

Market Dynamics of Risk-On and Risk-Off Incentives and Equilibrium Asset Prices

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Abstract

In today’s financial markets, the marginal investor is an asset manager. Compensation and career concerns generate an S-shaped objective over performance relative to the manager’s benchmark: convex below it, concave above it. We embed this objective in a dynamic equilibrium asset pricing model solved in closed form. Return predictability arises through two channels: incentive-driven hedging demand, a price effect producing reversal, and performance uncertainty, a quantity effect producing momentum. Their interaction yields momentum when the representative fund outperforms its benchmark and reversal when it underperforms—in both the time series and cross-sectional long-short portfolios. The predictability is fully rational, arising under complete information.

1 Introduction

Asset managers hold the majority of U.S. public equity and account for an even larger share of trading volume.¹ Given their central role in equity markets, the dominant marginal investor who determines prices in the U.S. stock market is an asset manager. Understanding its objectives and the incentives it faces is thus essential to understanding price dynamics.

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¹Institutional ownership of U.S. corporations rose from roughly 7–8% in 1950 to about 67% by 2010 (Blume and Keim, 2012). As of 2024, asset managers alone held about two-thirds (65%) of U.S. listed equity (OECD, 2025), while institutional ownership of large-capitalization indices such as the S&P 500 was higher still, on the order of 80%.

The rise of passive, index-tracking vehicles has made active managers even more dominant in determining price dynamics. These passive funds hold their portfolios largely irrespective of prices, and their presence withdraws shares from the traded float, which leaves price formation to active managers. Therefore, the mechanism we study is quantitatively more important over time as passive investing continues to grow.²

Since the 1990s, a central empirical regularity in the mutual fund literature has been the convex flow–performance relation: investors direct disproportionately large inflows to funds with strong past performance (Chevalier and Ellison (1997), Sirri and Tufano (1998)). This relation operates between investors and funds. It does not, however, identify how investment advisers compensate the portfolio managers who select fund holdings.

Evidence on this second layer of delegation has emerged from studies of realized compensation, which show that manager pay is a concave function of the fee revenue, or assets under management, attributable to the manager (Ibert, Kaniel, Van Nieuwerburgh, and Vestman (2018), Cen, Dou, Kogan, and Wu (2024)).³ Because outperformance raises assets under management, diminishing marginal compensation is especially relevant above the benchmark, when asset managers outperform. The convexity of investor flows therefore does not map mechanically into a convex managerial payoff. If anything, it points to a concave one above the benchmark.

Complementary evidence follows from the SEC’s 2005 requirement that funds disclose portfolio-manager compensation structures. Drawing on these disclosures, Ma, Tang, and Gómez (2019) show that performance-linked bonuses are widespread and asymmetric, tied to manager performance relative to a benchmark. Because such bonuses have a finite maximum, they remain flat when accumulated performance is well below the benchmark, rise as performance surpasses it, and flatten again once performance reaches the cap. On balance, this evidence implies a managerial payoff that is convex then concave (S-shaped) in benchmark-relative performance.⁴

The S-shape payoff evident in the compensation for flows and for performance is also consistent with the observed asymmetry in career outcomes and risk taking. Underperformance substantially increases the likelihood of demotion or termination, whereas outperformance

²Index mutual funds and index ETFs together held roughly 20% of U.S. equities at the end of 2025. This number has grown from 11% in 2015, (Investment Company Institute, 2026).

³Ibert et al. (2018) estimate a pay-revenue elasticity of roughly 0.15 in Swedish data, and Cen et al. (2024) estimate 0.12 for US managers. Cen et al. (2024) further show that fund flows affect total compensation primarily through their effect on fee revenue

⁴Ma et al. (2019) document that a subset of managers in their data report a maximum bonus opportunity. Bienz, Bonelli, Mjøs, and Santos (2026) provide more direct and comprehensive contract evidence: observing actual compensation contracts for Norwegian asset managers, they find that bonuses take the form of a bull spread—zero at low performance, increasing linearly over an intermediate range, and flat once performance is sufficiently above the benchmark.

produces no commensurate increase in the probability of promotion (Chevalier and Ellison (1999), Hu, Hall, and Harvey (2000)). Similarly, interim laggards take on risk relative to leaders, while managers with sufficiently large leads reduce risk to protect their advantage (Brown, Harlow, and Starks (1996), Chen, Hughson, and Stoughton (2025)). Together, these patterns support an effective objective that is S-shaped in benchmark-relative performance, with risk-on incentives below the benchmark and risk-off incentives above it.

The same S-shaped in the contractual incentive component appears in the compensation of internal managers at U.S. public pension funds and sovereign wealth funds. Public pension funds who disclose the compensation structure for their internal managers reveal that their internal managers are awarded for performance relative to predetermined benchmarks. For this component, the payout is zero below a threshold (typically set as the benchmark), increases with relative performance over an intermediate range, and is capped once performance reaches a specified maximum. Thus, the marginal sensitivity of compensation to relative performance rises at the threshold and falls to zero at the cap, producing a convex then concave payoff schedule.⁵

Related evidence from delegated pension management reinforces the asymmetric career-concern channel appearing in the mutual fund industry. Pension clients withdraw assets from poorly performing managers without disproportionately rewarding recent winners (Del Guercio and Tkac (2002)), while actual U.S. mandate terminations show that underperformance is an important reason plan sponsors dismiss external managers (Goyal and Wahal (2008)). More recently, (Nocera and Spotorno (2026)) find that negative active returns are associated with dismissal or non-renewal in Italian occupational-pension mandates, whereas positive active returns are not significantly associated with lower termination risk.

The manager’s effective incentives therefore combine direct compensation with career concerns and interim risk-taking, all of which depend on where cumulative performance

⁵Aon (2026), in its Radford McLagan Compensation Database, shows that 70% of US public pension funds with AUM of at least \$50 billion and 65% of funds with internal management offer their managers a performance base bonus. Aon states that the threshold for a bonus is set at the policy benchmark, and contains a cap: a maximum attainable bonus. Further, direct plan-level evidence is provided by California State Teachers’ Retirement System (2026) and the Teacher Retirement System of Texas (2025). Their plans specify benchmark-relative performance components, intermediate payout regions, and maximum awards. The equity holdings of their portfolio is benchmarked to the MSCI ACWI IMI, and the US equity portion in that benchmark is 63% as of June, 2026. Importantly, the US equity portion of MSCI ACWI IMI is almost fully correlated with US market index, which is our main focus. Related evidence from Norges Bank Investment Management (NBIM), which manages Norway’s Government Pension Fund Global, the largest sovereign wealth fund, establishes a similar payout structure. Investment managers receive a compensation based on the fund’s relative (to the benchmark) return, with a cap generally equal to 100% of fixed compensation and for some personnel the ceiling is 200%. The equity portfolio benchmark is FTSE Global All Cap Index, which is 62% as of June 2026. Similar to other world indices, the US equity portion is at near complete correlation with US market index, which is our main focus, see NBIM’s compensation principles and NBIM (2024) for further details .

stands relative to the benchmark. When cumulative returns fall short of the benchmark, the manager has strong incentives to catch up: continued underperformance erodes current compensation, damages reputation, and raises the likelihood of demotion or termination. When cumulative returns exceed the benchmark, the marginal value of further outperformance declines, while the value of protecting the accumulated lead rises. Together, these forces generate an S-shaped objective over cumulative performance relative to the benchmark.

Importantly, these incentive structures give rise to the same S-shaped objectives across a substantial volume of assets. Mutual funds and pension funds—among the largest actively managed vehicles in U.S. markets—typically tie manager compensation to performance against a broad market benchmark. Therefore, the dominant marginal investor determining prices in the financial markets is an asset manager with an S-shape objective.

Our main contribution is to study the asset allocation and general-equilibrium consequences of asset management frictions—‘risk-on’ during underperformance and ‘risk-off’ during outperformance relative to the benchmark performance—consequences that the literature has largely left unexplored. We show that these frictions offer a potential explanation for the momentum and reversal patterns observed in asset markets, both in the time series of individual assets and in the cross-section, through long-short strategies spanning multiple assets. Importantly, momentum and reversal arise here purely as equilibrium outcomes under symmetric, full information, unlike the leading mechanisms in the literature, which generate these patterns through asymmetric or incomplete information.

The analysis examines deviations of the representative fund from the market index. This is a first-order effect: the performance of funds accumulate at the aggregate level. It does not average out to the market return. Consequently, the representative fund exhibits a striking and previously undocumented stylized fact—the weighted-average return of all mutual funds deviates substantially from the S&P 500 in many months, see Figure 1.⁶ The benchmarks that active equity mutual funds use are highly concentrated. As of June 2025, the S&P 500 is by far the most widely used benchmark with roughly 60% of active equity mutual funds assets under management benchmarked to it. When combined with the Russell 1000 and Russell 3000 benchmarks, the figure climbs to 75%.⁷

Importantly, the evidence is indicative of a broader set of benchmarked managers: U.S. public pension funds and sovereign wealth funds award their internal managers under the

⁶Figure 1 shows that the value-weighted return deviates substantially from the S&P 500 return in many months, suggesting that larger funds tend to move together relative to the benchmark during those months. Because these larger funds exert a greater price impact, a representative-fund analysis offers a reasonable approximation.

⁷The percentage of assets under management (AUM) are computed from Morningstar Direct snapshot data. Total active mutual fund AUM is the combined AUM of US category groups belonging to U.S. Equity and Sector Equity.

same incentives plan as mutual fund managers, which produces the same S-shape objective with the benchmark as the inflection point. Their equity benchmarks carry a weight of roughly 62–63% for U.S. equity holdings, which is almost entirely correlated with broad U.S. equity benchmarks.

The model predicts momentum—both in the cross-section (long-short portfolios) and in the time series—when the representative fund outperforms its benchmark, and reversal—again in both the cross-section and the time series—when it underperforms the benchmark. To the best of our knowledge, this is the only framework that endogenously generates momentum following outperformance and reversal following underperformance in both the time series and the cross-section.⁸ In what follows, we omit “representative” and refer simply to “fund” or “asset manager” where doing so causes no confusion.

Given their time-varying incentives (to earn their bonus, keep their job, and demonstrate sustained performance), asset managers’ payoffs are convex when they underperform the benchmark but become concave as their returns exceed the benchmark. This results in an S-shaped objective similar to prospect theory, but the inflection (or reference) point is the benchmark rather than absolute wealth. Despite this added complexity, we characterize equilibrium prices in closed form.

The introduction of an S-shaped objective and its pricing implications is novel in the context of asset management.⁹ Originally, such objective functions were developed to explain seemingly paradoxical financial behaviors observed in households.¹⁰ The key distinction of our model is that the benchmark, not the value of assets under management, defines the reference point. Unlike this previous literature, the inflection point in the asset manager’s objective is disclosed in their contract, which is readily available.

Predictable asset returns are generally surprising because conventional wisdom suggests

⁸Models of over- and underreaction, such as Barberis, Shleifer, and Vishny (1998), also generate momentum and reversal. Models of slow-moving capital, such as Vayanos and Woolley (2013), do as well. The key distinction is the particular asymmetry conditional on performance relative to a benchmark.

⁹Our benchmark-as-reference-point formulation differs from Basak and Pavlova (2013), who likewise place a benchmarked manager at the margin in equilibrium. However, their specification induces a symmetric benchmark incentive response on either side of the benchmark point. Our S-shaped objective instead makes the intensity of incentives asymmetric: convex (risk-on) below the benchmark and concave (risk-off) above it. This asymmetry is what generates the momentum—reversal asymmetry at the core of our results.

¹⁰For example, Friedman and Savage (1948) show that for an agent to simultaneously purchase insurance and lottery tickets, their utility must be concave over certain wealth levels and convex over others. Levy (1969) demonstrates that an objective function dependent on the first three moments of the wealth’s distribution leads to an inverted S-shaped objective, concave below and convex above a reference point. Kahneman and Tversky (1979) introduce an S-shaped value function capturing differing risk attitudes toward gains and losses, where current wealth is the reference. More recently, Aristidou, Giga, Lee, and Zapatero (2025) developed a model of aspirational utility—where social interactions create target wealth levels that generate a utility jump if reached. This jump introduces a convex segment to the utility function, making individuals risk-seekers in the presence of aspirations.

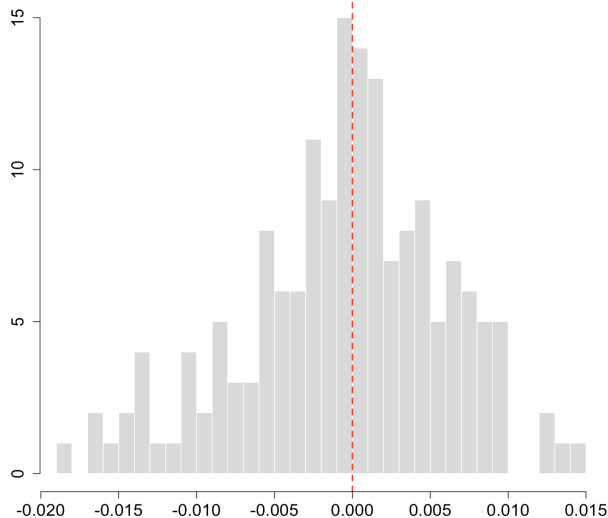


Figure 1. The figure presents a histogram of the monthly return differences between the representative mutual fund and the S&P 500: the weighted-average monthly return of all mutual funds minus the S&P 500 monthly return. It documents substantial dispersion in relative performance. In the top quartile of months, the representative mutual fund outperforms the S&P 500 by more than 0.39%. This outperformance corresponds to approximately 49% of the S&P 500’s average monthly return. In the bottom quartile of months, the representative mutual fund underperforms the index by less than -0.42%. This underperformance corresponds to approximately -53% of the S&P 500 average monthly return. The distribution is also left-skewed: the most negative monthly return difference, is -1.87%, compared with a largest positive monthly difference of +1.42%. The S&P 500 index monthly average is 0.8%. The figure uses active U.S. equity mutual funds, but the evidence is indicative of a broader set of benchmarked managers. A large proportion of U.S. public pension funds and sovereign wealth funds award their internal managers under the same contractual bonus plan as mutual fund managers, producing the same S-shaped objective with the benchmark as an inflection point. The U.S. equity component of their equity benchmarks — roughly 62–63% of those benchmarks — has a monthly return correlation of near one with the S&P 500.

that if market participants expect prices to rise, they will immediately buy the asset to capture the anticipated return—causing an instantaneous price increase that eliminates the expected gain. Similarly, if they anticipate a decline, they will sell immediately to avoid losses, triggering an instant price drop that neutralizes the expected fall. Consequently, traditional explanations for momentum and reversal typically invoke either asymmetric information (leading to gradual price adjustments) or irrational investor behavior. In both cases, these patterns emerge because market participants gradually update their positions as new information diffuses to everyone.

In contrast, the return predictability examined in this paper is fully rational and derives directly from no-arbitrage conditions, assuming all available information is instantaneously and completely reflected in prices. Predictability persists precisely because it is not optimal

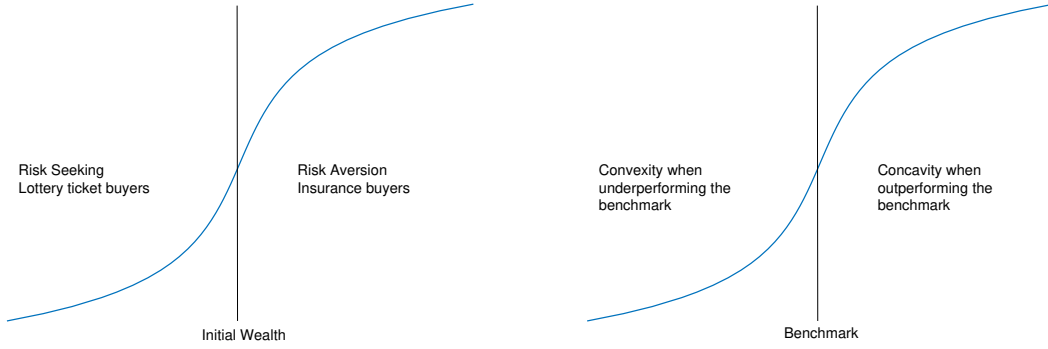


Figure 2. The figure on the left illustrates that an agent with a convex utility function over absolute wealth prefers engaging in a gamble, such as buying a lottery ticket. Conversely, when the utility function is concave, the agent is willing to pay a premium to avoid such a gamble. The figure on the right conveys a similar intuition for the asset manager: they are incentivized to take on risk and “gamble” when their wealth falls below the benchmark and they are underperforming, whereas they tend to seek insurance and preserve their wealth when it exceeds the benchmark.

for the representative asset manager to trade aggressively on it. While the representative asset manager responds promptly to information, its trading intensity depends on its accumulated performance relative to the benchmark. These performance-dependent responses generate return predictability patterns endogenously through equilibrium dynamics.

A single state variable determines equilibrium and drives predictability in asset markets: the ratio of accumulated market news to accumulated benchmark news. This state variable governs the likelihood that the representative fund will ultimately outperform or underperform.

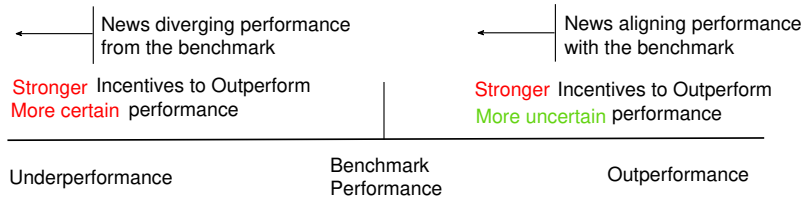


Figure 3. This figure illustrates the reactions to news during periods of underperformance and outperformance. The left side depicts scenarios where the asset manager underperforms, while the right side represents cases of outperformance. In both cases, lower accumulated performance intensifies incentives for managers to strive to outperform the benchmark. Conversely, news that aligns the accumulated performance of asset managers with the benchmark reduces outperformance for those ahead of the benchmark and worsens underperformance for those lagging behind. As a result, uncertainty of performance rises when accumulated performance converges toward the benchmark and declines when it diverges.

There are two equilibrium channels through which asset returns become predictable. The first is a price impact channel. The drive to outperform the benchmark prompts the representative asset manager to maintain a hedging position to avoid lagging behind. This hedging behavior increases demand for benchmark assets, independent of traditional risk and return considerations. Consequently, this incentive-driven demand raises the price of benchmark assets while depressing the prices of assets outside the benchmark. An unpredictable shock that worsens accumulated performance relative to the benchmark initially increases the benchmark’s price and lowers the prices of assets outside the benchmark (i.e., active assets) due to heightened demand for the benchmark. However, this shock subsequently reduces the benchmark’s expected return and increases the expected returns of the active asset, as the market anticipates stronger future price reactions to shocks when incentives to outperform the benchmark are intensified. Thus, the incentives channel ultimately leads to asset price reversals. This incentives channel—highlighted in red as “Stronger Incentives to Outperform” in Figure 3—produces symmetric reversal effects in both underperformance and outperformance regions, regardless of shock type.

The second is a quantity-effect channel. Performance uncertainty increases the sensitivity of assets to news. From an economics perspective, the representative asset manager’s portfolio is not perfectly aligned with the benchmark, holding both active and passive assets. Therefore, when news arrives that brings the portfolio’s accumulated performance closer to the benchmark, it raises performance uncertainty. This, in equilibrium, results in riskier asset markets.

When the representative asset manager outperforms, an unpredictable shock that worsens accumulated performance relative to the benchmark initially raises the benchmark’s price and depresses the prices of assets outside the benchmark. This shock subsequently increases the expected return of the benchmark asset and decreases the expected return of assets outside the benchmark, as the market anticipates greater uncertainty of performance when the asset manager’s accumulated performance aligns more closely with the benchmark. This uncertainty of performance channel thus drives momentum when the representative asset manager outperforms the benchmark.

In contrast, when the asset manager underperforms, an unpredictable shock that worsens accumulated performance relative to the benchmark increases the expected return of the benchmark asset and decreases the expected return of assets outside the benchmark, as the market anticipates lower uncertainty of performance when the representative asset manager’s accumulated performance diverges further from the benchmark. This uncertainty of performance channel thus drives reversal when the representative asset manager trails the benchmark.

The uncertainty-of-performance channel—highlighted in red as “More Certain Performance” below the benchmark and in green as “More Uncertain Performance” above the benchmark. As illustrated in Figure 3, it produces asymmetric return predictability: reversal following underperformance and momentum following outperformance, irrespective of shock type.

The concavity above the benchmark causes the quantity effect to dominate and induce momentum in asset returns when the performance of the representative asset manager exceeds the benchmark. In contrast, the convexity below the benchmark causes the price effect to dominate and induce reversal in asset markets when the performance of the representative asset manager falls below the benchmark. Since the benchmark asset and the active asset move opposite directions due to news, the time-series predictability appears also in the long-short portfolio, of the benchmark and active assets.

The remainder of the paper is organized as follows. Section 2 summarizes the related literature; Section 3 sets up the economy with the time-varying incentives to outperform and maintaining accumulated performance; Section 4 solves for the equilibrium in closed-form; Section 5 discusses momentum and reversal in the time-series; Section 6 discusses momentum and reversal in the cross-section; and Section 7 concludes.

2 Related Literature

This paper contributes to the asset pricing literature that investigates predictable patterns in stock returns, focusing in particular on the drivers of momentum and reversal. Jegadeesh and Titman (1993) are the first to document momentum and reversal effects in the cross section of the stock market, while Chan, Jegadeesh, and Lakonishok (1996) provide a thorough investigation of the phenomenon. They show that a portfolio that buys past winners and sells past losers generates significant positive returns over a short-term window of one year or less, with the strategy’s performance partially reversing over longer horizons.

There are several competing theories explaining momentum, which can be broadly categorized into a few key groups. One of the most prevalent explanations is behavioral. Within this category, Barberis et al. (1998) examine an agent who, due to behavioral biases, forms incorrect beliefs about stock earnings. Daniel, Hirshleifer, and Subrahmanyam (1998) analyze investors who overreact to confirming evidence while underreacting to refuting evidence regarding their prior signals. Hong and Stein (1999) propose a two-agent model in which agents face different constraints on how they process information. Barberis and Shleifer (2003) study an economy with two types of investors: one who groups assets into styles and progressively reallocates funds from the worst-performing style to the best-performing style,

and another who does not respond to this predictable pattern. Finally, Hillert, Jacobs, and Müller (2014) find that news coverage intensifies momentum by amplifying investors’ biases.

The most prevalent rational explanation for momentum is based on information acquisition. Holden and Subrahmanyam (2002) consider a model featuring sequential information acquisition that is reflected in prices. Allen, Morris, and Shin (2006) propose a framework in which agents forecast future forecasts rather than prices directly. Shin (2006) endogenizes firms’ disclosure decisions jointly with prices. Banerjee, Kaniel, and Kremer (2009) study a heterogeneous beliefs economy where agents agree to disagree about their beliefs. Biais, Bossaerts, and Spatt (2010) analyze an overlapping generations model comprising informed and uninformed traders. Cespa and Vives (2012) compare consensus opinions and prices, demonstrating that specific patterns of information flow generate both momentum and reversal. Albuquerque and Miao (2014) show that information about future earnings, independent of current earnings, induces momentum and reversal. Ottaviani and Sørensen (2015) explore a heterogeneous beliefs economy where traders face wealth constraints. Andrei and Cujean (2017) demonstrate that short-term momentum can arise in an economy where private information arrives at an increasing rate. Finally, Cujean and Hasler (2017) investigate an economy with heterogeneous agents, some of whom update their beliefs continuously while others do so intermittently.

More recent studies link the asset management industry to momentum and reversal, making this strand of literature particularly relevant to this paper. Dasgupta, Prat, and Verardo (2011) assume that investors observe the asset manager’s actions and form beliefs about its ability to perform. The asset manager, internalizing the reputational costs of underperformance, engages in herding behavior that positively predicts short-term returns. Vayanos and Woolley (2013) suggest that the slow-moving flow of capital between active and passive funds, driven by active funds’ performance, might be a key driver of momentum and reversal.

