 20.1             |
| US + PG                                             | 10.2        | 11.4      | 0.90   | 468 | 53.4                  | 20.1             |
| US + RG + PG                                        | 15.6        | 13.0      | 1.20   | 468 | 53.4                  | 20.1             |
| <i>Panel B: Pre-2005 (1986-01–2004-12)</i>          |             |           |        |     |                       |                  |
| US market only                                      | 8.1         | 16.0      | 0.51   | 228 | 100.0                 | 0.0              |
| World market (mcap-weighted)                        | 5.1         | 15.4      | 0.34   | 228 | 49.1                  | 18.8             |
| US + RG                                             | 8.3         | 11.7      | 0.71   | 228 | 58.2                  | 15.2             |
| US + PG                                             | 9.4         | 12.8      | 0.73   | 228 | 58.2                  | 15.2             |
| US + RG + PG                                        | 12.9        | 14.7      | 0.87   | 228 | 58.2                  | 15.2             |
| <i>Panel C: Post-2005 (2005-01–2024-12)</i>         |             |           |        |     |                       |                  |
| US market only                                      | 9.4         | 15.8      | 0.60   | 240 | 100.0                 | 0.0              |
| World market (mcap-weighted)                        | 7.5         | 15.9      | 0.47   | 240 | 48.9                  | 25.0             |
| US + RG                                             | 11.8        | 8.3       | 1.43   | 240 | 48.9                  | 24.8             |
| US + PG                                             | 11.0        | 9.9       | 1.11   | 240 | 48.9                  | 24.8             |
| US + RG + PG                                        | 18.2        | 11.0      | 1.65   | 240 | 48.9                  | 24.8             |
| <i>Panel D: All 25 pairs live (2010-08–2024-12)</i> |             |           |        |     |                       |                  |
| US market only                                      | 12.8        | 15.0      | 0.85   | 173 | 100.0                 | 0.0              |
| World market (mcap-weighted)                        | 9.2         | 14.5      | 0.63   | 173 | 50.8                  | 25.0             |
| US + RG                                             | 13.7        | 8.4       | 1.62   | 173 | 50.8                  | 25.0             |
| US + PG                                             | 13.9        | 9.7       | 1.43   | 173 | 50.8                  | 25.0             |
| US + RG + PG                                        | 21.2        | 10.8      | 1.96   | 173 | 50.8                  | 25.0             |

The Risk Gap combination raises the annualized mean return to 10.1% while reducing the standard deviation to 10.1%, for a Sharpe ratio of 1.00. The Price Gap combination earns 10.2% with a standard deviation of 11.4%, yielding a Sharpe ratio of 0.90. Combining both types of portfolios raises the mean return to 15.6% while maintaining a standard deviation of 13.0%. Its Sharpe ratio is 1.20, more than twice that of the US market and nearly three times that of the conventional world portfolio.

Market-capitalization weighting provides a relatively conservative assessment because it tilts the foreign allocation toward markets that comove most strongly with the United States. As an alternative allocation, we weight every component, including the US market, in proportion to its trailing Sharpe ratio. The resulting combined portfolio has a full-sample Sharpe ratio of 1.60, although its average weight on the US market is only 8.6%. Appendix Table 17 reports these results.

### 6.3 Time Variation in Risk Gaps and Price Gaps

A central concern motivating this paper is not that international diversification never worked, but that its gains have diminished over time. The rising comovement of country indices over recent decades is commonly offered as evidence of this decline. This is inherently a claim about a time trend rather than about full-sample averages. Because our country-pair samples begin in the late 1980s or 1990s, they are sufficiently long to examine whether the returns on the Risk Gap and Price Gap Portfolios have declined over time.

We first estimate

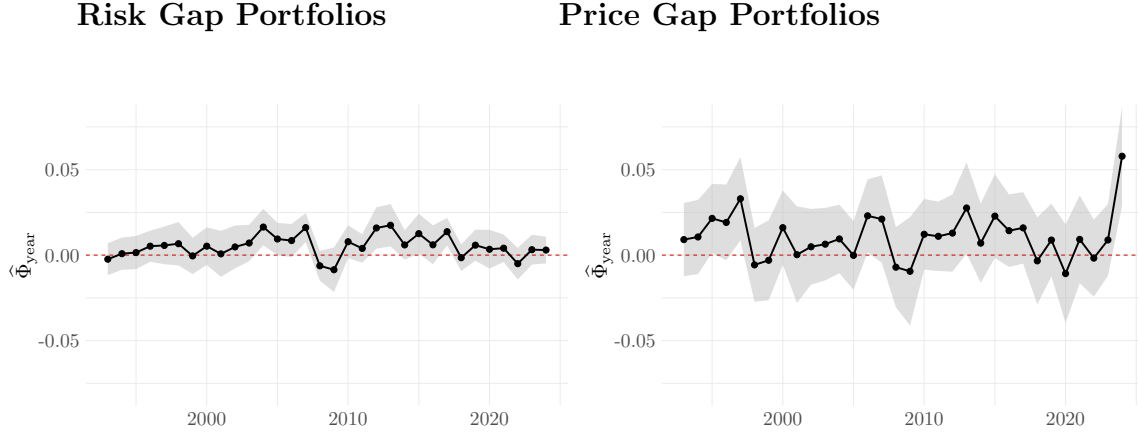
$$r_{ct} = a_c + \sum_{\text{year}=1992}^{2024} \phi_{\text{year}} \times 1(t = \text{year}) + u_{ct},$$

where  $r_{ct}$  denotes either a Risk Gap or a Price Gap portfolio return and  $c$  indexes the 25 country pairs. The country-pair fixed effect  $a_c$  absorbs persistent differences in average returns across pairs, while the coefficients  $\phi_{\text{year}}$  capture common year-specific variation. The year effects are normalized for identification. A downward path in  $\phi_{\text{year}}$  would indicate that the returns associated with exploiting the corresponding gap have eroded. Figure 6 shows no systematic downward pattern. For both portfolio types, the estimated year effects fluctuate around zero without a discernible declining trend.

Figure 6: Profitability persistence over time.

This figure reports the estimated coefficients  $\phi_{\text{year}}$  from the regression

$r_{ct} = a_c + \sum_{\text{year}=1992}^{2024} \phi_{\text{year}} \times 1(t = \text{year}) + u_{ct}$ , where  $c$  indexes the 25 country pairs,  $r_{ct}$  is the return on either a Risk Gap Portfolio (left) or a Price Gap Portfolio (right). The regression tests whether portfolio profitability has changed over the sample period; a flat coefficient path indicates no erosion. The solid line represents the estimated coefficients, and the gray area indicates the 95% confidence intervals, computed using time-clustered standard errors.



The same question can be examined from a US investor's perspective. Panels B and C of Table 6 split the sample at the end of 2004. The US market earns an annualized Sharpe ratio of 0.51 before 2005 and 0.60 afterward, while the corresponding Sharpe ratios of the world market portfolio are 0.34 and 0.47. Combining the Risk Gap and Price Gap portfolios produces Sharpe ratios of 0.87 before 2005 and 1.65 afterward. Panel D reports a Sharpe ratio of 1.96 over the period beginning in August 2010, during which all 25 country pairs are available. Thus, the investor-level gains are larger, rather than smaller, in the more recent part of the sample.

Part of this increase may be mechanical. The average number of available foreign countries rises from 15.2 before 2005 to 24.8 afterward, allowing greater diversification in the later period. Panel D therefore provides an additional assessment over a period in which the country coverage is fixed at 25 pairs. During this period, the US market has a Sharpe ratio of 0.85, the world market portfolio has a Sharpe ratio of 0.63, and the portfolio combining both gaps has a Sharpe ratio of 1.96. Whatever has happened to the comovement of country indices, the gains associated with the two gaps remain economically substantial when the available country set is held fixed.

Finally, we reproduce the main country-pair results using only observations after 2005. Tables 13 and 14 report the Risk Gap Portfolios, while Tables 15 and 16 report the

Price Gap Portfolios. The economic magnitudes and correlations with the US market are broadly comparable to their full-sample counterparts. Statistical significance is weaker for some country pairs, as expected in a sample roughly half as long, but the principal findings remain unchanged. We next examine the gap portfolio returns more closely, focusing on their relation to established factor strategies, their exposure to the US market during downturns, and their association with formal capital controls.

## 7 Interpreting Gap Portfolio Returns

Sections 5 and 6 document positive out-of-sample returns on the Risk Gap and Price Gap portfolios across a broad set of country pairs and show that these portfolios expand the investment opportunity set of a US investor. These findings establish the economic importance of the two gap portfolios, but a fuller assessment requires looking beyond average returns and unconditional correlations. We begin by benchmarking the portfolio returns against familiar observable risk factors and examining the portfolios' characteristic tilts. We next study their sensitivity to the US market during downturns, when diversification is most valuable. We then relate their profitability to formal restrictions on cross-border equity investment. Finally, we summarize additional robustness analyses reported in the Appendix. Together, these analyses address, at least in part, concerns about whether the portfolio returns reflect familiar observable risk exposures, whether their low unconditional correlation with the US market masks greater exposure during US market downturns, and whether their profitability is concentrated in markets subject to formal capital controls.

### 7.1 Exposure to Established Factor Strategies

Our factors are latent; that is, they are extracted from the cross-section rather than pre-specified. The resulting portfolios are therefore not constructed to target established factor strategies. Their returns and composition may nevertheless resemble those of such strategies. We examine this possibility from two complementary perspectives. From the return perspective, we benchmark the portfolios against local, US, and global equity factors. From the portfolio-composition perspective, we examine their characteristic tilts month by month to describe how the portfolio weights relate to familiar

firm characteristics and how these relations evolve over time.

**Local, US, and global factors.** An investor considering international investment can choose not only among country market portfolios but also among established factor strategies implemented domestically, within individual foreign markets, or across countries. These strategies provide natural benchmarks for assessing how the Risk Gap and Price Gap portfolios relate to familiar investment approaches. We focus on the market, size, value, and momentum factors because they are among the most widely used factor strategies and are constructed consistently across the countries in our sample. The US factors represent strategies available in the investor’s domestic market, the local factors represent familiar strategies implemented within the corresponding foreign market, and the global composites represent diversified international implementations of the same strategies. Our objective is not to specify an exhaustive asset-pricing model, but to evaluate the gap portfolios relative to a parsimonious and economically interpretable set of alternatives.

To implement this comparison, we pool the available country-month observations separately for the Risk Gap and Price Gap portfolios. For each foreign country, we regress its gap portfolio return on the country’s four local factor returns from the global factor library of Jensen et al. (2023), the corresponding four US factor returns, and global composites of the same factors. Each global composite is market-capitalization-weighted across the remaining foreign countries, excluding both the United States and the country corresponding to that observation. We estimate the regressions for all countries and separately for the G7 and non-G7 groups. Each specification includes country fixed effects, and standard errors are clustered by month. Tables 7 and 8 report the results for the Risk Gap and Price Gap portfolios, respectively.

Three findings emerge. First, established factor strategies account for little of what either portfolio earns. The within  $R^2$  ranges from 0.03 to 0.07, so the twelve benchmark returns together leave almost all of the month-to-month variation unexplained, and the average annualized alphas remain 7.5% for the Risk Gap portfolios and 8.1% for the Price Gap portfolios, both significant at the 1% level. An investor already holding the market, size, value, and momentum strategies of the domestic market, of the foreign market, and of a diversified international composite would not have captured these returns. Second, the one systematic overlap is with momentum in the foreign

market, where the loading is 0.25 for the Risk Gap portfolios and 0.24 for the Price Gap portfolios and is significant at the 1% level in every country group. No other loading is significant at conventional levels across both portfolio types and all three country groups. The gap portfolios thus lean toward recent winners in the foreign market; because the regressions hold that lean fixed, however, it does not account for the alphas above. Third, the two portfolios differ in their exposure to US strategies, in the direction their construction implies. For the Risk Gap portfolios the US factor block is jointly indistinguishable from zero in every country group, consistent with their targeting risks that the investor’s home market does not span; their only market exposure significant at conventional levels is a loading of 0.06 on the global composite in the all-country specification. The Price Gap portfolios instead load 0.14 on the US market, significant at the 1% level, and their US block is jointly significant at the 5% level in every country group. This is what the strategy does by construction: it trades matched positions in the two markets and, as Table 4 shows, frequently takes its long position in the United States. Nevertheless, the all-country within  $R^2$  of 0.035 and annualized alpha of 8.1% leave established strategies accounting for only a limited part of what it earns.

Table 7: Pooled panel factor model — Risk Gap Portfolios.

This table reports pooled regressions of each country’s Risk Gap Portfolio return on its own four local factors from the global factor library of Jensen et al. (2023), the corresponding US factors, and a global composite of the other countries’ local factors. The composite is market-capitalization-weighted and excludes both the US and, on each country’s own rows, that country. The specification has country but no time fixed effects, and months with fewer than five live countries are dropped, so the sample begins in 1987.  $t$ -statistics, in parentheses, use standard errors clustered on month. Stars mark significance at the 1% (\*\*\*), 5% (\*\*) and 10% (\*) levels. The average  $\alpha$  is the annualized intercept of the same specification without country fixed effects. The estimation window is 36 months.

|                              | All                | G7                 | non-G7             |
|------------------------------|--------------------|--------------------|--------------------|
| <i>Own local factors</i>     |                    |                    |                    |
| Market                       | 0.015<br>(1.02)    | −0.010<br>(−0.27)  | 0.019<br>(1.17)    |
| Size                         | 0.050**<br>(2.03)  | −0.010<br>(−0.16)  | 0.059**<br>(2.18)  |
| Value                        | 0.021<br>(0.80)    | 0.104<br>(1.36)    | 0.013<br>(0.45)    |
| Momentum                     | 0.253***<br>(9.32) | 0.433***<br>(7.11) | 0.224***<br>(7.79) |
| <i>US factors</i>            |                    |                    |                    |
| Market                       | −0.007<br>(−0.26)  | 0.061<br>(1.15)    | −0.033<br>(−1.02)  |
| Size                         | −0.043<br>(−1.27)  | −0.034<br>(−0.55)  | −0.045<br>(−1.16)  |
| Value                        | −0.008<br>(−0.33)  | −0.026<br>(−0.51)  | −0.004<br>(−0.12)  |
| Momentum                     | 0.030<br>(0.96)    | −0.010<br>(−0.16)  | 0.030<br>(0.82)    |
| <i>Global composite</i>      |                    |                    |                    |
| Market                       | 0.063**<br>(2.37)  | 0.049<br>(1.12)    | 0.075**<br>(2.44)  |
| Size                         | −0.038<br>(−0.56)  | 0.001<br>(0.01)    | −0.050<br>(−0.61)  |
| Value                        | 0.025<br>(0.39)    | −0.026<br>(−0.20)  | 0.021<br>(0.27)    |
| Momentum                     | 0.056<br>(1.02)    | 0.013<br>(0.13)    | 0.047<br>(0.66)    |
| Joint test US block = 0: $F$ | 0.91               | 0.53               | 1.05               |
| $p$ -value                   | 0.459              | 0.713              | 0.378              |
| Average $\alpha$ (% p.a.)    | 7.5<br>(8.57)      | 8.5<br>(5.04)      | 7.2<br>(6.29)      |
| Within $R^2$                 | 0.034              | 0.058              | 0.030              |
| Countries                    | 25                 | 6                  | 19                 |
| Observations                 | 9,408              | 2,556              | 6,852              |

Table 8: Pooled panel factor model — Price Gap Portfolios.

This table reports pooled regressions of each country's Price Gap Portfolio return on its own four local factors from the global factor library of Jensen et al. (2023), the corresponding US factors, and a global composite of the other countries' local factors. The composite is market-capitalization-weighted and excludes both the US and, on each country's own rows, that country. The specification has country but no time fixed effects, and months with fewer than five live countries are dropped, so the sample begins in 1987.  $t$ -statistics, in parentheses, use standard errors clustered on month. Stars mark significance at the 1% (\*\*\*), 5% (\*\*) and 10% (\*) levels. The average  $\alpha$  is the annualized intercept of the same specification without country fixed effects. The estimation window is 36 months.

|                              | All                | G7                 | non-G7              |
|------------------------------|--------------------|--------------------|---------------------|
| <i>Own local factors</i>     |                    |                    |                     |
| Market                       | 0.030<br>(1.23)    | -0.104*<br>(-1.79) | 0.046*<br>(1.79)    |
| Size                         | 0.097***<br>(2.79) | 0.086<br>(0.98)    | 0.103***<br>(2.92)  |
| Value                        | 0.013<br>(0.35)    | 0.172*<br>(1.70)   | -0.013<br>(-0.32)   |
| Momentum                     | 0.238***<br>(6.89) | 0.573***<br>(7.21) | 0.183***<br>(5.01)  |
| <i>US factors</i>            |                    |                    |                     |
| Market                       | 0.140***<br>(3.00) | 0.272***<br>(3.49) | 0.099**<br>(2.10)   |
| Size                         | -0.088<br>(-1.09)  | -0.112<br>(-0.90)  | -0.085<br>(-1.08)   |
| Value                        | 0.114*<br>(1.93)   | 0.082<br>(0.72)    | 0.124*<br>(1.96)    |
| Momentum                     | 0.087<br>(1.11)    | -0.002<br>(-0.01)  | 0.099<br>(1.26)     |
| <i>Global composite</i>      |                    |                    |                     |
| Market                       | -0.034<br>(-0.83)  | -0.048<br>(-0.71)  | -0.005<br>(-0.11)   |
| Size                         | -0.007<br>(-0.06)  | -0.135<br>(-0.78)  | 0.040<br>(0.27)     |
| Value                        | -0.219*<br>(-1.66) | -0.112<br>(-0.50)  | -0.293**<br>(-2.11) |
| Momentum                     | 0.183<br>(1.63)    | 0.102<br>(0.62)    | 0.166<br>(1.48)     |
| Joint test US block = 0: $F$ | 2.96               | 3.19               | 2.50                |
| $p$ -value                   | 0.020              | 0.013              | 0.042               |
| Average $\alpha$ (% p.a.)    | 8.1<br>(4.82)      | 5.6<br>(2.05)      | 9.0<br>(5.35)       |
| Within $R^2$                 | 0.035              | 0.070              | 0.031               |
| Countries                    | 25                 | 6                  | 19                  |
| Observations                 | 9,408              | 2,556              | 6,852               |

**Characteristic tilts.** The factor regressions characterize the portfolios through their realized returns. To complement this evidence with a direct view of their holdings, we examine the portfolios’ characteristic tilts month by month. This analysis describes the types of firms toward which the portfolios tilt and whether those tilts persist or change over time.

