

(Mis)Measurement of Long-Horizon Default Risk: Debt Dynamics in Structural Models*

Söhnke M. Bartram[†] Peter Feldhütter[‡] Dirk Hackbarth[§] Yerkin Kitapbayev[¶]

September 17, 2026

Abstract

Credit spreads are used to discipline pricing kernels, almost always at maturities of ten years or less. We study credit spreads based on bonds with an average maturity close to 30 years. Pricing them requires realistic long-horizon default probabilities, which we obtain in closed form from a structural model with refinancing, estimated on 70 years of default rates. A constant price of risk then matches the spread over 1953–2023, while five pricing kernels proposed to resolve the credit spread puzzle overpredict it, some severely. Our results show that long-maturity credit spreads are useful for distinguishing between pricing kernels.

JEL Classification Numbers: C23, G12, G13, G23, G33.

Keywords: Credit Risk, Debt Restructuring, Default Boundaries, Default Rates, Structural Models.

*We are grateful to Marc Chesney, Darrel Duffie, Douglas Dwyer, Stephen Figlewski, Jens Hilscher, David Lando, Alfred Lehar, Hayne Leland, Alessandro Sbuelz, Stephen Schaefer, Roger Stein, Ilya Strebulaev, and seminar participants at Collegio Carlo Alberto for comments and suggestions. Söhnke Bartram gratefully acknowledges the warm hospitality of the Department of Finance during a visit to the Wharton School, University of Pennsylvania. The implementations of the alternative pricing kernels in Section 5.4, the reconstruction of the Moody's index constituents, and parts of the empirical code were developed with the assistance of Claude (Anthropic). All modeling choices, validations, and results were checked by the authors, and the code is available on request.

[†]Research Fellow, Centre for Economic Policy Research (CEPR), London, UK, and Professor of Finance, University of Warwick, Warwick Business School, Finance Group, Coventry CV4 7AL, United Kingdom. Phone: +44 (24) 7657 4168, Fax: +1 (425) 952 10 70. Email: s.m.bartram@wbs.ac.uk. Webpage: <http://go.warwick.ac.uk/sbartram>.

[‡]Copenhagen Business School, Department of Finance, Denmark. Email: pf.fi@cbs.dk.

[§]Boston University, Questrom School of Business, USA. Email: dhackbar@bu.edu.

[¶]Department of Mathematics, Khalifa University of Science and Technology, Abu Dhabi, United Arab Emirates. Email: yerkin.kitapbayev@ku.ac.ae.

1 Introduction

Credit spreads have long been used to learn about risk premiums, not least because of an observation coined the credit spread puzzle: a structural model with a constant price of risk, calibrated to historical default and recovery rates, generates investment-grade spreads that appear below those observed. While the existence of the puzzle has been debated recently, a series of pricing kernels has been proposed to close the gap through long-run risk, habit formation, priced jump risk, rare disasters, or a variance risk premium. The literature typically evaluates these kernels at maturities of ten years or less (see Table 7).

We look at credit spreads based on bonds with maturity close to 30 years. We show that models giving similar predictions at short- and intermediate horizons give strongly different spread prediction at such long maturities. We find that all models proposed to explain the credit spread puzzle imply long-maturity credit spreads that are too high and in some cases severely too high. Our results imply that long-term credit spreads provide distinct information that is important when assessing pricing kernels.

The credit spread we use are based on Moody’s Aaa and Baa yields. Moody’s seasoned Aaa and Baa yield indices are the source of the most widely used credit spread series in finance and macroeconomics, available since 1919, and the Baa-Aaa spread constructed from them is a standard cyclical indicator and a common calibration target. Moody’s includes only bonds with remaining maturities of at least 20 years. We reconstruct the constituent portfolios for 2002–2025, match the published series closely and find that the average remaining maturity of the underlying bonds is 27 years. Yet the spread has routinely been used as an intermediate-maturity object: [Chen, Collin-Dufresne, and Goldstein \(2009\)](#) compare it with model spreads of four-year bonds, [Chen \(2010\)](#) with a ten-year bond, and [Gourio \(2013\)](#) with a one-year bond. We use the spread at its true maturity, and this provides a test of pricing kernels at a maturity at which, to the best of our knowledge, none of them has been evaluated.

Doing so requires a model with realistic default probabilities at long horizons. Structural models of default typically hold debt fixed over the prediction horizon, so that the default boundary is

constant. If debt is constant while asset value grows, leverage drifts toward zero, and the forward default rate eventually dies out. In the data, firms that grow re-lever: in our sample, nominal debt roughly doubles over ten years for the average surviving firm, and [Feldhütter and Schaefer \(2023\)](#) show that these debt dynamics matter for the pricing of corporate bonds. Infinite-horizon refinancing models are standard in the dynamic capital structure literature ([Fischer, Heinkel, and Zechner, 1989](#); [Goldstein, Ju, and Leland, 2001](#); [Hackbarth, Miao, and Morellec, 2006](#); [Strebulaev, 2007](#)), but while economically rich, they are not readily implementable for measuring default probabilities over finite horizons. We extend the literature by providing a simple closed-form solution for finite-horizon default probabilities in this setting, a solution that reduces to the Black-Cox model when the firm cannot refinance. Exploiting the scaling structure of the model and results on double-barrier passage times ([Lin, 1998](#); [Pelsser, 2000](#); [Sbuelz, 2005](#)), we show that the cumulative default probability solves a linear Volterra integral equation, and we solve the equation explicitly in Laplace-transform space as a ratio of hyperbolic functions. Our solution makes long-maturity default probabilities essentially costless to compute, a property we exploit when pricing 27-year bonds for thousands of firm-years. We show that the forward default rate converges to a strictly positive constant, whereas the forward rate in the commonly used Black-Cox model counterfactually converges to zero. This is the precise sense in which static-debt models understate long-horizon default risk.

We then estimate the model using historical default rates. Using firm-level data for more than 2,000 U.S. industrial firms over 1951–2023, we estimate a single default-boundary parameter and a single refinancing parameter common to all rating categories. Differences across ratings arise only through observable firm characteristics, leverage, asset volatility, and payout, as in [Feldhütter and Schaefer \(2018\)](#). With only two free parameters, the estimation is parsimonious and yet the model matches both the level and the slope of the historical term structures of default rates across ratings and does so substantially better than the standard Black-Cox model.

Default rates are only available up to a 20-year horizon from Moody’s, while the constituent bonds of the Baa-Aaa spread have an average remaining maturity of 27 years. We use the refinancing model to calculate default probabilities at the horizon of the Moody’s spread and first show that the

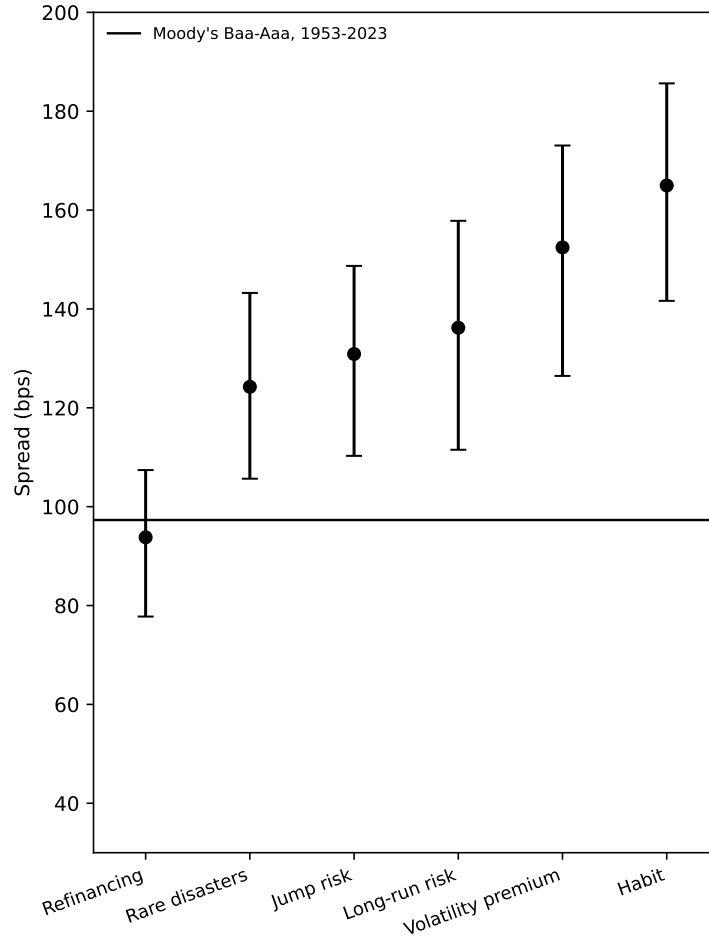
model matches the actual Baa-Aaa spread almost exactly over 1953–2023: the refinancing model implies an average Baa-Aaa spread of 95 bps against 97 bps in the data over 1953–2023. Put differently, at the true maturity of the underlying bonds of the Moody’s Baa-Aaa spread, and using well-established data and estimation methods, we find no evidence of a credit spread puzzle.

In contrast, the five pricing kernels proposed to explain the puzzle imply long-maturity Baa-Aaa spreads that are too high as Figure 1 shows. Using the same default probabilities as those implied by the refinancing model, every model overpredicts the Baa-Aaa spread. The Moody’s Aaa spread over Treasuries discriminates even more sharply, in that the habit formation and rare disaster models imply spreads that are much higher than what we observe empirically. The long-maturity spread thus brings out differences among pricing kernels that the short and intermediate maturities on which the literature has focused do not.

Our paper relates to three literatures. The first is the literature on the credit spread puzzle and the pricing kernels proposed to resolve it. [Huang and Huang \(2012\)](#) show that structural models calibrated to historical default rates, recovery rates, and equity risk premia explain only a small part of investment-grade spreads at four and ten years, and [Feldhütter and Schaefer \(2018\)](#), [Bai, Goldstein, and Yang \(2020\)](#), and [Huang, Nozawa, and Shi \(2025\)](#) debate how much of the puzzle survives once the estimation of the default boundary and the measurement of leverage are taken into account. The kernels proposed in response include option-implied jump risk premia ([Cremers, Driessen, and Maenhout, 2008](#)), learning about inflation and earnings regimes ([David, 2008](#)), habit formation ([Chen, Collin-Dufresne, and Goldstein, 2009](#)), long-run risk ([Chen, 2010](#); [Bhamra, Kuehn, and Strebulaev, 2010](#); [Chen, Cui, He, and Milbradt, 2018](#); [Chen, Xu, and Yang, 2021](#)), disaster risk ([Gourio, 2013](#); [Christoffersen, Du, and Elkamhi, 2017](#); [Seo and Wachter, 2018](#)), priced jump risk ([Bai, Goldstein, and Yang, 2020](#)), a variance risk premium ([Du, Elkamhi, and Ericsson, 2019](#)), and production-based kernels with investment, labor, or nominal frictions ([Bhamra, Fisher, and Kuehn, 2011](#); [Kuehn and Schmid, 2014](#); [Favilukis, Lin, and Zhao, 2020](#); [Gomes and Schmid, 2021](#)). As Table 7 documents, these models are evaluated against spreads with maturities of ten years or less, with the 20-year CDS spreads in [Bai, Goldstein, and Yang \(2020\)](#) the longest, and several of them against the Moody’s Baa-Aaa spread itself. Our contribution is to evaluate

Figure 1. The Long-Maturity Baa-Aaa Spread Across Pricing Kernels, 1953 to 2023

The figure plots the average model-implied Baa-Aaa spread over 1953 to 2023 for the refinancing model and the five alternative pricing kernels of Section 5.4, each with its 95% confidence band, against the average Moody’s Baa-Aaa spread over 1953 to 2023 (solid line). Every model, including the refinancing model, is evaluated for a representative Baa-rated and a representative Aaa-rated firm with the panel-average asset volatility and payout rate, whose physical 27-year default probability is anchored to the refinancing model’s firm-panel average over 1953 to 2023 (15.8% and 1.6%), and prices the same generic 27-year coupon bond in each year from 1953 to 2023 at that year’s average long-term Treasury yield and Moody’s yield with a recovery rate of 33.6%. Confidence bands scale the estimated default boundary by the 2.5% and 97.5% quantiles from the simulation of [Feldhütter and Schaefer \(2018\)](#), with every model re-anchored to the resulting firm-panel average default probabilities.



five of them, spanning the main mechanisms, at the 27-year maturity of that spread, holding the physical default probabilities fixed across models.

The second is the literature that uses the Moody’s Baa-Aaa and Aaa-Treasury series. [Krishna-](#)

[murthy and Vissing-Jorgensen \(2012\)](#) measure the convenience yield on Treasuries as the Moody’s Aaa yield minus the long-term Treasury yield and note that both correspond to maturities of about 20 years, while much of the subsequent macroeconomic literature measures it against the ten-year Treasury yield ([Luetticke, 2021](#); [Phillot, 2025](#)) or uses the Baa-Aaa spread as an intermediate-maturity indicator of credit conditions ([Del Negro, Giannone, Giannoni, and Tambalotti, 2017](#); [Bayer, Born, and Luetticke, 2023](#)). We reconstruct the constituents of the indices and document that their average remaining maturity is 27 years, which changes the Treasury benchmark against which the Aaa yield should be measured and the maturity at which models should be evaluated.

The third is the literature on default probabilities in structural models and on debt dynamics. The models that follow [Merton \(1974\)](#) and [Black and Cox \(1976\)](#) provide tractable default probabilities that are widely used in empirical work and in practice ([Leland and Toft, 1996](#); [Leland, 2004](#); [Eom, Helwege, and Huang, 2004](#); [Huang and Huang, 2012](#); [Feldhütter and Schaefer, 2018](#)), but their tractability comes from holding debt fixed. [Collin-Dufresne and Goldstein \(2001\)](#) first showed that mean-reverting leverage raises long-maturity credit spreads, and [Feldhütter and Schaefer \(2023\)](#) estimate a continuous mean-reverting debt process from panel data on debt issuance and show that it improves spread predictions at conventional maturities. The dynamic capital structure literature models refinancing as lumpy: the firm re-levers to a target when asset value reaches an upper boundary and defaults at a lower one ([Fischer, Heinkel, and Zechner, 1989](#); [Goldstein, Ju, and Leland, 2001](#); [Strebulaev, 2007](#)), and [Danis, Rettl, and Whited \(2014\)](#) and [Korteweg and Strebulaev \(2015\)](#) document that leverage adjustments in the data are indeed infrequent and large. These models are solved for the value of infinitely lived claims and do not deliver finite-horizon default probabilities. We connect the two literatures. Relative to the models with fixed debt, our default probability incorporates refinancing while remaining in closed form and relative to [Collin-Dufresne and Goldstein \(2001\)](#) and [Feldhütter and Schaefer \(2023\)](#) the solution is in closed-form.

The remainder of the paper is organized as follows. Section 2 describes the debt dynamics implied by static and dynamic leverage policies. Section 3 derives default probabilities for the constant-boundary benchmark and the dynamic refinancing model, including the closed-form Laplace transform and the limiting forward default rate. Section 4 estimates both models on his-

torical default rates. Section 5 reconstructs the Moody’s Aaa and Baa yield indices, shows that the firm panel prices them, evaluates the refinancing model’s out-of-sample spread predictions over 1953–2023, and compares them with the pricing kernels proposed to explain the credit spread puzzle. Section 6 concludes. Additional derivations, data details, and the implementation of the alternative pricing kernels are provided in the Appendix.

2 Debt Dynamics

To connect the literatures on leverage policies and default probabilities, we consider a simple setting for a firm’s debt dynamics that assumes that the firm’s leverage policy follows from a commonly-used, continuous-time capital structure model with an initial (static) or a dynamic leverage target. Given that this setting has been used widely in recent research for both dynamic and static models of capital structure, we do not reproduce the solutions—see, e.g., [Leland \(1994\)](#), [Goldstein, Ju, and Leland \(2001\)](#), [Hackbarth, Miao, and Morellec \(2006\)](#), [Strebulaev \(2007\)](#), and many others. We use this setting to motivate the boundaries of the default probability model; the boundary parameters themselves are estimated from default data in Section 4.

We consider a partial equilibrium model of corporate financing, in which time t is continuous and uncertainty is modeled by a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Under the actual probability measure $\mathbb{P}$, the value of the firm’s assets, $V(t)$, evolves according to

$$dV(t)/V(t) = \mu dt + \sigma dW_t \tag{1}$$

for $t > 0$, given a constant growth rate μ , a constant volatility σ , and an initial value $V(0) > 0$ at $t = 0$. In terms of asset value, the operating cash-flow rate is $(r - \hat{\mu})V_t$. Corporate taxes are paid on operating cash flows at a constant rate τ based on full loss offset provisions. There is a constant, risk-free rate r , and the economy’s pricing kernel, Λ_t , follows a geometric Brownian motion with volatility σ_Λ and correlation $\rho < 0$ with $V(t)$, characterizing the economy’s risk-reward trade-off. Hence, the risk-neutral drift is given by $\hat{\mu} = \mu + \rho \sigma \sigma_\Lambda$ and we assume $\hat{\mu} < r$. That is, under the physical measure $\mathbb{P}$, the asset value $V(t)$ has a risk premium of $\pi = -\rho \sigma \sigma_\Lambda > 0$. The default probability requires the physical measure $\mathbb{P}$, but at the same time, all optimal decisions of the firm are based on valuations under the risk-neutral measure $\mathbb{Q}$.

In the static version, the firm chooses an optimal capital structure of equity and perpetual, risky debt that balances tax benefits against bankruptcy and debt issuance costs. We consider debt contracts that are characterized by a perpetual flow of coupon payments c . The optimal time to default on coupon payments is endogenously determined (i.e. maximizes equity value). In the event of default, equityholders receive nothing, and debtholders assume ownership of the firm's assets net of bankruptcy costs. Bankruptcy triggers a loss of interest tax shields and a fractional loss α of assets.

In the dynamic version, the firm can, for example, balance tax benefits against bankruptcy and debt issuance costs at the initial date and at future restructuring dates. To stay in a simple time-homogeneous setting, debt is callable and issued at par. The proceeds from the debt issue are distributed pro rata to shareholders at the time of flotation. The firm can adjust their capital structure upward at any point in time by incurring a proportional cost q , but it can reduce its debt only in default, unless we also consider debt renegotiation. Under this assumption, the firm's initial debt level remains fixed until either the firm goes into default or the firm calls its debt and restructures with newly issued debt. That is, the firm selects a target leverage and infrequently returns to its target leverage until it defaults.

