

Identifying Investment Horizons Based on Investors' Beliefs

March 16, 2026

Abstract

This paper shows that a mismatch between investors' true horizons and the horizons assumed in the literature can cause their beliefs or risk preferences to violate Euler constraints, helping explain several asset pricing puzzles. By extracting investors' beliefs for pairs of short and long horizons and assuming that investors are rational in terms of transitivity, their true planning horizons are identified. The results suggest that the most likely investment horizon lies between eight and fourteen quarters under normal conditions, shortening to five to eight quarters during recessions. Investors are unlikely to invest with one- or two-quarter horizons regardless of risk aversion, and more risk-averse agents are even less likely to do so. Implied risk aversion decreases substantially for horizons longer than seven quarters, indicating that the equity premium puzzle is less pronounced for longer-horizon investors. Finally, in a portfolio with heterogeneous horizons, growth stocks are associated with shorter horizons, while value stocks correspond to longer horizons.

Keywords: Investment Horizons, Equity Premium Puzzle, Subjective Beliefs

1 Introduction

Most motivations for investing in the stock market emphasize a long-term outlook rather than short-term investment horizons, owing to the higher unpredictability and relative volatility of the latter. The majority of investors allocate their capital to the stock market as a form of hedging and saving, positioning themselves as long-term investors¹. The investment horizon is a critical piece of information that determines the period for which the asset pricing Euler equation should hold. Given that investors do not explicitly state their investment horizons and no alternative sources of this information are available, I aim to extract such data from current asset prices.

Historically, econometricians assumed that the investment horizon was a quarter or a year. The main reasons for this are the availability of data at that time and the ability to develop longer non-overlapping time series. The potential pitfalls lie in the possible mismatch between the investor's optimal investment horizon and the horizon that the econometrician assumes to be the investor's investment horizon. Therefore, if the investor's true investment horizon is longer, their beliefs and/or risk preferences may not satisfy the short-horizon Euler constraints typically assumed in the literature, and this mismatch may be a reason for the arising puzzles.

[Jensen \(1969\)](#) was the first to raise the 'horizon problem' for the evaluation of investment portfolios where investors have heterogeneous horizons. He shows that if the assumed investment horizon differs from the 'true' holding period, the linear relationship ('the market line') is distorted. [Levy \(1972\)](#) demonstrates that as long as the 'true' horizon does not coincide with the horizon assumed in empirical research, performance indexes contain a systematic bias.

[Levhari and Levy \(1977\)](#); [Levy \(1984\)](#) estimate CAPM with different horizons and find that the investment horizon for which data are collected plays a crucial role and has a great impact on the regression coefficients. They show that if the 'true' investment horizon is longer than assumed, then the estimation of systematic risk (beta) is biased closer to one, and the opposite is also true. [Handa et al. \(1989\)](#) provide evidence that the size effect becomes statistically insignificant when betas are estimated using annual returns.

[Gressis et al. \(1976\)](#) applied the mean-variance approach to multi-period investments and developed the relationship between the efficient frontiers of different investment horizons, showing that the investment horizon affects portfolio choice. Meanwhile, [Lee et al. \(1990\)](#) provide a generalized asset pricing model for the equilibrium risk-return relationship under

¹Yet, some investors are speculators who do not generate excess profits ([Barber and Odean, 2000](#)) and have no long-term impact on the market.

heterogeneous investment horizons and suggest that “some empirical anomalies that are inconsistent with the traditional CAPM have resulted from the misspecification of the CAPM by ignoring the discrepancy between observed data periods and true investment horizons”.

The literature suggests that many apparent asset pricing puzzles dissolve—or weaken considerably—once a longer investment horizon is assumed. [Parker and Julliard \(2005\)](#) show that a three-year horizon provides a significantly better explanation of portfolio variation than any smaller horizon. [Daniel and Marshall \(1997\)](#) conclude that the equity-premium and risk-free-rate puzzles are primarily problems for shorter-horizon returns. [Brainard et al. \(1991\)](#) demonstrate that the longer the horizon of the investor, the better the CCAPM performs relative to the CAPM.

While the term ‘investment horizon’ is widely used in the literature, it is mainly referenced as common knowledge without strict definitions. Broadly, an investment horizon can be defined as the planned duration for buying and holding an asset, without any obligation to keep this asset for precisely that period. The first part of the paper is dedicated to formally defining the investment horizon h , which can be applied to almost any model containing Euler equations, and to providing theoretical results on how the investment horizon could solve the equity premium puzzle.

In this paper, I make the first attempt, to the best of my knowledge, to extract information about the representative agent’s investment horizons from asset pricing, discover the parameters on which they depend, and examine differences across various time periods. These findings shed light on some puzzles and help deepen understanding of asset pricing, taking into account varying agent investment horizons.

In the next part of the paper, I extract the representative investor’s subjective beliefs and risk preferences that satisfy the Euler constraints for various investment horizons using the Smoothed Empirical Likelihood (SEL) methodology ([Chen et al., 2020](#); [Ghosh et al., 2021](#); [Ghosh and Roussellet, 2023](#)). At a first approximation, we conjecture that if for a particular horizon, the investor’s beliefs and/or risk aversion levels are unrealistic, then it is likely not a “true” investor horizon. This approach gives us some insights but not exact numbers.

In the main part of the paper, I propose a “quasi” test in which the agent chooses between two horizon types: short and long. The short-type agent invests two consecutive times. The long-type agent, with a horizon twice as long as the short type, invests only once. The test is constructed such that both agents live for the same period of time and face the same expected path for all conditional variables, with the only difference being their horizon type. Finally, the agent chooses the type that yields the higher utility of investing. I conduct this test for various possible horizons and construct boundaries for the likely true

investment horizons.

In the final part, I extract portfolio compositions for various horizons. As [Gressis et al. \(1976\)](#) and [Lee et al. \(1990\)](#) show, agents would keep different portfolios for various horizons. I obtain this information in two ways. First, I extract the portfolio composition for the given horizon h . Second, I find all possible allocations between two portfolios, the shorter and the longer horizons, for the representative investor who holds the entire market portfolio. Additionally, in this section, I explore the possibility that heterogeneous horizons could explain the differences in the investment portfolios of individual and institutional investors ([Gompers and Metrick, 2001](#); [Ferreira and Matos, 2008](#); [Barber and Odean, 2000](#); [Choi and Robertson, 2020](#)).

The main result is the following: the most likely investment horizon is between 8 and 14 quarters and between 5 and 8 quarters during a recession. Investors are unlikely to invest at one- and two-quarter horizons, regardless of risk aversion. More risk-averse agents are less likely to invest in these horizons, even when they have a shorter planning horizon. Additionally, the results highlight a significant reduction in implied risk aversion for investing beyond 7 quarters, providing insight that the equity premium puzzle is less pronounced for the longer-horizon investors. Auxiliary results from the analysis of portfolios with heterogeneous horizons support the main results and suggest that low B/M assets are located in shorter-horizon portfolios, while high B/M assets are in longer-horizon portfolios.

The paper is organized as follows: Section II (Literature Review) discusses how investment horizons appear in different areas of literature. Section III describes the methodology, including the model setup, identification approach, and method to identify subjective beliefs and risk preferences from current asset prices. Section IV presents the main results, including an overview of investors' beliefs and risk preferences over various horizons, and the identification of the true range of investment horizons. Section V explores investment portfolios with heterogeneous investment horizons and different points of view on how horizons, agents, and firms could correspond to each other. Finally, Section VI concludes. In addition, the appendix contains robustness check results and parsimonious numerical examples of the theoretical part.

2 Overview of Investment Horizons in Various Areas of Literature

In the introduction section, we covered the specific literature to which this paper primarily contributes. In this section, we provide a general overview of how the investment horizon

is reflected in different areas of finance and how these areas suggest long-term horizons.

At this moment, we assume a fairly broad definition of investment horizon as the planned time to buy and hold an asset, without any obligation to keep the asset for exactly that duration. The topic of investment horizon is widely covered in literature from various perspectives including behavioral factors, savings strategies, predictability, information acquisition, and equity term structure. To take a common example, if someone is planning to buy a house soon, he is unlikely to buy stocks over the next week, yet many use stocks as long-term savings. [Shiller \(2014\)](#) highlights that asset prices exhibit “instantaneous unpredictability” ([Sims, 1984](#)) when prediction time approaches zero, and “accords with long-standing advice that investors should be patient, that they cannot expect to see solid returns over short time intervals.” Thus, we can naturally link the investment horizon to the predictability of long-horizon returns, assuming that agents have an investment horizon longer than just the unpredictable immediate future.

One of the first papers to mention return predictability for the long horizon was [Fama and French \(1988\)](#), which showed that the power of dividend yields to forecast stock returns increases with the horizon. This result was followed by many other papers ([Campbell and Shiller, 1988](#); [Hodrick, 1992](#); [Fama and French, 2018](#)). [Mark \(1995\)](#) emphasizes the economic significance of exchange rate long-horizon predictability for 12-16 quarters, against a random walk dominated in the short-horizon over 2-4 quarters. [Cochrane \(1999\)](#) states “Returns are predictable. In particular: Variables including the dividend price ratio and term premium can predict substantial amounts of stock return variation... Daily, weekly, and monthly stock returns are still close to unpredictable.” Therefore, the long-horizon predictability could be one of the signs that the investment horizon should be longer than one month, and investors might even refrain from trading for some time.

[Barberis \(2000\)](#) demonstrates that “In the buy-and-hold case, we find that predictability in asset returns leads to strong horizon effects: an investor with a horizon of 10 years allocates significantly more to stocks than someone with a one-year horizon.” [Balduzzi and Lynch \(1999\)](#) establish that a long-horizon investor, who faces fixed and proportional transaction costs, may reduce the frequency of optimal rebalancing, leading to sizable no-trade regions. Thus, transaction costs are another reason for investors having various horizons.

It is worth mentioning that some long-horizon predictability is doubted, as [Kothari and Warner \(1997\)](#) show that tests for long-horizon abnormal security returns are severely misspecified and conclude that long-horizon studies require extreme caution. Similarly, [Pástor and Stambaugh \(2012\)](#) find that from an investor’s perspective, which recognizes that parameters are uncertain, stocks are substantially more volatile over long horizons. At the same time, [Barberis \(2000\)](#) states that “even after incorporating parameter uncertainty, there is

enough predictability in returns to make investors allocate substantially more to stocks, the longer their horizon.”

The investment horizon might depend on the agent’s ability to acquire information and learn from it. [Hirshleifer et al. \(2011\)](#) demonstrate that a subset of investors fails to fully incorporate the information contained in the most recent earnings surprises. [Coibion and Gorodnichenko \(2015\)](#) also show that individuals underreact to objective economic information. However, [Bordalo et al. \(2020\)](#) find that individual forecasters typically overreact to news, while consensus forecasters underreact relative to full-information rational expectations. Results are mixed, and both knowing the length of the investment horizon and understanding how information is learned will enhance each other. It is worth mentioning that price informativeness differs in good and bad times, and this may affect the length of the investment horizon depending on the state of the economy, as we will find below.

This paper naturally contributes to the large range of literature on multiperiod portfolio selection ([Li and Ng, 2000](#); [Basak and Chabakauri, 2010](#); [Cochrane, 2022](#)). In the classic i.i.d. world, portfolio allocation is independent of investment horizon, yet [Kandel and Stambaugh \(1996\)](#) are among the first to show how return predictability alters the optimal portfolio. [Campbell and Viceira \(1999\)](#) consider consumption and portfolio allocation and show that the long-horizon investors are free to rebalance their portfolio every period, even though the paper does not take into account transaction costs and portfolio restrictions. The same is also observed from real data; [Ameriks and Zeldes \(2004\)](#) report that in their sample data², individuals seldom change their portfolio allocation over time. Further, [Viceira \(2001\)](#) demonstrates, using the example of employed agents investing for retirement purposes, how horizon affects portfolio allocations and shows that longer horizons result in larger investments in stocks. Finally, in a recent working paper, [Gabaix et al. \(2024\)](#) explore the factors affecting the rebalancing of portfolios and found that the main one is responsible for the long-term equity risk premium.

Investment horizon is connected to the equity term structure literature, which has been growing lately ([Gonçalves, 2021](#); [Gormsen, 2021](#); [Bansal et al., 2021](#)). [Binsbergen et al. \(2012\)](#) show that long-term dividends have a lower risk premium than short-term dividends. [Cassella et al. \(2023\)](#) find that negative term premiums are associated with periods of above-average horizon bias, whereas periods of below-average horizon bias are associated with positive term premiums.

One of the behavioral factors affecting investment horizon is anxiety in the face of risk, or horizon-dependent risk aversion (HDRA), where agents tend to be more risk-averse when a risk is temporally close than when it is distant ([Eisenbach and Schmalz, 2016](#)). This

²Sample data is from the Survey of Consumer Finances and from TIAA-CREF.

preference generates a decreasing term structure of risk premia (Andries et al., 2014), and might be responsible for longer investment horizons. In addition, Daniel et al. (2020) provide other behavioral factors or psychological biases that are likely to affect asset prices, such as overconfidence and limited attention, which are responsible for longer and shorter horizon mispricing.

Finally, this paper contributes to the broad and growing literature on investors' subjective beliefs (Bianchi et al., 2022; Greenwood and Shleifer, 2014; Giglio et al., 2021; Chen et al., 2020; Ghosh and Roussellet, 2023). Survey data doesn't provide exact information about planned investment horizons, yet we can extract valuable insights from it. For instance, Giglio et al. (2021) show that even respondents who expect negative returns over the next year anticipate positive long-run returns. Therefore, we can make an assumption that an agent can make an investment decision based on long-horizon expectations, assuming they keep the stock for a longer period.

3 Investment horizon

3.1 Definition and Euler Equation

This section presents a formal definition of the investment horizon, a term widely used in the literature but rarely defined precisely. I assume that investors have long investment horizons³, as shown in the previous section and suggested by instantaneous unpredictability (Sims, 1984), long-horizon return predictability (Fama and French, 1988, 2018), transaction costs (Balduzzi and Lynch, 1999), behavioral factors (Eisenbach and Schmalz, 2016; Daniel et al., 2020), and subjective beliefs (Giglio et al., 2021).

I define the investment horizon h as the planned duration for holding an asset, though investors do not need to hold the asset for exactly this period. For this specific horizon h , the Euler Equation 1 is satisfied, where S denotes subjective beliefs. The horizon h likely falls within a certain interval $\mathcal{H}$ (Jensen, 1969) (e.g., approximately three years, plus or minus a quarter). Therefore, Equation 1 must hold for at least one $h \in \mathcal{H}$.

$$\mathbb{E}_t^S[M_{t,t+h}R_{t,t+h}|\mathcal{F}_t] = \mathbf{1}, \text{ for some } h \in \mathcal{H} \quad (1)$$

where $M_{t,t+h} \equiv \beta^h U'(c_{t+h})/U'(c_t)$ is the stochastic discount factor (SDF) between periods t

³Longer than one period of the model.

and $t + h$, and $R_{t,t+h}$ is the gross return over the same interval.⁴

At each time t , the investor chooses a horizon $h \in \mathcal{H}$ and makes decisions so that the Euler Equation 1 is satisfied over that horizon. Doing so requires forming expectations about consumption and returns over the full interval from t to $t + h$, accounting for both routine financial decisions and unpredictable events such as geopolitical shocks or career changes. When the next period $t + 1$ arrives, the investor re-optimizes: the new horizon may either reset to the same length h (a fresh plan) or count down to $h - 1$ (the same target date, now one period closer), provided the resulting horizon remains in $\mathcal{H}$.

Lee et al. (1990) assume investors hold their portfolio for the entire duration of their investment horizon without rebalancing or switching assets. In contrast, my approach does not impose such tight constraints and allows periodic rebalancing to be an integral part of the investment strategy. Conceptually, the investor primarily evaluates expected performance over their horizon (e.g., several years ahead) and reevaluates it in the next period.

The investment horizon framework aligns with time-inconsistent models, such as models employing quasi-hyperbolic preferences ($\beta\delta$ -discounting) (Laibson, 1997; O’donoghue and Rabin, 1999; Krusell and Smith, 2003). At time t , the investor commits to a plan for the near future, although when the next period arrives, they revise their plan and make new decisions about portfolio allocation.

The intuition parallels how a driver focuses on a specific range of the road ahead depending on speed⁵. Only what falls within that range primarily affects the driver’s actions: for instance, if they were going to make a right turn and found that it was already very close, they would miss it, as they would not have time to slow down. Similarly, a turn five miles ahead also does not affect their current actions. In contrast, if the turn is within the right range, it would affect their actions; they would slow down and prepare to turn. In the same way, investors concentrate on their investment horizon—the time range over which their Euler equation holds, since the very short-term market is too volatile compared to relatively stable consumption (income) in the near future, and what lies in the far future is heavily discounted and does not affect their decisions. This does not mean that instantaneous market events have no effect at all; it only means that investors mainly focus on the horizon range where their FOC must hold, just as a driver constantly focuses on the range where the car is heading and only rarely resorts to emergency braking.

⁴I use the double-index notation $M_{t,t+h}$ rather than M_{t+h} to emphasize that the SDF is defined over the *interval* from t to $t + h$, as we will later refer to the SDF between two periods in the future. This convention makes explicit both the base period and the evaluation period, which is important when the investment horizon h varies across agents or over time. The same convention applies to $R_{t,t+h}$.