For each country-pair portfolio, month, and characteristic, we calculate

$$\tau_{kt} = \sum_i w_{it} (x_{ikt} - \bar{x}_{kt}),$$

where  $w_{it}$  is the portfolio weight of firm  $i$ ,  $x_{ikt}$  is firm  $i$ ’s within-country rank for characteristic  $k$ , scaled to  $[0, 1]$ , and  $\bar{x}_{kt}$  is the cross-sectional mean rank in that country-month. The notation suppresses the country-pair and portfolio-side indices. A positive value of  $\tau_{kt}$  indicates a tilt toward firms with high values of characteristic  $k$ , while a negative value indicates a tilt toward firms with low values. We calculate the tilts separately for each country pair and then average them equally across the country pairs available in each month. For the Price Gap portfolios, we calculate the tilts separately for the foreign and US sides.

Figures 7 and 8 report the monthly tilts for momentum, profitability, beta, investment, size, and value, with one characteristic representing each broad characteristic theme. We evaluate persistence using both the fraction of months in which a tilt has the same sign as its time-series average and the ratio  $\frac{|\bar{\tau}_k|}{|\tau_{kt}|}$ , which measures how much of the average absolute monthly tilt remains after signed averaging. A value near one indicates a tilt that is stable in sign, whereas a value near zero indicates that positive and negative tilts largely offset over time.

The Risk Gap portfolios exhibit their clearest and most persistent tilt toward momentum. The average momentum tilt is 0.160, and it is positive in 98.9% of the 364 months. Profitability and beta also display relatively stable directions: the average profitability tilt is 0.083 and is positive in 86.0% of months, while the average beta tilt is  $-0.086$  and is negative in 83.2% of months. The remaining characteristics vary more substantially over time. For example, the average value tilt is only 0.017, has the same sign as its average in 62.4% of months, and has a persistence ratio of 0.238. Investment and size likewise alternate between positive and negative tilts. Thus, the Risk Gap portfolio has a persistent momentum orientation, together with less stable

positions along the other characteristic dimensions.

For the Price Gap portfolios, the foreign side displays patterns similar to those of the Risk Gap portfolios. It tilts toward momentum, with an average of 0.125 and a positive sign in 90.4% of months; toward low-beta firms, with an average of  $-0.122$  and a negative sign in 89.0% of months; and toward smaller firms, with an average size tilt of  $-0.071$  and a negative sign in 75.0% of months. The US side generally rotates more across characteristics. For beta, value, and size, the persistence ratios are 0.071, 0.089, and 0.123 on the US side, compared with 0.923, 0.283, and 0.709 on the foreign side. The corresponding US tilts have the same sign as their averages in only 52.7%, 59.6%, and 53.6% of months. Profitability is relatively more stable on the US side, with an average tilt of 0.062 that is positive in 69.5% of months.

We compare the two sides of the Price Gap portfolio in terms of persistence rather than magnitude. The characteristic ranks are formed within different cross-sections—a median of 195 firms in the foreign markets and 2,629 firms in the United States—so a rank-based tilt does not represent the same economic quantity on the two sides. Moreover, both sides are determined by the same estimated difference in premia, making part of any apparent opposition between them mechanical.

Overall, the holdings-based analysis shows that the gap portfolios have recognizable characteristic patterns, particularly the persistent momentum tilt on their foreign positions, while several other dimensions vary considerably over time. The momentum finding is consistent with the local momentum loadings in the return regressions. Because those regressions already control for the local momentum factor and continue to produce economically large alphas, however, the momentum tilt does not account for the portfolios' average returns.

Figure 7: Standing and rotating characteristic tilts: Risk Gap Portfolios.

Each panel plots the pooled monthly characteristic tilt of the Risk Gap Portfolios, one characteristic from each of six themes (momentum `ret_12_1`, profitability `ope_be`, beta `beta_60m`, investment `at_gr1`, size `market_equity`, value `be_me`), ordered by how much of the typical monthly tilt survives averaging,  $|\bar{\tau}|/|\tau|$ , from 0.998 for momentum down to 0.238 for value. Tilts are measured on ranks formed within each country-month, against a median cross-section of 195 securities. Window 1994-09 to 2024-12 (364 months), pooled within month across the live country pairs (17 in 1994-09, 25 from 2008-08). Both figures share one vertical scale. The series is filled to zero, blue where the tilt is positive and orange where it is negative, so the colour split within a panel is that characteristic's sign share; the dashed line is the panel mean. For size, a negative tilt is a lean toward small firms.

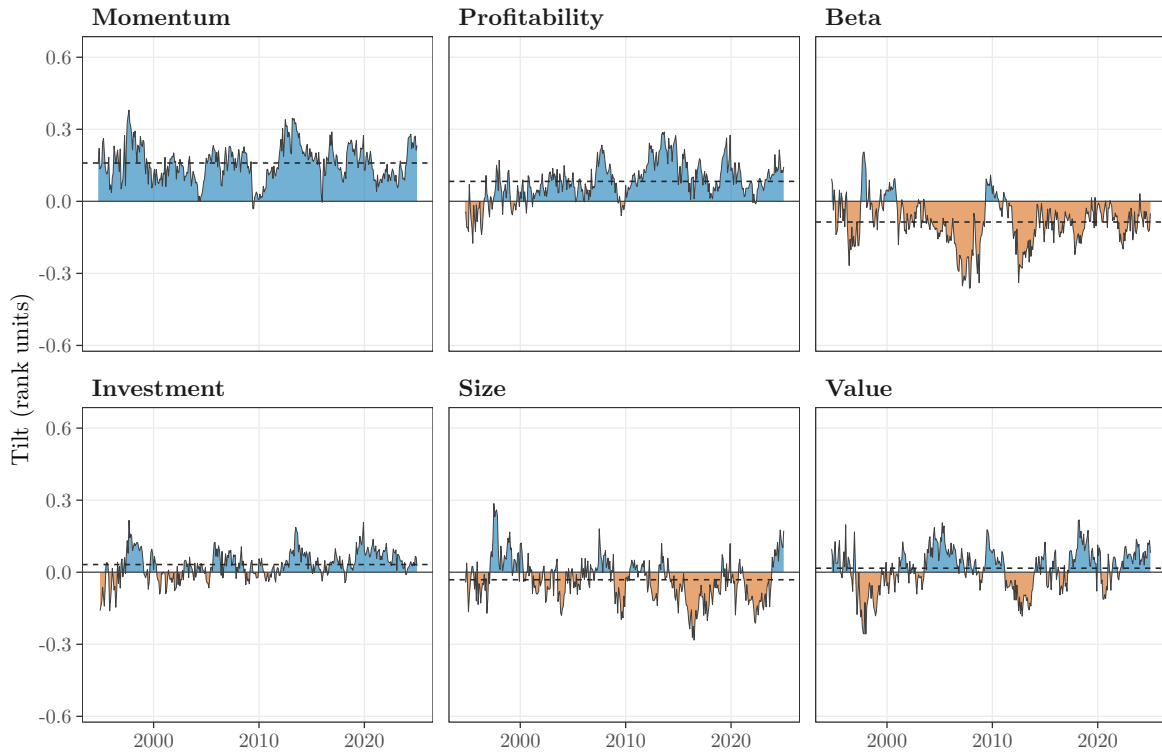
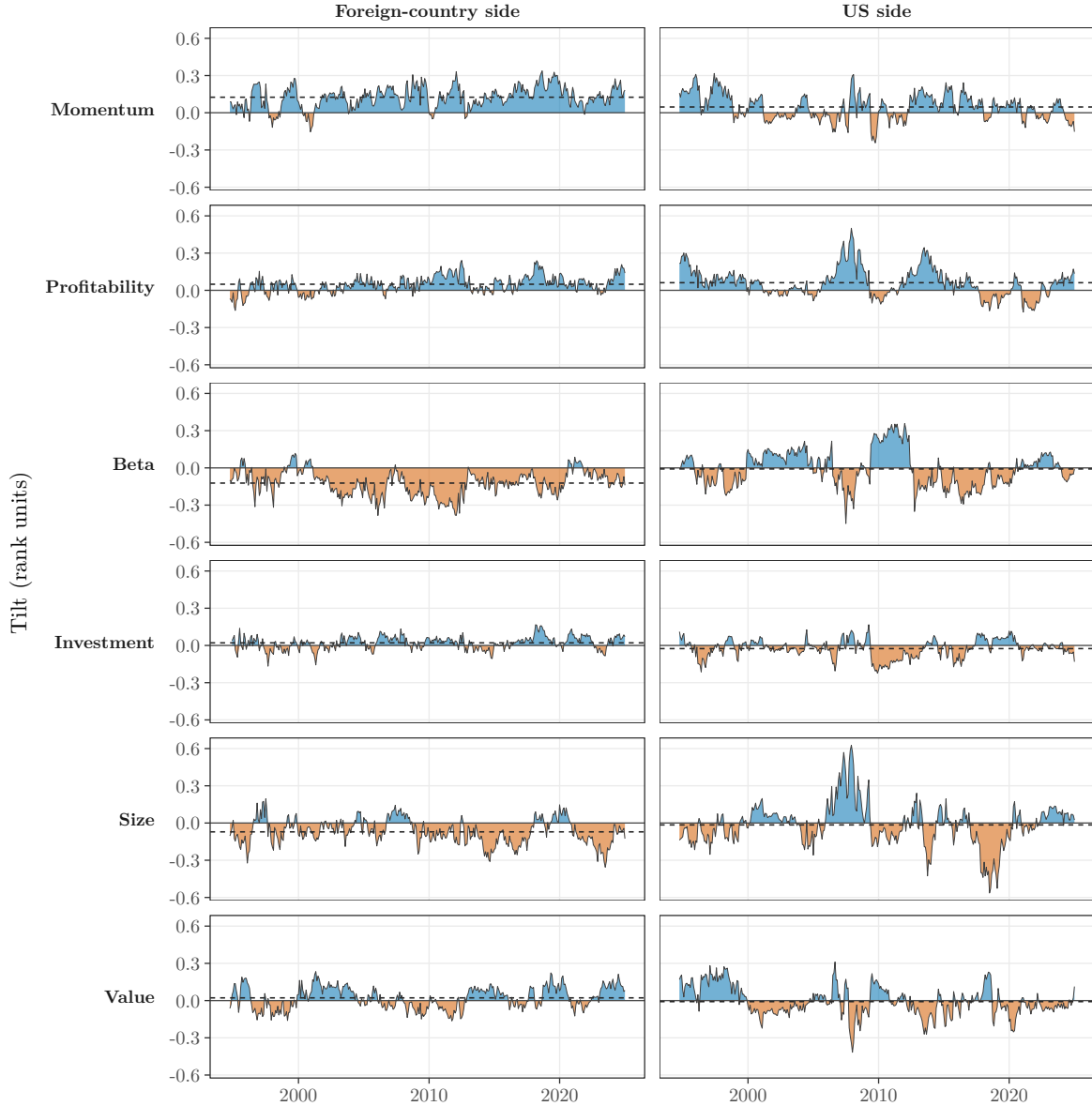


Figure 8: Standing and rotating characteristic tilts: Price Gap Portfolios, both sides. Each panel plots the pooled monthly characteristic tilt of one side of the Price Gap Portfolios — the foreign-country side (left column) and the US side (right column) — for the same six characteristics, window and vertical scale as Figure 7, and in that figure’s panel order. *The two columns are not commensurate and must not be read against each other:* ranks are formed within each country-month, against a median of 195 securities abroad but 2,629 in the United States, so a rank unit is not the same economic quantity on the two sides; and both sides are built from the same estimated premium difference, so any apparent opposition between them is in part mechanical. Fill and dashed mean line are as in Figure 7.



## 7.2 US Market Exposure during Downturns

The low unconditional exposure of the Risk Gap and Price Gap portfolios to the US market is central to the diversification gains documented in Section 6. Unconditional exposure may, however, mask greater sensitivity during US market downturns, when diversification is particularly valuable. We therefore examine whether the sensitivity of the gap portfolio returns to the US market changes when the market return falls below a specified threshold.

$$r_{ct} = a_c + \phi_1 \times \text{Mkt}_t + \phi_2 \times \text{Mkt}_t \times 1(\text{Mkt}_t < \text{threshold}) + u_{ct},$$

where  $r_{ct}$  is the return on the Risk Gap or Price Gap portfolio for country pair  $c$ , and  $\text{Mkt}_t$  is the US market excess return. The coefficient  $\phi_1$  measures the portfolio's sensitivity to the US market when the threshold is not crossed, while  $\phi_2$  measures the incremental sensitivity when the market return falls below the threshold. The total market sensitivity during these months is therefore  $\phi_1 + \phi_2$ . We consider thresholds of 0%, -5%, and -10%. Each specification includes country fixed effects, and standard errors are clustered by month.

Table 9 presents the results. Both portfolios do become more sensitive to the US market when it declines. At the zero threshold, total market sensitivity rises from -0.12 to 0.09 for the Risk Gap portfolios and from -0.22 to 0.23 for the Price Gap portfolios, and in both cases the increase is significant at the 1% level. Through this market-exposure channel, a 10% decline in the US market is associated with portfolio declines of approximately 0.9% and 2.3%, respectively, rather than the one-for-one decline of a fully exposed position. The change in sensitivity is therefore statistically clear but economically small.

The pattern does not intensify at more extreme thresholds, which is where a diversification benefit would be most likely to fail. At the -5% threshold, only the Price Gap portfolios show an incremental sensitivity distinguishable from zero at conventional levels, and total sensitivities are 0.02 and 0.15. At the -10% threshold the Risk Gap increment turns negative, significant only at the 10% level, and the Price Gap increment is indistinguishable from zero at conventional levels, leaving total sensitivities of -0.06 and 0.11. Successively deeper thresholds are identified from fewer months, so these estimates are correspondingly less precise. Nevertheless, across all six specifications the

US market accounts for almost none of the variation in either portfolio's returns. The within  $R^2$  never exceeds 0.007 and total down-market sensitivity stays between  $-0.06$  and  $0.23$ . The low US market exposure behind the diversification gains of Section 6 is therefore not confined to calm markets.

Table 9: Portfolio returns in down markets.

This table reports the estimated coefficients from the regression  $r_{ct} = a_c + \phi_1 \times \text{Mkt}_t + \phi_2 \times \text{Mkt}_t \times 1(\text{Mkt}_t < \text{threshold}) + u_{ct}$ , where  $c$  indexes the 25 country pairs,  $r_{ct}$  is the return on either a Risk Gap Portfolio (Panel A) or a Price Gap Portfolio (Panel B), and  $\text{Mkt}_t$  is the US market excess return. The coefficient  $\phi_2$  measures the additional sensitivity of portfolio returns to the US market when the market return falls below the threshold. Thresholds of 0,  $-5$  and  $-10$  percent are considered. The regression is estimated as a pooled panel across all 25 country pairs, with the intercepts  $a_c$  absorbing country fixed effects. Standard errors are clustered by time;  $t$ -statistics are reported in parentheses. Stars mark significance at the 1% (\*\*\*) , 5% (\*\*) and 10% (\*) levels. The estimation window is 36 months.

|                                           | Panel A: Risk Gap              |                          |                            | Panel B: Price Gap             |                              |                          |
|-------------------------------------------|--------------------------------|--------------------------|----------------------------|--------------------------------|------------------------------|--------------------------|
|                                           | (1)                            | (2)                      | (3)                        | (4)                            | (5)                          | (6)                      |
| $\text{Mkt}_t$                            | $-0.1221^{***}$<br>( $-3.27$ ) | $-0.0368$<br>( $-1.32$ ) | $-0.0046$<br>( $-0.21$ )   | $-0.2243^{***}$<br>( $-2.68$ ) | $-0.0933$<br>( $-1.60$ )     | $-0.0091$<br>( $-0.21$ ) |
| $\text{Mkt} \times 1(\text{Mkt} < 0)$     | $0.2079^{***}$<br>( $3.53$ )   |                          |                            | $0.4523^{***}$<br>( $3.83$ )   |                              |                          |
| $\text{Mkt} \times 1(\text{Mkt} < -5\%)$  |                                | $0.0542$<br>( $1.34$ )   |                            |                                | $0.2468^{***}$<br>( $3.21$ ) |                          |
| $\text{Mkt} \times 1(\text{Mkt} < -10\%)$ |                                |                          | $-0.0568^*$<br>( $-1.85$ ) |                                |                              | $0.1193$<br>( $1.48$ )   |
| Country fixed effects                     | Yes                            | Yes                      | Yes                        | Yes                            | Yes                          | Yes                      |
| Within $R^2$                              | 0.003                          | 0.000                    | 0.000                      | 0.007                          | 0.004                        | 0.001                    |
| Observations                              | 9,122                          | 9,122                    | 9,122                      | 9,122                          | 9,122                        | 9,122                    |

### 7.3 Capital Controls and Market Access

The investment gains documented above are relevant to a US investor only to the extent that the underlying positions can be implemented. One concern is that the portfolio returns arise primarily in markets or periods with tight capital controls, where cross-border equity positions may be difficult to establish. Such restrictions may also provide an institutional explanation for the Price Gap by limiting the capital flows that would otherwise narrow differences in the compensation for common risks. They may similarly restrict investor access to the foreign country-specific risks targeted by the

Risk Gap portfolios. We therefore examine whether the returns of both portfolios vary systematically with formal restrictions on cross-border equity investment.