This setting enables us to compare asset value paths over time for the different leverage policies and, in turn, to show what they imply for computations of default probabilities.¹ To begin, Figure 2 illustrates the static model by plotting time t on the horizontal axis and asset value V on the vertical axis. The firm's initial asset value is denoted by $V(0)$. In this version, the default barrier does not change and therefore V_B is constant forever. In addition, there is no upper boundary in the static version (or rather the upper barrier is infinitely large, i.e. $V_R = \infty$), so the firm's default probability satisfies the conventional single barrier solution. The firm resides only in one stage (stage 1), because the default barrier does not change over time. Moreover, if the firm does not default and its asset value grows over time, as in the data, leverage would tend to zero.

[Insert Figure 2 here.]

¹The valuation equations and optimal policies are well-known (see e.g. Goldstein, Ju, and Leland (2001) or Strebulaev (2007)) and hence only summarized in the Appendix.

By contrast, the dynamic model evolves over time as the firm moves through different restructuring stages. Figure 3 summarizes this case graphically. In stage 1, the firm begins at $V^1(0)$ and reaches the lower barrier V_B^1 , at which point the firm declares bankruptcy, or the firm reaches V_R^1 , at which point the debt is recalled at par value P and the firm again chooses an optimal capital structure, which will always be the same optimal (target) leverage ratio, because the firm is scaled by factor γ (to be specified endogenously) at each refinancing point. The asset value at the beginning of stage 2 is $V^2(0) = V_R^1 = \gamma V^1(0)$. Similarly, the default barrier scales up from stage 1 to stage 2 by the scaling factor γ , that is, $V_B^2 = \gamma V_B^1$. This scaling process continues forever, unless the firm defaults. In contrast to the static model, if the firm does not default and its asset value grows over time, as in the data, leverage would remain within an optimal range of non-trivial values.

[Insert Figure 3 here.]

For our sample from 1951 to 2023, we examine how future debt evolves relative to current debt. Figure 5 shows how the average ratio of a firm’s nominal debt, scaled by the first observation in the first year when the firm has leverage greater or equal to 5%. This ratio is tracked over the next ten consecutive years on a logarithmic scale for firms with non-missing data. The figure reveals a consistent and significant increase in debt ratios over time. Although firms’ asset values generally grow as well, this should not be surprising. However, without corresponding increases in nominal debt, a firm’s financial leverage would naturally decline toward zero (as discussed in the static model). Despite this, much of the credit risk literature often assumes constant debt levels.

[Insert Figure 5 here.]

To summarize, the graphs reveal that the assumption of a constant default boundary is an unrealistic simplification for firms that reoptimize their capital structures over time. However, the analytical complexity of computing the default probabilities for the dynamic case has hindered academics and practitioners from relying on more precise estimates. Against this backdrop, our paper makes three contributions. First, the next section derives a simple integral equation for the default probability over a finite time horizon of a company that makes coupon payments on

its debt and infrequently returns to its leverage target by increasing its debt unless it defaults on its debt. Second, we examine the quantitative significance of our new solution in a calibrated example. Third, the subsequent section takes the equation to the data to understand its empirical importance.

3 Default Probabilities

This section presents our main result. First, we revisit the conventional default probability formula for a static model of capital structure (Black-Cox formula) and then characterize the default probability for the dynamic model of capital structure. We assume here that the default (and restructuring) boundaries are given.

3.1 Static Case Without Restructuring

We start with the base case in which the firm has a static debt policy and hence only an option to default on its debt, which it may find optimal to exercise in a low asset value state. Let us first recall the [Merton \(1974\)](#) model, in which default cannot occur before the zero-coupon bond matures at time T . The default barrier is denoted by V_B . It is straightforward to show that the default probability under the physical measure $\mathbb{P}$ at time $t = 0$ is given by

$$DP_M(T) := \mathbb{P}(V(T) \leq V_B) = \Phi(-d) \quad (2)$$

for $V(0) > V_B > 0$, where $\Phi(\cdot)$ is the standard normal distribution function and d is the “distance to default”

$$d = \frac{\mathbb{E}[\log(V(T))] - \log(V_B)}{\sigma\sqrt{T}} = \frac{\log(V(0)/V_B) + (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \quad (3)$$

where $\mathbb{E}$ is the expectation under $\mathbb{P}$. To derive (2) one can simply transform the event of default in terms of the standard normal variable.

Now we assume that the default can happen before T , in particular, when the asset value process V reaches the constant level V_B from above. This can be the case if there are safety covenants or it is optimal for equity holders to do so. Let us define the default time T_B as the first time the process V hits the level V_B

$$T_B := \inf\{t > 0 : V(t) \leq V_B\} \quad (4)$$

and the cumulative default probability in the base case is defined as

$$DP_{BC}(T) = \mathbb{P}(T_B \leq T) = \mathbb{P}\left(\min_{0 \leq t \leq T} V(t) \leq V_B\right) \quad (5)$$

for a T -year horizon under the physical measure where we notice that the default event is equivalent to the event that the minimum of V on $[0, T]$ is smaller than or equal to V_B . In the [Black and Cox \(1976\)](#) first-passage specification, the physical-measure cumulative default probability at time $t = 0$ is given by

$$DP_{BC}(T) = \Phi(-d) + \left(\frac{V(0)}{V_B}\right)^{1-2\mu/\sigma^2} \Phi(-\hat{d}) \quad (6)$$

for $V(0) > V_B > 0$ where

$$\hat{d} = \frac{\log(V(0)/V_B) - (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}. \quad (7)$$

Note that the first term in (6) corresponds to Merton's probability (2) for one-time default at time $t = T$ and the second term represents paths that cross the default barrier before T but finish above it at T .² To obtain the base case formula (6), the well-known results on first-passage time problems (or distribution of running maximum/minimum) for Brownian motion with drift can be employed (see e.g. [Harrison \(1985\)](#)).

3.2 Dynamic Case With Restructuring

We present now the main result of this paper that allows for debt dynamics. In addition to the option to default on its debt, which the firm may find optimal to exercise in a low asset value state, the firm also possesses a refinancing right that may be optimal to trigger when asset values increase sufficiently. In the dynamic case, the default boundary is no longer constant over time and, in fact, the evolution of the default boundary is contingent on a particular sample path of the asset value process (see [Figure 3](#)). Thus, a company's cumulative default probability depends on the distribution of possible realizations of changes in default boundaries over time, which, in turn, depend on the distribution of the asset value process.

To derive the probability of default up to some time T , we employ the scaling feature of the geometric Brownian motion that allows for a stationary leverage target (refinancing) policy, the

²Even though [Black and Cox \(1976\)](#) assume a boundary that deterministically increases over time (e.g., due to a covenant), academics and practitioners typically use a constant boundary given by the firm's amount of book debt.

strong Markov property, time-homogeneity of the model, and certain results on the double-barrier passage time distributions (see [Lin \(1998\)](#) on the pricing of double-barrier options). We then apply a first-exit decomposition and the strong Markov property to obtain a defective renewal equation for the cumulative default probability.

Following an initial capital structure and default level $V_B(0) = V_B < V(0)$, the firm's debt increases the first time when V reaches $V_R > V(0) > V_B > 0$. We suppose that the leverage ratio becomes the same as the initial one at time $t = 0$. For this, we denote $\gamma = V_R/V(0)$ and define the sequence of stopping times

$$T_{R,n} = \inf\{t > T_{R,n-1} : V(t) = V_R \cdot \gamma^{n-1}\} \quad (8)$$

for stage $n \geq 1$ with $T_{R,0} = 0$ and the convention $\inf \emptyset = \infty$. Time $T_{R,n}$ represents the end of stage n for $n \geq 1$, i.e., when the company switches back to its optimal (target) leverage ratio. We then define the default boundary (contingent on the realization of V) as

$$V_B(t) = \sum_{n=1}^{\infty} V_B \gamma^{n-1} 1_{\{T_{R,n-1} \leq t < T_{R,n}\}} \quad (9)$$

for $t > 0$ where 1_A is an indicator function

$$1_A = \begin{cases} 1 & \text{for } \omega \in A \\ 0 & \text{for } \omega \notin A \end{cases} \quad (10)$$

of some event $A \in \mathcal{F}$. We note that for fixed t only one indicator will be distinct from zero and hence we pick up the corresponding number n (contingent on the path of V) of the current stage. The default time is then given by

$$T_B = \inf\{t > 0 : V(t) \leq V_B(t)\} \quad (11)$$

which differs from (4) in that $V_B(t)$ depends on time t . Given these definitions, we can express the cumulative default probability over a fixed time horizon $[0, T]$ for the dynamic case (*DC*)

$$DP_{DC}(T) := \mathbb{P}(T_B \leq T) \quad (12)$$

where $T > 0$. The main difficulty here is that the default boundary is not constant over time and, in fact, is stochastic. To tackle this problem, let us adopt the following definition to describe the mutually exclusive and collectively exhaustive (up to null events) model scenarios.

Definition 1. *For the default probability of the dynamic model, there are three mutually exclusive events:*

1. $A_1(T) = \{\omega \in \Omega : T_{R,1}(\omega) < \min(T, T_B(\omega))\}$, *which is that the first hitting time of V_R precedes the horizon of the probability of default T and the bankruptcy time T_B ;*
2. $A_2(T) = \{\omega \in \Omega : T_B(\omega) < \min(T, T_{R,1}(\omega))\}$, *which is that the bankruptcy time T_B precedes the horizon of the probability of default T and the first restructuring time $T_{R,1}$;*
3. $A_3(T) = \{\omega \in \Omega : T < \min(T_{R,1}(\omega), T_B(\omega))\}$, *which is that the time T occurs before V reaches V_R or V_B .*

Figure 4 illustrates three asset value paths that correspond to the three events or scenarios in Definition 1. The green line represents the restructuring scenario defined as $A_1(T) = \{\omega \in \Omega : T_{R,1}(\omega) < \min(T, T_B(\omega))\}$, in which the first restructuring time $T_{R,1}$ precedes the default probability's horizon T and the bankruptcy time T_B . The red line represents the bankruptcy scenario defined as $A_2(T) = \{\omega \in \Omega : T_B(\omega) < \min(T, T_{R,1}(\omega))\}$, in which the bankruptcy time T_B precedes the default probability's horizon T and the first restructuring time $T_{R,1}$. The blue line represents the remaining scenario defined as $A_3(T) = \{\omega \in \Omega : T < \min(T_{R,1}(\omega), T_B(\omega))\}$, in which the default probability's horizon T precedes the first restructuring time $T_{R,1}$ and the bankruptcy time T_B . We can summarize our main result for computing the default probability in the dynamic case as follows.

[Insert Figure 4 here.]

Proposition 1. *The dynamic default probability $DP_{DC}(T)$ satisfies the defective renewal equation, equivalently a convolution-type linear Volterra integral equation of the second kind,*

$$DP_{DC}(T) = \mathbb{P}(A_2(T)) + \int_0^T g^R(t) DP_{DC}(T-t) dt \quad (13)$$

for $T > 0$ with the initial condition $DP_{DC}(0) = 0$ where

$$\mathbb{P}(A_2(T)) = \exp(-\nu\ell) \sum_{n=0}^{\infty} [\Psi_{\nu}(T, \ell + 2nh) - \Psi_{\nu}(T, 2h - \ell + 2nh)], \quad T > 0, \quad (14)$$

$$g^R(t) = \exp\left(\nu u - \frac{1}{2}\nu^2 t\right) \sum_{n=0}^{\infty} [f(t; u + 2nh) - f(t; 2h - u + 2nh)], \quad t > 0, \quad (15)$$

$$\Psi_{\nu}(T, a) := \exp(-\nu a) \Phi\left(\frac{\nu T - a}{\sqrt{T}}\right) + \exp(\nu a) \Phi\left(-\frac{\nu T + a}{\sqrt{T}}\right), \quad T > 0, \quad a > 0,$$

$$f(t; a) := \frac{a}{\sqrt{2\pi t^3}} \exp\left(-\frac{a^2}{2t}\right), \quad t > 0, \quad a > 0,$$

$$\nu := \frac{\mu}{\sigma} - \frac{\sigma}{2}; \quad \ell := \frac{1}{\sigma} \log\left(\frac{V(0)}{V_B}\right);$$

$$u := \frac{1}{\sigma} \log\left(\frac{V_R}{V(0)}\right); \quad h := \ell + u = \frac{1}{\sigma} \log\left(\frac{V_R}{V_B}\right).$$

The intuition behind (13) is as follows. The first term, $\mathbb{P}(A_2(T))$, is the probability that the firm exits the initial stage through the default boundary before both the refinancing boundary and time T . The integral term accounts for paths that first exit the initial stage through the refinancing boundary. The possible first refinancing times are integrated with respect to the defective upper-exit density g^R . If the first refinancing occurs at time t , the strong Markov and scaling properties imply that the post-refinancing problem is identical in distribution to the original problem, but with the shorter remaining horizon $T - t$. The corresponding residual default probability is therefore $DP_{DC}(T - t)$.

The proof is given in Appendix A. The integral equation (13) can be easily solved numerically for the unknown function $DP_{DC}(\cdot)$, see the details of the numerical scheme in Appendix B.³

3.3 Laplace Transform

In this section, we present an alternative way to solve the integral equation (13). We apply Laplace transform techniques. Let us recall that the Laplace transform of some function $\varphi(T)$ is the function $\mathcal{L}[\varphi](s)$, which is a unilateral transform defined by

$$F(s) := \mathcal{L}[\varphi](s) = \int_0^{\infty} \exp(-sT) \varphi(T) dT \quad (17)$$

³We note that a simple change of variables in the integral term allows us to rewrite (13) as

$$DP_{DC}(T) = \mathbb{P}(A_2(T)) + \int_0^T g^R(T-t) DP_{DC}(t) dt. \quad (16)$$

for complex numbers s with a sufficiently large real part.

We now provide the Laplace transform of the default probability function, which is given in closed form. The proof is given in Appendix C.

Proposition 2. *The Laplace transform of DP_{DC} is given by*

$$\mathcal{L}[DP_{DC}](s) = \frac{\exp(-\nu\ell) \sinh(\kappa(s)u)}{s [\sinh(\kappa(s)h) - \exp(\nu u) \sinh(\kappa(s)\ell)]}. \quad (18)$$

We then compute $DP_{DC}(T)$ by applying a numerical inversion

$$DP_{DC}(T) = \mathcal{L}^{-1} \left[\frac{\exp(-\nu\ell) \sinh(\kappa(s)u)}{s [\sinh(\kappa(s)h) - \exp(\nu u) \sinh(\kappa(s)\ell)]} \right] (T) \quad (19)$$

for $T > 0$, where the inverse Laplace transform is defined as

$$\mathcal{L}^{-1}[F](T) = \frac{1}{2\pi i} \lim_{M \rightarrow \infty} \int_{\eta - iM}^{\eta + iM} \exp(sT) F(s) ds \quad (20)$$

where η is chosen such that the integration contour lies to the right of all singularities of F .

In summary, we can calculate $DP_{DC}(T)$ by applying a numerical inverse Laplace transform to the explicit expression in (18), without solving the integral equation (13).

3.4 The Forward Default Rate at Long Horizons

The economic content of the dynamic boundary is sharpest in the forward default rate,

$$h(T) := \frac{\partial DP(T)/\partial T}{1 - DP(T)}, \quad (21)$$

the default intensity at horizon T conditional on survival to T . In the Black-Cox model with $\nu > 0$, the firm's leverage drifts toward zero and default risk dies out: the probability of ever defaulting is $DP_{BC}(\infty) = \exp(-2\nu\ell) < 1$, and $h_{BC}(T) \rightarrow 0$. In the model with refinancing, the firm re-levers each time asset value reaches the upper boundary, leverage is stationary, and default occurs almost surely; the forward default rate converges to a strictly positive constant.

Proposition 3. *In the model with refinancing, $DP_{DC}(\infty) = 1$ and*

$$\lim_{T \rightarrow \infty} h(T) = \frac{1}{2} (\nu^2 + \kappa_1^2) > 0, \quad (22)$$

where κ_1 is the smallest positive root of

$$\sin(h\kappa) = \exp(\nu u) \sin(\ell\kappa). \quad (23)$$

The result follows from the closed-form Laplace transform (18): the transform of $1 - DP_{DC}$ is meromorphic with simple poles at $s_k = -\frac{1}{2}(\nu^2 + \kappa_k^2)$, where the κ_k solve (23), so the survival probability decays exponentially at the rate given by the rightmost pole. Inverting the transform by residues also yields an exact eigenvalue expansion for $DP_{DC}(T)$ in which two or three terms suffice beyond a ten-year horizon; we use this expansion when computing long-maturity default probabilities in Section 5.

The contrast between the two models is quantitatively large exactly where long-term bonds are priced, and it is visible directly in the default data. Figure 7 plots annual forward default rates computed from Moody’s 1920–2022 cumulative default rates against the two models’ averages at the pooled estimates of Section 4, for an investment-grade average (Aaa through Baa) and for Ba, B, and C separately. The speculative-grade panels are decisive because default counts are large and the two models separate within the twenty-year window: the empirical forward rates decline from their short-horizon peaks but flatten at strongly positive levels—about 150, 250, and 340 basis points per year for Ba, B, and C over years ten to twenty—which the restructuring model tracks closely, while the Black-Cox forward rate decays toward zero, reaching 56, 70, and 120 basis points by year twenty. The empirical crossing of the two models at a horizon of a few years also appears in the data: the Black-Cox model, whose fitted boundary is higher, implies more default risk at the shortest horizons. In the investment-grade panel, the empirical forward rate continues to rise through year thirteen, as only the restructuring model does, but the two models’ investment-grade forward rates separate slowly—the Black-Cox decay is a beyond-twenty-year phenomenon for high-grade firms—so the twenty-year window has limited discriminating power there; for a representative Baa firm, the restructuring model’s limiting forward rate of Proposition 3 is 124 basis points per year, while the Black-Cox forward rate at a 40-year horizon has fallen to 36 basis points and continues toward zero. Because a cohort’s cumulative default rate accumulates the forward rates of its surviving members, this figure is the direct evidence behind why constant-boundary models cannot match the slope of the default-rate term structure in Section 4, and why the two models price long-maturity bonds differently in Section 5.

[Insert Figure 7 here.]