Our paper is the first to demonstrate that a predictable pattern in the representative asset manager’s risk preferences generates momentum following outperformance of the benchmark and reversal following underperformance—both in the cross-section and the time series. We show that these predictable changes in risk preferences stem from the time-varying nature of its incentive contracts. Specifically, the representative asset manager who underperforms face stronger incentives to outperform the benchmark. However, as performance improves, the incentives to further exceed the benchmark diminish, and instead, the representative asset manager shifts its focus toward maintaining their performance to avoid losing their bonus.

Furthermore, our article contributes to the growing literature examining the impact of

professional asset management on asset prices. In particular, our research aligns with a strand of this literature that explores the asset pricing implications of benchmarking incentives.

The paper most closely related to ours is Sotes-Paladino and Zapatero (2019). Their model introduces a kink in the asset manager’s performance-based fees, which alters the sensitivity of the manager’s objective to wealth relative to the benchmark. Similar to our approach, the kink represents a shift in performance sensitivity between the end-of-period underperformance and outperformance regions. While this kink introduces convexity to the asset manager’s objective, the slope in the outperformance region can be concave, a feature consistent with our setup. They find that the asset manager overinvests in assets outside the benchmark and underinvest in the benchmark, which aligns with our predictions for periods when the asset manager is likely to outperform (the risk-off region). The primary distinction between their work and ours is that they analyze asset manager behavior given prices in a partial equilibrium framework, whereas our focus is on the pricing implications within an equilibrium setting.

By and large, the asset management literature assumes that the asset manager’s objective is always convex and does not become concave when outperformance is likely, as part of maintaining performance. The idea of embedding the benchmark into the asset manager’s objective was initiated by Brennan (1993) and Gómez and Zapatero (2003). Cuoco and Kaniel (2011) were the first to analyze the equilibrium implications of benchmarking incentives through performance-based fees. Subsequently, Basak and Pavlova (2013) and Basak and Pavlova (2016) introduced a reduced-form approach to incorporate the benchmark, allowing for greater tractability. More recently, Buffa and Hodor (2023) extended this framework to study the pricing implications of heterogeneous benchmarking incentives, and Hodor and Zapatero (2026) examined the pricing effects of heterogeneous investment horizons.

Unlike previous literature, we investigate the pricing implications of time-varying benchmarking incentives for the asset manager. While the asset manager has strong incentives to outperform the benchmark when underperformance is likely (consistent with existing literature), these incentives weaken and eventually reverse when outperformance becomes more likely, as the focus shifts from outperforming the benchmark to preserving performance.

Lastly, our paper relates to the early work of Friedman and Savage (1948), Levy (1969), Kahneman and Tversky (1979), and Kahneman and Tversky (1991) who examine a scenario in which the curvature of the objective function changes from convex below a reference wealth level to concave above a reference wealth level. We incorporate the key insights of these papers into an asset management framework with relative wealth concerns. Specifically, when the representative asset manager expects its wealth to fall below the benchmark index,

it behaves like poor individuals and exhibits risk-on incentives—akin to purchasing lottery tickets—with convex preferences that increases its marginal utility of wealth the further their wealth declines relative to the index. Conversely, when its wealth is expected to exceed the index, the representative asset manager behaves like a wealthy individual and faces risk-off incentives—similar to purchasing insurance—with concave preferences that reduces their marginal utility of wealth as their wealth increases above the index. More broadly, this paper fits into the reference-dependence literature, which originated with Rabin (2000) and Kőszegi and Rabin (2006) and was expanded more recently by DellaVigna and Pope (2017), Pagel (2017), and Khaw, Li, and Woodford (2020).

3 The Economy

This section lays out a simple and tractable model to study the joint equilibrium effect of risk-on and risk-off incentives of the representative asset manager. We consider a simple and tractable standard pure-exchange finite horizon economy in which time t is continuous and goes from zero to T . The economy is populated by two investors: a representative passive investor, $\mathcal{P}$, and a representative asset manager, $\mathcal{A}$.

While the economy could be generalized and accommodate multiple assets and different specifications of the index against which the representative asset manager’s accumulated performance is evaluated, the main results do not qualitatively change. Accordingly, uncertainty is driven by a (3)-dimensional Brownian motion $\mathbf{Z} = (Z_1, Z_2, Z_3)'$.

3.1 Investment Opportunities

Our economy has 3 risky assets and a single riskless bond. We set the bond as numeraire and normalize its interest rate to zero. The risky asset is denoted by S_{kt} , $k = 1, 2, 3$, and represents a claim on the dividends D_{kT} arriving at time T . We assume each risky asset is in unit supply and posit that it follows

$$dS_{kt} = S_{kt} (\mu_{kt}^S dt + \boldsymbol{\sigma}_{kt}^{S'} d\mathbf{Z}_t). \quad (1)$$

Prices (S_{kt}), the (instantaneous) vector of expected returns $\boldsymbol{\mu}_t^S \equiv (\mu_{1t}^S, \mu_{2t}^S, \mu_{3t}^S)'$, and the (instantaneous) volatility matrix $\boldsymbol{\Sigma}_t^S \equiv (\boldsymbol{\sigma}_{1t}^S, \boldsymbol{\sigma}_{2t}^S, \boldsymbol{\sigma}_{3t}^S)'$ are endogenous and determined in equilibrium. The bold symbols represent vectors and the $'$ represents the transpose throughout the analysis.

The dividends (D_{kT}) are determined by the dynamics of the processes

$$dD_{kt} = D_{kt} (\mu_k dt + \boldsymbol{\sigma}_k^{D'} d\mathbf{Z}_t), \quad D_{k0} > 0, \quad (2)$$

which we refer to as dividend news. The parameter μ_k and the vector $\boldsymbol{\sigma}_k^D$ are constants for the first two dividends. We follow Menzly, Santos, and Veronesi (2004), Basak and Pavlova (2013), and Buffa and Hodor (2023) and leave the remaining dividend news process unspecified, and, instead, we specify an aggregate news process as discussed below. The procedure substantially improves the tractability of the model and provides closed-form precise equilibrium characterization.

We assume that stocks' fundamentals are independent, implying that the vector $\boldsymbol{\sigma}_k^D$ (2) has a positive k th element equaling $\sigma_k > 0$, while the remaining entries are zero. This assumption provides clear interpretation to the type of shocks that affect prices. We assume that the distributional properties of news are the same, $\sigma_1 = \sigma_2 = \sigma_3$ and $\mu_1 = \mu_2 = \mu_3$.

3.2 The Index

We assume that the first asset is the index.

$$S_t^I \equiv S_{1t}. \quad (3)$$

The asset manager's accumulated performance is evaluated against the performance of this index and the passive investor invests its entire wealth in it.

The benchmark asset is a sufficient statistic for the benchmark payoff. Since the model studies incentives generated by relative performance, the internal composition of the index is secondary. The one-asset benchmark captures the aggregate benchmark risk, while the remaining asset captures the risk unrelated to the benchmark. We refer to the first as the benchmark asset (or index asset) and the second as the active asset (or non-index asset). In reality, any asset is a linear combination of the two securities in this economy.¹¹

For consistency of the analysis, we refer to I_t as the *index news*, which equals the news about the first asset,

$$I_t \equiv D_{1t}, \quad \boldsymbol{\sigma}^I \equiv \boldsymbol{\sigma}_1^D. \quad (4)$$

¹¹For example, one can identify the two components by conducting the factor decomposition, $R_i = \beta_i R_I + \varepsilon_i$. The first component is attributable the index asset and the second to the active asset.

3.3 Investors

The passive investors ($\mathcal{P}$) are endowed with $(1 - \lambda)$ units of the index at time 0

$$W_0^{\mathcal{P}} = (1 - \lambda) S_0^I, \quad (5)$$

and remain passive until time T .¹² The asset manager ($\mathcal{A}$) is endowed with the remaining (λ) units of the index and holds the entire supply of the other assets,

$$W_0^{\mathcal{A}} = \lambda S_0^I + S_{20} + S_{30} \equiv S_0^{\mathcal{A}}. \quad (6)$$

The asset manager's initial endowment identifies the active portion of the asset market, which is a claim on the total asset market dividends minus the passive index holdings' dividends,

$$V_T \equiv I_{1T} + D_{2T} + D_{3T} - (1 - \lambda) I_{1T}, \quad (7)$$

where we assume that this terminal value is determined by the process

$$dV_t = V_t (\eta dt + \boldsymbol{\nu}' d\mathbf{Z}_t), \quad V_0 > 0. \quad (8)$$

The parameter η is positive and the vector $\boldsymbol{\nu} \equiv (\nu_1, \nu_2, \nu_3)'$ has positive entries. We assume that $(\nu_1, \nu_2, \nu_3) = (\lambda\sigma_1/3, \sigma_2/3, \sigma_3/3)$ to mimic the aggregate active dividends and following the standard in the literature. We refer to V_t as the *market news*.

Remark 1 (Representative Asset Manager). *Figure 1 reveals a clear stylized fact: the representative fund tends to either outperform or underperform the market index. The figure demonstrates that the value-weighted return deviates significantly from the S&P 500 return in many months, which implies that larger funds tend to move in the same direction relative to the benchmark. Since these larger funds exert a greater price impact, a representative-fund analysis provides a reasonable approximation. This simplification does not change whether the whole sector is ahead of or behind the benchmark: the wealth-weighted aggregate that moves prices have the same sign to the sector's net position relative to the benchmark. Heterogeneity therefore cannot turn momentum following aggregate outperformance into reversal, or vice versa. We leave the cross-sectional dispersion across funds for future investigation.*

Remark 2 (Retail Investor). *Retail investors—those without benchmarking incentives—would not qualitatively change the predictability patterns generated by the model in this paper.*

¹²The size of the passive investor, $1 - \lambda$, does not change the predictions qualitatively and can be set to zero.

The logic is as follows. A shock that induces the asset manager to increase benchmark holdings creates additional demand, which is met through price adjustment—even with retail investors present—since rational retail investors refuse to sell at current equilibrium prices. These price movements align in direction with those in the economy absent retail investors, yielding identical predictability patterns as in our model without them. Since our focus is on qualitatively identifying novel predictability channels, we defer the inclusion of retail investors to future research.

3.4 Risk-On and Risk-Off Incentives

The effective incentives of asset managers reflect both compensation and career concerns, and each responds to the difference between accumulated performance and the benchmark. When performance lags the benchmark, the pull to catch up is strong, because continued underperformance reduce pay and reputation and heighten the threat of demotion or termination. When performance runs ahead of the benchmark, the payoff to pushing performance further shrinks, and the priority shifts toward defending the outperformance lead. This is evident in US mutual fund managers compensation (Ma et al. (2019)), in their pay for increased flow (Ibert et al. (2018), Cen et al. (2024)), and their career concerns (Chevalier and Ellison (1999), Hu et al. (2000)), and their risk taking activities (Brown et al. (1996), Chen et al. (2025)).

In a complete-market equilibrium, prices are determined by the marginal utility of wealth U_W . Therefore, the incentives of the asset manager affect prices only through how they shift the marginal utility of wealth. These changes are captured by the cross derivative U_{WS^I} .

When the manager is behind ($W^A < S^I$), the incentive to outperform the benchmark implies that, when the benchmark increases, it places greater value on an additional unit of wealth, because a wider underperformance gap makes catching up more valuable. That is, the marginal utility of wealth is increasing in the benchmark.

$$U_{WS^I} > 0. \tag{9}$$

In contrast, when the manager is ahead ($W^A > S^I$), it seeks to lock in the win and maintain performance. Its marginal utility of wealth is therefore highest in low-benchmark states, where it remains ahead and an extra dollar does the most to cement the outperformance. That is, the marginal utility of wealth is decreasing in the benchmark,

$$U_{WS^I} < 0. \tag{10}$$

As wealth falls toward the benchmark and the lead narrows, this preservation motive weakens and the cross derivative becomes less negative. Once the manager falls behind, the incentive reverses, and catching up again becomes valuable. The cross derivative thus switches sign at $(W^A = S^I)$,

$$U_{WS^I} > 0 \text{ when } W^A < S^I, \quad U_{WS^I} < 0 \text{ when } W^A > S^I. \quad (11)$$

Our specification is designed to be the smallest departure from the benchmarking-incentive objective in Basak and Pavlova (2013). Their model captures the incentive to outperform the benchmark in a reduced-form objective in which the manager's marginal utility of wealth is increasing in the benchmark. However, their reduced form specification does not model the maintaining outperformance part of the asset manager's objective. Therefore, our goal is to utilise the reduced-form approach of Basak and Pavlova (2013) while allow the sign of the cross derivative to switch with relative accumulated performance. The objective below is a parsimonious functional form that delivers exactly this sign switch while preserving monotonicity, concavity, Inada conditions, and analytical tractability.

We start with the risk-on component

$$U^{\text{On}}(W_T^A, S_T^I) = -\frac{(S_T^I)^{1-\gamma_1} (W_T^A)^{1-\gamma_2}}{(\gamma_2 - 1)(1 - \gamma_1)}, \quad \gamma_2 > 1 > \gamma_1 > 0. \quad (12)$$

This component satisfies

$$U_{WS^I}^{\text{On}} = (S_T^I)^{-\gamma_1} (W_T^A)^{-\gamma_2} > 0. \quad (13)$$

As a result, this incentives captures the asset manager risk seeking over performance relative to the benchmark. The risk-on component (12) nests the Basak and Pavlova (2013) benchmarking objective. The risk-on marginal utility (13) converges to the expression $\frac{S_T^I}{W_T^A}$, as $\gamma_1 \rightarrow 0$ and $\gamma_2 \rightarrow 1$, which is the marginal utility generated by the benchmark term in Basak and Pavlova (2013), $S_T^I \log W_T^A$.

Next, to capture the risk-off region, we introduce the symmetric counterpart

$$U^{\text{Off}}(W_T^A, S_T^I) = \frac{(W_T^A)^{1-\gamma_1} (S_T^I)^{1-\gamma_2}}{(\gamma_2 - 1)(1 - \gamma_1)}, \quad \gamma_2 > 1 > \gamma_1 > 0. \quad (14)$$

This component satisfies

$$U_{WS^I}^{\text{Off}} = - (S_T^I)^{-\gamma_2} (W_T^A)^{-\gamma_1} < 0. \quad (15)$$

In this region, the objective therefore encodes risk aversion over performance relative to the benchmark.

Combining the two components gives the asset manager's objective:

$$U(W_T^A, S_T^I) \equiv E \left[\mathbf{1}_{\text{Off}} \frac{(S_T^I)^{1-\gamma_2} (W_T^A)^{1-\gamma_1}}{(\gamma_2-1)(1-\gamma_1)} - \mathbf{1}_{\text{On}} \frac{(S_T^I)^{1-\gamma_1} (W_T^A)^{1-\gamma_2}}{(\gamma_2-1)(1-\gamma_1)} \right], \quad \gamma_2 > 1 > \gamma_1 > 0. \quad (16)$$

The indicators $\mathbf{1}_{\text{Off}}$ and $\mathbf{1}_{\text{On}}$ allow us to separate the asset-pricing implications of each incentive individually and their joint effect. A fully risk-on asset manager has only the second component, ($\mathbf{1}_{\text{Off}} = 0$, $\mathbf{1}_{\text{On}} = 1$), while a fully risk-off asset manager has only the first component, ($\mathbf{1}_{\text{Off}} = 1$, $\mathbf{1}_{\text{On}} = 0$). When both components are present, ($\mathbf{1}_{\text{Off}} = 1$, $\mathbf{1}_{\text{On}} = 1$), the relative strength of the two incentives depends on whether the asset manager's wealth is below or above the benchmark. Figure 4 plots the objective function on the wealth-benchmark plane. It illustrates the joint incentives and how the incentives change with performance relative to the benchmark.

In the joint case, the cross-partial derivative is

$$U_{WS^I} = (S_T^I)^{-\gamma_2} (W_T^A)^{-\gamma_2} \left[(S_T^I)^{\gamma_2-\gamma_1} - (W_T^A)^{\gamma_2-\gamma_1} \right]. \quad (17)$$

Since $\gamma_2 > \gamma_1$, it follows that (11) is satisfied: the benchmark is the reference point at which the manager's incentives switch from risk-on to risk-off.

Lastly, the asset manager's objective (16) remains increasing and concave in wealth:

$$U_W > 0, \quad U_{WW} < 0, \quad (18)$$

and it satisfies the Inada conditions:

$$\lim_{W \downarrow 0} U_W = \infty, \quad \lim_{W \uparrow \infty} U_W = 0. \quad (19)$$

The purpose of the functional form of the objective (16) is therefore to preserve the Basak and Pavlova (2013) objective while adding the minimal structure needed to study risk-on and risk-off incentives in a tractable equilibrium model. First, it adds a symmetric risk-off component to their risk-on benchmarking motive. Second, it replaces log utility with its CRRA analogue, which allows different curvature parameters below and above the benchmark. Third, it replaces the linear benchmark loading with the corresponding power loading so that the risk-on and risk-off components remain symmetric and the objective remains increasing and concave in own wealth.¹³

¹³The objective function in (16) differs in one important respect from the objective function in prospect theory. In this paper, the objective is concave everywhere ($U_{WW} < 0$), while the switch from convexity to concavity occurs solely with respect to wealth relative to the benchmark. In contrast, in prospect theory, concavity is with respect to absolute wealth changes. Specifically, when the agent is poor in absolute terms,

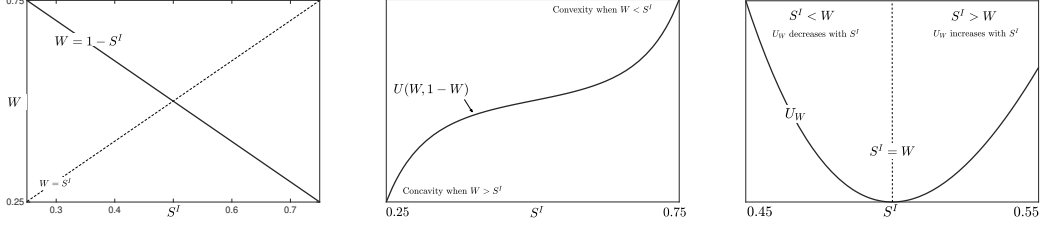


Figure 4. The left figure plots the $S^I - W$ plane. In the left panel, the diagonal $W = S^I$ (dashed) partitions it into two regions: in the upper-left triangle the manager is ahead of the benchmark ($W > S^I$) and the objective has a negative cross derivative ($U_{WS^I} < 0$), whereas in the lower-right triangle the manager is behind ($W < S^I$) and the objective has a positive cross derivative ($U_{WS^I} > 0$). The solid line $W = -S^I + 1$ is a transversal path running from the top-left corner to the bottom-right corner, crossing the diagonal once, at $(0.5, 0.5)$. Tracing this path, from the top left corner to the bottom right corner, the incentives pass from preserving outperformance, negative cross derivative, to the incentive to outperform, positive cross derivative. Correspondingly, the middle panel plots the asset manager objective, U , along the solid line path: the objective is concave and turn convex when crossing the dashed line. The right figure shows the cross derivative. When $W < S^I$ (right side of the figure), the marginal utility of wealth increases when S^I increases, indicating the asset manager desire to outperform. In contrast, when $W > S^I$ (left side of the figure), the marginal utility of wealth decreases when S^I increases, indicating the asset manager desire to maintain accumulated performance. The function U represents the asset manager's objective (16) with $\alpha_1 = 0.5$ and $\alpha_2 = 4.5$.

Starting with its initial endowments, the asset manager dynamically chooses a portfolio π_t^A , which represents the fraction of wealth invested in each of the risky assets. The wealth process of the asset manager, therefore, follows the dynamics

$$\frac{dW_t^A}{W_t^A} = \pi_t^{A'} (\mu_{S^I} dt + \Sigma_t^S dZ_t), \quad t \leq T. \quad (20)$$

Remark 3 (Alternative Objective Functions). *Let us denote, a general objective that satisfies the regularity conditions above, (11), (18), and (19). In a dynamically complete markets, the marginal value of wealth identifies the pricing kernel. We can isolate the benchmark-hedging motive by comparing two states that share a common value of the pricing kernel but differ in the benchmark realization. A total differentiation of the first-order condition yields*

$$\frac{\partial W_T^A}{\partial S_T^I} = -\frac{U_{WS^I}}{U_{WW}}. \quad (21)$$

Since $U_{WW} < 0$, the sensitivity $\partial W_T^A / \partial S_T^I$ inherits the sign of U_{WS^I} , positive for $W_T < S_T^I$

the objective function is convex ($U_{WW} > 0$), whereas when the agent is wealthy in absolute terms, it is concave ($U_{WW} < 0$).

and negative for $W_T > S_T^I$, as identified by (11).

Equilibrium responds to the sign of the cross-partial (U_{WS^I}) for any objective function that satisfies the conditions in our model setup. Therefore, any such objective leads the manager to tilt toward the benchmark when behind and away from it when ahead, and therefore produces qualitatively identical equilibrium dynamics: reversal following underperformance and momentum following outperformance. The curvature parameters (γ_1, γ_2) shape the magnitude of these effects, not their direction. We adopt (16) because it delivers the requisite sign structure in closed form.

Remark 4 (Risk Aversion). *The asset manager's risk-on and risk-off objectives are of a constant relative risk aversion type individually. However, when taken together, the relative risk aversion of the asset manager's objective (16) is a weighted average of the two risk aversion coefficients (γ_1, γ_2), with time-varying weights that depend on the likelihood of the representative asset manager to outperform or underperform. We denote the relative risk aversion by $RA(W, S^I)$. By applying the definition of relative risk aversion to the asset manager's objective function, we find that:*

$$RA(W, S^I) = \underbrace{\gamma_1 \left(1 - \frac{1}{1 + \frac{(1-\gamma_1)}{(\gamma_2-1)} (W/S^I)^{(\gamma_2-\gamma_1)}} \right)}_{\equiv 1-w} + \underbrace{\gamma_2 \left(\frac{1}{1 + \frac{(1-\gamma_1)}{(\gamma_2-1)} (W/S^I)^{(\gamma_2-\gamma_1)}} \right)}_{\equiv w}, \quad (22)$$

where $w \in (0, 1)$ since $0 < \gamma_1 < 1 < \gamma_2$. The formulation shows that risk aversion, $RA(W, S^I)$, is endogenous, time-varying, and dependent on the ratio of the asset manager's wealth to the index. When the representative asset manager's wealth is low relative to the index, it is in the risk-on region since $W < S^I$. In this case, the risk aversion aligns more closely to the risk-aversion coefficient in the risk-on region (γ_2) since the weight on the risk-off risk-aversion (γ_1) declines.

As the accumulated performance of the asset manager improves, its total wealth increases relative to the index. Consequently, the weight on the risk-off risk-aversion parameter rises while the weight on the risk-on risk-aversion parameter falls, resulting in an overall objective function that is decreasing in relative risk aversion: the risk aversion decreases as the asset manager becomes wealthier.

In the numerical illustrations below we calibrate $\gamma_1 = 0.5$ and $\gamma_2 = 4.5$. The the manager's effective relative risk aversion, $RA(W, S^I)$, ranges from 0.5 when the fund is far ahead of the benchmark ($W/S^I \rightarrow \infty$) to 4.5 when it is far behind ($W/S^I \rightarrow 0$), declining monotonically in relative performance, and equals exactly 4 at the benchmark $W = S^I$. The asset-pricing literature places relative risk aversion roughly between one and ten, (Mehra and Prescott (1985)). Our calibration also aligns with the time-varying aggregate risk aver-

sion of (Bekaert, Engstrom, and Xu (2022)), which ranges from about three in calm markets to a peak of eight. The momentum-in-outperformance and reversal-in-underperformance prediction therefore does not rely on unusual or finely tuned risk preferences.¹⁴

4 The Equilibrium

We define the equilibrium in a standard way: equilibrium prices and portfolio holdings are such that (i) the representative asset manager chooses its optimal portfolio for given prices, and (ii) stocks, bond and consumption-good markets clear.

In equilibrium, the likelihood that the representative asset manager's wealth will ultimately outperform the index level at maturity depends on the relative news of the market versus the index. In other words, the state variable that determines equilibrium quantities is V/I : states of the economy in which there is positive news about the market, $\uparrow V_t$, that is uncorrelated with news about the index, I_t , are states in which the representative asset manager is likely to outperform the benchmark. By contrast, states in which there is positive news about the index $\uparrow I_t$ that is uncorrelated with news about the market, V_t , are states in which the representative asset manager is likely to underperform the benchmark.