⁵Appendix C provides illustrations of driver-focused vision (Figure 23, Figure 24, Figure 25).

3.2 The Horizon Function

To incorporate the investment horizon directly into a model, I formalize it as a behavioral rule—a horizon function $H(\tau)$ Equation 2 that the investor plans to follow⁶—giving the share k_τ of invested wealth the agent plans to use at $t + \tau$:

$$H(\tau) = k_\tau; \quad \sum_{\tau} k_\tau = 1, \quad k_\tau \geq 0 \quad (2)$$

For instance, an investor with an exact two-quarter horizon sets $H(2) = 1$, planning to leave the investment untouched next period and use it entirely two periods ahead. The horizon function can also be interpreted as a probability distribution over future periods, capturing the likelihood that the investment is accessed at each date. A long-term investor concentrates probability mass several months or years into the future, assigning low probability to accessing the investment next period.

For simplicity, let us assume the maximum horizon range is two periods. The horizon function then takes the form Equation 3, stating that in the next period the investor plans to use only $H(1)a_t = ka_t$ of the asset currently invested, and $H(2)a_t = (1 - k)a_t$ will remain untouched until two periods ahead. Alternatively, the probabilistic approach states that in the next period, the investor will use the currently invested asset a_t with probability k and will keep it until two periods ahead with probability $1 - k$.

$$\begin{aligned} H(1) &= k \\ H(2) &= 1 - k \\ k &\in [0, 1] \end{aligned} \quad (3)$$

Consider a parsimonious example of an endowment economy where an agent has a stochastic endowment $\tilde{e}_t$ in each period, invests in a stock a_t with current price p_t and stochastic next-period price $\tilde{p}_{t+1}$, and has a horizon function Equation 3. For simplicity, bonds are not included in the model, though the framework can be extended to incorporate risk-free bonds with 1- and 2-period durations. The agent maximizes lifetime utility Equation 4.

At time t , the agent’s wealth consists of the current endowment and all past investments: $w_t = e_t + (a_{t-1} + a_{t-2})p_t$. Expected consumption next period depends on the stochastic endowment $\tilde{e}_{t+1}$, the asset price $\tilde{p}_{t+1}$, and the fraction $H(1)$ of currently invested assets the agent plans to use. The agent forms expectations about consumption two periods ahead in the same way, remaining consistent in applying the horizon function.

⁶Commonly recommended advice is “set it and forget it.”

$$\begin{aligned}
& \max_{\{a\}_t^\infty} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U(c_t) \\
& s.t. \quad c_t = e_t + (a_{t-1} + a_{t-2})p_t - a_t p_t \\
& \quad \mathbb{E}_t[c_{t+1}] = \mathbb{E}_t[\tilde{e}_{t+1} + H(1)a_t \tilde{p}_{t+1} - a_{t+1} \tilde{p}_{t+1}] \\
& \quad \mathbb{E}_t[c_{t+2}] = \mathbb{E}_t[\tilde{e}_{t+2} + H(2)a_t \tilde{p}_{t+2} + H(1)a_{t+1} \tilde{p}_{t+2} - a_{t+2} \tilde{p}_{t+2}]
\end{aligned} \tag{4}$$

Note that the budget constraint for c_t reflects the time-inconsistency of this framework: at time t , the agent has access to all available wealth from past investments $(a_{t-1} + a_{t-2})p_t$, regardless of any plans made in previous periods. That is, although at time $t-1$ the agent planned to keep a fraction $H(2) = (1-k)$ of a_{t-1} untouched until period $t+1$, upon reaching period t the agent re-optimizes and treats all past holdings as freely available. This is the same form of time-inconsistency as in the quasi-hyperbolic preference models discussed above (Laibson, 1997; O’donoghue and Rabin, 1999). The constraints for c_{t+1} and c_{t+2} , in contrast, represent the agent’s current forward-looking investment plan at time t .

The first-order condition Equation 5 depends on the horizon function constant k Equation 3 and is a convex combination of the short- and long-horizon “Euler equations”⁷; I refer to each such term $\mathbb{E}_t[M_{t,t+h}R_{t,t+h}]$ as a *horizon Euler term*. I consider this parsimonious model only for the purpose of presenting the Euler equation as a convex combination and have not stated any special assumption on the endowment process or the return. The following discussion could apply to a broad range of models.

$$\begin{aligned}
U'(c_t)p_t &= \mathbb{E}_t[k\beta U'(c_{t+1})p_{t+1} + (1-k)\beta^2 U'(c_{t+2})p_{t+2}] \\
\Rightarrow 1 &= k\mathbb{E}_t[M_{t,t+1}R_{t,t+1}] + (1-k)\mathbb{E}_t[M_{t,t+2}R_{t,t+2}]
\end{aligned} \tag{5}$$

3.3 Horizon Euler Terms

In Equation 5, either both terms exactly equal one, or one falls below one while the other exceeds one. If an investor’s true horizon is short, their one-period Euler equation holds ($\mathbb{E}_t[M_{t,t+1}R_{t,t+1}] = 1$) and, by the law of iterated expectations and the multiplicative structure of the SDF ($M_{t,t+2} = M_{t,t+1} \cdot M_{t+1,t+2}$), recursively extends to all longer horizons ($\mathbb{E}_t[M_{t,t+2}R_{t,t+2}] = 1$). In this scenario, Equation 5 holds trivially for any k assumed by the econometrician.

Conversely, if the investor places positive weight on the long horizon ($k < 1$), the one-period Euler equation no longer holds on its own, and the two horizon Euler terms deviate from one in opposite directions—since their weighted sum must still equal one. Note that

⁷I use quotation marks because the individual terms $\mathbb{E}_t[M_{t,t+h}R_{t,t+h}]$ do not themselves satisfy the standard Euler equation condition of equalling one.

Equation 5 always holds as a whole; what varies is the value of each individual horizon Euler term.

Suppose first that $\mathbb{E}_t[M_{t,t+1}R_{t,t+1}] < 1$ and $\mathbb{E}_t[M_{t,t+2}R_{t,t+2}] > 1$. The short-horizon term being less than one means that, considered in isolation, the investor would prefer to sell (or short) the asset⁸. The long-horizon term being greater than one means that, considered in isolation, the investor would prefer to buy the asset and hold it for two periods. The investor thus balances opposing short- and long-horizon incentives to satisfy the FOC. This configuration is empirically natural: the stock market is volatile enough that a short-term investor would avoid it⁹, yet the long-run expected return is attractive enough to justify holding.

The opposite configuration, $\mathbb{E}_t[M_{t,t+1}R_{t,t+1}] > 1$ and $\mathbb{E}_t[M_{t,t+2}R_{t,t+2}] < 1$, reverses these incentives: the asset is highly attractive to hold for the immediate period, yet its expected performance deteriorates over the longer horizon. A classic example is a momentum stock during a market rally: short-term expected returns are exceptionally high, but these are offset by the long-horizon Euler term falling below one.

The empirically relevant configuration is determined by how the horizon Euler term $F(h) \equiv \mathbb{E}_t[M_{t,t+h}R_{t,t+h}]$ depends on h : if $F(h)$ increases with h , the first configuration prevails; if it decreases, the second prevails. In general, $F(h)$ need not be monotone: it may increase over shorter horizons and decrease over longer ones, or vice versa, implying that the two configurations can apply to the same asset at different horizons. I next provide conditions under which $F(h)$ is increasing in h for moderate horizons, which I argue is the empirically relevant case.

To formalize this intuition, assume that the horizon Euler term $\mathbb{E}_t[M_{t,t+h}R_{t,t+h}]$ increases with the horizon h , at least for some $0 < h < \bar{h}$ (Daniel and Marshall, 1997; Campbell, 1996; Campbell and Cochrane, 1999). This assumption relies on departures from the random walk benchmark, which we formalize below. Using the decomposition $\mathbb{E}_t[M_{t,t+h}R_{t,t+h}] = \mathbb{E}_t[M_{t,t+h}] \mathbb{E}_t[R_{t,t+h}] + \text{Cov}_t(M_{t,t+h}, R_{t,t+h})$, each component depends on h . Consumption behaves approximately as a random walk, with the variance of consumption growth increasing with the horizon; the convexity of the SDF then implies that greater volatility raises $\mathbb{E}_t[M_{t,t+h}]$. Expected returns $\mathbb{E}_t[R_{t,t+h}]$ and return variance $\text{Var}_t(R_{t,t+h})$ also grow with h through compounding. The covariance $\text{Cov}_t(M_{t,t+h}, R_{t,t+h})$, while negative at all horizons, becomes less negative as h increases due to mean reversion in returns (Parker and Julliard, 2005). All three channels work in the same direction, pushing $\mathbb{E}_t[M_{t,t+h}R_{t,t+h}]$

⁸ $\mathbb{E}_t[M_{t,t+1}R_{t,t+1}] < 1 \Rightarrow U'(c_t) > \mathbb{E}_t[\beta U'(c_{t+1})R_{t,t+1}]$. The investor would want to increase current consumption and reduce investment to restore equality.

⁹Consider an example: if you are buying a house in one week, would you put your cash in the stock market for only that single week?

upward with h . Since the FOC [Equation 5](#) requires the weighted sum to equal one, if the short-horizon term falls below one, the longer-horizon term must exceed one.

Consider an econometrician who assumes a short investment horizon when the true horizon is longer. The econometrician would then observe [Equation 6](#), which is less than one under the agent's true beliefs and risk preferences. Estimating the model under the constraint that the short-horizon Euler equation equals one yields higher implied risk-aversion γ^{10} , leading to the well-known equity premium puzzle.

$$\mathbb{E}[M_{t,t+1}R_{t,t+1}] < 1 \quad (6)$$

3.4 Random Walk Dynamics and Horizon Irrelevance

To make the preceding argument precise, we now characterize when $F(h) \equiv \mathbb{E}_t[M_{t,t+h}R_{t,t+h}]$ is increasing in h within a tractable benchmark. Assume CRRA utility with risk aversion γ , random-walk cumulative log consumption growth $\ln(c_{t+h}/c_t) \sim \mathcal{N}(gh, \sigma_c^2 h)$, and cumulative log returns $r_{t,t+h} \sim \mathcal{N}(\mu_r h, \sigma_r^2 h)$, jointly normal with per-period covariance $\sigma_{c,r}$ ¹¹. Because $\ln(M_{t,t+h}R_{t,t+h}) = h \ln \beta - \gamma \ln(c_{t+h}/c_t) + r_{t,t+h}$ is normally distributed, the log-normal moment-generating property gives¹²:

$$F(h) = \exp(\psi \cdot h), \quad \psi \equiv \ln \beta - \gamma g + \mu_r + \frac{1}{2}\gamma^2 \sigma_c^2 + \frac{1}{2}\sigma_r^2 - \gamma \sigma_{c,r}. \quad (7)$$

The function $F(h)$ is monotonically increasing in h whenever $\psi > 0$. Substituting $F(h) = \exp(\psi \cdot h)$ into [Equation 5](#) yields

$$k \exp(\psi) + (1 - k) \exp(2\psi) = 1. \quad (8)$$

Under the random walk assumption for log consumption and log returns, ψ is a constant independent of the horizon, and this equation admits only the solution $\psi = 0$ ¹³, implying $F(1) = F(2) = 1$ regardless of k . The inconsistency between the empirical $\psi > 0$ and the

¹⁰Higher γ increases SDF curvature; by Jensen's inequality, this raises $\mathbb{E}[M_{t,t+1}]$.

¹¹These are the standard assumptions in the long-run risk and consumption-based asset pricing literatures ([Campbell, 1996](#); [Daniel and Marshall, 1997](#)).

¹²Let $x_h = h \ln \beta - \gamma \ln(c_{t+h}/c_t) + r_{t,t+h}$. Then $\mathbb{E}_t[x_h] = h(\ln \beta - \gamma g + \mu_r)$ and $\text{Var}_t(x_h) = h(\gamma^2 \sigma_c^2 + \sigma_r^2 - 2\gamma \sigma_{c,r})$, using $\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) + 2ab \text{Cov}(X, Y)$ and the fact that all variances and covariances scale linearly with h .

¹³Setting $y = \exp(\psi)$, the condition becomes $(1 - k)y^2 + ky - 1 = 0$, whose roots are $y = 1$ and $y = -1/(1 - k)$. Since $y > 0$, we must have $y = 1$ and hence $\psi = 0$. Crucially, the binding restriction stems from the random walk *dynamics*, not merely from log-normality of the marginal distribution at a given horizon: a process with richer temporal structure—such as persistent or mean-reverting consumption growth—can produce log-normal marginals at each horizon while still generating $\psi \neq 0$ and non-trivial horizon effects.

equilibrium requirement $\psi = 0$ echoes the equity premium puzzle from a horizon perspective. In other words, when log consumption and log returns follow random walks, the horizon Euler terms equal one at every horizon and the investment horizon is irrelevant—the standard one-period Euler equation suffices. Any model that maintains the random walk assumption therefore renders the investment horizon vacuous: introducing the horizon function [Equation 3](#) changes nothing, since $F(1) = F(2) = 1$ for every k , and the FOC [Equation 5](#) reduces to the standard one-period Euler equation regardless of the investor’s true horizon. Yet if investors do in fact have long horizons—as the evidence suggests—and the econometrician incorrectly imposes the random walk together with a short-horizon Euler equation, the model is misspecified: the true short-horizon Euler term $\mathbb{E}_t[M_{t,t+1}R_{t,t+1}] < 1$ (see [Equation 6](#)), and forcing it to equal one requires inflating the risk-aversion parameter γ , directly producing the equity premium puzzle.

Models that resolve the equity premium and risk-free rate puzzles relax the random walk assumption. [Parker and Julliard \(2005\)](#) demonstrate that consumption growth does not follow a random walk: its variance grows faster than linearly up to two years and slower after that. They show that a three-year horizon ($S = 11$ quarters following the return) provides a significantly better cross-sectional explanation of portfolio return variation than any shorter horizon.

More broadly, models that resolve these puzzles typically do so by modifying one of three ingredients: how investors evaluate consumption—as in the habit formation model of [Campbell and Cochrane \(1999\)](#); the dynamics of the consumption growth process—as in the long-run risk model of [Bansal and Yaron \(2004\)](#); or the distribution of shocks to consumption and returns—as in the rare disaster model of [Barro \(2006\)](#) and the rare-event framework of [Julliard and Ghosh \(2012\)](#). Using the habit formation model of [Constantinides \(1990\)](#), [Daniel and Marshall \(1997\)](#) conclude that the equity-premium and risk-free-rate puzzles are primarily problems for shorter-horizon returns.

3.5 Factors affecting investment horizon

In the remainder of the paper, for simplicity I assume a horizon function $H(h) = 1$ concentrated on a single horizon h , so that the FOC [Equation 5](#) reduces to the single-horizon [Equation 1](#). A reasonable assumption is that returns and consumption growth are log-normally distributed over this horizon h . Note that I do not make any assumption about the dynamics; I only assume a joint normal distribution at this slice—horizon h . Under these assumptions, [Equation 7](#) gives:

$$\mathbb{E}_t[M_{t,t+h}R_{t,t+h}] = \exp(\psi \cdot h) = 1, \quad \psi \equiv \ln \beta - \gamma g + \mu_r + \frac{1}{2}\gamma^2\sigma_c^2 + \frac{1}{2}\sigma_r^2 - \gamma\sigma_{c,r}. \quad (9)$$

Consider a hypothetical exercise. Equation 9 would hold in equilibrium; if some parameters change while portfolio allocation remains unchanged, how should h adjust to restore equilibrium? An alternative way to see this is as an individual investor whose SDF depends on aggregate consumption—since all idiosyncratic shocks can be insured—and who invests in the aggregate stock market¹⁴ (Brav et al., 2002). The following results echo the findings from the literature reviewed in the previous section.

We can conclude that more risk-averse investors prefer a longer investment horizon¹⁵; they are unwilling to bear the risk of short-horizon positions. Market returns and consumption growth work in opposite directions: if an investor expects the market to rise by more than expected consumption growth, the investor would prefer a shorter horizon; otherwise, the investor would prefer a longer horizon.

Higher volatility of market returns is associated with bad times and leads to a lower investment horizon. Higher volatility of consumption growth, also associated with recessions, has the same effect. If consumption and market returns become more negatively correlated, the investor can use the asset as a hedge and therefore prefers a shorter investment horizon.

Later in the paper, I present a method to identify a possible range of investor horizons based on the investor’s beliefs, and I expect to find similar results.

4 Methodology

4.1 Model Setup

We start with the class of consumption-based asset pricing models with a representative agent, who makes investment and consumption decisions at each time period. A representative agent framework is widely utilized because it provides analytical Euler equations for the joint behavior of asset returns and aggregate consumption. Even though the representative agent paradigm is sometimes criticized, it is still used due to its ability to explain the aggregate time series (Eichenbaum et al., 1988; Campbell and Cochrane, 1999).

We could state the problem from the individual agent’s point of view, as many papers do (Weil, 1989; Campbell and Cochrane, 1999), and later aggregate the results, assuming that the representative agent’s preferences could take the same form as the individual agents’

¹⁴I will discuss the representative agent framework in detail below.