4 Fitting the Term Structure of Default

In our empirical analysis, we use firm-level variables (for example, leverage and equity volatility) along with aggregate historical default rates, corporate bond prices, and risk-free rates. We focus on the U.S. market, and our main data sources are Compustat for firm fundamentals, CRSP for stock returns and market capitalization, Standard & Poor’s for entity credit ratings, Moody’s for historical default rates, Enhanced TRACE for corporate bond transactions, Mergent FISD for bond characteristics and bond ratings, and the Federal Reserve for Treasury yields. The data sources are well known and used in a large number of studies. In Appendix D we provide a description of each.

In this section we estimate the default and refinancing parameters of the two models by fitting model-implied default probabilities to historical default rates. We then compare how well the two models capture the term structure of default rates across rating categories.

We use a firm panel of rated U.S. industrial firms for the period 1951–2023: There are 29,495 firm-year observations from 2,234 firms, and details are described in Appendix D. Moody’s cumulative default rates are for horizons of one to 20 years from the period 1920–2022.

We estimate the parameters of the Black-Cox model and the model with refinancing by matching model-implied default rates to historical default rates, following the approach in [Feldhütter and Schaefer \(2018\)](#). The default probability in the Black-Cox model is given in equation (6) and the default probability in the refinancing model in equation (28).

In the Black-Cox model the only parameter is the default boundary V_B . In the Merton model the default boundary is the face value of debt D , but [Davydenko \(2013\)](#) shows that firms typically default when firm value is below the face value of debt. We therefore follow [Feldhütter and Schaefer \(2018\)](#) and let the firm default when firm value hits $d \times D$, where d is a fraction that we estimate.

The refinancing model has one additional parameter, the refinancing boundary V_R . In the estimation we assume that the firm has just refinanced, so that the ratio of the refinancing boundary to current firm value is the same for all firms at all times, and we write $V_R = u \times V_t$. Thus, $u - 1$ is the percentage increase in firm value at which the firm refinances.

Following [Feldhütter and Schaefer \(2018\)](#), we estimate the parameters jointly for all rating categories. That is, we assume that d in the Black-Cox model and (d, u) in the refinancing model are the same for all firms, and differences in default rates across ratings arise only through differences in leverage, asset volatility, and payout rates.⁴

For a given parameter vector Θ (where $\Theta = d$ in the Black-Cox model and $\Theta = (d, u)$ in the refinancing model) we compute model-implied default rates in the same way as Moody’s computes historical default rates. In each year we find all firms with rating a and calculate their default probabilities for horizons of one to 20 years. We average across firms within the year and then across the years 1951 to 2023. This gives the average model-implied default probabilities $\bar{\pi}_{a1}^P(\Theta), \bar{\pi}_{a2}^P(\Theta), \dots, \bar{\pi}_{a20}^P(\Theta)$. We denote the corresponding historical default rate by $\hat{\pi}_{at}^P$ and choose Θ to minimize the sum of weighted absolute differences between model-implied and historical default rates,

$$\min_{\{\Theta\}} \sum_a \sum_{t=1}^{20} \frac{1}{t} \left| \bar{\pi}_{at}^P(\Theta) - \hat{\pi}_{at}^P \right|, \quad (24)$$

where a runs over Aaa, Aa, A, Baa, Ba, B, and C.

Table 2 shows the parameter estimates and the sum of weighted absolute errors (SWAE) in equation (24) for both models. In the benchmark case we use default rates for cohorts formed in 1920 to 2022.

The Black-Cox model has a default boundary of $\hat{d} = 0.74$ which is lower than that reported in [Feldhütter and Schaefer \(2018\)](#) mainly because we use (a proxy for) market leverage rather than quasi-leverage in the estimation. In the refinancing model the default boundary is $\hat{d} = 0.68$, consistent with the evidence in [Davydenko \(2013\)](#) that firms default when asset value is around two thirds of the face value of debt. The estimated refinancing boundary is $\hat{u} = 1.65$, so firms refinance after firm value has increased by 65%. The estimated refinancing boundary is close to the upward restructuring points that dynamic trade-off models estimate: [Goldstein, Ju, and Leland \(2001\)](#) estimate the parameter to be 1.70 in the base case and between 1.64 and 1.76 times across their parameter variations, while [Chen \(2010\)](#) estimate it to be between about 1.5 and 2.4. Thus,

⁴As in [Feldhütter and Schaefer \(2018\)](#), the drift of firm value for firm j in year t is $\mu_{jt} = \theta \sigma_A^j + r_t - \delta_{jt}$, where σ_A^j is the asset volatility of firm j , δ_{jt} is the payout rate of firm j in year t , r_t is the 10-year Treasury yield in year t , and θ is the asset Sharpe ratio. As is common in the literature, we set $\theta = 0.22$.

an estimate of $\hat{u} = 1.65$ obtained from default rates alone is in line with the refinancing behaviour implied by these models.⁵

The last column in Table 2 shows that the fit to historical default rates are substantially better for the restructuring model compared to the Black-Cox model: The fitting errors SWAE are 3.25 times higher in the Black-Cox model compared to the restructuring model despite that the restructuring model only has one additional parameter to fit the cross-section of 140 default rates.

The remaining rows of Table 2 show that the estimates are robust to the default rate sample and to the weighting of rating categories. With default rates for cohorts formed in 1970 to 2022, the refinancing estimates are $(\hat{d}, \hat{u}) = (0.65, 1.35)$ and the SWAE ratio is 3.5. Bai, Goldstein, and Yang (2020) propose to weight each rating category by its share of observations and under their weights the refinancing estimates are $(0.64, 1.30)$ for 1920 to 2022 default rates and $(0.63, 1.25)$ for 1970 to 2022 default rates, the Black-Cox boundary is 0.77 and 0.74, and the SWAE ratio is 3.6 and 3.7. Thus, the estimates are fairly similar across different weighting schemes and different time periods over which historical default rates are measured.

[Insert Table 2 here.]

Figure 6 and Table 3 show actual and model-implied cumulative default rates for each rating category at the benchmark estimates. The improvement from refinancing is concentrated at ratings below high investment grade (A/Aa/Aaa) and at long horizons. At the 20-year horizon the refinancing model matches the historical default rates of Baa, Ba, B, and C rated firms closely (12% vs 11%, 27% vs. 28%, 45% vs. 47%, and 61% vs. 62%), while the Black-Cox model underestimates substantially (9%, 19%, 32%, and 48%). In fact, the refinancing model is closer at long horizons for every rating.

[Insert Figure 6 and Table 3 here.]

The difference between the models is clearest in the forward default rates. Figure 7 plots the annual forward default rate, the probability of defaulting in year t conditional on surviving

⁵The restructuring levels in Goldstein, Ju, and Leland (2001) are the column ‘Restructure Level’ (V_U/V_0) in their Table 2, and those in Chen (2010) are read from Figure 4, Panel D.

to year $t - 1$, computed from Moody’s cumulative default rates and from the two models at the benchmark estimates. To reduce the noise from having few defaults, we average the default rates of the investment grade categories Aaa, Aa, A, and Baa before computing forward rates.

[Insert Figure 7 here.]

The figure shows that forward rates in the Black-Cox are increasingly too low as the horizon increases compared to actual forward default rates. This deficiency is what is documented in the theoretical section, namely that forward default rates go to zero in the Black-Cox model while this is not the case in the data (i.e. old firms do not become riskless). In contrast, in the restructuring model leverage does not drifts to zero and the forward rate converges to a positive constant as shown in Section 3.4. This leads the model to track actual forward default rates well both at short horizons and long horizons. Thus, the refinancing model fits the term structure of default rates and the forward default rates behind it, while the Black-Cox model cannot generate the long-horizon default rates in the data without a boundary that is too high at short horizons. Therefore, in the remainder of the paper we use the refinancing model.

5 The Long-Maturity Credit Spread

Moody’s seasoned Aaa and Baa corporate bond yield indices, and the Baa-Aaa spread constructed from them, are among the most widely used credit market series in finance and macroeconomics. The series are available from 1919 on FRED, and the spread is a standard cyclical indicator and a common calibration target for structural models. Moody’s includes only bonds with remaining maturities of at least 20 years in the indices, but the literature routinely treats the series as reflecting maturities of around ten years or less. Moody’s does not provide detailed information on the constituents, so in this section we first document what the series measure by “backward engineering” the constituents. We then ask to what extent the restructuring model of Section 3, estimated on default rates alone, as well as other prominent models proposed in the literature can match the Moody’s spreads.

5.1 Replicating Moody’s Aaa and Baa Yields

Moody’s does not publish the constituent bonds of its yield indices, so we reconstruct them from the bond sample for the period 2002–2025 described in Section D. A bond-month enters the replication if the bond is seasoned (age of at least half a year), issued by an industrial firm, has a remaining maturity of at least 20 years, and has a Moody’s bond rating of Aaa (for the Aaa index) or Baa1 to Baa3 (for the Baa index). Moody’s drops bonds from the indices if their remaining life falls below 20 years, if they become susceptible to redemption, or if their rating changes, and generally excludes bonds trading at deep discounts or steep premiums to par.⁶ To implement the last two criteria we remove, within each rating category and month, yields more than 1.5 standard deviations from the median yield in that month. The replicated index yield is the equal-weighted average yield of the remaining bonds.

The composition of the actual index and our replication differs in one respect that our sample cannot reproduce. Moody’s computes separate averages for industrial and public utility bonds, and the corporate Baa yield is the unweighted average of the Baa industrial and the Baa public utility averages, while the corporate Aaa yield has been the Aaa industrial average alone since December 2001, when Moody’s discontinued its Aaa public utility average.⁷ Our firm panel excludes utilities because structural models are not well-suited for highly regulated utilities, so our replication of the Baa index covers its industrial component. As we show below, the omission of utilities matters for the month-to-month yields but little for averages over longer periods.

The replication is close. Over the 279 months from July 2002 to September 2025, the correlation between the replicated and the published series is 98.6% for the Aaa yield, 99.0% for the Baa yield, and 90.3% for the Baa-Aaa spread, and the average replicated spread of 1.02% equals the average

⁶Federal Reserve Board, H.15 Selected Interest Rates, series notes for Moody’s seasoned Aaa and Baa corporate bond yields for the vintages 1996 to 2017, archived at <https://alfred.stlouisfed.org/series?seid=BAA>, and Moody’s Investors Service, Long-Term Corporate Bond Yield Averages, methodology notes as reproduced in the Mergent Bond Record, for example [https://psc.ky.gov/pscecf/2014-00372/mkurtz@bkllawfirm.com/04062015050516/Corporate_Bond_Yield_Averages_\(January_2015\).pdf](https://psc.ky.gov/pscecf/2014-00372/mkurtz@bkllawfirm.com/04062015050516/Corporate_Bond_Yield_Averages_(January_2015).pdf).

⁷The Mergent Bond Record note (Corporate Bond Yield Averages, January 2015, [https://psc.ky.gov/pscecf/2014-00372/mkurtz@bkllawfirm.com/04062015050516/Corporate_Bond_Yield_Averages_\(January_2015\).pdf](https://psc.ky.gov/pscecf/2014-00372/mkurtz@bkllawfirm.com/04062015050516/Corporate_Bond_Yield_Averages_(January_2015).pdf)) states that each corporate average is “the unweighted average of the corresponding Average Industrial and Average Public Utility observations” and that the Aaa public utility average was discontinued on December 10, 2001. The Baa corporate yields in its tables equal the mean of the two group averages to the second decimal. Footnote 14 of the H.15 release states that Moody’s Aaa rates are averages of Aaa industrial bonds only from December 7, 2001, see <https://www.federalreserve.gov/releases/h15/20071231/h15.htm>.

of Moody’s spread. Figure 8 shows the replicated and the published series. Thus, we reproduce the level of the spread and most of its variation, and the differences that remain in the time series likely reflect the utility component of the Baa index and differences in constituent selection. Table 4 shows characteristics of the constituent bonds. The average remaining maturity is 26.7 years in the Aaa index and 27.3 years in the Baa index, and the average bond is around five years old. The Baa index contains 155 bonds in the average month while the Aaa index contains 12 bonds from a small number of issuers, reflecting that few industrial firms are rated Aaa in the sample period.

[Insert Figure 8 and Table 4 here.]

The maturity of the series matters for how it is used. [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) measure the convenience yield on Treasuries as the Moody’s Aaa yield minus the yield on Treasury bonds with maturities above ten years, note that both series correspond to maturities of approximately 20 years, and control for the remaining maturity mismatch with the slope of the yield curve. The subsequent literature often attributes a shorter maturity to the series. [Chen, Collin-Dufresne, and Goldstein \(2009\)](#) compare the Moody’s Baa-Aaa spread with model spreads of four-year bonds, [Chen \(2010\)](#) calibrates the spread of a ten-year Baa-rated bond to 105 basis points (bps) and notes that this is close to the average Moody’s Baa-Aaa spread of 101 bps, and [Gourio \(2013\)](#) compares it with the spread of a one-period bond in an annual model.

In the macroeconomic literature the convenience yield is often measured as the Moody’s Aaa yield minus the ten-year Treasury yield, for example in [Luetticke \(2021\)](#) and [Phillot \(2025\)](#), and FRED publishes this difference as a series of its own (AAA10Y). [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) and papers that follow them, such as [Del Negro, Giannone, Giannoni, and Tambalotti \(2017\)](#) and [Bayer, Born, and Luetticke \(2023\)](#), measure it against the 20-year Treasury yield. Both benchmarks are inaccurate matches for bonds with a remaining maturity of 27 years because the Treasury curve slopes upward on average with the 10-year Treasury yield being particularly poor. Over 2002–2025 the 30-year Treasury yield exceeds the ten-year yield by 63 bps and the 20-year yield by 10 bps on average. The Moody’s Aaa yield minus the ten-year Treasury yield averages 146 bps, minus the 20-year yield 92 bps, and minus the 30-year Treasury yield averages 82 bps. and the spread of the Aaa constituents over Treasury bonds with the same cash flows, which

we compute in the next subsection, averages 80 bps. Measuring the convenience yield against the ten-year Treasury yield thus overstates it by around 66 bps, or roughly 80% of its correctly measured level.

The Baa-Aaa spread is not affected by the choice of Treasury benchmark, but its interpretation is. Model-implied spreads rise with maturity, so a model that is asked to match the observed spread at a ten-year maturity needs a larger risk adjustment than a model that is asked to match it at the true maturity.

5.2 Pricing Moody’s Indices Without Their Constituents

The replication requires transaction data and is therefore available from 2002. The firm panel of Section D is available from 1951. If the average model-implied spread of a generic long-maturity bond, priced for every Baa and Aaa rated firm in the panel, is close to the average model-implied spread of the index constituents, then the firm panel can be used to compute the model-implied Moody’s spread over the much longer period it covers. In this subsection we test whether this is the case in the period where both calculations are available.

We compute model-implied spreads in two ways. In the first, we price every constituent bond-month in the replicated indices at its actual coupon and remaining maturity, as described in Appendix E, and we average spreads across bonds as in the replication. In the second, we price a generic 27-year bond, the average remaining maturity of the constituents, for every Baa-rated and every Aaa-rated firm-year in the firm panel, and we average spreads across firms within each year. The generic bond pays semiannual coupons, and we set the coupon to the Moody’s Baa yield for Baa-rated firms and to the Moody’s Aaa yield for Aaa-rated firms in the month of the fiscal year end, the coupon at which a new long-maturity bond of that rating would be issued near par. We discount at a flat Treasury curve at the 30-year constant maturity yield, since the [Gürkaynak, Sack, and Wright \(2007\)](#) curve does not extend to long maturities in the early part of the firm panel. Leverage is measured at the fiscal year end. Over 2002 to 2023 the firm panel contains 484 Baa-rated and 10 Aaa-rated firms, while the Baa and Aaa constituents are issued by 157 and 4 firms.

[Insert Table 5 here.]

Table 5 shows the average spreads for the whole period and for three subperiods of equal length. Over the whole period the two calculations give the same spreads. The average Baa spread is 148 bps in the firm panel and 147 bps for the constituents, the average Aaa spread is 27 and 25 bps, and the average Baa-Aaa spread is 122 bps in both. The Baa spreads also agree within each subperiod. They are 127 and 116 bps in 2002 to 2009, 145 and 148 bps in 2010 to 2017, and 181 and 178 bps in 2018 to 2025. Thus, the constituents of the Baa index are priced like the average Baa-rated firm. The Aaa spreads differ within subperiods. In 2002 to 2009 the firm panel gives 32 bps and the constituents 2 bps, and in 2018 to 2025 the firm panel gives 17 bps and the constituents 36 bps. The likely reason is that few industrial firms are rated Aaa, and the Aaa spread is determined by a handful of firms in both calculations. Importantly, the differences in the Aaa spread average out over the whole period.

The last row shows the correlation between the annual spread series of the two calculations. The correlation is 0.48 for the Baa spread, 0.11 for the Aaa spread, and 0.38 for the Baa-Aaa spread. The correlations are moderate in part because the firm panel measures leverage once a year at the fiscal year end while the constituent spreads are averages of monthly values, and because the firm panel contains many small Baa-rated firms whose leverage is more volatile than that of the constituents. For our purpose the agreement in average spreads is what matters and we conclude that the model-implied Baa index yield spread and, over periods long enough for differences in the Aaa spread to average out, the model-implied Baa-Aaa spread can be computed from the firm panel alone.

5.3 Model-Implied Spreads

We compare the model-implied spread with the published Moody’s spread over April 1953 to December 2023 using the firm panel, the period for which the model spreads are available (since the ten-year Treasury yield that enters the estimation begins in April 1953 and the firm panel ends in 2023). For every Baa-rated and Aaa-rated firm-year we price the generic 27-year bond of Section 5.2, with the Moody’s Baa or Aaa yield as coupon and a flat Treasury curve at the

long-term Treasury yield, at the parameters estimated from default rates in Section 4. We average spreads across firms within each year and then across years. Table 6 compares the averages with the published Moody’s spreads. The Moody’s Baa-Aaa spread averages 97 bps over 1953 to 2023 while the model-implied average spread is almost the same at 95 bps, with a 95% confidence band of 74 to 115 bps computed as in [Feldhütter and Schaefer \(2018\)](#)⁸. Within subperiods the model is within 20 bps of the data. It implies 69 bps against 77 bps in 1953 to 1975, 92 against 110 bps in 1976 to 2000, and 124 against 104 bps in 2001 to 2023.⁹

[Insert Table 6 here.]