We use the capital-control measures of Fernández et al. (2016), which are constructed from the IMF’s *Annual Report on Exchange Arrangements and Exchange Restrictions*. We focus on their measure of restrictions on cross-border equity transactions, which ranges from zero to one, with higher values indicating more restrictive capital controls. We regress the monthly Risk Gap and Price Gap portfolio returns, expressed in percentage terms, on the corresponding restriction measure for the foreign country. Standard errors are clustered by month. Taiwan is excluded because it is not covered by the capital-control dataset.

Table 10 presents the results. The estimated coefficient on the equity restriction measure is  $-0.216$  ( $t = -1.11$ ) for the Risk Gap portfolios and  $-0.013$  ( $t = -0.05$ ) for the Price Gap portfolios. Neither estimate is statistically distinguishable from zero, and the  $R^2$  is effectively zero in both specifications. In particular, the estimates provide no indication that either portfolio earns higher returns when formal restrictions on cross-border equity investment are more stringent. Thus, the profitability of the gap portfolios does not appear to be driven by formal capital controls.

Table 10: Capital flow restrictions and portfolio returns.

This table reports regressions of portfolio returns on the equity capital-flow restriction measure from Fernández et al. (2016). The dependent variable is the portfolio return expressed in percentage terms. The variable Equity Restriction captures restrictions on cross-border equity transactions for a given country 2 at time  $t$ . Columns (1) and (2) report the Risk Gap and Price Gap Portfolios, respectively. Observations corresponding to Taiwan are excluded because Taiwan is not available in the capital-control dataset. Standard errors are clustered by month;  $t$ -statistics are reported in parentheses. Stars mark significance at the 1% (\*\*\*) , 5% (\*\*) and 10% (\*) levels. The estimation window is 36 months.

|                    | (1)                     | (2)                     |
|--------------------|-------------------------|-------------------------|
|                    | Risk Gap                | Price Gap               |
| Equity Restriction | $-0.216$<br>( $-1.11$ ) | $-0.013$<br>( $-0.05$ ) |
| Fixed effects      | None                    | None                    |
| $R^2$              | 0.000                   | 0.000                   |
| Observations       | 6,745                   | 6,745                   |

## 7.4 Additional Robustness

Appendix F reports additional analyses of the portfolio construction and the interpretation of its returns. Appendix F.1 reconstructs the portfolios using a 60-month estimation window rather than the baseline 36-month window. The equal-weighted Risk Gap portfolio across all 25 foreign markets earns an annualized Sharpe ratio of 1.05, compared with 1.14 under the baseline specification, and remains nearly uncorrelated with the US market. The corresponding Sharpe ratio for the Price Gap portfolio declines from 1.02 to 0.74. Despite this attenuation, combining both gap portfolios with the US market produces a Sharpe ratio of 1.34, compared with 0.58 for the US market alone. Thus, the investor-level expansion of the opportunity set remains economically substantial under the longer estimation window.

Appendix F.2 examines whether the portfolio returns are related to bilateral exchange-rate movements or to the Carry and Dollar factors that capture systematic variation in currency markets. As reported in Table 21, none of these exposures is statistically significant for either portfolio. Appendix F.3 additionally fixes the number of common factors at four rather than selecting it using the bootstrap procedure. The resulting portfolio performance remains similar to the baseline results. Overall, the Risk Gap findings and the investor-level gains are stable across these alternative specifications, while the magnitude of the Price Gap returns is somewhat more sensitive to the length of the estimation window.

## 8 Monte Carlo Simulation

The empirical analyses above evaluate the gap portfolios using realized international equity returns. Because the true factor structure is unobserved in the data, however, they cannot directly establish whether the procedure correctly separates common and country-specific factors or whether the resulting portfolios isolate their intended sources of returns. We therefore turn to Monte Carlo simulations in which the underlying factor structure and risk premia are known. This allows us to evaluate the method under the correct specification and assess its sensitivity to misspecification of the factor dimensions.

We calibrate the return-generating process to the US–Canada pair. Each market contains six systematic factors, comprising three common factors and three country-

specific factors. We estimate factor loadings, residual variances, and factor processes using the 60-month period from 2018 to 2022 and treat these estimates as the true simulation parameters. We first examine how underestimating or overestimating the total number of systematic factors affects recovery of the common factor space. We then vary the allocation between common and country-specific factors and evaluate whether the Risk Gap and Price Gap portfolios capture their intended return opportunities.

## 8.1 Recovery of the Common Factor Space

The true return-generating process contains six systematic factors in each country, with three common and three country-specific factors ( $K^c = K^s = 3$ ). We estimate specifications with  $K = 5, 6$ , and  $7$  systematic factors in each market. For each specification, Table 11 reports the average estimated canonical correlations across 10,000 simulated samples.

Table 11: Canonical correlations for different numbers of estimated systematic factors. This table reports the average canonical correlations across 10,000 repetitions for varying numbers of estimated systematic factors ( $K$ ), comparing underestimation ( $K = 5$ ), correct estimation ( $K = 6$ ), and overestimation ( $K = 7$ ).

| Canonical Correlations    | 1      | 2      | 3      | 4      | 5      | 6      | 7      |
|---------------------------|--------|--------|--------|--------|--------|--------|--------|
| $K = 5$ , Underestimation | 0.9958 | 0.9372 | 0.7725 | 0.2332 | 0.0694 | -      | -      |
| $K = 6$ , Correct         | 0.9963 | 0.9644 | 0.9389 | 0.3345 | 0.1806 | 0.0562 | -      |
| $K = 7$ , Overestimation  | 0.9965 | 0.9661 | 0.9418 | 0.4121 | 0.2727 | 0.1545 | 0.0487 |

Under the correctly specified case of  $K = 6$ , the first three canonical correlations are 0.9963, 0.9644, and 0.9389, followed by a sharp decline to 0.3345 for the fourth. This break correctly identifies the three common factors in the return-generating process. A similar break appears when the total number of factors is underestimated or overestimated. With  $K = 5$ , the third and fourth canonical correlations are 0.7725 and 0.2332; with  $K = 7$ , they are 0.9418 and 0.4121. Underestimation therefore weakens the recovery of the third common factor, whereas overestimation largely preserves the leading common factor space. This asymmetry supports our baseline choice of seven systematic factors in the empirical analysis, where a relatively flexible specification is preferable to omitting a systematic factor.

## 8.2 Portfolio Performance

We next examine whether the two portfolio constructions respond to their intended sources of returns. We consider four configurations of country-specific and common-factor premia:

- (A) *Neither gap.* Common factors command the same premia in the two countries, and country-specific factors carry no premia. Neither portfolio should exhibit positive performance.
- (B) *Risk Gap only.* Common factors command the same premia in the two countries, while Country 2's country-specific factors carry positive premia. The Risk Gap portfolio should exhibit positive performance, whereas the Price Gap portfolio should not.
- (C) *Price Gap only.* Common factors command different premia across the two countries, while country-specific factors carry no premia. The Price Gap portfolio should exhibit positive performance, whereas the Risk Gap portfolio should not.
- (D) *Both gaps.* Common factors command different premia across the two countries, and Country 2's country-specific factors carry positive premia. Both portfolios should exhibit positive performance.

Table 12 reports the annualized Sharpe ratios of the two portfolios as we vary the numbers of factors classified as common,  $K^c$ , and country-specific,  $K^s$ , while holding their sum fixed at six. The return-generating process has  $K^c = K^s = 3$ , as in Table 11, but the factor premia vary across the four panels according to Scenarios A–D.

Table 12: Sharpe ratios of the Price Gap and Risk Gap Portfolios for varying common and country-specific factors.

This table reports simulated annualized Sharpe ratios (SR) for the Price Gap and Risk Gap Portfolios across combinations of common factors ( $K^c$ ) and country-specific factors ( $K_s$ ), with the total number of factors fixed at six, over 10,000 repetitions. Panel A imposes neither country-specific nor differential common-factor risk premia. Panel B allows country-specific risk premia in Country 2 only, so a Risk Gap is present and a Price Gap is not. Panel C allows the common factors to command different premia in the two countries, so a Price Gap is present and a Risk Gap is not. Panel D allows both.

| $K^c$ (Common Factors)           | 1       | 2      | 3       | 4       | 5       |
|----------------------------------|---------|--------|---------|---------|---------|
| $K_s$ (Country-Specific Factors) | 5       | 4      | 3       | 2       | 1       |
| <i>Panel A: Neither Gap</i>      |         |        |         |         |         |
| Price Gap Portfolio SR           | 0.0514  | 0.0443 | 0.0409  | 0.0374  | 0.0387  |
| Risk Gap Portfolio SR            | -0.0021 | 0.0062 | -0.0258 | -0.0360 | -0.0278 |
| <i>Panel B: Risk Gap Only</i>    |         |        |         |         |         |
| Price Gap Portfolio SR           | 0.0826  | 0.1033 | 0.1209  | 0.3193  | 0.4667  |
| Risk Gap Portfolio SR            | 0.7899  | 0.7925 | 0.7945  | 0.6268  | 0.4554  |
| <i>Panel C: Price Gap Only</i>   |         |        |         |         |         |
| Price Gap Portfolio SR           | 0.5899  | 0.8111 | 0.9085  | 0.8999  | 0.8927  |
| Risk Gap Portfolio SR            | 0.1624  | 0.0526 | -0.0302 | -0.0442 | -0.0408 |
| <i>Panel D: Both Gaps</i>        |         |        |         |         |         |
| Price Gap Portfolio SR           | 0.7215  | 0.9808 | 1.0729  | 1.1194  | 1.1784  |
| Risk Gap Portfolio SR            | 0.8218  | 0.7996 | 0.7962  | 0.6280  | 0.4473  |

Panel A confirms that the portfolio construction does not generate positive performance in the absence of either gap. The Sharpe ratios of both portfolios remain close to zero across the alternative allocations between common and country-specific factors.

Panel B introduces only a Risk Gap. When the number of common factors does not exceed its true value of three, the Price Gap portfolio has relatively low Sharpe ratios, consistent with the absence of differences in common-factor premia. When  $K^c$  is increased to four or five, however, its Sharpe ratio rises to 0.3193 and 0.4667. In these cases, priced country-specific factors are incorrectly classified as common, transferring some of the Risk Gap performance to the Price Gap portfolio. The Risk Gap portfolio's Sharpe ratio increases from 0.4554 when only one factor is classified as country-specific to 0.7945 under the correct specification of  $K^s = 3$ . Its performance remains similar when the number of country-specific factors is overestimated, with Sharpe ratios of 0.7925 and 0.7899 for  $K^s = 4$  and 5.

Panel C presents the corresponding results when only a Price Gap is present. The Price Gap portfolio’s Sharpe ratio increases from 0.5899 with one estimated common factor to 0.9085 under the correct specification of  $K^c = 3$  and remains high when the number of common factors is increased to four or five. The Risk Gap portfolio has near-zero Sharpe ratios when  $K^s$  does not exceed its true value of three. When  $K^s$  is increased to four or five, its performance rises modestly, reaching 0.1624 for  $K^s = 5$ , because common factors with differential premia are incorrectly classified as country-specific.

Panel D allows both gaps to be present. Under the correct allocation of three common and three country-specific factors, the Price Gap and Risk Gap portfolios have Sharpe ratios of 1.0729 and 0.7962, respectively. Both portfolios continue to perform positively under the alternative allocations, but factor misclassification shifts performance between them. Increasing  $K^c$  to four or five raises the Price Gap portfolio’s Sharpe ratio to 1.1194 and 1.1784 while reducing the Risk Gap portfolio’s Sharpe ratio. Conversely, increasing  $K^s$  to four or five raises the Risk Gap portfolio’s Sharpe ratio to 0.7996 and 0.8218 while reducing the Price Gap portfolio’s Sharpe ratio.

Overall, the simulations show that, under the correct factor dimensions, the two portfolios isolate their intended sources of returns. They also show that overestimating the total number of systematic factors is less consequential than underestimating it, while misallocating factors between the common and country-specific spaces can shift measured performance from one gap portfolio to the other. These findings support the flexible total-factor specification used in the empirical analysis and underscore the importance of distinguishing common from country-specific factors.

## 9 Conclusion

International market integration is commonly inferred from the increasing comovement of country indices. This paper shows why that inference is incomplete. Country indices can become highly correlated even when foreign markets contain systematic risks unavailable domestically and when risks shared across markets command different risk premia. Our framework separates these two failures of cross-border pricing, a Risk Gap and a Price Gap, and maps each into an investable portfolio.

Both gaps are economically large. Equally weighted across foreign markets, the Risk

Gap and Price Gap portfolios each earn an annualized Sharpe ratio above one, with low correlation to the US market. Added to the US market, the Risk Gap portfolio raises the market's Sharpe ratio from 0.55 to 1.00, the Price Gap portfolio raises it to 0.90, and conventional diversification through foreign indices lowers it to 0.41. Combining the two channels raises it further.

More generally, the framework applies to any pair of markets with large cross-sections of assets. It recovers the systematic risks one market contains that the other does not span, tests whether the risks the markets share command the same premia, and maps each departure from integration into a portfolio whose average return measures it.

## A Additional Tables

Table 13: Risk Gap Portfolio returns – post 2005.

This table reports the out-of-sample performance of the Risk Gap Portfolios, using G7 countries as Country 2. Mean returns (%), standard deviations (%) and Sharpe ratios are annualized;  $t$ -statistics of the average returns are Newey–West. The table also reports the worst and best monthly returns and the correlation with the US market ( $\rho_{US}$ ). The sample is restricted to months from January 2005 onward. The estimation window is 36 months.

| Country | Mean (%) | Standard<br>Deviation (%) | $t$  | Sharpe<br>Ratio | Worst<br>Month (%) | Best<br>Month (%) | $\rho_{US}$ |
|---------|----------|---------------------------|------|-----------------|--------------------|-------------------|-------------|
| CAN     | 8.19     | 18.05                     | 2.06 | 0.45            | −12.14             | 21.24             | 0.04        |
| DEU     | 21.11    | 16.93                     | 4.89 | 1.25            | −10.60             | 23.87             | −0.04       |
| FRA     | 13.63    | 18.27                     | 3.16 | 0.75            | −19.15             | 15.91             | 0.08        |
| GBR     | 16.01    | 22.90                     | 2.71 | 0.70            | −39.06             | 23.97             | −0.11       |
| ITA     | 7.96     | 17.97                     | 1.89 | 0.44            | −24.74             | 19.06             | 0.10        |
| JPN     | 8.44     | 18.97                     | 1.91 | 0.44            | −16.45             | 22.63             | −0.02       |

Table 14: Risk Gap Portfolio returns – post 2005, non-G7 countries.

This table reports the out-of-sample performance of the Risk Gap Portfolios, using non-G7 countries as Country 2. Mean returns (%), standard deviations (%) and Sharpe ratios are annualized;  $t$ -statistics of the average returns are Newey–West. The table also reports the worst and best monthly returns and the correlation with the US market ( $\rho_{US}$ ). The sample is restricted to months from January 2005 onward. The estimation window is 36 months.

| Country | Mean (%) | Standard<br>Deviation (%) | $t$   | Sharpe<br>Ratio | Worst<br>Month (%) | Best<br>Month (%) | $\rho_{US}$ |
|---------|----------|---------------------------|-------|-----------------|--------------------|-------------------|-------------|
| ARG     | −1.87    | 20.64                     | −0.42 | −0.09           | −23.44             | 21.00             | −0.04       |
| AUS     | 14.49    | 18.56                     | 2.92  | 0.78            | −23.70             | 20.81             | −0.13       |
| AUT     | 6.07     | 17.67                     | 1.49  | 0.34            | −16.90             | 13.64             | 0.05        |
| BEL     | 5.49     | 18.62                     | 1.30  | 0.29            | −33.56             | 13.86             | −0.05       |
| BRA     | 0.74     | 19.61                     | 0.15  | 0.04            | −25.99             | 13.59             | −0.02       |
| CHL     | 5.06     | 20.04                     | 1.30  | 0.25            | −14.67             | 24.43             | −0.03       |
| DNK     | 3.96     | 20.16                     | 0.77  | 0.20            | −22.64             | 25.01             | −0.04       |
| ESP     | 6.78     | 18.59                     | 1.85  | 0.36            | −17.72             | 16.52             | 0.01        |
| HKG     | 22.27    | 20.72                     | 4.65  | 1.07            | −26.10             | 32.75             | 0.00        |
| IND     | 25.19    | 21.61                     | 4.84  | 1.17            | −31.05             | 25.45             | −0.12       |
| KOR     | 1.62     | 18.13                     | 0.35  | 0.09            | −15.32             | 20.20             | −0.02       |
| MEX     | −0.79    | 19.15                     | −0.19 | −0.04           | −18.47             | 18.45             | −0.07       |
| NOR     | 9.65     | 20.47                     | 2.10  | 0.47            | −23.26             | 24.74             | −0.13       |
| POL     | 13.90    | 18.29                     | 3.07  | 0.76            | −16.01             | 23.66             | 0.04        |
| SGP     | 23.06    | 28.47                     | 3.45  | 0.81            | −20.89             | 92.29             | 0.05        |
| SWE     | 14.56    | 19.03                     | 2.72  | 0.77            | −18.40             | 22.75             | 0.07        |
| THA     | 14.41    | 17.27                     | 3.02  | 0.83            | −22.62             | 20.12             | −0.06       |
| TUR     | 7.21     | 19.44                     | 1.56  | 0.37            | −18.05             | 17.72             | 0.08        |
| TWN     | 8.96     | 18.89                     | 1.69  | 0.47            | −19.93             | 16.29             | 0.01        |

Table 15: Price Gap Portfolio returns – post 2005.