¹⁵Higher γ leads to lower ψ ; to maintain equilibrium, h must be higher.

preferences; or they could result from the aggregation of heterogeneous consumers with other preferences (Constantinides and Duffie, 1996). By assuming that the market is complete and hence households can insure their idiosyncratic shocks, the agent’s SDF will depend on aggregate consumption growth. Brav et al. (2002) state, “Full consumption insurance implies that heterogeneous consumers are able to equalize, state by state, their marginal rate of substitution. Therefore, the equilibrium in a heterogeneous-consumer, full-information economy is isomorphic in its pricing implications to the equilibrium in a representative-consumer.” Alternatively, to save space, we initially state the problem for a representative or single agent (Bansal and Yaron, 2004; Kocherlakota, 1990).

It is worth noting that at each time t , we assume that the market is in equilibrium in such a way that the representative agent’s Euler equation should be satisfied. The market reaches equilibrium through the adjustment of asset prices; thus, no direct trade is required in this framework. Typically, the production side is not considered, and it is assumed that the representative agent makes decisions about the allocation of consumption to firms and back.

4.2 Identification of Investment Horizon Range

In this subsection, we present an approach that is used to identify investment horizons. The approach is a sort of ‘quasi’ test, which compares the agent’s decisions made under different hypothetical beliefs and identifies the most likely one that maximizes his utility and matches his behavior. Section 4.3 covers the method to recover investor beliefs as a non-parametric joint distribution of consumption, returns, and other conditional variables.

First, we examine the fundamentals of analyzing an agent with different beliefs. Specifically, we explore what can be inferred about the agent’s decision-making when assuming alternative beliefs, while holding all other factors constant. We extract the beliefs in a manner that satisfies the first-order condition (10). The hat symbol denotes estimated beliefs.

$$U'(C_t) = \hat{\mathbb{E}}_t^S \beta U'(C_{t+1}) R_{t,t+1} \quad (10)$$

If we know the true subjective beliefs S and find that the first-order condition is not satisfied under these beliefs, it would mean that the agent, by solving his maximization problem, meets some constraints. If LHS of Equation 10 is less than RHS, the agent reaches the upper boundary solutions for the particular asset. He would like to move some of his current consumption to the future by buying this asset but could not. We do not know exactly the reasons for this; it could be explicit restrictions or any implicit behavior constrains¹⁶.

¹⁶For example, the first-order condition (FOC) for drug trading is not satisfied.

Conversely, if LHS of Equation 10 is greater than RHS, the agent's solution touches the opposite, lower boundary. In this case, he would like to short more of the particular asset than he could, or he just doesn't invest in this asset at all. For simplicity, we define Euler Equation Error as Equation 11. We can state that if Euler equation error $EER(S) > 0$, the agent would like to increase his investment. By contrast, if $EER(S) < 0$, the agent would like to decrease his investment, or he just does not invest at all.

$$EER(S) = \mathbb{E}_t^S \beta U'(C_{t+1}) R_{t,t+1} - U'(C_t) \quad (11)$$

We define the agent's beliefs S as a joint distribution of conditional variables such as consumption, market returns, and others, which affect the agent's expectations and SDF. Assume that we have two possible candidates for the true agent beliefs, $S1$ and $S2$. For each of them, we calculate $EER()$, and for simplicity, let's assume that $EER(S1) > 0 > EER(S2)$. Since the FOC is not satisfied for either of them, we can't definitively identify the true one¹⁷. Yet, we can infer which is more probable by considering the agent's decisions. If the agent is investing, $S1$ would be more probable, and the agent would reach an upper bound solution, as demonstrated above. Conversely, if the agent does not invest, it is likely that they hold the beliefs $S2$; otherwise, if the agent invests while $EER(S2) < 0$, they would be incentivized to disinvest until $EER(S2) = 0$ or until they liquidate their stock position entirely (assuming short selling is prohibited). Therefore, our inference about the agent's beliefs depends on their actual behavior, not only on the sign of EER. It is noteworthy that $EER()$ can be reformulated for excess return Equation 12 and interpreted similarly. In equilibrium, the agent should not be able to gain utility at the margin by selling bonds and investing in stocks. Otherwise, if $EER() > 0$, the agent would want to increase their investment.

$$EER(S) = \mathbb{E}_t^S \beta \frac{U'(C_{t+1})}{U'(C_t)} (R_{t,t+1} - R_{t,t+1}^{rf}) \quad (12)$$

Short vs Long horizon test

In this part, we describe the approach that we use to extract information about investment horizons. We conduct the test for the given investment horizon $2h$ and as a result, we can conclude which investment horizon is more probable: $2h$ or half of them - h , which we refer to as the long and short horizon types respectively. The test is based on the assumption that agents' beliefs are time-consistent in terms of transitivity, so the expectation about two

¹⁷At this moment, we consider the simplest model, without the possibility of having a different horizon, so we can state that $EER(S) = 0$ leads us to true beliefs.

periods ahead equals the multiplication of two sequential expectations¹⁸. We construct the test by comparing two identical agents with the same lifetime and the same expected paths for all variables, differing only in the investment horizon type.

We define the agent who maximizes his lifetime utility with a lifetime $2h$. The agent could have one of two investment horizons: h vs $2h$. Thus his beliefs are either S (short) or L (long), corresponding to each type of horizon. If the agent is the short type, he invests with a horizon of h and reinvests in the same horizon in the next period. If he is the long horizon type, he invests only with a horizon of $2h$. It is important to note that both types live the same time period - $2h$ and have the same expected path of all conditional variables, the only difference is the investment horizon type. For simplicity, further, we assume h equals one quarter.

Consider, if the agent is the short type horizon investor with beliefs S , then his first order condition should be satisfied:

$$U'(C_t) = \hat{\mathbb{E}}_t^S \beta U'(C_{t+1}) R_{t,t+1} \quad (13)$$

Since beliefs are the joint distribution of consumption and returns for all possible states, by obtaining expectations about C_{t+1} for each state T , the next period first-order conditions should also be satisfied:

$$U'(C_{t+1}) = \hat{\mathbb{E}}_{t+1}^S \beta U'(C_{t+2}) R_{t+1,t+2} \quad (14)$$

The agent solved his problem backward, thus substituting Equation 14 into Equation 13 to obtain the two-period Euler Equations. It is noteworthy that the two-period Euler Equation 15 includes all possible inner paths for all states. In addition, we would like to highlight that short beliefs are formed for one time ahead, therefore, for expectations two periods ahead, two symbols of expectation are used (the first one is for the time t and second expectations for each state T in the next period).

$$U'(C_t) = \beta^2 \mathbb{E}_t^S R_{t,t+1} \mathbb{E}_{t+1}^S U'(C_{t+2}) R_{t+1,t+2} \quad (15)$$

Consider, if the agent is the long type, his belief $\mathbb{E}^L$ is extracted such that the first-order condition (FOC) is satisfied:

$$U'(C_t) = \hat{\mathbb{E}}_t^L \beta^2 U'(C_{t+2}) R_{t,t+2} \quad (16)$$

¹⁸For simplicity, with only one possible state, if the agent expects 10% growth next month, they would expect 1.10^2 in two months. In the case of T possible states, the agent, two periods ahead, has T^*T possible paths, and the beliefs transitional matrix remains the same.

To sum up, we have an agent who lives time period $2h$. We have two candidates for his beliefs; he can be either long or short type horizon investor. Both candidate take into account all possible paths and states during this time. Thus, it is exactly the same agent; the only difference is in the beliefs he has used. We are going to test which beliefs are used by comparing the Euler Equation Error.

The Euler Equation Errors are given by Equation 17. If $EER(S) > EER(L)$ and if the agent actually invests, as we covered above, he likely has beliefs S , and we can conclude the agent is a short-horizon investor. On the other hand, If $EER(L) > EER(S)$, the agent is likely a long-horizon investor. Yet, we cannot state that it is the 'true' investment horizons, as we are only comparing two candidates. By comparing the sequences of candidate h and $2h$, we would be able to determine the range of most likely investment horizons.

It is worth noting that, due to the constraint, $EER(L)$ is zero, as shown in Equation 16. However, we cannot conclude that these are the true beliefs; we merely extract them, assuming they are true. Thus, we only compare the sign of $EER()$ for two beliefs' candidates and guess which of them is more probable if the agent invests. In addition, as we see above (12), the $EER()$ can easily be estimated by using excess returns.

$$\begin{aligned} EER(S) &= \beta^2 \hat{\mathbb{E}}_t^S R_{t,t+1} \hat{\mathbb{E}}_{t+1}^S U'(C_{t+2}) R_{t+1,t+2} - U'(C_t) \\ EER(L) &= \hat{\mathbb{E}}_t^L \beta^2 U'(C_{t+2}) R_{t,t+2} - U'(C_t) \end{aligned} \quad (17)$$

4.3 Recovery of Beliefs and Risk Preferences

In this subsection, we briefly explain how subjective beliefs are extracted. We follow the approach outlined in the papers by Ghosh and Roussellet (2023); Kitamura et al. (2004), which cover this topic in full detail. Beliefs are extracted for any given model so that this section can be viewed separately from matters concerning investment horizons.

The absence of arbitrage opportunities implies the existence of a strictly positive pricing kernel M_{t+1} such that the Euler equation (18) is satisfied. Capital S indicates that expectations are based on the subjective agent's beliefs. The SDF of economic agents can be identified as a parametric function (19) of consumption growth $G_{t+1} = C_{t+1}/C_t$, set of risk factors Y_{t+1} , and θ_0 as the true value of the vector of parameters driving the SDF.

$$\mathbb{E}^S [M_{t+1} \mathbf{R}_{t+1}^e \mid \mathcal{F}_t] = \mathbf{0}_k \quad (18)$$

$$M_{t+1} = M(G_{t+1}, Y_{t+1}; \theta_0), \quad (19)$$

Our goal is to extract these subjective beliefs S in a way that minimally distorts the

true Data Generating Process (DGP) and satisfies the conditional Euler equations (18). We define two matrices P^O and P^S for the objective and subjective joint distributions, such that $p_{i,j}^{(o)}$ and $p_{i,j}^{(s)}$ represent the objective and subjective conditional probabilities of transitioning from the state observed at time $t = i$ to that observed at $t' = j$. The number of states equals the number of observations T . Each state is defined by the vector x_t , which is a set of conditional variables $(g_t, r_t, y_t, \dots)$ over which joint distributions are defined. The Euler equation (18) can be rewritten as follows:

$$\mathbb{E}^S [M(G_{t+1}; \theta_0) \mathbf{R}_{t+1}^e \mid \mathcal{F}_t] = \mathbf{0}_k \Rightarrow \sum_{j=1}^T p_{t,j}^{(s)} \cdot M(g_j, y_j; \theta_0) \mathbf{r}_j = \mathbf{0}_k \quad (20)$$

The objective distribution is estimated as a non-parametric kernel:

$$\forall i, j \in \{1, \dots, T\}, \quad p_{i,j}^{(o)} = \frac{\mathcal{K}\left(\frac{x_i - x_{(j-1)}}{b_T}\right)}{\sum_{t=2}^T \mathcal{K}\left(\frac{x_i - x_{(t-1)}}{b_T}\right)} \quad (21)$$

To obtain the result, we use the Smoothed Empirical Likelihood (SEL) estimator which minimizes the Kullback-Leibler Information Criterion divergence (KLIC, or relative entropy) between the objective and subjective measures:

$$\hat{p}_{t,\cdot}^{(s)} = \arg \min_{\{p_{t,\cdot}\} \in \Delta} \sum_{j=1}^T p_{t,j}^{(o)} \cdot \log \left(\frac{p_{t,j}^{(o)}}{p_{t,j}^{(s)}} \right) \text{ s.t. } \sum_{j=1}^T p_{t,j}^{(s)} \cdot M(g_j, y_j; \theta_0) \mathbf{r}_j = \mathbf{0}_k \quad (22)$$

and, for a given $\forall t$ and $j \in \{1, \dots, T\}$ the closed-form solution is as follows:

$$\hat{p}_{t,j}(\theta_0) = \frac{1}{1 + \hat{\lambda}_t(\theta_0)' [M(g_j, y_j; \theta_0) \mathbf{r}_j - \mathbf{1}_k]} \times p_{t,j}^{(o)},$$

where $\hat{\lambda}_t(\theta_0) \in \mathbb{R}^k$ are the Lagrange multipliers associated with the conditional Euler equation constraints, and solve the following unconstrained maximization problem:

$$\hat{\lambda}_t(\theta_0) = \arg \max_{\lambda_t \in \mathbb{R}^k} \sum_{j=1}^T p_{t,j}^{(o)} \log [1 + \lambda_t' (M(g_j, y_j; \theta_0) \mathbf{r}_j - \mathbf{1}_k)]$$

The recovered conditional probabilities $\{\hat{p}_{t,j}(\theta_0)\}_{t,j=1}^T$ represent the beliefs of the representative investor that are consistent with observed asset prices at each time. It is worth noting that beliefs are recovered for the particular moment, a time t , and are independent of previous and future periods. Thus, the approach does not have drawbacks such as those

which occur from overlapping periods in time series econometric analysis.

5 Results for the Homogeneous Investment Horizon

This section is dedicated to homogeneous investment horizons, as covered in the previous section. The first part presents an overview of subjective beliefs and risk preferences for a representative investor with various investment horizons, which provides insights into the true investment horizon. The second part presents the results of identifying the investment horizon range, as described in Section 4.2.

Data

We utilize data spanning from 1961:3Q to 2016:4Q.¹⁹ For the stock market, we use real market returns computed as the value-weighted index of a broad set of stocks provided by CRSP.²⁰ As for aggregate consumption, we use real personal expenditure on nondurable goods and services, and for risk-free rates, we use the deflated nominal yield curves (ranging from 1 year to 5 years). The set of state variables (or conditional variables) consists of exponentially weighted moving averages²¹ of past real excess returns, consumption growth, three factors from PCA on nominal risk-free yield curves, and the first difference of the conditional variables. We use a Gaussian kernel as a baseline.

5.1 Overview of Subjective Beliefs and Risk Preferences over Varying Investment Horizons

In this subsection, we extract subjective beliefs for the CRRA utility agent over possible horizons h , ranging from 1 to 20 quarters.

Firstly, we extract implied risk aversion for different investment horizons by minimizing Smoothed Empirical Likelihood (SEL) (22) over γ , for a given h :

$$\gamma_h = \min_{\gamma} SEL(\gamma, h) \quad (23)$$

Figure (1) shows the results for various investment horizons. The long-run risk aversion is about $\gamma = 50$, which is six times lower than that of the 1Q horizon. It stabilizes for horizons

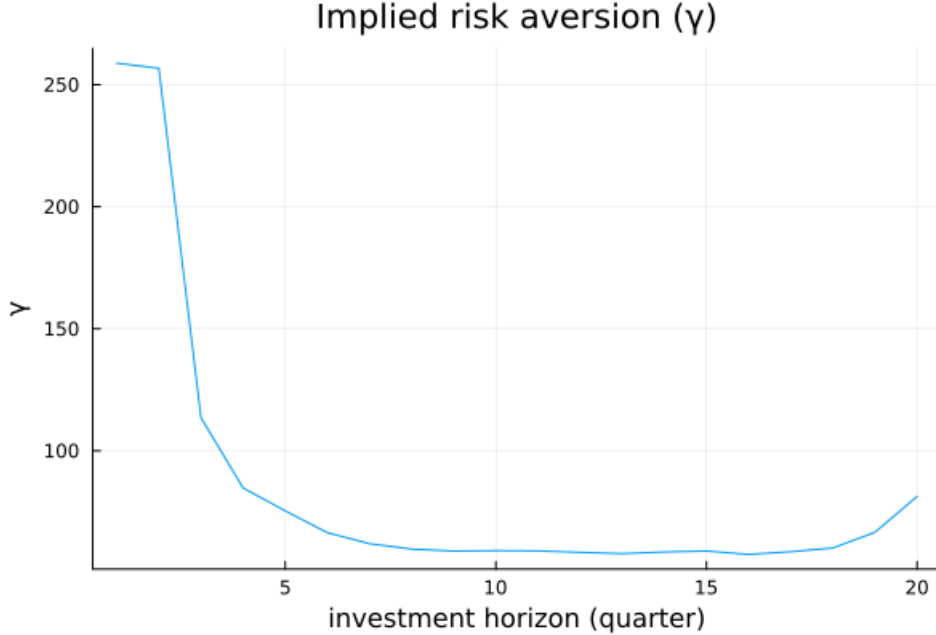
¹⁹The paper is currently being updated to cover a broader time span—1947:2Q to 2019:4Q. All results remain the same.

²⁰In the fourth quarter of 2019, the index included 7,729 stocks.

²¹Our exponentially weighted variables are computed as $X_t^{(EW)} = \alpha X_t + (1 - \alpha)X_{t-1}^{(EW)}$. We set $\alpha = 0.28$, whereby the past 13 quarters receive 99% of the weight. Our results are unaffected by the value of α .

longer than 8Q, which significantly reduces the severity of the equity premium puzzle. All of the above suggests that the risk aversion for the shorter horizon is unrealistically high. Since we can assume that risk aversion is a stable and constant trait of the investor, for the rest of the paper we will use $\gamma = 50$, unless otherwise specified.