Before commencing the equilibrium analysis, we introduce a new representation for the state variable V/I to simplify the equilibrium expressions. We term this representation the *risk-on hedging demand*, $\mathcal{H}_{it}$, which is given by

$$\mathcal{H}_{it} \equiv \frac{1}{1 + \frac{1-\gamma_1}{\gamma_2-1} (V_t/I_t)^{\gamma_2-\gamma_1} \mathcal{E}_{it}}, \quad i = 1, 2, 3, 4, \quad (23)$$

where i refers to four different deterministic functions of time, $\mathcal{E}_{1t}$, $\mathcal{E}_{2t}$, $\mathcal{E}_{3t}$, $\mathcal{E}_{4t}$, given in (A.3), (A.6), (A.9), (A.12), respectively. We refer to $\mathcal{H}_{it}$ as the risk-on hedging demand because the closer it is to one, the more correlated the equilibrium quantities of the joint incentives case are with those of the purely risk-on incentives case. The deterministic functions $\mathcal{E}_{it}$ determines the speeds at which the hedging demand changes with the market-index ratio, $\partial \mathcal{H}_{it} / \partial (V/I)$. Thus, we refer to $\mathcal{H}_{it}$ for $i = 1, 2, 3, 4$ as the risk-on hedging demand. The market-index ratio governs both the risk-on hedging demand and the risk aversion parameter, and it serves as the key state variable that determines all equilibrium quantities.

We begin by analyzing the partial equilibrium effect of how shocks impact the wealth of the asset manager, given prices. This approach allows us to identify the different shock propagation channels for risk-on and risk-off incentives, both individually and jointly. To

¹⁴The admissible set of γ_1 and γ_2 that generate momentum and reversal are given in Figure 8.

facilitate this, let us denote the (equilibrium) discount factor by ξ_t and its dynamics by

$$d\xi_t = -\xi_t \left(\boldsymbol{\theta}'_t d\mathbf{Z}_t \right). \quad (24)$$

By definition of the discount factor, the vector of market prices of risk process, $\boldsymbol{\theta}_t$, can be represented as

$$\boldsymbol{\theta}_t = (\boldsymbol{\Sigma}_t^S)^{-1} \boldsymbol{\mu}_t^S, \quad (25)$$

which in a single asset economy simplifies to the ratio of an asset's (excess) expected return and its standard deviation.

Lemma 1 (Risk Exposure). *The asset manager's risk exposure due to risk-on incentives is given by*

$$\boldsymbol{\Sigma}_t^{S'} \boldsymbol{\pi}_t^A = \boldsymbol{\theta}_t - (\gamma_2 - 1) \boldsymbol{\nu} + (1 - \gamma_1) \boldsymbol{\sigma}^I, \quad (26)$$

while the risk exposure due to risk-off incentives is given by

$$\boldsymbol{\Sigma}_t^{S'} \boldsymbol{\pi}_t^A = \boldsymbol{\theta}_t + (1 - \gamma_1) \boldsymbol{\nu} - (\gamma_2 - 1) \boldsymbol{\sigma}^I. \quad (27)$$

In the presence of both risk-on and risk-off incentives the risk exposure is given by

$$\begin{aligned} & \boldsymbol{\Sigma}_t^{S'} \boldsymbol{\pi}_t^A \\ &= \boldsymbol{\theta}_t + [(1 - \gamma_1)(1 - \mathcal{H}_{2t}) - (\gamma_2 - 1)\mathcal{H}_{2t}] \boldsymbol{\nu} + [(1 - \gamma_1)\mathcal{H}_{2t} - (\gamma_2 - 1)(1 - \mathcal{H}_{2t})] \boldsymbol{\sigma}^I. \end{aligned} \quad (28)$$

The function $\mathcal{H}_{2t}$ is defined in (23).

There are three shock propagation channels. The first is the myopic mean-variance channel, where the asset manager seeks a risk exposure proportional to the assets' risk-return tradeoff, correlating their wealth with the market price of risk, $\boldsymbol{\theta}_t$. Lemma 1 demonstrates that this channel remains unaffected regardless of the type of incentives. However, the exact exposure will be determined in equilibrium once the market prices of risk are identified.

The second channel is a risk-aversion adjustment channel, and Lemma 1 shows that it has different implications for risk-on and risk-off incentives. In the pure risk-on incentives case (26), the representative asset manager is risk averse with respect to the market and desires to hedge against adverse market moves (buy insurance on the market). The asset manager does so by aspiring to higher wealth following adverse market movements, as the risk aversion coefficient exceeds one, $\gamma_2 > 1$. The logic reverses for risk-off incentives. In pure risk-off incentives (27), the asset manager is a risk takers with respect to the market

(gambles on the market) and correlate their wealth with the market, since the risk aversion coefficient is less than one, $\gamma_1 < 1$.

The third channel is the benchmark consideration. Similar to the risk aversion adjustment channel, it has opposing implications for risk-on and risk-off incentives. When the asset manager trails the index and aim to outperform it (26), they act as risk-takers with respect to the index, seeking higher returns when the benchmark is high compared to when it is low — an effort to surpass the benchmark, as evident in their optimal positive risk exposure, $(1 - \gamma_1) \sigma^I$. In particular, when the benchmark is high and the asset manager trails it, their marginal utility of wealth is convex with respect to the benchmark, $U_{WS^I} > 0$. This convexity induces the asset manager to take larger risk when they are further below the benchmark, similar to the findings in the asset management literature.

Conversely, when the asset manager outperforms the index and aim to maintain their lead (27), they are risk-averse relative to the benchmark, seeking higher returns when the benchmark is low compared to when it is high — an effort to maintain their lead over the benchmark, as evident in their optimal negative risk exposure, $-(\gamma_2 - 1) \sigma^I$. Specifically, when the benchmark is low and the asset manager leads, its marginal utility of wealth is concave with respect to the benchmark, $U_{WS^I} < 0$. This concavity induces the asset manager to protect their wealth more when they are outperforming the benchmark by a larger margin.

The joint effect is a time-varying weighted average of the incentives to outperform the benchmark and maintain accumulated performance. Without factoring in equilibrium pricing effects, when the asset manager is far below the benchmark ($\mathcal{H}_{2t} \rightarrow 1$), they act as risk-takers and aim to achieve higher returns by gaining significant exposure to the benchmark (for example, through leverage). As the asset manager's accumulated performance improves, they more closely balance risk-on and risk-off portfolio strategies ($0 < \mathcal{H}_{2t} < 1$), which reduces their optimal benchmark exposure (for example, fully replicating the benchmark). Finally, when the accumulated performance of the asset manager continues to improve ($\mathcal{H}_{2t} \rightarrow 0$), they become risk-averse relative to the benchmark and seek to take large risks negatively correlated with the benchmark (for example, by taking a short benchmark position).

The incentives of the asset manager to leverage, replicate, or short the benchmark induce demand pressures that are counteracted in equilibrium by changes in the risk-return trade-offs of assets, which are captured by the market price of risk. We proceed by deriving closed-form expressions for the equilibrium quantities, beginning with the market price of risk.

Proposition 1 (Market Price of Risk). *The market price of risk in the presence of risk-on incentives is given by*

$$\boldsymbol{\theta}_t = -(1 - \gamma_1) \boldsymbol{\sigma}^I + \gamma_2 \boldsymbol{\nu}, \quad (29)$$

while in the presence of risk-off incentives, it is given by

$$\boldsymbol{\theta}_t = (\gamma_2 - 1) \boldsymbol{\sigma}^I + \gamma_1 \boldsymbol{\nu}. \quad (30)$$

In the presence of both risk-on and risk-off incentives the market prices of risk are given by

$$\boldsymbol{\theta}_t = [\gamma_1 (1 - \mathcal{H}_{1t}) + \gamma_2 (\mathcal{H}_{1t})] \boldsymbol{\nu} + [-(1 - \gamma_1) \mathcal{H}_{1t} + (\gamma_2 - 1) (1 - \mathcal{H}_{1t})] \boldsymbol{\sigma}^I. \quad (31)$$

The function $\mathcal{H}_{1t}$ is defined in (23).

In the risk on incentives case, (29), the asset manager trails the benchmark. In this case, the market price of risk aligns with findings documented in the literature over the past decade: equilibrium counteracts the heightened demand for assets correlated with the benchmark by decreasing the market price of risk associated with those assets, making them less attractive from a risk-return trade-off perspective, $-(1 - \gamma_1) \boldsymbol{\sigma}^I$.

The logic reverses for pure risk-off incentives (30). In this case, when the asset manager's wealth exceeds the benchmark, they maintain their lead over the benchmark by taking large risky positions negatively correlated with the benchmark. Equilibrium counteracts the heightened demand for assets negatively correlated with the benchmark by increasing the market price of risk associated with those assets, making them less attractive instruments from a risk-return trade-off perspective, $(\gamma_2 - 1) \boldsymbol{\sigma}^I$.

The joint effect is a time-varying weighted average of the market price of risk during underperformance and that during outperformance, with $\mathcal{H}_{1t}$ determining the weights. When the asset manager underperforms by a large margin ($\mathcal{H}_{1t} \rightarrow 1$), the market price of risk in underperformance dominates, leading to a lower market price of risk for assets correlated with the benchmark. As accumulated performance improves and aligns more closely with the benchmark, the asset manager balances the risk-on and risk-off market prices of risk more closely ($0 < \mathcal{H}_{1t} < 1$). The reduced demand for the benchmark leads to a recovery in the market price of risk correlated with the benchmark. Finally, when the asset manager outperforms and significantly exceeds the benchmark ($\mathcal{H}_{1t} \rightarrow 0$), the market price of risk associated with outperformance dominates, leading to an increase in the market price of risk for assets correlated with the benchmark.

The market price of risk associated with the market exposure is also time-varying. In the underperformance region, the asset manager is risk-averse relative to the market, seeking to negatively correlate their wealth with the market. Equilibrium prices respond by increasing the market price of risk for assets correlated with the market, $\gamma_2 \boldsymbol{\nu}$. As accumulated per-

formance improves, the demand for assets negatively correlated with the market decreases, and equilibrium responds by reducing the market price of risk associated with the market, $\gamma_1 \boldsymbol{\nu}$. When accumulated performance aligns more closely with the benchmark, equilibrium balances the risk-on and risk-off market prices of risk, with the weights determined by $\mathcal{H}_{1t}$.

We proceed by characterizing the risk exposures.

Proposition 2 (Volatility). *The return volatilities in the presence of either risk-on incentives or risk-off incentives, separately, are given by*

$$\boldsymbol{\sigma}_{1t}^S = \boldsymbol{\sigma}_{1t}^D, \quad \boldsymbol{\sigma}_{2t}^S = \boldsymbol{\sigma}_{2t}^D. \quad (32)$$

The return risk exposures in the presence of joint risk-on and risk-off incentives are given by

$$\boldsymbol{\sigma}_{1t}^S = \boldsymbol{\sigma}_1^D + (\gamma_2 - \gamma_1) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) (\mathcal{H}_{3t} - \mathcal{H}_{1t}), \quad (33)$$

$$\boldsymbol{\sigma}_{2t}^S = \boldsymbol{\sigma}_2^D - (\gamma_2 - \gamma_1) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) (\mathcal{H}_{1t} - \mathcal{H}_{4t}), \quad (34)$$

where $\mathcal{H}_{1t}$, $\mathcal{H}_{3t}$, $\mathcal{H}_{4t}$ are given in (23), and $\mathcal{H}_{3t} - \mathcal{H}_{1t} > 0$, $\mathcal{H}_{1t} - \mathcal{H}_{4t} > 0$.

The index asset sensitivity to news is the largest when $\mathcal{H}_{1t} + \mathcal{H}_{3t} = 1$, and for the active asset is when $\mathcal{H}_{1t} + \mathcal{H}_{4t} = 1$.

Positive index news increases the index price, while positive active news decreases the index price:

$$\boldsymbol{\sigma}_{1t}^S(1) > 0, \quad \boldsymbol{\sigma}_{1t}^S(2) < 0, \quad \boldsymbol{\sigma}_{1t}^S(3) < 0. \quad (35)$$

In contrast, positive index asset news decreases the active asset price, while positive active news increases the active asset price:

$$\boldsymbol{\sigma}_{2t}^S(1) < 0, \quad \boldsymbol{\sigma}_{2t}^S(2) > 0, \quad \boldsymbol{\sigma}_{2t}^S(3) > 0. \quad (36)$$

In a traditional endowment economy without benchmark incentives, prices only reflect news about the assets' fundamentals. In such environments, optimal portfolios remain unchanged by the arrival of news, and, therefore, equilibrium does not alter the exposure of prices to future news, leading to the immediate reflection of news in prices.

Proposition 2 reveals that pure incentives cause the asset manager to respond to news differently than market participants in a traditional setup. However, similar to a traditional endowment economy, optimal portfolios (and risk exposures) remain unchanged by the arrival of news. Thus, equilibrium does not change the sensitivity of prices to future news.

In contrast, in previous pure risk-on models, such as those by Basak and Pavlova (2013), and Buffa and Hodor (2023), the asset manager desires to change their portfolios with the

arrival of news, since the incentives to outperform the benchmark increases as accumulated performance deteriorates (the marginal utility of wealth becomes more convex with respect to the index). As a result, equilibrium changes how the index price will react to future news. In these pure risk-on incentive models, the incentives to outperform always exist and never shift to the incentives to maintain performance—the marginal utility of wealth is always convex.

In the joint incentives case, equilibrium reacts to changes in the incentives to outperform the benchmark relative to the incentives to maintain accumulated performance. As the accumulated performance of the asset manager converges to the benchmark ($\mathcal{H}_{1t} + \mathcal{H}_{3t}$ and $\mathcal{H}_{1t} + \mathcal{H}_{4t}$ get closer to one), assets become riskier due to the higher uncertainty of performance.

When positive index news arrives, the index price increases, making the asset manager more likely to fall behind the benchmark and underperform. Anticipating this, the asset manager aims to buy more of the benchmark as positive news arrives, increasing the demand for the index. Consequently, equilibrium causes the index price to overreact to index news. In contrast, the active asset becomes less attractive to the asset manager because it does not fulfil the role of risk-on incentives. Consequently, positive index news translates into a negative price impact on the active asset.

Lastly, when positive active news arrives, the active asset price increases, boosting the overall market relative to the index. In this scenario, the asset manager is more likely to outperform. Anticipating this, the asset manager buys less of (or sell) the benchmark as positive active news arrives, decreasing the demand for the index. Consequently, equilibrium causes the index price to underreact to positive active news. In contrast, the active asset becomes more attractive to the asset manager because it fulfils the role of risk-off incentives. Consequently, positive active news translates into a positive price impact on the active asset.

We proceed with analyzing the predictability of asset returns.

5 Time-Series Predictability

This section examines how an asset’s past returns can predict its current return. Positive autocorrelation is known as time-series *momentum*, while negative autocorrelation is referred to as time-series *reversal*. Predictability arises from predictable changes to both quantities of risk and market prices of risk. We separate these two predictability channels throughout the analysis.

To deepen our understanding of the equilibrium mechanism, we analyze how the market

prices of risk and the quantities of risk react to news.¹⁵

5.1 Equilibrium Response to Frictions

Prices not only react to information about the fundamentals but also react to news about management frictions.

To illustrate the intuitions clearly, we rewrite the vector of market prices of risk, (31), as a sum of the *index news exposure* ($\boldsymbol{\sigma}^I$) and the *market news exposure* ($\boldsymbol{\nu}$),

$$\boldsymbol{\theta}_t = \kappa_t^I \boldsymbol{\sigma}^I + \kappa_t^\nu \boldsymbol{\nu}, \quad (37)$$

where

$$\kappa_t^I \equiv (\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t}), \quad \kappa_t^\nu \equiv \gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t}). \quad (38)$$

When the asset manager is purely risk-on or purely risk-off, the market prices of risk do not reflect news about the management frictions because the risk-on hedging demand ($\mathcal{H}_{1t}$) is a constant that equals one or zero, respectively, and never changes. As a result, prices fully incorporate fundamental information. However, when the asset manager balances both strategies, market prices of risk reflect not only fundamentals but also news about asset management frictions. Note that from a purely fundamental perspective, the price reaction to management friction might appear as incorrect pricing, or an overreaction.

As the asset manager's accumulated performance deteriorates, risk-on incentives become more pronounced than risk-off incentives, leading the asset manager to invest more in the benchmark and less in assets outside the benchmark. To restore equilibrium, market prices of risk associated with the benchmark are depressed to make the index assets less attractive and more expensive. Conversely, the market prices of risk associated with assets outside the benchmark increase to make these assets more attractive and less expensive.

Similar to the market price of risk, we rewrite the vector of return exposures (quantities of risk) as a sum of index news exposure ($\boldsymbol{\sigma}^I$) and market news exposure ($\boldsymbol{\nu}$),

$$\boldsymbol{\sigma}_{1t}^S = \boldsymbol{\sigma}_1^D + (\gamma_2 - \gamma_1) (\mathcal{H}_{3t} - \mathcal{H}_{1t}) \boldsymbol{\sigma}^I - (\gamma_2 - \gamma_1) (\mathcal{H}_{3t} - \mathcal{H}_{1t}) \boldsymbol{\nu}, \quad (39)$$

$$\boldsymbol{\sigma}_{2t}^S = \boldsymbol{\sigma}_2^D - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t} - \mathcal{H}_{4t}) \boldsymbol{\sigma}^I + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t} - \mathcal{H}_{4t}) \boldsymbol{\nu}. \quad (40)$$

The representations reveal that when the asset manager is purely risk-on or purely risk-off, asset prices react only to news about fundamentals since the risk-on hedging demands are

¹⁵Appendix C provides preliminary yet compelling evidence indicating that mutual fund returns are more likely to exhibit momentum when they outperform their benchmark, and reversal when they underperform it.

constant, either equaling one or zero, respectively. In contrast, when the asset manager weighs both strategies, assets incorporate news about management frictions in addition to the news about fundamentals.

The difference between the risk-on hedging demands attain its maximum when the sum of the risk-on hedging demands equals one, $\mathcal{H}_{it} + \mathcal{H}_{jt} = 1$, $i, j = 1, 2, 3, 4$. The maximum values for different i and j are reached close to each other and identify the state at which the asset manager has an equal likelihood of outperforming or underperforming the benchmark.

This likelihood influences prices. The arrival of news can cause the asset manager's accumulated performance to either align more closely with the benchmark or deviate away from the benchmark. The uncertainty about accumulated performance is strongest when the asset manager's accumulated performance closely matches the benchmark, compared to when their performances diverge. Consequently, the price sensitivity to news is the largest in this scenario.

When the asset manager's accumulated performance diverges towards underperformance, their portfolios weigh more towards the risk-on strategy relative to the risk-off strategy. In this case, it becomes more likely that the asset manager will underperform. Equilibrium reflects this higher certainty of underperformance by reducing the risks in the asset markets. Conversely, when the asset manager's accumulated performance diverges towards outperformance, their portfolios weigh more towards the risk-off strategy relative to the risk-on strategy. Equilibrium reflects this higher certainty of outperformance by reducing the risks in the asset markets.

The equilibrium response to news about asset management frictions, through both the market prices of risk and the quantities of risk, reveals two predictability channels, leading to predictable returns in time series and in long-short portfolios. The following section discusses these two predictability channels and how they jointly determine the time-series predictability of returns.

5.2 Time-Series Predictability

So far, we have shown that market prices of risk and quantities of risk react to news about asset management frictions.

Before we delve into why these responses are predictable, let us first establish how we identify predictability. Let's begin by defining the expected return dynamics for both the index and active assets as follows

$$d\mu_{1t}^S = \alpha_{1t}^\mu dt + \mathbf{b}_{1t}^{\mu'} d\mathbf{Z}_t, \quad d\mu_{2t}^S = \alpha_{2t}^\mu dt + \mathbf{b}_{2t}^{\mu'} d\mathbf{Z}_t. \quad (41)$$

The covariance between the expected returns in time t and $t + \Delta$ (for $\Delta > 0$) is given by

$$\begin{aligned}\mathbb{Cov}_t\left(\frac{dS_{kt}}{S_{kt}}, \frac{dS_{kt+\Delta}}{S_{kt+\Delta}}\right) &= \mathbb{Cov}_t\left((\mu_{kt}^S dt + \sigma_{kt}^{S'} d\mathbf{Z}_t), (\mu_{kt+\Delta}^S d(t+\Delta) + \sigma_{kt+\Delta}^{S'} d\mathbf{Z}_{t+\Delta})\right) \\ &= \mathbb{Cov}_t\left(\sigma_{kt}^{S'} d\mathbf{Z}_t, \mu_{kt+\Delta}^S d(t+\Delta)\right) = \sigma_{kt}^{S'} \mathbb{E}_t\left[(d\mathbf{Z}_t) \mu_{kt+\Delta}^S\right] d(t+\Delta).\end{aligned}\quad (42)$$

This covariance indicates that the predictability of asset k depends on how today's news ($d\mathbf{Z}_t$) influences the asset's expected return in $t + \Delta$. It reflects the response of the expected return $\mu_{kt+\Delta}^S$ to the news $d\mathbf{Z}_t$.

To obtain a closed-form characterization, we examine predictability as Δ approaches 0. Plugging the dynamics of μ_{kt}^S and taking the covariance leads to

$$\mathbb{Cov}_t\left(\frac{dS_{kt}}{S_{kt}}, \frac{dS_{kt+\Delta\downarrow 0}}{S_{kt+\Delta\downarrow 0}}\right) = \sigma_{kt}^{S'} \mathbf{b}_{kt}^\mu dt, \quad (43)$$

where $\mathbf{b}_{kt}^\mu$ is the exposure of asset k expected return to news, (41). The following definition characterizes time-series momentum and reversal.

Definition 1 (Time-Series Momentum and Reversal). *Asset k exhibits momentum if both the asset return and its expected return move in the same direction,*

$$\sigma_{kt}^{S'} \mathbf{b}_{kt}^\mu > 0, \quad (44)$$

and exhibits reversal if the asset return and the expected return move in opposite directions,

$$\sigma_{kt}^{S'} \mathbf{b}_{kt}^\mu < 0, \quad (45)$$

where $k = 1, 2$ and identifies the index and active assets, respectively.

The definition is straightforward. We identify momentum when returns rise (fall) and the equilibrium expectation is that they will continue to rise (fall). Conversely, we identify reversal when returns rise (fall) but the equilibrium expectation is that they will fall (rise) in the future. The direction of predictability in equilibrium depends on the average effect across all shocks, which makes the analysis particularly challenging.

There are two potential predictable channels that emerge from the no arbitrage condition, $\theta'_t \sigma_{it}^S = \mu_{it}^S$. The first channel is a *pricing effect*, which is driven by the predictable change in the market prices of risk, θ_t . The second channel is a *quantity-of-risk effect*, which is driven by the predictable change in the asset's risk exposures, σ_{it}^S .

To disentangle these two channels and examine how shocks in each channel propagate to expected returns, we adopt the approach of Buffa and Hodor (2023). This approach

introduces the concept of elasticity between two quantities within a dynamic system, akin to an impulse-response function.¹⁶ Accordingly, we define the shock elasticity at time t of an equilibrium quantity X at time t as the time t expectation of the Malliavin derivative $\mathcal{D}_t$ of X_t , normalized by the absolute value of the time t expectation of X_t ,

$$\varepsilon_t(X, \ell) \equiv \frac{\mathbb{E}_t[\mathcal{D}_t^\ell X_t]}{|\mathbb{E}_t[X_t]|} = \frac{\mathcal{D}_t^\ell X_t}{|X_t|} = \frac{\sigma_t^X(\ell)}{|X_t|}, \quad (46)$$

where $\sigma_t^X(\ell)$ is entry ℓ of the diffusion term of dX_t . The shock elasticity $\varepsilon_t(X, \ell)$ is well-defined for $X_t \neq 0$. News in the economy can be separated into two mutually exclusive types: index news and active news, which implies that there are two shock elasticities, one for each type of news. By applying the chain rule, we find that

$$\varepsilon_t(\mu_{it}^S, \ell) = \varepsilon_t(\boldsymbol{\sigma}_{it}^{S'} \boldsymbol{\theta}_t, \ell) = \frac{\boldsymbol{\sigma}_{it}^{S'}}{|\mu_{it}^S|} \left(\underbrace{(\mathcal{D}_t^\ell \boldsymbol{\theta}_t)}_{\text{benchmarking incentives}} \right) + \frac{\boldsymbol{\theta}_t'}{|\mu_{it}^S|} \left(\underbrace{(\mathcal{D}_t^\ell \boldsymbol{\sigma}_{it}^S)}_{\text{uncertainty of performance}} \right), \quad (47)$$

where the vector of Malliavin derivatives is the vector of derivatives for every entry. For example, $\mathcal{D}_t^\ell \boldsymbol{\theta}_t$ is the Malliavin derivative of each entry of the market price of risk vector with respect to news ℓ , which could be either index news or active news.