This table reports the out-of-sample performance of the Price Gap Portfolios, using G7 countries as Country 2. Mean returns (%), standard deviations (%) and Sharpe ratios are annualized;  $t$ -statistics of the average returns are Newey–West. The table also reports the worst and best monthly returns and the correlation with the US market ( $\rho_{US}$ ). The sample is restricted to months from January 2005 onward. The estimation window is 36 months.

| Country | Mean (%) | Standard<br>Deviation (%) | $t$   | Sharpe<br>Ratio | Worst<br>Month (%) | Best<br>Month (%) | $\rho_{US}$ |
|---------|----------|---------------------------|-------|-----------------|--------------------|-------------------|-------------|
| CAN     | −9.33    | 27.17                     | −1.63 | −0.34           | −45.81             | 20.82             | 0.21        |
| DEU     | 9.37     | 26.98                     | 1.71  | 0.35            | −34.17             | 22.24             | −0.05       |
| FRA     | 12.96    | 31.28                     | 1.80  | 0.41            | −50.42             | 31.49             | 0.01        |
| GBR     | 13.19    | 27.25                     | 2.17  | 0.48            | −55.75             | 21.40             | 0.07        |
| ITA     | 11.72    | 29.76                     | 1.69  | 0.39            | −26.10             | 29.88             | −0.10       |
| JPN     | 20.18    | 27.43                     | 2.63  | 0.74            | −22.41             | 25.37             | −0.03       |

Table 16: Price Gap Portfolio returns – post 2005, non-G7 countries.

This table reports the out-of-sample performance of the Price Gap Portfolios, using non-G7 countries as Country 2. Mean returns (%), standard deviations (%) and Sharpe ratios are annualized;  $t$ -statistics of the average returns are Newey–West. The table also reports the worst and best monthly returns and the correlation with the US market ( $\rho_{US}$ ). The sample is restricted to months from January 2005 onward. The estimation window is 36 months.

| Country | Mean (%) | Standard<br>Deviation (%) | $t$   | Sharpe<br>Ratio | Worst<br>Month (%) | Best<br>Month (%) | $\rho_{US}$ |
|---------|----------|---------------------------|-------|-----------------|--------------------|-------------------|-------------|
| ARG     | −2.71    | 26.51                     | −0.51 | −0.10           | −32.61             | 30.21             | 0.08        |
| AUS     | 2.37     | 25.95                     | 0.40  | 0.09            | −26.81             | 26.40             | −0.07       |
| AUT     | 18.56    | 26.03                     | 3.17  | 0.71            | −25.69             | 28.73             | −0.02       |
| BEL     | 20.57    | 26.19                     | 3.89  | 0.79            | −27.05             | 24.61             | 0.05        |
| BRA     | 17.72    | 24.52                     | 3.11  | 0.72            | −16.15             | 32.16             | −0.02       |
| CHL     | 8.29     | 30.42                     | 1.25  | 0.27            | −49.20             | 33.12             | 0.09        |
| DNK     | 14.85    | 28.24                     | 2.27  | 0.53            | −24.63             | 31.82             | 0.04        |
| ESP     | 28.28    | 27.83                     | 4.41  | 1.02            | −25.13             | 34.70             | −0.07       |
| HKG     | 6.61     | 26.69                     | 1.04  | 0.25            | −20.47             | 38.34             | −0.00       |
| IND     | 17.36    | 28.77                     | 2.32  | 0.60            | −65.92             | 20.53             | 0.00        |
| KOR     | 19.31    | 25.54                     | 3.26  | 0.76            | −30.78             | 23.16             | −0.01       |
| MEX     | −1.27    | 29.34                     | −0.20 | −0.04           | −34.24             | 27.13             | −0.10       |
| NOR     | 4.39     | 26.07                     | 0.66  | 0.17            | −26.34             | 29.98             | 0.08        |
| POL     | 7.34     | 25.34                     | 1.39  | 0.29            | −29.42             | 26.56             | −0.10       |
| SGP     | 10.96    | 27.04                     | 1.69  | 0.41            | −36.48             | 42.61             | 0.02        |
| SWE     | 6.25     | 26.08                     | 1.06  | 0.24            | −43.99             | 23.03             | 0.14        |
| THA     | 10.40    | 29.11                     | 1.35  | 0.36            | −32.50             | 23.35             | 0.04        |
| TUR     | 16.69    | 26.53                     | 2.73  | 0.63            | −18.68             | 29.69             | 0.05        |
| TWN     | 10.18    | 26.23                     | 1.54  | 0.39            | −27.87             | 18.79             | −0.03       |

Table 17: Diversification benefits — Sharpe-weighted combinations.

Every component, *including the US market*, is weighted in proportion to its Sharpe ratio, and the weights are renormalized to sum to one. Each component’s weight is its *expanding-window* Sharpe ratio, computed on every month from 1986-01 up to  $t - 1$  (at least 24 months) and annualized; components with a negative Sharpe are clipped to zero weight. The country portfolios enter raw, as in Table 6, with no volatility pre-scaling. The US market is the Fama–French *mktrf* factor, whereas the market-cap table uses a rebuilt value-weighted US market ( $\rho = 0.999$ ), so the “US market only” rows differ marginally between the two tables. The sample starts 1988-01 rather than 1986-01 because no weight is defined until a component has 24 months of history.  $\bar{n}_{ctry}$  counts foreign components carrying a strictly positive weight, so it falls below the number live whenever a component’s trailing Sharpe is negative. Because every component is weighted the same way, the US market carries only a small share of these portfolios, as the  $\bar{w}_{US}$  column shows, and the combinations are close to portfolios of the foreign components alone. The rows measure what Sharpe-proportional weighting achieves across the full set of components; for the gain to an investor who holds the US market and adds foreign assets, see Table 6. The estimation window is 36 months.

| Portfolio                                           | Mean<br>(%) | SD<br>(%) | Sharpe | $N$ | $\bar{w}_{US}$<br>(%) | $\bar{n}_{ctry}$ |
|-----------------------------------------------------|-------------|-----------|--------|-----|-----------------------|------------------|
| <i>Panel A: Full sample (1988-01–2024-12)</i>       |             |           |        |     |                       |                  |
| US market only                                      | 9.1         | 15.1      | 0.60   | 444 | 100.0                 | 0.0              |
| US + country markets                                | 6.6         | 17.7      | 0.37   | 444 | 8.1                   | 20.1             |
| US + RG                                             | 10.6        | 7.8       | 1.37   | 444 | 12.9                  | 16.9             |
| US + PG                                             | 13.6        | 10.9      | 1.25   | 444 | 8.9                   | 18.2             |
| US + RG + PG                                        | 23.3        | 14.6      | 1.60   | 444 | 8.6                   | 18.3             |
| <i>Panel B: Pre-2005 (1988-01–2004-12)</i>          |             |           |        |     |                       |                  |
| US market only                                      | 8.3         | 14.6      | 0.57   | 204 | 100.0                 | 0.0              |
| US + country markets                                | 6.0         | 16.3      | 0.37   | 204 | 11.3                  | 14.9             |
| US + RG                                             | 9.5         | 9.7       | 0.98   | 204 | 23.3                  | 10.3             |
| US + PG                                             | 14.1        | 11.1      | 1.27   | 204 | 14.4                  | 12.8             |
| US + RG + PG                                        | 23.4        | 15.3      | 1.53   | 204 | 15.0                  | 12.2             |
| <i>Panel C: Post-2005 (2005-01–2024-12)</i>         |             |           |        |     |                       |                  |
| US market only                                      | 9.7         | 15.5      | 0.63   | 240 | 100.0                 | 0.0              |
| US + country markets                                | 7.1         | 18.9      | 0.38   | 240 | 5.5                   | 24.6             |
| US + RG                                             | 11.6        | 5.7       | 2.04   | 240 | 4.1                   | 22.6             |
| US + PG                                             | 13.2        | 10.7      | 1.24   | 240 | 4.2                   | 22.9             |
| US + RG + PG                                        | 23.3        | 14.0      | 1.66   | 240 | 3.2                   | 23.5             |
| <i>Panel D: All 25 pairs live (2010-08–2024-12)</i> |             |           |        |     |                       |                  |
| US market only                                      | 13.4        | 14.9      | 0.90   | 173 | 100.0                 | 0.0              |
| US + country markets                                | 6.1         | 16.5      | 0.37   | 173 | 5.8                   | 24.9             |
| US + RG                                             | 13.1        | 5.4       | 2.42   | 173 | 4.1                   | 23.1             |
| US + PG                                             | 13.7        | 10.2      | 1.34   | 173 | 4.3                   | 23.5             |
| US + RG + PG                                        | 24.5        | 12.9      | 1.89   | 173 | 3.2                   | 23.9             |

## B Additional Figures

Figure 9: Correlation with the US market, country by country.

The figure repeats the right panel of Figure 1 for each country separately. Correlations between monthly US dollar MSCI returns and the US market are estimated with a one-sided Epanechnikov kernel of bandwidth 120 months; dashed lines are OLS trends. Panels are ordered G7, other developed, then emerging. A country appears once its index has at least 24 effective monthly observations within the kernel window, which is why the emerging panels begin later.

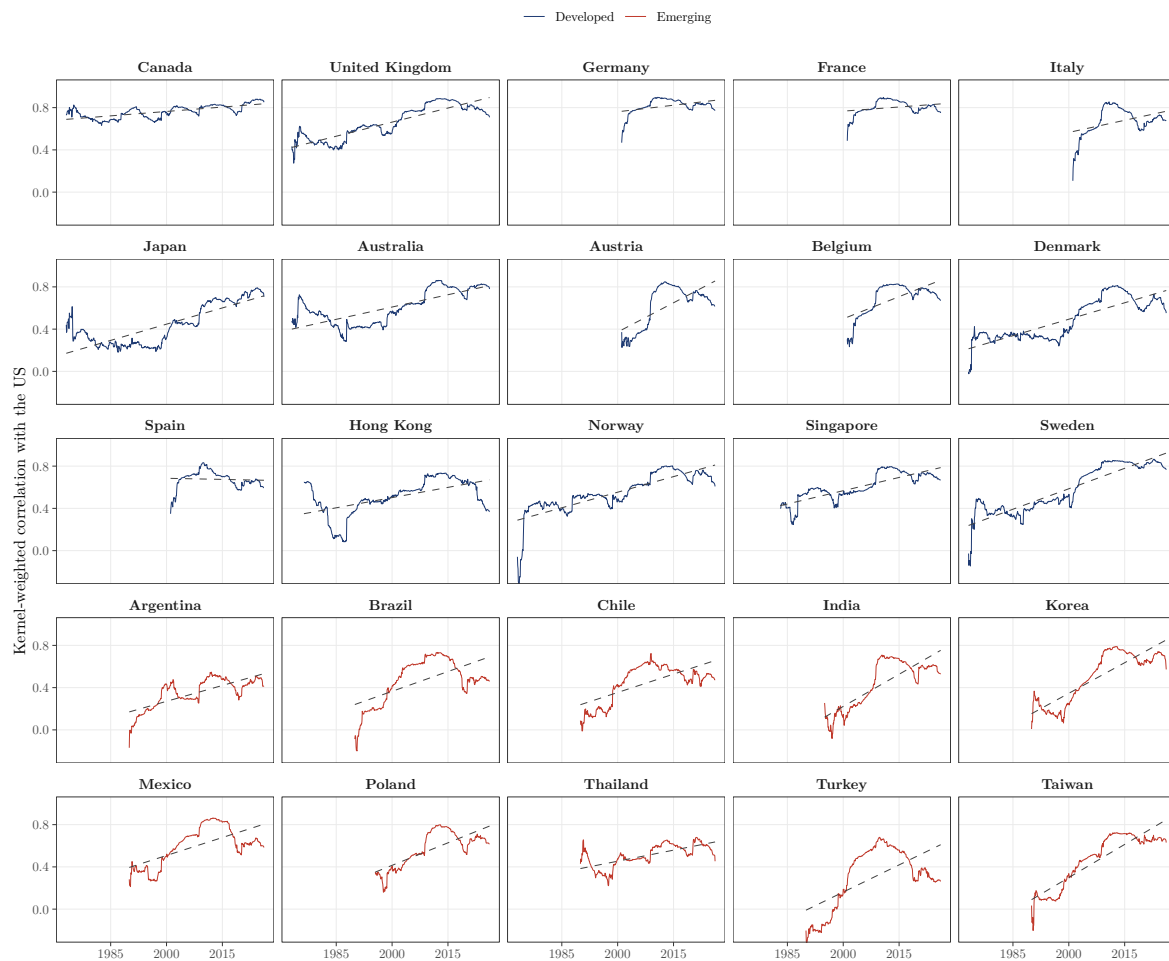
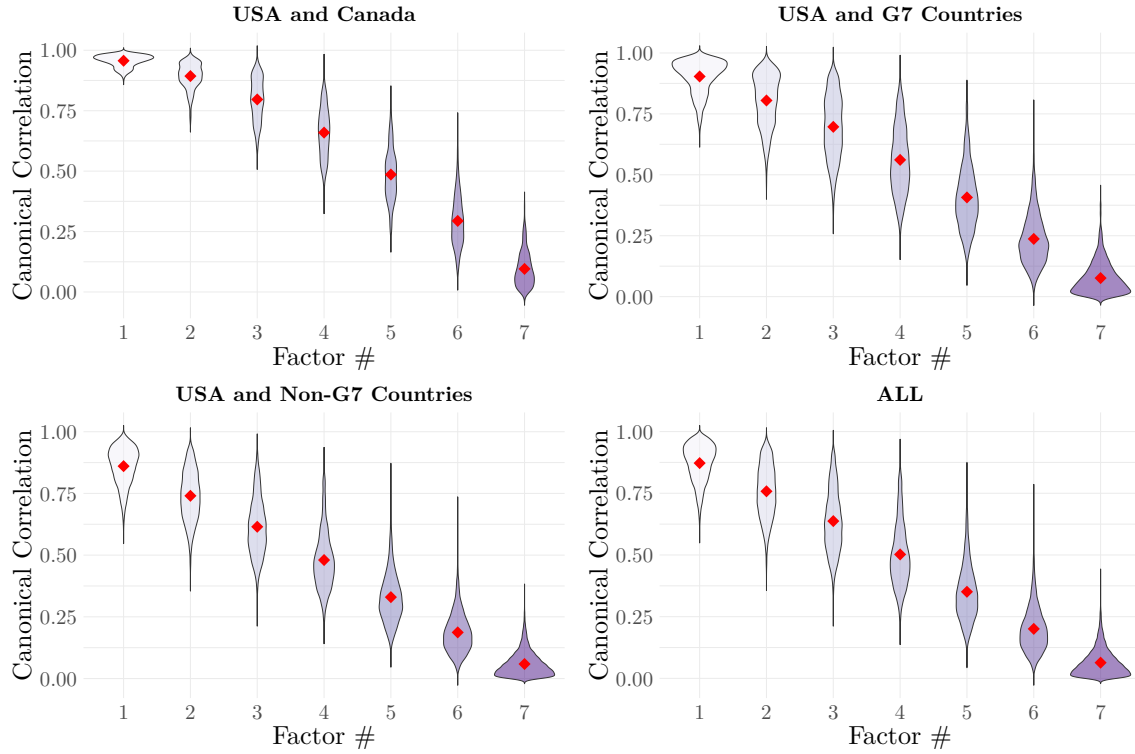


Figure 10: Canonical correlations.

Each violin plot shows the distribution of canonical correlations across rolling estimation windows between the systematic factors of Country 1 (USA) and Country 2. Wider sections indicate higher density; red dots mark the mean. The top-left panel presents results with Canada as Country 2. The top-right panel shows results for G7 countries, the bottom-left for non-G7 countries, and the bottom-right for all 25 countries in the sample.



## C Proofs

**Lemma C.1.** *Let  $\mathbf{L}_g$  be a lower triangular matrix such that  $\mathbf{H}'_g \mathbf{J}_T \mathbf{H}_g = \mathbf{L}_g \mathbf{L}'_g$ . From the eigendecomposition of  $\mathbf{L}'_g \mathbf{V}_{\Theta_g} \mathbf{L}_g$ , find  $\mathbf{U}_g$  such that  $\mathbf{L}'_g \mathbf{V}_{\Theta_g} \mathbf{L}_g = \mathbf{U}_g \mathbf{D} \mathbf{U}'_g$ , where  $\mathbf{D}$  is a diagonal matrix and  $\mathbf{U}'_g \mathbf{U}_g = \mathbf{I}_{K_g}$ . Define  $\mathcal{O}_g$  as  $\mathbf{L}'_g^{-1} \mathbf{U}_g$ . Then, the following hold:*

- (i)  $\mathcal{O}'_g \mathbf{H}'_g \mathbf{J}_T \mathbf{H}_g \mathcal{O}_g$  is an identity matrix, and
- (ii)  $\mathcal{O}_g^{-1} \mathbf{V}_{\Theta_g} \mathcal{O}_g^{-1'}$  is a diagonal matrix.

**Proof** First, we show (i)  $\mathcal{O}'_g \mathbf{H}'_g \mathbf{J}_T \mathbf{H}_g \mathcal{O}_g$  is an identity matrix. Note that

$$\mathcal{O}'_g \mathbf{H}'_g \mathbf{J}_T \mathbf{H}_g \mathcal{O}_g = \mathbf{U}'_g \mathbf{L}'_g^{-1} \mathbf{L}_g \mathbf{L}'_g \mathbf{L}'_g^{-1} \mathbf{U}_g = \mathbf{U}'_g \mathbf{U}_g = \mathbf{I}_{K_g},$$

where the last equality is from the property of  $\mathbf{U}_g$ .