Figure 1



Notes: The figure plots implied risk aversion γ for various investment horizons. Excess market returns are used in the Euler equation. The result is obtained by minimizing SEL over γ . The pricing kernels correspond to CRRA. Real market returns correspond to S&P 500 index, consumption to the real personal expenditure on nondurable goods and services, and risk-free rates to the deflated nominal yield curves for each quarter from 1 to 20. Sample is 1947:2Q-2019:4Q.

To support our findings, we extract risk aversion parameters for various setups. Figure (2)(a) presents results for gross returns on a five-asset portfolio and bonds. Figure (2)(b) shows the estimation of implied risk aversion preferences of the Epstein-Zin utility function on market returns and bonds. Finally, Figure (3) shows cross-sectional results, with estimations conducted separately for six non-overlapping portfolios. All these results demonstrate similar patterns of declining risk aversion values around the middle of investment horizons. Estimations, which include separate Euler equations for risk-free asset (gross returns Euler equations), show a slightly rising value at the longer tail. We do not delve deeply into this section; one hypothesis is that fixed income could have different investment horizons. We will speculate about this later.

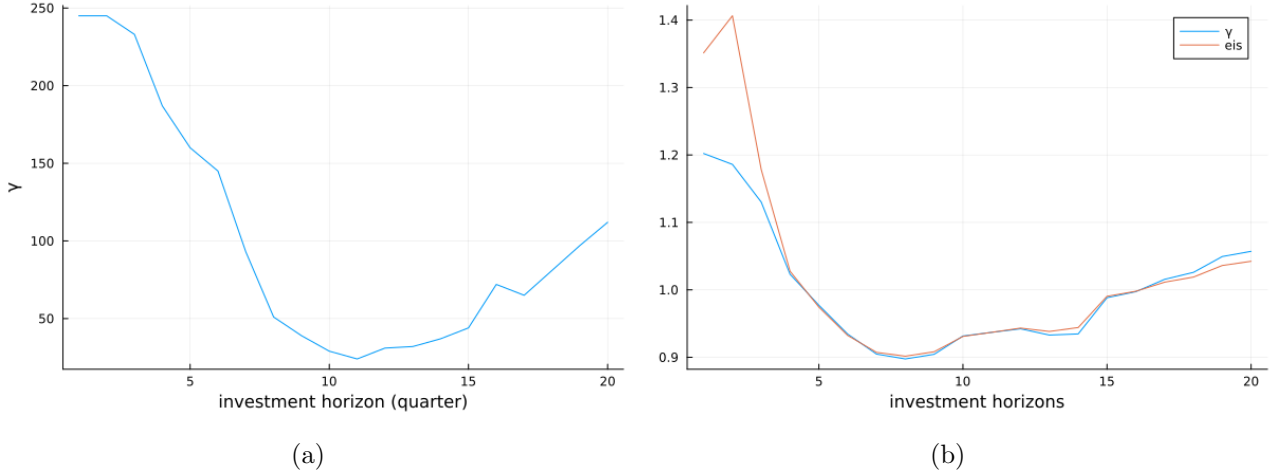
In the rest of this section, we will compare subjective and objective beliefs, exploring the differences and drawing insights about the 'true' investment horizon. The figure (4) presents

the results of the divergence between objective and subjective excess returns for various horizons. Distortions between objective and subjective returns are much more pronounced at shorter horizons and subside by 8 quarters. Figure (5) shows annualized objective and subjective standard deviation for 1, 4, 8, and 10 quarters horizons. The first observation (which is not new) is that annualized standard deviation is declining over time. Objective and subjective standard deviations diverge up to 8Q, but the level isn't economically significant, so we can't conclude that it's not a realistic divergence. I want to highlight two additional observations. The standard deviation is countercyclical after 10Q, suggesting that during a recession, agents are more certain that in the period two and a half years, some growth is likely expected, but they are not sure about the nearest future. Second, there are two time periods, before and after 1992, where 4Q subjective and objective standard deviations coincide before and diverges after. We will see the similar result later in the paper and discuss it there.

Figures (6 - 7) present the results of the divergence of objective and subjective Sharpe ratios for different investment horizons. Firstly, we observe that the Sharpe Ratio is growing over time, which is reasonable. However, as depicted in Figure 7, the Sharpe Ratio grows at a higher rate for up to 8 quarters and then stabilizes. Second, objective and subjective Sharpe Ratios significantly diverge, up to 5 times, for short horizons, which is not realistic and may be a sign that this horizon is not reasonable. Third, belief distortions are more pronounced at shorter horizons and less pronounced during a recession. This could be a sign that the horizon is shorter in 'bad' times. And finally, we observe the same pattern as for standard deviation, but much more pronounced; after 5Q horizons, there are several stages: before 1992, after 1992, and after 2009. The reasons for this are discussed below.

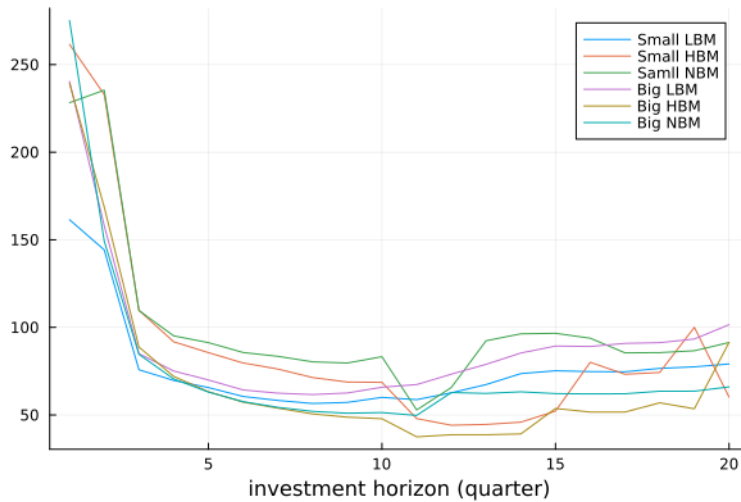
To conclude this section, we summarize all findings and highlight the main intuition. We examine subjective beliefs and attempt to identify whether they are plausible or unrealistic. The term 'unrealistic' is somewhat ambiguous and not strictly defined, but insights can be gained by comparing subjective and objective beliefs. Based on the findings in this section, we can make some inferences about the 'true' investment horizon. First, we observe that implied risk aversion declines rapidly and stabilizes around 8Q. Second, the subjective return and Sharpe ratio are approximately five times lower for 1Q, and only converge around 7-8Q. Finally, the Sharpe ratio exhibits two distinct patterns of growth, before and after 8Q. Something significant appears to happen around 8Q, leading us to infer that the true investment horizon may be close to this number.

Figure 2: Implied risk aversion and EIS. Gross returns.



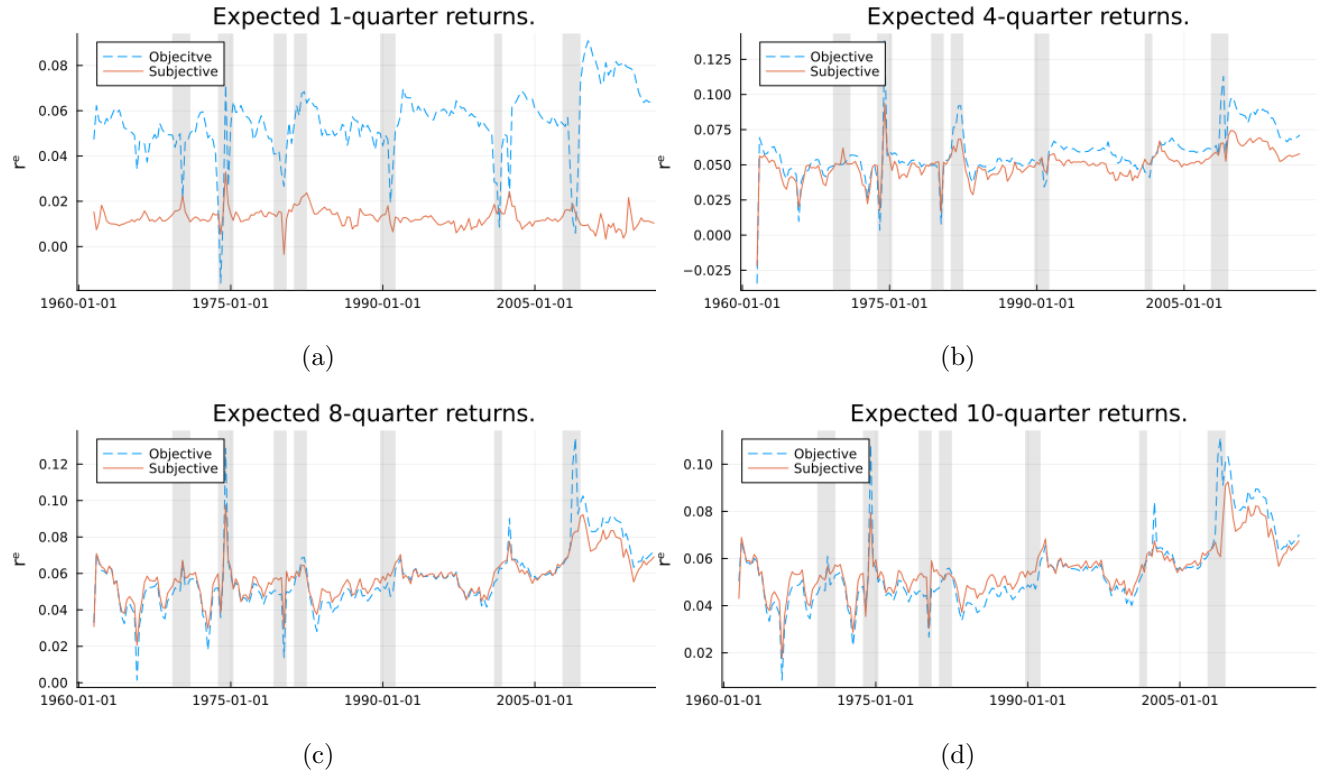
Notes: The figure plots implied risk aversion γ and *EIS* for various investment horizons. Assets: Market, Low and High B/M, Large and Small sizes. Panel (a) presents results for CRRA utility function and 5 assets and RF gross returns. Panel (b) presents results for Epstein-Zin utility function and gross market return and RF. The result is obtained by minimizing SEL. The pricing kernels correspond to CRRA. Real market returns correspond to S&P 500 index, consumption to the real personal expenditure on nondurable goods and services, and risk-free rates to the deflated nominal yield curves for each quarter from 1 to 20. Sample is 1961:3Q-2016:4Q.

Figure 3: Implied risk aversion. Each asset class separately.



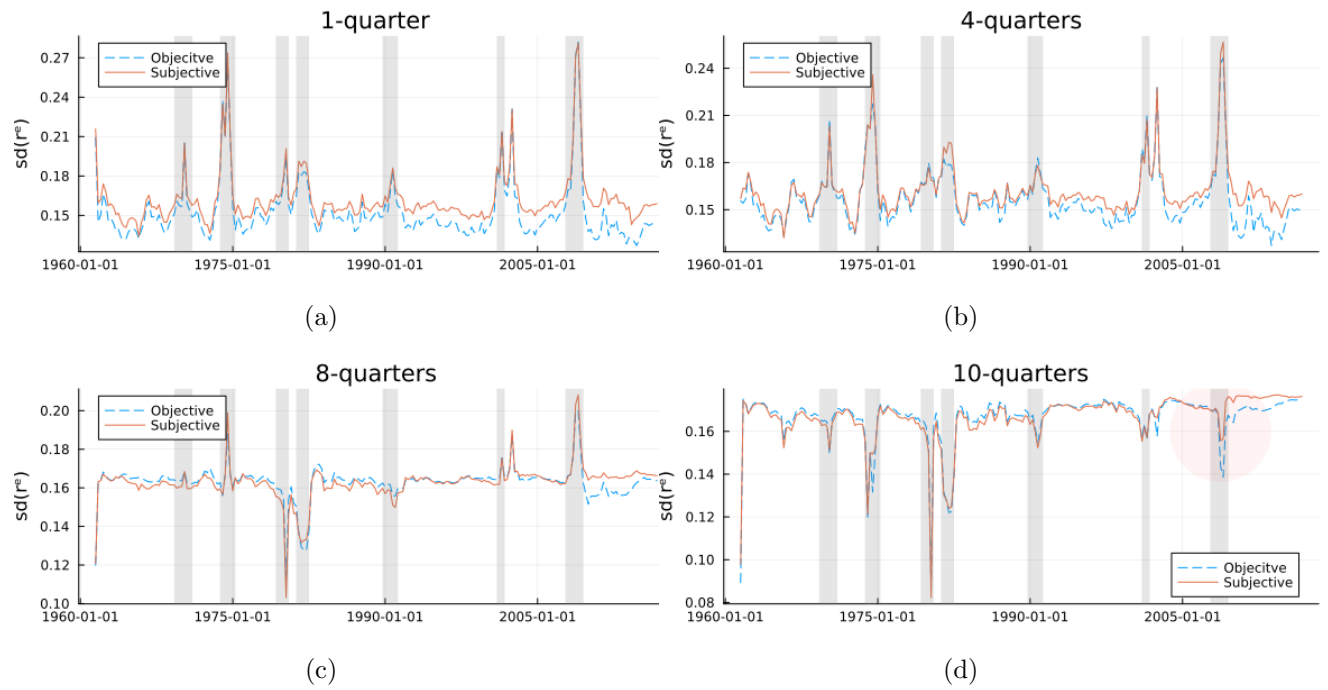
Notes: The figure plots implied risk aversion γ for various investment horizons. Excess returns on six, non overlapping portfolios are used in the Euler equation. The result is obtained by minimizing SEL over γ . The pricing kernels correspond to CRRA. Real market returns correspond to S&P 500 index, consumption to the real personal expenditure on nondurable goods and services, and risk-free rates to the deflated nominal yield curves for each quarter from 1 to 20. Sample is 1961:3Q-2016:4Q.

Figure 4: Objective and Expected Excess Returns for Various Investment Horizons



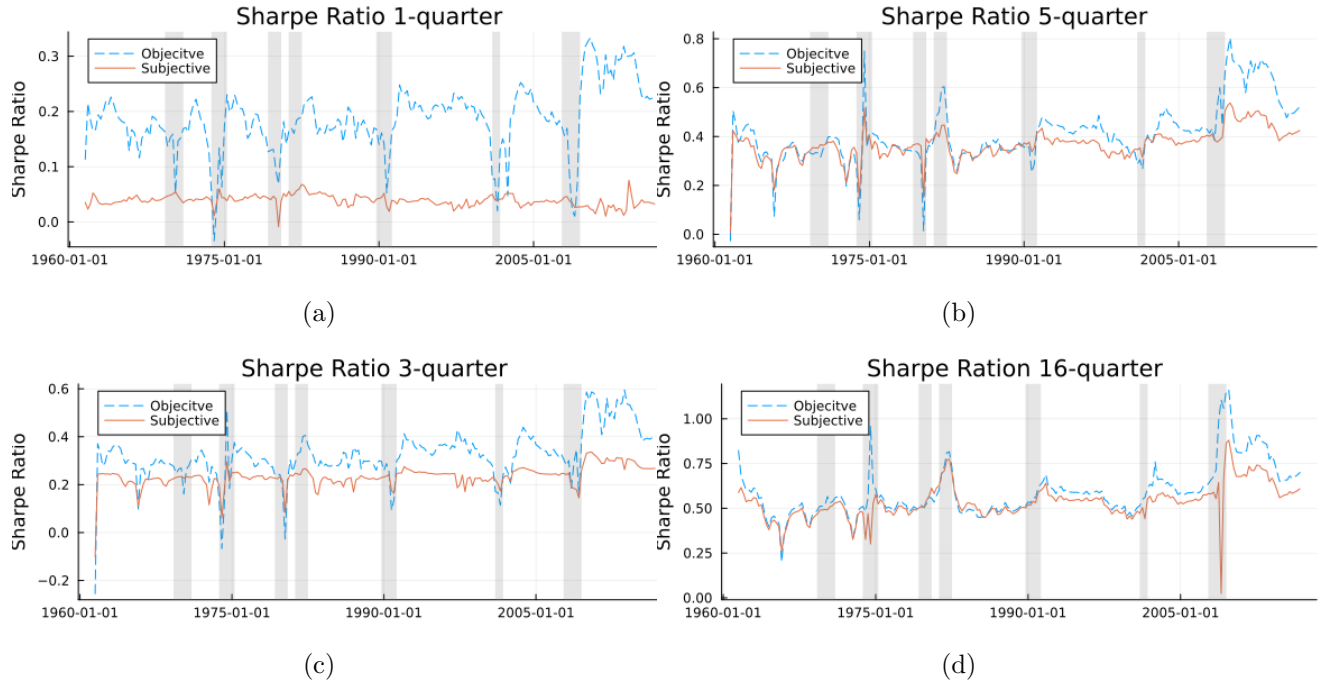
Notes: The figure plots annualized Objective and Subjective Expected Excess returns for various investment horizons. The pricing kernels correspond to CRRA, $\gamma = 50$. Real market returns correspond to S&P 500 index, consumption to the real personal expenditure on nondurable goods and services, and risk-free rates to the deflated nominal yield curves for each quarter from 1 to 20. Sample is 1961:3Q-2016:4Q.