Table 6 also reports the Aaa spread over Treasuries, which is informative on its own because the default probabilities of Aaa-rated firms are small and the spread is bounded by the convenience yield on Treasuries. We measure the Moody’s Aaa yield against the long-term Treasury yield, detailed in Appendix F. The raw Moody’s Aaa-Treasury spread averages 69 bps over 1953 to 2023. [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) show that investors value the liquidity and the safety of Treasuries beyond their expected payoff, and they estimate the liquidity component of the convenience yield on long-term Treasuries to be 46 bps.¹⁰ We subtract the 46 bps, so the default component of the Aaa spread is 23 bps over the whole period. The model implies an Aaa spread that is 10 bps lower at 13 bps and the difference is statistically significant.

We conclude that the restructuring model matches 70 years of long-term corporate credit spreads well. At the actual maturity of the Moody’s constituents, default risk evaluated with historically estimated default probabilities, a standard asset Sharpe ratio, and Moody’s average recovery rate

⁸Model spreads inherit the statistical uncertainty of the historical default rates on which the models are estimated. We compute confidence bands following [Feldhütter and Schaefer \(2018\)](#). We simulate default histories in a Black-Cox economy with a common factor. There are 103 annual cohorts, corresponding to 1920 to 2022, with Moody’s average cohort size in each rating category, firms within a rating category are identical, and the leverage of each rating category is set such that the ten-year default probability equals Moody’s historical rate. Asset volatility, drift, and payout rate are the averages of the firm panel, and the pairwise correlation of asset values is 0.20, the estimate in [Feldhütter and Schaefer \(2018\)](#). For each of 2,400 simulated histories we compute average cumulative default rates for horizons of one to 20 years and re-estimate the default boundary with the objective in equation (24). The 2.5% and 97.5% quantiles of the estimated boundary relative to the true boundary are 0.84 and 1.13. Since spreads increase in the default boundary, the confidence band for the model spreads is obtained by scaling the estimated boundary by these quantiles, with the refinancing boundary u held fixed, and recomputing the firm-panel spreads.

⁹The overprediction in the last subperiod reflects the low riskless rates of the period which raise the model’s default probabilities above the historical rates through a low drift.

¹⁰[Krishnamurthy and Vissing-Jorgensen \(2012\)](#) estimate of 46 bps is for the period 1926–2008. [Feldhütter and Lando \(2008\)](#) obtain a similar estimate, 57 bps for ten-year Treasuries, over the shorter period 1996–2005.

accounts for the observed Baa-Aaa spread over 1953 to 2023, and comes within 10 bps of the Aaa spread net of the convenience yield on Treasuries. The model does so with a default boundary consistent with the direct evidence in [Davydenko \(2013\)](#) and with a term structure of default rates that matches the historical one.

5.4 Long-Maturity Spreads and the Pricing Kernel

Credit spreads have long been used to distinguish between pricing kernels with different specifications of risk premiums. The credit spread puzzle is the finding that a Merton-type model with a constant price of risk underpredicts investment-grade spreads at maturities of ten years or less once it is calibrated to historical default and recovery rates.¹¹ In response to the puzzle, a number of pricing kernels have been proposed that raise spreads.

While the typical target has been spreads with a maturity of ten years or less, credit spreads with longer maturity have received much less attention, see [Table 7](#). This is noteworthy because the Moody’s Baa-Aaa spread that is frequently used to characterize the credit spread puzzle is itself a long-maturity spread, constructed from bonds with at least 20 years remaining maturity and maturities targeted toward 30 years. The influential 109 bps Baa-Aaa spread reported by [Chen, Collin-Dufresne, and Goldstein \(2009\)](#) is the average of the Moody’s spread for the period 1970–2001, whereas their model comparisons are conducted for four-year and ten-year bonds. Pricing kernels proposed to explain the credit spread puzzle are typically evaluated at a maturity of 10 years or less, and to the best of our knowledge none has been evaluated at the 27-year average maturity of the Moody’s constituents.

[Section 5.3](#) shows that the refinancing model, which has a constant price of risk, matches the Moody’s Baa-Aaa spread when calibrated to historical default rates. This section asks what the proposed kernels imply for the same spread when they are calibrated to the default probabilities of the refinancing model.

We compute the average Baa-Aaa spread and Aaa spread over 1953 to 2023 implied by five

¹¹It is well-documented that such a model underpredict investment grade spreads with a maturity of a few years or less, while the extent to which it underpredicts spreads at maturities around ten years is debated ([Huang and Huang, 2012](#); [Feldhütter and Schaefer, 2018](#); [Bai, Goldstein, and Yang, 2020](#)).

models, using the calibration in each paper. The models represent three primary mechanisms through which the literature raises spreads relative to a diffusion model with a constant price of risk. The first is a countercyclical price of consumption risk. In the long-run risk model of [Chen \(2010\)](#), risk prices vary with expected consumption growth and consumption volatility, while in the habit formation model of [Chen, Collin-Dufresne, and Goldstein \(2009\)](#) they vary with the surplus consumption ratio. In both models, defaults tend to occur in states in which risk prices are high, and this risk adjustment is persistent because the underlying state variables are persistent. The second mechanism is priced jump risk. In the jump-diffusion model of [Bai, Goldstein, and Yang \(2020\)](#), firm value is exposed to downward jumps whose intensity and size are priced, and in the rare disaster pricing kernel used by [Seo and Wachter \(2018\)](#) to explain CDX tranche spreads, consumption is subject to disasters whose probability varies over time. The third mechanism is priced stochastic volatility. In [Du, Elkamhi, and Ericsson \(2019\)](#), asset variance is stochastic and carries a risk premium, and we use their stochastic volatility model without the jumps they add to explain short-maturity spreads.

We fix the physical default risk across models at the refinancing model’s average 27-year default probabilities of Baa-rated and Aaa-rated firms over 1953 to 2023, 15.8% and 1.6%, which extend Moody’s historical default rates from the 20-year horizon of the data to the 27-year horizon of the constituents through the fitted model. Each model has one free parameter per rating class, the initial distance to the default boundary of a representative Baa-rated and a representative Aaa-rated firm, and we set it such that the model’s physical default probability at the 27-year horizon equals these averages. The pricing kernel of each model then maps the representative firm’s physical default probabilities into risk-neutral ones, which are the same in every year, and we price the generic 27-year bond of [Section 5.3](#) for the representative firm in each year from 1953 to 2023 at that year’s average long-term Treasury yield and Moody’s yield, with the same recovery rate of 33.6%, and average across years. The bond, the discount curve, the coupon, and the recovery rate are those under which the refinancing model matches the observed spread in [Table 6](#), where it is priced firm by firm. [Appendix G](#) describes the implementation of each model including how confidence bands are calculated.

Figure 1 plots each model’s Baa-Aaa spread with its confidence band against the observed average of 97 bps. To make the comparison like for like, the refinancing model is evaluated in the figure in the same way as the alternative models, for a representative Baa-rated and a representative Aaa-rated firm with the panel-average asset volatility and payout rate whose 27-year default probabilities equal the firm-panel averages. It implies 94 bps with a confidence band of 78 to 107 bps, against 95 bps when priced firm by firm in Table 6, so the observed spread of 97 bps is well within the band. Every alternative model overpredicts the spread, by 27 bps in the rare disaster model, 34 bps in the jump risk model, 39 bps in the long-run risk model, 55 bps in the volatility risk premium model, and 68 bps in the habit formation model, and in each case the observed spread lies below the lower bound of the confidence band. The reason is what the alternative kernels were built to do. In each of them, defaults cluster in states in which investors are worst off, when consumption growth is low, volatility is high, or a disaster has struck, so bondholders demand more compensation per unit of expected loss than a constant Sharpe ratio of 0.22 provides. Because every model is given the same default probabilities, this larger compensation is the only difference between them and the refinancing model, and it raises the Baa spread more than the Aaa spread.

[Insert Figure 1.]

Using the Baa-Aaa spread rather than the spread between the corporate yield to the Treasury rate has the advantage that differences due to taxes, convenience yield of Treasuries and illiquidity are differenced out. Nevertheless, it is informative to investigate the Aaa-Treasury spread on its own, since it allows us to discriminate stronger between models. In Figure 9 we plot each model’s Aaa spread against the observed Moody’s spread of 69 bps and its default component of 23 bps after the convenience yield on Treasuries is removed. There are different estimates of the non-default component of the Aaa-Treasury spread and we subtract the convenience yield estimated in [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) as discussed in Section 5.3. There are other estimates in the literature of the non-default component, some bigger and some smaller, but it is reasonable to conclude that over the period 1953–2023 it is positive and thus the model-implied AAA-Treasury spread should be smaller than the raw spread of 69bps.

[Insert Figure 9 here.]

The refinancing model implies an Aaa spread of 23 bps, equal to the adjusted spread and 46 bps below the raw spread. The long-run risk, volatility risk premium, and jump risk models imply 41, 45, and 47 bps, above the adjusted spread but below the observed spread, so they are consistent with the Aaa spread if the non-default component is smaller than the estimate of [Krishnamurthy and Vissing-Jorgensen \(2012\)](#). The habit formation and rare disaster models imply 100 and 138 bps, significantly above the raw Aaa-Treasury spread, i.e both models attribute more than the entire Aaa spread over Treasuries to default risk. [Chen, Collin-Dufresne, and Goldstein \(2009\)](#) note that their model, evaluated at four years, explains only a small part of the Aaa-Treasury spread and suggest that much of the spread is due to factors other than credit risk. Evaluated at a 27-year maturity the same model implies an Aaa spread that exceeds the entire observed spread substantially. Based on the Aaa-Treasury spread, even when there are non-default factors that increase the actual spread, we can reject the habit formation and rare disaster kernels.

There are two implementation choices that are relevant to highlight. The first is the use of a representative firm in Figures 1 and 9 rather than firm-level default probabilities. The map from physical to risk-neutral default probabilities is concave, so evaluating it at the average default probability can overstate spreads. For the refinancing model we can compute both. The representative Baa-rated and Aaa-rated firms imply a Baa spread of 116 bps against 108 bps firm by firm and an Aaa spread of 23 against 13 bps, and the Baa-Aaa spread is 94 against 95 bps. The representative-firm approximation does not affect the comparison of Baa-Aaa spreads, while it overstates the Aaa leg by around 10 bps in the refinancing model, and presumably by a similar amount in the alternative models, so the Aaa spreads in Figure 9 are, if anything, slightly high for every model. The second is the symmetric treatment of the Aaa and Baa firms. If the Aaa firms were considered default-free, a common practice, the Baa-Aaa spread of each model would rise by the size of its Aaa leg, 41 to 138 bps, and every alternative model would appear to overpredict the spread by 80 bps or more.

The comparison does not depend on which estimate of the refinancing model from Table 2 is used. Figure A.1 in the Appendix repeats Figure 1 for the benchmark estimate and the three

alternative estimates, from default rates for cohorts formed in 1970 to 2022 and with observation weights. The observed spread remains inside the band of the refinancing model and below the lower bound of every alternative model under all four estimates.

To summarize, when the maturity and the coupons of the Moody’s constituents are measured correctly and every model is matched to the historical default rates, the average Baa-Aaa spread over 1953 to 2023 is accounted for by default risk under a constant price of risk, and every alternative kernel overpredicts it. Based on the Aaa-Treasury spread we can furthermore reject the habit formation and rare disaster models more sharply. The long-maturity spread thus brings out differences among pricing kernels that the short and intermediate maturities on which the literature has focused do not.

6 Conclusion

Credit spreads have long been used to discipline pricing kernels, and the credit spread puzzle has been the main motivation for kernels in which the price of risk is high in the states in which defaults cluster. The evidence for and against these kernels comes from spreads at maturities of ten years or less. We study a spread at close to 30 years. Moody’s Baa-Aaa spread, the most widely used credit spread series in finance and macroeconomics, is built from bonds with an average remaining maturity of 27 years, although it has routinely been treated as a four-year, ten-year, or even one-year object.

Pricing a 27-year bond requires default probabilities that are realistic at long horizons, which models with fixed debt do not deliver, since their forward default rate dies out as leverage drifts to zero. We derive closed-form default probabilities for a firm that refinances to a leverage target, in a model that reduces to the Black-Cox model without refinancing and whose forward default rate converges to a positive constant. Estimated on 70 years of default rates with one default and one refinancing parameter, the model fits the term structure of default rates remarkably well and tracks the forward default rates in the data.

At the true maturity of the Moody’s constituents, default risk with a constant price of risk accounts for the Baa-Aaa spread: the model implies 95 bps against 97 bps in the data over 1953

to 2023, with no parameter chosen to fit spreads. The five pricing kernels proposed to resolve the credit spread puzzle, given the same default probabilities, overpredict the spread by 27 to 68 bps, and the habit formation and rare disaster kernels attribute more than the entire Aaa spread over Treasuries to default risk. The long-maturity spreads thus separate pricing kernels in a way that the shorter end of the curve does not allow.

References

- Bai, Jennie, Robert S. Goldstein, and Fan Yang, 2020, Is the credit spread puzzle a myth?, *Journal of Financial Economics* 137, 297–319.
- Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.
- Barro, Robert J., and José F. Ursúa, 2008, Macroeconomic crises since 1870, *Brookings Papers on Economic Activity* 2008, 255–350.
- Bartram, Söhnke M., Mark Grinblatt, and Yoshio Nozawa, 2025, Book-to-Market, Mispricing, and the Cross-Section of Corporate Bond Returns, *Journal of Financial and Quantitative Analysis* 60, 1185–1233.
- Bayer, Christian, Benjamin Born, and Ralph Luetticke, 2023, The liquidity channel of fiscal policy, *Journal of Monetary Economics* 134, 86–117.
- Bhamra, Harjoat S., Adlai J. Fisher, and Lars-Alexander Kuehn, 2011, Monetary policy and corporate default, *Journal of Monetary Economics* 58, 480–494.
- Bhamra, Harjoat S., Lars-Alexander Kuehn, and Ilya A. Strebulaev, 2010, The levered equity risk premium and credit spreads: A unified framework, *Review of Financial Studies* 23, 645–703.
- Black, Fischer, and John Cox, 1976, Valuing corporate securities: Some effects of bond indenture provisions, *Journal of Finance* 31, 351–67.
- Bretscher, Lorenzo, Peter Feldhütter, Andrew Kane, and Lukas Schmid, 2021, Marking to Market Corporate Debt, *Working Paper*.
- Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.
- Carr, Peter, Robert Jarrow, and Ravi Myneni, 1992, Alternative characterizations of american put options, *Mathematical Finance* 2, 87–106.

- Chen, Hui, 2010, Macroeconomic conditions and the puzzles of credit spreads and capital structure, *Journal of Finance* 65, 2171–2212.
- , Rui Cui, Zhiguo He, and Konstantin Milbradt, 2018, Quantifying liquidity and default risks of corporate bonds over the business cycle, *Review of Financial Studies* 31, 852–897.
- Chen, Hui, Yu Xu, and Jun Yang, 2021, Systematic risk, debt maturity, and the term structure of credit spreads, *Journal of Financial Economics* 139, 770–799.
- Chen, Long, Pierre Collin-Dufresne, and Robert S. Goldstein, 2009, On the relation between the credit spread puzzle and the equity premium puzzle, *Review of Financial Studies* 22, 3367–3409.
- Christoffersen, Peter, Du Du, and Redouane Elkamhi, 2017, Rare disasters, credit, and option market puzzles, *Management Science* 63, 1341–1364.
- Collin-Dufresne, Pierre, and Robert S. Goldstein, 2001, Do credit spreads reflect stationary leverage ratios?, *Journal of Finance* 56, 1929–1957.
- Cook, Timothy Q., and Patric H. Hendershott, 1978, The impact of taxes, risk and relative security supplies on interest rate differentials, *Journal of Finance* 33, 1173–1186.
- Cremers, K.J. Martijn, Joost Driessen, and Pascal Maenhout, 2008, Explaining the level of credit spreads: Option-implied jump risk premia in a firm value model, *Review of Financial Studies* 21, 2209–2242.
- Culp, Christopher L., Yoshio Nozawa, and Pietro Veronesi, 2018, Option-based credit spreads, *American Economic Review* 108, 454–488.
- Danis, Andras, Daniel Rettl, and Toni Whited, 2014, Refinancing, profitability, and capital structure, *Journal of Financial Economics* 114, 424–443.
- David, Alexander, 2008, Inflation uncertainty, asset valuations, and the credit spreads puzzle, *Review of Financial Studies* 21, 2487–2534.
- Davydenko, Sergei A., 2013, When do firms default? A Study of the Default Boundary, *Working paper*.

- , and Ilya A. Strebulaev, 2007, Strategic actions and credit spreads: An empirical investigation, *Journal of Finance* 62, 2633–2671.
- Del Negro, Marco, Domenico Giannone, Marc P. Giannoni, and Andrea Tambalotti, 2017, Safety, liquidity, and the natural rate of interest, *Brookings Papers on Economic Activity* 2017, 235–316.
- Detemple, Jerome, 2006, *American-Style Derivatives: Valuation and Computation* (Chapman and Hall/CRC).
- Dick-Nielsen, Jens, 2009, Liquidity Biases in TRACE, *Journal of Fixed Income* 19, 43–55.
- , 2014, How to Clean Enhanced TRACE Data, *Working Paper*.
- , Peter Feldhütter, Lasse Heje Pedersen, and Christian Stolborg, 2025, Corporate Bond Factors: Replication Failures and a New Framework, *Review of Financial Studies* forthcoming.
- Du, Du, Redouane Elkamhi, and Jan Ericsson, 2019, Time-varying asset volatility and the credit spread puzzle, *Journal of Finance* 74, 1841–1885.
- Duffee, Gregory R., 1998, The relation between treasury yields and corporate bond yield spreads, *Journal of Finance* 53, 2225–2241.
- Eom, Young Ho, Jean Helwege, and Jing-Zhi Huang, 2004, Structural models of corporate bond pricing: An empirical analysis, *Review of Financial Studies* 17, 499–544.
- Favilukis, Jack, Xiaoji Lin, and Xiaofei Zhao, 2020, The elephant in the room: The impact of labor obligations on credit markets, *American Economic Review* 110, 1673–1712.
- Feldhütter, Peter, and David Lando, 2008, Decomposing swap spreads, *Journal of Financial Economics* 88, 375–405.
- Feldhütter, Peter, and Stephen Schaefer, 2018, The myth of the credit spread puzzle, *Review of Financial Studies* 31, 2897–2942.
- , 2023, Debt dynamics and credit risk, *Journal of Financial Economics* 149, 497–535.