We move to the next claim. Note that  $\mathcal{O}_g^{-1} = \mathbf{U}_g^{-1} \mathbf{L}'_g = \mathbf{U}'_g \mathbf{L}'_g$ , which in turn gives,

$$\mathcal{O}_g^{-1} \mathbf{V}_{\Theta_g} \mathcal{O}_g^{-1'} = \mathbf{U}'_g \mathbf{L}'_g \mathbf{V}_{\Theta_g} \mathbf{L}_g \mathbf{U}_g = \mathbf{U}'_g \mathbf{U}_g \mathbf{D} \mathbf{U}'_g \mathbf{U}_g = \mathbf{D},$$

where the second equality is from the eigendecomposition of  $\mathbf{L}'_g \mathbf{V}_{\Theta_g} \mathbf{L}_g$  and the last equality is from the property of  $\mathbf{U}_g$ . This completes the proof of the lemma.  $\square$

**Proof of Theorem 3.1** The following three steps complete the proof.

Step 1.  $\frac{\widehat{\mathbf{R}}'_g \widehat{\mathbf{R}}_g}{N_g} \xrightarrow{p} \mathbf{J}_T \mathbf{H}_g \mathbf{V}_{\Theta_g} \mathbf{H}'_g \mathbf{J}_T$ : From (3.2), we have that

$$\widehat{\mathbf{R}}_g = j_1 + j_2 + j_3 + j_4, \tag{C.1}$$

where  $j_1 = \mathbf{X}_g \Theta_g \mathbf{H}'_g \mathbf{J}_T$ ,  $j_2 = \mathbf{P}_g \Gamma_g^c \mathbf{F}' \mathbf{J}_T$ ,  $j_3 = \mathbf{P}_g \Gamma_g^s \mathbf{G}'_g \mathbf{J}_T$ , and  $j_4 = \mathbf{P}_g \mathbf{E}_g \mathbf{J}_T$ . Then,

$$\frac{\widehat{\mathbf{R}}'_g \widehat{\mathbf{R}}_g}{N_g} = \sum_{k,l=1,2,3,4} \frac{j'_k j_l}{N_g}. \tag{C.2}$$

Note that

$$\frac{j'_1 j_1}{N_g} = \mathbf{J}_T \mathbf{H}_g \frac{\Theta'_g \mathbf{X}'_g \mathbf{X}_g \Theta_g}{N_g} \mathbf{H}'_g \mathbf{J}_T \xrightarrow{p} \mathbf{J}_T \mathbf{H}_g \mathbf{V}_{\Theta_g} \mathbf{H}'_g \mathbf{J}_T, \tag{C.3}$$

where the limit is from Assumption 2(i), and that

$$\frac{j_2' j_2}{N_g} = \frac{\mathbf{J}_T \mathbf{F} \Gamma_g^{c'} \mathbf{P}_g \Gamma_g^c \mathbf{F}' \mathbf{J}_T}{N_g} = \mathbf{J}_T \mathbf{F} \frac{\Gamma_g^{c'} \mathbf{X}_g}{N_g} \left( \frac{\mathbf{X}_g' \mathbf{X}_g}{N_g} \right)^{-1} \frac{\mathbf{X}_g' \Gamma_g^c}{N_g} \mathbf{F}' \mathbf{J}_T \xrightarrow{p} \mathbf{0}_{T \times T}, \quad (\text{C.4})$$

where the limit is from Assumptions 1(i) and 1(ii), and that

$$\frac{j_3' j_3}{N_g} = \frac{\mathbf{J}_T \mathbf{G}_g \Gamma_g^{s'} \mathbf{P}_g \Gamma_g^s \mathbf{G}_g' \mathbf{J}_T}{N_g} = \mathbf{J}_T \mathbf{G}_g \frac{\Gamma_g^{s'} \mathbf{X}_g}{N_g} \left( \frac{\mathbf{X}_g' \mathbf{X}_g}{N_g} \right)^{-1} \frac{\mathbf{X}_g' \Gamma_g^s}{N_g} \mathbf{G}_g' \mathbf{J}_T \xrightarrow{p} \mathbf{0}_{T \times T}, \quad (\text{C.5})$$

where the limit is from Assumptions 1(i) and 1(ii), and that

$$\frac{j_4' j_4}{N_g} = \frac{\mathbf{J}_T \mathbf{E}_g' \mathbf{P}_g \mathbf{E}_g \mathbf{J}_T}{N_g} = \mathbf{J}_T \frac{\mathbf{E}_g' \mathbf{X}_g}{N_g} \left( \frac{\mathbf{X}_g' \mathbf{X}_g}{N_g} \right)^{-1} \frac{\mathbf{X}_g' \mathbf{E}_g}{N_g} \mathbf{J}_T \xrightarrow{p} \mathbf{0}_{T \times T}, \quad (\text{C.6})$$

where the limit is from Assumptions 1(i) and 1(ii). From (C.3)-(C.6) and the submultiplicativity of Frobenius norm, we have that

$$\left\| \frac{j_1' j_l}{N_g} \right\| \leq \left\| \frac{j_1}{\sqrt{N_g}} \right\| \left\| \frac{j_l}{\sqrt{N_g}} \right\| = \sqrt{\text{tr} \left( \frac{j_1' j_1}{N_g} \right) \text{tr} \left( \frac{j_l' j_l}{N_g} \right)} \xrightarrow{p} 0 \quad (\text{C.7})$$

for  $l = 2, 3, 4$  and that

$$\left\| \frac{j_k' j_l}{N_g} \right\| \leq \left\| \frac{j_k}{\sqrt{N_g}} \right\| \left\| \frac{j_l}{\sqrt{N_g}} \right\| = \sqrt{\text{tr} \left( \frac{j_k' j_k}{N_g} \right) \text{tr} \left( \frac{j_l' j_l}{N_g} \right)} \xrightarrow{p} 0 \quad (\text{C.8})$$

for  $k, l = 2, 3, 4$ . By plugging (C.3)-(C.8) into (C.2), we confirm the claim of Step 1.

Step 2. The  $k$ -th column of  $\mathbf{J}_T \mathbf{H}_g \mathcal{O}_g$ , where  $\mathcal{O}_g$  is given by Lemma C.1, is the  $k$ -th eigenvector of  $\mathbf{J}_T \mathbf{H}_g \mathbf{V}_{\Theta_g} \mathbf{H}_g' \mathbf{J}_T$ : Note that

$$\mathbf{J}_T \mathbf{H}_g \mathbf{V}_{\Theta_g} \mathbf{H}_g' \mathbf{J}_T = \mathbf{J}_T \mathbf{H}_g \mathcal{O}_g (\mathcal{O}_g^{-1} \mathbf{V}_{\Theta_g} \mathcal{O}_g^{-1'}) \mathcal{O}_g' \mathbf{H}_g' \mathbf{J}_T. \quad (\text{C.9})$$

Given the properties of  $\mathcal{O}_g$  in Lemma C.1, the claim directly follows. Step 3.  $\widehat{\mathbf{H}}_g \xrightarrow{p} \mathbf{J}_T \mathbf{H}_g \mathcal{O}_g$ : The claim holds due to the continuity of the eigendecomposition. This completes the proof of the theorem.  $\square$

**Lemma C.2.** Define  $\widehat{\Sigma}_{H_g}$  and  $\widehat{\Sigma}_{H_{12}}$  as  $\frac{\widehat{\mathbf{H}}_g' \mathbf{J}_T \widehat{\mathbf{H}}_g}{T}$  and  $\frac{\widehat{\mathbf{H}}_1' \mathbf{J}_T \widehat{\mathbf{H}}_2}{T}$ , respectively, where  $\widehat{\mathbf{H}}_g$  is given in Theorem 3.1. Let  $\widehat{\mathbf{L}}_{H_g}$  be a lower triangular matrix such that  $\widehat{\Sigma}_{H_g} = \widehat{\mathbf{L}}_{H_g} \widehat{\mathbf{L}}_{H_g}'$

for  $g = 1, 2$ . From the singular value decomposition of  $\widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\mathbf{L}}_{H_2}^{-1}$ , we have  $\widehat{\mathbf{V}}_1$ ,  $\widehat{\mathbf{V}}_2$ , and  $\widehat{\Sigma}$  such that

$$\widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\mathbf{L}}_{H_2}^{-1} = \widehat{\mathbf{V}}_1 \widehat{\Sigma} \widehat{\mathbf{V}}_2', \quad (\text{C.10})$$

where  $\widehat{\mathbf{V}}_1' \widehat{\mathbf{V}}_1 = \mathbf{I}_{K_1}$ ,  $\widehat{\mathbf{V}}_2' \widehat{\mathbf{V}}_2 = \mathbf{I}_{K_2}$ , and  $\widehat{\Sigma}$  is a rectangular diagonal matrix with nonnegative diagonal elements in a decreasing order. Let  $\widehat{\Sigma}_{kk}$  be the  $k$ -th diagonal element of  $\widehat{\Sigma}$ . Then, it holds that

- (i)  $\widehat{\mathbf{W}}_1 = \widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\mathbf{V}}_1$ , and the  $k$ -th largest eigenvalue of  $\widehat{\Sigma}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\Sigma}_{H_2}^{-1} \widehat{\Sigma}_{H_{12}}'$  is  $\left(\widehat{\Sigma}_{kk}\right)^2$ ,
- (ii)  $\widehat{\mathbf{W}}_2 = \widehat{\mathbf{L}}_{H_2}^{-1} \widehat{\mathbf{V}}_2$ , and the  $k$ -th largest eigenvalue of  $\widehat{\Sigma}_{H_2}^{-1} \widehat{\Sigma}_{H_{12}}' \widehat{\Sigma}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}}$  is  $\left(\widehat{\Sigma}_{kk}\right)^2$ ,
- (iii) the correlation between  $\widehat{\mathbf{H}}_1 \widehat{\mathbf{W}}_1^{(m)}$  and  $\widehat{\mathbf{H}}_2 \widehat{\mathbf{W}}_2^{(n)}$  is the  $(m, n)$ -th element of  $\widehat{\Sigma}$ , where  $\widehat{\mathbf{W}}_1^{(m)}$  is the  $m$ -th column of  $\widehat{\mathbf{W}}_1$  and  $\widehat{\mathbf{W}}_2^{(n)}$  is the  $n$ -th column of  $\widehat{\mathbf{W}}_2$ .

**Proof** First, we prove the claim (i). Recall that  $\widehat{\mathbf{W}}_1$  is defined by the two conditions: (a) each column of  $\widehat{\mathbf{W}}_1$  is an eigenvector of  $\widehat{\Sigma}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\Sigma}_{H_2}^{-1} \widehat{\Sigma}_{H_{12}}'$  with the corresponding eigenvalues in the diagonal element of  $\widehat{\Sigma} \widehat{\Sigma}'$  and (b)  $\widehat{\mathbf{W}}_1' \widehat{\Sigma}_{H_1} \widehat{\mathbf{W}}_1 = \mathbf{I}_{K_1}$ .

From  $\widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\mathbf{L}}_{H_2}^{-1} = \widehat{\mathbf{V}}_1 \widehat{\Sigma} \widehat{\mathbf{V}}_2'$ , we have that

$$\left(\widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\mathbf{L}}_{H_2}^{-1}\right) \left(\widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\mathbf{L}}_{H_2}^{-1}\right)' = \left(\widehat{\mathbf{V}}_1 \widehat{\Sigma} \widehat{\mathbf{V}}_2'\right) \left(\widehat{\mathbf{V}}_1 \widehat{\Sigma} \widehat{\mathbf{V}}_2'\right)',$$

which, in conjunction with  $\widehat{\mathbf{V}}_2' \widehat{\mathbf{V}}_2 = \mathbf{I}_{K_2}$ , gives

$$\widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\mathbf{L}}_{H_2}^{-1} \widehat{\mathbf{L}}_{H_2}' \widehat{\Sigma}_{H_{12}}' \widehat{\mathbf{L}}_{H_1}^{-1} = \widehat{\mathbf{V}}_1 \widehat{\Sigma} \widehat{\Sigma}' \widehat{\mathbf{V}}_1',$$

yielding, along with  $\widehat{\mathbf{V}}_1' \widehat{\mathbf{V}}_1 = \mathbf{I}_{K_1}$ , that

$$\widehat{\Sigma}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\Sigma}_{H_2}^{-1} \widehat{\Sigma}_{H_{12}}' \left(\widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\mathbf{V}}_1\right) = \widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\mathbf{V}}_1 \widehat{\Sigma} \widehat{\Sigma}' \widehat{\mathbf{V}}_1' \widehat{\mathbf{V}}_1 = \left(\widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\mathbf{V}}_1\right) \widehat{\Sigma} \widehat{\Sigma}',$$

This shows that the  $k$ -th column of  $\widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\mathbf{V}}_1$  is the eigenvector of  $\widehat{\Sigma}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\Sigma}_{H_2}^{-1} \widehat{\Sigma}_{H_{12}}'$  corresponding to the eigenvalue in the  $k$ -th diagonal element of  $\widehat{\Sigma} \widehat{\Sigma}'$ . This confirms the condition (a) for  $\widehat{\mathbf{W}}$ . The condition (b) is satisfied due to that  $\widehat{\mathbf{V}}_1' \widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\Sigma}_{H_1} \widehat{\mathbf{L}}_{H_1}' \widehat{\mathbf{V}}_1 = \widehat{\mathbf{V}}_1' \widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\mathbf{L}}_{H_1} \widehat{\mathbf{L}}_{H_1}' \widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\mathbf{V}}_1 = \widehat{\mathbf{V}}_1' \widehat{\mathbf{V}}_1 = \mathbf{I}_{K_1}$ . Hence, the claim (i) holds. The claim (ii) follows due to symmetry.

Next, we move to the claim (iii). From the claims (i) and (ii), we showed that

$\widehat{\mathbf{W}}_g' \widehat{\Sigma}_{H_g} \widehat{\mathbf{W}}_g = \mathbf{I}_{K_g}$ , implying the variances of  $\widehat{\mathbf{H}}_1 \widehat{\mathbf{W}}_1^{(m)}$  and  $\widehat{\mathbf{H}}_2 \widehat{\mathbf{W}}_2^{(n)}$  are 1. Hence, the correlation of the two would be the covariance of the two,  $\widehat{\mathbf{W}}_1^{(m)'} \widehat{\Sigma}_{H_{12}} \widehat{\mathbf{W}}_2^{(n)}$ . Note that

$$\widehat{\mathbf{W}}_1' \widehat{\Sigma}_{H_{12}} \widehat{\mathbf{W}}_2 = \widehat{\mathbf{V}}_1' \widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\mathbf{L}}_{H_2}'^{-1} \widehat{\mathbf{V}}_2 = \widehat{\Sigma}, \quad (\text{C.11})$$

which confirms that the correlation of the two is the  $(m, n)$ -th element of  $\widehat{\Sigma}$ . This completes the proof of the lemma.  $\square$

The next lemma reveals the  $N$ -limit behavior of CCA in our setup.

**Lemma C.3.** *Let  $\mathbf{V}_g$ ,  $\check{\mathbf{L}}_{H_g}$ ,  $\check{\Sigma}_{H_{12}}$  and  $\Sigma$  be the  $N$ -limits of  $\widehat{\mathbf{V}}_g$ ,  $\widehat{\mathbf{L}}_{H_g}$ ,  $\widehat{\Sigma}_{H_{12}}$  and  $\widehat{\Sigma}$ , respectively, where  $\widehat{\mathbf{V}}_g$ ,  $\widehat{\mathbf{L}}_{H_g}$ ,  $\widehat{\Sigma}_{H_{12}}$  and  $\widehat{\Sigma}$  are defined in Lemma C.2. Let  $\mathbf{L}_F$  and  $\mathbf{L}_{G_g}$  be lower triangular matrices such that  $\frac{\mathbf{F}' \mathbf{J}_T \mathbf{F}}{T} = \mathbf{L}_F \mathbf{L}_F'$  and  $\frac{\mathbf{G}_g' \mathbf{J}_T \mathbf{G}_g}{T} = \mathbf{L}_{G_g} \mathbf{L}_{G_g}'$ . From the singular value decomposition of  $\mathbf{L}_{G_1}^{-1} \frac{\mathbf{G}_1' \mathbf{J}_T \mathbf{G}_2}{T} \mathbf{L}_{G_2}'^{-1}$ , we have  $\mathbf{Z}_1$ ,  $\mathbf{Z}_2$ , and  $\Delta$  such that*

$$\mathbf{L}_{G_1}^{-1} \frac{\mathbf{G}_1' \mathbf{J}_T \mathbf{G}_2}{T} \mathbf{L}_{G_2}'^{-1} = \mathbf{Z}_1 \Delta \mathbf{Z}_2', \quad (\text{C.12})$$

where  $\mathbf{Z}_1' \mathbf{Z}_1 = \mathbf{I}_{K_1^s}$ ,  $\mathbf{Z}_2' \mathbf{Z}_2 = \mathbf{I}_{K_2^s}$ , and  $\Delta$  is a rectangular diagonal matrix with nonnegative diagonal elements in decreasing order. Then, the following hold:

- (i) Any  $(k, k)$ -th element of  $\Delta$  is less than 1,
- (ii)  $\mathbf{V}_g = \check{\mathbf{L}}_{H_g}^{-1} \mathcal{O}_g' \begin{bmatrix} \mathbf{L}_F \mathcal{R}' & \mathbf{0}_{K_c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K_c} & \mathbf{L}_{G_g} \mathbf{Z}_g \end{bmatrix}$ , where  $\mathcal{R} \mathcal{R}' = \mathbf{I}_{K^c}$ ,
- (iii)  $\Sigma = \begin{bmatrix} \mathbf{I}_{K^c} & \mathbf{0}_{K^c \times K_2^s} \\ \mathbf{0}_{K_1^s \times K_c} & \Delta \end{bmatrix}$ .