Figure 5: Objective and Subjective Standard Deviation of Returns for Various Investment Horizons



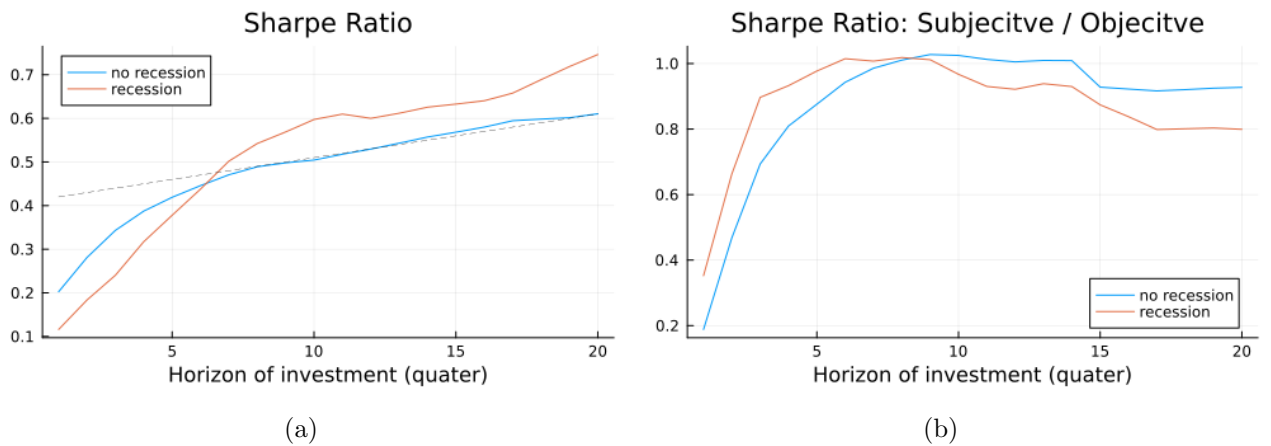
Notes: The figure plots annualized Objective and Subjective Standard Deviations of returns for various investment horizons. The pricing kernels correspond to CRRA, $\gamma = 50$. Real market returns correspond to S&P 500 index, consumption to the real personal expenditure on nondurable goods and services, and risk-free rates to the deflated nominal yield curves for each quarter from 1 to 20. Sample is 1961:3Q-2016:4Q.

Figure 6: Objective and Subjective Sharpe Ratio for Various Investment Horizons



Notes: The figure plots Objective and Subjective Sharpe Ratio for various investment horizons. The pricing kernels correspond to CRRA, $\gamma = 50$. Real market returns correspond to S&P 500 index, consumption to the real personal expenditure on nondurable goods and services, and risk-free rates to the deflated nominal yield curves for each quarter from 1 to 20. Sample is 1961:3Q-2016:4Q.

Figure 7: Mean Sharpe Ratio for each Investment Horizon



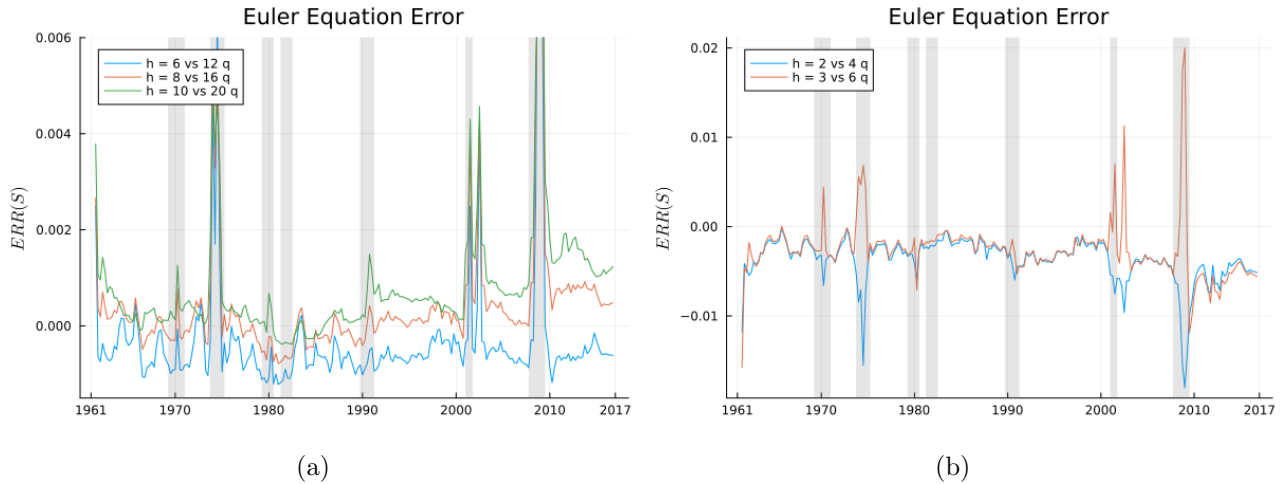
Notes: The left panel (a) shows objective Sharpe Ratio for various investment horizons. The right panel (b) shows the subjective Sharpe ratio relative to the objective one.

5.2 Identification Investment Horizon Range

In this subsection, we follow the methodology outlined in Section 4.2 and conduct a "quasi" test to identify the most likely investment horizon, h versus $2h$. We estimate $EER(S)$, as defined in equation (17), for various horizons h and present the results below, utilizing the Euler equation for excess market returns. If $EER(S) > 0$, the agent is more likely to have a short horizon h ; otherwise, a long horizon $2h$ is indicated. This process is repeated for all possible values of h . For instance, as we will see below, when considering the pair 1Q versus 2Q, if the agent invests, they likely have a horizon of 2Q. Similarly, when comparing the pair 8Q versus 16Q, the agent likely has an investment horizon of 8Q.

Figure 8 presents a time series of $EER(S)$ for various values of h . Several observations can be made. Generally, during a recession, $EER(S)$ is elevated for horizons greater than 2Q, implying that the agent prefers a shorter horizon over a longer one during this unfavorable period. Conversely, $EER(S)$ is counter-cyclical for horizons up to 3Q. Thus, during a recession, the agent is less likely to invest in a short horizon of up to 3Q compared to a more favorable period. In summary, during a recession, the investment horizon tends to be shorter, yet farther from 1Q.

Figure 8

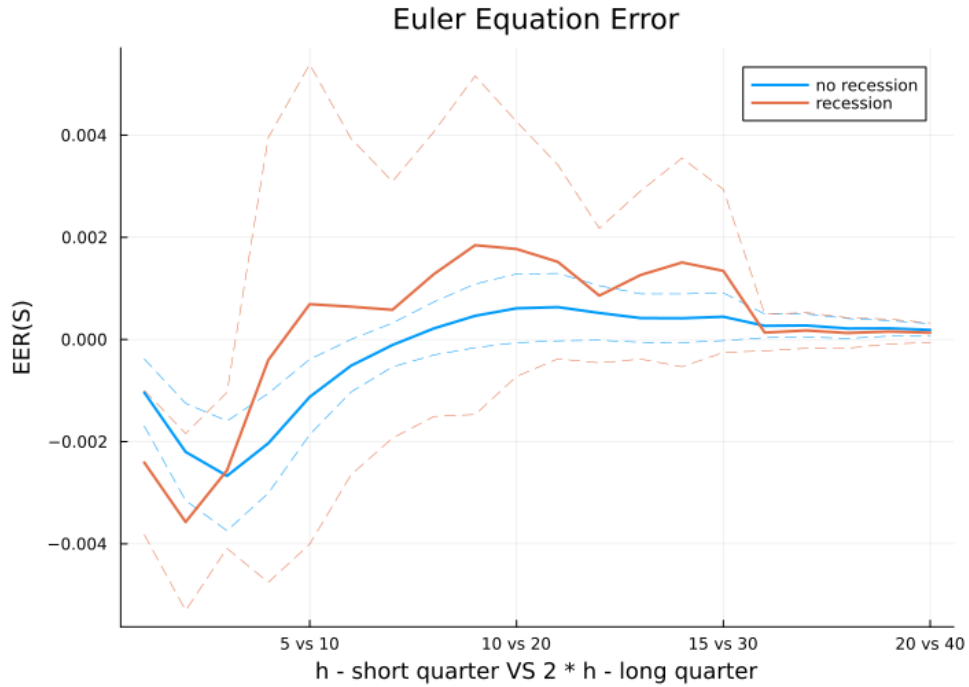


Notes: The figure plots Euler Equation errors (17) by applying short-horizon beliefs to twice the long-horizon investment. The value below zero means the long horizon is more likely to be the true investment horizon. A positive value means the opposite. The pricing kernels correspond to CRRA, $\gamma = 50$.

The primary result can be derived from Figure 9, which illustrates the mean of EER for each possible horizon. When EER is below 0, it indicates that the agent is more likely to have a horizon of $2h$; conversely, if EER is positive, the agent is likely to have a short

horizon. The key findings are as follows: (1) The most likely investment horizon ranges from 8 to 14 quarters. (2) During a recession, the most likely investment horizon narrows to between 5 and 8 quarters. (3) During a recession, the agent is highly unlikely to invest for a horizon of up to 3 quarters. To further support our findings, we conduct the same test using gross returns. Figure 12 presents results that show similar values. This raises important questions and hypotheses regarding the differing investment horizons between equity and fixed-income assets, which aligns with the concluding section of this paper and suggests the need for further research.

Figure 9



Notes: The figure plots the mean Euler Equation errors (17) by applying short-horizon beliefs to twice the long-horizon investment. The value below zero means the long horizon is more likely to be the true investment horizon. A positive value means the opposite. The pricing kernels correspond to CRRA, $\gamma = 50$.

Figure 10: EER for various Risk Aversion

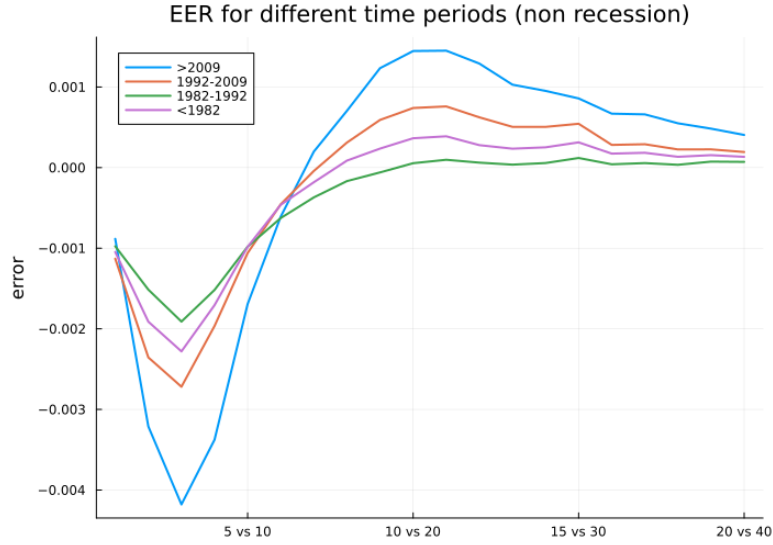


Notes: The figure plots the mean Euler Equation errors (17) by applying short-horizon beliefs to twice the long-horizon investment, for various levels of risk aversion γ . The left panel (a) shows results for non-recessionary times, while the right panel (b) shows results for recessionary times.

The next question we address is how the investment horizon depends on risk aversion. To explore this, we conduct the same test for various risk aversion parameters, with the results summarized in Figure 10. The key findings are as follows: (1) In non-recessionary periods, the investment horizon remains unchanged, regardless of the level of risk aversion. (2) More risk-averse agents exhibit a shorter investment horizon during recessions. (3) Additionally, more risk-averse agents are less likely to invest in a short horizon (1–2 quarters). The first finding is particularly noteworthy. It suggests that an agent’s risk aversion influences the investment horizon in conjunction with market-wide risks, which are more pronounced during recessions but less so during favorable periods. Consequently, risk aversion does not impact the investment horizon in good times but becomes influential during recessions.

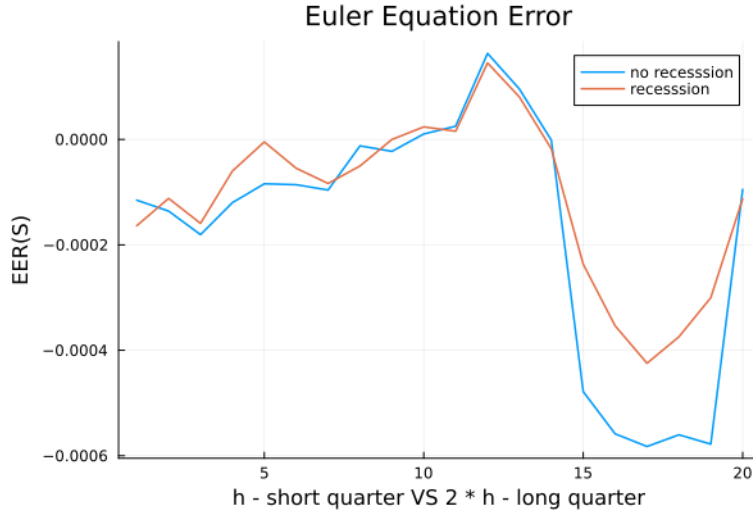
Finally, the results from Figure 11, along with the time series in Figure 8, demonstrate that the investment horizon shifts slightly across different time periods. For example, as shown in the figures, after the 2009 recession, agents are more likely to adopt a short-horizon approach compared to the period from 1982 to 1992. This trend, which began in 1992, can be attributed to easier access to the stock market for household investors, many of whom are (1) first-time participants in the stock market, focusing on shorter horizons, and (2) less experienced, leading to lower risk aversion, which makes them more inclined to favor short horizons of 1Q to 2Q.

Figure 11



Notes: The figure plots the mean Euler Equation errors (17) by applying short-horizon beliefs to twice the long-horizon investment for different time periods. The value below zero means the long horizon is more likely to be the true investment horizon. A positive value means the opposite. The pricing kernels correspond to CRRA, $\gamma = 50$.

Figure 12: Short vs Long test for Gross Returns Euler Equation



Notes: The figure plots the mean Euler Equation errors (17) by applying short-horizon beliefs to twice the long-horizon investment. Gross returns for Market and RF. EER is sum of two. The value below zero means the long horizon is more likely to be the true investment horizon. A positive value means the opposite. The pricing kernels correspond to CRRA, $\gamma = 50$.

6 Heterogeneous Horizons Portfolio

The cross-sectional analysis for implied risk aversion demonstrates heterogeneity among different asset classes. A question that arises is whether investment portfolio compositions differ for various horizons.

In this section, we start by examining the portfolio composition for each given horizon separately. In a representative investor setup, this question is not particularly relevant since it is assumed that the investor holds the entire market. However, it can provide insight, reveal useful information, and prompt other possible research questions.

Additionally, from another perspective, we can consider an individual investor who is fully insured against idiosyncratic shocks and whose SDF depends on aggregate consumption growth. We can extract information about their possible portfolio composition for particular investment horizons. [Gressis et al. \(1976\)](#) show that agents with the same utility function but different investment horizons will choose different portfolios in the mean-variance approach. [Lee et al. \(1990\)](#) also state that each investor with an h -period horizon holds an identical portfolio, which should differ across different values of h , in the generalized CAPM.

In the second part, we examine a more restrictive model, where the representative investor holds the entire market, yet different asset classes in their portfolio have various investment horizons.

Overview of portfolio composition for a given investment horizon

Consider an agent's investment portfolio P as an asset; then, for a given asset/portfolio, the Euler equation [Equation 24](#) should also be satisfied. Portfolio P consists of J assets with weight w_{jh} . For each investment horizon, portfolio P has a different composition.

$$\begin{aligned} \mathbb{E}^S [M_{t+h} \mathbf{R}_{t+h}^P \mid \mathcal{F}_t] &= \mathbf{1} \\ \mathbf{R}_{t+h}^P &= \sum_{j=1}^J w_{jh} R_{t+h}^j \end{aligned} \tag{24}$$

We follow the same procedure as before and extract portfolio weights for different investment horizons by minimizing the Smoothed Empirical Likelihood (SEL) [\(22\)](#) over w_j , for a given h :

$$w_{jh} = \min_{\{w_{jh}\}} SEL(w_h, h) \tag{25}$$

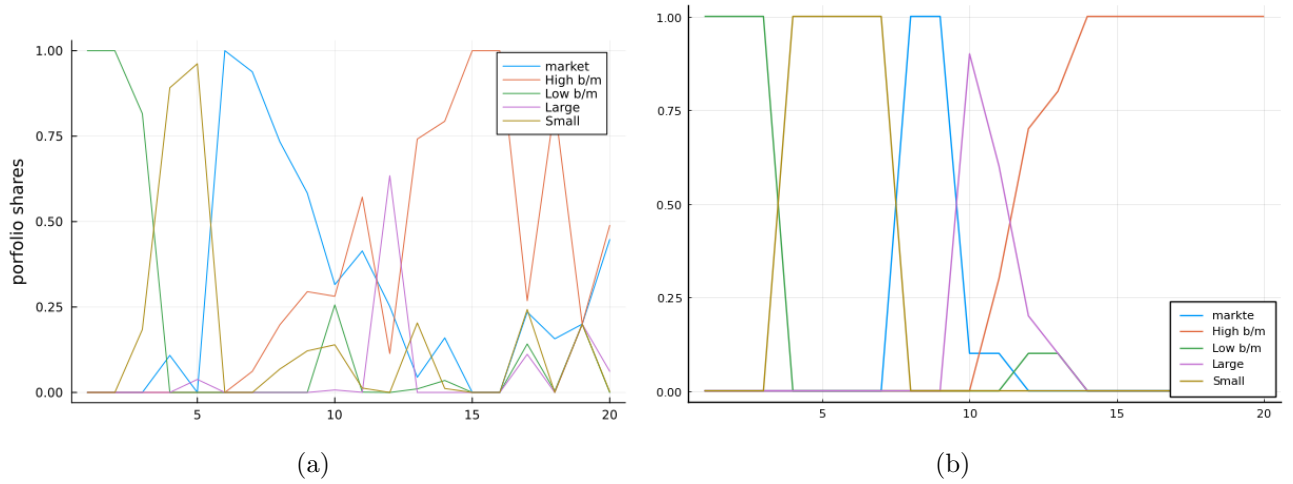
We use a five-asset investment portfolio, which consists of the following assets: total market, high and low B/M, and large and small sizes. The model is estimated for different setups. First, we use excess returns and thus one Euler equation. Second, we use gross

returns and two Euler equations: one for the portfolio and one for the risk-free asset.