- Fischer, Edwin O, Robert Heinkel, and Josef Zechner, 1989, Dynamic capital structure choice: Theory and tests, *Journal of Finance* 44, 19–40.
- Garbade, Kenneth D., 2007, The emergence of “regular and predictable” as a treasury debt management strategy, *Federal Reserve Bank of New York Economic Policy Review* 13, 53–71.
- Goldstein, Robert, Nengjiu Ju, and Hayne Leland, 2001, An EBIT-based model of dynamic capital structure, *Journal of Business* 74, 483–512.
- Gomes, João F., and Lukas Schmid, 2021, Equilibrium asset pricing with leverage and default, *Journal of Finance* 76, 977–1018.
- Gourio, François, 2013, Credit risk and disaster risk, *American Economic Journal: Macroeconomics* 5, 1–34.
- Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2007, The U.S. Treasury yield curve: 1961 to the present, *Journal of Monetary Economics* 54, 2291–2304.
- Hackbarth, Dirk, Jianjun Miao, and Erwan Morellec, 2006, Capital structure, credit risk, and macroeconomic conditions, *Journal of Financial Economics* 82, 519–550.
- Harrison, Michael, 1985, *Brownian Motion and Stochastic Flow Systems* (Wiley, New York).
- Huang, Jing-Zhi, and Ming Huang, 2012, How much of the corporate-treasury yield spread is due to credit risk?, *Review of Asset Pricing Studies* 2, 153–202.
- Huang, Jing-Zhi, Yoshio Nozawa, and Zhan Shi, 2025, The global credit spread puzzle, *Journal of Finance* 80, 101–162.
- Korteweg, Arthur, and Ilya A. Strebulaev, 2015, An Empirical Target Zone Model of Dynamic Capital Structure, Working Paper.
- Kou, S. G., and Hui Wang, 2003, First passage times of a jump diffusion process, *Advances in Applied Probability* 35, 504–531.

- Krishnamurthy, Arvind, and Annette Vissing-Jorgensen, 2012, The aggregate demand for Treasury debt, *Journal of Political Economy* 120, 233–267.
- Kuehn, Lars-Alexander, and Lukas Schmid, 2014, Investment-based corporate bond pricing, *Journal of Finance* 69, 2741–2776.
- Leland, Hayne, 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.
- , 2004, Predictions of default probabilities in structural models of debt, *Journal of Investment Management* 2, 16–40.
- , and Klaus Bjerre Toft, 1996, Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads, *Journal of Finance* 51, 987–1019.
- Lin, X. Sheldon, 1998, Double barrier hitting time distributions with applications to exotic options, *Insurance: Mathematics and Economics*, 21, 45–58.
- Longstaff, Francis A., Sanjay Mithal, and Eric Neis, 2005, Corporate yield spreads: Default risk or liquidity? New evidence from the credit default swap market, *Journal of Finance* 60, 2213–2253.
- Luetticke, Ralph, 2021, Transmission of monetary policy with heterogeneity in household portfolios, *American Economic Journal: Macroeconomics* 13, 1–25.
- Merton, Robert, 1974, On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* 29, 449–70.
- Moody’s, 2023, Annual Default Study: Corporate Default and Recovery Rates, 1920-2022, *Moody’s Investors Service* pp. 1–60.
- Pelsser, Antoon, 2000, Pricing double barrier options using Laplace transforms, *Finance and Stochastics* 4, 95–104.
- Phillot, Maxime, 2025, US treasury auctions: A high-frequency identification of supply shocks, *American Economic Journal: Macroeconomics* 17, 245–273.

- Rossi, Marco, 2014, Realized Volatility, Liquidity, and Corporate Yield Spreads, *Quarterly Journal of Finance* 4, 1450004.
- Sbuelz, Alessandro, 2005, Hedging double barriers with singles, *International Journal of Theoretical and Applied Finance* 08, 393–407.
- Seo, Sang Byung, and Jessica A. Wachter, 2018, Do rare events explain CDX tranche spreads?, *Journal of Finance* 73, 2343–2383.
- Strebulaev, Ilya, 2007, Do tests of capital structure theory mean what they say?, *Journal of Finance* 62, 1747–1787.

A Proof of Proposition 1

Proof. Since $A_1(T)$, $A_2(T)$, and $A_3(T)$ form a partition of Ω up to null events, and since

$$A_2(T) \subseteq \{T_B \leq T\}, \quad A_3(T) \cap \{T_B \leq T\} = \emptyset,$$

we obtain

$$DP_{DC}(T) = \mathbb{P}(A_2(T)) + \mathbb{P}(T_B \leq T, A_1(T)). \quad (25)$$

The function g^R is the defective density of the first refinancing time, that is,

$$g^R(t) dt = \mathbb{P}(T_{R,1} \in dt, T_{R,1} < T_B). \quad (26)$$

On the event $\{T_{R,1} < T_B\}$, define the rescaled post-refinancing process

$$\tilde{V}(s) := \frac{V(T_{R,1} + s)}{\gamma}, \quad s \geq 0.$$

Since

$$V(T_{R,1}) = V_R = \gamma V(0),$$

we have $\tilde{V}(0) = V(0)$. By the strong Markov property, time homogeneity, and the multiplicative scaling property of geometric Brownian motion, $\tilde{V}$ has the same law as the original asset-value process starting from $V(0)$. Moreover, after division by γ , the post-refinancing default and refinancing boundaries coincide with the original boundaries V_B and V_R . Consequently,

$$\begin{aligned} \mathbb{P}(T_B \leq T, A_1(T)) &= \mathbb{E}[1_{A_1(T)} DP_{DC}(T - T_{R,1})] \\ &= \int_0^T g^R(t) DP_{DC}(T - t) dt. \end{aligned} \quad (27)$$

Substituting (27) into (25) gives

$$DP_{DC}(T) = \mathbb{P}(A_2(T)) + \int_0^T g^R(t) DP_{DC}(T - t) dt. \quad (28)$$

Now we need to calculate $\mathbb{P}(A_2(T))$ and $g^R(t)$. As they contain double barrier passage times, unlike in the simple, single barrier case, there are no trivial expressions, and one has to rely, for instance, on series representation expressions. To this end, we employ the formulas derived by [Lin \(1998\)](#) (see equations (28)–(29) in the paper) and obtain (14) and (15) above.

■

B Numerical scheme for the integral equation

The equation (13) is a linear integral equation of Volterra type with initial condition $DP_{DC}(0) = 0$. This type of equation is well known in the integral equation theory from both theoretical and numerical points of view. In the finance literature, Volterra-type integral equations (though non-linear) are used to determine the optimal exercise boundary for finite maturity American call/put options (see e.g. Carr, Jarrow, and Myneni (1992) for the derivation and Chapter 8 in Detemple (2006) for a detailed description of the numerical scheme). More precisely, the exercise boundary of the American option (as a function of time t) is the solution to a nonlinear integral equation of Volterra type. In our case, the integral equation is simpler, because it is linear in $DP_{DC}(\cdot)$.

We now describe a numerical scheme for solving equation (13). Let us fix $T > 0$ and discretize the interval $[0, T]$ with grid step size $\Delta = T/N$ for large N . Therefore, we have the grid $0 = t_0 < t_1 < \dots < t_N = T$. We approximate the integral term in (28) by the following quadrature scheme

$$\int_0^{t_j} g^R(t) DP_{DC}(t_j - t) dt \approx \Delta \sum_{i=1}^j g^R(t_i) DP_{DC}(t_j - t_i) \quad (29)$$

for $j = 1, \dots, N$. We then recursively define the sequence $\pi_j \approx DP_{DC}(t_j)$ according to (13)

$$\pi_j = \mathbb{P}(A_2(t_j)) + \Delta \sum_{i=1}^j g^R(t_i) \pi_{j-i} \quad (30)$$

for $j = 1, \dots, N$ with the initial condition $\pi_0 = 0$. We note that we use the time t_j instead of T in (13) as it holds for all $T > 0$.

At the end, we will obtain the approximation π_N of $DP_{DC}(T)$. After defining $g^R(0) = 0$, both g^R and $\mathbb{P}(A_2(\cdot))$ are continuous on every bounded interval. Standard convergence results for quadrature methods applied to linear Volterra integral equations therefore imply that, for fixed T ,

$$\max_{0 \leq j \leq N} |\pi_j - DP_{DC}(t_j)| \longrightarrow 0$$

as $\Delta = T/N \longrightarrow 0$. We also note that by the nature of the algorithm above, we recover along the way the whole term structure of default probabilities $\{DP_{DC}(t_i), 0 \leq i \leq N\}$. This numerical scheme is easy to implement and requires $O(N^2)$ operations for $\{DP_{DC}(t_i), 0 \leq i \leq N\}$.

C Proof of Proposition 2

Proof. We apply the Laplace transform to both sides of (13):

$$\mathcal{L}[DP_{DC}](s) = \mathcal{L}[\mathbb{P}(A_2(\cdot))](s) + \mathcal{L}[g^R](s)\mathcal{L}[DP_{DC}](s) \quad (31)$$

where we use the fact that the Laplace transform of a convolution is the product of the corresponding Laplace transforms. We then obtain

$$\mathcal{L}[DP_{DC}](s) = \frac{\mathcal{L}[\mathbb{P}(A_2(\cdot))](s)}{1 - \mathcal{L}[g^R](s)}. \quad (32)$$

To derive the corresponding Laplace transforms, let $g^B(t)$ denote the defective lower-exit density in the first stage, so that

$$\mathbb{P}(A_2(T)) = \int_0^T g^B(t) dt. \quad (33)$$

Define

$$\kappa(s) := \sqrt{\nu^2 + 2s}, \quad \Re(s) > 0, \quad (34)$$

where the square-root branch is chosen such that $\Re(\kappa(s)) > 0$. Using the definitions of $\mathbb{P}(A_2(T))$, $g^R(t)$, and $g^B(t)$, we obtain the corresponding Laplace transforms (Lin, 1998; Pelsser, 2000; Sbuelz, 2005):

$$\mathcal{L}[\mathbb{P}(A_2(\cdot))](s) = \frac{\exp(-\nu\ell)}{s} \frac{\sinh(\kappa(s)u)}{\sinh(\kappa(s)h)}, \quad (35)$$

$$\mathcal{L}[g^R](s) = \exp(\nu u) \frac{\sinh(\kappa(s)\ell)}{\sinh(\kappa(s)h)}, \quad (36)$$

$$\mathcal{L}[g^B](s) = \exp(-\nu\ell) \frac{\sinh(\kappa(s)u)}{\sinh(\kappa(s)h)}. \quad (37)$$

In particular, equation (35) follows from (33) and

$$\mathcal{L}[\mathbb{P}(A_2(\cdot))](s) = \frac{1}{s} \mathcal{L}[g^B](s).$$

Substituting (35) and (36) into (32), we obtain the explicit Laplace transform of (18) of $DP_{DC}(T)$. Finally, we recover $DP_{DC}(T)$ by applying standard numerical inversion methods to (18). ■

D Data

D.1 Firm variables

For a given firm and calendar year, we obtain interest payments (INTPN), dividend payments (DVT), net stock repurchases (PRSTKC), debt in current liabilities (DLC), total long-term debt (DLTT) as well as industry classification from the CRSP/Compustat Merged Database. We assign annual firm fundamentals to the calendar with the most overlap (i.e., fiscal years ending in January to May are assigned to the prior calendar year) and use the last annual filing of a firm in a calendar year. We employ historical 4-digit standard industry classification codes (SICH) where available and otherwise current industry codes (SIC). The sample is limited to industrial firms and thus excludes utilities (SIC codes 4900–5000) and financials (SIC codes 6000–6999). The nominal amount of debt is calculated as the sum of debt in current liabilities and total long-term debt.

We obtain firm-level entity credit ratings for U.S. firms from Standard & Poor’s. Ratings are long-term issuer credit ratings (RatingTypeCode = “STDLONG” and OrgDebtTypeCode = “ICR”). We aggregate the available 33 rating levels into 7 broader rating categories, which we label with Moody’s letter grades, i.e., A (A, A+, +/A–1, A–, A/A, A/A–1), Aa (AA, AA+, AA+/A–1+, AA–, AA–/A–1+), Aaa (AAA, AAA/A–1+), B (B, B+, B+/B, B–), Ba (BB, BB+, BB–), Baa (BBB, BBB+, BBB+/A–2, BBB–, BBB–/A–3, BBB/A–2), and C (C, CC, CCC, CCC+, CCC–). For each firm and rating date, we use the last available rating.

To calculate the leverage ratio of firm j at time t , we first compute the quasi-leverage ratio at time t as $L_{q,t}^j = (\text{nominal amount of debt}) / (\text{market value of equity} + \text{nominal amount of debt})$, where the market value of equity is calculated as the number of shares outstanding (SHROUT) times the (absolute value of the) closing share price at the fiscal year end date (PRC), summed across all common stocks of an issuer (share codes 10 and 11) traded on NYSE, Nasdaq and Amex (exchange codes 1, 2, and 3). Due to limited data on market values of debt, this is the measure of leverage commonly used in the literature, but theoretically it is correct to use the market leverage ratio, $L_{m,t}^j = (\text{nominal amount of debt}) / (\text{market value of equity} + \text{market value of debt})$, see, for example, [Merton \(1974\)](#). Given that we do not have complete data on market values

of debt (we have corporate bond data but do not have market values for bank debt), we use the approximation in [Bretscher, Feldhütter, Kane, and Schmid \(2021\)](#) (BFKS) and calculate market leverage as $L_{m,t}^j = 0.871 \times L_{q,t}^j + 0.311 \times (L_{q,t}^j)^2$.

Equity volatility $\sigma_{E,t}^j$ is computed using daily stock returns (RET) from CRSP for the period between two fiscal year-end dates. We limit observations to common stock (share codes 10 and 11) traded on NYSE, Nasdaq and Amex (exchange codes 1, 2, and 3). For firms with several share classes, we use prior day market capitalization (calculated as the product of the absolute value of share price (PRC) and shares outstanding (SHROUT)) of each share class as weights to obtain a single daily return series for the issuer. The calculation of equity volatility requires at least 180 observations during the fiscal year; it is annualized by multiplying by the square root of 255.

We use the approximation in BFKS and calculate asset volatility $\sigma_{A,t}^j$ at time t as $\sigma_{A,t}^j = [1 - 1.129 \times L_{q,t}^j + 0.354 \times (L_{q,t}^j)^2] \sigma_{E,t}^j$. For each firm, we calculate a constant asset volatility parameter by computing the average asset volatility for the years the firm is in the sample.

The payout rate is the sum of interest payments, dividend payments, and net stock repurchases during the year divided by the market value of the firm. The payout rate is capped at 0.13 as in [Feldhütter and Schaefer \(2018\)](#), and we use the payout rate lagged one year.

For a given firm and year, we use the leverage ratio, asset volatility, and rating at the end of the fiscal year, as well as the payout ratio at the end of the prior fiscal year. We restrict our attention to firm-years with available credit rating and positive values for leverage, equity volatility, and (constant) asset volatility, and a non-negative prior-year payout rate. We set leverage ratios below 0.01 equal to 0.01. The number of firm-year observations for which we can calculate all firm values is 29,495, and the number of firms is 2,234. Firm summary statistics are in [Table 1](#).

[Insert [Table 1](#) here.]

D.2 Default and recovery rates

We use Moody’s average cumulative issuer-weighted default rates by letter rating for horizons of 1 to 20 years ([Moody’s, 2023](#)). Our benchmark series covers cohorts formed in 1920–2022 (Exhibit 38 of [Moody’s \(2023\)](#)) and we also use the postwar series 1970–2022 (Exhibit 39) for robustness.

We set the bond recovery rate to 33.6%, which is Moody’s dollar-weighted average recovery rate on senior unsecured bonds over 1983–2022, measured by post-default trading prices (Moody’s, 2023), and the same choice as in Feldhütter and Schaefer (2018) with the updated number.

D.3 Corporate bond data

We use corporate bond transaction data from Enhanced TRACE for the period July 2002 to September 2025 and bond characteristics from the Mergent Fixed Income Securities Database (FISD). We match bonds to firms on the six-digit issuer CUSIP. From FISD we keep bonds issued by U.S.-domiciled firms that are denominated in U.S. dollars and are not asset-backed, convertible, or variable-coupon bonds. For each bond, FISD provides the coupon rate, the coupon frequency, the maturity date, the offering date and amount, and the redemption features of the bond.

From TRACE we first remove cancelled and corrected trades following Dick-Nielsen (2009, 2014). We then keep secondary-market trades in corporate bonds and delete when-issued, locked-in, and special-price trades, reversals of trades reported on a prior day, and the buy side of interdealer trades, which are reported twice. Erroneous prices remain in TRACE after these steps. Dick-Nielsen, Feldhütter, Pedersen, and Stolborg (2025) test a range of filters against manually identified errors and find that a 100% reversal filter followed by the filter of Rossi (2014) performs best, and we apply both. The reversal filter deletes a trade whose price is more than twice the higher, or less than half the lower, of the prices of the preceding and the following trade in the same bond, provided the median of the three prices is at least 50. The Rossi filter deletes a trade whose price deviates from the centered moving median of the surrounding 20 trades by more than five times the corresponding moving median absolute deviation plus 10 dollars per 100 dollars of face value. Bartram, Grinblatt, and Nozawa (2025) apply a related filter to consecutive monthly returns. We use the yield reported in TRACE. For each bond and trading day we keep the last trade of the day, and for each bond and month we keep the last trading day of the month, so the bond sample is a monthly panel of end-of-month yields. Bond ratings are Moody’s bond ratings from the FISD ratings history, and we assign to each observation the rating in effect on the trade date. When we price bonds, we update the issuer’s leverage monthly using the CRSP market value of equity.

Finally, we obtain Moody’s monthly Aaa and Baa corporate bond yields from FRED.

D.4 Risk-free rates

We use the market yield on U.S. Treasury securities at 10-year constant maturity (GS10) from the FRED database as risk-free rate when we compute default probabilities. We use the value for the month of the fiscal year-end date. When we price bonds, we discount cash flows using the zero-coupon Treasury curve of [Gürkaynak, Sack, and Wright \(2007\)](#) and we use the curve parameters on the trade date.