**Proof** We start with the claim (i). Given  $\mathbf{Z}_g' \mathbf{L}_{G_g}^{-1} \frac{\mathbf{G}_g' \mathbf{J}_T \mathbf{G}_g}{T} \mathbf{L}_{G_g}'^{-1} \mathbf{Z}_g = \mathbf{I}_{K_g^s}$ , any linear combination of  $\mathbf{G}_g$ , denoted as  $\mathbf{G}_g \mathbf{x}_g$ , will have unit variance if  $\mathbf{x}_g = \mathbf{L}_{G_g}'^{-1} \mathbf{Z}_g \mathbf{y}_g$  where  $\mathbf{y}_g' \mathbf{y}_g = 1$ . Hence, set  $\mathbf{y}_g' \mathbf{y}_g = 1$  for  $g = 1, 2$ . Then, the covariance of  $\mathbf{G}_1 \mathbf{x}_1$  and  $\mathbf{G}_2 \mathbf{x}_2$  would be their correlation. Note that

$$\begin{aligned} \mathbf{x}_1' \frac{\mathbf{G}_1' \mathbf{J}_T \mathbf{G}_2}{T} \mathbf{x}_2 &= \mathbf{y}_1' \mathbf{Z}_1' \mathbf{L}_{G_1}^{-1} \frac{\mathbf{G}_1' \mathbf{J}_T \mathbf{G}_2}{T} \mathbf{L}_{G_2}'^{-1} \mathbf{Z}_2 \mathbf{y}_2 \\ &= \mathbf{y}_1' \mathbf{Z}_1' \mathbf{Z}_1 \Delta \mathbf{Z}_2' \mathbf{Z}_2 \mathbf{y}_2 = \mathbf{y}_1' \Delta \mathbf{y}_2, \end{aligned} \quad (\text{C.13})$$

where the second equality is from (C.12). It follows that the maximum of  $\mathbf{y}_1' \Delta \mathbf{y}_2$  would be the maximum element of  $\Delta$ . Because  $\mathbf{y}_1' \Delta \mathbf{y}_2$  is a correlation between  $\mathbf{G}_1 \mathbf{x}_1$  and  $\mathbf{G}_2 \mathbf{x}_2$ ,

the maximum of  $\mathbf{y}'_1 \Delta \mathbf{y}_2$  would be less than one due to Assumption 2(iii), thus proving the claim (i).

Next, we show the claims (ii) and (iii). From Lemma C.2, recall  $\widehat{\mathbf{L}}_{H_1}^{-1} \widehat{\Sigma}_{H_{12}} \widehat{\mathbf{L}}_{H_2}^{-1} = \widehat{\mathbf{V}}_1 \widehat{\Sigma} \widehat{\mathbf{V}}'_2$ . Hence, it should hold that

$$\check{\mathbf{L}}_{H_1}^{-1} \check{\Sigma}_{H_{12}} \check{\mathbf{L}}_{H_2}^{-1} = \mathbf{V}_1 \Sigma \mathbf{V}'_2. \quad (\text{C.14})$$

We prove the claims (ii) and (iii) by verifying the equation above along with  $\mathbf{V}'_g \mathbf{V}_g = \mathbf{I}_{K_g}$ . First, note that

$$\begin{aligned} \mathbf{V}'_g \mathbf{V}_g &= \begin{bmatrix} \mathcal{R} \mathbf{L}'_F & \mathbf{0}_{K_c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K_c} & \mathbf{Z}'_g \mathbf{L}'_{G_g} \end{bmatrix} \mathcal{O}_g \check{\mathbf{L}}_{H_g}^{-1} \check{\mathbf{L}}_{H_g}^{-1} \mathcal{O}'_g \begin{bmatrix} \mathbf{L}_F \mathcal{R}' & \mathbf{0}_{K_c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K_c} & \mathbf{L}_{G_g} \mathbf{Z}_g \end{bmatrix} \\ &= \begin{bmatrix} \mathcal{R} \mathbf{L}'_F & \mathbf{0}_{K_c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K_c} & \mathbf{Z}'_g \mathbf{L}'_{G_g} \end{bmatrix} \begin{bmatrix} (\mathbf{L}_F \mathbf{L}'_F)^{-1} & \mathbf{0}_{K_c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K_c} & (\mathbf{L}_{G_1} \mathbf{L}'_{G_1})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{L}_F \mathcal{R}' & \mathbf{0}_{K_c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K_c} & \mathbf{L}_{G_g} \mathbf{Z}_g \end{bmatrix} = \mathbf{I}_{K_g}. \end{aligned} \quad (\text{C.15})$$

Next, note that

$$\begin{aligned} \check{\mathbf{L}}_{H_1}^{-1} \check{\Sigma}_{H_{12}} \check{\mathbf{L}}_{H_2}^{-1} &= \check{\mathbf{L}}_{H_1}^{-1} \mathcal{O}'_1 \frac{\mathbf{H}'_1 \mathbf{J}_T \mathbf{H}_2}{T} \mathcal{O}_2 \check{\mathbf{L}}_{H_2}^{-1} \\ &= \check{\mathbf{L}}_{H_1}^{-1} \mathcal{O}'_1 \begin{bmatrix} \frac{\mathbf{F}' \mathbf{J}_T \mathbf{F}}{T} & \mathbf{0}_{K_c \times K_2^s} \\ \mathbf{0}_{K_1^s \times K_c} & \frac{\mathbf{G}'_1 \mathbf{J}_T \mathbf{G}_2}{T} \end{bmatrix} \mathcal{O}_2 \check{\mathbf{L}}_{H_2}^{-1} \\ &= \check{\mathbf{L}}_{H_1}^{-1} \mathcal{O}'_1 \begin{bmatrix} \mathbf{L}_F \mathcal{R}' & \mathbf{0}_{K_c \times K_1^s} \\ \mathbf{0}_{K_1^s \times K_c} & \mathbf{L}_{G_1} \mathbf{Z}_1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{K_c} & \mathbf{0}_{K_c \times K_2^s} \\ \mathbf{0}_{K_1^s \times K_c} & \Delta \end{bmatrix} \begin{bmatrix} \mathcal{R} \mathbf{L}'_F & \mathbf{0}_{K_c \times K_2^s} \\ \mathbf{0}_{K_2^s \times K_c} & \mathbf{Z}'_2 \mathbf{L}'_{G_2} \end{bmatrix} \mathcal{O}_2 \check{\mathbf{L}}_{H_2}^{-1} \\ &= \mathbf{V}_1 \Sigma \mathbf{V}'_2, \end{aligned}$$

which confirms (C.14), which along with (C.15) proves the claims (ii) and (iii). This completes the proof of the lemma.

**Proof of Theorem 3.2** From Lemmas C.2 and C.3,

$$\begin{aligned}
\widehat{\mathbf{W}}_g &\xrightarrow{p} \check{\mathbf{L}}_{H_g}^{\prime-1} \check{\mathbf{L}}_{H_g}^{-1} \mathcal{O}_g' \begin{bmatrix} \mathbf{L}_F \mathcal{R}' & \mathbf{0}_{K_c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K_c} & \mathbf{L}_{G_g} \mathbf{Z}_g \end{bmatrix} = \mathcal{O}_g^{-1} \left( \frac{\mathbf{H}_g' \mathbf{J}_T \mathbf{H}_g}{T} \right)^{-1} \begin{bmatrix} \mathbf{L}_F \mathcal{R}' & \mathbf{0}_{K_c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K_c} & \mathbf{L}_{G_g} \mathbf{Z}_g \end{bmatrix} \\
&= \mathcal{O}_g^{-1} \begin{bmatrix} (\mathbf{L}_F \mathbf{L}_F')^{-1} & \mathbf{0}_{K_c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K_c} & (\mathbf{L}_{G_g} \mathbf{L}_{G_g}')^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{L}_F \mathcal{R}' & \mathbf{0}_{K_c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K_c} & \mathbf{L}_{G_g} \mathbf{Z}_g \end{bmatrix} \\
&= \mathcal{O}_g^{-1} \begin{bmatrix} \mathbf{L}_F'^{-1} \mathcal{R}' & \mathbf{0}_{K_c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K_c} & \mathbf{L}_{G_g}'^{-1} \mathbf{Z}_g \end{bmatrix},
\end{aligned}$$

which in conjunction with Theorem 3.1, gives

$$\begin{aligned}
\widehat{\mathbf{H}}_g \widehat{\mathbf{W}}_g &= \widehat{\mathbf{H}}_g \begin{bmatrix} \widehat{\mathbf{W}}_g^c & \widehat{\mathbf{W}}_g^s \end{bmatrix} \xrightarrow{p} \mathbf{J}_T \mathbf{H}_g \mathcal{O}_g \mathcal{O}_g^{-1} \begin{bmatrix} \mathbf{L}_F'^{-1} \mathcal{R}' & \mathbf{0}_{K_c \times K_g^s} \\ \mathbf{0}_{K_g^s \times K_c} & \mathbf{L}_{G_g}'^{-1} \mathbf{Z}_g \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{J}_T \mathbf{F} \mathbf{L}_F'^{-1} \mathcal{R}' & \mathbf{J}_T \mathbf{G}_g \mathbf{L}_{G_g}'^{-1} \mathbf{Z}_g \end{bmatrix}.
\end{aligned}$$

Finally, set  $\mathcal{S}^c = \mathbf{L}_F'^{-1} \mathcal{R}'$  and  $\mathcal{S}_g^s = \mathbf{L}_{G_g}'^{-1} \mathbf{Z}_g$ . Note that  $\mathcal{S}^c \mathcal{S}^c' = \mathbf{L}_F'^{-1} \mathcal{R}' \mathcal{R} \mathbf{L}_F^{-1} = \mathbf{L}_F'^{-1} \mathbf{L}_F^{-1} = \Sigma_F^{-1}$  and that  $\mathcal{S}_g^s = \mathbf{L}_{G_g}'^{-1} \mathbf{Z}_g \mathbf{Z}_g' \mathbf{L}_{G_g}^{-1} = \Sigma_{G_g}^{-1}$ . This completes the proof of the theorem.  $\square$

**Proof of Theorem 3.3** Note that

$$\begin{aligned}
&\|\widehat{\mathbf{R}}_g - (\mathbf{X}_g \Theta_g^c) \widehat{\mathbf{F}}_g' - (\mathbf{X}_g \Theta_g^s) \widehat{\mathbf{G}}_g'\| = \|\widehat{\mathbf{R}}_g - \mathbf{X}_g \begin{bmatrix} \Theta_g^c & \Theta_g^s \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{F}} & \widehat{\mathbf{G}} \end{bmatrix}'\| \\
&= \|\text{vec}(\widehat{\mathbf{R}}_g) - \left( \begin{bmatrix} \widehat{\mathbf{F}} & \widehat{\mathbf{G}} \end{bmatrix} \otimes \mathbf{X}_g \right) \text{vec}(\begin{bmatrix} \Theta_g^c & \Theta_g^s \end{bmatrix})\|,
\end{aligned}$$

where the second equality is from the properties of the vectorization operator. Hence, the solution  $(\widehat{\Theta}_g^c, \widehat{\Theta}_g^s)$  will be determined by

$$\text{vec} \left( \begin{bmatrix} \widehat{\Theta}_g^c & \widehat{\Theta}_g^s \end{bmatrix} \right) = (\mathbf{X}' \mathbf{X})^{-1} \mathbf{X}' \mathbf{y}, \quad (\text{C.16})$$

where  $\mathbf{y} = \text{vec}(\widehat{\mathbf{R}}_g)$  and  $\mathcal{X} = \left( \begin{bmatrix} \widehat{\mathbf{F}} & \widehat{\mathbf{G}} \end{bmatrix} \otimes \mathbf{X}_g \right)$ .

Furthermore, note that

$$\begin{aligned}
\widehat{\mathbf{R}}_g &= \mathbf{P}_g \mathbf{R}_g \mathbf{J}_T \\
&= \mathbf{X}_g \Theta_g^c \mathcal{S}_g^{c'-1} \mathcal{S}_g^{c'} \mathbf{F}' \mathbf{J}_T + \mathbf{X}_g \Theta_g^s \mathcal{S}_g^{s'-1} \mathcal{S}_g^{s'} \mathbf{G}' \mathbf{J}_T + \mathbf{P}_g \Gamma_g^c \mathbf{F}' \mathbf{J}_T + \mathbf{P}_g \Gamma_g^s \mathbf{G}' \mathbf{J}_T + \mathbf{P}_g \mathbf{E}_g \mathbf{J}_T \\
&= \mathbf{X}_g \begin{bmatrix} \Theta_g^c \mathcal{S}_g^{c'-1} & \Theta_g^s \mathcal{S}_g^{s'-1} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{F}} & \widehat{\mathbf{G}} \end{bmatrix}' + \mathcal{U}_g,
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{U}_g &= \mathbf{X}_g \Theta_g^c \mathcal{S}_g^{c'-1} \left( \mathbf{J}_T \mathbf{F} \mathcal{S}_g^c - \widehat{\mathbf{F}} \right)' + \mathbf{X}_g \Theta_g^s \mathcal{S}_g^{s'-1} \left( \mathbf{J}_T \mathbf{G} \mathcal{S}_g^s - \widehat{\mathbf{G}} \right)' \\
&\quad + \mathbf{P}_g \Gamma_g^c \mathbf{F}' \mathbf{J}_T + \mathbf{P}_g \Gamma_g^s \mathbf{G}' \mathbf{J}_T + \mathbf{P}_g \mathbf{E}_g \mathbf{J}_T.
\end{aligned}$$

We rewrite  $\mathbf{y}$  as follows:

$$\begin{aligned}
\mathbf{y} &= \text{vec} \left( \widehat{\mathbf{R}}_g \right) = \text{vec} \left( \mathbf{X}_g \begin{bmatrix} \Theta_g^c \mathcal{S}_g^{c'-1} & \Theta_g^s \mathcal{S}_g^{s'-1} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{F}} & \widehat{\mathbf{G}} \end{bmatrix}' + \mathcal{U}_g \right) \\
&= \left( \begin{bmatrix} \widehat{\mathbf{F}} & \widehat{\mathbf{G}} \end{bmatrix} \otimes \mathbf{X}_g \right) \text{vec} \left( \begin{bmatrix} \Theta_g^c \mathcal{S}_g^{c'-1} & \Theta_g^s \mathcal{S}_g^{s'-1} \end{bmatrix} \right) + \text{vec} (\mathcal{U}_g) \\
&= \mathbf{X} \text{vec} \left( \begin{bmatrix} \Theta_g^c \mathcal{S}_g^{c'-1} & \Theta_g^s \mathcal{S}_g^{s'-1} \end{bmatrix} \right) + \text{vec} (\mathcal{U}_g),
\end{aligned}$$

which in conjunction with (C.16) gives

$$\text{vec} \left( \begin{bmatrix} \widehat{\Theta}_g^c & \widehat{\Theta}_g^s \end{bmatrix} \right) = \text{vec} \left( \begin{bmatrix} \Theta_g^c \mathcal{S}_g^{c'-1} & \Theta_g^s \mathcal{S}_g^{s'-1} \end{bmatrix} \right) + (\mathbf{X}' \mathbf{X})^{-1} \mathbf{X}' \text{vec} (\mathcal{U}_g). \quad (\text{C.17})$$

Hence, it suffices to show that  $\left( \frac{\mathbf{X}' \mathbf{X}}{N_g} \right)^{-1} \frac{\mathbf{X}' \text{vec}(\mathcal{U}_g)}{N_g} \xrightarrow{p} \mathbf{0}_{L_g}$ . First, note that

$$\begin{aligned}
\frac{\mathbf{X}' \mathbf{X}}{N_g} &= \frac{\left( \begin{bmatrix} \widehat{\mathbf{F}} & \widehat{\mathbf{G}} \end{bmatrix} \otimes \mathbf{X}_g \right)' \left( \begin{bmatrix} \widehat{\mathbf{F}} & \widehat{\mathbf{G}} \end{bmatrix} \otimes \mathbf{X}_g \right)}{N_g} \\
&= \left( \begin{bmatrix} \widehat{\mathbf{F}} & \widehat{\mathbf{G}} \end{bmatrix}' \begin{bmatrix} \widehat{\mathbf{F}} & \widehat{\mathbf{G}} \end{bmatrix} \right) \otimes \frac{\mathbf{X}_g' \mathbf{X}_g}{N_g} \\
&\xrightarrow{p} \left( \begin{bmatrix} \mathbf{J}_T \mathbf{F} \mathcal{S}_g^c & \mathbf{J}_T \mathbf{G} \mathcal{S}_g^s \end{bmatrix}' \begin{bmatrix} \mathbf{J}_T \mathbf{F} \mathcal{S}_g^c & \mathbf{J}_T \mathbf{G} \mathcal{S}_g^s \end{bmatrix} \right) \otimes \mathbf{V}_{X_g}, \quad (\text{C.18})
\end{aligned}$$

where the last limit is from Assumption 1(i) and Theorem 3.2. Next, using the property

of vectorize operator, we rewrite  $\text{vec}(\mathcal{U}_g)$  as follows:

$$\begin{aligned}\text{vec}(\mathcal{U}_g) &= \left( \left[ \left( \mathbf{J}_T \mathbf{F} \mathcal{S}_g^c - \widehat{\mathbf{F}} \right) \left( \mathbf{J}_T \mathbf{G} \mathcal{S}_g^c - \widehat{\mathbf{G}} \right) \right] \otimes \mathbf{X}_g \right) \text{vec} \left( [\Theta_g^c \mathcal{S}_g^{c'-1} \ \Theta_g^s \mathcal{S}_g^{s'-1}] \right) \\ &\quad + (\mathbf{I}_T \otimes (\mathbf{P}_g \Gamma_g^c)) \text{vec}(\mathbf{F}' \mathbf{J}_T) + (\mathbf{I}_T \otimes (\mathbf{P}_g \Gamma_g^s)) \text{vec}(\mathbf{G}' \mathbf{J}_T) \\ &\quad + (\mathbf{I}_T \otimes (\mathbf{P}_g \mathbf{E}_g)) \text{vec}(\mathbf{J}_T),\end{aligned}$$

which in turn gives

$$\begin{aligned}& \frac{\mathbf{X}' \text{vec}(\mathcal{U}_g)}{N_g} \\ &= \left( \left( \left[ \widehat{\mathbf{F}} \ \widehat{\mathbf{G}} \right]' \left[ \left( \mathbf{J}_T \mathbf{F} \mathcal{S}_g^c - \widehat{\mathbf{F}} \right) \left( \mathbf{J}_T \mathbf{G} \mathcal{S}_g^c - \widehat{\mathbf{G}} \right) \right] \right) \otimes \frac{\mathbf{X}'_g \mathbf{X}_g}{N_g} \right) \text{vec} \left( [\Theta_g^c \mathcal{S}_g^{c'-1} \ \Theta_g^s \mathcal{S}_g^{s'-1}] \right) \\ &\quad + \left( \left[ \widehat{\mathbf{F}} \ \widehat{\mathbf{G}} \right]' \otimes \left( \frac{\mathbf{X}'_g \mathbf{P}_g \Gamma_g^c}{N_g} \right) \right) \text{vec}(\mathbf{F}' \mathbf{J}_T) + \left( \left[ \widehat{\mathbf{F}} \ \widehat{\mathbf{G}} \right]' \otimes \left( \frac{\mathbf{X}'_g \mathbf{P}_g \Gamma_g^s}{N_g} \right) \right) \text{vec}(\mathbf{G}' \mathbf{J}_T) \\ &\quad + \left( \left[ \widehat{\mathbf{F}} \ \widehat{\mathbf{G}} \right]' \otimes \left( \frac{\mathbf{X}'_g \mathbf{P}_g \mathbf{E}_g}{N_g} \right) \right) \text{vec}(\mathbf{J}_T) \xrightarrow{p} \mathbf{0}_{L_g},\end{aligned}\tag{C.19}$$

where the last limit is from Assumptions 1(i) and 1(ii) and Theorem 3.2.