Let $\boldsymbol{\varepsilon}_t(\mu_{it}^S)$ be the vector of shock-elasticities. The definition of momentum implies a positive cross product between the shock elasticities of the expected return and the shock elasticities of the asset. Conversely, the definition of reversal is indicated by a negative cross product,

$$\text{Momentum} \Rightarrow \boldsymbol{\varepsilon}_t(\mu_{it}^S)' \boldsymbol{\varepsilon}_t(S_{it}) > 0, \quad \text{Reversal} \Rightarrow \boldsymbol{\varepsilon}_t(\mu_{it}^S)' \boldsymbol{\varepsilon}_t(S_{it}) < 0. \quad (48)$$

5.3 Incentives Channel

This is a price effect channel that causes a reversal in asset returns. Benchmarking incentives induce strong (weak) demand for assets inside (outside) the benchmark, regardless of their risk-return profile. This differential demand causes news about the benchmark asset to have larger price impacts compared to news about the active asset. As cumulative performance of the representative asset manager deteriorates, the difference in price impacts intensifies — stronger for benchmark assets and weaker for active asset. Because expected returns account

¹⁶This methodology was originally presented by Borovička, Hansen, Hendricks, and Scheinkman (2011) and Borovička, Hansen, and Scheinkman (2014), which, in continuous time, builds on Malliavin calculus. Please refer to Ocone and Karatzas (1991), Øksendal (1997), Detemple, Garcia, and Rindisbacher (2003) for further discussion on Malliavin calculus with Markovian processes in finance.

for future price impacts, this channel generates return reversals in asset markets.

To see the mechanism, suppose news arrives that increases the likelihood of underperformance, whether it is positive index news or negative active news. The asset manager responds by hedging against further declines in accumulated performance, as their portfolio is not fully correlated with the benchmark. Consequently, the asset manager seeks to increase their allocation to the benchmark at the expense of other assets. To maintain equilibrium, the benchmark asset must become less attractive (and the active asset more attractive) to offset the asset manager's demand. This reduces the market prices of risk correlated with the benchmark and increasing the market prices of risk negatively correlated with the benchmark. As a result, the expected return of the index asset falls, while the expected return of the active asset rises.

Let's examine this channel more closely, which is derived by the Malliavin derivative of the market prices of risk with respect to a particular shock ℓ , which is given by

$$\mathcal{D}_t^\ell \boldsymbol{\theta}_t = -(\gamma_2 - \gamma_1)^2 \mathcal{H}_{1t} (1 - \mathcal{H}_{1t}) (\boldsymbol{\sigma}^I(\ell) - \boldsymbol{\nu}(\ell)) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}). \quad (49)$$

Since $\boldsymbol{\theta}_t$ is a vector, the Malliavin derivative with respect to shock ℓ of this vector results in a vector of Malliavin derivatives for each entry. Let $\mathcal{D}_t \boldsymbol{\theta}_t$ be the vector Malliavin derivatives with respect for all shocks, $\ell = 1, 2, 3$. The resulting Malliavin derivative has a matrix form identified by

$$\mathcal{D}_t \boldsymbol{\theta}_t = -(\gamma_2 - \gamma_1)^2 \mathcal{H}_{1t} (1 - \mathcal{H}_{1t}) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) (\boldsymbol{\sigma}^I - \boldsymbol{\nu})'. \quad (50)$$

The signs of the each of the entries in this matrix are given by

$$\text{sign}(\mathcal{D}_t \boldsymbol{\theta}_t) = \begin{pmatrix} - & + & + \\ + & - & - \\ + & - & - \end{pmatrix}, \quad (51)$$

and they do not change sign with the state variable V/I . That is, the sensitivities do not change with the manager's accumulated performance relative to the benchmark.

Each column represents the market price of risk with respect to particular news. Each row represents how this market price of risk change with arrival of particular news. For example, the entry $(2, 1)$ represent how the market price of risk that is associated with index news ($\ell = 1$) changes with the arrival of active news ($\ell = 2$). The $+$ sign implies that the market price of this risk increases.

The changes in the expected returns of the index and active assets due to the incentives channel are given by

$$\text{sign}(\boldsymbol{\varepsilon}_t(\mu_{1t}^S)') = \text{sign}(\boldsymbol{\sigma}_{1t}^{S'}(\mathcal{D}_t \boldsymbol{\theta}_t)) = \begin{pmatrix} + & - & - \end{pmatrix} \times \begin{pmatrix} - & + & + \\ + & - & - \\ + & - & - \end{pmatrix} = \begin{pmatrix} - & + & + \end{pmatrix}, \quad (52)$$

$$\text{sign}(\boldsymbol{\varepsilon}_t(\mu_{2t}^S)') = \text{sign}(\boldsymbol{\sigma}_{2t}^{S'}(\mathcal{D}_t \boldsymbol{\theta}_t)) = \begin{pmatrix} - & + & + \end{pmatrix} \times \begin{pmatrix} - & + & + \\ + & - & - \\ + & - & - \end{pmatrix} = \begin{pmatrix} + & - & - \end{pmatrix}. \quad (53)$$

This cross multiplication reveals that for any entry whereby $\mathcal{D}_t \boldsymbol{\theta}_t$ is negative, $\boldsymbol{\sigma}_{1t}^S$ is positive, and vice versa. That is, index news ($\ell = 1$) decreases the expected return of the index, while active news ($\ell = 2$) increases it. The opposite occurs for the active asset.

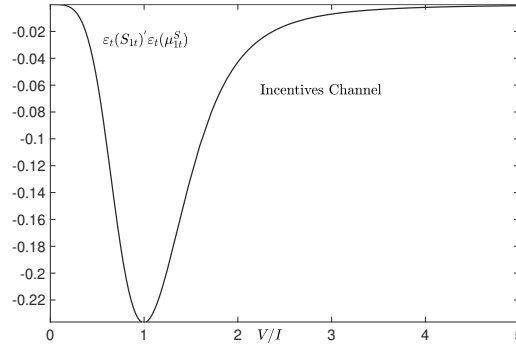


Figure 5. The figure shows that the overall effect coming from the *incentives* channel produces a reversal in asset returns. The parameters are $\gamma_2 = 4.5$, $\gamma_1 = 0.5$, $\mu_1 = \mu_2 = 0.05$, $\sigma_1 = \sigma_2 = 0.25$, $\lambda = 0.5$, $V_t = 5$, $V_0 = 2.5$, $I_0 = 1$, $T = 5$, $t = 2$. The risk-aversion calibration $\gamma_1 = 0.5$, $\gamma_2 = 4.5$ is discussed in Remark 4.

To determine whether the *incentives* channel generates price momentum or reversal, we identify the sign of the index and active assets shock elasticities. These signs are given by

$$\text{sign}(\boldsymbol{\varepsilon}_t(S_{1t})') = \begin{pmatrix} + & - & - \end{pmatrix}, \quad \text{sign}(\boldsymbol{\varepsilon}_t(S_{2t})') = \begin{pmatrix} - & + & + \end{pmatrix}, \quad (54)$$

which reveals that positive index news boosts the index return, while positive active news reduces it. Conversely, positive index news reduces the active asset return, whereas positive active news increases it.

By cross multiplying the signs of the two shock elasticities,

$$\text{sign}(\epsilon_t(\mu_{1t}^S)') \text{sign}(\epsilon_t(S_{1t})) = \begin{pmatrix} - & + & + \end{pmatrix} \times \begin{pmatrix} + \\ - \\ - \end{pmatrix} = \begin{pmatrix} - \end{pmatrix} \Rightarrow \text{Reversal}, \quad (55)$$

$$\text{sign}(\epsilon_t(\mu_{2t}^S)') \text{sign}(\epsilon_t(S_{2t})) = \begin{pmatrix} + & - & - \end{pmatrix} \times \begin{pmatrix} - \\ + \\ + \end{pmatrix} = \begin{pmatrix} - \end{pmatrix} \Rightarrow \text{Reversal}. \quad (56)$$

The analysis reveals that, in sum, the *incentives* channel induce an opposite reaction on the expected return and asset return shock elasticities: whenever one shock elasticity is positive the other is negative, and vice versa. Therefore, this channel causes asset price *reversals* during periods of both underperformance and outperformance.

5.4 Uncertainty of Performance Channel

This is a quantity-effect channel that causes either momentum or reversal depending on the accumulated performance of the representative manager. Specifically, when the accumulated performance of the representative asset manager converges toward the benchmark, uncertainty about future performance relative to the benchmark increases, which affects asset markets in equilibrium. As this convergence intensifies, so too does the uncertainty, inducing assets to have larger positive exposures and smaller negative exposures. Expected returns account for these future price impacts, which increase the expected return for the benchmark asset and decrease it for the active asset.

Proximity to the benchmark increases uncertainty about the asset manager's future performance, which propagates to asset markets through general equilibrium effects. Convergence toward the benchmark amplifies this uncertainty. The resulting uncertainty of performance channel, interacting with asymmetric price impacts, drives momentum in asset returns when performance exceeds the benchmark but induces reversal when performance lags. Momentum resumes with severe underperformance relative to the benchmark due the level of prices, as the following analysis identifies.

To see the mechanism, notice that higher uncertainty of performance implies that positive asset sensitivities become more positive and negative asset sensitivities become more negative. However, since market prices of risk are always positive and do not change signs, the sign of the expected return is determined by the average effect.

The average effect is dominated by the index news sensitivity. When accumulated performance converges to the benchmark, it becomes less certain whether the representative asset

manager will outperform or underperform the benchmark. This uncertainty increases the assets response to news because the asset manager balances risk-on and risk-off strategies more closely. Equilibrium market clearing compensates the asset manager for the heightened uncertainty by increasing expected returns.

When the asset manager outperforms the benchmark, news that brings their accumulated performance more closely in line with the benchmark boosts the benchmark asset return and due to the heightened uncertainty also its expected return. Therefore, the benchmark asset returns exhibit momentum when the asset manager outperforms the benchmark. In contrast, when the asset manager underperforms the benchmark, news that aligns their accumulated performance more closely with the benchmark decreases the benchmark asset return, but due to the heightened uncertainty its expected return increases. As a result, the benchmark asset returns exhibit reversal when the representative asset manager underperforms the benchmark.

In cases of severe underperformance, the average effect is determined by active news sensitivity instead of index news sensitivity. The demand for the benchmark asset surges to such an extent that, to maintain equilibrium market-clearing, the market price of risk associated with index news drops significantly. As a result, the pricing effect of uncertainty of performance is determined by the negative active sensitivity, which reduces the expected return of the benchmark asset. Overall, when the asset manager severely underperforms, the benchmark asset returns exhibit momentum.

Before analyzing the mechanism more rigorously, we first define the outperformance region—the set of states in which the asset manager is more likely to outperform—as states satisfying

$$\text{Outperformance} \Rightarrow \mathcal{H}_{1t} + \mathcal{H}_{3t} < 1, \quad \& \quad \mathcal{H}_{1t} + \mathcal{H}_{4t} < 1. \quad (57)$$

Similarly, we define the underperformance region—the set of states in which the asset manager is more likely to underperform—as states satisfying

$$\text{Underperformance} \Rightarrow \mathcal{H}_{1t} + \mathcal{H}_{3t} > 1, \quad \& \quad \mathcal{H}_{1t} + \mathcal{H}_{4t} > 1. \quad (58)$$

The uncertainty of performance channel is driven by the predictable change in assets' risk exposures, (47), which is given by the derivatives

$$\mathcal{D}_t^\ell \sigma_{1t}^S = (\gamma_2 - \gamma_1)^2 (\mathcal{H}_{3t} - \mathcal{H}_{1t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) (\sigma^I(\ell) - \nu(\ell)) (\sigma^I - \nu)', \quad (59)$$

$$\mathcal{D}_t^\ell \sigma_{2t}^S = -(\gamma_2 - \gamma_1)^2 (\mathcal{H}_{1t} - \mathcal{H}_{4t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) (\sigma^I(\ell) - \nu(\ell)) (\sigma^I - \nu)'. \quad (60)$$

Since the return exposures, σ_{1t}^S and σ_{2t}^S , are vectors, the Malliavin derivative with respect

to shock ℓ of these vectors result in a vector of Malliavin derivatives for each entry. Let $\mathcal{D}_t \boldsymbol{\sigma}_{it}^S$, $i = 1, 2$, be the vector Malliavin derivatives with respect to all shocks, $\ell = 1, 2, 3$. The resulting Malliavin derivatives have a matrix form identified by

$$\mathcal{D}_t \boldsymbol{\sigma}_{1t}^S = (\gamma_2 - \gamma_1)^2 (\mathcal{H}_{3t} - \mathcal{H}_{1t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) (\boldsymbol{\sigma}^I - \boldsymbol{\nu})', \quad (61)$$

$$\mathcal{D}_t \boldsymbol{\sigma}_{2t}^S = -(\gamma_2 - \gamma_1)^2 (\mathcal{H}_{1t} - \mathcal{H}_{4t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) (\boldsymbol{\sigma}^I - \boldsymbol{\nu})'. \quad (62)$$

Different from the incentives channel, the signs of each of the entries in these matrices (representing the uncertainty of performance) change with the manager's accumulated performance relative to the benchmark. This is because in outperformance, the risk-on hedging demands ($\mathcal{H}_{1t}$, $\mathcal{H}_{3t}$, and $\mathcal{H}_{4t}$) are close to zero, whereas in underperformance, they are close to one, which flip the signs of all the entries in these matrices. Therefore, the signs of these matrices are given by

$$\text{Outperformance} \Rightarrow 1 > \mathcal{H}_{1t} + \mathcal{H}_{3t} \Rightarrow \text{sign}(\mathcal{D}_t \boldsymbol{\sigma}_{1t}^S) = \begin{pmatrix} + & - & - \\ - & + & + \\ - & + & + \end{pmatrix}, \quad (63)$$

$$\text{Underperformance} \Rightarrow 1 < \mathcal{H}_{1t} + \mathcal{H}_{3t} \Rightarrow \text{sign}(\mathcal{D}_t \boldsymbol{\sigma}_{1t}^S) = \begin{pmatrix} - & + & + \\ + & - & - \\ + & - & - \end{pmatrix}. \quad (64)$$

Each column represents the return exposure with respect to particular news. Each row represents how this return exposure changes with arrival of particular news. For example, the entry (2,1) represent how the benchmark asset return exposure to index news ($\ell = 1$) changes with the arrival of active news ($\ell = 2$). The “ $-$ ” sign in outperformance implies that the return exposure shrinks when the asset manager outperforms the benchmark. In contrast, “ $+$ ” sign in underperformance implies that return exposure increases in the underperformance region.

The signs flip for the active asset,

$$\text{Outperformance} \Rightarrow 1 > \mathcal{H}_{1t} + \mathcal{H}_{4t} \Rightarrow \text{sign}(\mathcal{D}_t \boldsymbol{\sigma}_{2t}^S) = \begin{pmatrix} - & + & + \\ + & - & - \\ + & - & - \end{pmatrix}, \quad (65)$$

$$\text{Underperformance} \Rightarrow 1 < \mathcal{H}_{1t} + \mathcal{H}_{4t} \Rightarrow \text{sign}(\mathcal{D}_t \boldsymbol{\sigma}_{2t}^S) = \begin{pmatrix} + & - & - \\ - & + & + \\ - & + & + \end{pmatrix}. \quad (66)$$

Overall, the signs in these matrices, (63), (64), (65), (66), determine whether the arrival of news increases or decreases the assets' sensitivities. However, the changes in the expected

returns of the index and active assets due to the uncertainty of performance channel cannot immediately be determined because the market prices of risk are positive and do not change signs,

$$\text{sign}(\boldsymbol{\theta}'_t) = \begin{pmatrix} + & + & + \end{pmatrix}. \quad (67)$$

To determine whether this channel induces momentum or reversal we apply (48) and find that

$$\begin{aligned} \text{sign}(\boldsymbol{\varepsilon}_t(\mu_{1t}^S)') \text{sign}(\boldsymbol{\varepsilon}_t(S_{1t})) &= \text{sign}(\boldsymbol{\theta}'_t(\mathcal{D}_t \boldsymbol{\sigma}_{1t}^S)) \text{sign}(\boldsymbol{\varepsilon}_t(S_{1t})) \\ &= \begin{pmatrix} + & + & + \end{pmatrix} \times \underbrace{\begin{pmatrix} + & - & - \\ - & + & + \\ - & + & + \end{pmatrix}}_{\text{outperformance}} \begin{pmatrix} + \\ - \\ - \end{pmatrix} = \begin{pmatrix} + & + & + \end{pmatrix} \times \begin{pmatrix} + \\ - \\ - \end{pmatrix} = \pm, \end{aligned} \quad (68)$$

$$\begin{aligned} &= \begin{pmatrix} + & + & + \end{pmatrix} \times \underbrace{\begin{pmatrix} - & + & + \\ + & - & - \\ + & - & - \end{pmatrix}}_{\text{underperformance}} \begin{pmatrix} + \\ - \\ - \end{pmatrix} = \begin{pmatrix} + & + & + \end{pmatrix} \times \begin{pmatrix} - \\ + \\ + \end{pmatrix} = \mp. \end{aligned} \quad (69)$$

By plugging $\boldsymbol{\theta}'_t$, $(\mathcal{D}_t \boldsymbol{\sigma}_{1t}^S)$, and $\boldsymbol{\varepsilon}_t(S_{1t})$, respectively given in (37), (61), and (39), we find that the predictability pattern due to the uncertainty of performance channel is characterized by

$$\begin{aligned} \boldsymbol{\theta}'_t(\mathcal{D}_t \boldsymbol{\sigma}_{1t}^S) \boldsymbol{\varepsilon}_t(S_{1t}) &= (\gamma_2 - \gamma_1)^3 (\mathcal{H}_{3t} - \mathcal{H}_{1t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2 (\bar{\mathcal{H}}_1 - \mathcal{H}_{1t}) \times \\ &\quad [(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu}) + (\gamma_2 - \gamma_1) (\mathcal{H}_{3t} - \mathcal{H}_{1t}) \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2], \end{aligned} \quad (70)$$

where

$$\bar{\mathcal{H}}_1 \equiv \frac{(\gamma_2 - 1) \boldsymbol{\sigma}^{I'} (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) - \gamma_1 \boldsymbol{\nu}' (\boldsymbol{\nu} - \boldsymbol{\sigma}^I)}{(\gamma_2 - \gamma_1) \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2} > 0. \quad (71)$$

The signs of the shock elasticities, (68) and (69), show that index news and active news contribute to opposite predictability patterns: whenever the index news implies positive predictability, the active news implies negative predictability, and vice versa. The sum of these two contributions is determined by whether the hedging demand $\mathcal{H}_{1t}$ is above or below the parameter $\bar{\mathcal{H}}_1$, given in (71).

In other words, the predictability pattern is determined by the relative magnitudes of the market prices of risk for index and active news, as follows

$$\bar{\mathcal{H}}_1 - \mathcal{H}_{1t} = \frac{\kappa_t^I (\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu}) - \kappa_t^\nu (\|\boldsymbol{\nu}\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu})}{(\gamma_2 - \gamma_1) \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2}. \quad (72)$$

When $\mathcal{H}_{1t} < \bar{\mathcal{H}}_1$, the numerator in (72) is positive. In this case, the overall predictability pattern is determined by the predictability pattern of the index news (the first entries in the

last cross products in (68) and (69)).

In outperformance, the hedging demands $\mathcal{H}_{1t}$ and $\mathcal{H}_{3t}$ are small, which implies that the benchmark asset exhibits momentum because $1 - \mathcal{H}_{1t} - \mathcal{H}_{3t} > 0$ and the predictability pattern is determined by index news ($\bar{\mathcal{H}}_1 > \mathcal{H}_{1t}$), implying that “+” in (68) determines the predictability. Therefore, the benchmark asset return exhibits momentum when the asset manager outperforms the benchmark.

In contrast, in underperformance, the hedging demands become larger, leading $1 - \mathcal{H}_{1t} - \mathcal{H}_{3t} < 0$. As long as the underperformance is not too severe, the predictability pattern is determined by the index news ($\bar{\mathcal{H}}_1 > \mathcal{H}_{1t}$), implying that “−” in (69) determines the predictability. Therefore, the benchmark asset return exhibits reversal when the asset manager underperforms the benchmark.

In severe underperformance, the benchmark asset becomes so desired that its market prices of risk κ_t^I drops so low that the numerator in (72) becomes negative. In this case, the predictability pattern is determined by the active news ($\bar{\mathcal{H}}_1 < \mathcal{H}_{1t}$), implying that “+” in (69) determines the predictability. Therefore, the benchmark asset return exhibits momentum when the asset manager severely underperforms the benchmark.

Like the benchmark asset, we determine whether the uncertainty of performance channel drives momentum or reversal in the active asset using (48) as follows.

$$\begin{aligned} \text{sign}(\varepsilon_t(\mu_{2t}^S)') \text{sign}(\varepsilon_t(S_{2t})) &= \text{sign}(\theta_t'(\mathcal{D}_t \sigma_{2t}^S)) \text{sign}(\varepsilon_t(S_{2t})) \\ &= \underbrace{\begin{pmatrix} + & + & + \end{pmatrix} \times \begin{pmatrix} - & + & + \\ + & - & - \\ + & - & - \end{pmatrix}}_{\text{outperformance}} \begin{pmatrix} - \\ + \\ + \end{pmatrix} = \begin{pmatrix} + & + & + \end{pmatrix} \times \begin{pmatrix} + \\ - \\ - \end{pmatrix} = \pm, \end{aligned} \quad (73)$$

$$\begin{aligned} &= \underbrace{\begin{pmatrix} + & + & + \end{pmatrix} \times \begin{pmatrix} + & - & - \\ - & + & + \\ - & + & + \end{pmatrix}}_{\text{underperformance}} \begin{pmatrix} - \\ + \\ + \end{pmatrix} = \begin{pmatrix} + & + & + \end{pmatrix} \times \begin{pmatrix} - \\ + \\ + \end{pmatrix} = \mp. \end{aligned} \quad (74)$$

By plugging θ_t' , $(\mathcal{D}_t \sigma_{2t}^S)$, and $\varepsilon_t(S_{2t})$, respectively given in (37), (62), and (40), we find that the active asset has similar predictability pattern to the benchmark asset due to the uncertainty of performance, which is characterized by

$$\begin{aligned} \theta_t'(\mathcal{D}_t \sigma_{2t}^S) \varepsilon_t(S_{2t}) &= (\gamma_2 - \gamma_1)^3 (\mathcal{H}_{1t} - \mathcal{H}_{4t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) \|\sigma^I - \nu\|^2 (\bar{\mathcal{H}}_1 - \mathcal{H}_{1t}) \times \\ &\quad [(\|\sigma^I\|^2 - \sigma^{I'} \nu) + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t} - \mathcal{H}_{4t}) \|\sigma^I - \nu\|^2]. \end{aligned} \quad (75)$$

where $\bar{\mathcal{H}}_1$ is given in (71).

The characterization reveals that overall, the predictability pattern in the benchmark

asset are similar to those in the active asset.

In outperformance, the hedging demands $\mathcal{H}_{1t}$ and $\mathcal{H}_{4t}$ are small, which implies that the benchmark asset exhibits momentum because $1 - \mathcal{H}_{1t} - \mathcal{H}_{4t} > 0$ and the predictability pattern is determined by index news ($\bar{\mathcal{H}}_1 > \mathcal{H}_{1t}$), implying that “+” in (73) determines the predictability. Therefore, the active asset return exhibits momentum when the asset manager outperforms the benchmark.

In contrast, in underperformance, the hedging demands become larger, leading $1 - \mathcal{H}_{1t} - \mathcal{H}_{4t} < 0$. As long as the underperformance is not too severe, the predictability pattern is determined by the index news ($\bar{\mathcal{H}}_1 > \mathcal{H}_{1t}$), implying that “−” in (74) determines the predictability. Therefore, the benchmark asset return exhibits reversal when the asset manager underperforms the benchmark.