It is worth noting that the asset portfolios are overlapping. This is done intentionally to easily identify where the investment portfolio coincides with the whole market; otherwise, we would see the composition of the market portfolio, and the results would be much noisier. In further work, we would use non-overlapping asset portfolios.

We expect that for the representative investor, there should be some horizons for which the investment portfolio coincides with the total market, as expected, since the representative agent holds the whole market. Second, since assets demonstrate different joint behavior with consumption growth, we expect that portfolio composition will differ over horizons.

Figure 13: Portfolio composition for a given investment horizon



Notes: The figure shows asset weights in the investment portfolio if the agent invested for the given horizons (quarter, x-axis). Panel (a) represents results for excess returns, while Panel (b) represents results for gross returns.

Figure 13 shows weights for assets in the investment portfolio for various horizons. The left and right panels represent results for excess and gross returns, respectively²². First, we see that both results are quite similar and support each other. The main result is that, as expected, in the middle horizons, the investment portfolio coincides with the whole market. We can argue that this is highly likely to be the range for "true" investment horizons, and this result supports the previously obtained range. Third, we see that if the agent had shorter investment horizons, they would prefer low B/M assets, and if the agent had longer horizons, they would invest in high B/M stocks. This seems quite reasonable and raises the hypothesis that an investor can have a portfolio with different horizons for various assets.

²²Results for gross returns are obtained on the grid, so they are less smooth.

We will explore this in more depth in the next part.

Investment portfolio with heterogeneous horizons

As we see above, the composition of the optimal portfolio depends on the investment horizon, as various asset classes exhibit different joint distributions with consumption over different time spans. That analysis was not scientifically rigorous and had the sole purpose of providing insight for further research. In this part, we assume that the representative agent holds a total market portfolio, as expected, and has different investment horizons for each class of assets. This setup can also be easily generalized to a heterogeneous-agents setup, where agents have different horizons. For instance, private and institutional investors could have different horizons and hold different portfolios.

The model setup is as follows: an agent has two possible investment horizons—short and long. The market consists of three asset classes: low book-to-market (B/M), neutral B/M, and high B/M (alternatively, six classes: separated into small and large sizes). The representative agent holds the total market and invests in each asset class for a specific investment horizon (an asset class cannot be divided between two horizons). The agent’s FOC for each horizon should be satisfied:

$$\begin{aligned}\mathbb{E} \left[M_{t+S} \mathbf{R}_{t+S}^{Short} - \mathbf{R}_{t+S}^{rf} \mid \mathcal{F}_t \right] &= \mathbf{0} \\ \mathbb{E} \left[M_{t+L} \mathbf{R}_{t+L}^{Long} - \mathbf{R}_{t+L}^{rf} \mid \mathcal{F}_t \right] &= \mathbf{0}\end{aligned}\tag{26}$$

$$\begin{aligned}\mathbf{R}_{t+S}^{Short} &= (w_S^h R_{t+S}^h + w_S^n R_{t+S}^n + w_S^l R_{t+S}^l) / \sum (w_s) \\ \mathbf{R}_{t+L}^{Long} &= (w_L^h R_{t+L}^h + w_L^n R_{t+L}^n + w_L^l R_{t+L}^l) / \sum (w_l) \\ w_S + w_L &= 1 \\ w &\in 0, 1\end{aligned}\tag{27}$$

where the superscripts are as follows:

h - high B/M, n - neutral B/M, l - low B/M

We do not know the exact "true" two horizons, short and long, so we estimate over all available pairs $\{S, L\}$. For given horizons S and L , we extract subjective beliefs for these horizons and portfolio weights w by minimizing the sum of $SEL^{S,L}$.

For each possible pair of S and L from 2 to 18 quarters, we extract portfolio compositions and present the results as follows: a subplot labeled 'short' represents each pair where the S -short horizon is on the x-axis, and the y-axis shows the proportion of each particular asset in the short portfolios among all portfolios for this S as a short horizon. For instance,

as Figure 14(a) shows, if the short horizon is three, all possible pairs $\{S, L\}$ are: $\{3, 4\}$, $\{3, 5\}, \dots, \{3, 18\}$. Among all these pairs, low B/M assets are present in approximately half of the cases in short horizon portfolios, while high B/M assets are never present in short horizon portfolios.

The subplot labeled 'long' represents the opposite scenario, where each pair with the L -long horizon is on the x-axis. Figure 14(a) shows that if the long horizon is twelve, all possible pairs $\{S, L\}$ are: $\{2, 12\}, \{3, 12\}, \dots, \{11, 12\}$. Among all these pairs, low B/M assets never appear in long horizon portfolios. Together, these two results suggest that if three quarters were a short horizon, it is very likely that for each possible pair, low B/M assets would always be in short portfolios.

Figure 14 shows results for three asset classes for normal and abnormal times, and Figure 15 shows results for six asset classes. First, we can state that during a recession, the investment horizon for all asset classes is shorter since the probability that an asset class is in a short-horizon portfolio is higher during a recession. This result is consistent with what we see in the previous sections.

Second, low B/M assets are located in short-horizon portfolios, regardless of their size, and high B/M assets are in long-horizon portfolios. In et al. (2011) support these results²³.

Third, considering sizes, large neutral and high B/M assets are more likely to be in short-horizon portfolios compared to small-sized assets, yet size has a quite subtle marginal effect. Therefore, we can conclude that more risky assets, such as small assets and undervalued assets (high B/M), are located in long-horizon portfolios, and that investors expect significant returns for currently undervalued assets and small assets in the long run. This can also be connected to momentum effects, since low B/M stocks are likely fast-growing and highly valued. Thus, they are located in short-horizon portfolios with the expectation of maintaining this momentum for a while.

We can consider this model from another perspective. Various asset classes have different horizons, and the short and long portfolios are held by different agents, such as individual investors and institutional investors. Both types of investors are fully insured against idiosyncratic shocks, so their SDF depends on the same aggregate consumption growth. This point of view can provide an additional reason why, in general, even in the case of many agents, some do not hold market portfolios: they may have various horizons.

The differences in investment approaches between institutional and individual investors are widely covered in the literature. Ferreira and Matos (2008) and Gompers and Metrick (2001) show that U.S. institutions have a preference for value stocks (high B/M), whereas

²³The paper uses the wavelet approach and adopts mean-variance frontiers with different investment horizons.

Barber and Odean (2000) show that households bias their investments toward high-beta stocks (low B/M can be considered as such stocks (Fama and French, 1993; Campbell and Vuolteenaho, 2004)). As we can see in our results, high B/M assets are in long portfolios and low B/M assets are in short ones. Many studies state that individual investors trade more often (Barber and Odean, 2000; Barber et al., 2009; Cohn et al., 1975). This aligns with our findings as short-horizon portfolios are likely to be traded more often than long-horizon portfolios. In addition, Basak and Pavlova (2013) find that institutional investors demand riskier stocks than retail investors. All these findings support our results, which suggest that individual investors could hold shorter-horizon portfolios, whereas institutional investors could hold longer-horizon portfolios.

Investment horizons as asset characteristics, not investors'

We would like to consider another point of view in a broad manner. We constructed an investment portfolio with different horizons for various assets and connected it to agent investment horizons, and even to heterogeneous agents. It is a plausible connection, yet we would like to return to the initial problem and propose that the horizon may depend not on the agent but on the asset characteristics. We see a strong separation between low and high B/M assets, where the former are associated with short-term portfolios and the latter are associated with long-term portfolios. Thus, some of the inherent characteristics of the asset may affect this allocation.

High B/M stocks are typically utilities, manufacturing, and financial services companies that constantly pay dividends. We can therefore characterize them as companies from stable sectors promising returns over long periods. Conversely, low B/M assets include technology companies, startups, and firms in rapidly expanding industries that rarely pay dividends. These companies are much less stable and promise returns in the relatively near future²⁴. We can see how the distribution of expected returns over time is one of the key characteristics of these companies. Lettau and Wachter (2007) present firms as long-lived assets distinguished by the timing of cash flows and show that growth and value firms covary differently with the discount rate and cash flows, which explains their pricing. Along with this, Van Binsbergen (2020) presents an empirical paper on duration-based stock valuation, using returns from dividend strips. Recently, Gormsen and Lazarus (2023) introduces a duration factor and shows that cash-flow duration is an important determinant of the value premium.

As we see above, 'implied equity duration'/'duration factor' is an inherent characteristic of an asset. Therefore, we can state that the investment horizons of the Euler equation²⁵

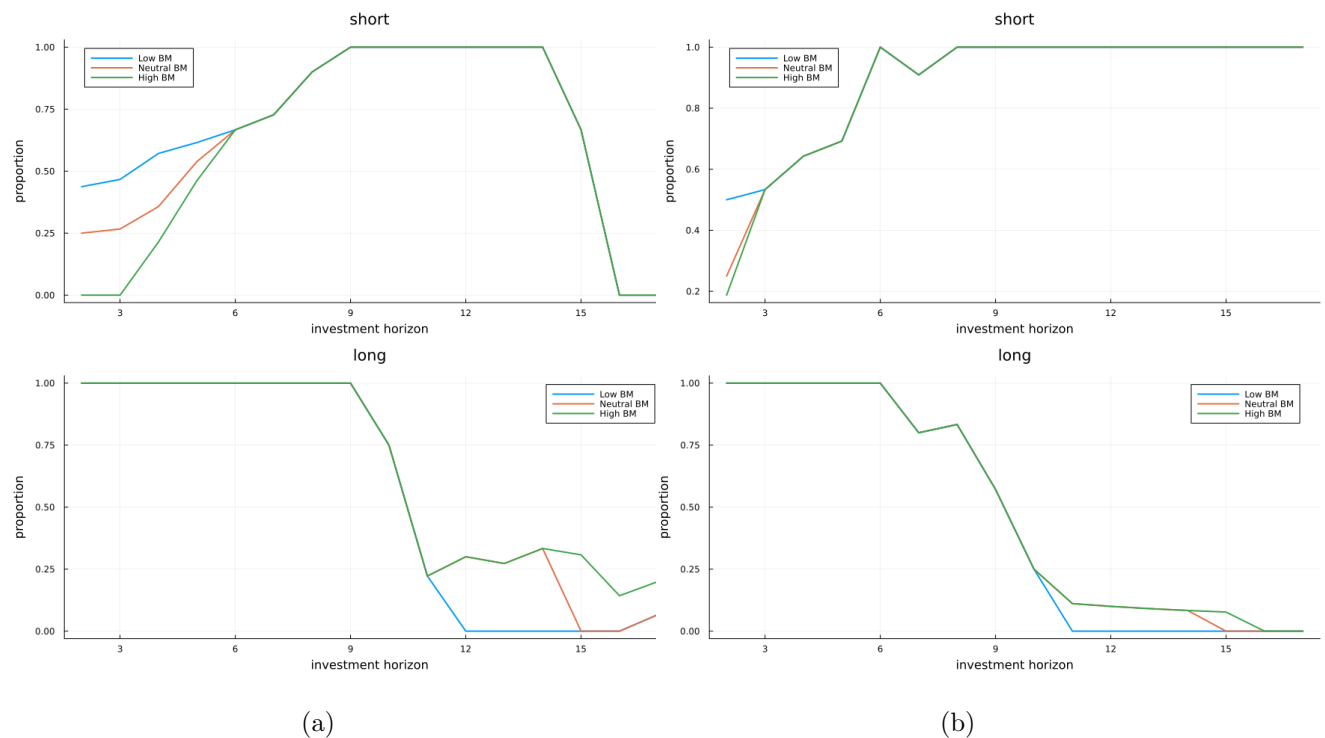
²⁴In this case, returns are typically generated by the fast-growing values of firms.

²⁵We do not say 'agent's investment horizons' intentionally, since an agent can have many of them.

should match this expected duration of the asset. For simplicity, we will call these short-horizon and long-horizon assets.

This point of view, seeing investment horizons as an asset characteristic, can explain some of the features of book-to-market portfolios. High B/M stocks have a lower market beta than low B/M stocks; beta is typically computed on monthly data, and it is clear that current economic fluctuations²⁶ have a greater effect on short SDF than on long SDF. Thus, all else equal, long-horizon stocks would have a lower market beta. [Campbell and Vuolteenaho \(2004\)](#) present another point of view, the two-beta model, and show that value stocks have considerably higher cash-flow betas. This model is supported by and aligns with the view of value stocks as long-horizon stocks, since their cash flows are distributed in the distant future, and therefore their values are much more sensitive to these cash flows.

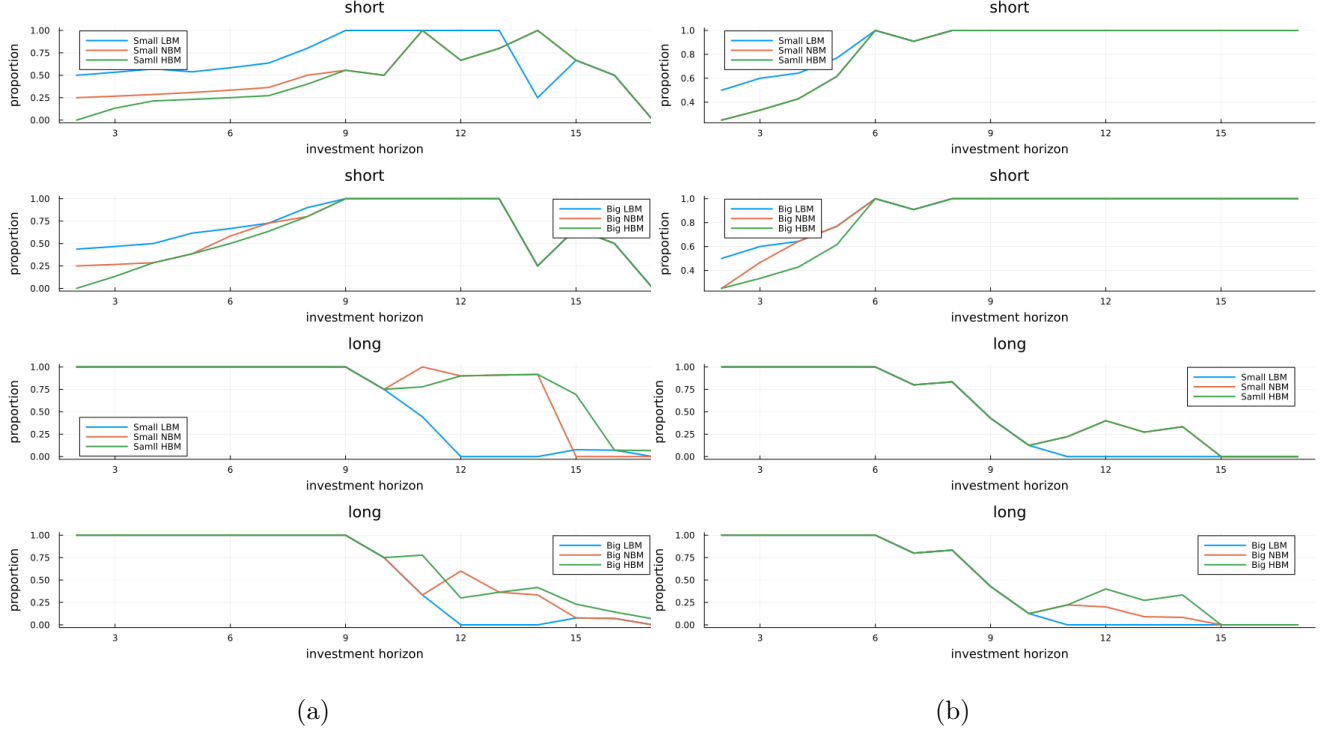
Figure 14: Short vs Long Portfolio. Three asset classes.



Notes: The figures show the portfolio composition for all possible short and long horizon pairs. Panel (a) represents good times, while Panel (b) represents a recession. A subplot labeled 'short' represents each pair where the S -short horizon is on the x-axis, and the y-axis shows the proportion of each particular asset in the short portfolios among all portfolios for this S as a short horizon. The subplot labeled 'long' represents the opposite scenarios.