E Model-Implied Bond Spreads

This appendix describes how we compute the model-implied spread of a constituent bond-month in Section 5.2. We price each bond as the coupon bond it is, with its FISD coupon c , semiannual coupon payments, and principal at its remaining maturity T . The model price discounts each cash flow on the [Gürkaynak, Sack, and Wright \(2007\)](#) Treasury zero-coupon curve at the trade date, weights it by the model’s risk-neutral survival probability, and pays a recovery of 33.6% of face value at the time of default,

$$P = \sum_k \frac{c}{2} Z(t_k)(1 - DP^Q(t_k)) + Z(T)(1 - DP^Q(T)) + 0.336 \sum_k Z(t_k)(DP^Q(t_k) - DP^Q(t_{k-1})), \quad (38)$$

where $Z(t)$ is the Treasury discount factor and $DP^Q(t)$ is the model’s cumulative default probability under the risk-neutral measure. We obtain DP^Q by replacing the drift μ_{jt} with $r_t - \delta_{jt}$, which is equivalent to subtracting the asset risk premium implied by the Sharpe ratio of 0.22. The recovery rate of 33.6% is the one described in Section D. The model yield spread is the yield to maturity implied by P minus the yield to maturity of a Treasury bond with the same cash flows priced on the same curve. The actual yield spread is the bond’s yield in TRACE minus the same Treasury yield. Thus, data and model are compared on a basis that is matched on both maturity and coupon. We update the issuer’s leverage monthly using the CRSP market value of equity and the latest annual amount of debt, and we use the firm-level asset volatility and payout rate from Section 4.

F The Long-Term Treasury Yield

The Aaa-Treasury spread in Table 6 requires a Treasury yield whose maturity matches that of the Moody’s constituents of approximately 27 years over 1953 to 2023. From April 1953 to January 1977 we use the 20-year constant maturity yield (GS20), and from February 1977 we use the 30-year constant maturity yield (GS30). The Treasury discontinued the 30-year constant maturity series on February 18, 2002 and reintroduced it on February 9, 2006, and in between it published a factor for adjusting the 20-year constant maturity yield to estimate a 30-year rate.¹² The same spliced series is the flat discount curve at which the generic 27-year bond of Section 5.2 is priced, so the Moody’s spread and the model spread are measured against the same benchmark.

The Federal Reserve’s average yield on long-term U.S. government securities (LTGOVTBD), available from 1925 to June 2000, is the long-term Treasury yield used in the literature on the convenience yield. Krishnamurthy and Vissing-Jorgensen (2012) measure the Aaa-Treasury spread against LTGOVTBD from 1919 to 1999 and against GS20 from 2000 to 2008, and Bayer, Born, and Luetticke (2023) follow them. The Federal Reserve’s H.15 release defines LTGOVTBD as the unweighted average of the yields on all outstanding Treasury bonds neither due nor callable in less than ten years, ‘including several very low yielding “flower” bonds’.¹³ The Third Liberty Bond Act of 1918 capped the coupon on Treasury bonds at 4.25%, and once long-term rates rose above that level in 1965 the Treasury could not issue bonds. Garbade (2007) documents that the ceiling kept the Treasury out of the bond market from mid 1965 to 1973, when Congress granted a limited exemption, and that the ceiling was eliminated only in 1988, so the population of outstanding bonds aged and shortened over the late 1960s and 1970s. In addition, Treasury bonds issued before 1971 were redeemable at par in payment of federal estate taxes. Cook and Hendershott (1978) show that these flower bonds, which sold at deep discounts, and the capital gains tax treatment of discount bonds pushed the Federal Reserve’s average long-term government bond yield below the yield required by taxable investors, increasingly so over 1973 to 1976. The two provisions reinforced

¹²Footnote 9 of the H.15 release, <https://www.federalreserve.gov/releases/h15/>. The GS30 series on FRED has values for every month of the gap, which we take to be the Treasury’s estimated 30-year rate.

¹³Footnote 7 of the H.15 release of June 28, 1982, <https://fraser.stlouisfed.org/title/h15-selected-interest-rates-86/june-28-1982-67104>.

each other, since the ceiling kept new bonds without the feature out of the basket while rising rates pushed the old bonds to the discounts at which the feature was valuable.

Over the months in which GS20 and LTGOVTBD overlap, April 1953 to January 1977, GS20 exceeds LTGOVTBD by 28 bps on average, but the gap is 6 bps from 1953 to 1966, 27 bps from 1967 to 1972, and 104 bps from 1973 to January 1977. Thus, the two series measure the same long-term yield until the coupon ceiling became binding, and diverge by around a percentage point in the 1970s.

G Implementation of the Alternative Pricing Kernels

This appendix describes how the five alternative models of Section 5.4 are implemented. Each model is taken from the paper that proposes it, with the calibration reported there, and is used to generate the term structure of physical and risk-neutral default probabilities of a representative Baa-rated and a representative Aaa-rated firm. We validate each implementation against the results reported in the paper before using it, and the validation is reported at the end of each subsection. The common procedure is as follows. We simulate the model’s state variables and the value of a representative firm under the physical and under the risk-neutral measure from the same initial state distribution, and we record the running minimum of the log firm value at horizons of 2 to 45 years (5 to 50 years in the habit and rare disaster models). The default boundary of each leg is the level of the running minimum such that the fraction of physical paths that cross it by the 27-year horizon equals the anchor, 15.8% for the Baa leg and 1.6% for the Aaa leg. The risk-neutral default probability at each horizon is the fraction of risk-neutral paths that cross the same level. We interpolate linearly between horizons, with zero at time zero, and price the generic 27-year semiannual coupon bond of Section 5.3 with the resulting risk-neutral survival probabilities, a recovery of 33.6% of face value paid at default, and, for each year from 1953 to 2023, that year’s average long-term Treasury yield as flat discount rate and Moody’s Baa or Aaa yield as coupon. The confidence bands are computed as in Section 5.3. For the alternative models we anchor each model to the refinancing model’s average default probabilities at the scaled boundaries, which are 12.2% and 1.2% at the lower and 18.9% and 2.0% at the upper quantile. The simulated economy is the Black-Cox model, so the bands for the other models assume that the sampling uncertainty of the default boundary is the same as in the Black-Cox model. This is an approximation, but the uncertainty comes from the default rate data and not from the model, and the approximation keeps the bands comparable across models.

Long-run risk, [Chen \(2010\)](#)

The economy is his nine-state Markov chain for expected consumption growth and consumption volatility (his Table A.I), with the generator rebuilt from the monthly parameters of [Bansal and](#)

Yaron (2004) by quarterly Tauchen-Hussey discretization, Epstein-Zin preferences with risk aversion 7.5, elasticity of intertemporal substitution 1.5, and time preference 0.015, and the state-dependent riskless rate, price of diffusion risk, and jump risk premia of his equations (A6) to (A8). The firm's cash flow follows his equations (10) to (16) with the parameters of his Table II and an idiosyncratic volatility of 0.235. Two features of his model beyond the pricing kernel matter for spreads, and we include both. The default boundary is countercyclical, since equity holders default at a higher cash flow level in states with low expected growth and high volatility. It is the equity holders' optimal boundary in each state, obtained by solving the equity valuation problem as an obstacle problem on a log cash flow grid under the risk-neutral dynamics of his economy; relative to the median state the boundary ranges from 0.80 in the high-growth, low-volatility state to 1.32 in the low-growth, high-volatility state, the pattern of his Figure 4. The recovery rate is procyclical, since the fraction of assets recovered and the value of assets at default are both lower in bad states. The recovery rate of face value at default is proportional to his estimated firm recovery rate in the state of default, $\alpha(s)$ from his Table III, times the unlevered value of a unit of cash flow in that state times the boundary, and it is scaled so that its average over defaults under the physical measure over 27 years equals 33.6%, so the model has the same average recovery as the other models but a lower recovery in the states that carry a high risk-neutral weight; the ratio between the recovery rates in his best and worst states is 3.3, against 3.5 in his Table V. We simulate 60,000 paths of the state and the cash flow under each measure with a time step of 0.05 years. Table III of Chen (2010) prints the recovery function with signs under which it is negative in high-growth states, and we use the signs implied by his text and his Figure 3, under which the recovery rate increases in expected growth and is lowest in the low-growth, high-volatility state. At his own calibration, a physical ten-year default probability of 4.9% and his mean recovery rate, the implementation gives a ten-year Baa spread of 122 bps against his 104.5 bps, so it overstates his spread by about 15%; without the two state-dependent features it gives 96 bps.

Habit formation, Chen, Collin-Dufresne, and Goldstein (2009)

The economy is the Campbell and Cochrane (1999) habit model with their parameters, risk aversion 2.45, persistence of the log surplus consumption ratio $e^{-0.138}$ per year, consumption growth 1.89%

with volatility 1.5%, and a riskless rate of 0.94%. The price-output ratio of the aggregate claim is obtained by iterating on the Euler equation on a grid for the surplus consumption ratio, as in their Section 3, with output growth 1.89%, output volatility 6.3%, and a correlation between output and consumption shocks that reproduces the moments of the aggregate claim in their Table 3. Firm value follows their equation (22), the return on the aggregate claim plus an idiosyncratic component with volatility 0.208. The risk-neutral measure is obtained by the exact Girsanov shift of the consumption shock implied by the habit kernel, so that no likelihood ratio weighting is needed. Following their coupon bond setup, in which bondholders receive the recovery rate at default and all later coupons are lost, the firm defaults at the first annual date at which its value is below the boundary, and the boundary is the countercyclical one of their Section 3.5, $\psi^*[0.52 - 0.61S(0)][1 - 4(S(t) - \bar{S})]$ in their equations (28) and (29), which depends on the surplus consumption ratio at issuance and at the current date; ψ^* is the free parameter that we anchor. The simulation uses annual steps and 400,000 paths from the stationary distribution of the surplus consumption ratio. At their calibration, with ψ^* set to their four-year default rates of 1.55% and 0.04%, the implementation gives four-year risk-neutral default probabilities of 7.2% and 0.42% against the 8.2% and 0.59% of their Table 7 and a four-year Baa-Aaa spread of 95 against their 107 bps, and with ψ^* re-matched to their ten-year default rates of 4.89% and 0.63%, as in their Table 8, it gives ten-year risk-neutral default probabilities of 28% and 8% against their 31% and 11%.

Priced jump risk, [Bai, Goldstein, and Yang \(2020\)](#)

Firm value follows the [Kou and Wang \(2003\)](#) jump diffusion with downward exponential jumps, the special case of their Section 5, with the parameters of their Table 8: diffusion volatility 0.21, physical jump intensity 0.13 and risk-neutral intensity 0.20, risk-neutral mean jump size $1/2.53$, price of diffusion risk 0.11, and jump risk price 2.76, which implies a physical mean jump size of $1/3.97$ through their equations (17) to (22). The correlations between the firm and the market are 0.543 for Baa-rated and 0.603 for Aaa-rated firms. Default probabilities are first-passage probabilities computed by numerical inversion of the Kou-Wang Laplace transform, validated against simulation. At the boundaries that match Moody's ten-year default rates the implementation reproduces their model CDS spreads for Baa-rated firms at five, ten, and 20 years within about 10% (86, 125, and

148 against 96, 132, and 158 bps).

Rare disasters, [Seo and Wachter \(2018\)](#)

The economy is their two-factor model in which consumption is subject to disasters whose intensity follows a square-root process with a time-varying mean that itself follows a square-root process, with risk aversion 3 and unit elasticity of intertemporal substitution. Their statement that the discriminant of the price-dividend equation is zero pins the disaster size distribution, which we impose on the multinomial approximation to the [Barro and Ursúa \(2008\)](#) distribution; under the risk-neutral measure the disaster intensity is multiplied by 2.88, disaster sizes are tilted by the kernel, and both intensity factors have the explosive drift implied by the zero discriminant. Dividend and firm strips are priced by their ordinary differential equations, and firm value is the dividend claim of a firm with dividend leverage 1.3, idiosyncratic jumps, and sector jumps as in their Table 3. The firm defaults at the first monthly observation at which its value falls below the boundary. We simulate 150,000 paths under each measure with exact sampling of the square-root processes. The implementation matches the moments of the disaster probability and of the aggregate market in their Tables 1 and 2, and at their median disaster probability it gives a five-year CDX spread of 48 bps against their 41 bps; at higher disaster probabilities it overstates their spreads, so its spreads in Figures 1 and 9 should be read as an upper bound.

Priced volatility risk, [Du, Elkamhi, and Ericsson \(2019\)](#)

Firm value follows a Heston process with the calibration of their Table I for a representative Baa-rated firm: mean reversion of variance 4, volatility of variance 0.30, correlation between value and variance shocks -0.15 , and initial and long-run asset volatility 29%. The expected asset return exceeds the riskless rate by $(\sqrt{1 - \rho^2}\lambda_D + \rho\lambda_V)V_t$, where V_t is the asset variance, with the mix of diffusive and variance risk premia that matches the historical ten-year Baa-Aaa spread of 131 bps in their table, $\lambda_D = 0.30$ and $\lambda_V = -3.08$, so that the asset Sharpe ratio is 0.22 at the long-run variance. Under the risk-neutral measure the mean reversion of variance is $\kappa + \sigma\lambda_V = 3.08$ and the long-run volatility 33.1%. The riskless rate net of payout is the panel mean of 0.74%. Default is the first passage of firm value below the boundary, and we simulate 60,000 paths under each measure

with a time step of 0.02 years. We use their stochastic volatility model without the jumps they add to explain short-maturity spreads. At their calibration, a physical ten-year default probability of 4.9%, the implementation gives a risk-neutral ten-year default probability of 23.4% against their 22.9% and a ten-year spread of 122 bps against their 131 bps.

Figure 2. Asset Value Path and Default Barrier in Base Case of Static Capital Structure

The figure plots a hypothetical trajectory of firm value $V(t)$ and the default threshold V_B in the base case of static capital structure. Default happens when firm value drops to V_B at time T_B .

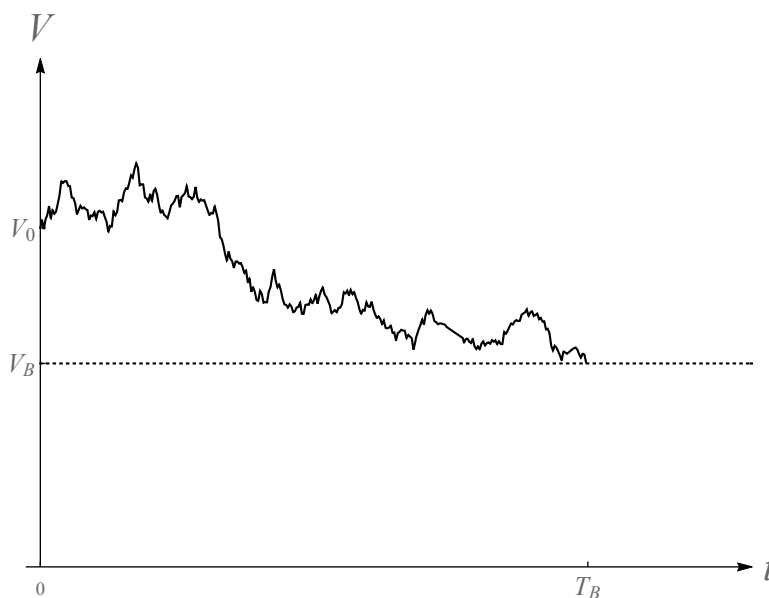


Figure 3. Asset Value Path and Barriers for Default and Restructuring in Case of Dynamic Capital Structure

The figure plots a hypothetical trajectory of firm value $V(t)$ and the evolution of the restructuring boundary V_R and default threshold V_B in the case of dynamic capital structure. When firm value increases to the restructuring boundary, the firm can issue additional debt, and the restructuring boundary and default threshold increase. Default happens when firm value drops to V_B at time T_B .

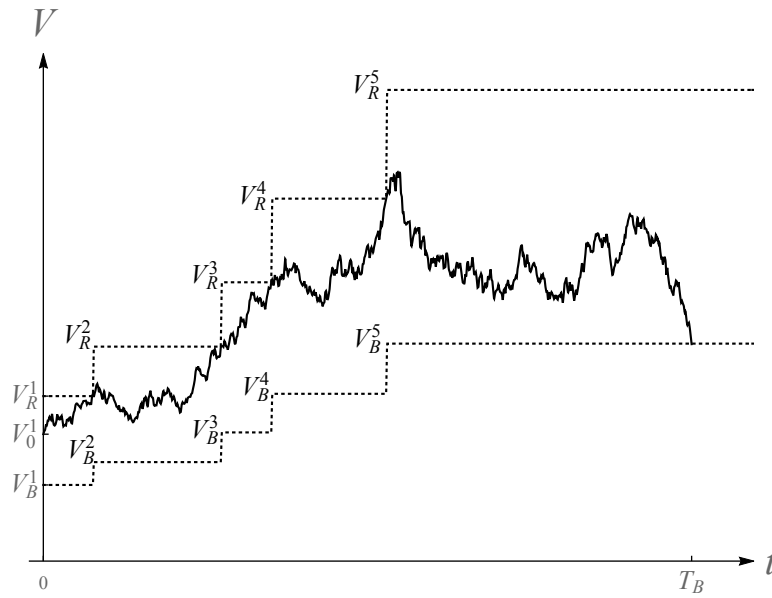


Figure 4. Possible Scenarios of Asset Value for Dynamic Default Probability

The figure displays possible scenarios, which are mutually exclusive and collectively exhaustive. The green line represents the restructuring scenario (i.e., the first restructuring time $T_{R,1}$ precedes the default probability's horizon T and the bankruptcy time T_B). The red line represents the bankruptcy scenario (i.e., the bankruptcy time T_B precedes the default probability's horizon T and the first restructuring time $T_{R,1}$). The blue line represents the remaining scenario (i.e., the default probability's horizon T precedes the first restructuring time $T_{R,1}$ and the bankruptcy time T_B).