In severe underperformance, the benchmark asset becomes so desired that its market prices of risk κ_t^I drops so low that the numerator in (72) becomes negative. In this case, the predictability pattern becomes determined by the active news ($\bar{\mathcal{H}}_1 < \mathcal{H}_{1t}$), implying that “+” in (74) determines the predictability. Therefore, the benchmark asset return exhibits momentum when the asset manager severely underperforms the benchmark.

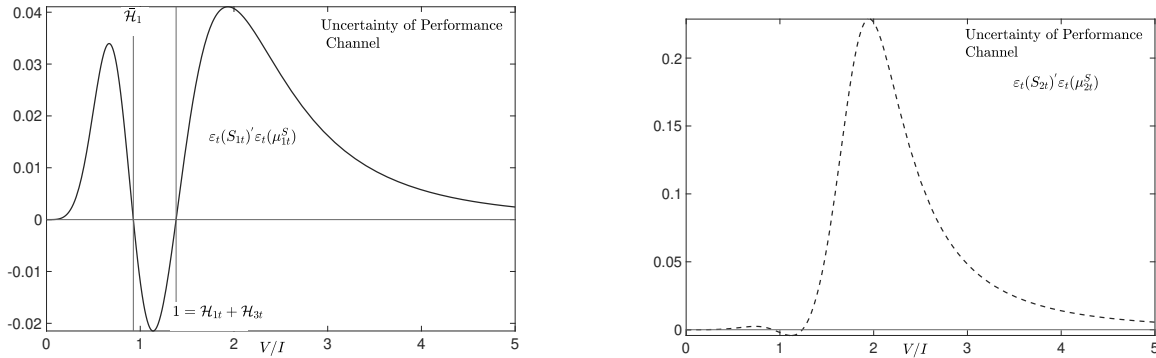


Figure 6. The figures show the predictability arising from the *uncertainty of performance* channel. When V/I is above the vertical line $\bar{\mathcal{H}}_1$, the overall predictability patterns is determined by the predictability in index news. In this case, the *uncertainty of performance* generates momentum when V/I is above the vertical line $1 = \mathcal{H}_{1t} + \mathcal{H}_{3t}$ (indicating the outperformance region), and reversal when it is below (indicating the underperformance region). However, when V/I is below the vertical line $\bar{\mathcal{H}}_{1t}$, the predictability patterns is determined by the predictability of the active news. In this case, the *uncertainty of performance* generates momentum in asset returns. The parameters are as in Figure 5.

To summarize, when the asset manager outperforms the benchmark or severely underperforms the benchmark, the *uncertainty of performance* induces the same reaction in the expected return and the asset return, leading to momentum regions. In mild underperformance, this channel induces an opposite reaction in the expected return and the asset return,

leading to a reversal region.

$$\text{Outperformance: } \Rightarrow \boldsymbol{\varepsilon}_t(\mu_{it}^S)' \boldsymbol{\varepsilon}_t(S_{it}) > 0, \Rightarrow \text{Momentum}, \quad (76)$$

$$\text{Underperformance: } (\mathcal{H}_{1t} \leq \bar{\mathcal{H}}_1) \Rightarrow \boldsymbol{\varepsilon}_t(\mu_{it}^S)' \boldsymbol{\varepsilon}_t(S_{it}) < 0, \Rightarrow \text{Reversal}, \quad (77)$$

$$\text{Underperformance: } (\mathcal{H}_{1t} > \bar{\mathcal{H}}_1) \Rightarrow \boldsymbol{\varepsilon}_t(\mu_{it}^S)' \boldsymbol{\varepsilon}_t(S_{it}) > 0 \Rightarrow \text{Momentum}. \quad (78)$$

5.5 The Joint Effect of the Two Predictable Channels

The concavity above the benchmark causes the quantity effect to dominate and induce momentum in asset returns when the performance of the representative asset manager exceeds the benchmark. In contrast, the convexity below the benchmark causes the price effect to dominate and induce reversal in asset markets when the performance of the representative asset manager falls below the benchmark.

The two predictable channels, *incentives* and *uncertainty of performance*, work in opposite directions when V/I is large such that $\mathcal{H}_{1t} + \mathcal{H}_{3t} < 1$ and $\mathcal{H}_{1t} + \mathcal{H}_{4t} < 1$, but also when V/I is low so much that $\mathcal{H}_{1t} > \bar{\mathcal{H}}_1$.

When taken together, the sum of the two predictable channels lead to an asymmetric price reaction in these two regions. When the representative asset manager underperforms the benchmark, the benchmarking incentives channel dominates and lead to price reversals. However, when the asset manager outperforms the benchmark, the uncertainty of performance channel dominates and leads to price momentum.

In the following proposition, we identify a parametric restriction on the risk-on risk aversion parameter, γ_2 . This ensures that the *benchmarking incentives* is at least as significant as the *uncertainty of performance*. Specifically, when the risk-on risk-aversion parameter falls within the specified range, $\underline{\gamma}_2 \leq \gamma_2 \leq \bar{\gamma}_2$, prices show overall momentum during outperformance periods and reversal in underperformance periods.

The same two mechanisms apply similarly to the active asset, though with a slightly different parametric restriction on the risk-on risk-aversion parameter. When the asset manager underperforms, the reversal channel is more dominant. Conversely, when outperformance is more likely, the momentum channel is stronger under the parametric restriction $\underline{\gamma}_2 \leq \gamma_2 \leq \bar{\gamma}_2$. The following proposition summarizes our main findings.

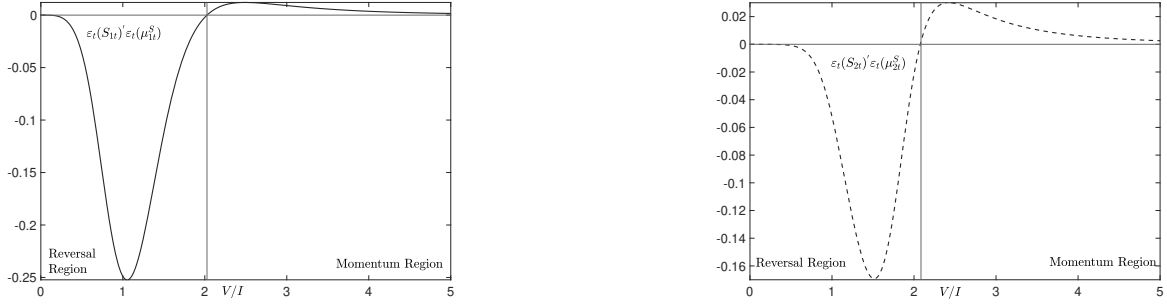


Figure 7. These figures show that momentum, which arises from the uncertainty-of-performance channel, dominates when the asset manager outperforms. Conversely, reversal, which stems from the benchmarking-incentives channel, dominates when the asset manager underperforms. The parameters are as in Figure 5.

Proposition 3 (Time-Series Momentum and Reversal). *The index asset exhibits momentum in the outperformance region reversal in the underperformance region, if and only if*

$$\underline{\gamma}_2 < \gamma_2 < \bar{\gamma}_2. \quad (79)$$

Similarly, the active asset exhibits momentum in the outperformance region and reversal in the underperformance region, if and only if

$$\underline{\underline{\gamma}}_2 < \gamma_2 < \bar{\bar{\gamma}}_2. \quad (80)$$

The risk-on risk-aversion parameter thresholds $\underline{\gamma}_2$, $\bar{\gamma}_2$, $\underline{\underline{\gamma}}_2$, and $\bar{\bar{\gamma}}_2$ depend on the risk-off risk aversion parameter, γ_1 , and the active management share λ . For a given γ_1 , there always exist an active management share λ such that (79) holds. The same is true for the active asset under the parametric restriction in (A.74). The thresholds are given in (A.81), (A.83), (A.86), and (A.87), respectively.

As Figure 8 shows, the baseline pair $(\gamma_1, \gamma_2) = (0.5, 4.5)$ calibrated in Remark 4 lies in the interior of the admissible region for both assets: for the index asset the region admits γ_2 between roughly 3 and 8 at $\gamma_1 = 0.5$ for the benchmark asset, and between roughly 3 and 6 at $\gamma_1 = 0.5$ for the active asset. The momentum-in-outperformance and reversal-in-underperformance prediction therefore does not rely on unusual or finely tuned risk preferences.

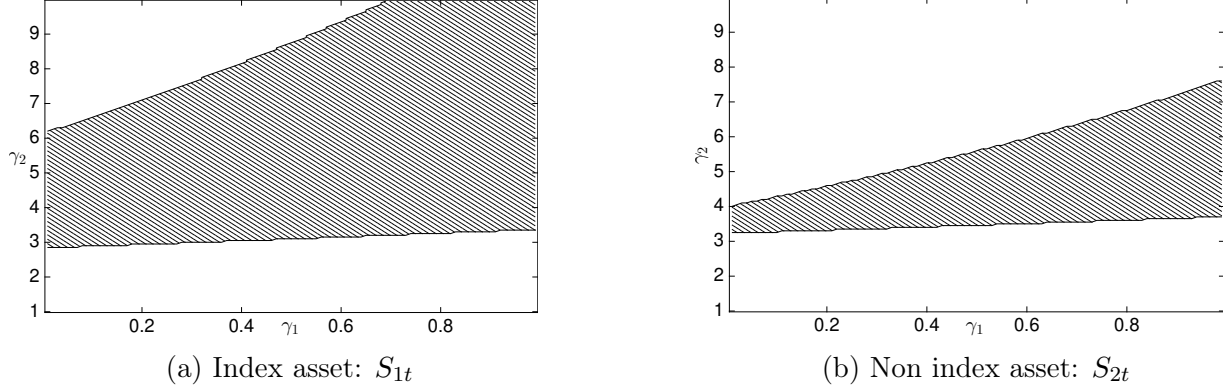


Figure 8. These figures illustrate the risk aversion values γ_1 and γ_2 that satisfy the inequalities (79) and (80) for the index and active asset, respectively. In particular, the shaded regions denote where assets exhibit momentum in outperformance and reversal in underperformance. The figures demonstrate that the parameter space satisfying the model's primary predictions is extensive and encompasses reasonable values. The rest of the parameter values are as in Figure 5.

6 Predictability in Long-Short Portfolio Strategies

Since the work of Jegadeesh and Titman (1993), the analysis of predictability in asset returns has involved constructing long-short portfolios by purchasing assets with high returns and selling those with low returns. According to this framework, momentum is identified when the long-short portfolio yields a positive return, whereas reversal is identified when the portfolio yields a negative return.

In this section, we show that the predictability patterns observed in the time-series of each asset separately also exist in the long-short portfolio. Our theory implies momentum in the long-short portfolio when the asset manager outperforms the benchmark index and reversal when they underperform it.

It is neither obvious nor immediate that time-series predictability would manifest in the long-short portfolio. This phenomenon occurs because index and active asset sensitivities are opposite to one another. Positive news for one implies negative news for the other. The long-short portfolio buys the asset that has risen and sells the asset that has fallen, as the former has a positive return and the latter a negative return. When the asset manager outperforms the benchmark, momentum causes the rising asset to continue rising and the falling asset to continue falling. Consequently, this strategy generates a positive return, aligning with traditional long-short portfolio momentum.

Conversely, when the representative asset manager underperforms the benchmark, a reversal causes the rising asset to fall in the future and the falling asset to rise in the future. As a result, this strategy generates a negative return, aligning with traditional long-short

portfolio reversal. The following proposition summarizes our key findings.

Proposition 4 (Momentum and Reversal in the Cross-Section). *The shock sensitivity of the index asset is positive if and only if the shock sensitivity of the active asset is negative,*

$$\varepsilon_t(S_{1t}, \ell) > 0 \iff \varepsilon_t(S_{2t}, \ell) < 0, \quad \ell = 1, 2, 3. \quad (81)$$

Suppose that the risk-on risk aversion parameter satisfies the conditions (79) and (80), which guarantee time-series momentum in outperformance and time-series reversal in underperformance.

A long-short portfolio that takes a long position on the asset with positive shock sensitivity and a short position on the asset with negative shock sensitivity generates momentum when the asset manager outperforms the benchmark and reversal when it underperforms the benchmark.

7 Conclusion

Asset managers are the marginal investors in U.S. equity markets, and their contracts make their effective preferences depend on where accumulated performance stands relative to a benchmark. This paper takes that institutional fact seriously. Evidence on managers' compensation, career concerns, and risk-taking points to an *S*-shaped managerial objective: Managerial pay is convex below the benchmark and concave above it. Demotion and termination risk rises with underperformance, without a symmetric reward for outperformance. Interim laggards increase risk relative to leaders. We embed this *S*-shaped objective in a dynamic, continuous-time, complete-markets equilibrium.

We further document a stylized fact and, to our knowledge, has not been emphasized before: the wealth-weighted average return of all active U.S. equity mutual funds departs materially from the S&P 500 in a large share of months. Because larger funds tend to move together relative to the benchmark, their aggregate position does not average away to the index. At any point the sector is meaningfully ahead of or behind its benchmark. This fact is what licenses a representative-fund analysis, and our theory is built on it.

Our central result is that this single friction—risk-on incentives when the manager trails the benchmark, risk-off incentives when it leads—endogenously generates momentum following outperformance and reversal following underperformance, and does so simultaneously in the time series of individual assets and in the cross-sectional long-short portfolio. To the best of our knowledge, no existing mechanism delivers this pairing, and in both time-series

and cross-sectional dimensions at once. Equally important, the predictability is fully rational. It survives complete information and no-arbitrage: prices incorporate all available information instantaneously. Predictability persists precisely because it is not optimal for the representative manager to trade it away. The manager responds promptly to news, but its trading intensity depends on accumulated performance relative to the benchmark, and those performance-dependent responses generate the patterns through equilibrium market clearing. This distinguishes our channel from behavioral accounts and from rational theories that rely on asymmetric or slowly diffusing information.

The equilibrium is available in closed form and is characterized by a single state variable: the ratio of accumulated market news to accumulated benchmark news, which governs the likelihood of ultimate out- or underperformance. Two channels emerge from it. The incentives channel is a price effect: benchmark-hedging demand raises the price of benchmark assets and depresses that of active assets. Because expected returns internalize future price impacts, this channel produces reversal symmetrically on both sides of the benchmark. The uncertainty-of-performance channel is a quantity effect: performance uncertainty peaks when accumulated performance sits near the benchmark, which amplifies the sensitivity of prices to news and, interacting with the asymmetric price impacts, produces momentum. Concavity above the benchmark lets the quantity effect dominate and yields momentum in outperformance. Convexity below the benchmark lets the price effect dominate and yields reversal in underperformance. Because the index and active assets load on news with opposite signs, the same asymmetry that governs each asset in the time series carries through to the long-short portfolio. The model’s central prediction is directly testable, and the preliminary fund-level evidence is consistent with it.

The mechanism does not rely on unusual or finely tuned risk preferences. The set of curvature parameters that deliver momentum in outperformance and reversal in underperformance is broad, and the pure risk-on benchmarking objective of Basak and Pavlova (2013) and Buffa and Hodor (2023) is obtained as a limiting case of our specification. More generally, what drives the results is the sign switch of the cross-partial derivative at the benchmark rather than the particular functional form: any objective with that sign structure produces the same qualitative equilibrium dynamics, with the curvature parameters shaping magnitudes rather than directions.

Two features of the environment suggest the mechanism is becoming more, not less, relevant. First, the continued growth of index-tracking vehicles withdraws shares from the traded float and concentrates price formation among benchmarked active managers. Second, from a purely fundamental perspective, the price responses we characterize would be read as overreaction or mispricing, when in fact they are the equilibrium response to a contractual

friction. This has implications for how anomalies are interpreted, and it suggests that features of managerial contracts often treated as internal to the fund—benchmark choice, bonus thresholds, and caps—carry asset-pricing externalities.

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A Appendix – Proofs

Before we begin with the formal proofs, we solve the following expected values.

$$\begin{aligned}\mathcal{E}_{1t}^{\text{Off}} &= (\mathbf{1}_{\text{Off}}) \mathcal{E}_{1t}(1 - \gamma_2, -\gamma_1) \equiv \frac{E_t \left[(I_T)^{1-\gamma_2} (V_T)^{-\gamma_1} \right]}{(I_t)^{1-\gamma_2} (V_t)^{-\gamma_1}} \\ &= e^{((- \gamma_1)(\eta) - (\gamma_2 - 1)(\mu_1) + (\sigma_1 \nu_1)(\gamma_1)(\gamma_2 - 1) + \frac{1}{2}(\sigma_1)^2(\gamma_2)(\gamma_2 - 1) + \frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_1)(1 + \gamma_1))(T-t)},\end{aligned}\tag{A.1}$$

$$\begin{aligned}\mathcal{E}_{1t}^{\text{On}} &= (\mathbf{1}_{\text{On}}) \mathcal{E}_{1t}(1 - \gamma_1, -\gamma_2) \equiv \frac{E_t \left[(I_T)^{1-\gamma_1} (V_T)^{-\gamma_2} \right]}{(I_t)^{1-\gamma_1} (V_t)^{-\gamma_2}} \\ &= e^{((- \gamma_2)(\eta) + (1 - \gamma_1)(\mu_1) - (\sigma_1 \nu_1)(\gamma_2)(1 - \gamma_1) - \frac{1}{2}(\sigma_1)^2(\gamma_1)(1 - \gamma_1) + \frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_2)(1 + \gamma_2))(T-t)},\end{aligned}\tag{A.2}$$

$$\frac{\mathcal{E}_{1t}^{\text{Off}}}{\mathcal{E}_{1t}^{\text{On}}} \equiv \mathcal{E}_{1t} = e^{((\gamma_2 - \gamma_1)(\eta - \mu_1 + \sigma_1 \nu_1 + \frac{1}{2}(\sigma_1)^2(\gamma_1 + \gamma_2 - 1) - \frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_1 + \gamma_2 + 1)))(T-t)},\tag{A.3}$$

$$\begin{aligned}\mathcal{E}_{2t}^{\text{Off}} &= (\mathbf{1}_{\text{Off}}) \mathcal{E}_{2t}(1 - \gamma_2, 1 - \gamma_1) \equiv \frac{E_t \left[(I_T)^{1-\gamma_2} (V_T)^{1-\gamma_1} \right]}{(I_t)^{1-\gamma_2} (V_t)^{1-\gamma_1}} \\ &= e^{((1 - \gamma_1)(\eta) - (\gamma_2 - 1)(\mu_1) - (\sigma_1 \nu_1)(1 - \gamma_1)(\gamma_2 - 1) + \frac{1}{2}(\sigma_1)^2(\gamma_2)(\gamma_2 - 1) + \frac{1}{2}\|\boldsymbol{\nu}\|^2(-\gamma_1)(1 - \gamma_1))(T-t)},\end{aligned}\tag{A.4}$$

$$\begin{aligned}\mathcal{E}_{2t}^{\text{On}} &= (\mathbf{1}_{\text{On}}) \mathcal{E}_{2t}(1 - \gamma_1, 1 - \gamma_2) \equiv \frac{E_t \left[(I_T)^{1-\gamma_1} (V_T)^{1-\gamma_2} \right]}{(I_t)^{1-\gamma_1} (V_t)^{1-\gamma_2}} \\ &= e^{((1 - \gamma_2)(\eta) + (1 - \gamma_1)(\mu_1) - (\sigma_1 \nu_1)(\gamma_2 - 1)(1 - \gamma_1) - \frac{1}{2}(\sigma_1)^2(\gamma_1)(1 - \gamma_1) + \frac{1}{2}\|\boldsymbol{\nu}\|^2(\gamma_2)(\gamma_2 - 1))(T-t)},\end{aligned}\tag{A.5}$$

$$\frac{\mathcal{E}_{2t}^{\text{Off}}}{\mathcal{E}_{2t}^{\text{On}}} \equiv \mathcal{E}_{2t} = e^{((\gamma_2 - \gamma_1)(\eta - \mu_1 + \frac{1}{2}[(\sigma_1)^2 - \|\boldsymbol{\nu}\|^2](\gamma_1 + \gamma_2 - 1)))(T-t)},\tag{A.6}$$

$$\begin{aligned}\mathcal{E}_{3t}^{\text{Off}} &= (\mathbf{1}_{\text{Off}}) \mathcal{E}_{3t}(2 - \gamma_2, -\gamma_1) \equiv \frac{E_t \left[(I_T)^{2-\gamma_2} (V_T)^{-\gamma_1} \right]}{(I_t)^{2-\gamma_2} (V_t)^{-\gamma_1}} \\ &= e^{(-\gamma_1\eta + (2 - \gamma_2)(\mu_1) - (\sigma_1 \nu_1)\gamma_1(2 - \gamma_2) + \frac{1}{2}(\sigma_1)^2(\gamma_2 - 1)(\gamma_2 - 2) + \frac{1}{2}\|\boldsymbol{\nu}\|^2\gamma_1(1 + \gamma_1))(T-t)},\end{aligned}\tag{A.7}$$

$$\begin{aligned}\mathcal{E}_{3t}^{\text{On}} &= (\mathbf{1}_{\text{On}}) \mathcal{E}_{3t}(2 - \gamma_1, -\gamma_2) \equiv \frac{E_t \left[(I_T)^{2-\gamma_1} (V_T)^{-\gamma_2} \right]}{(I_t)^{2-\gamma_1} (V_t)^{-\gamma_2}} \\ &= e^{(-\gamma_2\eta + (2 - \gamma_1)(\mu_1) - (\sigma_1 \nu_1)\gamma_2(2 - \gamma_1) + \frac{1}{2}(\sigma_1)^2(\gamma_1 - 1)(\gamma_1 - 2) + \frac{1}{2}\|\boldsymbol{\nu}\|^2\gamma_2(1 + \gamma_2))(T-t)},\end{aligned}\tag{A.8}$$

$$\frac{\mathcal{E}_{3t}^{\text{Off}}}{\mathcal{E}_{3t}^{\text{On}}} \equiv \mathcal{E}_{3t} = e^{(\gamma_2 - \gamma_1)(\eta - \mu_1 + 2(\sigma_1 \nu_1) + \frac{1}{2}(\sigma_1)^2(\gamma_2 + \gamma_1 - 3) - \frac{1}{2}\|\nu\|^2(\gamma_2 + \gamma_1 + 1))^{(T-t)}}, \quad (\text{A.9})$$

$$\begin{aligned} \mathcal{E}_{4t}^{\text{Off}} &= (\mathbf{1}_{\text{Off}}) \mathcal{E}_{4t} (1 - \gamma_2, -\gamma_1) \equiv \frac{E_t \left[(I_T)^{1-\gamma_2} (V_T)^{-\gamma_1} D_{2T} \right]}{(I_t)^{1-\gamma_2} (V_t)^{-\gamma_1} D_{2t}} \\ &= e^{[-(\gamma_2 - 1)\mu_1 - \gamma_1\eta + \mu_2 + (\sigma_1 \nu_1)(\gamma_1)(\gamma_2 - 1) - (\sigma_2 \nu_2)(\gamma_1) + \frac{1}{2}(\sigma_1)^2(\gamma_2 - 1)(\gamma_2) + \frac{1}{2}\|\nu\|^2\gamma_1(1 + \gamma_1)]^{(T-t)}}, \end{aligned} \quad (\text{A.10})$$

$$\begin{aligned} \mathcal{E}_{4t}^{\text{On}} &= (\mathbf{1}_{\text{On}}) \mathcal{E}_{4t} (1 - \gamma_1, -\gamma_2) \equiv \frac{E_t \left[(I_T)^{1-\gamma_1} (V_T)^{-\gamma_2} D_{2T} \right]}{(I_t)^{1-\gamma_1} (V_t)^{-\gamma_2} D_{2t}} \\ &= e^{[-(\gamma_1 - 1)\mu_1 - \gamma_2\eta + \mu_2 + (\sigma_1 \nu_1)(\gamma_2)(\gamma_1 - 1) - (\sigma_2 \nu_2)(\gamma_2) + \frac{1}{2}(\sigma_1)^2(\gamma_1 - 1)(\gamma_1) + \frac{1}{2}\|\nu\|^2\gamma_2(1 + \gamma_2)]^{(T-t)}}, \end{aligned} \quad (\text{A.11})$$

$$\frac{\mathcal{E}_{4t}^{\text{Off}}}{\mathcal{E}_{4t}^{\text{On}}} \equiv \mathcal{E}_{4t} = e^{(\gamma_2 - \gamma_1)[- \mu_1 + \eta + (\sigma_1 \nu_1) + (\sigma_2 \nu_2) + \frac{1}{2}(\sigma_1)^2(\gamma_2 + \gamma_1 - 1) - \frac{1}{2}\|\nu\|^2(\gamma_2 + \gamma_1 + 1)]^{(T-t)}}. \quad (\text{A.12})$$