Finally, substituting (C.18) and (C.19) into (C.17), we have that

$$\text{vec} \left( \left[ \widehat{\Theta}_g^c \ \widehat{\Theta}_g^s \right] \right) \xrightarrow{p} \text{vec} \left( [\Theta_g^c \mathcal{S}_g^{c'-1} \ \Theta_g^s \mathcal{S}_g^{s'-1}] \right),$$

which completes the proof of the theorem.  $\square$

**Proof of Theorem 3.4** We rewrite  $\mathbf{R}_g$  in (2.6) as follows:

$$\begin{aligned}\mathbf{R}_g &= \mathbf{X}_g \left[ \Theta_g^c \mathcal{S}_g^{c'-1} \ \Theta_g^s \mathcal{S}_g^{s'-1} \right] \begin{bmatrix} \mathcal{S}_g^{c'} (\boldsymbol{\lambda}_g^c \mathbf{1}'_T + \mathbf{F}') \\ \mathcal{S}_g^{s'} (\boldsymbol{\lambda}_g^s \mathbf{1}'_T + \mathbf{G}') \end{bmatrix} \\ &\quad + \Gamma_g^c (\boldsymbol{\lambda}_g^c \mathbf{1}'_T + \mathbf{F}') + \Gamma_g^s (\boldsymbol{\lambda}_g^s \mathbf{1}'_T + \mathbf{G}') + \mathbf{E}_g.\end{aligned}$$

Set  $\Xi = \left( \begin{bmatrix} \widehat{\Theta}_g^c & \widehat{\Theta}_g^s \end{bmatrix}' \frac{\mathbf{X}_g' \mathbf{X}_g}{N} \begin{bmatrix} \widehat{\Theta}_g^c & \widehat{\Theta}_g^s \end{bmatrix} \right)^{-1}$ , which gives  $\widehat{\mathbf{w}}_g^b = \frac{\mathbf{X}_g}{N} \begin{bmatrix} \widehat{\Theta}_g^c & \widehat{\Theta}_g^s \end{bmatrix} \Xi$ . Note that

$$\begin{aligned} \widehat{\mathbf{w}}_g^{b'} \mathbf{R}_g &= \Xi \begin{bmatrix} \widehat{\Theta}_g^c & \widehat{\Theta}_g^s \end{bmatrix}' \frac{\mathbf{X}_g' \mathbf{R}_g}{N_g} \\ &= \Xi \begin{bmatrix} \widehat{\Theta}_g^c & \widehat{\Theta}_g^s \end{bmatrix}' \frac{\mathbf{X}_g' \mathbf{X}_g}{N_g} \begin{bmatrix} \Theta_g^c \mathcal{S}^{c'-1} & \Theta_g^s \mathcal{S}_g^{s'-1} \end{bmatrix} \begin{bmatrix} \mathcal{S}^{c'} (\boldsymbol{\lambda}_g^c \mathbf{1}_T' + \mathbf{F}') \\ \mathcal{S}_g^{s'} (\boldsymbol{\lambda}_g^s \mathbf{1}_T' + \mathbf{G}_g') \end{bmatrix} \\ &\quad + \Xi \begin{bmatrix} \widehat{\Theta}_g^c & \widehat{\Theta}_g^s \end{bmatrix}' \left( \frac{\mathbf{X}_g' \Gamma_g^c}{N_g} (\boldsymbol{\lambda}_g^c \mathbf{1}_T' + \mathbf{F}') + \frac{\mathbf{X}_g' \Gamma_g^s}{N_g} (\boldsymbol{\lambda}_g^s \mathbf{1}_T' + \mathbf{G}_g') + \frac{\mathbf{X}_g' \mathbf{E}_g}{N_g} \right) \\ &\xrightarrow{p} \begin{bmatrix} \mathcal{S}^{c'} (\boldsymbol{\lambda}_g^c \mathbf{1}_T' + \mathbf{F}') \\ \mathcal{S}_g^{s'} (\boldsymbol{\lambda}_g^s \mathbf{1}_T' + \mathbf{G}_g') \end{bmatrix}, \end{aligned}$$

where the last limit is from Assumptions 1(i) and 1(ii) and Theorem 3.3. This completes the proof of the theorem.  $\square$

**Proof of Theorem 3.5** From Theorem 3.4, we have that  $\widehat{\mathbf{w}}_1^{c'} \mathbf{R}_1 \xrightarrow{p} \mathcal{S}^{c'} (\boldsymbol{\lambda}_1^c \mathbf{1}_T' + \mathbf{F}')$  and  $\widehat{\mathbf{w}}_2^{c'} \mathbf{R}_2 \xrightarrow{p} \mathcal{S}^{c'} (\boldsymbol{\lambda}_2^c \mathbf{1}_T' + \mathbf{F}')$ . Hence, in the regression of the  $(T \times K^c)$  matrix  $\mathbf{R}_2' \widehat{\mathbf{w}}_2^c$  on a constant and the  $(T \times K^c)$  matrix  $\mathbf{R}_1' \widehat{\mathbf{w}}_1^c$ , the intercept vector converges to  $\mathcal{S}^{c'} (\boldsymbol{\lambda}_2^c - \boldsymbol{\lambda}_1^c)$ , while the coefficient matrix on  $\mathbf{R}_1' \widehat{\mathbf{w}}_1^c$  converges to the identity matrix.  $\square$

## D Bootstrap Method for the Number of Common Factors

For the reader's convenience, we outline the bootstrap inference procedure proposed by Gonçalves et al. (2025) to determine the number of common factors in our two-country model.

### Model and Estimation

For expositional simplicity, we slightly abuse notation; it does not exactly match that used in the main text. We consider two sets of time-series returns,  $\mathbf{y}_{1t}$  ( $N_1 \times 1$ ) and  $\mathbf{y}_{2t}$  ( $N_2 \times 1$ ), for  $t = 1, \dots, T$ , modeled as

$$\mathbf{y}_{1t} = \mathbf{B}_1 \mathbf{f}_t + \mathbf{D}_1 \mathbf{g}_{1t} + \mathbf{e}_{1t}, \quad \mathbf{y}_{2t} = \mathbf{B}_2 \mathbf{f}_t + \mathbf{D}_2 \mathbf{g}_{2t} + \mathbf{e}_{2t},$$

where  $\mathbf{f}_t$  denotes the  $K^c$  common factors,  $\mathbf{g}_{1t}$  and  $\mathbf{g}_{2t}$  represent country-specific factors of dimensions  $K_1^s$  and  $K_2^s$ , respectively, and  $\mathbf{e}_{1t}$  and  $\mathbf{e}_{2t}$  are errors. The total number of factors for each group is therefore  $K_1 = K^c + K_1^s$  and  $K_2 = K^c + K_2^s$ .

Regarding the data  $\widehat{\mathbf{R}}_g$  in (3.2), we define  $\mathbf{Y}_g = [\mathbf{y}_{g1}, \mathbf{y}_{g2}, \dots, \mathbf{y}_{gT}]' = \widehat{\mathbf{R}}_g'$ , which is a  $(T \times N_g)$  matrix for  $g = 1, 2$ . The factors for country  $g$  are estimated using principal component analysis (PCA) applied to the data matrix  $\mathbf{Y}_g$ . Specifically, the eigenvectors of the covariance matrix  $\mathbf{Y}_g \mathbf{Y}_g' / N_g$  provide the estimated factor space  $\widehat{\mathbf{H}}_g$ .

Next, the canonical correlations  $\widehat{\rho}_k$  for  $k = 1, \dots, \min(K_1, K_2)$  are computed as the square roots of the eigenvalues of the matrix  $\left(\frac{\widehat{\mathbf{H}}_1' \widehat{\mathbf{H}}_1}{T}\right)^{-1} \left(\frac{\widehat{\mathbf{H}}_1' \widehat{\mathbf{H}}_2}{T}\right) \left(\frac{\widehat{\mathbf{H}}_2' \widehat{\mathbf{H}}_2}{T}\right)^{-1} \left(\frac{\widehat{\mathbf{H}}_2' \widehat{\mathbf{H}}_1}{T}\right)$ . Using the canonical correlation analysis, the common factors  $\widehat{\mathbf{f}}_t$  and the residuals  $\widehat{\mathbf{e}}_{1t}$  and  $\widehat{\mathbf{e}}_{2t}$  are obtained, along with the group-specific factors  $\widehat{\mathbf{g}}_{1t}$  and  $\widehat{\mathbf{g}}_{2t}$ .

### Test Statistic and Bootstrap Inference

We set the null hypothesis as  $H_0 : K^c = K_0^c$ . The test statistic is defined as

$$\widehat{\tau} = \sum_{k=1}^{K_0^c} \widehat{\rho}_k - K_0^c.$$

The number of common factors is determined through the following procedure:

1. As previously described, compute  $\hat{\tau}$  and fit the model

$$\mathbf{y}_{gt} = \hat{\mathbf{B}}_g \hat{\mathbf{f}}_t + \hat{\mathbf{D}}_g \hat{\mathbf{g}}_{gt} + \hat{\mathbf{e}}_{gt}, \quad g = 1, 2,$$

where  $\hat{\mathbf{e}}_t = [\hat{\mathbf{e}}'_{1t}, \hat{\mathbf{e}}'_{2t}]'$  denotes the stacked residuals.

2. For  $b = 1, \dots, B$ :

- Generate bootstrap residuals  $\hat{\mathbf{e}}_t^{*(b)} = [\hat{\mathbf{e}}_{1t}^{*(b)'}; \hat{\mathbf{e}}_{2t}^{*(b)'}]' = \hat{\mathbf{e}}_t \odot \boldsymbol{\eta}_t^{(b)}$ , where each element of  $\boldsymbol{\eta}_t^{(b)}$  is independently drawn from  $\mathcal{N}(0, 1)$ .
- Construct the bootstrap data:

$$\mathbf{y}_{gt}^{*(b)} = \hat{\mathbf{B}}_g \hat{\mathbf{f}}_t + \hat{\mathbf{D}}_g \hat{\mathbf{g}}_{gt} + \hat{\mathbf{e}}_{gt}^{*(b)}, \quad g = 1, 2.$$

- Recompute the test statistic  $\hat{\tau}^{*(b)}$  using the bootstrap data.
3. Calculate the bootstrap  $p$ -value as  $p = \frac{1}{B} \sum_{b=1}^B \mathbf{1}(\hat{\tau}^{*(b)} < \hat{\tau})$ .
  4. If  $p < 0.05$ , reject  $H_0$  and increment  $K_0^c$  by one; otherwise, stop and accept the current  $K_0^c$  as the estimated number of common factors.

The test is performed annually, once every 12 months, using the same rolling estimation window as the portfolio construction itself — 36 months in the body, and 60 months in the robustness appendix of Section F.1.

## E Data construction and return cleaning

This appendix documents how the working panel is built from the raw data, in the order the steps are applied. Individual stock data are constructed with the code of Jensen et al. (2023). Steps 1–2 define the universe, step 3 cleans returns, steps 4–5 build the regressors, and steps 6–7 define what enters an estimation window.

1. **Stock universe.** We retrieve monthly observations for common shares in each of the 26 countries. We keep a security only if it is the firm’s primary security, is listed on the country’s main exchange, and is flagged as a common share, and we deduplicate to one observation per security-month.
2. **Returns.** Returns are monthly, converted to US dollars, and taken in excess of the one-month US Treasury bill rate  $r_{f,t}$  from Ken French’s data library.
3. **Return cleaning.** International vendor return data contain errors large enough to move portfolio moments, so we apply three adjustments in sequence. Earlier steps take precedence over later ones.
  - (a) *Seed corrections.* A hand-authored list of specific security-months, one row per (security, date), addressing corporate actions — principally stock splits — that the vendor did not apply.
  - (b) *Spike-reversal filter.* We apply the filter of Ince and Porter (2006): if a monthly return exceeds 300% and is reversed within one month, so that the compounded two-month return is below 50%, we remove *both* months rather than the spike alone.
  - (c) *Vendor cross-check.* Every remaining flagged observation is compared with an independent Datastream value on the same excess-return basis. We keep the panel value if the two agree within 2 percentage points; otherwise we replace it with the Datastream value, except that we delete the month when the replacement is itself larger than 100 percentage points *and* fails to corroborate the panel. Where a Datastream dollar return is unavailable we repeat the comparison using the Datastream local return times an independently sourced exchange rate, and where no vendor value exists at all we delete the month. Deletion always removes a single security-month, never a security.

Because US returns come from CRSP, they are exempt from all three steps. We retain both a cleaned return series and an uncorrected series that has had only the seed corrections and the spike-reversal filter applied; all results in the paper use the cleaned series. Should we show them?

4. **Firm characteristics.** We construct up to 153 characteristics per security-month. Characteristics are lagged one period relative to the return they predict. Missing values are imputed cross-sectionally within each country-month.
5. **Fitted returns.** For each country-month we form  $\hat{y}$ , the fitted value from a cross-sectional regression of returns on characteristics.
6. **Estimation universe.** Within each estimation window we keep the largest 80% of firms, ranked on company market capitalization in US dollars as of December of the preceding year, and apply the same screen to the out-of-sample month. Anchoring the screen to a December value prevents a firm from entering or leaving the universe because of the return being studied.
7. **Sample start.** Each country’s sample begins at the first month from which its cross-section sustains at least 20 usable securities for every estimation window length we report. Start dates therefore differ across countries and are a property of the data rather than of the estimator.

## F Robustness

### F.1 60 Months Estimation Window

The results in the body use a rolling 36-month estimation window. This appendix reproduces the diversification summary and the two headline portfolio tables on a 60-month window. The longer window consumes 24 additional months of burn-in — exactly 24 for every one of the 25 countries — so each series is correspondingly shorter.

The Risk Gap result is robust to the change. The equal-weighted Risk Gap portfolio across all 25 foreign markets earns an annualized Sharpe ratio of 1.05 with the 60-month window (Table 18), compared with 1.14 under the 36-month baseline, and retains very low correlation to the US market. The Price Gap result is more sensitive: the

corresponding Price Gap portfolio's Sharpe ratio declines from 1.02 to 0.74, and the cross-country ranking is only loosely preserved across the two windows. We report this attenuation because it delineates the sensitivity of the Price Gap evidence to the estimation-window length.

Nevertheless, the diversification conclusion survives under the longer window. Combining the US market with the world Risk Gap and Price Gap portfolios raises the annualized Sharpe ratio from 0.58 for the US market alone to 1.34, compared with 1.62 under the 36-month baseline.

Table 18: Properties of the equal-weighted Risk Gap and Price Gap Portfolios: robustness, 60-month estimation window.

This table reports summary statistics for the out-of-sample returns of equal-weighted gap portfolios and of their combinations with the US market. Panel A reports the equal-weighted portfolios of the Risk Gap Portfolios (RG) and the Price Gap Portfolios (PG), averaged each month across the country pairs available at that date, for the G7 markets, the non-G7 markets, and all 25 foreign markets; the RG + PG rows sum the two gap portfolios. Panel B reports the corresponding statistics for combinations with the US market: for a single country group, an equal-weighted investment in the US market portfolio and the group's gap portfolio; for all countries, an equal-weighted investment in three assets — the US market portfolio, the G7 gap portfolio, and the non-G7 gap portfolio. Panel B uses the months in which the US market factor and all group portfolios are available. The reported statistics are the annualized mean excess return (Mean), annualized standard deviation (SD), Newey–West  $t$ -statistic for the mean return, annualized Sharpe ratio, correlation with the US market factor (Fama–French  $mktrf$ ,  $\rho_{US}$ ), and number of monthly observations ( $T$ ). As a robustness check, the estimation window is 60 months rather than the baseline 36.

| Portfolio                                       | Mean<br>(%) | SD<br>(%) | $t$  | Sharpe | $\rho_{US}$ | $T$ |
|-------------------------------------------------|-------------|-----------|------|--------|-------------|-----|
| <i>Panel A: Equal-weighted gap portfolios</i>   |             |           |      |        |             |     |
| RG (G7)                                         | 8.2         | 11.1      | 4.26 | 0.74   | −0.17       | 444 |
| RG (non-G7)                                     | 7.9         | 6.4       | 6.14 | 1.22   | −0.07       | 407 |
| RG (all countries)                              | 7.4         | 7.1       | 5.49 | 1.05   | −0.11       | 444 |
| PG (G7)                                         | 9.7         | 13.8      | 3.68 | 0.70   | −0.02       | 444 |
| PG (non-G7)                                     | 5.8         | 9.1       | 3.89 | 0.64   | 0.02        | 407 |
| PG (all countries)                              | 7.9         | 10.5      | 3.89 | 0.74   | −0.01       | 444 |
| RG + PG (G7)                                    | 17.9        | 19.8      | 4.93 | 0.90   | −0.11       | 444 |
| RG + PG (non-G7)                                | 13.6        | 12.7      | 5.44 | 1.08   | −0.02       | 407 |
| RG + PG (all countries)                         | 15.3        | 13.9      | 5.52 | 1.10   | −0.07       | 444 |
| <i>Panel B: Combinations with the US market</i> |             |           |      |        |             |     |
| US market                                       | 8.9         | 15.3      | 3.40 | 0.58   | 1.00        | 395 |
| US + RG (G7)                                    | 8.5         | 8.5       | 5.10 | 1.00   | 0.78        | 395 |
| US + RG (non-G7)                                | 8.4         | 8.1       | 5.64 | 1.04   | 0.92        | 395 |
| US + RG (all)                                   | 8.3         | 6.2       | 5.97 | 1.34   | 0.69        | 395 |
| US + PG (G7)                                    | 8.6         | 9.7       | 4.56 | 0.88   | 0.77        | 395 |
| US + PG (non-G7)                                | 7.3         | 8.9       | 4.48 | 0.82   | 0.86        | 395 |
| US + PG (all)                                   | 7.6         | 7.6       | 5.18 | 1.00   | 0.66        | 395 |
| US + RG + PG (G7)                               | 12.6        | 11.4      | 5.71 | 1.11   | 0.57        | 395 |
| US + RG + PG (non-G7)                           | 11.2        | 9.8       | 5.68 | 1.15   | 0.76        | 395 |
| US + RG + PG (all)                              | 12.9        | 9.6       | 6.28 | 1.34   | 0.43        | 395 |

Table 19: Properties of the Risk Gap Portfolios: robustness, 60-month estimation window.