²⁶Economic fluctuations include both market and consumption changes; thus, they can be applied to different models.

Figure 15: Short vs Long Portfolio. Six asset classes.



Notes: The figures show the portfolio composition for all possible short and long horizon pairs. Panel (a) represents good times, while Panel (b) represents a recession. A subplot labeled 'short' represents each pair where the S -short horizon is on the x-axis, and the y-axis shows the proportion of each particular asset in the short portfolios among all portfolios for this S as a short horizon. The subplot labeled 'long' represents the opposite scenarios.

7 Conclusion

The investment horizon is an essential characteristic of investors that certainly needs to be taken into account, yet it has not received sufficient attention. We have made one of the first attempts to estimate the investment horizon range of the representative investor. The most likely investment horizon falls between 8 and 14 quarters and between 5 and 8 quarters during a recession; additionally, investors are unlikely to invest for one- and two-quarter horizons, regardless of risk aversion. All these findings raise the concern that the most commonly used horizon range in empirical finance is substantially shorter than true investment horizons, which could lead to model misspecification and resulting puzzles.

On the theoretical side, a secondary contribution is the formal introduction of the horizon function $H(\tau)$, which embeds investment horizons directly into financial models. Interpreted as a probability distribution over future withdrawal dates, $H(\tau)$ yields a first-order

condition that is a convex combination of short- and long-horizon Euler terms rather than a single one-period equation. A key theoretical result is horizon irrelevance: under random-walk dynamics for log consumption and log returns, all horizon Euler terms equal one at every horizon, so the investment horizon has no effect on asset pricing. Conversely, when these random-walk assumptions are relaxed—as the empirical evidence requires—explicitly incorporating investor horizons into the model helps to explain the equity premium puzzle.

Additionally, it is worth mentioning that the investment horizon of a representative investor has been gradually shortening. This decline—likely attributable to the entry of less experienced household investors—suggests that financial market development and investor literacy are important determinants of the aggregate investment horizon, raising the question of how financial development and literacy programs can shape them.

The second part of the paper addresses the question of heterogeneous investment horizon portfolios. This perspective, while novel, aligns with common sense: investors purchase different asset classes for their portfolios with varying intended holding periods. Specifically, our findings indicate that growth stocks are associated with shorter horizons, while value stocks correspond to longer horizons. Our results support the hypothesis that the investment horizon can also be a characteristic of assets (for instance, ‘duration factor’ or ‘implied equity duration’). This characteristic should align with portfolios featuring heterogeneous horizons, and in a multi-investor environment, it should be matched to a distinct agent with that specific horizon.

From a policy perspective, investment horizons shorten during recessions and depend on financial access.²⁷ This means that policies encouraging retail investors to participate in markets—such as financial education programs or default savings plans—could shift aggregate horizons and affect asset prices. These findings open broad directions for future research, both theoretical and empirical, and may inform policy design aimed at stock markets and investor behavior.

²⁷The post-1992 shortening of the estimated horizon is attributable to easier stock market access for household investors, many of whom were first-time participants with shorter horizons and lower risk aversion (see [Figure 11](#) and [Figure 8](#)).

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A Appendix: Robustness Alternative Conditioning Sets and Test Assets

This appendix demonstrates the robustness of our results to the choice of conditioning set and test assets. We consider four conditioning sets:

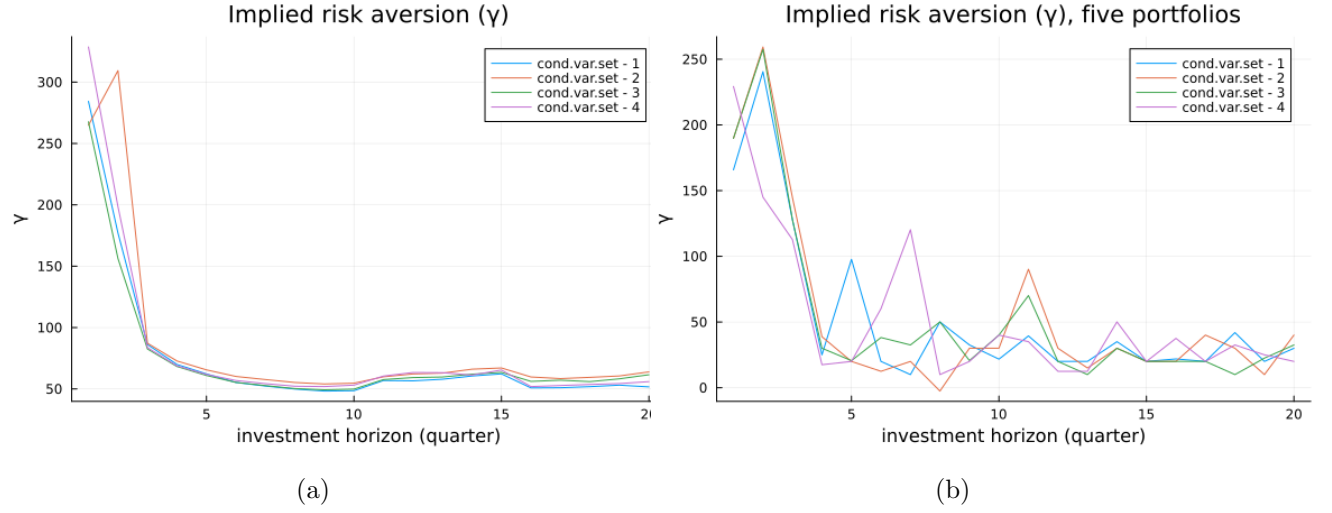
1. The first conditioning set, which is used in the main part, includes 1-quarter real excess returns, 1-year nominal yield, 1-quarter consumption growth, and changes in consumption growth.
2. The second conditioning set contains 1-quarter consumption growth and 1-quarter gross real returns.
3. The third conditioning set contains 1-quarter consumption growth, 1-quarter gross real returns, and 1-quarter bond yield.
4. The fourth conditioning set contains 1-quarter consumption growth, 1-quarter real excess returns, and 1-quarter inflation.

Figure 16 shows the implied risk aversion γ for various investment horizons. Panel (a) shows results when only excess market returns are used in the Euler equation. Panel (b) presents results from five Euler equations: excess return for the total market, high and low B/M, and small and large size portfolios. The main pattern and results remain consistent regardless of the conditioning set and number of test assets.

Figures 17, 18, and 19 present comparisons of objective and subjective expected returns, standard deviations, and Sharpe ratios for various horizons. For all conditioning sets, results are similar, with conditioning sets 1 and 3 demonstrating subjective beliefs much closer to objective data.

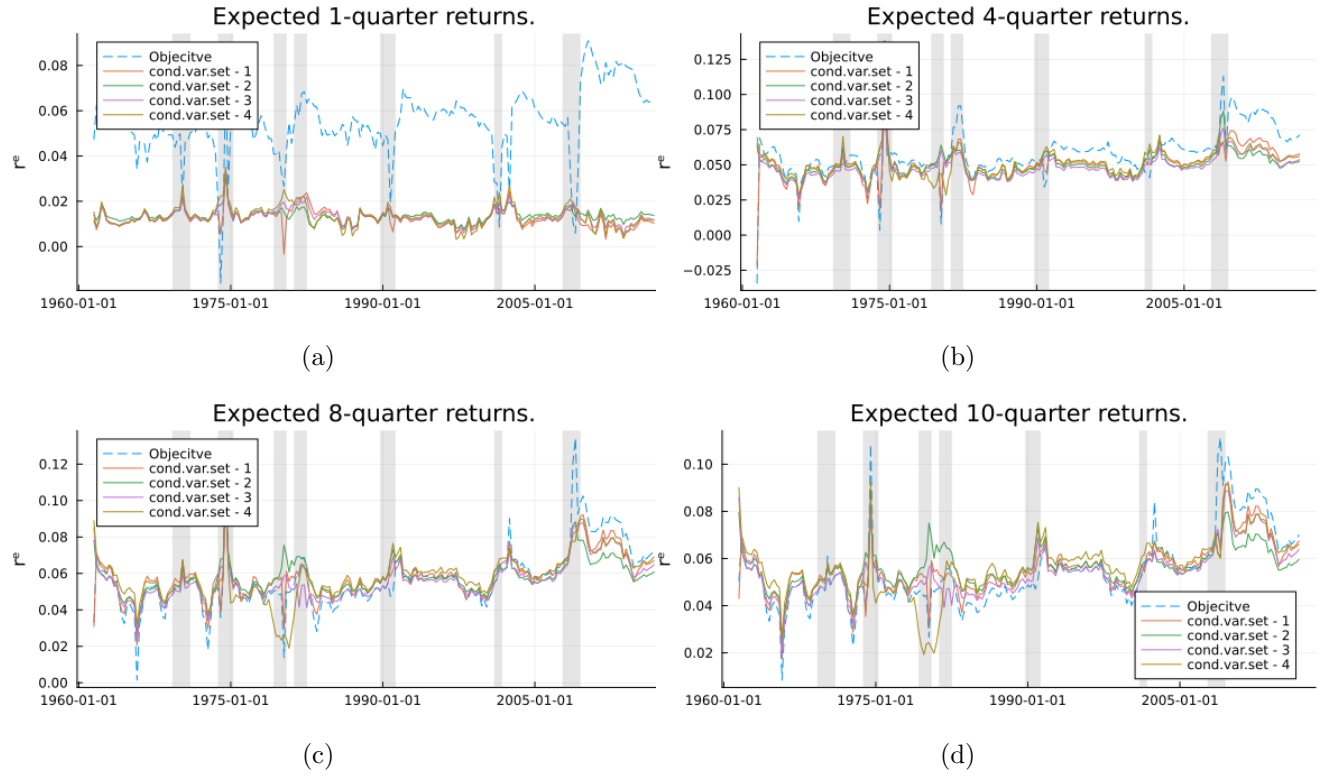
Figures 20 and 21 show the Euler Equation error to compare whether short or long horizons are more likely. The general pattern remains consistent across all conditioning sets. Conditioning sets 2 and 4 demonstrate less volatility and do not exhibit a slight trend over the past 20 years. The reason for this is that these sets do not contain bond returns (although the 4th conditioning set includes excess returns, thus not explicitly incorporating the risk-free rate). Finally, Figure 22 presents a comparison of short and long horizon results for five portfolios separately. The results support the main findings from the second part of the paper.

Figure 16



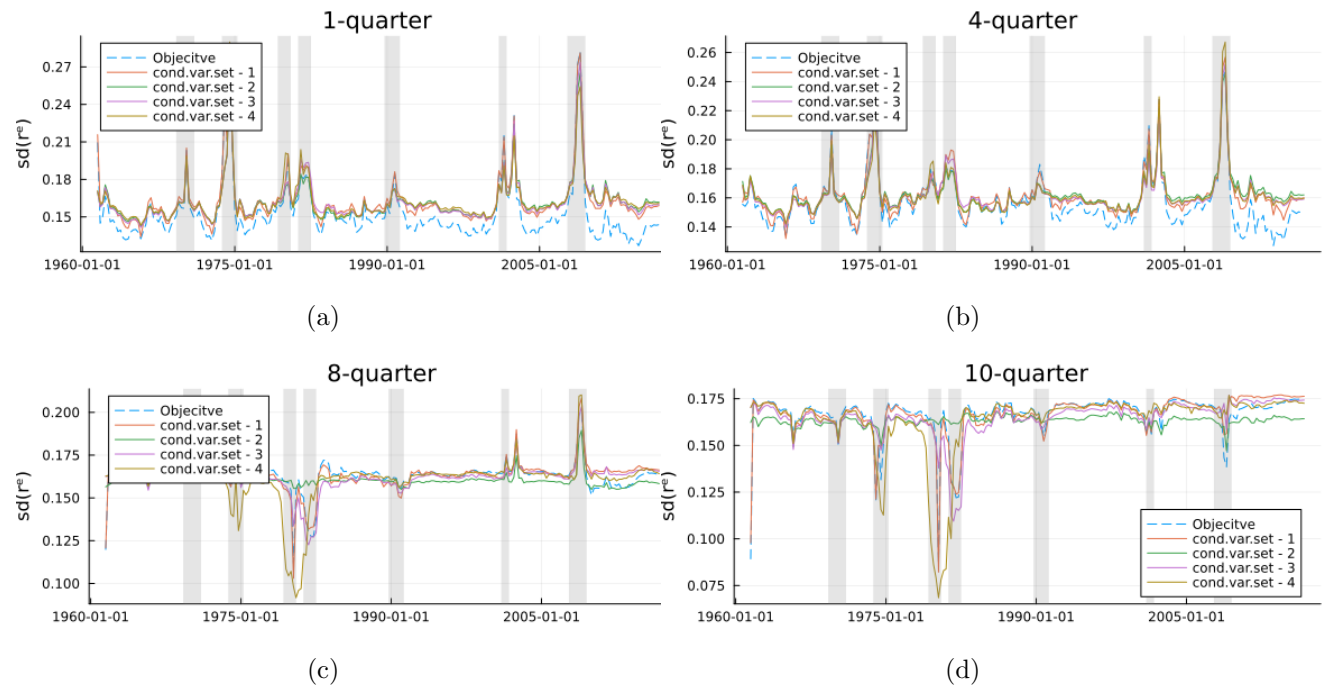
Notes: The figure plots implied risk aversion γ for various investment horizons. Panel (a) shows results when excess market returns are used in the Euler equation. Panel (b) presents results from five Euler equations: excess return for total market, high and low B/M, and small and large size portfolios. The results are obtained by minimizing SEL over γ . The pricing kernels correspond to CRRA utility. Real market returns are based on the S&P 500 index, consumption refers to real personal expenditure on nondurable goods and services, and risk-free rates are derived from deflated nominal yield curves for each quarter from 1 to 20. The sample period is 1961:3Q-2016:4Q.

Figure 17: Objective and Expected Excess Returns for Various Investment Horizons



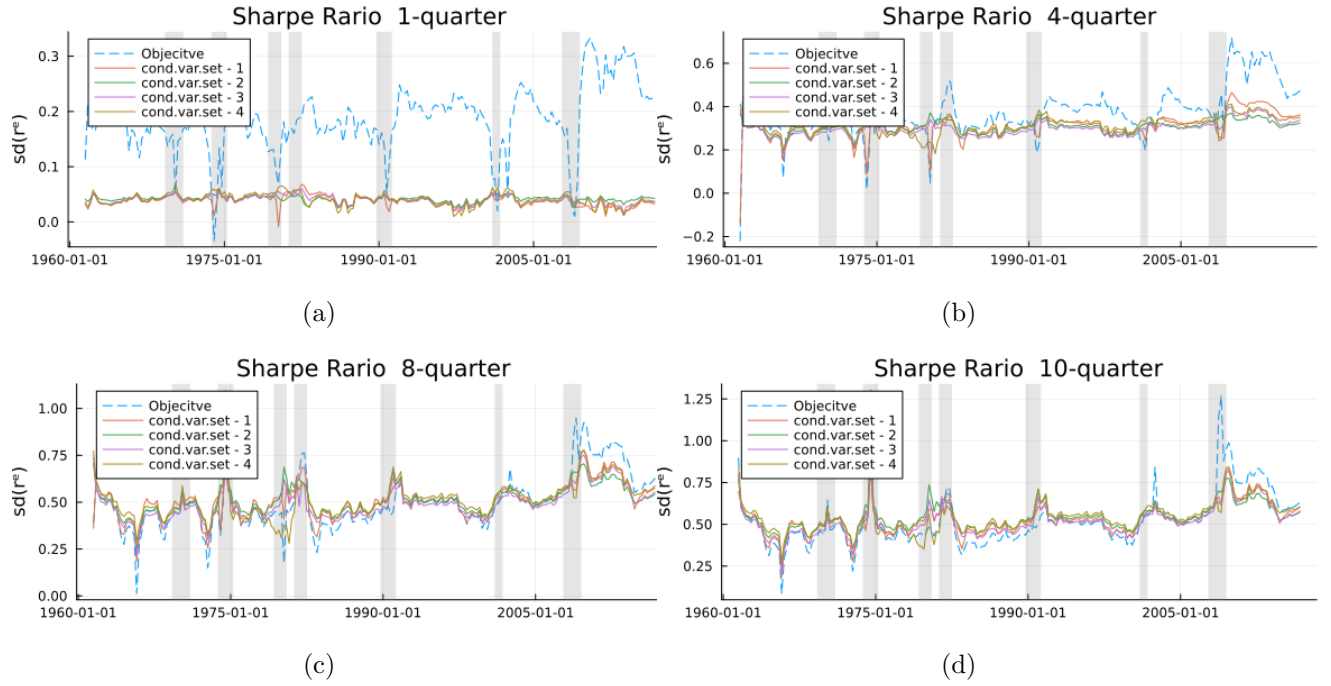
Notes: The figure plots annualized Objective and Subjective Expected Excess returns for various investment horizons. The pricing kernels correspond to CRRA, $\gamma = 50$. Real market returns correspond to S&P 500 index, consumption to the real personal expenditure on nondurable goods and services, and risk-free rates to the deflated nominal yield curves for each quarter from 1 to 20. Sample is 1961:3Q-2016:4Q.