We reference to these expected values throughout the proofs. Note that $\mathcal{E}_{it}$, $i = 1, 2, 3, 4$ is not well defined in the risk-off incentive case, since $\mathcal{E}_{it}^{\text{On}} = 0$. In pure risk-on incentives $\mathcal{E}_{it} = 0$, $i = 1, 2, 3, 4$ since $\mathcal{E}_{it}^{\text{Off}} = 0$. By plugging the definition of $\mathcal{E}_{it}$ into $\mathcal{H}_{it}$ we find that

$$\mathcal{H}_{it} \equiv \frac{1}{1 + \frac{1-\gamma_1}{\gamma_2-1} (V_t/I_t)^{\gamma_2-\gamma_1} \mathcal{E}_{it}} = \frac{\mathcal{E}_{it}^{\text{On}}}{\mathcal{E}_{it}^{\text{On}} + \frac{1-\gamma_1}{\gamma_2-1} (V_t/I_t)^{\gamma_2-\gamma_1} \mathcal{E}_{it}^{\text{Off}}}, \quad (\text{A.13})$$

implying that $\mathcal{H}_{it} = 0$ in the pure risk-off incentives and $\mathcal{H}_{it} = 1$ in the pure risk-on incentives. Notice that

$$\|\sigma^I\|^2 - \sigma^{I'} \nu = \frac{\sigma_1^2}{N} (N - \lambda) > 0, \quad (\text{A.14})$$

$$\|\nu\|^2 - \nu' \sigma^I = \frac{\sigma_1^2}{N^2} (1 - \lambda) ((N - 1) - \lambda), \quad (\text{A.15})$$

implying that $\|\nu\|^2 - \nu' \sigma^I$ approached zero as $\lambda \rightarrow 1$, and that

$$\|\sigma^I\|^2 - \sigma^{I'} \nu > \|\nu\|^2 - \nu' \sigma^I. \quad (\text{A.16})$$

By using the inequalities (A.14) and (A.15), and the that positive news have a positive

impact on fundamentals, $\sigma_2 > 0$ and $\nu_2 > 0$, we find that

$$\mathcal{H}_{3t} > \mathcal{H}_{1t} \iff \mathcal{E}_{1t} > \mathcal{E}_{3t} \iff e^{(\gamma_2 - \gamma_1)(\|\sigma^I\|^2 - \sigma^{I'}\nu)(T-t)} > 1, \quad (\text{A.17})$$

$$\mathcal{H}_{1t} > \mathcal{H}_{4t} \iff \mathcal{E}_{4t} > \mathcal{E}_{1t} \iff e^{(\gamma_2 - \gamma_1)(\sigma_2\nu_2)(T-t)} > 1, \quad (\text{A.18})$$

$$\mathcal{H}_{1t} > \mathcal{H}_{2t} \iff \mathcal{E}_{2t} > \mathcal{E}_{1t} \iff e^{(\gamma_2 - \gamma_1)(\|\nu\|^2 - \nu'\sigma^I)(T-t)} > 1, \quad (\text{A.19})$$

$$\mathcal{H}_{3t} > \mathcal{H}_{2t} \iff \mathcal{E}_{2t} > \mathcal{E}_{3t} \iff e^{(\gamma_2 - \gamma_1)(\|\sigma^I\|^2 - \nu'\sigma^I)(T-t)} > 1. \quad (\text{A.20})$$

The definition of the risk-on hedging demand (23) and the ensuing analysis implies that in extreme underperformance the ratio of market news to index news approaches zero, $V/I \downarrow 0$, while in extreme outperformance the ratio approaches infinity, $V/I \downarrow \infty$, implying that

$$\lim_{V/I \rightarrow 0} \mathcal{H}_{it} - \mathcal{H}_{jt} \rightarrow 0, \quad \lim_{V/I \rightarrow \infty} \mathcal{H}_{it} - \mathcal{H}_{jt} \rightarrow 0. \quad (\text{A.21})$$

The derivative of the risk-on hedging demand with respect to V/I equals

$$\frac{d\mathcal{H}_{it}}{d\frac{V}{I}} = -(\gamma_2 - \gamma_1) \mathcal{H}_{it} (1 - \mathcal{H}_{it}) (V_t/I_t)^{-1} < 0, \quad (\text{A.22})$$

and the derivative of $\mathcal{H}_{it} - \mathcal{H}_{jt}$ with respect to V/I is given by

$$\frac{d\mathcal{H}_{it}}{d\frac{V}{I}} - \frac{d\mathcal{H}_{jt}}{d\frac{V}{I}} = -(\gamma_2 - \gamma_1) (V_t/I_t)^{-1} (\mathcal{H}_{it} - \mathcal{H}_{jt}) (1 - \mathcal{H}_{it} - \mathcal{H}_{jt}), \quad (\text{A.23})$$

which approaches zero at extreme outperformance and underperformance due to (A.21). Next, we find that $\mathcal{H}_{3t} - \mathcal{H}_{1t}$ converges to zero as fast as $\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})$ converges to zero, both when $\mathcal{H}_{it} \rightarrow 0$ and $\mathcal{H}_{it} \rightarrow 1$, and the limits are given by

$$\lim_{\mathcal{H}_{it} \rightarrow 0} = \lim_{V/I \rightarrow \infty} \frac{\mathcal{H}_{jt} - \mathcal{H}_{1t}}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} = \frac{\mathcal{E}_{1t} - \mathcal{E}_{jt}}{\mathcal{E}_{jt}} > 0, \quad j = 3, 4, \quad (\text{A.24})$$

$$\lim_{\mathcal{H}_{it} \rightarrow 1} = \lim_{V/I \rightarrow 0} \frac{\mathcal{H}_{jt} - \mathcal{H}_{1t}}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} = \frac{\mathcal{E}_{1t} - \mathcal{E}_{jt}}{\mathcal{E}_{1t}} > 0. \quad (\text{A.25})$$

Lastly,

$$\lim_{V/I \rightarrow 0} \frac{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})}{V/I} = \lim_{V/I \rightarrow \infty} \frac{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})}{V/I} = 0, \quad (\text{A.26})$$

$$\lim_{V/I \rightarrow 0} \frac{\mathcal{H}_{jt} - \mathcal{H}_{1t}}{V/I} = \lim_{V/I \rightarrow \infty} \frac{\mathcal{H}_{jt} - \mathcal{H}_{1t}}{V/I} = 0, \quad j = 3, 4. \quad (\text{A.27})$$

Proof of Proposition 5 (Discount Factor). The security market is dynamically complete. As such, there exists a unique state price density process, ξ , and the no arbitrage relations

$$\xi_t S_{kt} = \mathbb{E}_t [\xi_T D_{kT}], \quad t \in [0, T], k = 1, 2, 3, \quad (\text{A.28})$$

is always satisfied. In our setup, we set $r = 0$ for $t \leq T$, and thus, the state price density evolves according to

$$d\xi_t = -\xi_t (\boldsymbol{\theta}'_t d\mathbf{Z}_t), \quad t \leq T. \quad (\text{A.29})$$

The vector process $\boldsymbol{\theta}_t$ is the cash-flow news market risk prices. Restating the dynamic budget constraint as

$$\xi_t W_t^A = \mathbb{E}_t [\xi_T W_T^A], \quad t \in [0, T], \quad (\text{A.30})$$

and maximizing the asset manager's objective function (16), subject to (A.30) at time $t = 0$, we obtain the first order condition

$$\mathbf{1}_{\text{Off}} \frac{(S_T^I)^{1-\gamma_2} (W_T^A)^{1-\gamma_1}}{(\gamma_2 - 1)(1 - \gamma_1)} - \mathbf{1}_{\text{On}} \frac{(S_T^I)^{1-\gamma_1} (W_T^A)^{1-\gamma_2}}{(\gamma_2 - 1)(1 - \gamma_1)} \quad (\text{A.31})$$

$$y^A \xi_T = \mathbf{1}_{\text{Off}} \frac{(S_T^I)^{1-\gamma_2} (W_T^A)^{-\gamma_1}}{(\gamma_2 - 1)} + \mathbf{1}_{\text{On}} \frac{(S_T^I)^{1-\gamma_1} (W_T^A)^{-\gamma_2}}{(1 - \gamma_1)}, \quad (\text{A.32})$$

$$y^A \xi_T = \mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (W_T^A)^{-\gamma_1}}{(\gamma_2 - 1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (W_T^A)^{-\gamma_2}}{(1 - \gamma_1)}, \quad (\text{A.33})$$

where y^A is the Lagrange multiplier, and due to no-arbitrage condition at time T , $S_T^I = I_T$.

To clear markets, the asset manager must hold the entire active supply S_t^A , which at time T equals V_T , leading to

$$y^A \xi_T = \mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{-\gamma_1}}{(\gamma_2 - 1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{-\gamma_2}}{(1 - \gamma_1)}. \quad (\text{A.34})$$

Since ξ_t is a martingale, we have

$$\xi_{0,t} = \frac{\mathbb{E}_t \left[\mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{-\gamma_2}}{(1-\gamma_1)} \right]}{\mathbb{E} \left[\mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{-\gamma_2}}{(1-\gamma_1)} \right]}. \quad (\text{A.35})$$

By plugging (A.1) and (A.2) into (A.35) we find that

$$\xi_{0,t} = \frac{\frac{\mathbf{1}_{\text{Off}}}{\gamma_2-1} (I_t)^{1-\gamma_2} (V_t)^{-\gamma_1} \mathcal{E}_{1t}^{\text{Off}} + \frac{\mathbf{1}_{\text{On}}}{1-\gamma_1} (I_t)^{1-\gamma_1} (V_t)^{-\gamma_2} \mathcal{E}_{1t}^{\text{On}}}{\frac{\mathbf{1}_{\text{Off}}}{\gamma_2-1} (I_0)^{1-\gamma_2} (V_0)^{-\gamma_1} \mathcal{E}_{10}^{\text{Off}} + \frac{\mathbf{1}_{\text{On}}}{1-\gamma_1} (I_0)^{1-\gamma_1} (V_0)^{-\gamma_2} \mathcal{E}_{10}^{\text{On}}}. \quad (\text{A.36})$$

To find the Lagrange multiplier, we plug the equilibrium discount factor (A.34) into the dynamic budget constraint (A.30) evaluated at time $t = 0$, apply the market clearing condition so that the asset manager must hold the entire active supply and find that

$$\xi_0 S_0^A = \mathbb{E} \left[\frac{1}{y^A} \left(\mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{1-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{1-\gamma_2}}{(1-\gamma_1)} \right) \right], \quad (\text{A.37})$$

which eventually leads to

$$\begin{aligned} y^A &= \frac{1}{\xi_0 S_0^A} \mathbb{E} \left[\left(\mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{1-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{1-\gamma_2}}{(1-\gamma_1)} \right) \right] \\ &= \frac{1}{\xi_0 S_0^A} \left[\frac{\mathbf{1}_{\text{Off}}}{\gamma_2-1} (I_0)^{1-\gamma_2} (V_0)^{1-\gamma_1} \mathcal{E}_{20} (1-\gamma_2, 1-\gamma_1) + \frac{\mathbf{1}_{\text{On}}}{1-\gamma_1} (I_0)^{1-\gamma_1} (V_0)^{1-\gamma_2} \mathcal{E}_{20} (1-\gamma_1, 1-\gamma_2) \right], \end{aligned} \quad (\text{A.38})$$

where the second equality is due to (A.4) and (A.5) evaluated at time $t = 0$.

We conclude the proof with the discount factor derivatives. It is straightforward to show that $\xi_{0,t}$ is decreasing in V_t and increasing in I_t in the risk-on incentives case, (B.1), and $\xi_{0,t}$ is decreasing in V_t and I_t in the risk-off incentives case, (B.2). The derivative of the discount factor with respect to I_t in the joint incentives case equals

$$\frac{\partial \xi_{0,t}}{\partial I_t} = (1-\gamma_1) \frac{1}{I_t} \xi_{0,t} - \xi_{0,t} \frac{1}{\mathcal{H}_{1t}} \frac{\partial \mathcal{H}_{1t}}{\partial I_t}, \quad (\text{A.39})$$

where the derivative of the risk-on hedging demand with respect to I_t equals

$$\frac{\partial \mathcal{H}_{1t}}{\partial I_t} = (\mathcal{H}_{1t})^2 \left(\frac{1-\gamma_1}{\gamma_2-1} (\gamma_2-\gamma_1) (I_t)^{\gamma_1-\gamma_2-1} (V_t)^{\gamma_2-\gamma_1} \mathcal{E}_{1t} \right) > 0. \quad (\text{A.40})$$

By plugging the hedging demand derivative (A.40) into the discount factor derivative (A.39), and by removing strictly positive terms and rearranging, we find that $\partial \xi_{0,t} / \partial I_t < 0$ if and

only if

$$(1 - \gamma_1) - (\gamma_2 - \gamma_1)(-\mathcal{H}_{1t} + 1) < 0, \quad (\text{A.41})$$

which leads to our desired result in (B.4). Similarly, the derivative of the discount factor with respect to V_t in the joint incentives case equals

$$\frac{\partial \xi_{0,t}}{\partial V_t} = (-\gamma_2) \frac{1}{V_t} \xi_t - \xi_t \frac{1}{\mathcal{H}_{1t}} \frac{\partial \mathcal{H}_{1t}}{\partial V_t}, \quad (\text{A.42})$$

where the derivative of the risk-on hedging demand with respect to V_t equals

$$\frac{\mathcal{H}_{it}}{\partial V_t} = -(\mathcal{H}_{it})^2 \left(\frac{1 - \gamma_1}{\gamma_2 - 1} (\gamma_2 - \gamma_1) (I_t)^{\gamma_1 - \gamma_2} (V_t)^{\gamma_2 - \gamma_1 - 1} \mathcal{E}_{1t} \right) < 0, \quad i = 1, 2, 3, 4. \quad (\text{A.43})$$

By plugging the hedging demand derivative (A.43) into the discount factor derivative (A.42) and rearranging, we find that $\partial \xi_{0,t} / \partial V_t < 0$ if and only if

$$(\gamma_2 - \gamma_1) \mathcal{H}_{1t} + \gamma_1 > 0, \quad (\text{A.44})$$

which is always true because $\gamma_2 > \gamma_1$ and $\mathcal{H}_{1t} > 0$. □

Proof of Lemma 1 (Risk Exposures). We utilize the no-arbitrage condition (A.30) at time t , the market clearing condition, and write

$$\xi_t W_t^{\mathcal{A}} = \mathbb{E}_t [\xi_T V_T] = \frac{1}{y^{\mathcal{A}}} \mathbb{E}_t \left[\left(\mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{1-\gamma_1}}{(\gamma_2 - 1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{1-\gamma_2}}{(1 - \gamma_1)} \right) \right]. \quad (\text{A.45})$$

By plugging (A.4) and (A.5) we obtain

$$\xi_t W_t^{\mathcal{A}} = \frac{1}{y^{\mathcal{A}}} \left[\frac{\mathbf{1}_{\text{Off}}}{\gamma_2 - 1} (I_t)^{1-\gamma_2} (V_t)^{1-\gamma_1} \mathcal{E}_{2t}^{\text{Off}} + \frac{\mathbf{1}_{\text{On}}}{1 - \gamma_1} (I_t)^{1-\gamma_1} (V_t)^{1-\gamma_2} \mathcal{E}_{2t}^{\text{On}} \right] \quad (\text{A.46})$$

Applying Itô's Lemma on both sides of the above equation leads to the desired result. □

Proof of Proposition 1 (Market Price of Risk). Applying Itô's Lemma on both sides of (A.36) leads to the desired result. □

Proof of Proposition 6 (Stock Prices). We start from the no arbitrage condition (A.28), plug $\xi_{0,T}$ (A.36), and find that the deflated index price satisfies

$$S_t^I \xi_{0,t} = \frac{\mathbb{E}_t \left[\left(\mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{-\gamma_2}}{(1-\gamma_1)} \right) I_T \right]}{\mathbb{E} \left[\mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{-\gamma_2}}{(1-\gamma_1)} \right]}. \quad (\text{A.47})$$

By plugging the expected values from (A.7) and (A.8), $\xi_{0,t}$ (A.36), and rearranging we obtain the desired result. Next, we characterize the active price. Similar to the index price, we find that the deflated active price satisfies

$$S_{2t} \xi_{0,t} = \frac{\mathbb{E}_t \left[\left(\mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{-\gamma_1} D_{2T}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{-\gamma_2} D_{2T}}{(1-\gamma_1)} \right) \right]}{\mathbb{E} \left[\mathbf{1}_{\text{Off}} \frac{(I_T)^{1-\gamma_2} (V_T)^{-\gamma_1}}{(\gamma_2-1)} + \mathbf{1}_{\text{On}} \frac{(I_T)^{1-\gamma_1} (V_T)^{-\gamma_2}}{(1-\gamma_1)} \right]}. \quad (\text{A.48})$$

By plugging the expected values from (A.10) and (A.11) and $\xi_{0,t}$ (A.36), and rearranging we obtain the desired result. We conclude the proof by taking the derivative of the index and active price dividend ratios, and find that

$$\partial (S_t^I / I_t) / \partial V_t = \frac{\mathcal{E}_{3t}^{\text{On}}}{\mathcal{E}_{1t}^{\text{On}} (\mathcal{H}_{3t})^2} [(\partial \mathcal{H}_{1t} / \partial V_t) \mathcal{H}_{3t} - (\partial \mathcal{H}_{3t} / \partial V_t) \mathcal{H}_{1t}] \quad (\text{A.49})$$

$$\partial (S_{2t} / D_{2t}) / \partial V_t = \frac{\mathcal{E}_{4t}^{\text{On}}}{\mathcal{E}_{1t}^{\text{On}} (\mathcal{H}_{4t})^2} [(\partial \mathcal{H}_{1t} / \partial V_t) \mathcal{H}_{4t} - (\partial \mathcal{H}_{4t} / \partial V_t) \mathcal{H}_{1t}]. \quad (\text{A.50})$$

By plugging the derivative of $\mathcal{H}_{it}$, $i = 1, 3, 4$, (23) with respect to V_t , into the price-dividend ratios (A.49) and (A.50), and by removing strictly positive terms and rearranging, we find that

$$\partial (S_t^I / I_t) / \partial V_t < 0 \iff \mathcal{E}_{3t} - \mathcal{E}_{1t} < 0, \quad (\text{A.51})$$

$$\partial (S_{2t} / D_{2t}) / \partial V_t > 0 \iff \mathcal{E}_{4t} - \mathcal{E}_{1t} > 0. \quad (\text{A.52})$$

The right hand side inequalities are true due to (A.17) and (A.18), respectively, implying that the left hand side inequality are also true, which is our desire result. In similar fashion, we obtain

$$\partial (S_t^I / I_t) / \partial I_t > 0 \iff \mathcal{E}_{3t} - \mathcal{E}_{1t} < 0, \quad (\text{A.53})$$

$$\partial (S_{2t} / D_{2t}) / \partial I_t < 0 \iff \mathcal{E}_{4t} - \mathcal{E}_{1t} > 0. \quad (\text{A.54})$$

The right hand side is always satisfied due to (A.17) and (A.18), respectively, implying that the left hand side inequality are also true, which is our desire result.

We continue with the comparison of the index price-dividend ratio and the active price-dividend ratio. Dividing the index price-dividend ratio by the active price-dividend ratio (B.5) in the pure risk-on incentives, we obtain

$$\begin{aligned}\frac{S_t^I/I_t}{S_{2t}/D_{2t}} &= \frac{\mathcal{E}_{3t}^{\text{On}}}{\mathcal{E}_{4t}^{\text{On}}} = e^{(\mu_1 - \mu_2 - (\sigma_1 \nu_1) \gamma_2 + (\sigma_2 \nu_2) \gamma_2 + (\sigma_1)^2 (1 - \gamma_1))(T-t)} \\ &= e^{-(\lambda \sigma^2/3) \gamma_2 + (\sigma^2/3) \gamma_2 + (\sigma)^2 (1 - \gamma_1)(T-t)} > 1.\end{aligned}\quad (\text{A.55})$$

We obtain the second equality by plugging the definitions of $\mathcal{E}_{3t}^{\text{On}}$ (A.8) and $\mathcal{E}_{4t}^{\text{On}}$ (A.11) and cancelling similar terms, and the last equality by using the assumptions stating that $\mu_1 = \mu_2$, $\sigma_1 = \sigma_2$, and $\nu_1 = \lambda \sigma/3$, $\nu_2 = \sigma/3$. In similar fashion, dividing the index price-dividend ratio by the active price-dividend ratio (B.6) in the pure risk-off incentives, we obtain

$$\begin{aligned}\frac{S_t^I/I_t}{S_{2t}/D_{2t}} &= \frac{\mathcal{E}_{3t}^{\text{Off}}}{\mathcal{E}_{4t}^{\text{Off}}} = e^{(\mu_1 - \mu_2 - (\sigma_1 \nu_1) \gamma_1 + (\sigma_2 \nu_2) \gamma_1 - (\sigma_1)^2 (\gamma_2 - 1))(T-t)} \\ &= e^{-(\lambda \sigma^2/3) \gamma_1 + (\sigma^2/3) \gamma_1 - (\sigma)^2 (\gamma_2 - 1)(T-t)} < 1.\end{aligned}\quad (\text{A.56})$$

We obtain the second equality by plugging the definitions of $\mathcal{E}_{3t}^{\text{Off}}$ (A.7) and $\mathcal{E}_{4t}^{\text{Off}}$ (A.10) and cancelling similar terms, and the last equality by using the assumptions stating that $\mu_1 = \mu_2$, $\sigma_1 = \sigma_2$, and $\nu_1 = \lambda \sigma/3$, $\nu_2 = \sigma/3$. The inequality is true because the exponent simplifies to $\sigma^2 \left(\frac{\gamma_1}{3} (1 - \lambda) + 1 - \gamma_2 \right) < 0$, which is negative because we assume that $\gamma_2 - \gamma_1 > 1$.