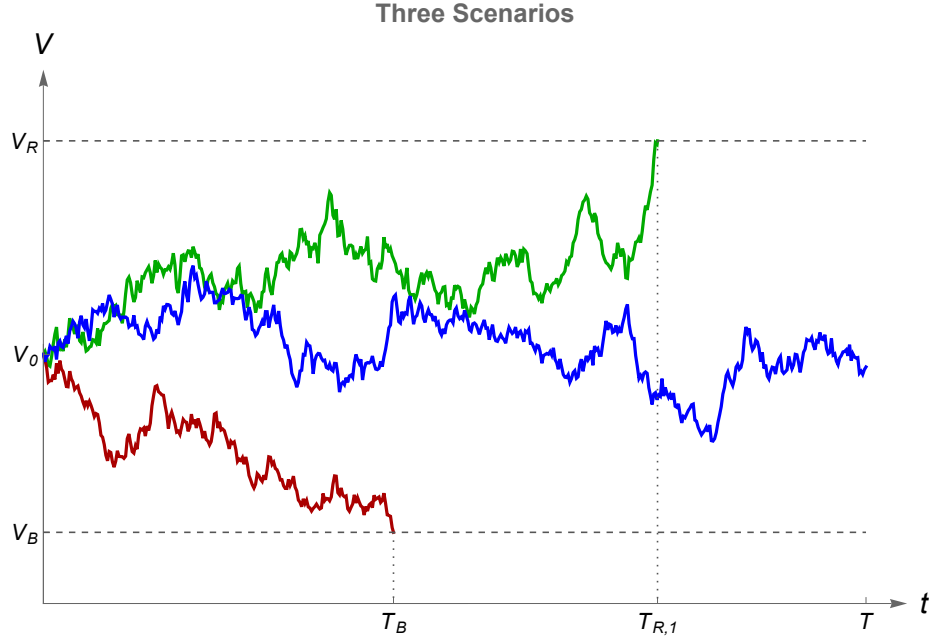


Figure 5. Future Debt Relative to Current Debt

The figure plots the average ratio of nominal debt of a firm scaled by the first observation in the first year where the firm has leverage greater or equal to 5% for our sample from 1951 to 2023. The graph plots the average nominal debt ratio for all firms with non-missing data for the first 10 consecutive years on a logarithmic scale.

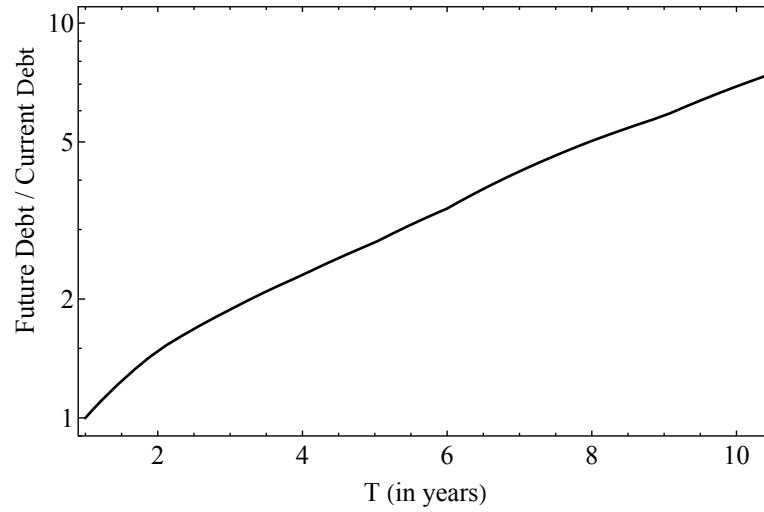


Figure 6. Average Cumulative Actual and Model-Implied Default Rates

The figure shows average cumulative actual and model-implied default rates by rating category for horizons of 1 to 20 years. For each rating category from Aaa to C, it reports the average actual historical default rates from Moody's for cohorts formed in 1920–2022 ('Actual'). In addition, it shows average model-implied default probabilities for the Black-Cox model and the restructuring model at the pooled parameter estimates of Table 2 ($\hat{d} = 0.74$ for Black-Cox; $\hat{d} = 0.68$, $\hat{u} = 1.65$ for the restructuring model), common to all rating categories. Model-implied default probabilities are calculated for a sample of 29,495 firm-years over the period 1951–2023.

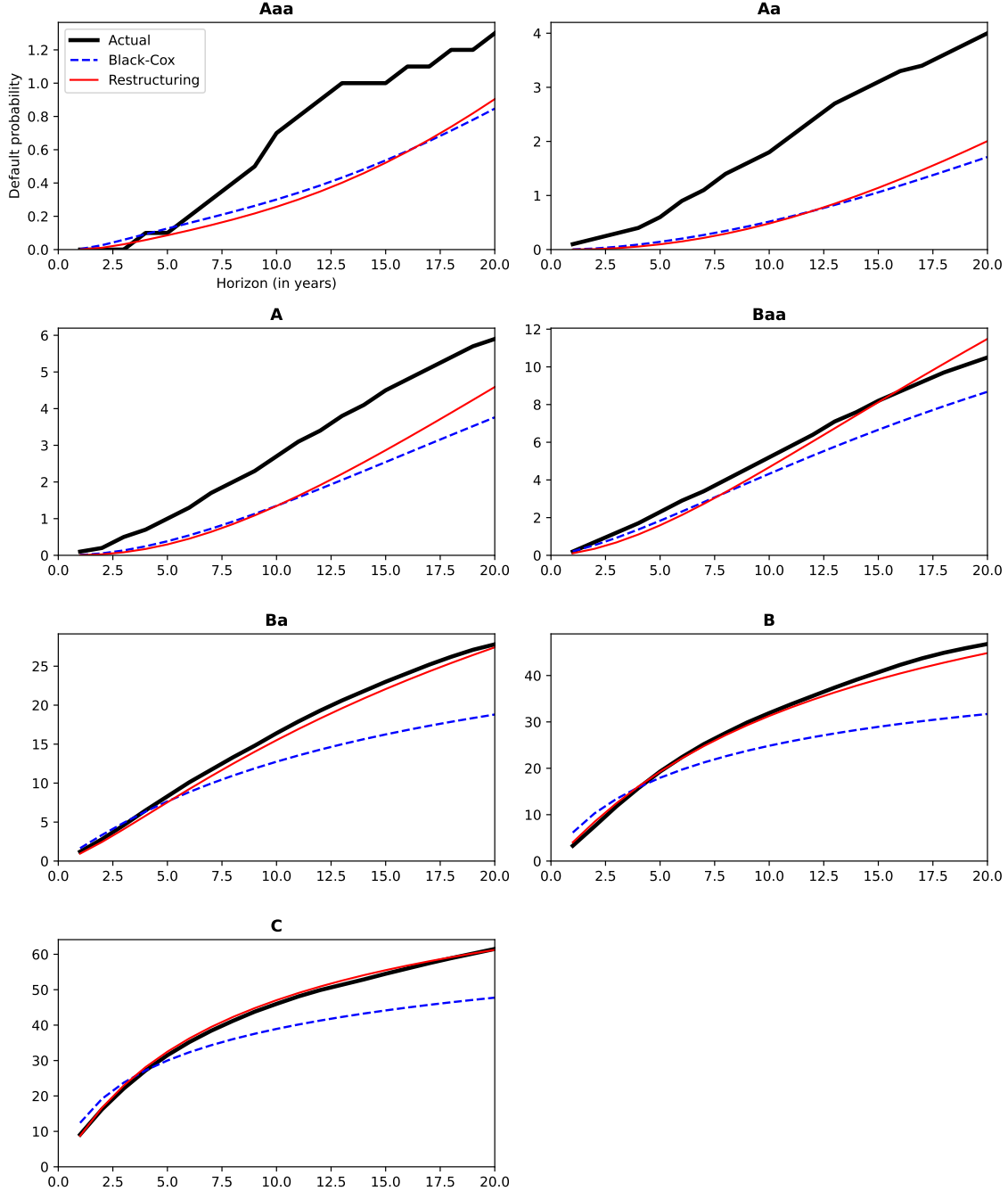


Table 1. Summary Statistics of Firm Variables

The table shows summary statistics for the panel of rated U.S. industrial firms for the period 1951–2023. There are 29,495 firm-year observations from 2,234 firms. Calculation details are in Section D.1

Credit Rating	Number of Firms	Mean	10th	25th	Median	75th	90th
Panel A: Market Leverage ratio							
Aaa	38	0.12	0.02	0.04	0.07	0.16	0.29
Aa	135	0.16	0.04	0.07	0.13	0.21	0.33
A	384	0.20	0.05	0.09	0.16	0.27	0.40
Baa	700	0.27	0.07	0.13	0.22	0.36	0.54
Ba	1,169	0.36	0.09	0.17	0.30	0.51	0.73
B	1,217	0.53	0.12	0.27	0.49	0.77	0.99
C	272	0.57	0.04	0.20	0.58	0.93	1.09
All firms	2,234	0.34	0.06	0.13	0.26	0.48	0.75
Panel B: Equity volatility							
Aaa	38	0.23	0.16	0.18	0.22	0.27	0.34
Aa	135	0.27	0.17	0.21	0.25	0.31	0.38
A	384	0.30	0.19	0.22	0.27	0.34	0.44
Baa	700	0.34	0.20	0.25	0.31	0.40	0.51
Ba	1,169	0.45	0.25	0.31	0.40	0.53	0.71
B	1,217	0.63	0.33	0.40	0.53	0.73	1.04
C	272	0.81	0.39	0.51	0.68	0.95	1.36
All firms	2,234	0.43	0.21	0.27	0.36	0.51	0.72
Panel C: Asset volatility							
Aaa	38	0.20	0.17	0.17	0.20	0.22	0.23
Aa	135	0.22	0.17	0.19	0.21	0.24	0.28
A	384	0.24	0.18	0.20	0.23	0.26	0.32
Baa	700	0.26	0.19	0.21	0.24	0.29	0.35
Ba	1,169	0.30	0.20	0.24	0.28	0.35	0.43
B	1,217	0.34	0.21	0.25	0.31	0.41	0.51
C	272	0.37	0.22	0.27	0.35	0.44	0.57
All firms	2,234	0.28	0.19	0.22	0.26	0.32	0.43
Panel D: Payout ratio							
Aaa	38	0.034	0.016	0.022	0.032	0.042	0.056
Aa	135	0.034	0.012	0.020	0.030	0.042	0.058
A	384	0.038	0.010	0.020	0.032	0.049	0.072
Baa	700	0.040	0.009	0.018	0.032	0.053	0.082
Ba	1,169	0.038	0.006	0.014	0.029	0.053	0.084
B	1,217	0.045	0.005	0.015	0.037	0.066	0.099
C	272	0.055	0.007	0.022	0.049	0.083	0.124
All firms	2,234	0.040	0.007	0.017	0.032	0.054	0.085

Table 2. Parameter Estimates and Model Fit

The table shows pooled parameter estimates and model fit for the Black-Cox model and the model with restructuring. For each model, a single parameter vector is estimated jointly for all rating categories by minimizing the sum of weighted absolute errors, $SWAE = \sum_a \sum_{t=1}^{20} \frac{1}{t} |\bar{\pi}_{at}^P(\Theta) - \hat{\pi}_{at}^P|$, where $\bar{\pi}_{at}^P(\Theta)$ is the model-implied average default probability for rating a and horizon t and $\hat{\pi}_{at}^P$ is the corresponding historical default frequency. The first row uses Moody’s average cumulative default rates for cohorts formed in 1920–2022. The second row uses cohorts formed in 1970–2022. The third and fourth rows weight each rating category by its share of sample observations, $w_a = N_a / \sum_a N_a$, instead of equally, so that the objective is $\sum_a w_a \sum_{t=1}^{20} \frac{1}{t} |\bar{\pi}_{at}^P(\Theta) - \hat{\pi}_{at}^P|$. Because the rating weights sum to one rather than seven, the SWAE levels in these rows are not comparable to the other rows, but the ratio is. The last column shows the ratio of the Black-Cox SWAE to the restructuring-model SWAE.

	Black-Cox Model		Restructuring Model			Model Comparison
	$\hat{d}$	$SWAE_{BC}$	$\hat{d}$	$\hat{u}$	$SWAE_{RES}$	$\frac{SWAE_{BC}}{SWAE_{RES}}$
<i>Benchmark case</i>						
Default rates 1920–2022	0.74	0.454	0.68	1.65	0.140	3.25
<i>Alternative cases</i>						
Default rates 1970–2022	0.73	0.454	0.65	1.35	0.128	3.54
1920–2022, observation weights	0.77	0.066	0.64	1.30	0.018	3.58
1970–2022, observation weights	0.74	0.070	0.63	1.25	0.019	3.72

Table 3. Average Cumulative Actual and Model-Implied Default Rates

The table shows average cumulative actual and model-implied default rates by rating category for horizons of 1 to 20 years. For each rating category from Aaa to C, it reports the average actual historical default rates from Moody's for cohorts formed in 1920–2022 ('Actual'). In addition, it shows average model-implied default probabilities for the Black-Cox model and the restructuring model at the pooled parameter estimates of Table 2, common to all rating categories. Model-implied default probabilities are calculated for a sample of 29,495 firm-years over the period 1951–2023.

horizon (years)	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
Aaa															
Actual, 1920-2022	0.000	0.000	0.000	0.001	0.001	0.002	0.003	0.004	0.005	0.007	0.009	0.010	0.011	0.012	0.013
Black-Cox	0.000	0.000	0.001	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.004	0.005	0.006	0.007	0.008
Restructuring	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.002	0.002	0.003	0.003	0.005	0.006	0.007	0.009
Aa															
Actual, 1920-2022	0.001	0.002	0.003	0.004	0.006	0.009	0.011	0.014	0.016	0.018	0.024	0.029	0.033	0.036	0.040
Black-Cox	0.000	0.000	0.001	0.001	0.001	0.002	0.003	0.003	0.004	0.005	0.007	0.009	0.012	0.014	0.017
Restructuring	0.000	0.000	0.000	0.001	0.001	0.002	0.002	0.003	0.004	0.005	0.007	0.010	0.013	0.016	0.020
A															
Actual, 1920-2022	0.001	0.002	0.005	0.007	0.010	0.013	0.017	0.020	0.023	0.027	0.034	0.041	0.048	0.054	0.059
Black-Cox	0.000	0.001	0.001	0.002	0.004	0.005	0.007	0.009	0.011	0.014	0.018	0.023	0.028	0.033	0.038
Restructuring	0.000	0.000	0.001	0.002	0.003	0.005	0.006	0.009	0.011	0.013	0.019	0.025	0.032	0.039	0.046
Baa															
Actual, 1920-2022	0.002	0.007	0.012	0.017	0.023	0.029	0.034	0.040	0.046	0.052	0.064	0.076	0.087	0.097	0.105
Black-Cox	0.002	0.005	0.009	0.014	0.018	0.023	0.028	0.033	0.038	0.043	0.053	0.062	0.071	0.079	0.087
Restructuring	0.001	0.003	0.007	0.011	0.016	0.021	0.027	0.034	0.040	0.047	0.060	0.074	0.088	0.102	0.115
Ba															
Actual, 1920-2022	0.012	0.028	0.046	0.065	0.083	0.101	0.117	0.133	0.148	0.164	0.193	0.218	0.241	0.262	0.278
Black-Cox	0.016	0.033	0.049	0.063	0.077	0.089	0.100	0.110	0.119	0.127	0.143	0.156	0.168	0.179	0.188
Restructuring	0.010	0.024	0.041	0.058	0.075	0.092	0.109	0.125	0.140	0.155	0.183	0.209	0.232	0.254	0.274
B															
Actual, 1920-2022	0.033	0.075	0.118	0.157	0.193	0.224	0.252	0.276	0.299	0.319	0.356	0.391	0.423	0.449	0.468
Black-Cox	0.061	0.103	0.134	0.159	0.180	0.197	0.212	0.226	0.238	0.249	0.267	0.283	0.296	0.307	0.317
Restructuring	0.040	0.085	0.125	0.161	0.193	0.222	0.248	0.271	0.293	0.313	0.348	0.378	0.405	0.428	0.448
C															
Actual, 1920-2022	0.091	0.162	0.221	0.272	0.315	0.352	0.384	0.412	0.438	0.460	0.499	0.529	0.560	0.589	0.615
Black-Cox	0.124	0.192	0.238	0.272	0.300	0.323	0.343	0.360	0.376	0.389	0.413	0.433	0.450	0.464	0.478
Restructuring	0.088	0.167	0.230	0.282	0.325	0.362	0.395	0.423	0.448	0.470	0.509	0.541	0.568	0.592	0.612

Figure 7. Empirical and Model-Implied Forward Default Rates

The figure shows annual forward default rates, $h(t) = (\pi(t) - \pi(t-1))/(1 - \pi(t-1))$ in basis points per year, computed from cumulative default rates. Dots show Moody's average cumulative default rates for cohorts formed in 1920–2022. Lines show forward default rates computed from the model-implied average cumulative default rates in Table 3 for the restructuring model and the Black-Cox model. The investment-grade panel averages the Aaa, Aa, A, and Baa cumulative default rates before computing forward rates.