This table reports summary statistics for the out-of-sample returns of the Risk Gap Portfolio constructed for each pairing of the United States (Country 1) with a foreign market (Country 2). Panel A presents the G7 markets and Panel B presents the non-G7 markets. Foreign markets are identified by the three-letter country codes reported in Table 1. The reported statistics are the annualized mean excess return (Mean), annualized standard deviation (SD), Newey–West  $t$ -statistic for the mean return, annualized Sharpe ratio, worst and best monthly returns, correlation with the US market factor (Fama–French  $mktrf$ ,  $\rho_{US}$ ), and number of out-of-sample monthly observations ( $T_{OOS}$ ). As a robustness check, the estimation window is 60 months rather than the baseline 36.

| Country 2                      | Mean<br>(%) | SD<br>(%) | $t$   | Sharpe | Worst<br>(%) | Best<br>(%) | $\rho_{US}$ | $T_{OOS}$ |
|--------------------------------|-------------|-----------|-------|--------|--------------|-------------|-------------|-----------|
| <i>Panel A: G7 markets</i>     |             |           |       |        |              |             |             |           |
| CAN                            | 6.2         | 19.9      | 1.78  | 0.31   | −17.5        | 26.7        | −0.09       | 444       |
| DEU                            | 19.4        | 21.0      | 5.39  | 0.92   | −28.7        | 19.7        | −0.03       | 396       |
| FRA                            | 5.0         | 19.1      | 1.35  | 0.26   | −20.6        | 15.4        | 0.05        | 396       |
| GBR                            | 11.9        | 23.6      | 2.72  | 0.51   | −40.2        | 22.1        | −0.18       | 396       |
| ITA                            | 10.1        | 20.8      | 2.75  | 0.49   | −43.5        | 16.5        | −0.11       | 396       |
| JPN                            | 1.7         | 21.2      | 0.44  | 0.08   | −26.5        | 15.8        | −0.14       | 396       |
| <i>Panel B: non-G7 markets</i> |             |           |       |        |              |             |             |           |
| ARG                            | −2.2        | 20.4      | −0.38 | −0.11  | −18.0        | 26.6        | 0.01        | 211       |
| AUS                            | 16.7        | 18.9      | 4.33  | 0.88   | −22.4        | 18.3        | −0.11       | 407       |
| AUT                            | 12.2        | 19.6      | 2.81  | 0.62   | −18.4        | 24.6        | −0.01       | 294       |
| BEL                            | 13.0        | 20.8      | 2.73  | 0.63   | −23.8        | 23.4        | −0.11       | 357       |
| BRA                            | 3.0         | 21.7      | 0.53  | 0.14   | −32.4        | 19.3        | 0.13        | 173       |
| CHL                            | −2.2        | 21.8      | −0.51 | −0.10  | −28.5        | 20.8        | 0.00        | 319       |
| DNK                            | 6.9         | 21.4      | 1.65  | 0.32   | −20.7        | 19.4        | 0.03        | 348       |
| ESP                            | 10.9        | 21.3      | 2.91  | 0.51   | −22.7        | 25.4        | −0.05       | 396       |
| HKG                            | 8.6         | 22.1      | 2.00  | 0.39   | −26.5        | 37.9        | 0.01        | 396       |
| IND                            | 14.5        | 22.6      | 2.65  | 0.64   | −38.7        | 18.7        | −0.06       | 372       |
| KOR                            | 7.4         | 22.0      | 1.78  | 0.34   | −29.0        | 20.7        | −0.00       | 377       |
| MEX                            | 5.9         | 21.0      | 1.32  | 0.28   | −20.4        | 19.9        | −0.13       | 296       |
| NOR                            | 12.9        | 19.2      | 3.42  | 0.68   | −17.2        | 21.2        | −0.10       | 367       |
| POL                            | 17.7        | 17.7      | 4.01  | 1.00   | −14.8        | 16.2        | 0.12        | 291       |
| SGP                            | 11.6        | 20.1      | 2.76  | 0.57   | −21.8        | 24.1        | −0.01       | 396       |
| SWE                            | 4.6         | 23.2      | 0.96  | 0.20   | −49.4        | 20.6        | 0.11        | 336       |
| THA                            | 2.8         | 22.9      | 0.68  | 0.12   | −72.1        | 21.7        | −0.15       | 377       |
| TUR                            | 7.7         | 20.1      | 1.62  | 0.39   | −19.0        | 27.0        | 0.05        | 313       |
| TWN                            | 0.7         | 19.8      | 0.19  | 0.03   | −21.7        | 17.2        | 0.01        | 372       |

Table 20: Properties of the Price Gap Portfolios: robustness, 60-month estimation window.

This table reports summary statistics for the out-of-sample returns of the Price Gap Portfolio constructed for each pairing of the United States (Country 1) with a foreign market (Country 2). Panel A presents the G7 markets and Panel B presents the non-G7 markets. Foreign markets are identified by the three-letter country codes reported in Table 1. The reported statistics are the annualized mean excess return (Mean), annualized standard deviation (SD), Newey–West  $t$ -statistic for the mean return, annualized Sharpe ratio, worst and best monthly returns, correlation with the US market factor (Fama–French  $mktrf$ ,  $\rho_{US}$ ), and number of out-of-sample monthly observations ( $T_{OOS}$ ). As a robustness check, the estimation window is 60 months rather than the baseline 36.

| Country 2                      | Mean<br>(%) | SD<br>(%) | $t$   | Sharpe | Worst<br>(%) | Best<br>(%) | $\rho_{US}$ | $T_{OOS}$ |
|--------------------------------|-------------|-----------|-------|--------|--------------|-------------|-------------|-----------|
| <i>Panel A: G7 markets</i>     |             |           |       |        |              |             |             |           |
| CAN                            | 1.1         | 22.9      | 0.30  | 0.05   | −25.4        | 22.1        | −0.02       | 444       |
| DEU                            | 12.8        | 23.6      | 2.53  | 0.54   | −24.4        | 31.4        | 0.07        | 396       |
| FRA                            | 3.0         | 25.2      | 0.73  | 0.12   | −62.0        | 23.3        | −0.10       | 396       |
| GBR                            | 12.4        | 23.1      | 2.86  | 0.54   | −33.9        | 16.2        | 0.04        | 396       |
| ITA                            | 9.9         | 23.8      | 2.24  | 0.42   | −32.7        | 22.0        | −0.01       | 396       |
| JPN                            | 12.1        | 26.4      | 2.24  | 0.46   | −27.0        | 24.9        | 0.01        | 396       |
| <i>Panel B: non-G7 markets</i> |             |           |       |        |              |             |             |           |
| ARG                            | −12.4       | 28.3      | −1.69 | −0.44  | −27.8        | 25.9        | 0.16        | 211       |
| AUS                            | 5.2         | 22.2      | 1.41  | 0.23   | −21.5        | 30.0        | −0.00       | 407       |
| AUT                            | 2.9         | 25.8      | 0.61  | 0.11   | −25.7        | 25.8        | 0.03        | 294       |
| BEL                            | 4.4         | 26.9      | 0.81  | 0.16   | −41.8        | 22.8        | −0.08       | 357       |
| BRA                            | 4.5         | 24.6      | 0.74  | 0.18   | −18.2        | 29.0        | −0.01       | 173       |
| CHL                            | 1.9         | 27.0      | 0.36  | 0.07   | −30.0        | 29.0        | −0.07       | 319       |
| DNK                            | 4.1         | 26.1      | 0.81  | 0.16   | −29.1        | 20.6        | −0.02       | 348       |
| ESP                            | 9.0         | 27.3      | 1.98  | 0.33   | −42.4        | 40.9        | −0.05       | 396       |
| HKG                            | −0.2        | 22.8      | −0.04 | −0.01  | −20.3        | 25.6        | −0.02       | 396       |
| IND                            | 12.2        | 24.4      | 2.40  | 0.50   | −54.5        | 23.5        | 0.00        | 372       |
| KOR                            | 7.9         | 23.5      | 1.67  | 0.34   | −19.0        | 22.2        | 0.13        | 377       |
| MEX                            | 0.9         | 27.0      | 0.18  | 0.03   | −32.3        | 25.9        | 0.03        | 296       |
| NOR                            | 4.1         | 25.5      | 0.86  | 0.16   | −30.4        | 22.6        | −0.10       | 367       |
| POL                            | 10.6        | 21.9      | 2.38  | 0.48   | −14.9        | 45.9        | 0.03        | 291       |
| SGP                            | 11.8        | 24.0      | 2.67  | 0.49   | −26.1        | 29.3        | 0.09        | 396       |
| SWE                            | −0.6        | 24.6      | −0.12 | −0.02  | −29.1        | 30.5        | −0.04       | 336       |
| THA                            | 4.7         | 24.6      | 0.92  | 0.19   | −29.9        | 32.3        | −0.05       | 377       |
| TUR                            | 10.7        | 22.2      | 2.62  | 0.48   | −21.2        | 40.5        | 0.03        | 313       |
| TWN                            | 14.6        | 22.4      | 3.33  | 0.65   | −28.9        | 25.5        | 0.00        | 372       |

## F.2 Exchange Rate Risk

In this section, we examine if the currency invariance of Solnik (1983) and Ikeda (1991) holds empirically. We first estimate,

$$r_{ct} = a_c + \beta \times \Delta(\text{Exchange Rate vs. USD})_{ct} + u_{ct},$$

where  $c$  indexes country pairs. Table 21 shows that the coefficient on bilateral exchange-rate changes is economically small and portfolios (Panel A) and the Price Gap portfolios (Panel B).

Following Verdelhan (2018), we also examine exposure to two systematic currency factors:

$$r_{ct} = \alpha + \phi_1 \times \text{Carry}_t + \phi_2 \times \text{Dollar}_t + u_{ct},$$

where  $\text{Carry}_t$  denotes the difference in exchange rate changes between high- and low-interest-rate countries, and  $\text{Dollar}_t$  represents the average change in exchange rates. The results, reported in Table 21, are again not significant for either portfolio type, suggesting that the profitability of our portfolios does not arise from systematic exposure to currency risk factors.

Table 21: Currency exposure of the Risk Gap and Price Gap Portfolios.

This table reports the estimated coefficients from the following regressions:  $r_{ct} = a_c + \beta \times \Delta(\text{Exchange Rate vs. USD})_{ct} + u_{ct}$  and  $r_{ct} = a_c + \phi_1 \times \text{Carry}_t + \phi_2 \times \text{Dollar}_t + u_{ct}$ , where  $c$  indexes the 25 country pairs.  $\text{Carry}_t$  denotes the difference in exchange rate changes between high- and low-interest-rate countries, and  $\text{Dollar}_t$  represents the average change in exchange rates. Standard errors are clustered by time;  $t$ -statistics are reported in parentheses. Stars mark significance at the 1% (\*\*\*), 5% (\*\*) and 10% (\*) levels. The estimation window is 36 months.

|                                        | Panel A: Risk Gap |                   | Panel B: Price Gap |                   |
|----------------------------------------|-------------------|-------------------|--------------------|-------------------|
|                                        | (1)               | (2)               | (3)                | (4)               |
| $\Delta(\text{Exchange Rate vs. USD})$ | -0.008<br>(-0.17) |                   | -0.058<br>(-0.90)  |                   |
| Carry                                  |                   | 0.053<br>(1.32)   |                    | -0.079<br>(-1.05) |
| Dollar                                 |                   | -0.029<br>(-0.49) |                    | 0.097<br>(0.86)   |

### **F.3 Fixed Number of Common Factors**

A key parameter in our application is the number of common factors. Although we typically select this number using the bootstrap method of Gonçalves et al. (2025), here we intentionally fix it at four – an arbitrary choice – to test robustness. Table 22 reports the resulting performance of the Risk Gap and Price Gap portfolios for each country pair. The results closely mirror the main findings in Tables 2 and 3.

Table 22: Properties of the Risk Gap and Price Gap Portfolios with a fixed number of common factors.

This table reports summary statistics for the out-of-sample returns of the Risk Gap and Price Gap Portfolios constructed for each pairing of the United States (Country 1) with a foreign market (Country 2). Panel A presents the G7 markets and Panel B presents the non-G7 markets. Foreign markets are identified by the three-letter country codes reported in Table 1. The reported statistics are the annualized mean excess return (Mean), Newey–West  $t$ -statistic for the mean return, annualized Sharpe ratio, and correlation with the US market factor (Fama–French  $mktrf$ ,  $\rho_{US}$ ). As a robustness check, the number of common factors is fixed at four rather than selected by the bootstrap procedure; the number of country-specific factors is three for each country. The estimation window is 36 months.

| Country 2                      | Risk Gap Portfolio |       |        |             | Price Gap Portfolio |       |        |             |
|--------------------------------|--------------------|-------|--------|-------------|---------------------|-------|--------|-------------|
|                                | Mean<br>(%)        | $t$   | Sharpe | $\rho_{US}$ | Mean<br>(%)         | $t$   | Sharpe | $\rho_{US}$ |
| <i>Panel A: G7 markets</i>     |                    |       |        |             |                     |       |        |             |
| CAN                            | 9.2                | 2.90  | 0.48   | 0.05        | 0.0                 | 0.00  | 0.00   | 0.11        |
| DEU                            | 15.2               | 4.05  | 0.82   | −0.07       | 8.7                 | 1.90  | 0.33   | −0.02       |
| FRA                            | 13.0               | 3.41  | 0.68   | 0.11        | 12.3                | 2.37  | 0.44   | −0.00       |
| GBR                            | 16.8               | 4.11  | 0.79   | −0.07       | 14.1                | 3.33  | 0.52   | 0.00        |
| ITA                            | 8.7                | 2.51  | 0.42   | 0.05        | 12.8                | 2.55  | 0.45   | −0.12       |
| JPN                            | 5.2                | 1.61  | 0.26   | −0.03       | 17.3                | 2.71  | 0.62   | −0.00       |
| <i>Panel B: non-G7 markets</i> |                    |       |        |             |                     |       |        |             |
| ARG                            | −2.6               | −0.58 | −0.13  | −0.04       | −2.0                | −0.39 | −0.08  | 0.08        |
| AUS                            | 16.9               | 4.80  | 0.88   | −0.04       | 12.7                | 2.38  | 0.46   | −0.05       |
| AUT                            | 13.9               | 3.61  | 0.75   | 0.06        | 20.1                | 4.19  | 0.79   | 0.05        |
| BEL                            | 8.4                | 2.41  | 0.43   | −0.02       | 17.9                | 4.49  | 0.71   | 0.05        |
| BRA                            | 2.3                | 0.46  | 0.12   | −0.03       | 16.9                | 2.98  | 0.70   | 0.00        |
| CHL                            | 4.0                | 1.13  | 0.20   | −0.05       | 9.7                 | 1.86  | 0.34   | 0.12        |
| DNK                            | 4.7                | 1.08  | 0.23   | 0.05        | 14.2                | 2.69  | 0.50   | 0.01        |
| ESP                            | 10.0               | 3.80  | 0.53   | 0.02        | 25.9                | 5.50  | 0.98   | −0.01       |
| HKG                            | 17.5               | 4.75  | 0.77   | 0.02        | 8.2                 | 1.69  | 0.32   | 0.02        |
| IND                            | 14.8               | 3.59  | 0.70   | −0.09       | 13.8                | 2.59  | 0.50   | 0.02        |
| KOR                            | −1.2               | −0.34 | −0.06  | −0.04       | 17.0                | 3.98  | 0.70   | 0.03        |
| MEX                            | 1.7                | 0.47  | 0.09   | −0.10       | 2.6                 | 0.53  | 0.10   | −0.02       |
| NOR                            | 9.0                | 2.61  | 0.45   | −0.09       | 4.4                 | 0.88  | 0.17   | 0.11        |
| POL                            | 12.2               | 3.31  | 0.66   | −0.07       | 8.1                 | 1.69  | 0.33   | −0.08       |
| SGP                            | 16.1               | 3.51  | 0.61   | 0.04        | 13.9                | 3.02  | 0.53   | −0.01       |
| SWE                            | 15.3               | 3.70  | 0.77   | 0.06        | 7.1                 | 1.39  | 0.27   | 0.04        |
| THA                            | 6.9                | 1.71  | 0.34   | −0.06       | 11.8                | 2.31  | 0.42   | 0.05        |
| TUR                            | 4.0                | 1.08  | 0.21   | 0.07        | 14.0                | 2.94  | 0.56   | 0.04        |
| TWN                            | 6.2                | 1.51  | 0.31   | −0.03       | 8.2                 | 1.75  | 0.33   | −0.02       |

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