Lastly, dividing the index price-dividend ratio by the active price-dividend ratio (B.7) in the joint incentives case, we obtain

$$\frac{S_t^I/I_t}{S_{2t}/D_{2t}} = \frac{\frac{\mathcal{E}_{3t}^{\text{On}}}{\mathcal{H}_{3t}}}{\frac{\mathcal{E}_{4t}^{\text{On}}}{\mathcal{H}_{4t}}} = \frac{\left(\mathcal{E}_{3t}^{\text{On}} + \frac{1-\gamma_1}{\gamma_2-1} (V_t/I_t)^{\gamma_2-\gamma_1} \mathcal{E}_{3t}^{\text{Off}} \right)}{\left(\mathcal{E}_{4t}^{\text{On}} + \frac{1-\gamma_1}{\gamma_2-1} (V_t/I_t)^{\gamma_2-\gamma_1} \mathcal{E}_{4t}^{\text{Off}} \right)}, \quad (\text{A.57})$$

where the last equality is obtained by plugging the definitions of $\mathcal{E}_{3t}^{\text{On}}$ (A.8) and $\mathcal{E}_{4t}^{\text{On}}$ (A.11) and the definitions of $\mathcal{H}_{3t}$ and $\mathcal{H}_{4t}$, (23), and cancelling similar terms. Thus, we find that $\frac{S_t^I/I_t}{S_{2t}/D_{2t}} > 1$ if and only if V/I is below the deterministic threshold in (B.8). The threshold is well defined because $\mathcal{E}_{3t}^{\text{On}} > \mathcal{E}_{4t}^{\text{On}}$ and $\mathcal{E}_{4t}^{\text{Off}} > \mathcal{E}_{3t}^{\text{Off}}$ since $\gamma_2 - \gamma_1 > 1$. $\square$

Proof of Proposition 2 (Risk Exposures). We start with the joint incentives case. By applying Itô's Lemma on both sides of the deflated index price (A.47), given (A.7) and (A.8), we obtain

$$\sigma_{1t}^S = \theta_t + (2 - \gamma_2) \sigma^I - \gamma_1 \nu + (\gamma_2 - \gamma_1) (\mathcal{H}_{3t}) (\sigma^I - \nu). \quad (\text{A.58})$$

And, plugging θ_t (31) and rearranging leads to the desired result.

$$\boldsymbol{\sigma}_{1t}^S = \boldsymbol{\sigma}_1^D + (\gamma_2 - \gamma_1) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) (\mathcal{H}_{3t} - \mathcal{H}_{1t}). \quad (\text{A.59})$$

Note that $\mathcal{H}_{3t} > \mathcal{H}_{1t}$ since $\mathcal{E}_{1t} > \mathcal{E}_{3t}$, as (A.17) shows. Therefore, the first entry is always positive, $\boldsymbol{\sigma}_{1t}^S(1) > 0$ since $\nu_1 = \lambda\sigma/3 < \sigma$, while the second and third entries are always negative because $\boldsymbol{\sigma}^I \equiv \boldsymbol{\sigma}_1^D = 0$ for these two entries. In similar fashion, by applying Itô's Lemma on both sides of the deflated active price (A.48), given (A.7) and (A.8), we obtain

$$\boldsymbol{\sigma}_{2t}^S = \boldsymbol{\theta}_t + \boldsymbol{\sigma}_2^D - (\gamma_2 - 1) \boldsymbol{\sigma}^I - \gamma_1 \boldsymbol{\nu} + (\gamma_2 - \gamma_1) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) (\mathcal{H}_{4t}). \quad (\text{A.60})$$

And, plugging θ_t (31) and rearranging leads to the desired result. Note that $\mathcal{H}_{4t} < \mathcal{H}_{1t}$ since $\mathcal{E}_{1t} < \mathcal{E}_{4t}$, as (A.18) shows. Therefore, the first entry is always negative, $\boldsymbol{\sigma}_{2t}^S(1) < 0$, because $\nu_1 = \lambda\sigma/3 < \sigma$ and $\boldsymbol{\sigma}_2^D(1) = 0$, while the second and third entries are always positive because $\boldsymbol{\sigma}^I = 0$ and $\boldsymbol{\nu} > 0$ for the second and third entries, and $\boldsymbol{\sigma}_2^D(2) = \sigma > 0$.

$$\boldsymbol{\sigma}_{2t}^S = \boldsymbol{\sigma}_2^D + (\gamma_2 - \gamma_1) (\boldsymbol{\nu} - \boldsymbol{\sigma}^I) (\mathcal{H}_{1t} - \mathcal{H}_{4t}). \quad (\text{A.61})$$

The pure incentives cases are two knife-edge cases of the joint incentives case. In the pure risk-off incentives ($\mathbf{1}_{\text{Off}} = 1, \mathbf{1}_{\text{On}} = 0$), the risk-on hedging demand equals zero, $\mathcal{H}_{it} = 0$, which leads to the simplifications in (32). Similarly, in the pure risk-on incentives ($\mathbf{1}_{\text{Off}} = 0, \mathbf{1}_{\text{On}} = 1$), the risk-on hedging demand equals one, $\mathcal{H}_{it} = 1$, which leads to the same simplifications. $\square$

Proof of Proposition 3 (Momentum and Reversal). By taking Itô's Lemma of the index asset expected return (μ_{1t}^S), given in (B.9), and matching the $d\mathbf{Z}_t$ terms with (41) we find that

$$\begin{aligned} b_{1t}^\mu &= (\gamma_2 - \gamma_1)^2 [(\mathcal{H}_{3t} - \mathcal{H}_{1t})(1 - \mathcal{H}_{1t} - \mathcal{H}_{3t})] [(\gamma_2 - 1) - (\gamma_2 - \gamma_1)(\mathcal{H}_{1t})] \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) \\ &\quad - (\gamma_2 - \gamma_1)^2 [(\mathcal{H}_{3t} - \mathcal{H}_{1t})(1 - \mathcal{H}_{1t} - \mathcal{H}_{3t})] [\gamma_1 + (\gamma_2 - \gamma_1)(\mathcal{H}_{1t})] \left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I \right) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) \\ &\quad - (\gamma_2 - \gamma_1)^3 (\mathcal{H}_{3t} - \mathcal{H}_{1t}) [\mathcal{H}_{1t}(1 - \mathcal{H}_{1t})] \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2 (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) \\ &\quad - \left[(\gamma_2 - \gamma_1)^2 \mathcal{H}_{1t}(1 - \mathcal{H}_{1t}) \right] \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) (\boldsymbol{\sigma}^I - \boldsymbol{\nu}). \end{aligned} \quad (\text{A.62})$$

By taking the inner product of $\mathbf{b}_{1t}^\mu$ and $\boldsymbol{\sigma}_{1t}^S$, we find that

$$\begin{aligned} \boldsymbol{\sigma}_{1t}^{S'} \mathbf{b}_{1t}^\mu &= (\gamma_2 - \gamma_1)^2 \mathcal{H}_{1t} (1 - \mathcal{H}_{1t}) \times \left\{ \right. \\ &\quad (\gamma_2 - \gamma_1) \frac{(\mathcal{H}_{3t} - \mathcal{H}_{1t})^2}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) [(\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2 \\ &\quad - (\gamma_2 - \gamma_1) \frac{(\mathcal{H}_{3t} - \mathcal{H}_{1t})^2}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) [\gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I \right) \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2 \\ &\quad - (\gamma_2 - \gamma_1)^2 (\mathcal{H}_{3t} - \mathcal{H}_{1t})^2 \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2 \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2 \\ &\quad - (\gamma_2 - \gamma_1) (\mathcal{H}_{3t} - \mathcal{H}_{1t}) \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2 \\ &\quad - \frac{(\mathcal{H}_{3t} - \mathcal{H}_{1t})}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) [(\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right)^2 \\ &\quad - \frac{(\mathcal{H}_{3t} - \mathcal{H}_{1t})}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) [\gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I \right) \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) \\ &\quad - (\gamma_2 - \gamma_1) (\mathcal{H}_{3t} - \mathcal{H}_{1t}) \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2 \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) \\ &\quad \left. - \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right)^2 \right\}. \end{aligned}$$

The function multiplying the curly brackets is always positive and approaches zero as $V/I \rightarrow 0$ and $V/I \rightarrow \infty$ due to (A.26) and (A.27). Therefore, to show that the momentum condition is satisfied as $V/I \rightarrow \infty$, we establish that the function inside the curly brackets is positive as $V/I \rightarrow \infty$ ($\mathcal{H}_{it} \rightarrow 0$). Accordingly, the limit $V/I \rightarrow \infty$ ($\mathcal{H}_{it} \rightarrow 0$) of the function inside the curly brackets is positive if and only if

$$\gamma_2 > \underline{\gamma}_2. \quad (\text{A.63})$$

Similarly, the reversal condition is satisfied as $V/I \rightarrow 0$ ($\mathcal{H}_{it} \rightarrow 1$), if and only if

$$\gamma_2 < \bar{\gamma}_2. \quad (\text{A.64})$$

We conclude by showing that there always exist an active share λ such that

$$\underline{\gamma}_2 < \bar{\gamma}_2. \quad (\text{A.65})$$

First, comparing the factors multiplying γ_1 in (A.81) and (A.83), it is clear that

$$\gamma_1 \frac{(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu})}{(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I)} > \gamma_1 \frac{(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I)}{(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu})},$$

due to (A.16). Second, comparing the remaining factors in (A.81) and (A.83), it is clear that

$$\frac{(\|\sigma^I\|^2 - \sigma'^I \nu)}{(\|\nu\|^2 - \nu' \sigma^I)} \frac{\mathcal{E}_{3t}}{\mathcal{E}_{1t} - \mathcal{E}_{3t}} > \frac{\mathcal{E}_{1t}}{\mathcal{E}_{1t} - \mathcal{E}_{3t}} \quad (\text{A.66})$$

because the denominator shrinks to zero as the active increases, (A.15). Therefore, there is always exist a γ_2 such that index asset exhibits momentum and reversal.

We continue with the active asset. By taking Itô's Lemma of the active asset expected return (μ_{2t}^S) , given in (B.10), and matching the $d\mathbf{Z}_t$ terms with (41) we find that

$$\begin{aligned} \mathbf{b}_{2t}^\mu = & -(\gamma_2 - \gamma_1)^2 (\mathcal{H}_{1t} - \mathcal{H}_{4t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) [(\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\sigma^I\|^2 - \nu' \sigma^I \right) (\sigma^I - \nu) \\ & + (\gamma_2 - \gamma_1)^2 (\mathcal{H}_{1t} - \mathcal{H}_{4t}) (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) [\gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\nu\|^2 - \sigma'^I \nu \right) (\sigma^I - \nu) \\ & + (\gamma_2 - \gamma_1)^3 (\mathcal{H}_{1t} - \mathcal{H}_{4t}) \mathcal{H}_{1t} (1 - \mathcal{H}_{1t}) \|\sigma^I - \nu\|^2 (\sigma^I - \nu) \\ & + (\gamma_2 - \gamma_1)^2 (\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})) \nu' \sigma_2^D (\sigma^I - \nu). \end{aligned} \quad (\text{A.67})$$

By taking the inner product of $\mathbf{b}_{2t}^\mu$ and σ_{2t}^S , we find that

$$\begin{aligned} \sigma_{2t}^{S'} \mathbf{b}_{2t}^\mu = & (\gamma_2 - \gamma_1)^2 \mathcal{H}_{1t} (1 - \mathcal{H}_{1t}) \times \left\{ \right. \\ & (\gamma_2 - \gamma_1) \frac{(\mathcal{H}_{1t} - \mathcal{H}_{4t})^2}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) [(\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\sigma^I\|^2 - \nu' \sigma^I \right) \|\sigma^I - \nu\|^2 \\ & - (\gamma_2 - \gamma_1) \frac{(\mathcal{H}_{1t} - \mathcal{H}_{4t})^2}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) [\gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\nu\|^2 - \sigma'^I \nu \right) \|\sigma^I - \nu\|^2 \\ & - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t} - \mathcal{H}_{4t}) \nu' \sigma_2^D \|\sigma^I - \nu\|^2 \\ & - (\gamma_2 - \gamma_1)^2 (\mathcal{H}_{1t} - \mathcal{H}_{4t})^2 \|\sigma^I - \nu\|^2 \|\sigma^I - \nu\|^2 \\ & - \frac{(\mathcal{H}_{1t} - \mathcal{H}_{4t})}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) [(\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\sigma^I\|^2 - \nu' \sigma^I \right) \sigma_2^{D'} (\sigma^I - \nu) \\ & + \frac{(\mathcal{H}_{1t} - \mathcal{H}_{4t})}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) [\gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\nu\|^2 - \sigma'^I \nu \right) \sigma_2^{D'} (\sigma^I - \nu) \\ & \left. + \left(\nu' \sigma_2^D \right) \sigma_2^{D'} (\sigma^I - \nu) + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t} - \mathcal{H}_{4t}) \|\sigma^I - \nu\|^2 \sigma_2^{D'} (\sigma^I - \nu) \right\}. \end{aligned} \quad (\text{A.68})$$

The function multiplying the curly brackets is always positive and approaches zero as $V/I \rightarrow 0$ and $V/I \rightarrow \infty$ due to (A.26) and (A.27). Therefore, to show that the reversal condition is satisfied as $V/I \rightarrow \infty$, we establish that the function inside the curly brackets is negative as $V/I \rightarrow \infty$ ($\mathcal{H}_{it} \rightarrow 0$). Accordingly, the limit $V/I \rightarrow \infty$ ($\mathcal{H}_{it} \rightarrow 0$) of the function inside the curly brackets is positive if and only if

$$\gamma_2 > \underline{\gamma}_2. \quad (\text{A.69})$$

Similarly, the reversal condition is satisfied as $V/I \rightarrow 0$ ($\mathcal{H}_{it} \rightarrow 1$), if and only if

$$\gamma_2 < \bar{\gamma}_2 \quad (\text{A.70})$$

We conclude the proof by showing that there always exist an active share λ such that

$$\underline{\gamma}_2 < \bar{\gamma}_2. \quad (\text{A.71})$$

First, comparing the factors multiplying γ_1 in (A.86) and (A.87), it is clear that

$$\gamma_1 \frac{(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu})}{(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I)} > \gamma_1 \frac{(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I)}{(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu})}, \quad (\text{A.72})$$

due to (A.16). Second, comparing the remaining factors in (A.86) and (A.87), it is clear that

$$1 + \frac{1}{(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu})} \left[\frac{\mathcal{E}_{4t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}} \boldsymbol{\nu}' \boldsymbol{\sigma}_2^D \right] < \frac{1}{(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I)} \left[\frac{\mathcal{E}_{1t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}} \boldsymbol{\nu}' \boldsymbol{\sigma}_2^D - (\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu}) \right] \quad (\text{A.73})$$

is true if

$$\frac{\mathcal{E}_{1t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}} \geq \frac{(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu})}{\boldsymbol{\nu}' \boldsymbol{\sigma}_2^D}. \quad (\text{A.74})$$

The denominator in the right hand side of (A.73) shrinks to zero as the active increases due to (A.15). Therefore, there is always exist a γ_2 such that the active asset exhibits momentum in outperformance and reversal in underperformance. $\square$

Proof of Proposition 4 (Momentum and Reversal in the Cross-Section). The condition in (81) is verified in the text in (54). In the outperformance region, both the index asset and the active asset exhibit momentum, and in the underperformance region the two assets exhibit reversal.

Suppose news arrive, the asset with a positive sensitivity will rise, and the other asset will fall. In the outperformance region, both assets exhibit momentum. By buying the asset with positive sensitivity and selling the asset with negative sensitivity, the long short-portfolio generates a positive return. That is, momentum in the long-short portfolio. In contrast, in the underperformance region, both assets exhibit reversal. By buying the asset with positive sensitivity and selling the asset with negative sensitivity, the long short-portfolio generates a negative return. That is, reversal in the long-short portfolio. $\square$

Proof of Proposition 7 (Expected Return). The index and active expected returns are given by (B.9) and (B.10), respectively, and obtained by taking the inner product between the market price of risk (37) and the return exposures (39) and (40). The six channels are identified by the six combinations of inner products between the two components of the

market price of risk (σ^I, ν) and the three components of the asset exposures (39) and (40) $(\sigma_i^D, \sigma^I, \nu)$. By multiplying the characterizations for θ_t and σ_{1t}^S , we obtain our desired result in (B.9) and (B.10)

Next, we show that the expected return in extreme underperformance is strictly lower than the expected return in extreme outperformance. First, notice that the term,

$$(\gamma_2 - \gamma_1) (\mathcal{H}_{3t} - \mathcal{H}_{1t}), \quad (\text{A.75})$$

in the first four channels is strictly positive (A.17) and approaches zero in extreme outperformance and underperformance (A.21). Therefore, in extreme outperformance the index asset expected return becomes

$$\mu_{1t}^S |_{\mathcal{H}_{it} \downarrow 0} = (\gamma_2 - 1) \sigma^{I'} \sigma_1^D + \gamma_1 \nu' \sigma_1^D, \quad (\text{A.76})$$

while in extreme underperformance it becomes

$$\mu_{1t}^S |_{\mathcal{H}_{it} \uparrow 1} = -(1 - \gamma_1) \sigma^{I'} \sigma_1^D + \gamma_2 \nu' \sigma_1^D.$$

The difference between extreme outperformance and underperformance is strictly positive,

$$\mu_{1t}^S |_{\mathcal{H}_{it} \downarrow 0} - \mu_{1t}^S |_{\mathcal{H}_{it} \uparrow 1} = (\gamma_2 - \gamma_1) (\sigma^{I'} \sigma_1^D - \nu' \sigma_1^D) > 0, \quad (\text{A.77})$$

because $\gamma_2 - \gamma_1 > 0$, and (A.14). In similar fashion, the difference between extreme outperformance and underperformance for the active asset is

$$\mu_{2t}^S |_{\mathcal{H}_{it} \downarrow 0} - \mu_{2t}^S |_{\mathcal{H}_{it} \uparrow 1} = -(\gamma_2 - \gamma_1) \nu' \sigma_2^D < 0. \quad (\text{A.78})$$

Next, we establish that the index expected return decreases in outperformance and increases in underperformance. We achieve it by showing the derivative of the expected return with respect to V/I is negative as $V/I \rightarrow \infty$ ($\mathcal{H}_{it} \rightarrow 0$) and positive as $V/I \rightarrow 0$ ($\mathcal{H}_{it} \rightarrow \infty$). We derive the derivative, $d\mu_{1t}^S/d(V/I)$, by plugging the derivatives of $\mathcal{H}_{it}$ and $\mathcal{H}_{it} - \mathcal{H}_{jt}$, given in (A.22) and (A.23), respectively, and grouping similar terms, which leads to

$$\begin{aligned} d\mu_{1t}^S/d(V/I) = & (V_t/I_t)^{-1} (\gamma_2 - \gamma_1)^2 \mathcal{H}_{1t} (1 - \mathcal{H}_{1t}) \times \left\{ \right. \\ & - \frac{(\mathcal{H}_{3t} - \mathcal{H}_{1t})}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) [(\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\sigma^I\|^2 - \sigma^{I'} \nu \right) \\ & + \frac{(\mathcal{H}_{3t} - \mathcal{H}_{1t})}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} (1 - \mathcal{H}_{1t} - \mathcal{H}_{3t}) [\gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\nu\|^2 - \nu' \sigma^I \right) \\ & \left. + (\gamma_2 - \gamma_1) (\mathcal{H}_{3t} - \mathcal{H}_{1t}) \|\sigma^I - \nu\|^2 + \left(\|\sigma^I\|^2 - \sigma^{I'} \nu \right) \right\}. \quad (\text{A.79}) \end{aligned}$$

The function multiplying the curly brackets is always positive and approaches zero as $V/I \rightarrow 0$ and $V/I \rightarrow \infty$ due to (A.26) and (A.27). Therefore, to show that the derivative is negative as $V/I \rightarrow \infty$, we establish that the function inside the curly brackets is negative as $V/I \rightarrow \infty$. Accordingly, the limit $V/I \rightarrow \infty$ of the function inside the curly brackets equals

$$-\frac{\mathcal{E}_{1t} - \mathcal{E}_{3t}}{\mathcal{E}_{3t}} (\gamma_2 - 1) \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) + \frac{\mathcal{E}_{1t} - \mathcal{E}_{3t}}{\mathcal{E}_{3t}} \gamma_1 \left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I \right) + \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) \quad (\text{A.80})$$

because the limit of $\frac{(\mathcal{H}_{3t} - \mathcal{H}_{1t})}{\mathcal{H}_{1t}(1 - \mathcal{H}_{1t})}$ is a strictly positive constant (A.25). Simplifying the term, we find that the derivative is negative if and only if

$$\underline{\gamma}_2 \equiv \frac{\mathcal{E}_{1t}}{\mathcal{E}_{1t} - \mathcal{E}_{3t}} + \gamma_1 \frac{(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I)}{(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu})} < \gamma_2. \quad (\text{A.81})$$

Similarly, the limit $V/I \rightarrow 0$ of the function inside the curly brackets equals

$$-\frac{\mathcal{E}_{1t} - \mathcal{E}_{3t}}{\mathcal{E}_{1t}} (1 - \gamma_1) \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) - \frac{\mathcal{E}_{1t} - \mathcal{E}_{3t}}{\mathcal{E}_{1t}} \gamma_2 \left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I \right) + \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right). \quad (\text{A.82})$$

Simplifying the term, we find that the derivative is positive if and only if

$$\gamma_2 < \frac{(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu})}{(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I)} \left(\frac{\mathcal{E}_{3t}}{\mathcal{E}_{1t} - \mathcal{E}_{3t}} + \gamma_1 \right) \equiv \bar{\gamma}_2. \quad (\text{A.83})$$

Similar to $d\mu_{1t}^S/d(V/I)$, we plug the derivatives of $\mathcal{H}_{it}$ and $\mathcal{H}_{it} - \mathcal{H}_{jt}$, given in (A.22) and (A.23), respectively, and group similar terms, leading to

$$\begin{aligned} d\mu_{2t}^S/d(V/I) = (V_t/I_t)^{-1} (\gamma_2 - \gamma_1)^2 \mathcal{H}_{1t} (1 - \mathcal{H}_{1t}) \times \Big\{ \\ \frac{(\mathcal{H}_{1t} - \mathcal{H}_{4t})}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) [(\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) \\ - \frac{(\mathcal{H}_{1t} - \mathcal{H}_{4t})}{\mathcal{H}_{1t} (1 - \mathcal{H}_{1t})} (1 - \mathcal{H}_{1t} - \mathcal{H}_{4t}) [\gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I \right) \\ - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t} - \mathcal{H}_{4t}) \|\boldsymbol{\sigma}^I - \boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}_2^D \Big\}. \end{aligned} \quad (\text{A.84})$$

The function multiplying the curly brackets is always positive and approaches zero as $V/I \rightarrow 0$ and $V/I \rightarrow \infty$ due to (A.26) and (A.27). Therefore, to show that the derivative is positive as $V/I \rightarrow \infty$, we establish that the function inside the curly brackets is positive as $V/I \rightarrow \infty$.

Accordingly, the limit $V/I \rightarrow \infty$ of the function inside the curly brackets equals

$$\frac{\mathcal{E}_{4t} - \mathcal{E}_{1t}}{\mathcal{E}_{4t}} (\gamma_2 - 1) \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) - \frac{\mathcal{E}_{4t} - \mathcal{E}_{1t}}{\mathcal{E}_{4t}} \gamma_1 \left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I \right) - \boldsymbol{\nu}' \boldsymbol{\sigma}_2^D, \quad (\text{A.85})$$

which is positive if and only if

$$\gamma_2 > 1 + \frac{1}{(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu})} \left[\gamma_1 \left(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I \right) + \frac{\mathcal{E}_{4t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}} \boldsymbol{\nu}' \boldsymbol{\sigma}_2^D \right] \equiv \underline{\underline{\gamma}}_2. \quad (\text{A.86})$$

Similarly, the limit $V/I \rightarrow 0$ of the function inside the curly brackets is negative if and only if

$$\gamma_2 < \frac{1}{(\|\boldsymbol{\nu}\|^2 - \boldsymbol{\nu}' \boldsymbol{\sigma}^I)} \left[\frac{\mathcal{E}_{1t}}{\mathcal{E}_{4t} - \mathcal{E}_{1t}} \boldsymbol{\nu}' \boldsymbol{\sigma}_2^D - (1 - \gamma_1) \left(\|\boldsymbol{\sigma}^I\|^2 - \boldsymbol{\sigma}^{I'} \boldsymbol{\nu} \right) \right] \equiv \bar{\bar{\gamma}}_2 \quad (\text{A.87})$$

□

B Internet Appendix – Further Discussion

Proposition 5 (Discount Factor). *The discount factor in the presence of risk-on incentives is given by*

$$\xi_{0,t} = (I_t/I_0)^{1-\gamma_1} (V_t/V_0)^{-\gamma_2} \mathcal{E}_{1t}^{On}/\mathcal{E}_{10}^{On}, \quad (\text{B.1})$$

while it is given by

$$\xi_{0,t} = (I_t/I_0)^{-(\gamma_2-1)} (V_t/V_0)^{-\gamma_1} \mathcal{E}_{1t}^{Off}/\mathcal{E}_{10}^{Off} \quad (\text{B.2})$$

in presence of risk-off incentives. In the presence of both risk-on and risk-off incentives, the discount factor is given by

$$\xi_{0,t} = (I_t/I_0)^{1-\gamma_1} (V_t/V_0)^{-\gamma_2} \mathcal{E}_{1t}^{On}/\mathcal{E}_{10}^{On} \frac{1}{\mathcal{H}_{1t}/\mathcal{H}_{10}}, \quad (\text{B.3})$$

where $\mathcal{E}_{1t}^{Off}$ and $\mathcal{E}_{1t}^{On}$ are deterministic functions of time given in (A.1) and (A.2), respectively, and $\mathcal{H}_{1t}$ is given in (23). The discount factor exhibits the following characteristics:

- It decreases following market news: $\partial \xi_t / \partial V_t < 0$.
- It decreases following index news in the risk-off incentives case: $\partial \xi_t / \partial I_t < 0$.
- It increases following index news in the risk-on incentives case: $\partial \xi_t / \partial I_t > 0$.
- It has a U-shape with index news in the joint incentives case. It decreases with the index news ($\partial \xi_t / \partial I_t < 0$) if and only if the risk-on hedging demand is below a threshold,

$$\mathcal{H}_{1t} < \frac{\gamma_2 - 1}{\gamma_2 - \gamma_1} < 1, \quad (\text{B.4})$$

and increases with the index news ($\partial \xi_t / \partial I_t > 0$) when $\mathcal{H}_{1t}$ passes the threshold.