Figure 18: Objective and Subjective Standard Deviation of Returns for Various Investment Horizons



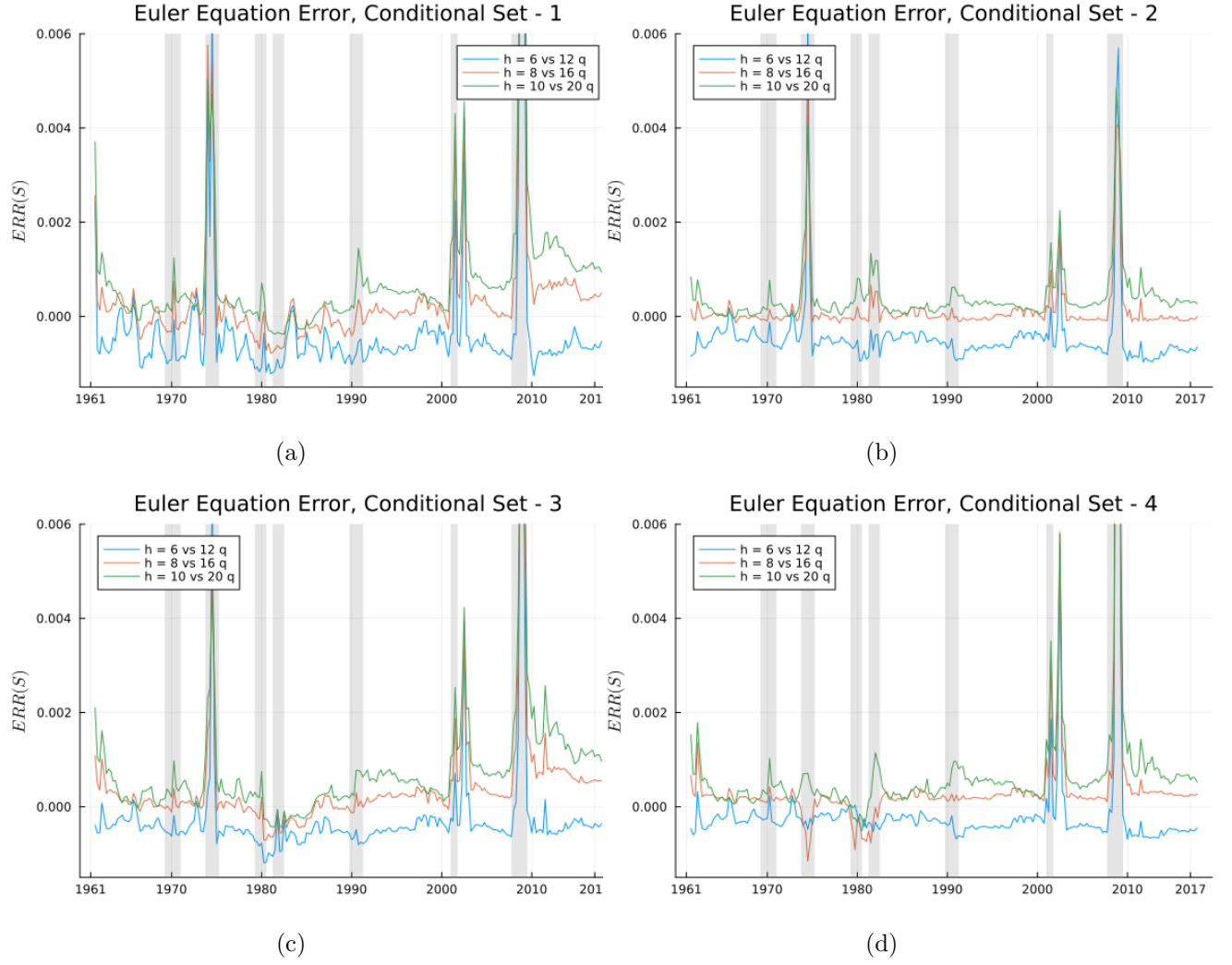
Notes: The figure plots annualized Objective and Subjective Standard Deviations of returns for various investment horizons. The pricing kernels correspond to CRRA, $\gamma = 50$. Real market returns correspond to S&P 500 index, consumption to the real personal expenditure on nondurable goods and services, and risk-free rates to the deflated nominal yield curves for each quarter from 1 to 20. Sample is 1961:3Q-2016:4Q.

Figure 19: Objective and Subjective Sharpe Ratio for Various Investment Horizons



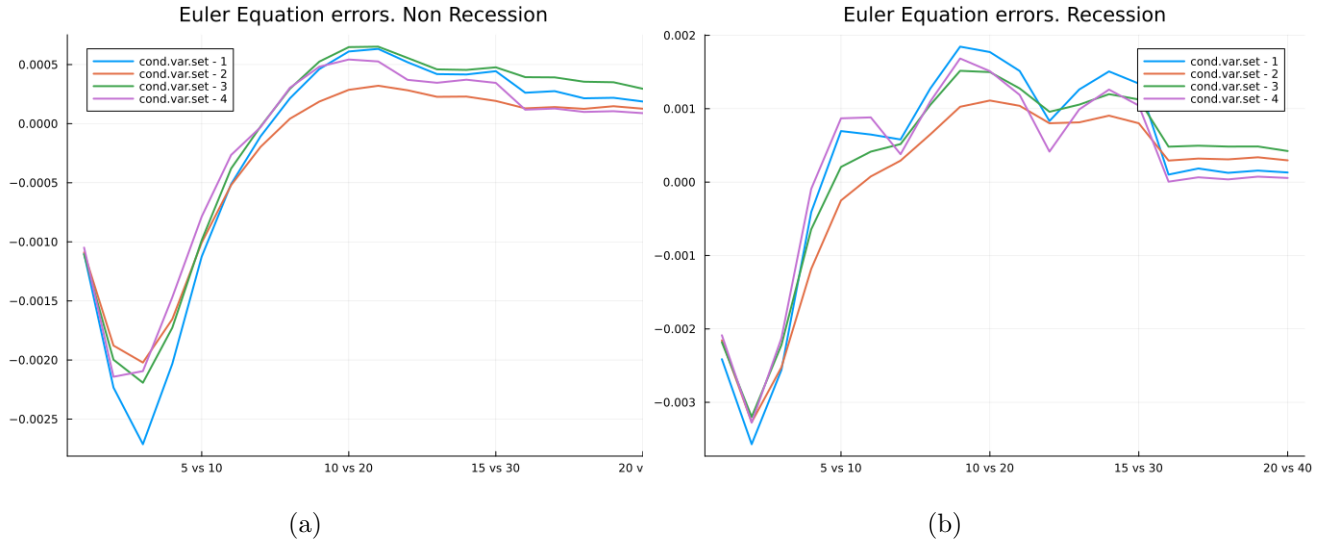
Notes: The figure plots Objective and Subjective Sharpe Ratio for various investment horizons. The pricing kernels correspond to CRRA, $\gamma = 50$. Real market returns correspond to S&P 500 index, consumption to the real personal expenditure on nondurable goods and services, and risk-free rates to the deflated nominal yield curves for each quarter from 1 to 20. Sample is 1961:3Q-2016:4Q.

Figure 20



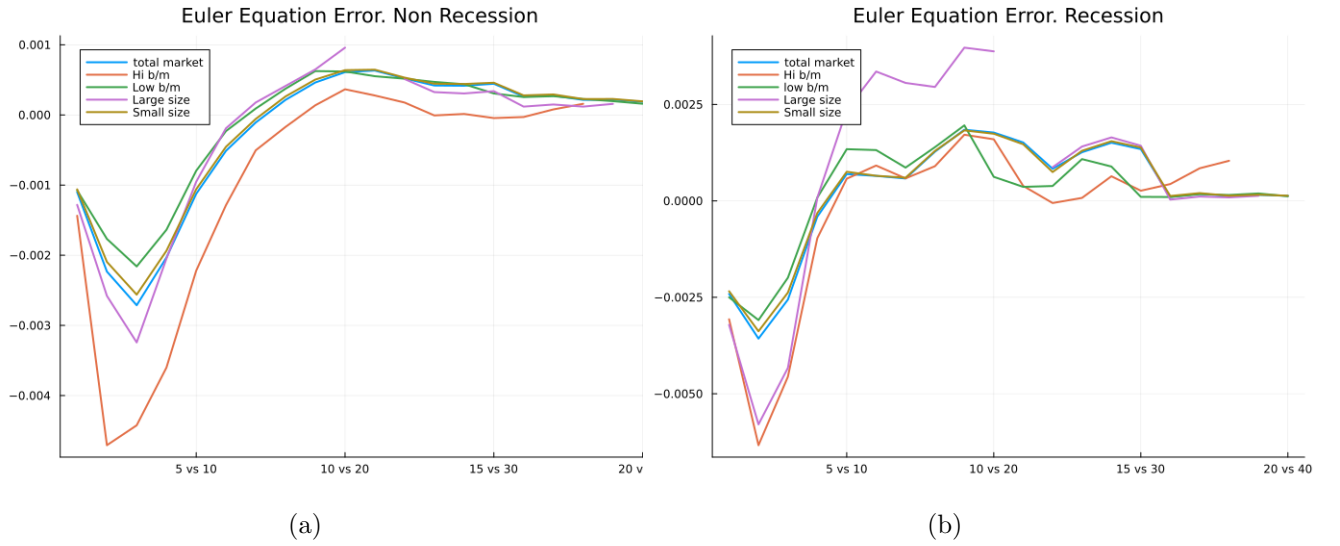
Notes: The figure plots Euler Equation errors (17) by applying short-horizon beliefs to twice the long-horizon investment. The value below zero means the long horizon is more likely to be the true investment horizon. A positive value means the opposite. The pricing kernels correspond to CRRA, $\gamma = 50$.

Figure 21



Notes: The figure plots the mean Euler Equation errors (17) by applying short-horizon beliefs to twice the long-horizon investment. The value below zero means the long horizon is more likely to be the true investment horizon. A positive value means the opposite. The pricing kernels correspond to CRRA, $\gamma = 50$.

Figure 22



Notes: The figure plots the mean Euler Equation errors (17) by applying short-horizon beliefs to twice the long-horizon investment. The value below zero means the long horizon is more likely to be the true investment horizon. A positive value means the opposite. The pricing kernels correspond to CRRA, $\gamma = 50$.

B Numerically Parsimonious Example

To illustrate asset allocation with different horizons, we provide a simple, parsimonious example, which demonstrates that if returns demonstrate a mean reversion pattern, an agent would not invest if he is a short-term investor and would invest if he has a long horizon, while they both would have the same expected utility. The investor has either a short horizon (1 period, $h(1) = 1$) or a long horizon (2 periods, $h(2) = 1$). We consider a three-period model (29) ($t = 0, 1, 2$) with two states at $t = 1$, each of which branches into two states at $t = 2$. All states have equal probabilities of occurrence. At each state, endowments e_t are uniform at 1; an agent allocates investment in stock a_t with stock prices p_t . The prices exhibit a pattern similar to mean reversion, with higher volatility at shorter horizons, as follows:

$$\begin{aligned} e_0 &= 1; \quad p_0 = 1 \\ e_1 &= [1; 1]; \quad p_1 = [1.3; .6] \\ e_2 &= [1; 1; 1; 1]; \quad p_2 = [1.5; 1.2; .9; .8] \end{aligned} \tag{28}$$

$$\begin{aligned} &\max_{\{a_t\}_t^\infty} \mathbb{E} \sum_{t=0}^2 \beta^t U(c_t) \\ &s.t. \end{aligned} \tag{29}$$

$$\text{short type: } c_t = e_t - a_t p_t + a_{t-2} p_t$$

$$\text{long type: } c_t = e_t - a_t p_t + a_{t-1} p_t$$

The short-horizon investor has the following solutions: $a_0 = 0.00$, $a_{11} = 0.00$, $a_{12} = 0.11$. The agent does not invest at the initial time due to high short-period volatility. The sum of expected utility is $\sum_{t=1}^3 u(c_t) = -2.96$ (using a CRRA utility function with $\gamma = 2$). In contrast, the long-horizon investor's solution is to invest at the initial time with $a_0 = 0.17$ and has a slightly higher sum of expected utility.

The FOC and Euler equation for the first period of the long-horizon investor are not satisfied (30). Consequently, if the econometrician assumes that the investor is a short-horizon type whose Euler equation should hold, a high value of the risk aversion parameter γ is required to elevate²⁸ the left-hand side and achieve equality with 1.

$$\begin{aligned} U'(c_0)p_0 &= 1.035 > \beta \mathbb{E}_t U'(c_1)p_1 = 0.94 \\ \Rightarrow \quad \mathbb{E}_t[M_1 R_1] &< 1 \end{aligned} \tag{30}$$

²⁸SDF is a convex function, and the expectation with high curvature will be higher.

C Investor's investment horizon illustration figures

Figure 23

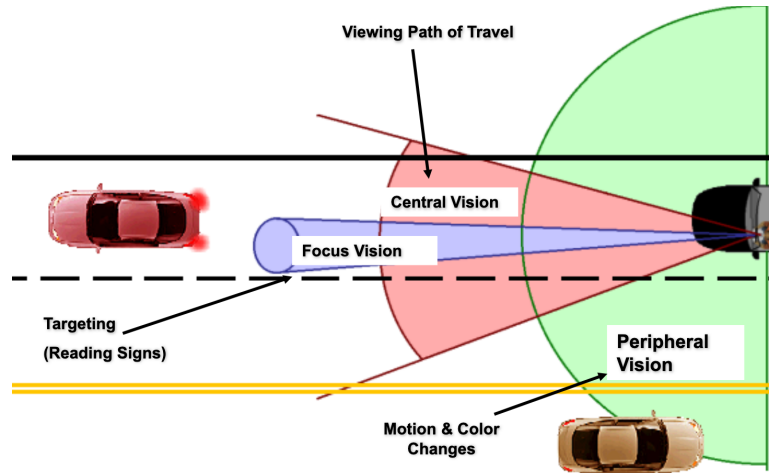
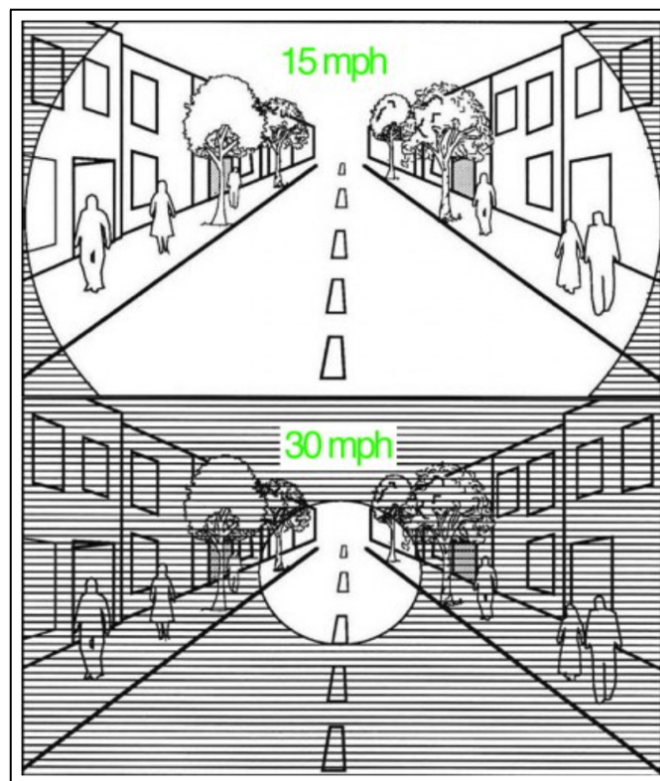
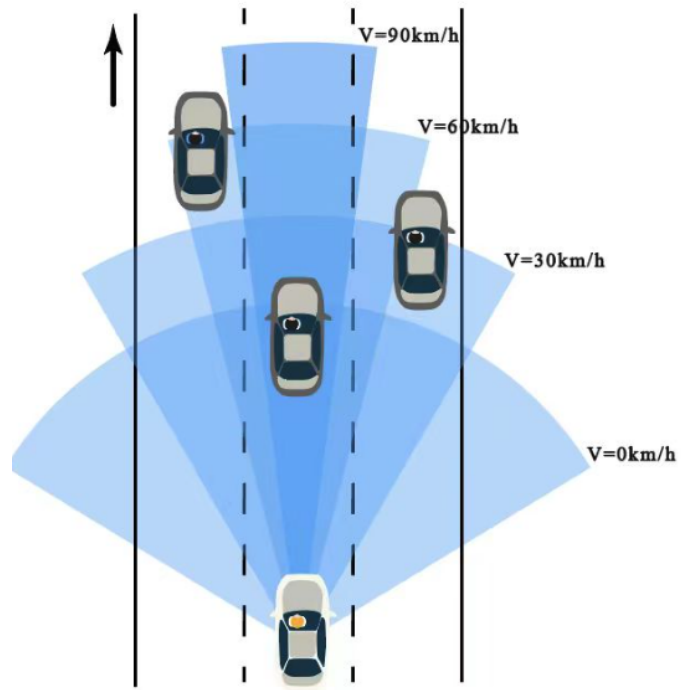


Figure 24



Notes: source "<https://americawalks.org/part-2-reducing-vehicle-speeds-pedestrian-safety/>"

Figure 25



Notes: source "<https://arxiv.org/html/2402.04318v2>"