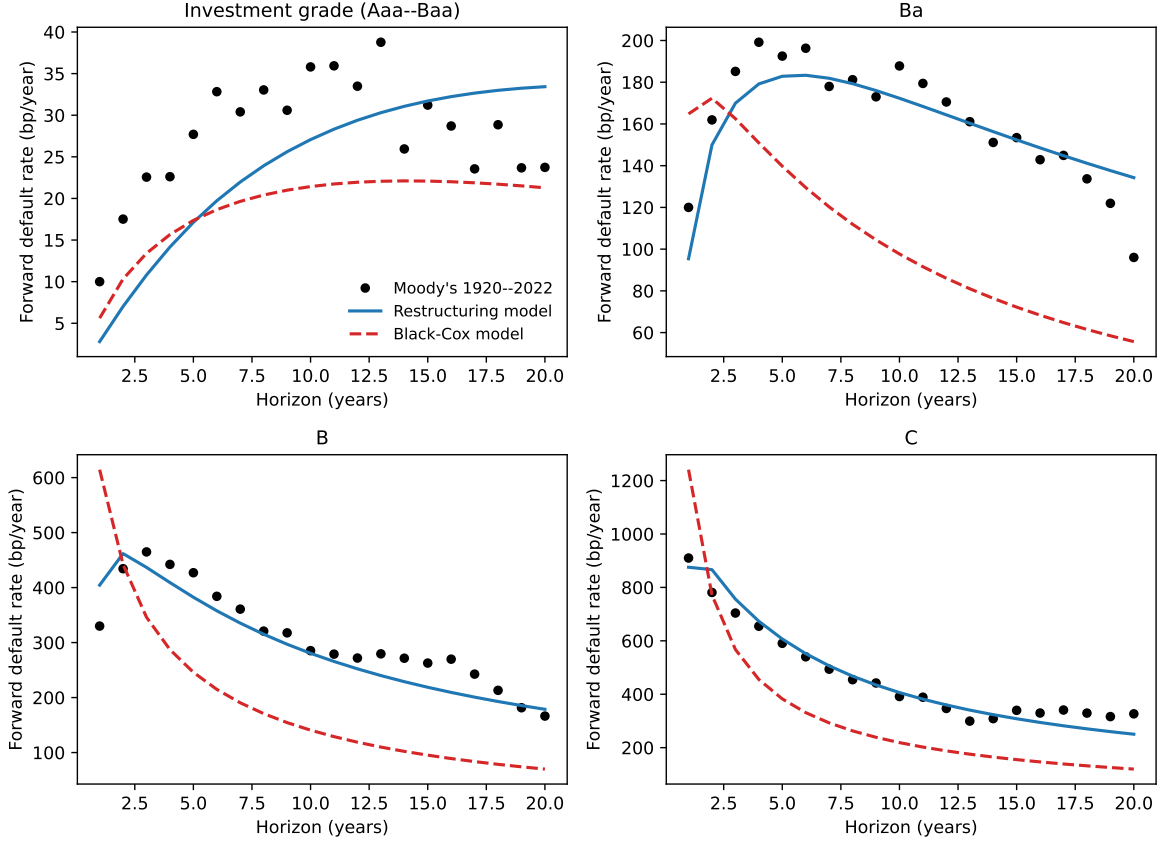


Figure 8. Moody's and Replicated Corporate Bond Yields

The figure compares Moody's published seasoned corporate bond yield indices with our replication from FISD and TRACE over July 2002 to September 2025. The top left panel shows the Aaa yield, the top right panel the Baa yield, and the bottom panel the Baa-Aaa spread. Replicated yields are equal-weighted averages of transaction yields of seasoned bonds with remaining maturity of at least 20 years and the corresponding Moody's bond rating, after removing, within each month and rating, yields more than 1.5 standard deviations from the monthly median. Correlations between the replicated and published series are 98.6% (Aaa), 99.0% (Baa), and 90.8% (spread).

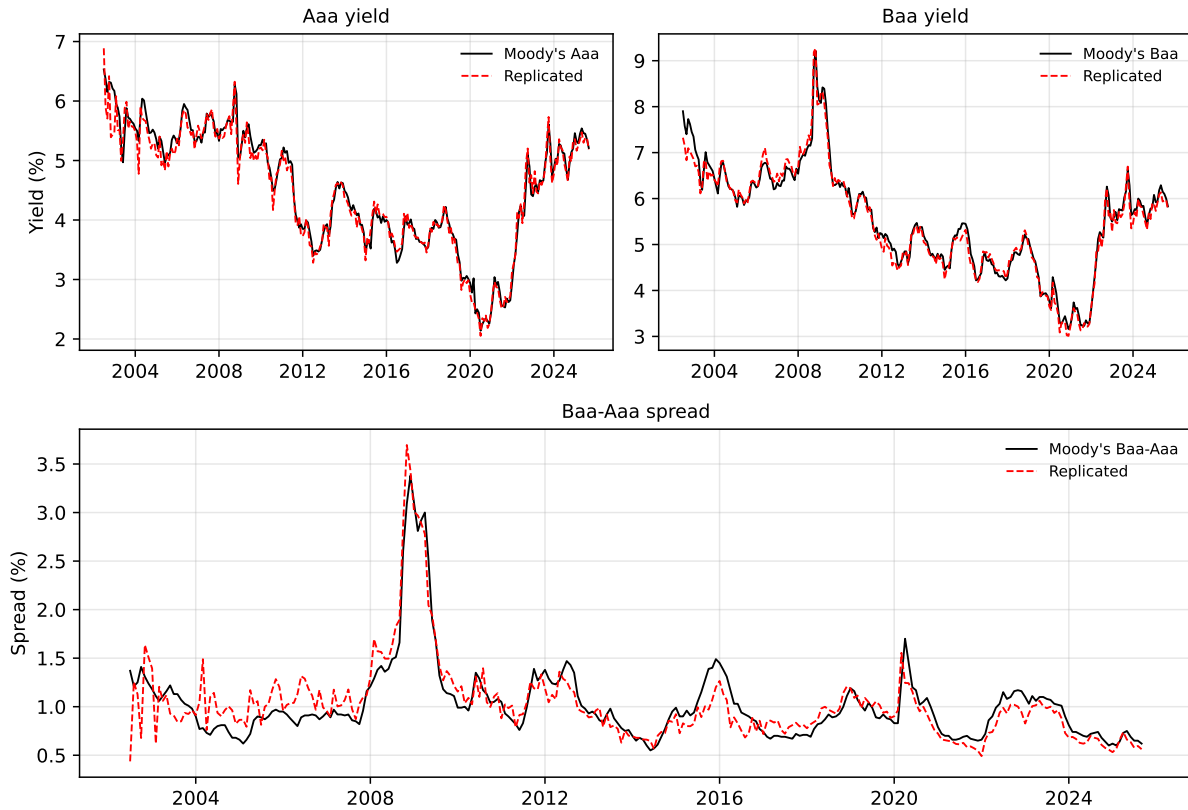


Table 4. Characteristics of Replicated Index Constituents

The table shows characteristics of the bond-month observations underlying the replicated Moody's Aaa and Baa yield indices over July 2002 to September 2025, after all filters. Yields are transaction yields from TRACE while maturity, age, coupon, and offering amount are from FISD.

	Mean	10th	Median	90th	Min	Max
Panel A: Aaa constituents (3,271 bond-months)						
Yield (%)	4.19	2.79	4.16	5.36	1.91	7.62
Remaining maturity (years)	26.7	21.5	26.0	34.8	20.0	40.5
Age (years)	4.9	1.4	4.7	8.7	0.5	10.6
Coupon (%)	4.20	2.80	4.00	5.30	2.25	8.72
Offering amount (\$ million)	1462	500	1000	3000	175	6250
Bonds per month	11.7	2.0	8.0	29.0	2.0	35.0
Number of unique bonds	58					
Panel B: Baa constituents (43,143 bond-months)						
Yield (%)	5.09	3.42	5.13	6.47	2.16	12.03
Remaining maturity (years)	27.3	21.5	25.9	29.5	20.0	99.5
Age (years)	5.0	1.2	4.5	8.8	0.5	58.9
Coupon (%)	5.27	3.80	5.00	7.20	2.12	10.50
Offering amount (\$ million)	923	250	600	1750	50	15000
Bonds per month	154.6	60.0	119.0	291.4	52.0	309.0
Number of unique bonds	823					

Table 5. Model-Implied Spreads From the Firm Panel and From the Index Constituents

The table compares model-implied spreads (in basis points) in the refinancing model computed from all Baa-rated and Aaa-rated firms in the firm panel with model-implied spreads computed from the constituents of the replicated Moody’s Baa and Aaa indices. In the firm panel, a generic 27-year semiannual coupon bond is priced for every Baa-rated and Aaa-rated firm-year, with the coupon set to the Moody’s Baa yield (Baa) or Aaa yield (Aaa) in the month of the fiscal year end, discounting at a flat curve at the 30-year Treasury yield, and spreads are averaged across firms within each year and then across years. For the constituents, every bond-month is priced at its actual coupon and remaining maturity on the [Gürkaynak, Sack, and Wright \(2007\)](#) Treasury curve, as in [Appendix E](#), and spreads are averaged as in the replication. Both calculations use a recovery rate of 33.6% and the parameter estimates of the refinancing model in [Table 2](#). The firm panel ends in 2023, so the last subperiod covers 2018 to 2023 for the firm panel and 2018 to 2025 for the constituents. The last row reports the correlation between the annual averages of the two series over 2002 to 2023.

	Baa spread		Aaa spread		Baa-Aaa spread	
	Firm panel	Constituents	Firm panel	Constituents	Firm panel	Constituents
2002–2009	127	116	32	2	95	114
2010–2017	145	148	28	37	117	110
2018–2025	181	178	17	36	164	142
2002–2025	148	147	27	25	122	122
Correlation	0.48		0.11		0.38	

Table 6. Moody’s Spreads and Model-Implied Spreads From Firm Panel 1953–2023

The table compares the average Moody’s Baa-Aaa spread and Aaa-Treasury spread (in basis points) with the spreads implied by the refinancing model. The Moody’s spreads are annual averages of the monthly Aaa and Baa yields, averaged across years. ‘Net of convenience yield’ subtracts the 46 bps liquidity component of the convenience yield on long-term Treasuries estimated by [Krishnamurthy and Vissing-Jorgensen \(2012\)](#). Model spreads price a generic 27-year semiannual coupon bond for every Baa-rated and Aaa-rated firm-year in the firm panel, with the coupon set to the Moody’s Baa or Aaa yield in the month of the fiscal year end, discounting at a flat curve at the long-term Treasury yield, a recovery rate of 33.6%, and the parameter estimates of the refinancing model in Table 2. Spreads are averaged across firms within each year and then across years. Brackets below the model spreads contain [Feldhütter and Schaefer \(2018\)](#) 95% confidence bands that reflect the sampling uncertainty of the historical default rates on which the model is estimated. One, two, and three asterisks mark a model spread whose 95%, 99%, and 99.9% confidence band does not contain the Moody’s spread, the spread net of the convenience yield in the case of the Aaa-Treasury spread. The last two columns report the number of Baa-rated and Aaa-rated firm-years in the period.

	Baa-Aaa spread		Aaa-Treasury spread			Firm-years	
	Moody’s	Model	Moody’s	Net of conv. yield	Model	Baa	Aaa
1953–1975	77	69 [49, 89]	47	1	8*** [6, 10]	502	146
1976–2000	110	92 [68, 117]	72	26	5*** [3, 7]	2,379	314
2001–2023	104	124* [105, 140]	87	41	26*** [21, 31]	4,322	108
1953–2023	97	95 [74, 115]	69	23	13*** [10, 16]	7,203	568

Table 7. Maturities, Spread Data, and Sample Periods in the Literature on Pricing Kernels and Credit Spreads

The table lists, for structural models of credit spreads with a pricing kernel calibrated to explain the level of spreads and for the constant-price-of-risk benchmarks, the maturity at which model-implied spreads are compared with data, the spread data used, and the sample period of those data. Maturities in parentheses are secondary comparisons.

Paper	Pricing kernel	Maturity	Spread data	Period
<i>Time-varying prices of risk</i>				
Cremers, Driessen, and Maenhout (2008)	Option-implied jump risk premia	10 years (5-year CDS; 1 to 11 years)	Lehman index Baa-Treasury from Huang and Huang (2012); Lehman indices; Longstaff, Mithal, and Neis (2005) CDS; Bloomberg bond prices	1973–93; 1983–2002; 2001–02; 1996–2002
David (2008)	Learning about inflation and earnings regimes	10 years	Moody’s Baa-Aaa	1960–2001
Chen, Collin-Dufresne, and Goldstein (2009)	Habit formation	4 years (10 years)	Moody’s composite Baa-Aaa; Lehman noncallable industrial Baa-Aaa, 2 to 7 years	1970–2001; 1985–95
Chen (2010)	Long-run risk	10 years	Duffee (1998) Baa-Treasury and Aaa-Treasury, industrial; Moody’s Baa-Aaa	1985–95; 1920–2009
Bhamra, Kuehn, and Strebulaev (2010)	Recursive preferences with regime switching	5 and 10 years	Baa-Aaa from Duffee (1998), Huang and Huang (2012), David (2008), and Davydenko and Strebulaev (2007)	1985–95 and others
Bhamra, Fisher, and Kuehn (2011)	Nominal debt and monetary policy	Perpetual	Moody’s Baa-Aaa	1919–2010
Gourio (2013)	Disaster risk, production economy	1 year	Moody’s Baa-Aaa	1947–2011
Kuehn and Schmid (2014)	Investment-based, recursive preferences	5 years (1 and 10 years)	CMA CDS; 10-year spreads from Huang and Huang (2012)	2004–11; 1973–93
Christoffersen, Du, and Elkamhi (2017)	Disaster risk with habit	5 years (1 to 10 years)	Markit CDS, Baa and Aaa/Aa	2001–14
Chen, Cui, He, and Milbradt (2018)	Long-run risk with bond illiquidity	10 years	FISD and TRACE bond spreads, Aaa/Aa to Ba	1994–2010
Seo and Wachter (2018)	Rare disasters	5 years (3 years)	CDX investment-grade index and tranches	2005–08
Du, Elkamhi, and Ericsson (2019)	Variance risk premium	4 and 10 years (1 to 10 years CDS)	10-year Baa-Aaa from Huang and Huang (2012); Markit CDS	1973–93; 2001–13
Bai, Goldstein, and Yang (2020)	Priced jump risk	1 to 20 years	Markit CDS; bond spreads from Feldhütter and Schaefer (2018)	2002–17; 1987–2012
Favilukis, Lin, and Zhao (2020)	Labor obligations, recursive preferences	10 years	Moody’s Baa-Aaa	1948–2014
Chen, Xu, and Yang (2021)	Regime switching with endogenous maturity	5 and 10 years	Markit CDS, 1, 5, and 10 years	2007–08
Gomes and Schmid (2021)	Production economy with default	8 years	Moody’s Baa-Aaa	1951–2014
<i>Constant price of risk and empirical benchmarks</i>				
Huang and Huang (2012)	Constant Sharpe ratio, several structural models	4 and 10 years	Lehman index spreads over Treasury; Duffee (1998) noncallable	1973–93; 1985–95
Culp, Nozawa, and Veronesi (2018)	Option-implied pseudo bonds	1 month to 2 years	SPX and single-stock put options; Lehman and TRACE bonds	1996–2015
Feldhütter and Schaefer (2018)	Constant Sharpe ratio, Black-Cox	3 to 20 years	Lehman and Merrill Lynch bond quotes	1987–2012
Huang, Nozawa, and Shi (2025)	Constant Sharpe ratio, Black-Cox and He-Milbradt	Portfolio averages of 4 to 17 years	Merrill Lynch bond prices, eight countries; Markit CDS	1997–2017; 2002–17
This paper	Constant Sharpe ratio, refinancing	27 years	Moody’s Baa-Aaa and Aaa-Treasury; reconstructed index constituents	1953–2023; 2002–25

Figure 9. The Long-Maturity Aaa-Treasury Spread Across Pricing Kernels, 1953 to 2023

The figure plots the average model-implied Aaa spread over Treasuries over 1953 to 2023 for the refinancing model and the five alternative pricing kernels of Section 5.4, each with its 95% confidence band, against the average Moody's Aaa-Treasury spread over 1953 to 2023 (solid line) and the same spread net of the 46 bps convenience yield on Treasuries (dashed line). Models, anchoring, pricing convention, and confidence bands are as in Figure 1.

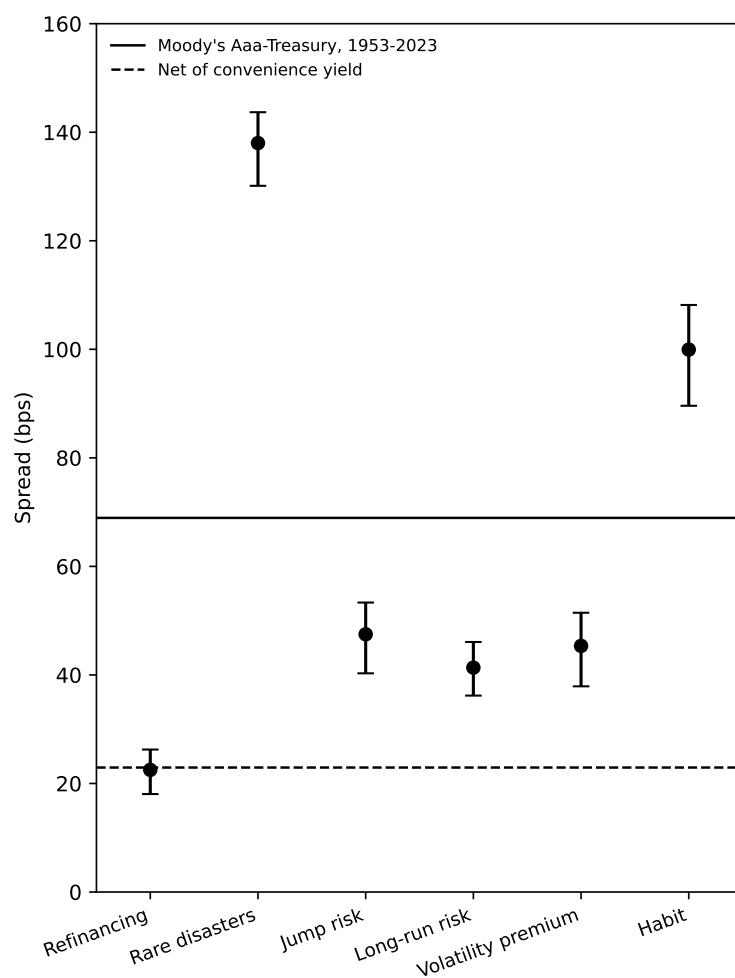


Figure A.1. The Long-Maturity Baa-Aaa Spread Across Pricing Kernels Under the Four Estimates of Table 2

The figure repeats Figure 1 for each of the four estimates of the refinancing model in Table 2: the benchmark estimate from default rates for cohorts formed in 1920 to 2022 (filled circles), and the alternative estimates from cohorts formed in 1970 to 2022 (squares), from 1920 to 2022 with observation weights (triangles), and from 1970 to 2022 with observation weights (diamonds). For each estimate every model, including the refinancing model, is evaluated for a representative Baa-rated and Aaa-rated firm whose physical 27-year default probability equals the refinancing model's firm-panel average over 1953 to 2023 at that estimate, 15.8% and 1.6% in the benchmark and 16.5% and 1.7% in the three alternatives, and prices the generic 27-year coupon bond in each year as in Figure 1. The solid line is the average Moody's Baa-Aaa spread over 1953 to 2023. Confidence bands scale the estimated default boundary by the 2.5% and 97.5% quantiles from the simulation of [Feldhütter and Schaefer \(2018\)](#), with every model re-anchored to the resulting firm-panel average default probabilities. The order of the models is that of Figure 1.