Regardless of the representative asset manager's incentives, Proposition 5 reveals that the state price density is inversely related to the market news, V_t — a feature that is similar to a traditional asset pricing model: an asset that pays off in bad states get a high value.

The risk-on incentives imply that assets that pay off when the index is high get a high value because the asset manager with risk-on incentives are concerned about underperforming the index and are willing to pay high prices for assets that pay in these states. This feature is typical to an asset pricing model with benchmarking concerns, such as Basak and Pavlova (2013), Buffa and Hodor (2023), Hodor and Zapatero (2026), among others.

The logic flips with risk-off incentives: assets that pay off when the index is high get a

low value. With risk-off incentives, the asset manager is outperforming the index and do not desire to hold assets that pay off when the benchmark is high. Instead of hedging unexpected increases in the benchmark, outperforming asset manager hedges unexpected falls by taking a short position on the benchmark and investing the proceeds in other assets — a feature unique to risk-off incentives.

When the asset manager faces both risk-on and risk-off incentives, the joint incentives discount factor fluctuates between risk-on and risk-off pricing environments. When the benchmark news (I) is low relative to the market news (V), the risk-on hedging demand ($\mathcal{H}_{1t}$) is small, and the asset prices correlate with the purely risk-off prices. In these states, the representative asset manager is more likely to outperform, and prices adjust to accommodate their risk-off portfolios. As the benchmark news (I) increases, the risk-on hedging demand ($\mathcal{H}_{1t}$) becomes more pronounced, and prices adjust to accommodate the heightened likelihood that the asset manager will underperform. Eventually, the risk-on hedging demand ($\mathcal{H}_{1t}$) cross the threshold in (B.4) and becomes so pronounced that prices correlate with the purely risk-on prices. Figure 11 illustrates this logic.

B.1 Prices and Expected Returns

To clarify the mechanism, this section characterizes the index and active prices and their expected returns. This analysis provides further intuitions for why the long-short portfolio generates momentum in the outperformance region and reversal in the underperformance region. We start by characterizing the index and active prices.

Proposition 6 (Stock Prices). *The index and active price-dividend ratios in the presence of risk-on incentives are given by*

$$S_t^I/I_t = \frac{\mathcal{E}_{3t}^{On}}{\mathcal{E}_{1t}^{On}}, \quad S_{2t}/D_{2t} = \frac{\mathcal{E}_{4t}^{On}}{\mathcal{E}_{1t}^{On}}, \quad (\text{B.5})$$

while in the presence of risk-off incentives, they are given by

$$S_t^I/I_t = \frac{\mathcal{E}_{3t}^{Off}}{\mathcal{E}_{1t}^{Off}}, \quad S_{2t}/D_{2t} = \frac{\mathcal{E}_{4t}^{Off}}{\mathcal{E}_{1t}^{Off}}. \quad (\text{B.6})$$

The index and active prices in the presence of both risk-on and risk-off incentives are given by

$$S_t^I/I_t = \frac{\mathcal{E}_{3t}^{On}}{\mathcal{E}_{1t}^{On}} \left[\frac{\mathcal{H}_{1t}}{\mathcal{H}_{3t}} \right], \quad S_{2t}/D_{2t} = \frac{\mathcal{E}_{4t}^{On}}{\mathcal{E}_{1t}^{On}} \left[\frac{\mathcal{H}_{1t}}{\mathcal{H}_{4t}} \right]. \quad (\text{B.7})$$

The comparison between the index asset and active asset price-dividend ratios reveals that the index asset price-dividend ratio

- is higher than the active asset one with pure risk-on incentives.
- is lower than the active asset one with pure risk-off incentives, if the difference between risk-on and risk-off risk aversion parameters is greater than one: $\gamma_2 - \gamma_1 \geq 1$.

In the joint incentives case, if $\gamma_2 - \gamma_1 \geq 1$, the index price-dividend ratio is higher than the active asset one if and only if the ratio of the market news to the index news is below a deterministic threshold,

$$\frac{V_t}{I_t} \leq \left[\frac{(\mathcal{E}_{3t}^{On} - \mathcal{E}_{4t}^{On}) (\gamma_2 - 1)}{(\mathcal{E}_{4t}^{Off} - \mathcal{E}_{3t}^{Off}) (1 - \gamma_1)} \right]^{\frac{1}{\gamma_2 - \gamma_1}}. \quad (\text{B.8})$$

Further, in the joint incentives case, the index price-dividend ratio S_t^I/I_t

- decreases following news about the market: $\partial (S_t^I/I_t) / \partial V_t < 0$,
- increases following news about the index: $\partial (S_t^I/I_t) / \partial I_t > 0$.

The results flip for the active asset price-dividend ratio, S_{2t}/D_{2t} ; it

- increases following news about the market: $\partial (S_{2t}/D_{2t}) / \partial V_t > 0$,
- decreases following news about the index: $\partial (S_{2t}/D_{2t}) / \partial I_t < 0$,

where $\mathcal{H}_{1t}$, $\mathcal{H}_{3t}$, and $\mathcal{H}_{4t}$ are given in (23), and $\mathcal{E}_{1t}^{Off}$, $\mathcal{E}_{1t}^{On}$, $\mathcal{E}_{3t}^{Off}$, $\mathcal{E}_{3t}^{On}$, $\mathcal{E}_{4t}^{Off}$, $\mathcal{E}_{4t}^{On}$ are given in (A.1), (A.2), (A.7), (A.8), (A.10), and (A.11), respectively.

In pure risk-on incentives, the asset manager desires to post higher returns when the benchmark is high rather than when it is low. These risk-on incentives induce the asset manager to buy the benchmark to hedge against unexpected increases in the benchmark, which, in turn, creates extra demand for the benchmark. Equilibrium is restored by raising the prices of assets within the benchmark and lowering the prices of assets outside the benchmark, thereby diminishing the benchmark's attractiveness relative to active assets.

In pure risk-off incentives, the asset manager desires to maintain their accumulated performance. In this case, the asset manager is risk-averse and protect their portfolio against scenarios where the benchmark declines. To achieve that goal, the asset manager substantially reduces their benchmark position, which increases the available supply for the benchmark. Equilibrium is restored by decreasing the prices of benchmark assets and increasing the prices of active assets, which in turn, enhances the attractiveness of the benchmark relative to the market.

In the joint incentives case, the asset manager weighs the risk-on and risk-off strategies, depending on the likelihood that they will outperform the benchmark. Since their portfolio is not fully correlated with the benchmark, the likelihood of outperformance is influenced

by the ratio of market news to index news, V/I . When V is large relative to I , the asset manager is likely to outperform, whereas when V is small relative to I , the asset manager is likely to underperform.

Thus far, we have demonstrated a negative correlation between index asset and active asset prices, an increase in the index price leads to a decrease in the active asset price, and vice versa. Next, we discuss the implications for expected returns.

By applying the covariaion between between the discount factor exposure (37) and the assets exposure, (33) and (34), we find that the expected return (per unit of time) of the index is given by

$$\begin{aligned} \mu_{1t}^S = & (\gamma_2 - \gamma_1) (\mathcal{H}_{3t} - \mathcal{H}_{1t}) \times \\ & \{ [(\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \boldsymbol{\sigma}^{II} (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) - [\gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \boldsymbol{\nu}' (\boldsymbol{\nu} - \boldsymbol{\sigma}^I) \} \\ & + \{ [(\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \boldsymbol{\sigma}^{II} (\boldsymbol{\sigma}_1^D) + [\gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \boldsymbol{\nu}' (\boldsymbol{\sigma}_1^D) \}, \end{aligned} \quad (\text{B.9})$$

and the expected return (per unit of time) of the active asset is given by

$$\begin{aligned} \mu_{2t}^S = & (\gamma_2 - \gamma_1) (\mathcal{H}_{1t} - \mathcal{H}_{4t}) \times \\ & \{ - [(\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \boldsymbol{\sigma}^{II} (\boldsymbol{\sigma}^I - \boldsymbol{\nu}) + [\gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \boldsymbol{\nu}' (\boldsymbol{\nu} - \boldsymbol{\sigma}^I) \} \\ & + \{ [(\gamma_2 - 1) - (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \boldsymbol{\sigma}^{II} (\boldsymbol{\sigma}_2^D) + [\gamma_1 + (\gamma_2 - \gamma_1) (\mathcal{H}_{1t})] \boldsymbol{\nu}' (\boldsymbol{\sigma}_2^D) \}. \end{aligned} \quad (\text{B.10})$$

There are two curly brackets in each representation. The bottom curly bracket in each representation identifies the contribution to the expected return due to the fundamental risk exposures $(\boldsymbol{\sigma}_1^D)$ and $(\boldsymbol{\sigma}_2^D)$. The top curly brackets identify the contribution to the expected return due to the changes in the market prices of risk, which reflects the *incentives* channel. As asset manager's accumulated performance declines, equilibrium lowers the market prices of risk that correlate with the benchmark and raises the market prices of risk orthogonal to the benchmark. Consequently, this channel reduces the expected return of benchmark assets and increases the expected return of assets outside the benchmark.

The leading factor multiplying the first curly bracket in each representation highlights the assets' response to the *uncertainty of performance*. As asset manager's accumulated performance aligns more closely with the benchmark, assets become increasingly sensitive to news in equilibrium, which increases the expected returns of both assets.

Overall, the expected returns are influenced by the two channels and the contribution of the fundamental effect. Figure 9 illustrates the effects of the two channels, and Figure 10 illustrates the combined effect of the two channels and the fundamental effect. The fact that the expected returns of the index and active assets work in opposite direction is the key driver of the cross-sectional predictability: if the arrival of news increases the expected

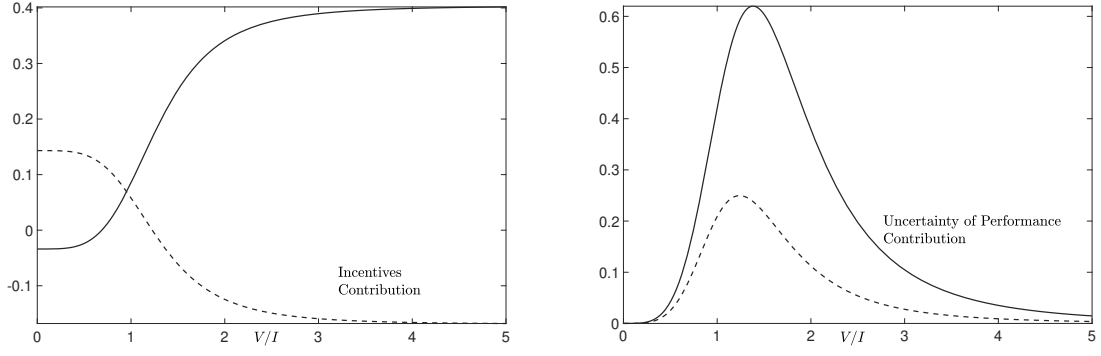


Figure 9. The solid line in the left figure plots the top curly brackets for the index asset (B.9), while the dashed line represents the curly brackets for the active asset (B.10). The figure shows that as accumulated performance deteriorates, the benchmarking incentives (changes to the market prices of risk) lead the benchmark asset's expected return to decline and the active asset's expected return to rise. The solid line in the right figure plots the leading factor multiplying the first curly bracket for the index asset (B.9), while the dashed line represents the leading factor for the active asset (B.10). First, the figure shows that the expected returns of both assets rise due to the uncertainty of performance. Second, it shows that as accumulated performance deteriorates, the expected return of the benchmark asset falls, while that of the active asset rises. The parameters are as in Figure 5.

return of the index, it also decreases the expected return of assets outside the index. Figure 10 also illustrates the predictability patterns in the cross-section.

The following proposition summarizes our key findings.

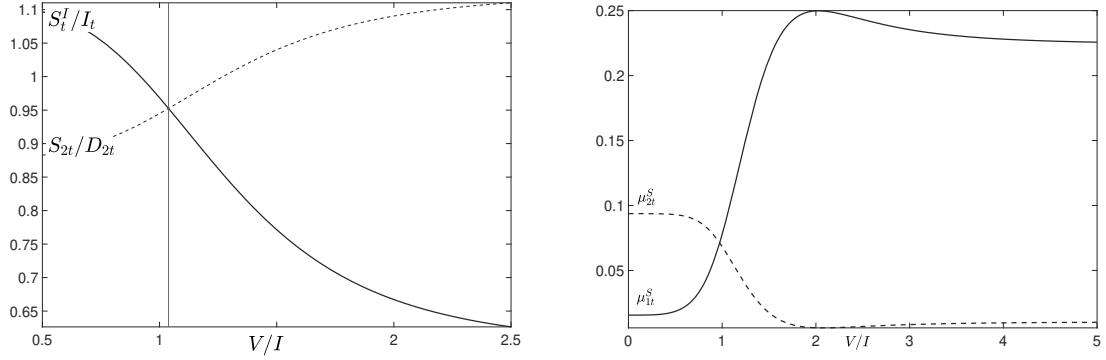


Figure 10. The left figure plots the index asset and active asset price-dividend ratios in the joint incentives case as a function of (V/I) , the state variable governing the likelihood of accumulated performance. The vertical line is given in (B.8) and captures the location at which the price-dividend ratios is at the threshold. The figure shows that when the arrival of news causes one asset to rise, it also causes the other asset to fall. The right figure plots the expected returns of index asset and active asset as a function of (V/I) . The long-short portfolio buys the asset that has risen (positive slope in the left figure) and sells the asset that has fallen (negative slope in the left figure). When the asset manager outperforms the benchmark, momentum causes the rising asset to continue rising (positive slope in the right figure) and the falling asset to continue falling (negative slope in the right figure). Consequently, this strategy results in a positive return, aligning with traditional cross-sectional momentum. Conversely, when the asset manager underperforms the benchmark, reversal causes the rising asset to fall (negative slope in the right figure) in the future and the falling asset to rise (positive slope in the right figure). As a result, this strategy leads to a negative return, aligning with traditional cross-sectional reversal. The parameters are as in Figure 5.

Proposition 7 (Expected Return). *The expected returns for the index asset and active asset are given by (B.9) and (B.10), respectively. The expected return of the index in the case of extreme outperformance is strictly higher than in the case of extreme underperformance ($\mu_{1t}^S |_{\mathcal{H}_{it} \downarrow 0} - \mu_{1t}^S |_{\mathcal{H}_{it} \uparrow 1} > 0$). In contrast, the expected return of the active asset in extreme outperformance is strictly lower than in the case of extreme underperformance ($\mu_{2t}^S |_{\mathcal{H}_{it} \downarrow 0} - \mu_{2t}^S |_{\mathcal{H}_{it} \uparrow 1} < 0$). Furthermore, the index expected return*

- *decreases in outperformance if and only if $\underline{\gamma}_2 \leq \gamma_2$,*
- *increases in underperformance if and only if $\bar{\gamma}_2 \geq \gamma_2$.*

Conversely, the active asset expected return

- *increases in outperformance if and only if $\underline{\underline{\gamma}}_2 \leq \gamma_2$,*
- *decreases in underperformance if and only if $\bar{\bar{\gamma}}_2 \geq \gamma_2$,*

where $\underline{\gamma}_2$, $\bar{\gamma}_2$, $\underline{\underline{\gamma}}_2$, and $\bar{\bar{\gamma}}_2$ are given in (A.81), (A.83), (A.86), and (A.87), respectively.

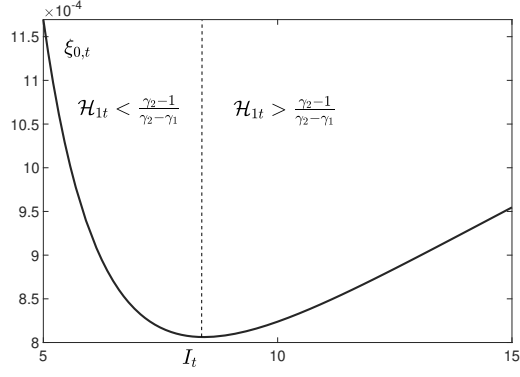


Figure 11. This figure plots the discount factor as a function of index news in the joint incentives case. The discount factor sets prices similar to the risk-off environment when the risk-on hedging demand is below a threshold. In contrast, the discount factor sets prices similar to the risk-on environment when the risk-on hedging demand crosses that threshold. The parameters are as in Figure 5.

C Internet Appendix – Preliminary Evidence

This section empirically validates the predictions of the model at the portfolio level of mutual funds. Table 1 reports the average probability of correctly predicting the direction of next month’s mutual fund return, conditional on whether the fund outperformed or underperformed the benchmark in the previous month. The results indicate that momentum during outperformance and reversal during underperformance can be correctly predicted with 53% to 62% accuracy. These averages are computed across time and across all active U.S. equity funds with broad U.S. mandates.

Additionally, Table 2 presents the estimated coefficients from a conditional logistic regression, including time and fund fixed effects, demonstrating that the predictive accuracy remains robust. For instance, the log-odds of momentum in the following month increase by 84% when a fund outperforms the market by at least 2%, compared to when it underperforms by the same margin. This corresponds to a 19.8% higher probability of momentum next month relative to reversal.

Table 1. Average Probability of Successful Predictions

This table presents the average probability of correctly predicting the direction of next month’s mutual fund return based on the fund’s performance in the previous month. If a mutual fund’s return exceeded the market return by at least a given threshold (ranging from 0% to 2%) last month, we predict momentum: a positive return last month forecasts a positive return this month, while a negative return last month forecasts a negative return this month. Conversely, if the fund’s return was below the market return by at least the threshold, we predict reversal: a positive return last month forecasts a negative return this month, and a negative return last month forecasts a positive return this month. The table demonstrates that mutual fund performance in the prior month can effectively predict the direction of returns in the following month. As the difference between the fund’s return and the market return widens, the average accuracy of predicting next month’s return improves for both outperforming funds (momentum) and underperforming funds (reversal). Our analysis focuses on U.S. all-equity mutual funds with broad investment mandates. The predictability is robust to different specifications. The (***) indicates significance below the 1% level. A complete description of the data and methodology is provided in Section C.1.

<i>Funds with at least one instance of underperformance and one instance of outperformance in prior months</i>			
	Both Detected	Momentum Detected	Reversal Detected
$ \text{Portfolio} - \text{Index} > 0$	53.4% (***)	53.6% (***)	53.0% (***)
$ \text{Portfolio} - \text{Index} > 1.0\%$	55.5% (***)	54.8% (***)	55.7% (***)
$ \text{Portfolio} - \text{Index} > 2.0\%$	58.0% (***)	55.7% (***)	58.8% (***)

This literature often documents positive return autocorrelation (time-series momentum) at high frequencies, such as daily, weekly and monthly, which reverts to negative autocorrelation (time-series reversal) at lower frequencies, such as annually and over several years.

See for example, Fama and French (1988), Lo and MacKinlay (1988), James M. Poterba (1988), and Cutler, Poterba, and Summers (1991). More recently, Moskowitz, Ooi, and Pedersen (2012) documented time series momentum in equity index returns. However, Huang, Li, Wang, and Zhou (2020) question the significance of these findings, claiming that the evidence for time series momentum in the stock level and index level is weak.

Our results indicate that time-series momentum is strongly present in mutual fund returns and stock level returns—but only when the fund outperforms the market. When it underperforms the market, strong return reversal is observed. This evidence supports the main findings of our theory.

In particular, when the fund’s monthly return underperforms the market return, the return next month is significantly more likely to exhibit reversal: a positive return this month predicts a negative return next month, while a negative return this month predicts a positive return next month. In contrast, when the fund’s monthly return outperforms the market return, the next monthly return is significantly more likely to exhibit momentum: a positive return this month predicts a positive return next month, while a negative return this month predicts a negative return next month.

Table 2 presents the estimated coefficient of a conditional logistic regression: Panel A includes fund fixed effects, and Panel B includes both fund and year fixed effects. Each column represents a different threshold for outperformance and underperformance: in the first column, the threshold is 0%; in the second, it is 1%; and in the third, it is 2%. In the second and third columns, performance may fall between the thresholds for underperformance and outperformance. In such cases, we exclude the month from the dataset. In the first column, there are no exclusions, and the dataset contains 295,583 fund-month observations. The restrictions in columns two and three exclude fund-month observations where the funds are less active and generate performance within the threshold (1% or 2%) relative to the market. Indeed, the estimated coefficients in columns two and three are economically larger than in column one.

Each estimated coefficient presents the change in the log odds from predicting reversal (predicted direction=0) to predicting momentum (predicted direction=1). In panel A, in column three, the performance threshold is 2% and the coefficient is 0.84. This means that the log odds of momentum next month increase by 84% when a fund outperforms the market by at least 2% relative to when that fund underperforms the market by at least 2%. In other words, the probability of momentum next month is 19.8% higher than reversal if the fund outperforms the market by at least 2% this month. In the lower section of Panel A, we control for year fixed effects. The results indicate that the log odds increase by 80%. The year fixed effects indicate that the effect we identify appears both cross-sectionally, across

underperforming and outperforming funds within a given year, and within funds, between months of outperformance and underperformance.

C.1 Data

We obtain monthly mutual fund return data from the CRSP Mutual Fund database, which contains 9,580,005 fund-month observations. We merge this dataset with the CRSP Mutual Fund Summary data based on CRSP fund number. This data has 675,790 fund-year observations. The merge leads to 7,844,130 fund-month observations. We removed index funds (resulting in 7,338,637), ETF and ETN funds (resulting in 7,191,153), and Variable Annuity funds (resulting in 6,493,363). We identify all-equity mutual funds by the ‘Morningstar Category Group’ equaling ‘Equity’, and the broad market mandate by the ‘U.S. Category Group’ equaling ‘U.S. Equity’.¹⁷ We merge these two datasets based on the mutual fund CUSIP code, and exclude all funds that do not match. Our analysis starts at 1/1/2010 and end at 31/12/2023. The market’s monthly return is proxied by SPY, the SPDR ETF that tracks the performance of the S&P 500 Index, with data downloaded from CRSP. The merge between SPY, Morningstar, based on the above characteristics leads to 295,583 fund-month observations and 2,326 unique mutual funds.

¹⁷This identification is robust to several other specifications, such as requiring the prospectus benchmark to be a broad benchmark.

Table 2. Conditional Logistic Regression: Direction of Next Month Return

This table presents the estimated coefficients of a conditional logistic panel regression. The dependent variable is the direction of the monthly return, and the independent variable is the predicted return direction based on whether the mutual fund outperformed or underperformed the market in the previous month. Both the dependent and independent variables take the value 0 for a negative return and 1 for a positive return. If a mutual fund's return was above the market return (plus the threshold, ranging from 0% to 2%) last month, we predict momentum in returns: positive return last month predict a positive return this month; negative return last month predict a negative return this month. Conversely, if a mutual fund's return was below the market return (minus the threshold) last month, we predict reversal in returns: positive return last month predict a negative return this month; negative return last month predict a positive return this month. The t-value is reported in parenthesis. Our analysis focuses on U.S. all-equity mutual funds with a broad category. A complete description of the data and methodology is provided in Section C.1.

<i>Panel A: Funds with at Least One Instance of Outperformance and One Instance of Underperformance in Prior Month</i>			
	$ \text{Portfolio} - \text{Index} > 0$	$ \text{Portfolio} - \text{Index} > 0.01$	$ \text{Portfolio} - \text{Index} > 0.02$
Predicted direction ($t - 1$)	0.30 (39.89)	0.56 (53.38)	0.84 (56.67)
Fund Fixed Effects	Yes	Yes	Yes
Year Fixed Effects	No	No	No
Concordance	54.0%	58.0%	61.0%
Unique Funds	2,326	2,310	2,153
Monthly Time Period	11–167	2–139	2–104
N	295,583	152,536	77,381
Predicted direction ($t - 1$)	0.32 (41.59)	0.56 (52.07)	0.80 (52.41)
Fund Fixed Effects	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes
Concordance	64.0%	64.0%	66.0%
Unique Funds	2,326	2,310	2,153
Monthly Time Period	11–167	2–139	2–104
N	295,583	152,536	77,381